ADAPTIVE CONTROL OF FISHING SYSTEMS

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September 1975                          WP-75-114

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## Abstract

This paper discusses some formal techniques for deciding how harvesting policies should be modified in the face of uncertainty. Parameter estimation and dynamic optimization methods are combined for the Ricker stock recruitment model to show how exploitation rates should be manipulated to give more information about the model parameters. In general, harvesting rates should be lower than would be predicted by the best fitting recruitment curve unless this curve predicts that the stock is very productive. A decision procedure is developed for comparing alternative stock recruitment models; when applied to the Fraser River sockeye salmon, the procedure indicates that an experimental increase in escapements would be worthwhile. It appears that there is considerable promise for extending these methods and procedures to cases where the stock size is unknown and where fishing effort is poorly controlled.

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I. Introduction

A variety of dynamic models have been used in recent years to establish fishery regulations and to catch quotas; increasingly, elaborate monitoring systems have been developed to provide sound statistical estimates of model parameters. Most models are used only to predict optimum equilibrium harvest rates, though there have been a few recent attempts to develop harvest strategy curves or "control laws" that specify optimum harvest rates for non-equilibrium situations (Allen, [1]; Walters, [5]). Given a time series from which model parameter estimates have been derived, it has often been assumed that the best management strategy is to act as though these estimates were actually correct: that is, insert the estimates into the model (or into several alternative models), generate a yield curve or an isopleth diagram that reveals an apparently optimum harvest policy, and recommend that this policy be followed.

Little attention has been paid to the problem that, by following the apparently optimum policy, the fishery might be brought to an equilibrium that is neither truly optimal nor productive of the type of date necessary to determine the true optimum. Luckily, most fisheries have gone through a period of more or less uncontrolled development of the point of obvious overexploitation, allowing us to interpolate an optimum regime from data on a broad range of stock sizes. Another problem subsequently arises: when a fishery has been held near some equilibrium for a long period of time, how much confidence can we have in older

data from the non-equilibrium period? Environmental carrying capacities may have changed, selection by the fishery may have produced new genetic types capable of responses, or the old data may be simply unreliable. Implicit in many research programs is the assumption that detailed biological studies on populations near equilibrium will allow a priori determination of optimum harvesting policies, thus making it unnecessary to introduce trial and error changes or large scale experiments in harvest rates; this assumption appears to be naive and unjustified at present.

This paper addresses the question of how harvesting decisions should be modified to take account of statistical uncertainty. In seeking a formal framework for dealing with this question, we have been drawn to the literature on control system theory, where the problem is addressed under the heading of "adaptive" or "dual" control (Larson, [8]). To simplify the discussion, we will concentrate primarily on situations where the stock-recruitment relationship (as opposed to growth and natural mortality) is the critical determinant of potential yield. The analysis is divided into two parts: a) we look at the case where a simple model, the Ricker stock-recruitment curve, is assumed to be the correct functional form and only the model parameters are uncertain; and b) we examine more general cases where the form or shape of the stock-recruitment function is uncertain , stock sizes are not directly measurable, and fishing effort is poorly controlled.

## II. Adaptive Control with the Ricker Model

As indicated by several authors with many examples in Parrish [11], the simple model developed by Ricker [14] has been used often in the analysis of stock-recruitment relationships:

$$R_t = S_{t-1} \exp \left\{ \alpha - \beta S_{t-1} + \nu_t \right\} \tag{1}$$

where

$R_t$ = recruits (adults) at end of generation t ;

$S_{t-1}$ = spawners at the start of generation t ;

$\alpha$ = a stock production parameter ;

$\beta$ = equilibrium stock parameter (equilibrium stock in absence of fishing is equal to $1/\beta$);

$\nu_t$ = a random environmental factor normally distributed with mean 0.0 and variance $\sigma^2$.

For the discussion that follows, it is critical that $\nu_t$, the noise factor, be normally distributed. There is good empirical evidence for this assumption for a sockeye salmon population on the Skeena River (Allen, [1]). A theoretical justification can be constructed by noting that $e^{\nu_t}$ can be viewed as a random survival factor resulting from several independent and multiplicative environmental factors operating in series. (Thus $\nu_t$ represents a sum of several random factors and should be normally distributed by the central limit theorem.)

Let us assume that the management objcetive is to maximize the sum of discounted catches over time:

$$\max \sum_{t=0}^{\infty} C_t e^{-\delta t} \tag{2}$$

where

$$C_t = \text{catch} = R_t - S_t \; ;$$

$$\delta = \text{a discount rate} \; .$$

The discount rate is critical in adaptive control problems, since without $\delta = 0$ we would put all management emphasis on obtaining improved information for the distant future, regardless of the cost in terms of lost yields in the near future. It is known (Allen [1]; Walters [5]) that, for the objective in equation (2) when $\alpha$ and $\beta$ are not uncertain, the optimum management policy is to allow a fixed escapement each year: choose

$$C_t = \begin{cases} R_t - \tilde{S} & \left(\text{if } R_t > \tilde{S}\right) \\ 0 & \left(\text{if } R_t \leq \tilde{S}\right), \end{cases} \tag{3}$$

where $\tilde{S}$ is the optimum escapement, computed from $\alpha$ and $\beta$ (Ricker [15]).

Ordinarily, we would recommend that management actions be based on estimates of $\tilde{S}$ computed from regression estimates $\hat{\alpha}_t$ and $\hat{\beta}_t$. The Ricker model can be rewritten (after Dahlberg, [3]) in the form:

$$\ln\left(\frac{R_t}{S_{t-1}}\right) = \alpha - \beta S_{t-1} + \nu_t \; ; \tag{4}$$

this is a linear regression for $(y = \alpha + \beta x)$ with:

$y = \ln\left(\dfrac{R_t}{S_{t-1}}\right)$ and $x = -S_{t-1}$. We would probably ignore some

useful information that comes from regression analysis,

namely, the parameter covariance matrix:

$$\hat{P}_t = \begin{bmatrix} \hat{\sigma}_\alpha^2 & \hat{\sigma}_{\alpha\beta} \\ \hat{\sigma}_{\alpha\beta} & \hat{\sigma}_\beta^2 \end{bmatrix} \quad , \tag{5}$$

that measures our uncertainty about the parameter estimates

given data to time t. Further, under the assumption that

the $\nu_t$ are normally distributed and independent of one another,

it can be shown that $\dot{\hat{\alpha}}_t$, $\hat{\beta}_t$, and the elements of $\hat{P}_t$, constitute a

set of "sufficient statistics". That is, there is no other function

or manipulation of data that can give additional information

about the underlying true Ricker parameters.

The objective of adaptive control analysis in this case

is to show how the choice of escapement $S_t$ should be related

to $\hat{P}$ as well as to $\hat{\alpha}$ and $\hat{\beta}$. The analysis can be formulated

as a problem in stochastic dynamic optimization (Walters [5]):

given the system state at any time as measured by $\left\{ R_t, \hat{\alpha}_t, \hat{\beta}_t, \right.$

$\left. \hat{P}_t \right\}$ , what choice of $C_t$ will give the best expected combination

of present return and future returns, recognizing that a variety of

possible future states may occur because of random events?

To solve problems of this type, we must be able to formulate

a model that specifies how each of the state variables (R, $\hat{\alpha}$, etc.)

will change in relation to the variety of stochastic outcomes

that may occur between times t and t + 1. We must also be

able to assign probabilities to each of these stochastic out-

comes. Future recruitment states $(R_{t+1})$ can be predicted

with the Ricker model, but analogous predictive

formulae are required for the statistical parameters. The following section shows how these formulae can be derived from a special form of regression analysis.

## A. Recursive or Adaptive Parameter Estimation

Suppose we begin at time t = 0 no data but with prior estimates $\hat{\alpha}_o$ and $\hat{\beta}_o$. We might wish to assign no confidence to these estimates, which is equivalent to saying that we recognize $\sigma_\alpha^2$ and $\sigma_\beta^2$ to be very large, or:

$$\hat{P}_o = \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix} , \tag{6}$$

where L is some large number (e.g. $10^6$). In Bayesian statistical terms, we are in effect assigning a "diffuse prior" distribution for $\alpha$ and $\beta$ (Raiffa and Schlaifer, [13]). With starting conditions such as these, it can be shown that oridinary regression analysis can be written in a special "recursive" format (Young [22]). The general format is presented here, since it may be of interest outside the adaptive control context.

In general, linear regression equations are written in the form:

$$y_i = \sum_{j=1}^{m} a_j x_{ij} + e_i$$

where

$y_i$ = dependent observations;

$x_{ij}$ = independent variables; and

$e_i$ = error terms.

This form can be written more compactly in vector notation. That is:

$$y_i = \underline{a}^T \underline{x}_i + e_i , \tag{7}$$

where $\underline{a}$ and $\underline{x}_i$ represent the vectors $(a_1, a_2, \ldots, a_m)$ and $(x_{i1}, x_{i2}, \ldots, x_{im})$, respectively. Using this notation, we could write the common regression formulae in recursive form as:

$$\hat{\underline{a}}_n = \hat{\underline{a}}_{n-1} - \frac{\hat{P}_{n-1} \underline{x}_n}{\sigma^2 + \underline{x}_n^T \hat{P}_{n-1} \underline{x}_n} (\hat{\underline{a}}_{n-1}^T x_n - y_n) \quad , \quad (8a)$$

and

$$\hat{P}_n = \hat{P}_{n-1} - \frac{\hat{P}_{n-1} \underline{x}_n \underline{x}_n^T \hat{P}_{n-1}}{\sigma^2 + \underline{x}_n^T \hat{P}_{n-1} \underline{x}_n} \quad , \quad (8b)$$

where $\hat{\underline{a}}_n$ and $\hat{P}_n$ refer to the parameter and parameter error covariance estimators, respectively, after the $n^{\text{th}}$ data point is acquired, and $\sigma^2$ is the regression error variance. These formulae allow new data points to be added to a regression analysis without tedious computations involving matrix inversion. Estimation for the Ricker model can be written in the recursive form with:

$$y_n = \ln\left(\frac{R_t}{S_{t-1}}\right)$$

$$\underline{x}_n = \begin{bmatrix} 1 \\ -S_{t-1} \end{bmatrix} \quad (9)$$

$$\hat{\underline{a}}_n = \begin{bmatrix} \hat{\alpha}_t \\ \hat{\beta}_t \end{bmatrix} \quad ;$$

similar transformations can be developed for a variety of other fisheries models.

Equations (8a) and (8b) are critical for the adaptive control formulation developed in Section III. Note that the change in parameter uncertainty from any observation or time step to the next, as measured by $\hat{P}_n - \hat{P}_{n-1}$, depends on $\sigma^2$, $\hat{P}_{n-1}$, and on the choice of $\underline{x}_n$, i.e. on the choice of $S_{t-1}$ for the Ricker model. Changes in the parameter estimates as measured by $\hat{\underline{a}}_n - \hat{\underline{a}}_{n-1}$ depend on:

a) the level of uncertainty as measured by $\hat{P}_{n-1}$ and $\sigma^2$;

b) the choice of $\underline{x}_n$; and

c) the a priori prediction error, $D_n = (\underline{a}_{n-1}^T \underline{x}_n - y_n)$.

The a priori prediction error $D_n$ is the difference between the observed $y_n$ and its predicted value using the latest $\underline{x}_n$ data and the older or prior parameter estimates, $\hat{\underline{a}}_{n-1}$. This prediction error (that is the only uncontrolled or stochastic input into the $\hat{\underline{a}}$ and $\hat{P}$ changes for any time step), can be rewritten as two error components:

$$D_n = (\underline{a}^T \underline{x}_n - y_n) + (\hat{\underline{a}}_{n-1}^T - \underline{a}^T)\underline{x}_n \quad . \tag{10}$$

The first component is the deviation of $y_n$ from the true model, while the second component represents deviation of the parameter estimates from the true value. If the regression errors $\nu_t$ are normally distributed, both of these error components are normally distributed; thus $D_n$ should have a normal distribution with mean zero and variance. That is:

$$\sigma_{D_n}^2 = \sigma^2 + \underline{x}_n^T \hat{P}_{n-1} \underline{x}_n \quad . \tag{11}$$

With data available up to any time step, we can compute proba-
bilities for different values of $D_n$, and thus have different
parameter estimates at the next time step. This is known
as preposterior analysis in statistical decision theory
(Raiffa [12]). For the Ricker model, $D_n$ is interpreted as:

$$D_n = \hat{\alpha}_{t-1} - \hat{\beta}_{t-1}S_{t-1} - \ln\left(\frac{R_t}{S_{t-1}}\right) . \qquad (12)$$

Having chosen a value for $D_n$, with its associated probability,
we can predict $R_t$ by solving equation (12) as:

$$\hat{R}_t = S_{t-1}\exp\left\{\hat{\alpha}_{t-1} - \beta_{t-1}S_{t-1} - D_n\right\} . \qquad (13)$$
$$\text{(given } D_n)$$

This is the original Ricker model, with an error component
that reflects not only the noise $\nu_t$ but also the uncertainty
about $\alpha$ and $\beta$.

The adaptive regression equations (8) can be modified to
"forget" older data. There are two simple types of modifi-
cations:   a) exponential past weighting  of data, based on
the assumption that all data become progressively less reliable
as they become older; and  b) parameter variance incrementation,
based on the more specific assumption that the parameters
do vary in some random or unspecifiable systematic
way over time.  For exponential past weighting, we define a
discount factor $V_d$ that represents the value of any obser-
vation relative to the next one that is obtained; for example,
if we want to assume that an observation at time $t - 1$ is

worth 90% as much as an observation at time t, then $V_d = 0.9$. Using this discount factor, equations (8) are simply modified by changing the denominator terms $\sigma^2 + x_n^T \hat{P}_{n-1} x_n$ to:

$$V_d \sigma^2 + \underline{x}_n^T \hat{P}_{n-1} \underline{x}_n \quad . \tag{14}$$

When there is reason to believe that one or more parameters are changing over time, the estimation is modified by introducing a parameter variation matrix Q, where the elements of the matrix are chosen to reflect the expected rate of change in the parameters. For example, if we believe that the Ricker $\beta$ parameter may change about 10% per year from an average value of about $10^{-6}$ (while the $\alpha$ parameter is stable), we could set:

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 10^{-14} \end{bmatrix} \quad . \tag{15}$$

In statistical terms, the elements of Q are interpreted as variances on a random walk process; thus a 10% change from a base of $10^{-6}$ represents a standard deviation of $10^{-7}$, or a variance of $(10^{-7})^2$. The Q matrix is introduced into equations (8) simply by replacing every $\hat{P}_{t-1}$ with $\hat{P}_{t-1}^*$ where

$$\hat{P}_{t-1}^* = \hat{P}_{t-1} + Q \quad . \tag{16}$$

The choice of $V_d$ or Q is not particularly critical; the major effect in both cases is to prevent $\hat{P}_t$ from going to zero over time, so that new observations can continue to effect changes in $\hat{\underline{a}}$.

### B. Adaptive Decision Structure and Optimization

The problem of adaptive control and optimization for the Ricker model can be visualized in terms of the decision tree shown in Figure 1. At any point in time, the manager is faced with a recruitment $R_t$, a summary of past data in terms of $\hat{\alpha}_t$ and $\hat{\beta}_t$, and with uncertainties in terms of $\sigma^2$ and the elements of $\hat{P}_t$. He must choose a harvest $C_t$; there are many possible choices, but the optimization problem can be approximated by looking at a reduced, discrete set of possibilities (e.g. $C_t = 0$, $C_t = 0.1R_t$, $C_t = 0.2R_t$).

Given any choice of $C_t$, there are many possible random outcomes; these can be summarized in terms of discrete deviations $D_t$ from the regression predictors of $y = \ln(R_{t+1}/S_t)$. The reasoning is as follows:

a) Given $C_t$, $S_t$ is calculated as $R_t - S_t$;

b) $\hat{\alpha}$, $\hat{\beta}_t$, and $S_t$ are used to make a regression prediction $y_{t+1}$ [equations (7) and (9)];

c) Probabilities for different outcomes $y_{t+1} = \hat{y}_{t+1} + D_t$ are computed from the probability distribution for $D_t$, that is normal with mean zero and variance given by equation (11);

d) Each outcome $y_{t+1}$ is inserted, along with $C_t$, into the recursive regression equations (8) to obtain new estimates $\hat{\alpha}_{t+1}$, $\hat{\beta}_{t+1}$, $\hat{P}_{t+1}$; and

e) Since $y_{t+1} = \ln(R_{t+1}/S_t)$, $R_{t+1}$ for each outcome $y_{t+1}$ is given as $S_t e^{y_{t+1}}$ [equivalent to equation (13)].

If we know the total future value for being in any next state $\left\{ R_{t+1}, \hat{\alpha}_{t+1}, \hat{\beta}_{t+1}, \hat{P}_{t+1} \right\}$, we can calculate the expected value for making a $C_t$ decision. This expected value is the sum of products of probabilities of next states, times the values associated with these next states, plus the value of $C_t$ itself. The difficulty is that we cannot immediately assign a value for each of the next states, since that state is itself a starting point for another decision tree similar to Figure 1. If we look ahead a few time steps, the number of branching possibilities becomes essentially infinite. There is a partial way out of this problem using the "backward recursion" procedure of dynamic programming. A simplified discussion of this procedure is given in Walters [5]; the basic idea is that we begin the optimization calculations at some time point far enough in the future that the discounted values after that point can be neglected. We then move backwards towards the present, evaluating decisions at each of the time steps in terms of future values that have just been computed for the next time steps forward.

Unfortunately, even dynamic programming involves formidable computation problems. If at each of the time steps we examine only ten discrete values for each of the six state variables $(R_t, \hat{\alpha}_t, \hat{\beta}_t, \sigma_\alpha^2, \sigma_\beta^2, \sigma_{\alpha\beta})$, 10 catch levels and 10 values of $D_t$, we must compute about $10^8$ solutions for equations (8) and (13). The problem can be reduced somewhat by using special computation procedures (Larson [8]), but there is a clear need for different ways of looking at the problem. The optimization need only be carried out for a few representative

values of the environmental variance $\sigma^2$ and the discount rate $\delta$ in order to get a complete adaptive picture for the Ricker model. Given $\sigma^2$ and $\delta$, the optimization automatically arrives at best harvest rates for <u>all</u> stock size-parameter value-parameter uncertainty combinations, in the form of a multidimensional "control law." (Allen [1] and Walters [5] have referred to one dimensional versions of this control law as "strategy curves.")

A further point worth noting is the size of adaptive optimization problems. Let us suppose that instead of the Ricker model we wish to analyze some model with three parameters (say, $\alpha_1$, $\alpha_2$, and $\alpha_3$). Even if we can put this model into the linear regression form with normally distributed errors, the number of state variables for the dynamic optimization is 10 ($R_t$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$, $\sigma^2_{\alpha_1}$, $\sigma^2_{\alpha_2}$, $\sigma^2_{\alpha_3}$, $\sigma_{\alpha_1\alpha_2}$, $\sigma_{\alpha_1\alpha_3}$, $\sigma_{\alpha_2\alpha_3}$). This is too large a problem for even the best modern computers to handle.

## C.  Solutions for Special Cases

Instead of carrying out the tedious and expensive computations for the full adaptive optimization, we elected to examine two special cases that appear to be of management interest and that should reveal the general flavor of the full solution.  These cases are shown in Figure 2, and reflect two extreme situations:

<u>Case 1</u>:  The fishery is just beginning, and the stock is near natural equilibrium; $\beta$ can be treated as known and the main uncertainty is about $\alpha$.

Case 2: The fishery has been holding spawning stocks
at low levels for many years; α is well known and the
main uncertainty is about β. Many Pacific salmon
fisheries seem to fit this case; environmental carrying
capacities may have changed conisderably in recent
years.

In either of the cases the size of dynamic optimization problem
is reduced considerably by treating one parameter as known. In
Case 1, the stock and recruitment data can be expressed in
stock units relative to the natural equilibrium: the Ricker
model can be written in the form:

$$R_{t+1} = S_t e^{\alpha(1-S_t)+\nu_t} \quad ,$$

where the system state vector for optimization becomes
$\left\{ R_t, \hat{\alpha}_t, \sigma_\alpha^2 \right\}$, and the variables in the adaptive regression
equations become:

$$y_t = \ln\left( \frac{R_t}{S_{t-1}} \right)$$

$$x_t = (1 - S_{t-1}) \tag{17}$$

$$\hat{P}_t = \hat{\sigma}_\alpha^2 \quad .$$

In Case 2, the Ricker model is assumed to maintain its usual
form, the optimization state vector becomes $\left\{ R_t, \hat{\beta}_t, \sigma_\beta^2 \right\}$,
and the adaptive regression variables become:

$$y_t = \ln\left(\frac{R_t}{S_{t-1}}\right) - \tilde{\alpha}$$

$$x_t = -S_{t-1}$$

(18)

$$\hat{P}_t = \hat{\sigma}_\beta^2 \quad,$$

where $\tilde{\alpha}$ is the reasonably certain estimate of $\alpha$.

Several dynamic programming solutions for the simplified cases were carried out on a PDP 11/45 computer system. Each of the solutions required about five hours of computer time, which is not excessive considering the broad range of stock-parameter-uncertainty combinations that must be evaluated. By trial and error, we discovered that it was necessary to use 10 discrete levels for each of the variables ($R_t$, $\hat{\alpha}_t$ or $\hat{\beta}_t$, $\hat{\sigma}_\alpha^2$ or $\hat{\sigma}_\beta^2$), and to move backwards in time around 20 steps (generations); finer state intervals and more time steps did not change the solutions.

Representative results for Case 1 ($\alpha$ uncertain) are shown in Figure 3. Each of the isopleth diagrams show optimal harvest rates for a cross section through the $R_t - \hat{\alpha}_t$ plane at one uncertainty ($\sigma_\alpha^2$) level. The most striking feature of these results is that optimal harvest rates are nearly independent of $\hat{\alpha}$ for large $\sigma_\alpha^2$. What we expected to see was some indication that spawning populations should be reduced (high exploitation) when $\alpha$ is uncertain; by equations (8) and (17), we would expect the greatest reduction in uncertainty by conducting such an "experiment." As the cross section for high uncertainty ($\sigma_\alpha^2 = 4.0$) in Figure 3 shows, experiments involving high exploitation rates are optimal only if $\hat{\alpha}_t$ is also large; indeed it appears that the best strategy is to avoid high harvest experiments

when $\alpha$ is low and the stock size is large. The optimization
also takes into account the possiblity that low spawning
stocks will reveal $\alpha$ to be small; thus a period of recovery
without harvest will be necessary. Examining the low un-
certainty $(\sigma_\alpha^2 < 0.1)$ diagram in Figure 3, it is apparent
that the optimal harvest rate for any stock size is insen-
sitive to $\alpha$, no matter what the environmental noise $\sigma^2$ (Walters
[5] obtained a similar result). We should not expect the
optimal harvest strategy to depend greatly on $\sigma_\alpha^2$ if this strategy
is nearly independent of $\alpha$ in the first place.

The results for Case 2 ($\beta$ uncertain) indicate a similar pat-
tern; the optimum harvest strategy is quite insensitive to $\hat{\beta}$ when
$\sigma_\beta^2$ is high (Figure 4). Examing equations (8) and (18), we would
expect high spawning stocks to produce the greatest reduction in
uncertainty about $\beta$; yet the optimization balances the value of
low exploitation (high $S_t$) experiments against the loss in immed-
iate yields that such experiments would entail. Low harvest ex-
periments are called for only when there is intermediate uncer-
tainty about $\beta$.

III. Selection Among Alternative Models.

The analysis in Section II took two sources of
uncertainty into account: random environmental variation, and
uncertainty about production parameters. Section III explores
a third type of problem: uncertainty about the basic functional
form of the stock recruitment relationship. As an example, con-
sider the data in Figure 5 on "off-cycle" runs of sockeye salmon
(Oncorhynchus nerka) in the Fraser River. Several subpopulations
of sockeye in this river system exhibit cyclic dominance (Ward
and Larkin, [20]), with very large "cycle" runs every four years

(1962, 1966,....) that apparently follow a different stock-recruitment relationship from the off-cycle runs. Escapement levels in the off-cycle years have apparently been chosen under the assumption that high spawning populations may result in lowered recruitment because of overutilization (space, oxygen, etc.) of spawning areas; Figure 5 does not support this assumption, at least when the whole river system is treated as a single population unit. Also, the overspawning phenomenon should have resulted in damping or destruction of higher cycle years during the early development of the fishery. However, there is no evidence of this (Ward and Larkin [20]); the off-cycle years sustained annual catches of around 4 million fish until the destructive Hell's Gate Slides of 1911. The best fitting Ricker curve for the data (curve $\eta_1$ in Figure 5; $\alpha = 1.9$, $\beta = 0.44$) does predict that production would decline for spawning stock above 2 million, but it seems equally reasonable to assume that the correct relationship is a saturating curve of the "Beverton Holt" type (Ricker [15]). That is:

$$R_t = \frac{e^{\nu_t}}{\beta + \alpha/S_{t-1}} \, , \tag{19}$$

where

$\alpha = 1/$(maximum recruits per spawner);

$\beta = 1/$(maximum recruits ever possible);

$e^{\nu_t}$ = random environmental survival factor as in equation (1). A visual fit to this relationship is shown in Figure 5 as curve $\eta_2$; the parameters ($\alpha = 0.1237$, $\beta = 0.1025$) were chosen so as to:

(a)  closely match the Ricker curve through the available data; and

(b)  predict an equilibrium stock (8.5 million) that seems reasonable considering early catch records.

Whatever the fitting precedure and even allowing for decrease in production for high spawning stocks (dotted lines off curve $\eta_2$ in

Figure 5), significant improvements in yield could be obtained if
the $\eta_2$ curve is correct. The question is: should an experiment
(reduced harvests for one or more years) be conducted to test
this possiblity?

In principle this question could be addressed with the
optimization approach introduced in Section II. The stock-
recruitment relationship can be written in the form:

$$R_t = \theta_1 f_1 (S_{t-1}) + \theta_2 f_2 (S_{t-1}) + \cdots \cdots \tag{20}$$

where $\theta_i$ represents model selection parameters that take only the
values 0 or 1 and are constrained as $\Sigma \theta_i = 1$ (so that all but one
of the $\theta_i$ must equal 0), and the $f_i$ are alternative models such
as the Ricker [equation (1)] and the Beverton-Holt [equation (19)].
Wood [21], Smallwood [17] and others have shown that it is possible
to calculate the probability that each $\theta_i = 1$ (model i is correct),
given that the true model is among the alternatives represented. These
probabilities along with parameter estimates and measures of un-
certainty for each of the alternative models can be formed into an ex-
tended vector of state variables. Unfortunately, the number of vari-
ables involved makes dynamic programming optimization imprac-
tical. Thus some drastic simplifications and approximations are
necessary in order to trace the most likely statistical outcomes and
the most promising decision possibilites.

Since full adaptive control analysis is not feasible, the
remainder of this section attempts to develop a simplified pro-
cedure for designing and evaluating experimental harvesting re-
gimes intended to discriminate between alternative production models.
The procedure is modified from a general approach suggested by
Bard [2], and involves the following five basic steps.

(a)  <u>Identification of a series of models</u>, or of alternative

"states of nature" $\eta_1, \eta_2, \ldots \eta_m$ that are to be compared.
Alternative $\eta_1$ might be the Ricker model, $\eta_2$ might be the
Beverton-Holt model, $\eta_3$ might be a simple free hand curve
extrapolating from existing data, and so forth.

(b) <u>Assignment of prior or judgemental probabilities</u>
$p^*(\eta_1)$, $p^*(\eta_2)$... to each of the alternative states of nature.
These probabilities might be derived through some statistical
procedure, or they may represent simple intuition.  Reflect-
ing on the Fraser River data, we might for example assign p*
( Ricker model) = 0.7, and p*(Beverton-Holt model) = 0.3. Though
both models fit the data about  equally well, this probability
assignment would give some weight to the common arguments about
overspawning.

(c) <u>Identification of a series of alternative harvesting</u>
experiments $\mathscr{A}_1, \mathscr{A}_2, \ldots, \mathscr{A}_n$, each of which would be reasonably
certain to discriminate between the alternative models but would
require different lengths of time to complete.  For the Frazer
River example, some reasonable alternatives are

$\mathscr{A}_1$ — Continue the present escapement policy ($S_t$ around 1.0
  million/yr.) indefinitly. That is, do not experiment and
  hope that luck will eventually provide the necessary data.

$\mathscr{A}_2$ — Allow escapements intermediate between the optima
  for the alternative models; considering all en-
  vironmental variability, this experiment would
  probably not give definite results for at least
  20 years.

$\mathscr{A}_3$ — Allow the optimum escapement ($S_t$ = 2.0 million) for the
  Beverton-Holt model. At this escapement level, any ten-
  dency for overspawning should be apparent within five

years while it is unlikely that environmental circum-

stances would combine for that long to give consistently

low recruitments if the saturating model was true.

In general, the experiments should reflect tradeoff

between small harvest manipulations that require a long time

to yield definitive results and large harvest manipulations

that yield results quickly. The length of experiment required

at any escapement or stock level can be assessed by examining

expected variability around the alternative stock-recruitment

models at that escapement level.

(d) Calculations of expected long-term returns for each of
the combinations of experiment and state of nature. The elements
of Table 1 below must be evaluated:

<div align="center">

Table 1

</div>

| | | EXPERIMENT | | | |
| | | $\mathscr{A}_1$ | $\mathscr{A}_2$ | $\cdots\cdots$ | $\mathscr{A}_n$ |
|---|---|---|---|---|---|
| | $\eta_1$ | $V_{11}$ | $V_{12}$ | $\cdots\cdots$ | $V_{1n}$ |
| TRUE STATE | $\eta_2$ | $V_{21}$ | $V_{22}$ | | |
| | $\eta_m$ | $V_{m1}$ | $\cdots\cdots\cdots\cdots$ | | $V_{mn}$ |

Here $V_{ij}$ represents the expected total value of all future

harvests, given that experimental strategy $\mathscr{A}_j$ is applied and

the true state of nature is $\eta_i$. Let us suppose that strategy $\mathscr{A}_j$

involves allowing a certain escapement $\tilde{E}_j$ for $\tau_j$ years. If

$\tau_j$ is chosen carefully, we should be reasonably certain of

detecting that $\eta_i$ is the true state of nature after the $\tau_j$

years; we should be unwilling to accept $\eta_i$ as the true

state until the $\tau_j$ years have elapsed. (Any experiment not

meeting these qualifications should not be included).

Thus, $V_{ij}$ can be calculated as the expected value of

two discounted sums: (a) the sum of catches during the

$\tau_j$ experiment years, given that excapement $\tilde{E}_j$ is applied

and model $\eta_i$ is correct; and (b) the sum of catches after

the $\tau_j$'th year, given that the optimum escapement for model

$\eta_i$ is followed thereafter. The first component reflects the

short-term impacts of the experiment, while the second compon-

ent reflects long term benefits. The simplest way to cal-

culate $V_{ij}$ is to do a whole series of simulation trials, each

using model $\eta_i$ and a different sequence of random environmental

inputs. For each of the trials we calculate:

$$V'_{ij} = \sum_{t=1}^{\tau_j} (C_t | \tilde{E}_j) e^{-\delta t} + \sum_{t=\tau_j}^{T} (C_t | \hat{E}_i) e^{-\delta t} \ ,$$

where $(C_t | \tilde{E}_j)$ is defined as the catch in year t using experi-

mental escapement $\tilde{E}_j$, and $(C_t | \hat{E}_i)$ as the catch in year t

given the optimum escapement $\hat{E}_i$ for model $\eta_i$ ($\delta$ is

the discount rate). It should not be necessary to perform

more than about 20 trials of length T ≈ 50 generations for

reasonable discount rates. $V_{ij}$ is found as the average of $V'_{ij}$

across these trials. To develop the entire strategy-experi-

ment table, it is necessary to do about m x n x 20 simulation

trials. This is a trivial computing exercise.

(e) <u>Selection of the experiment with maximum expected</u>

<u>benefits</u>. Each of the columns of the strategy-experiment Table 1

gives the returns to be expected from one experiment for each of the

possible states of nature. The overall value for the experi-

ment is simply the sum of these returns weighted by the prior

probabilities for the $\eta_i$. That is:

$$\text{expected value of experiment } j = \sum_{i=1}^{m} p^*(\eta_i) V_{ij} \quad \cdot \qquad (21)$$

The best experiment is that which has the maximum expected

value. The key point about this selection procedure is that

it takes into account all possible states of nature in

evaluating each of the proposed experiments.

The five steps outlined above lend themselves well to

a gaming situation in which the resource manager is asked to de-

vise alternative recruitment models, to asses their probabilities,

and to evaluate alternative experimental schemes. The most critical

point in the analysis is the identification of appropriate

durations for alternative experiments. If the manager is overly

pessimistic (e.g. if he assumes that some experiment will require

$\tau_j$ years to be certain which model is correct when in fact

fewer years are required), perfectly good experiments may appear

poor in relation to those states of nature for which the

experimental escapement $\tilde{E}_j$ is far from optimal. On the other

hand, the manager may be overconfident, and may suggest a short

experiment that in reality would simply result in loss of yield

with no improvement in understanding about the system. In a

gaming situation, the best $\tau_j$ for any proposed $\tilde{E}_j$ can be

evaluated quickly by facing the manager with several stochastic

simulations from each of the possible true models while noting

how long it takes to be sure which model is being used in each

of the simulations. More precisely, the analyst sets up a series of trials. For each of the trials the manager chooses an escapement $\tilde{E}_j$, and the analyst secretly chooses a model $\eta_i$. A stochastic simulation with $\eta_i$ is then initiated and carried forward in time until the manager positively identifies the results as coming from $\eta_i$. The number of simulation steps required for the various trials can be plotted in relation to the choices of $\tilde{E}_j$. This plot will reflect the manager's subjective definition of "positive" results, analogous to his choice of confidence limit probabilities (e.g. 90% vs. 95%) in ordinary statistical problems. The degree of random variation introduced in each of the trials should reflect uncertainty about the model parameters as well as expected environmental variation by using the variance relationship in equation (11) or its subjective equivalent.

To test the procedure, we carried out a gaming analysis on the Fraser River problem with one of the authors acting as manager and the other as analyst. Two alternative models were considered:

$\eta_1$ = Ricker curve from Figure 5;

$\eta_2$ = Beverton-Holt curve from Figure 5.

By examining the data and following the trial procedure for $\tau_j$ outlined in the previous paragraph, we arrived at the following set of experiments:

$\mathscr{A}_1$ – allow an escapement of 1.0 million indefinitly. (Thus do not experiment);

$\mathscr{A}_2$ – allow an escapement of 1.5 million for 15 years ;

$\mathscr{A}_3$ – allow an escapement of 2.0 million for 5 years ;

$\mathscr{A}_4$ – allow an escapement of 3.0 million for 3 years .

Simulation trials to evaluate the $V_{ij}$ were performed,

assuming a discount rate of 4% per generation (i.e. 1% per year for Fraser sockeye). The results are given below in Table 2.

### Table 2

Expected Future Values for Alternative Harvesting Experiments on Off-Cycle Fraser River Sockeye.*

| | | Experiment |
| | | $\mathscr{A}_1$ | $\mathscr{A}_2$ | $\mathscr{A}_3$ | $\mathscr{A}_4$ |
| --- | --- | --- | --- | --- | --- |
| TRUE STATE | $\eta_1$ | 77.2 | 77.8 | 75.4 | 71.7 |
| | $\eta_2$ | 92.4 | 108.2 | 110.9 | 110.6 |

*Elements of the table are discounted sums of catches, in millions of fish.

When only two or three alternative states of nature are to be compared (as in the Fraser example), the analysis can be presented in an elegant form that simplifies the problem of assigning subjective probabilities to the alternative models. Suppose we make a graph where the abscissa is $V_{1j}$ (expected value of experiment j given that $\eta_1$ is true) and the ordinate is $V_{2j}$ (expected value of experiment j given that $\eta_2$ is true). Each of the experiments can be plotted as a point on this graph (Figure 6). Points that are close to the ordinate represent experiments or policies that are good if $\eta_2$ is true, and poor if $\eta_1$ is true; points near the abscissa represent policies that are good if $\eta_1$ is true, and poor if $\eta_2$ is true. If we designate those experiments that can distinguish between the $\eta_i$ as "effective experiments," then the graphical representation allows us to sort out a smaller subset of "efficient experiments" that are best for at least some values of $p*(\eta_i)$. In Figure 6, experiments $\mathscr{A}_2$ and $\mathscr{A}_3$ are efficient, while experiments $\mathscr{A}_1$ and $\mathscr{A}_4$ are inefficient since $\mathscr{A}_2$ and $\mathscr{A}_3$ have better expected values for

all assignments of $p^*(\eta_i)$. We are trying to find that experiment $\mathscr{A}_j$ which maximizes the "objective function":

$$y = p^*(\eta_i)V_{1j} + (1 - p^*(\eta_1))V_{2j} \quad .$$

This objective function can be represented as a series of straight lines on Figure 6, with higher lines representing larger y values and greater slopes representing larger values of $p^*(\eta_1)$. To find the optimum experiment, we move the lines downward (choose lower y values) until it first touches a "feasible point" representing some $\mathscr{A}_j$. They key point is that we can identify ranges of $p^*(\eta_1)$ for which any efficient $\mathscr{A}_j$ is optimal, simply by changing $p^*(\eta_1)$ so as to steepen or flatten the objective function line. The results of the process on the Fraser River example are given in Table 3 below.

### Table 3
Range of p*(Ricker) For Which $\mathscr{A}_j$ is Best

| Experiment | | Range |
|---|---|---|
| | $\mathscr{A}_1$ | none |
| | $\mathscr{A}_2$ | 0.4 - 1.0 |
| | $\mathscr{A}_3$ | 0.0 - 0.4 |
| | $\mathscr{A}_4$ | none |

Thus, the manager does not need to precisely specify his judgement about $p^*(\eta_1)$ as a single number.

The Fraser River test results suggest the following conclusions:

(a)  Present management policy for off-cycle year is not optimal for either of the models shown in Figure 5; some alternative (and perhaps unclearly specified)

model is in use, or the real management objectives are unrelated to maximization of discounted long-term catches.

(b) A modest experiment involving increased escapements (1.5 million) should cause no serious problems if the Ricker model is correct; it may, in fact, result in considerably higher yields over the long run.

(c) A more drastic experimental policy involving escapements of 2.0 million for several years would be more advisable if there is considerable confidence that the true stock-recruit relationship is similar to the curve $\eta_2$ in Figure 5.

To test the effect of discounting rate on these conclusions, we reevaluated Table 2 for $\delta = 1\%$, $\delta = 10\%$, $\delta = 20\%$, and $\delta = 30\%$ per generation. For $\delta$ greater than 20%, the tests suggested that the modest experiment is the best unless p*(Ricker) is less than 0.2. For $\delta = 1\%$, the drastic experiment becomes the best alternative unless p*(Ricker) is greater than 0.6.

IV. Extensions and Generalizations.

Previous sections have dealt mostly with uncertainty about stock recrutiment relationships. Two major assumptions have been the discussion: a) stock size is directly measurable without error; and b) fishing effort is fully controllable to conform with biological recommendations. Since these assumptions are often not justified, Section IV attempts to show how the concepts and methods introduced previously could be extended to include these additional sources of uncertainty.

## A. Schaeffer Production Model

The idea of using logistic population growth assumptions as a basis for production modelling was first made popular by Schaeffer [16] in his studies of Pacific tunas. In its simplest form, the "Schaeffer Model" can be written as:

$$N_{t+1} = N_t + \alpha N_t (1 - \beta N_t) - C_t \quad , \tag{22}$$

where

$N_t$ = stock size, usually in biomass units;

$\alpha, \beta$ = production parameters with similar definitions as in the Ricker Model;

$C_t$ = total catch.

Noting that equation (22) is not directly usable (since $N_t$ is not observable for most populations), Schaeffer and others have assumed a simple "observation model" to accompany the dynamic model:

$$N_t \doteq \frac{q_t}{c} = \frac{C_t/E_t}{c} \quad , \tag{23}$$

where

$q_t$ = catch per unit effort;

$E_t$ = some effort measure having units (boats) x (time fishing per boat);

$c$ = catchability coefficient.

Substituting the observation model (23) for (22), we obtain an expression containing only observable quantities and parameters:

$$\frac{q_{t+1}}{c} = \frac{q_t}{c} + \alpha\frac{q_t}{c}\left(1 - \beta\frac{q_t}{c}\right) - C_t \quad ;$$

this can be simplified to give:

$$q_{t+1} - q_t = \alpha q_t - \frac{\alpha\beta}{c} q_t^2 - cC_t \quad . \tag{24}$$

For parameter estimation and adaptive control analysis, this version of the Schaeffer Model can be cast into the recursive regression format with:

$$y_t = q_t - q_{t-1} \qquad (\text{or } y_t = q_t)$$

$$\underline{x}_t = \begin{bmatrix} q_{t-1} \\ q_{t-1}^2 \\ C_{t-1} \end{bmatrix} \tag{25}$$

$$\underline{\hat{a}}_t = \begin{bmatrix} \hat{\alpha}_t \\ \widehat{\alpha\beta/c}_t \\ \hat{c}_t \end{bmatrix} \qquad \left( \text{or } \underline{\hat{a}}_t = \begin{bmatrix} \widehat{(1 + \alpha)}_t \\ \widehat{\alpha\beta/c}_t \\ \hat{c}_t \end{bmatrix} \right) .$$

Though there is no reason here to expect such statistical properties as normally distributed errors, the regression format at least provides a unified framework for evaluating parameter uncertainties. Also, the format automatically provides a

simple state estimator, $\hat{N}_t = q_t/\hat{c}_t$. More complicated,
statistically nonlinear versions of equation (24) can be
devised using more realistic observation models than
equation (23). Recursive nonlinear estimation techniques
are beginning to appear in the literature under the general
heading "extended Kalman filters" (Young, [22]).

The Schaeffer Model gives a remarkably good fit to histor-
ical data for many large fisheries, as shown in Figure 7. In
fitting these data, we used the linear regression scheme in
equations (24) to obtain estimates of $\alpha, \beta$, and c. In all
cases the data had already been corrected for changing vessel
efficiency (changing c); thus it was not necessary to introduce
discounting of old data or a parameter variation matrix Q
(see Section II) into the regression equations. It is clear
that further adaptive control work for the Schaeffer Model is
justified, and we intend to develop more complete analyses in
a future paper.

### B.   Incomplete Control of Fishing Effort

The fisheries literature abounds with biological models
and equilibrium yield analyses; almost no attention has
been paid to the dynamics of the predator-prey system that
results from incomplete control of economic investment. Fishery
fleets have basic "reproductive" (investment) and "mortality"
(disinvestment) relationships that in principle make them similar
to any predator population (Smith [18]; Gatto, et al. [5]).
In the absence of investment control, many fishing fleets
have developed to the point where pressure for short-term eco-
nomic and social welfare benefits has made it virtually im-

possible to implement biologically sound long-term policies;
the current state of the International Whaling Commission is
a good example.

To pursue the predator-prey analogy, we may find it useful to
think of management controls directed at fishing effort as
generating a "reachable region" of stock size and investment
combinations (Figure 8) around the "nominal trajectory" of develop-
ment that would occur without management. Investment control may
occur as subsidies to increase the rate of investment,
or as taxes and direct regulations to reduce it. If
the fishery fleet can operate economically at very low stock
sizes, and if only small control decreases in effort are
possible each year, it may be impossible to move the fishery
to a state where maximum sustained yield is possible. This
problem may become serious if the first incremental
controls are not applied until the fishery is well developed.
One is reminded of the adage about ounces of protection and
pounds of cure.

Limitations on control changes from one time step to
the next may be represented in dynamic optimization by including
the  control level (effort, harvest rate, etc.) as an additional
state variable. For example, if the system state without effort
limitation was represented as $(R_t, \hat{\alpha}_t, \sigma_\alpha^2)$, the state
with limitation would be $(U_{t-1}, R_t, \hat{\alpha}_t, \sigma_\alpha^2)$ where $U_{t-1}$ is
the exploitation rate from the previous time step. Instead of
looking at all possible harvest rates at each of the time steps for
each of the $(R_t, \hat{\alpha}_t, \sigma_\alpha^2)$ combinations, the optimization would only

examine harvest rates $U_t$ over an interval:

$$U_{t-1} - k_1 \leq U_t \leq U_{t-1} + k_2$$

where

$k_1$ = maximum permissable annual <u>decrease</u> in exploitation rate;

$k_2$ = maximum permissable annual <u>increase</u> in exploitation rate.

Similar constraints can be applied in generating experimental harvest regimes for the analyses mentioned in Section III.

A series of interesting issues arise concerning the selection of appropriate values for the control limits $k_1$ and $k_2$. The maximum rate of increase in harvest, $k_2$, will depend on private and public willingness to invest in the fishery and on the availability of fishing gear to be transferred from alternative fisheries. The maximum rate of decrease, $k_1$, will depend on the regulatory power vested in the management agency, on the willingness of the agency to accept responsibility for immediate economic and social hardships, and on the expected profitability of the fishery. Fisheries agencies are beginning to face these political and economic issues, and optimization formulations may provide a useful focus for formal debate even if no quantitative solutions are attempted.

## V. Summary.

This paper moved from formal optimization analysis of trivial models to broader approaches for experimental management. The formal analysis was conducted in order

to discover simple principles that might be applied in more complex and realistic fisheries situations where uncertainty is a key factor. The principles that emerged are intuitively reasonable: (a) when production parameters are uncertain, the harvest rates used should be lower than would be supposed if only the available parameter estimates were considered; and (b) when the equilibrium stock size is uncertain but production rates are well understood, the harvest rates should be lowered. When the general form of the production relationship is uncertain and when management control is limited, formal optimization becomes impossible: however, a gaming procedure may help to devise and to evaluate alternative management strategies. The gaming procedure involves defining a series of possible models, selecting a series of effective experiments, and calculating the optimum experiment under subjective prior probabilities for all models. This technique is an immediately useable solution for complex fisheries problems. Future work is continuing to overcome the computational obstacles encountered in formal optimization of complex models.
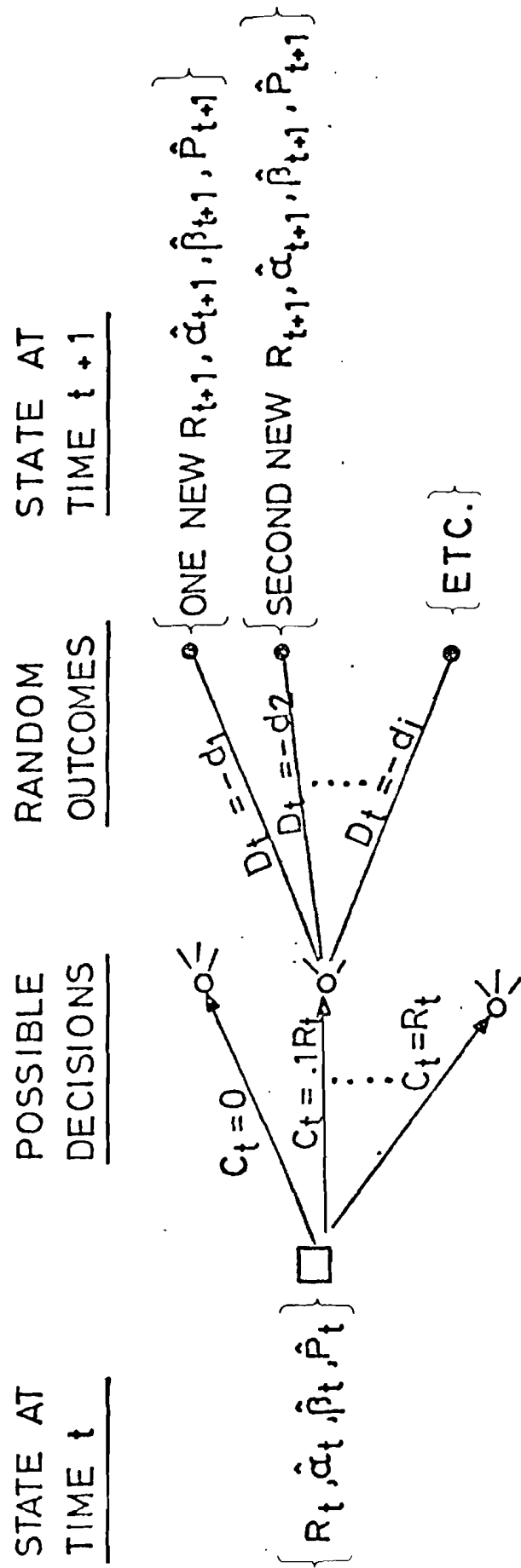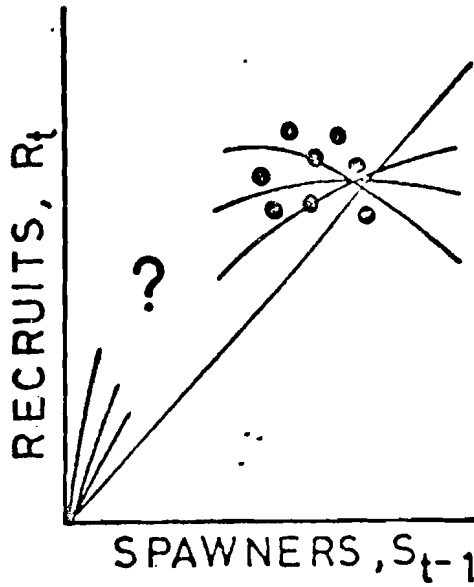
Figure 1. Decision possibilities and random outcomes for each of the time steps in adaptive control analysis for Ricker model.

CASE 1: DEVELOPING FISHERY, $\beta$ KNOWN AND
$\alpha$ UNCERTAIN.



CASE 2: OLDER FISHERY WITH UNRELIABLE DATA
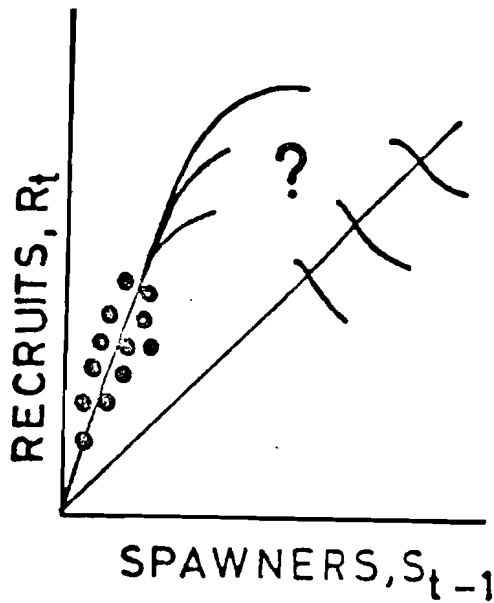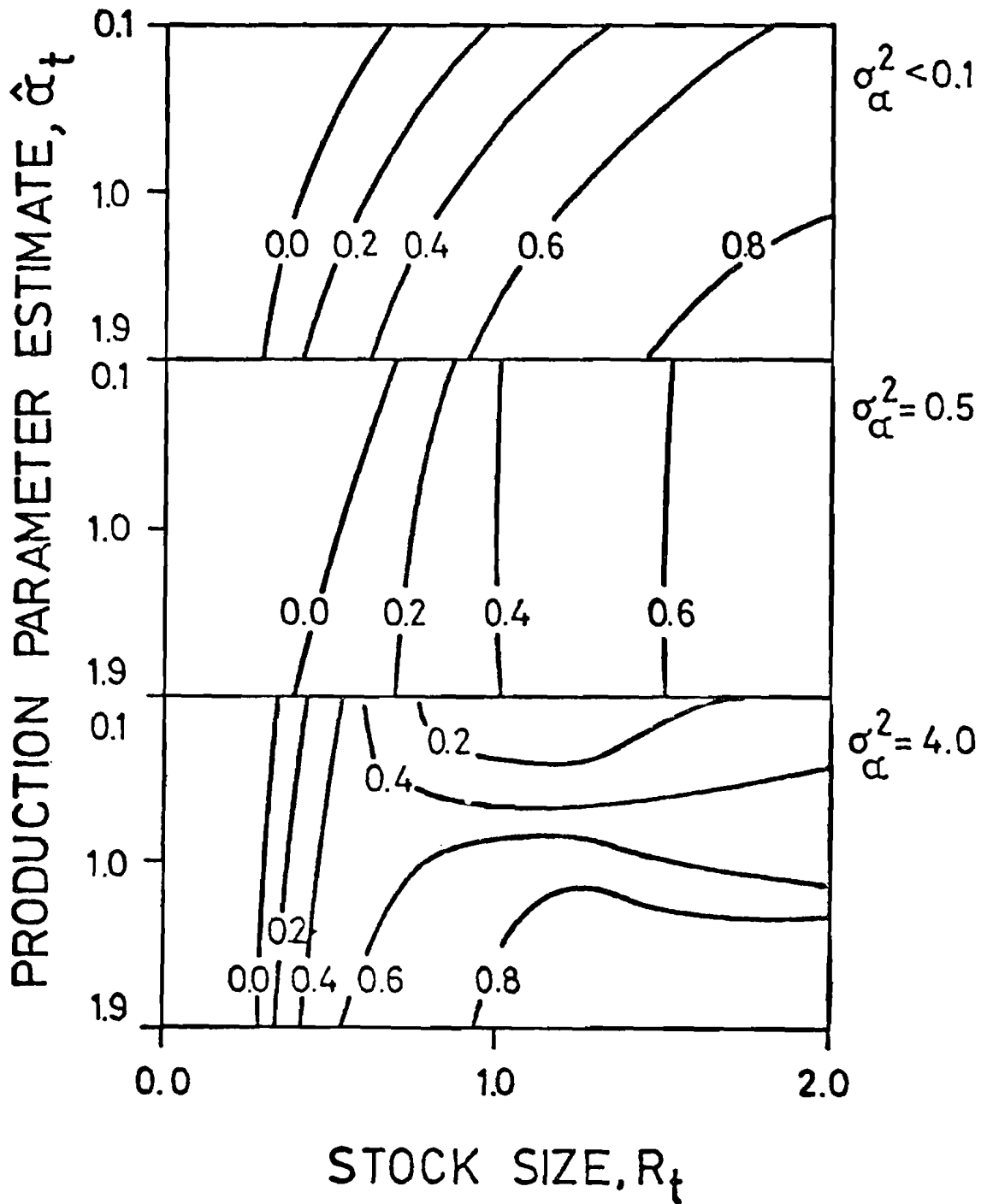ON NATURAL STOCKS, $\alpha$ KNOWN AND $\beta$ UNKNOWN.



Figure 2.   Management situations that permit
simplified adaptive optimization.

Note: Results obtained with environmental variance

$\sigma^2 = 0.5$ and discount rate $\delta = 4\%$ per generation.

Figure 3. Optimal exploitation rates for various stock sizes, production rate estimates ($\hat{\alpha}_t$), and uncertainties about $\alpha$ ($\sigma_\alpha^2$) assuming the Ricker model form is correct and equilibrium stock is known (see case 1, Figure 2).
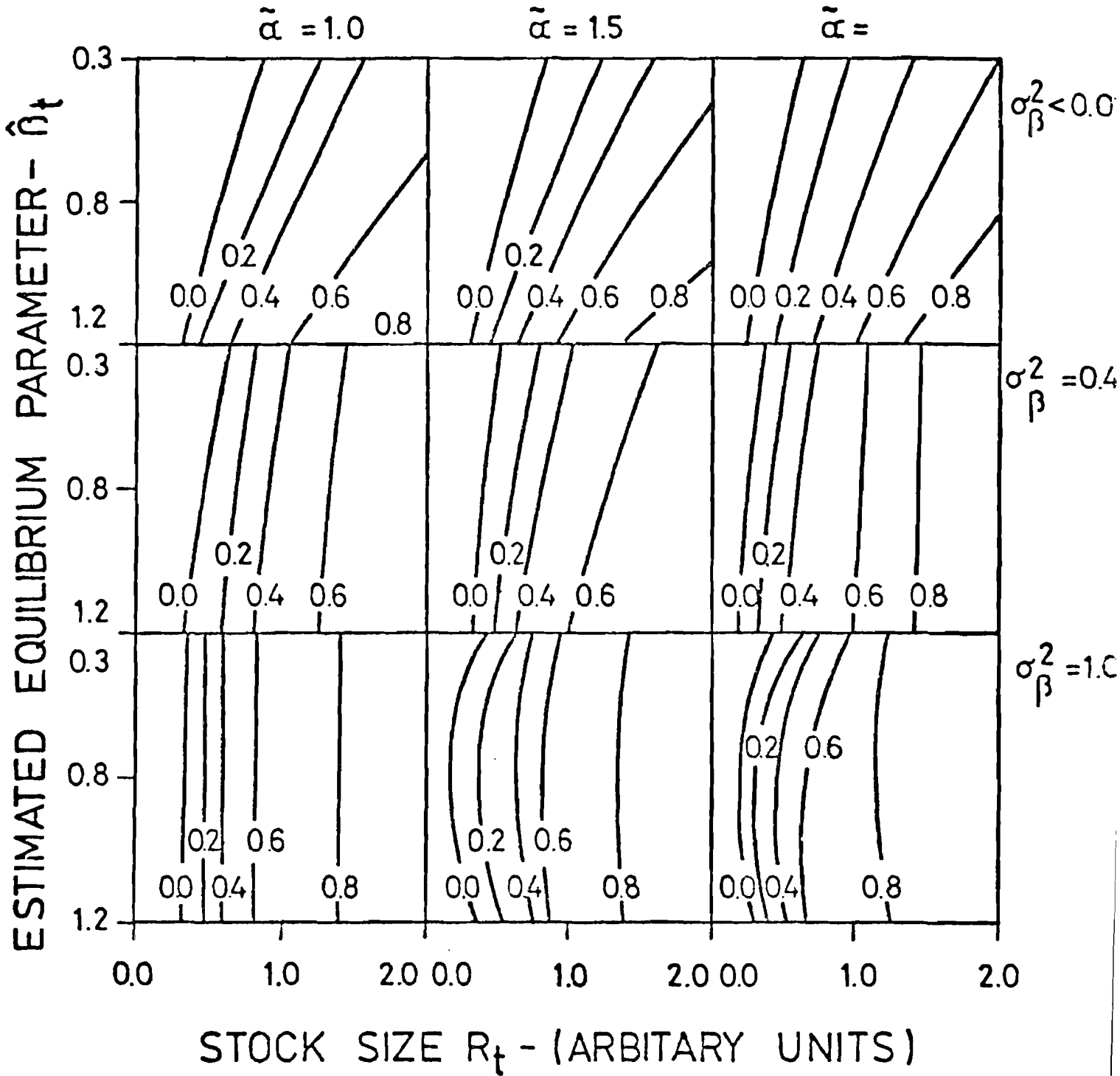
Note: Results obtained with environmental variance $\sigma^2 = 0.5$ and discount rate $\delta = 4\%$ per generation.

Figure 4. Optimal exploitation rates for various stock sizes, equilibrium population sizes ($1/\hat{\beta}_t$), and uncertainties about $\hat{\beta}$ ($\sigma_{\hat{\beta}}^2$), assuming Ricker form is correct and $\hat{\alpha}$ is known (see case 2, Figure 2).
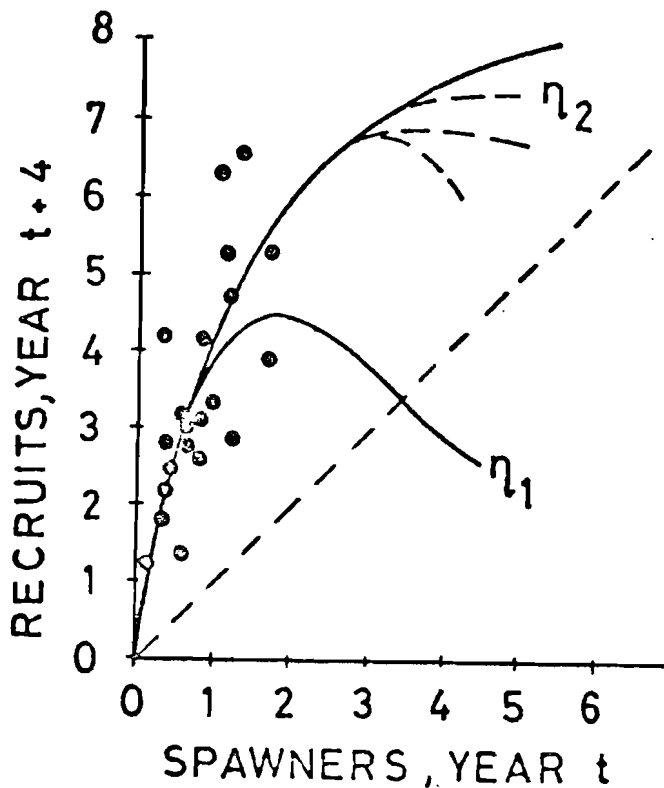
Note: Data shown are for 1939-1973, omitting every fourth or cycle year beginning in 1942. $\eta_1$--least squares fit to Ricker model; $\eta_2$--visual fit to Beverton-Holt model. Graph axis in millions of fish.

Figure 5. Alternative stock-recruitment models for Fraser River Sockeye Salmon, off-cycle years.
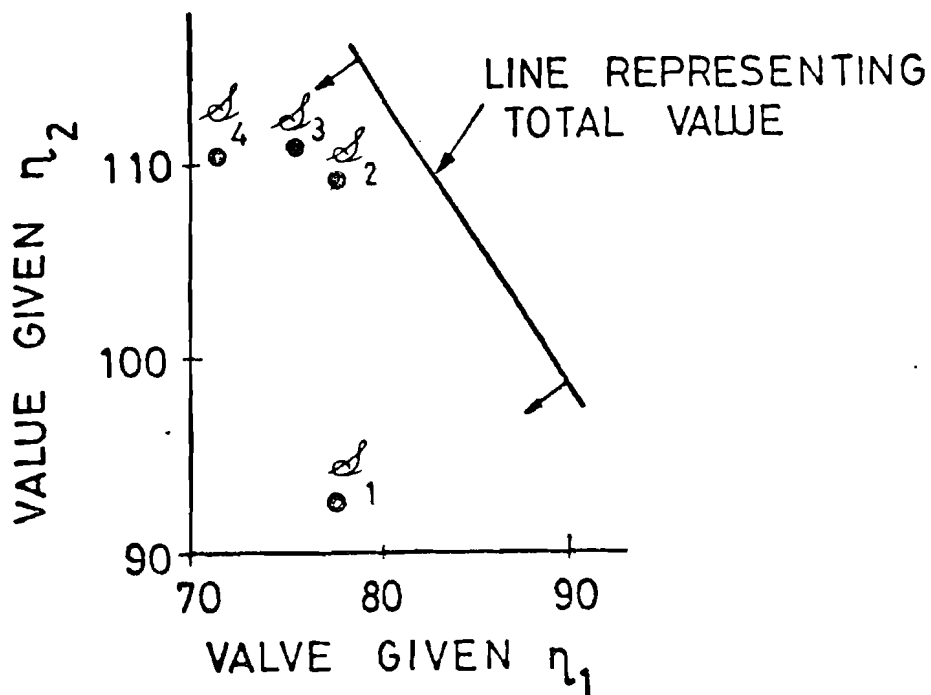
Note: Graph units are discounted (4%/generation) sums of catches, in millions of fish.

$\mathcal{E}_1$ - $\mathcal{E}_4$ denote escapement experiments defined in text. Total value line is given by value = $p^*(\eta_1)$ [value given $\eta_1$] + $p^*(\eta_2)$ [value given $\eta_2$].

Figure 6. Values of alternative Fraser River harvesting experiments if the alternative models $\eta_1$ and $\eta_2$ in Figure 5 are correct.
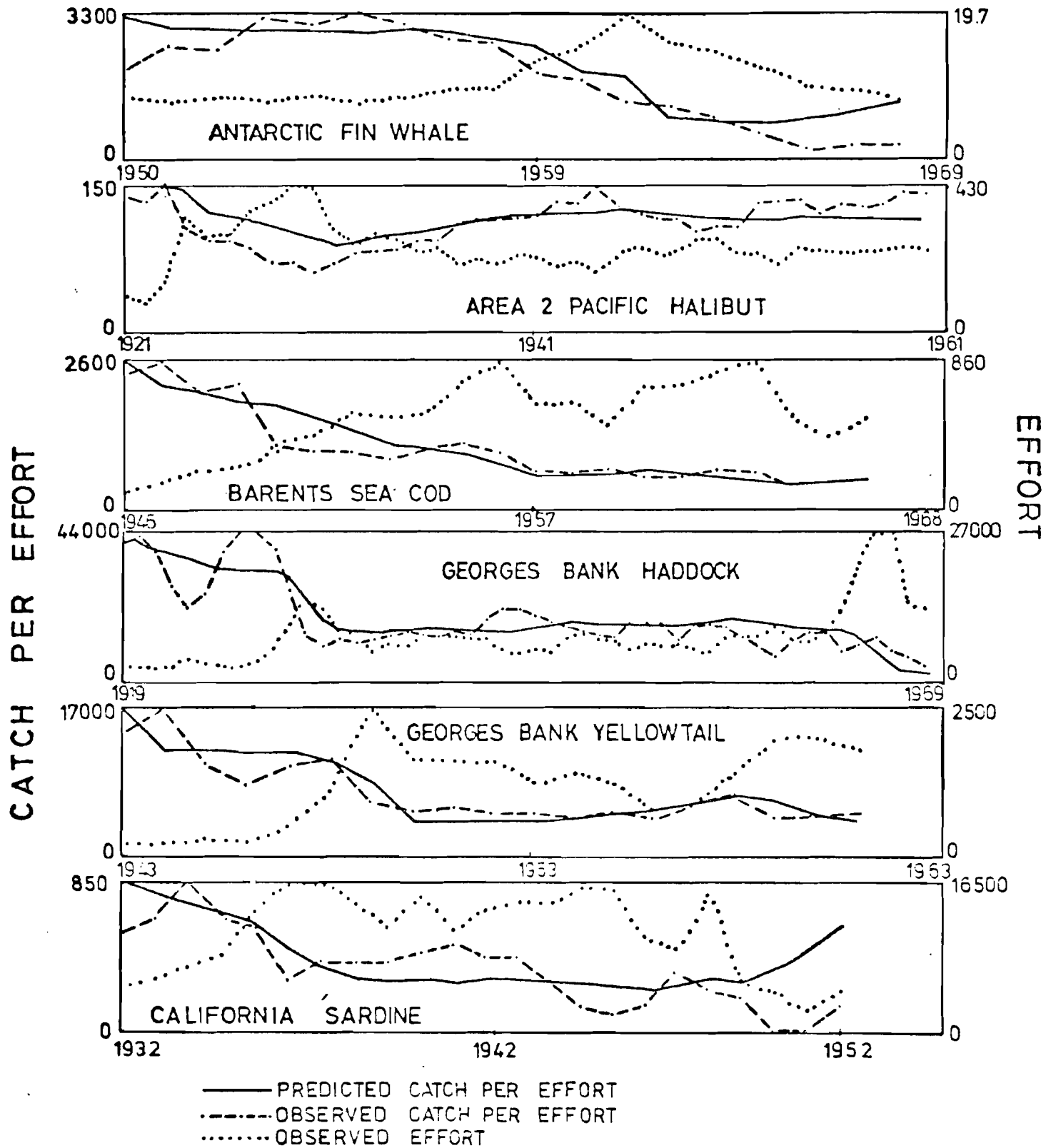
Note: The entire data set for each case was used to obtain para-
meter estimates; simulated value for the previous year
(rather than from previous year's data). Data were obtained
from following sources: fin whales - International Whaling
Commission [7]; halibut - Southward [19]; cod - Garrod [4];
haddock - Grosslein and Hennemuth [6]; yellowtail - Lux [9];
sardine - Marr [10];

Figure 7. Observed changes in some fisheries compared to
simulated trends predicted by the Schaeffer model
(equations 13, 23) using parameter estimator from
regression analysis.
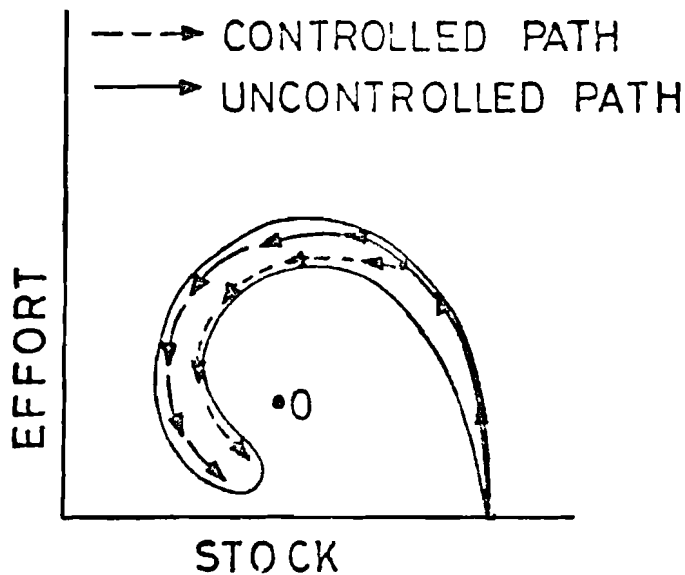
# GOOD SITUATION
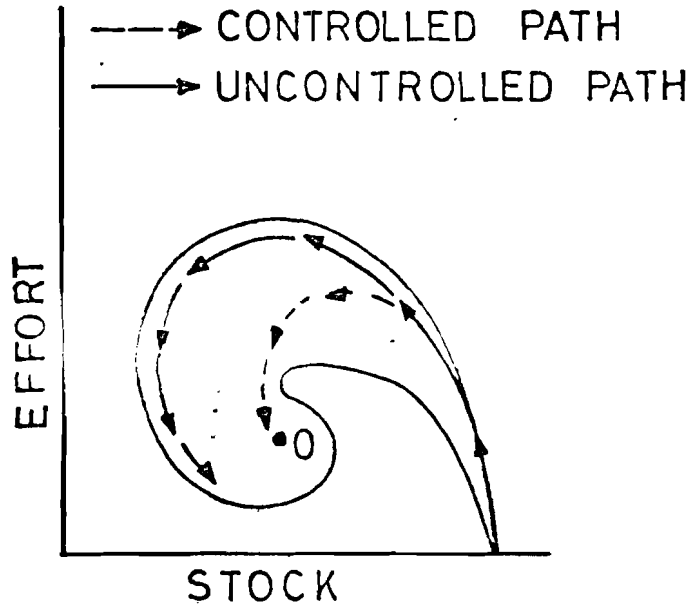


# PATHOLOGICAL SITUATION



Figure 8.  Imperfect control of fishing effort may result in a restricted "reachable region" of stock size and effort combinations as a fishery develops. The point "0" denotes optimum equilibrium state.

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