PARAMETERS CALCULATION OF SOLAR – AND WIND-ELECTRIC WATER LIFTING SYSTEMS

S.P. Urias'ev
G.K. Kutliev

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A-2361 Laxenburg, Austria

FOREWORD

This paper describes results of the application of stochastic programming to parameters calculation of solar- and wind-electric water lifting systems. It provides an example of both a realistic problem with inherent stochasticity, and a valuable test problem for the algorithms under development. It also gives some insights into the nature of solutions of certain classes of stochastic programming problems, and the justication for the consideration of randomness in decision models.

Alexander B. Kurzhanski
Chairman
System and Decision Sciences Program

AUTHORS

Senior research scientist Stanislav Urias'ev from the Institute of Cybernetics Academy of Sciences of the Ukr.SSR (252207 Kiev, USSR).

Research scientist Geldu Kutliev from the Scientic Production Unit SOLNCE Academy of Sciences the Tur. SSR (744032 Ashchabad, USSR)

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1. INTRODUCTION

The development of energy and resource saving technologies is very important because of shortage in traditional energy supply resources (gas, oil, coal). A combined use of the wind and solar energy for power and water supply of small isolated consumers is one of the promising tendencies in the world economy. The efficiency of a joint use of these power sources under conditions of desert and mountain regions feature high power characteristics of wind-driven and solar plants which mutually complement each other when combined.

The purpose of the present calculation is to determine optimum values of parameters of solar- and wind-electric water supply systems. These systems are intended for use in pasture and irrigated regions of the Middle Asia, Kazakhstan and in steppe regions of the USSR European territory. The optimum parameters are selected depending on the presence of natural power resources in the region and the preset water consumption schedule, with minimum costs of capital construction and operation of the system for the whole planning period.

This article follows the article by Kutliev and Urias'ev [1]. In [1] it was proposed to use stochastic quasi-gradient algorithms for calculation of water lifting systems. The following main parameters of the system are calculated: the wind-electric plant (WEP) blade-swept area, the solar-power plant (SPP) collector area and the water reservoir volume.

The problem under consideration is reduced to the stochastic programming problem [2] [3]. A stochastic quasi-gradient algorithm with adaptive step size control is used for solving the problem [4], [5]. This method was included into the ADO/SDS library stochastic optimization codes developed in IIASA and package NSO developed in V.M. Glushkov Institute of Cybernetics (Kiev, USSR).

The water lifing system being studied comprises a water source, a WEP and a SPP with pump equipment, a water reservoir and a water trough system. The water consumption schedule is preset and the lifted water is supplied from the water source primarily for everyday necessities and for watering of cattle. In ensuring the watering schedule and filling the accumulating water reservoir, water is fed for irrigation of gardens and fields of melons or for irrigation of trees, shrubbery, green plantations etc. It is assumed that the water source output is unlimited.

By virtue of the fact that the inflow of the wind and solar energy is a random quantity, the output of the power plants, the amount of water in the reservoir at any instant of time as well as the values of water deficit and excess are random values.

2. STOCHASTIC MODEL OF THE SOLAR- AND WIND-ELECTRIC WATER LIFTING SYSTEM

Let us consider the model of the water supply system for some independent station using WEP and SPP. The modeling period (a season, a year) is subdivided into daily periods $k = 1, 2, \dots N$. The output of the water lifing system is determined by the parameters

$$a = (a_1, a_2, a_3) \ ,$$

where a_1 is the WEP blade-swept area, a_2 is the SPP collector area, a_3 is the water reservoir volume.

Vector a meets the natural restrictions:

$$a_1 \geq 0, \ a_2 \geq 0, \ a_3 \geq 0 \ . \tag{1}$$

Let us assume that all random values are defined in probability space (Ω, F, P).

For each period $k (k = 1, 2, \dots, N)$, let us introduce the following designations: $b(k)$ is demand for water, $V(k)$ is wind velocity daily average value, $E_1(k)$ is daily total solar radiation, $Q(a_1, a_2, V(k), E_1(k))$ is quantity of electric power generated by the power plants, $W(k)$ is the amount of water lifted by the electric pump, $x(k)$ is the amount of water in the reservoir, $Z_1(k)$ and $Z_2(k)$ are water deficit and excess, respectively.

The state of the system parameters at instant k is characterized by the following relations:

$$Z_1(k) = \max\{0, b(k) - x(k-1) - W(k)\}, \ k = 1, \ldots, N \ , \tag{2}$$

$$Z_2(k) = \max\{0, x(k-1) + W(k) - b(k) - a_3\}, \ k = 1, \ldots, N \ ,$$

where values $x(k)$, $W(k)$ are determined by relations:

$$x(k) = \max\{0, \min\{a_3, x(k-1) + W(k) - b(k)\}\}, \ k = 1, \ldots, N \ , \tag{3}$$

$$x(0) = 0 \ ,$$

$$W(k) = R_1 Q(a_1, a_2, V(k), E_1(k)), \ R_1 > 0, \ k = 1, \ldots, N \tag{4}$$

$$Q(a_1, a_2, V(k), E_1(k)) = G_1(V(k))a_1 + G_2(E_1(k))a_2, \ k = 1, \ldots, N \tag{5}$$

where $G_1(V(k))$, $G_2(E_1(k))$ are specific electric outputs of the WEP and SPP, respectively. Value R_1 is a positive constant depending on the water lifting level H. Below, the methods will be described for calculating the values

$$R_1, \ G_1(V(k)) \quad \text{and} \quad G_2(E_1(k)), \ k = 1, \ldots, N \ .$$

The total cost minimum is accepted as the optimization criterion:

$$F = \Theta + \varepsilon A + E(y_b) - E(\Pi_b) \tag{6}$$

where Θ are operation expenditures for calculation period, A are one-time investments, ε is the relative efficiency coefficient ($\varepsilon = 0.15$), $E(y_b)$ is the mathematical expectation of the cost of water deficit, $E(\Pi_b)$ is the mathematical expectation of profit due to water excess.

The solar- and wind-power system is characterized by large initial investments and small operation cost, of the order of 10% of the investments

$$\Theta = 0,1\,\varepsilon A$$

Investments A required for construction of the systems comprise the following components: A_b, A_c – investments required for WEP and SPP with pump equipment: A_H – investments for elevated water reservoir, A_1 – investments for the head part, foundation and watering system, A_2 – expenditures for the equipment transportation, A_3 – those for the system assembly and erecting. Therefore

$$A = A_B + A_c + A_H + A_1 + A_2 + A_3 \ .$$

Let us assume [6] that $A_1 + A_2 + A_3 =$ const. Let us present values $A_б$ and A_c in the following way

$$A_б = c_1 a_1, \; A_c = c_2 a_2 \; ,$$

where c_1 and c_2 is specific cost of WEP and SPP, respectively. To calculate the cost of the elevated water reservoir, we use the following relation [7]:

$$A = 260 + 43.8\, a_3^{0.785} \; .$$

Denote: $d_1(k)$ – specific cost of water deficit, $d_2(k)$ – specific profit due to the use of excessive water. Therefore:

$$E(y_б) = E \sum_{k=1}^{N} d_1(k) Z_1(k), \; E(\Pi_б) = E \sum_{k=1}^{N} d_2(k) Z_2(k) \; .$$

Then from (6) we obtain

$$F = 0.1 \cdot 0.15 A + 0.15 A + E(y_б) - E(\Pi_б) = 0.165(A_1 + A_2 + A_3 + c_1 a_1$$
$$+ c_2 a_2 + 260 + 43.8\, a_3^{0.785}) + E \sum_{k=1}^{N} d_1(k) Z_1(k) - E \sum_{k=1}^{N} d_2(k) Z_2(k) \; .$$

Denote $c = 0.165(A_1 + A_2 + A_3 + 260)$ then

$$F = c + 0.165(c_1 a_1 + c_2 a_2) + 7.227\, a_3^{0.785} \tag{7}$$
$$+ E \sum_{k=1}^{N} (d_1(k) Z_1(k) - d_2(k) Z_2(k))$$

It is required to find vector $a = (a_1, a_2, a_3)$ which minimizes function (7) under conditions (1)–(5).

Denote

$$\varphi(k) = b(k) - x(k-1) - W(k) \; . \tag{8}$$

Then problem (7), (1)–(5) can be written in the following form:

$$F(a) = c + 0.165(c_1 a_1 + c_2 a_2) + 7.227\, a_3^{0.785} \tag{9}$$
$$+ E \sum_{k=1}^{N} (d_1(k) \max\{0, \varphi(k)\} - d_2(k) \max\{0, -\varphi(k) - a_3\}) \rightarrow \min$$

provided that

$$\varphi(k) = b(k) - W(k) - \max\{0, \min\{a_3, -\varphi(k-1)\}\}, \ \varphi(0) = 0 \ , \tag{10}$$

$$W(k) = R_1 Q(a_1, a_2, V(k), E(k)) \ , \tag{11}$$

$$Q(a_1, a_2, V(k), E(k)) = G_1(V(k))a_1 + G_2(E(k))a_2 \ , \tag{12}$$

$$a_1 \geq 0, \ a_2 \geq 0, \ a_3 \geq 0 \ . \tag{13}$$

3. STOCHASTIC QUASI-GRADIENT ALGORITHM

For solving the problem (9)–(13) the stochastic quasi-gradient algorithm with adaptively controlled parameters was used [4], [5]. For improving the practical convergence rate of this algorithm the scaling procedure suggested by Saridis [8] was implemented. Scaling procedure contains changes taking into account the projection operation and adaptive step size control

$$a^{s+1} = \Pi_D(a^s - \rho_s H^s \xi^z) \ ,$$

where s is the iteration number; Π_D is orthoprojection operation on the set $D = \{a : a_1 \geq 0, \ a_2 \geq 0, \ a_3 \geq 0\}$; $\rho_s \geq 0$ is the positive step size; H^s is the scaling matrix; ξ^s is the stochastic quasi-gradient, i.e.

$$E(\xi^z / a^s) = \nabla F(a^s) \ .$$

The scaling matrix is calculated as follows

$$H^{s+1} = \begin{bmatrix} h_1(s+1) & & 0 \\ & \ddots & \\ 0 & & h_n(s+1) \end{bmatrix}, \ H^0 = \begin{bmatrix} 1/n & & 0 \\ & \ddots & \\ 0 & & 1/n \end{bmatrix},$$

$$h_i(s+1) = \alpha h_i(s) + \lambda_i(s+1)(1-\alpha) \ ,$$

$$\lambda_i(s+1) = \begin{cases} 0, & \text{if } \xi_i^{s+1}(x_i^s - x_i^{s+1}) \leq 0, \ k(s+1) \neq 0 \ , \\ 1/n, & \text{if } k(s+1) = 0 \ , \\ 1/k(s+1), & \text{if } \xi_i^{s+1}(x_i^s - x_i^{s+1}) > 0, \ k(s+1) \neq 0 \ , \end{cases}$$

where $k(s+1)$, $0 \leq k(s+1) \leq n$ is the quantity of numbers i for which $\xi_i^{s+1}(x_i^s - x_i^{s+1}) > 0$.

Step size ρ_s is given by following recursive relations

$$T_s = \langle H^{s+1}, \Delta x^{s+1} \rangle \ .$$

$$z_z = z_{s-1} + (|T_s| - z_{s-1})\Lambda, \; s_{s-1} = 0 \ ,$$

$$\rho_{s+1} = \rho_s \, a^{T_s/z_s} \begin{cases} 1 \ , & \text{if } T_s > 0, \\ U \ , & \text{if } T_s \le 0 \ . \end{cases}$$

The recommended values of parameters are

$$a = 2, \; u = 0.8, \; \Lambda = 0.2, \; \alpha = 0.5 \ .$$

4. STOCHASTIC QUASI-GRADIENT CALCULATION

Function $F(a)$ comprises of two components

$$F(a) = f(a) + Ey(a, w) \ , \tag{14}$$

where

$$f(a) = c + 0.165(c_1 a_1 + c_2 a_2) + 7.227\, a_3^{0.785} \ , \tag{15}$$

$$y(a, \omega) = \sum_{k=1}^{N} (d_1(k) \max\{0, \varphi(k)\} - d_2(k) \max\{0, -\varphi(k) - a_3\}) \ . \tag{16}$$

Here, $f(a)$ is a smooth function. Function $y(a, \omega)$ with fixed $\omega \in \Omega$, generally speaking, is non-smooth with respect to a. Nevertheless, if random values $W(k)$, $k = 1, \ldots, N$ have probability densities then the mathematical expectation smoothness the integrand.

THEOREM1 *If random values $W(k)$, $k = 1, 2, \ldots, N$ have probability densities for all $a_1 \ge 0$, $a_2 \ge 0$ other than $a_1 = a_2 = 0$, then function $F(a)$ defined by (9)–(12) is differentiable on the set $a_1 \ge 0$, $a_2 \ge 0$, $a_3 \ge 0$ except for ray $a_1 = a_2 = 0$, $a_3 \ge 0$ and*

$$F_a(a) = \begin{bmatrix} 0.165\, c_1 \\ 0.165\, c_2 \\ 5.673195\, a_3^{-0.215} \end{bmatrix} + E \sum_{k=1}^{N} [\varphi_a(k)(u_1(k) \tag{17}$$

$$+ u_2(k)) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(k)] \ ,$$

$$\varphi_a(k) = \begin{bmatrix} -R_1G_1(V(k)) \\ -R_1G_2(E_1(k)) \\ 0 \end{bmatrix} + \begin{cases} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, & \text{if } -\varphi(k-1) \le 0 , \\ \varphi_a(k-1) , & \text{if } 0 < -\varphi(k-1) < \alpha_3 , \\ \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, & \text{if } -\varphi(k-1) \ge \alpha_3 , \end{cases} \tag{18}$$

$$u_1(k) = \begin{cases} d_1(k) , & \text{if } \varphi(k) > 0 , \\ 0, & \text{otherwise} , \end{cases}$$
$$u_2(k) = \begin{cases} d_2(k) , & \text{if } \varphi(k) + \alpha_3 < 0 , \\ 0 , & \text{otherwise} . \end{cases} \tag{19}$$

PROOF On substitution of recurrent relations (10), (11) and (12), function $y(\alpha, \omega)$ becomes differentiable on α with probability 1 for any α which belongs to the set $\alpha_1 \ge 0$, $\alpha_2 \ge 0$, $\alpha_3 \ge 0$ except for the ray $\alpha_1 = \alpha_2 = 0$, $\alpha_3 \ge 0$. The gradient of the function $y(\alpha, \omega)$ in the points where it is differentiable has the form:

$$y_a(\alpha, \omega) = \sum_{k=1}^{N} [\varphi_a(k)u_1(k) + (\varphi(k) + \alpha_3)_a u_2(k)] \tag{20}$$

$$= \sum_{k=1}^{N} \left[\varphi_a(k)(u_1(k) + u_2(k)) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(k) \right] ,$$

where $\varphi_a(k)$, $u_1(k)$, $u_2(k)$, $k = 1, \dots, N$ are defined by relations (10)–(12), (18), (19).

From relation (18), (19), (20) it is easy to see, that the mathematical expectation of vector function $y_a(\alpha, \omega)$ exists since the following estimates hold;

$$|y_{a_1}(\alpha,\omega)| \le \max_k (|d_2(k)| + |d_2(k)|)R_1 \sum_{k=1}^{N} \sum_{l=1}^{k} |G_1(V(l))| , \tag{21}$$

$$|y_{a_2}(\alpha, \omega)| \le \max_k (|d_1(k)| + |d_2(k)|)R_1 \sum_{k=1}^{N} \sum_{l=1}^{k} |G_1(V(l))| , \tag{22}$$

$$|y_{a_3}(\alpha, \omega)| \le \sum_{k=1}^{N} (|d_1(k)| + |d_2(k)|) \tag{23}$$

and functions of right-hand sides of the inequalities have mathematical expectation.

Denote unit vectors e_i, $i = 1, 2, 3$. The differentiability of function $Ey(a, \omega)$ follows from Lebeque theorem on limit transition inside the integral sign [9]

$$(Ey(a, \omega))_{a_i} = \lim_{h \to 0} E\frac{y(a + he_i, \omega) - y(a, \omega)}{h} = Ey_{a_i}(a, \omega) \ ,$$

$i = 1, 2, 3$ since

$$\left|\frac{y(a + he_1, \omega) - y(a, \omega)}{h}\right| \le \max_k (|d_1(k)| + |d_2(k)|)R_1 \sum_{k=1}^{N} \sum_{l=1}^{k} |G_1(V(l))| \ ,$$

$$\left|\frac{y(a + he_2, \omega) - y(a, \omega)}{h}\right| \le \max_k (|d_1(k)| + |d_2(k)|)R_1 \sum_{k=1}^{N} \sum_{l=1}^{k} |G_2(V(l))| \ ,$$

$$\left|\frac{y(a + he_3, \omega) - y(a, \omega)}{h}\right| \le \sum_{k=1}^{N} (|d_1(k)| + |d_2(k)|)$$

for any $h \in R$. Functions in right-hand sides of the inequalities have mathematical expectations. The last three inequalities follow from estimates for $|y_{a_i}(a, \omega)|$, $i = 1, 2, 3$. The theorem is proved.

From the theorem we obtain the following formula for stochastic quasi-gradient calculation:

$$\xi^s = \begin{pmatrix} \xi_1^s \\ \xi_2^s \\ \xi_2^s \end{pmatrix} = \begin{pmatrix} 0.165\, c_1 \\ 0.165\, c_2 \\ 5.673195 (a_2^s)^{-0.215} \end{pmatrix} + \sum_{k=1}^{N} \left[\varphi^s(k)(u_1^s(k) + u_2^s(k)) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u_2(k) \right] \qquad (24)$$

where $\varphi_a^s(k)$, $u_1^s(k)$, $u_2^s(k)$ are obtained from (10)–(12), (18), (19) by substituting argument a by approximation as on the s-th iteration.

5. MODELING OF WEP AND SPP OUTPUT

Expression for R_1 is set as follows:

$$R_1 = \bar{\omega}\eta_1\eta_2 / H \ , \qquad (25)$$

where $\bar{\omega}$ is the dimensions conversion coefficient, η_1 is the efficiency of the generated energy transfer to the consumer, η_2 is the pump efficiency, H is the specified head.

Random values $G_1(V(k))$, $k = 1, 2, \ldots N$ are modeled according to the formula [10], [11]

$$G_s(V(k)) = 0.5\tau(k) j \eta_p \eta_r \rho_b V^3(k)(1 + 3C_v^2 + C_s^3) \tag{26}$$

where wind velocity daily average values are specified by continuous distributions with densities:

$$g(V(k)) = b\left[\frac{\Delta V}{\bar{V}}\right]\left[\frac{V(k)}{\bar{V}}\right]_c \exp\left[-d\left[\frac{V(k)}{\bar{V}}\right]^c\right] . \tag{27}$$

where $\tau(k)$ is the number of the WEP operation hours in a day; j is the wind energy utilization factor; η_p, η_r is the efficiency of the double reduction gear and synchronous wind-driven electric generator; ρ_b is air density, $v(k)$ is the daily average values of the wind velocity reduced to height h = 12m; c_v, c_s are coefficients of variation and asymmetry, respectively; ΔV is the wind velocity variation interval ($\Delta V = 1$); $\bar{v}$ is the average wind velocity for the time period under consideration; b, c, d, e are the parameters of the wind velocity probability density function.

For $\tau(k)$, similarly to [12], it was found that

$$\tau(k) = \begin{cases} 0 , & \text{if } V(k) < 3 , \\ 5V(k) - 13 , & \text{if } 3 \le V(k) \le 7 , \\ 24 , & \text{if } V(k) > 7 , \; k = 1, \ldots, N . \end{cases} \tag{28}$$

Generation of wind velocity average daily values $V(k)$, $k = 1, 2, \ldots, N$ is carried out according to (27) by the climination method [13]. Random numbers α_1, α_2 uniformly distributed over [0, 1] are generated by referring to the random numbers generator. When assuming V^0 and V^{max} as the initial and the maximum daily average wind velocities, we obtain for given location

$$\nu = V^0 + \alpha_1(V^{max} - V^0) ,$$

$$g(\nu) = \frac{B}{\bar{V}}\left[\frac{\nu}{\bar{V}}\right]^c \exp\left[-d\left[\frac{\nu}{\bar{V}}\right]^c\right] ,$$

$$\eta = \alpha_2 B , \; B = \max_{v \in [0, \infty[} g(v) .$$

Then let us compare the values η and $g(\nu)$. If $\eta > g(\nu)$ then the available numbers ν and η are omitted and the process is repeated with new values α_1 and α_2. Otherwise, the ν value is accepted as the $V(k)$. Then $\tau(k)$ is determined by relations (28) and specific daily power output by the electric plants in k-th day is calculated using (26).

To calculate $G_2(k)$ – specific output of the SPP for $k = 1, \ldots, N$, used are the following relations [14], [15]:

$$G_2(E(k)) = \eta_c \tilde{Q}(E(k)) \ ,$$

$$\tilde{Q}(E(k)) = F'(\overline{\tau\alpha})_e \tilde{Q}_0(k) \max\{0, K_c(k) - K_c'(k)\} \ ,$$

$$\tilde{Q}_0(k) = l_1 + l_2 \cos\frac{2\pi}{365}(k - 123) \ ,$$

$$K_c(k) = K_c^{\min}(k) + \alpha_3(K_c^{\max}(k) - K_c^{\min}(k)) \ , \tag{29}$$

$$K_c^{\min}(k) = l_3\left[1 - \cos\frac{2\pi k}{365}\right] \ ,$$

$$K_c'(k) = \frac{\bar{\tau}(k) K_t (T_c(k) - T_b(k))}{\bar{r}\tilde{Q}_0(k)(\overline{\tau\alpha})_e} \ ,$$

$$T_c(k) = (323 + T_b(k))/2, \ \ k = 1, \ldots, N \ ,$$

where η_c is the total efficiency of solar energy conversion to electrical one according to Brayton thermodynamic cycle; $\tilde{Q}(E(k))$ is specific effective heat drawn of the solar energy collector; F' is coefficient of the heat withdrawal; $(\overline{\tau\alpha})_e$ is the reduced effective absorptive capacity of the collector; $\tilde{Q}_0(k)$ is extra-atmospheric flow of solar energy; $K_c(k)$ is the clearness coefficient; $K_c'(k)$ is the threshold value of the clearness coefficient; $K_c^{\min}(k)$ and $K^{mad}(k)$ are the minimum and the maximum limit of the clearness coefficient, respectively; K_t is the collector coefficient of heat losses; $T_c(k)$ is the heat-transfer agent temperature; $T_b(k)$ is the ambient air temperature; $\bar{\tau}(k)$ is the collector operation time; $\bar{r}$ is the slope; l_1, l_2, l_3 are parameters depending on the place latitude φ; α_3 is a random number uniformly distributed over [0, 1].

On the basis of available experimental data, the following analytical relationships for determining the values of parameters l_1, l_2, l_3 and the values of $K_c'(k)$ have been found with the aid of interpolation polynomials:

$$l_1(d) = -0.41 \cdot 10^{-2} d^5 + 0.537 \cdot 10^{-1} d^4 - 0.2385\, d^3 + 0.361\, d^2$$

$$- 1.5119\, d + 31.04 \ ,$$

$$l_2(d) = -0.3 \cdot 10^{-3} d^5 + 0.42 \cdot 10^{-3} d^4 + 0.02 d^3 - 0.0142\, d^2$$

$$+ 1.8304\, d + 10.92 \ ,$$

$$l_3(\varphi) = 0.195 \cdot 10^{-3} \varphi^2 - 0.321 \cdot 10^{-1} \varphi + 1.24 \qquad (30)$$

$$K'_c\ (k) = K'_c\ (\delta(k)) = -0.534 \cdot 10^{-3} \delta^5(k) + 0.8674 \cdot 10^{-2} \delta^4(k)$$

$$- 0.04186 \delta^3(k) + 0.09037\, \delta^2(k) - 0.1616\, \delta(k) + 0.4540 \ ,$$

$$k = 1, \ldots, N$$

where

$$d = 0.2(\varphi - 30),\ \delta(k) = (k - 29)/61 \ .$$

For given place, we assume

$$K_c^{\max}(k) = K_c^{\max} = \text{const},\ k = 1, \ldots, N \ .$$

Then the modeling algorithm $G_2(F(k))$, $k = 1, \ldots, N$ will be presented with the aid of formulae (29)–(30) as follows.

Specified are the values F', $(\overline{\tau\alpha})$, $K_c^{\max}$, η_c and the place latitude φ. The values of parameters l_1, l_2, l_3 are determined from relations (30). With $k = 1$, calculated one after another are the values $K'_c\ (k)$, $K_c^{\min}(k)$ and $\tilde{Q}_0(k)$. Modeling of α_3 is performed by reference to the random numbers generator and then $K_c(k)$, $\tilde{Q}(E_1(k))$ and $G_2(E_1(k))$ are calculated. Similar operations are performed for the next values of k, $k = 2, \ldots, N$.

6. CALCULATIONS RESULTS

In performing calculations, data for Turkmen SSR territory and Central Kara Kum region have been used [6], [7]. Data for modeling average daily wind velocities and specific output of WEP are presented in Table 1.

In calculating $G_1(V(k))$, $k = 1, \ldots, N$, the following values have been used in addition to the Table data: $\Delta V = 1.0\, m/s$;

$$V^0 = 0;\ V^{\max} = 8.5\, m/s,\ j = 0.36;\ \eta_p = 0.95,\ \eta_r = 0.80 \ .$$

TABLE 1

Season Quantity	Winter	Spring	Summer	Autumn
b	2.917	1.812	0.871	4.532
c	0.590	0.500	0.380	0.750
d	1.868	1.324	0.389	2.303
l	0.880	1.100	2.280	0.800
$\bar{v}(m/s)$	3.200	3.900	3.800	3.100
$\rho_b(kg/m^3)$	1.294	1.211	1.133	1.221
C_v	0.835	0.784	0.618	0.855
C_s	1.141	1.079	0.602	1.172

For modeling $G_2(E_1(k))$, $k = 1, \ldots, N$, the following values have been accepted: the place latitude $\varphi = 38$; $F' = 0.982$; $(\overline{\tau\alpha})_e = 0.738$; $K_c^{\max} = 0.850$ and $\eta_c = 0.03$.

The considered control period is one year ($N = 365$). Daily water demand for two shepherd teams (each consisting of three men), two to three camels and 850 sheep are determined by the following quadratic relation:

$$b(k) = -0.2566 \cdot 10^{-3} k^2 + 0.09392k + 6.80634\,(m^3),\ k = 1, \ldots, N \ ,$$

where maximum value ($15.4m^3$) is observed in the middle of July.

The program is written in FORTRAN. The following parameters values are used in the algorithm; $a = 2$, $u = 0.8$, $\Lambda = 0.8$, $\rho_0 = 0.01$, $\alpha = 0.5$. The initial approximation is as follows: $a_1^0 = 0.5\,m^2$, $a_2^0 = 3\,m^2$, $a_3^0 = 30\,m^3$. The R_1 value is determine from formula (25). Calculations are performed with the following values of the model parameters: $\omega = 1$; $\eta_1 = 0.9$; $\eta_2 = 0.7$; $H = 5\,m$; $c = 0$; $c_1 = 200\,wb/m$; $c_2 = 250\,rub/m$; $d_1(k) = d_1 = 20\ rub/m^3$; $d_2(k) = d_2 = 0$, $k = 1, \ldots, N$.

To provide a stable operation of the algorithm, its feasible area D is narrowed: $D = \{a : a_1 \geq 0.5,\ a_2 \geq 3,\ a_3 \geq 30\}$. The computing experiments results show that these restrictions are not active therefore such contraction does not change optimum point. In case of small values of parameters a_1, a_2, and a_3, stochastic quasi-gradient assumes very large absolute values which result in a loss of computation process stability. To eliminate such effect, the feasible area is narrowed.

The calculation results are presented in Table 2.

The last column presents averaged estimates of the objective function obtained in the following way: with fixed values a_1, a_2, a_3, the performance of the plant was simulated a hundred times and estimates of the objective function have

TABLE 2 Results of calculation of main parameters of a water supply system using WEP and SPP

Iteration No. S	Parameter values			Averaged values		
	a_1^s, m^2	a_2^s, m^2	a_3^s, m^2	Total water deficit $\sum_1^{365} z_1(k)$, m^3	Total water excess $\sum_1^{365} z_2(k)$, m^3	Function value $\bar{F}(a)$, $rub/year$
0	0.50	3.00	30.0	62.4	20700	1490
20	1.02	14.50	36.0	9.66	67800	946
40	0.99	5.55	38.2	13.8	44200	664
60	1.09	4.89	42.8	12.7	46300	630
90	1.10	4.89	42.8	12.7	46600	630

been calculated:

$$\bar{F}(a) = 0.165(c_1a_1 + c_2a_2) + 7.227\,a_3^{0.785} + \sum_{k=1}^{N} (d_1(k)z_1(k) - d_2(k)z_2(k)) ,$$

and then the arithmetic mean for these 100 random realizations are calculated. The annual average total water deficit and surplus are calculated similarly. Such calculations are performed in the initial approximation point and after 20 and 40 iterations and in the point which is believed to be the best approximations. According to norms, a 90% water provision is usually required. Calculation results show that the total water deficit for one year is 13m^3 in the optimum version. When it is remembered that the total water demand is 4581m^3, then it follows that the water supply system with optimum parameters provides 99% of the water demand. Total amount of excess water is 10 times greater than the total water demand. If necessary, the energy spent for lifting excess water can be used to meet demand for electric energy.

Calculations performed for increasing water lifting level H show that the optimum parameters values increase with increasing H (Table 3).

TABLE 3 Optimum parameter values of water lifing system using WEP and SPP for different H, for Central Kara Kum region, Turkmen Soviet Socialist Republic (φ = 38°).

H, m	a_1^*, m^2	a_2^*, m^2	a_3^*, m^3
5	1.10	4.89	42.8
10	2.02	8.59	44.3
15	2.22	9.98	44.7
20	3.12	12.01	46.4
25	3.32	14.04	48.1
30	4.25	15.90	50.2

In these calculations, the estimates of the parameters optimum values obtained with one H value are used as initial approximation for the next one. Consequently satisfactory approximations of the optimum points are found for a rather small number of iterations (about 40 to 45).

Calculations also were made for the water supply system using only SPP as a source of power.

For the system using only WEP we have Table 5.

TABLE 4

H, m	a_2^*, m^2	a_3^*, m^3
5	12.9	65.87
10	19.26	70.00
15	26.54	82.22
20	33.00	86.86
25	39.72	92.11
30	46.46	99.70

TABLE 5

H, m	a_1^*, m^2	a_3^*, m^3
5	5.025	106.1
15	5.84	122.6
30	11.81	135.93

The calculation experiments results show that stochastic quasi-gradient algorithms are effective means for finding optimum parameters of solar- and wind-electric water supply systems.

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