WORKING PAPER

PENETRATION MECHANISM OF NEW TECHNOLOGIES: INTEGRATION FORMULATION FROM THE COMPANY LEVEL TO THE MACROECONOMIC LEVEL

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FOREWORD

One of the most important tasks of the CIM project is to understand the diffusion mechanism of advanced manufacturing technologies such as CIM and its components.

The author developed a macroeconometric forecasting model of in his first paper, entitled "Future Penetration of Advanced Industrial Robots in the Japanese Manufacturing Industry". In addition, his second paper pointed out the importance of the company size factor in the diffusion of technologies by means of quantitative analyses. The present paper proposes an integrated formulation of the diffusion mechanism of costs/benefits from the company level (micro-level) to the macroeconomic level by introducing the factors related to company size. This is quite an interesting quantitative and methodological approach, which includes a new kind of elements. However, it should be kept in mind that so far the model is only valid for the technologies, which are mainly driven by labor substitution.

It is hoped that the approach proposed in this paper will be applied to build up the forecasting model of CIM diffusion.

Prof. Jukka Ranta
Project Leader
Computer Integrated Manufacturing

Summary

A new approach to formulate the penetration mechanism of advanced technological equipment into industry is proposed in this paper. This approach aims at the integration of the company level (micro-level) and the industry level (macro-level) into the penetration mechanism.

Our model is formulated on the basis of several empirical laws observed in the real world. As a result, the following factors are introduced into the model:

a) Cost/benefit judgement at the company level;

b) Economy of scale in user costs;

c) Wage rate gap between large and small companies;

d) Company size distribution;

e) Decreasing price of advanced technological equipment;

f) Increase of the wage rate.

Although we introduce many parameters related to the above factors, we can derive the final mathematical form of our model as a kind of Gompertz curves.

New sophisticated implications can be added to the parameters of the Gompertz curves through our model.

This model focuses on the benefits of labor-saving. However, the formulation itself is also applicable to other kinds of benefits, if the benefits can be expressed as a function of company size.

Contents

1. Introduction

One of the most important tasks of the CIM project is to understand the diffusion mechanism of advanced manufacturing technologies, such as CIM and its components.

Generally speaking, there are two different approaches to the diffusion problems. One is called the micro-level approach, which focuses on the decision-making problem in introducing the advanced technologies at the company[1] (or factory) level. Cost/benefit analyses are carried out through a lot of case studies. The other is called the macro-level approach, which deals with the diffusion process of the technologies into the industries by means of statistical data and methods. Macroeconomic models are developed to forecast the future diffusion [Tani, 1987a; Mori, 1987; Tchijov, 1987b]. However, there are many difficulties concerning the interrelation between these models.

The present paper proposes an integrated approach from the micro-level to the macro-level as a mathematical model. In his earlier paper on the "Enterprise Size and its Impact on Penetration of Industrial Robots - Application of Econometric Analysis" [Tani, 1987b] the author shows that the distribution of company size is one of the most important factors affecting the penetration. Maly [1987a] has also pointed out the importance of the company size.

By introducing the company size factor, the author makes an effort, in this paper, to build a bridge between cost/benefit analyses at the company level and the diffusion curve at the macro-level.

2. Empirical Laws in Technology Diffusion

When we review the past history the technology, we can observe some kinds of broad, general trends in the diffusion processes of new technological equipment into industries. These general trends might be called "empirical laws" in technology diffusion. The following "empirical laws" are used in our formulation of penetration mechanisms.

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[1] In this paper, "company" means establishment (or factory in most cases of manufacturers) rather than enterprise.

(1) Decreasing price of advanced equipment.
The price has a tendency to decrease year by year, although it is high at the initial stage of diffusion. The reasons generating this tendency could be classified into the following factors:

a) The technological innovation effect (see Figure 1);
b) The "Economy of scale" effect in production (see Figure 2);
c) The "Experience curve" (or learning curve) effect in production (see Figure 3), [Tani, 1987b; Ayres & Funk, 1987].

However, it is difficult to extract these three effects separately from the statistical data because they work simultaneously to reduce the costs of production in the real diffusion process.

(2) Large companies (or factories) introduce the advanced equipment earlier than smaller companies in terms of the statistical (macro-level) diffusion rate (see Figure 4). In other words, this says that most of the advanced equipment is introduced in large companies at an early stage of diffusion in terms of share of equipment number installed. This does not mean that large companies are simply more <u>innovative</u> in technology adoption.

For example, the following should be noted:

	large companies	small companies
Number of companies	2	10
Company size (employees)	1000	30
Units/company	100	3
Number of units	200 (= 2 x 100)	30 (= 10 x 3)

In the above cases, small companies are more <u>innovative</u> (10 to 2), while most units are introduced in large companies (200 to 30).

(3) Decision-making to introduce the advanced equipment at the company level depends mainly on the cost/benefit evaluation criterion [JIRA, 1984; Maly, 1987b; Sheinin & Tchijov, 1987; ECE 1986]. If the major benefit is a labor-saving effect, the relative cost of equipment to wage rate becomes the most

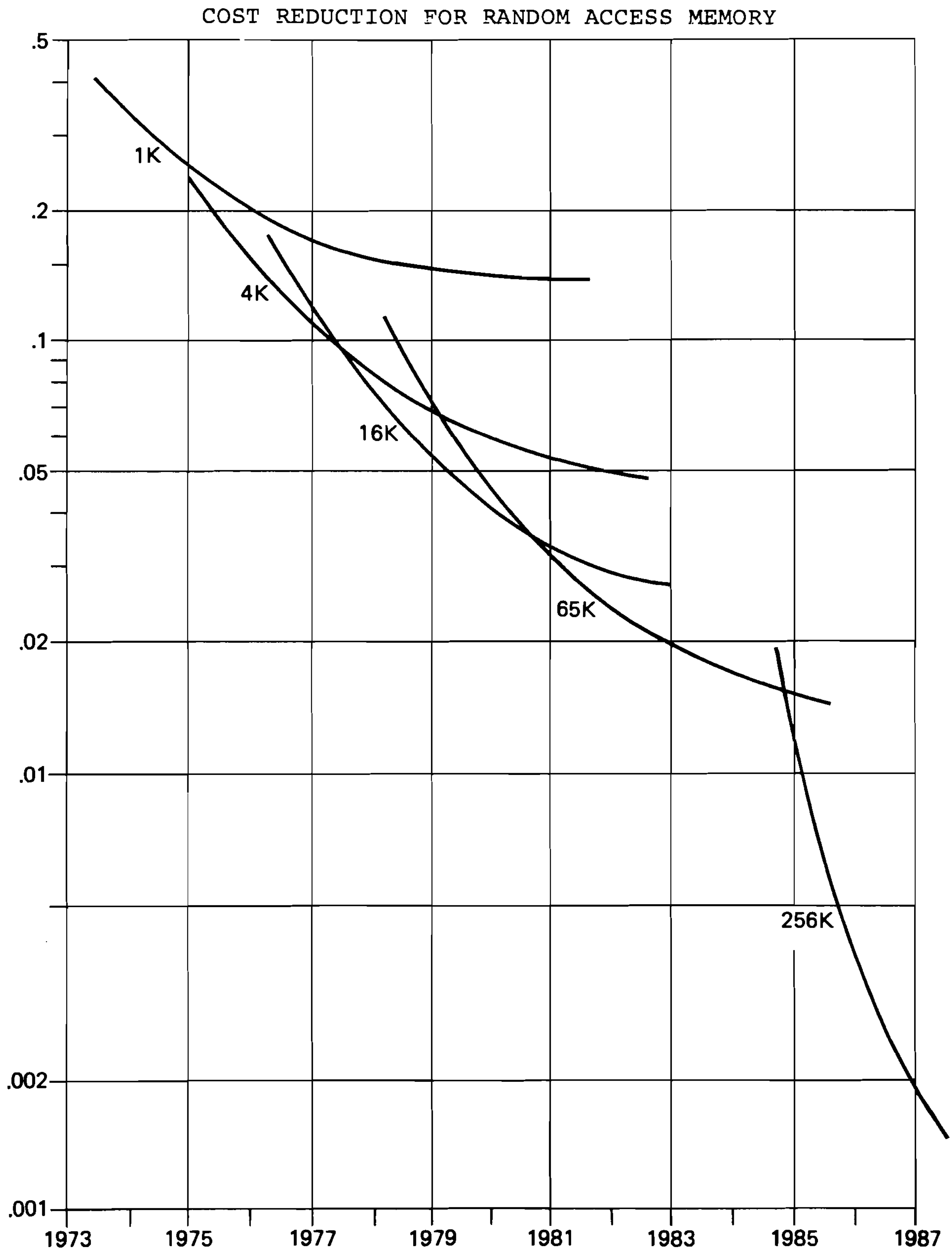


Figure 1. Technological innovation effect
Source [Ayres, 1987].

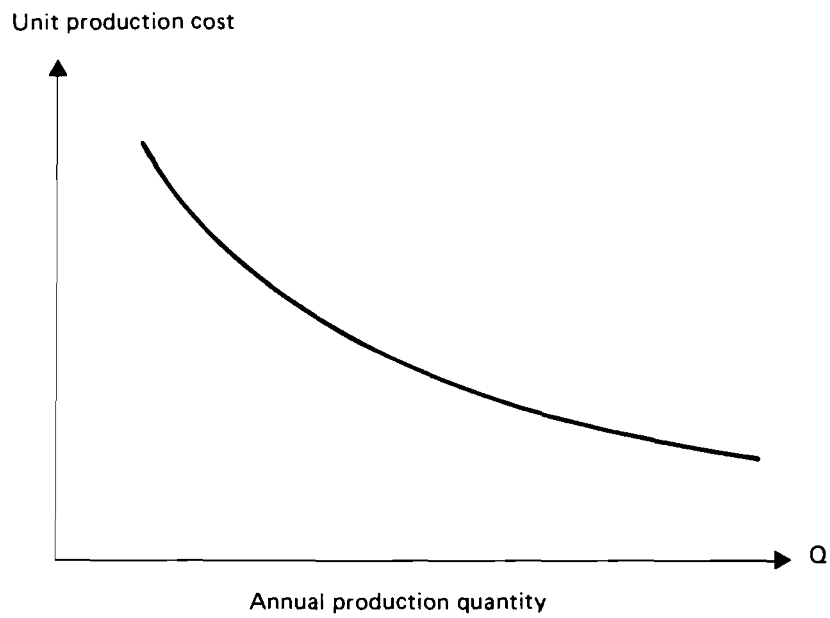


Figure 2. Economy of scale effect

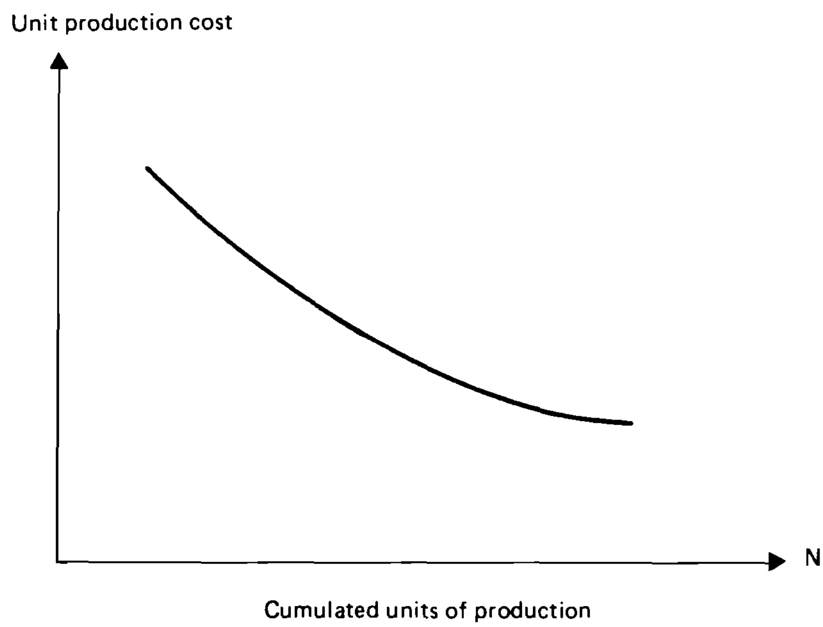


Figure 3. Experience curve effect

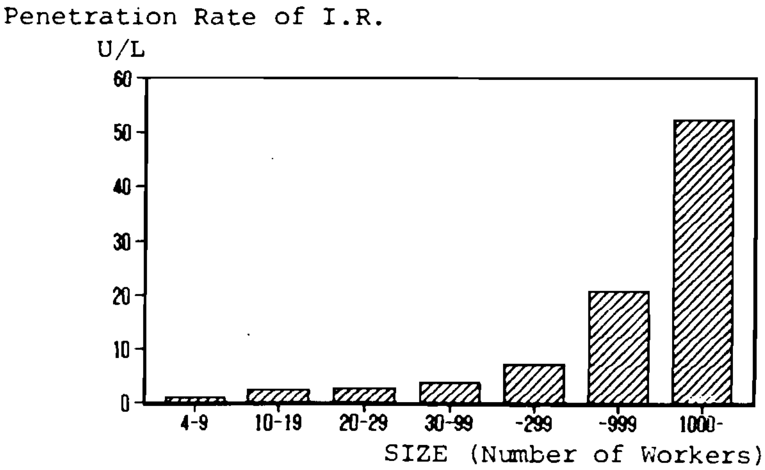


Figure 4. Penetration rate of I.R. by size of establishments in Japanese whole manufacturing industry (final estimation)

Source: [Tani 1987b].

important factor in decision-making [Mori, 1987; Tani, 1987a; Ayres, Brautzsch & Mori, 1987].

(4) "Economy of scale" in user costs.
The cost-performance of advanced equipment has a tendency to be better in larger companies than in smaller companies (see Figure 5). Small companies must pay not only the fixed costs, but also additional costs to improve their capabilities to use advanced equipment effectively.

(5) The wage gap between large and small companies (see Figure 6).

(6) Company size distribution (see Figure 7).
Small and medium-sized companies have a great share of labor in industry.

The above two tendencies are clearly observed in Japan. The same patterns are also seen in the U.S.A., although the differences among company sizes are smaller than in Japan.

(7) As the diffusion proceeds with technological progress and price reduction, various applications of advanced equipment appear; the equipment is of higher quality and is used in the broader industrial sectors, which accelerates the further diffusion of technologies.

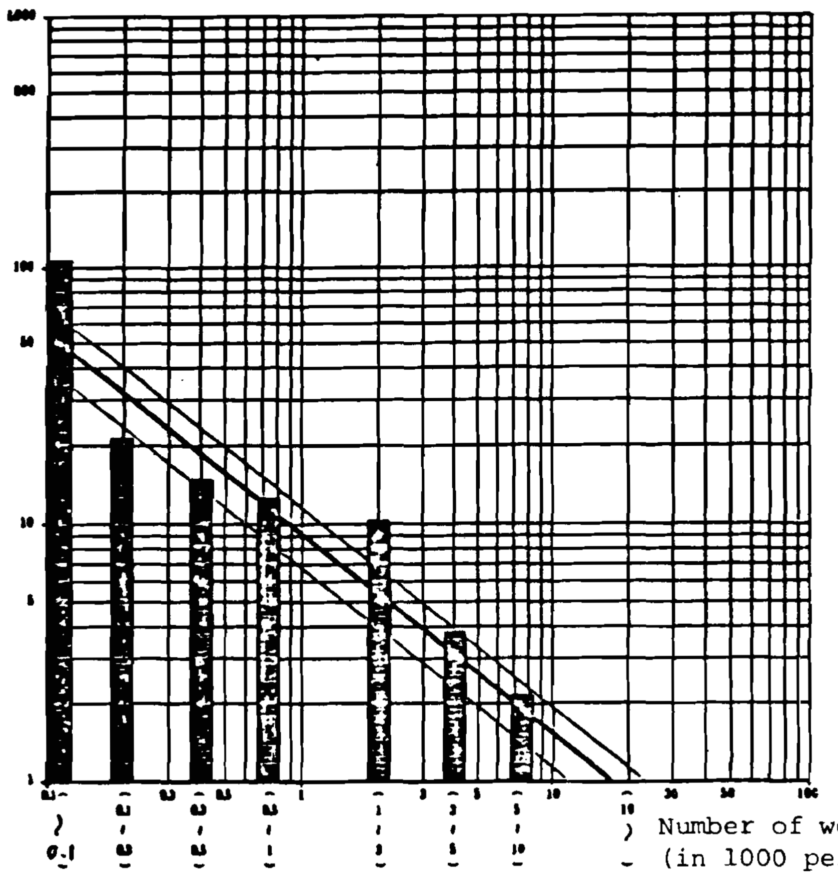
3. Formulation

This chapter formulates the penetration mechanism of advanced technological equipment into industry, based upon the empirical laws discussed in the previous chapter. As explained before, various applications and user-oriented industrial sectors of the equipment are potentially considered [JIRA, 1985; Tani, 1987c]. Therefore, we should first make a kind of matrix (sectors x applications). Then, the penetration mechanism for each cell of the matrix should be generally formulated. After that we will be able to apply the penetration model for each cell to other cells, and finally obtain the total diffusion process by summing up the results of these cells.

In other words, it is important to keep in mind the following three-dimensional diffusion of technologies into industry.

a) Diffusion from large companies to smaller companies;
b) Diffusion to other sectors having the same application;

Average Monthly
EDP Expenses
per Worker
(in 1000 yen
per month)



Number of workers
(in 1000 persons)

R=0.98
Samples: 236

$$\frac{c}{x} = 8.66.x^{-0.7785}$$

c: EDP expenses in million yen/month
x: Number of workers in thousands
Batch Processing System

Figure 5. Economy of scale in EDP expenses (company size and EDP expenses

Source: [JIPDEC, 1987].

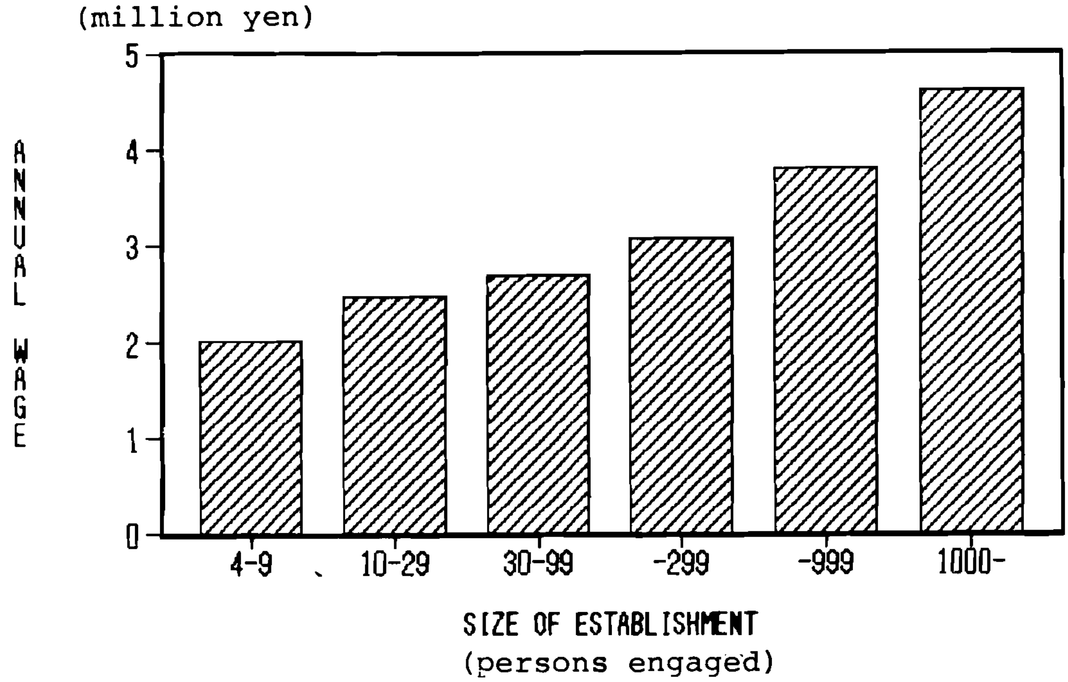


Figure 6. Wage gap by size in Japanese manufacturers (1984)
Source: [MITI, 1986].

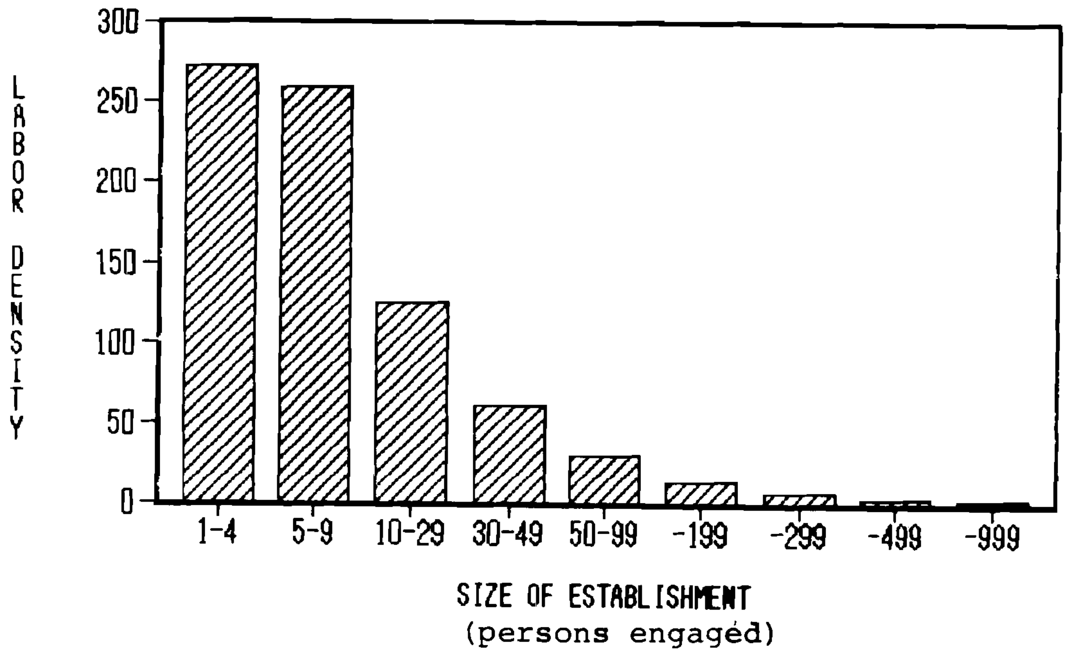


Figure 7. Labor distribution density by size in manufacturers (1981)
Source: [MCA, 1983].

c) Diffusion to other applications within the same sector.

This concept is illustrated in Figure 8. Therefore we will hereafter focus on the formulation of the penetration mechanism for a given cell.

(1) Distribution of Company Size

First, we introduce the number of employees in some company (x) as an index of company size.

Labor distribution density corresponding to company size x in a given industrial sector is denoted as f(x), which satisfies the following equation:

$$\int_0^\infty f(x)dx = L \tag{1}$$

where

L denotes the total number of labor in a given industry, and f(x)dx gives the number of employees in the companies whose size is within the range [x, x+dx].

In our model the exponential function is assumed as a distribution density function

$$f(x) = A \cdot e^{-\lambda \cdot x} \tag{2}$$

where parameters λ and A are positive.

By substituting equation (2) into (1), parameter A is expressed in the following equation:

$$A = \lambda \cdot L \tag{3}$$

(2) Decision-making mechanism at the company level

Firstly, it is assumed that the number of units [u(x)] introduced in company of size x is proportional to the number of employees (x) as shown below.

$$u(x) = u_o \cdot x \tag{4}$$

where parameter u_o denotes the density of advanced equipment in terms of units, per employee.

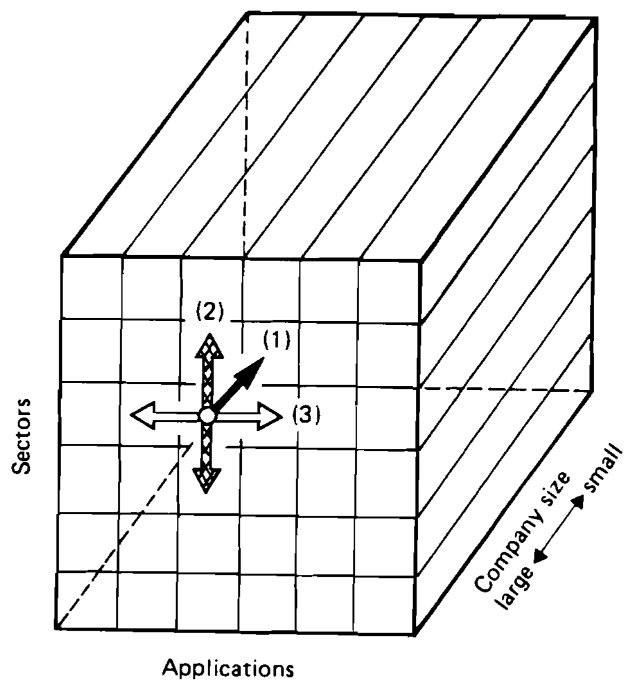


Figure 8. Three-dimensional diffusion of high technological equipment.

Secondly, we introduce l(x) as labor saved by using the advanced equipment in company of size x. Then, l(x) is assumed proportional to u(x) as shown in equation (5).

$$l(x) = l_0 \cdot u(x) \tag{5}$$

where parameter l_0 denotes the number of workers that can be reduced by introducing one unit of the equipment.

Thirdly, investment cost [C(x,t)] for introducing u(x) units of equipment is assumed proportional to the number of units, u(x). C(x,t) can be expressed in the following equation:

$$C(x,t) = p(x,t) \cdot u(x) \tag{6}$$

where p(x,t) denotes the unit cost.

According to the laws of technological progress and economy of scale in user cost, as explained in the previous chapter, unit cost p(x,t) can be expressed as follows:

$$p(x,t) = P_0 \cdot x^{-a} \cdot e^{-\alpha t} \tag{7}$$

where x^{-a} and $e^{-\alpha t}$ denote the effects of "economy of scale" in user cost, and the decreasing price by technological progress, respectively. Parameter P_0 is a constant coefficient.

On the other hand, the benefits [B(x,t)] of labor saving can be expressed as shown below.

$$B(x,t) = w(x,t) \cdot l(x) \tag{8}$$

where w(x,t) denotes the annual wage in company of size x at time t.

According to the empirical laws of the wage gap and wage increase explained in the previous chapter, the annual wage w(x,t) can be expressed as follows:

$$w(x,t) = W_0 \cdot x^{b} \cdot e^{\beta t} \tag{9}$$

where x^b and $e^{\beta t}$ denote the effects of the wage gap and wage increase, respectively. Parameter W_0 is a constant coefficient.

According to the assumptions stated above, we can obtain the cost/benefit ratio (C/B) as shown below.

$$(C/B) = \left[\frac{P_0}{W_0 \cdot l_0} \right] \cdot x^{-(a+b)} \cdot e^{-(\alpha+\beta)\cdot t} \qquad (10)$$

It is assumed that the company decides to introduce advanced equipment if the ratio of investment costs to benefits (C/B) is less than n years.[2]

$$(C/B) \leq n \qquad (11)$$

By substituting equation (11) for equation (10), the above condition of introducing advanced equipment can be transformed into the condition for company size x as follows.

$$x \geq X(t) \qquad (12)$$

where

$$X(t) = \left[\frac{P_0}{W_0 \cdot l_0 \cdot n} \right]^{\frac{1}{(a+b)}} \cdot e^{-\frac{(\alpha+\beta)}{(a+b)} \cdot t} \qquad (13)$$

Condition (12) means that companies of a larger number of employees than X(t) introduce the equipment.

(3) Diffusion at the macroeconomic level

The population of advanced equipment at the macroeconomic level [U(t)] can be obtained by the following equation.

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[2]With regard to the pay-back time, see [Sheinin & Tchijov, 1987] for FMS, and [JIRA, 1984] for Industrial Robots.

$$U(t) = \int_{X(t)}^{\infty} u_0 \cdot f(x)dx = U_\infty \cdot e^{-\lambda \cdot x(t)} \qquad (14)$$

where U_∞ (= $u_0 \cdot L$) denotes the upper limit of the population.

Finally, diffusion rate R(t) [= $U(t)/U_\infty$] can be expressed explicitly as follows:

$$R(t) = EXP(-q \cdot e^{-rt}) \qquad (15)$$

$$q = \lambda \left[\frac{P_0}{W_0 \cdot l_0 \cdot n} \right]^{\frac{1}{a+b}} \qquad (16)$$

$$r = \left[\frac{\alpha + \beta}{a + b} \right] \qquad (17)$$

The conditions for the parameters are shown as below.

$$\lambda,\ P_0,\ W_0,\ l_0,\ n,\ a,\ b > 0 \qquad (18)$$

$$\alpha,\ \beta \geq 0 \qquad (19)$$

The whole structure of this model is illustrated in Figure 9.

4. Implications

(1) Type of diffusion curve

By the differentials of R(t) in equation (15) we can ascertain that our model R(t) has a kind of S-shaped curves.[3]

$$\frac{dR}{dt} > 0 \qquad (20)$$

[3]The Meaning of S-shaped curves in diffusion rates is discussed in [Tchijov, 1987a].

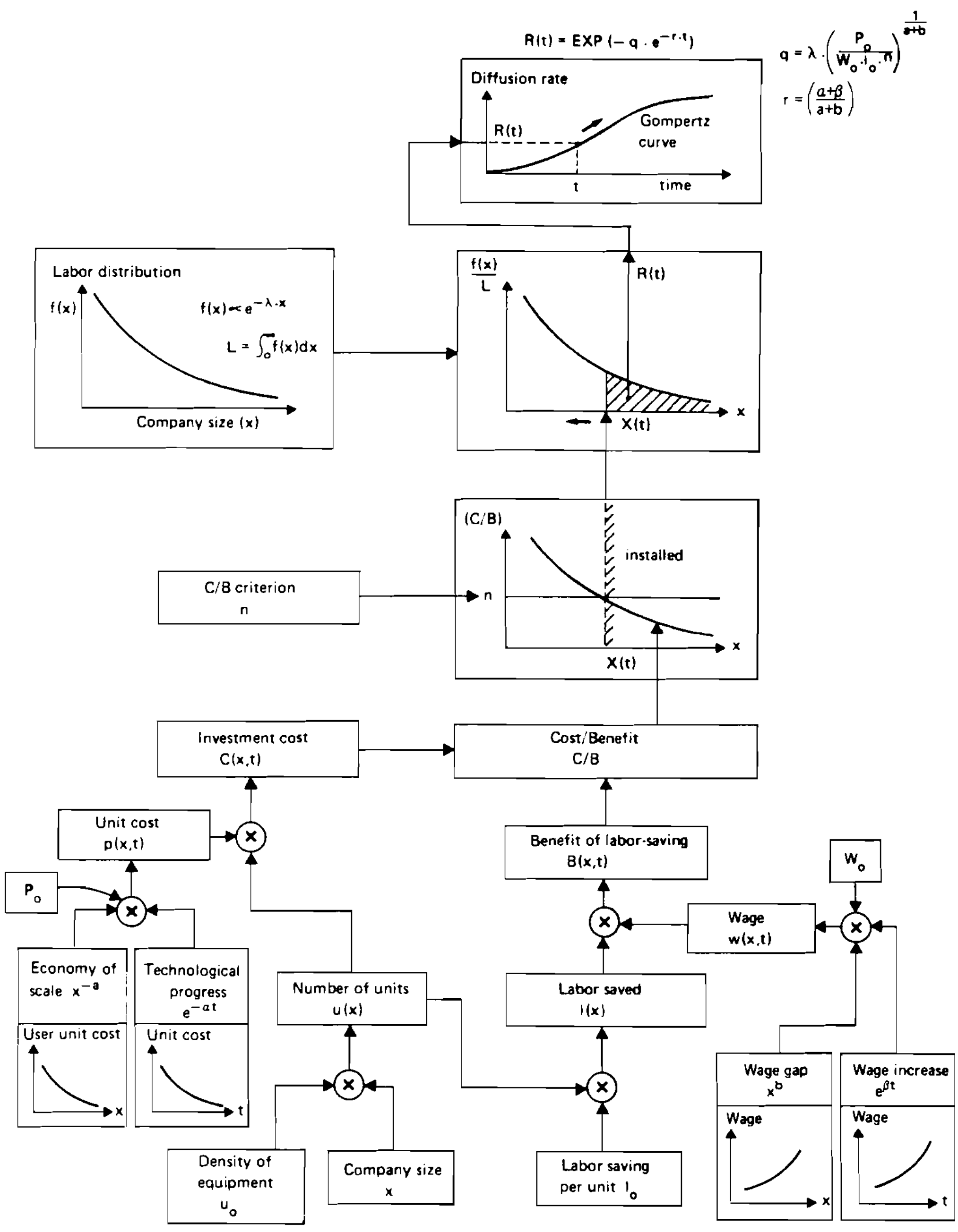

Figure 9. Structure of the model.

$$\frac{d^2R}{dt^2} = q \cdot r \cdot R \cdot e^{-rt} \cdot (q \cdot e^{-rt} - 1) \tag{21}$$

A point of inflection (t*) can be calculated by solving the equation

$$\frac{d^2R}{dt^2} = 0$$

as shown below.

$$t^* = \frac{\ln(q)}{r} \tag{22}$$

$$\left.\begin{array}{lll} \dfrac{d^2R}{dt^2} > 0 & & \text{when } t < t^* \\[2ex] \dfrac{d^2R}{dt^2} = 0 & & \text{when } t = t^* \\[2ex] \dfrac{d^2R}{dt^2} < 0 & & \text{when } t > t^* \end{array}\right\} \tag{23}$$

$$R(t^*) = 1/e \tag{24}$$

In addition, R(t) has the following values at infinite points:

$$R(-\infty) = 0 \tag{25}$$

$$R(+\infty) = 1 \tag{26}$$

By using t* instead of q, equation (15) can be transformed into the following form:

$$R(t) = EXP(-e^{-r(t-t*)}) \tag{27}$$

The parameters t* and r can be interpreted as a time shift and a diffusion speed, respectively.

Finally, it can be ascertained that our model becomes a Gompertz curve [Kotz & Johnson, 1983; Kurtz, 1984] which is well known as a growth curve, as follows:

$$R(t) = Y^{Z^t} \tag{28}$$

where

$$\left.\begin{array}{l} Y = e^{-q} \\ Z = e^{-r} \\ 0 < Y < 1, \quad 0 < Z < 1 \end{array}\right\} \tag{29}$$

(2) Implications

In order to clarify the meaning of our model equation (15) in more detail, we analyze the impacts of the parameters in our diffusion model. Partial derivatives of R with respect to the parameters satisfy the following conditions:

$$\left.\begin{array}{l} \left(\frac{\partial R}{\partial \lambda}\right), \left(\frac{\partial R}{\partial P_c}\right) < 0 \\ \\ \left(\frac{\partial R}{\partial W_c}\right), \left(\frac{\partial R}{\partial l_c}\right), \left(\frac{\partial R^4}{\partial \alpha}\right), \left(\frac{\partial R^4}{\partial \beta}\right) > 0 \end{array}\right\} \tag{30}$$

Each parameter in (30) gives a monotonous impact on the diffusion. The equations in (30) show us the conditions for promoting the diffusion as follows:

1) greater share of large companies (smaller λ)

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[4]when $t > 0$.

2) lower price of equipment (smaller P_o)
3) higher wage rate (larger W_o)
4) higher labor-saving effect (larger l_o)
5) more rapid price decrease (larger α)
6) more rapid wage increase (larger β).

On the other hand, the parameters a and b give a non-monotonous impact on the diffusion. In order to investigate the impact by a and b, we introduce the following parameter y instead of a and b for reasons of simplicity.

$$y = 1/(a+b) \tag{31}$$

Equation (15) can be expressed in the following form by using parameter y:

$$R(t) = EXP(-\lambda \cdot e^{(\ln V - h)\cdot y}) \tag{32}$$

where

$$V = \frac{P_o}{W_o \cdot l_o \cdot n} \tag{33}$$

$$h = (\alpha + \beta) \cdot t \tag{34}$$

The resulting partial derivative of R with respect to y is shown below.

$$\left.\begin{aligned} \left(\frac{\partial R}{\partial y}\right) &< 0 \quad \text{when } t < t_k \\ \left(\frac{\partial R}{\partial y}\right) &= 0 \quad \text{when } t = t_k \\ \left(\frac{\partial R}{\partial y}\right) &> 0 \quad \text{when } t > t_k \end{aligned}\right\} \tag{35}$$

where

$$t_k = \frac{\ln V}{(\alpha + \beta)} \tag{36}$$

The diffusion rate at time t_k, $R(t_k)$, has the following value, which depends only on λ, namely the parameter of company size distribution.

$$R(t_k) = e^{-\lambda} \tag{37}$$

According to the results shown above, we can derive the impact of the parameters of "economy of scale in user costs" (parameter a) and "wage gap between large and small companies" (parameter b) on the diffusion R(t) as follows:

1) Larger a and b promote the diffusion until the diffusion rate reaches $e^{-\lambda}$.
2) However, larger a and b slow down the further diffusion after that point.
3) The turning point (t_k) appears at a lower diffusion rate ($R(t_k) = e^{-\lambda}$) if the share of large companies is smaller (larger λ).

Although the impact of a+b is theoretically complex, the actual impact is easy to understand because $\lambda \ll 1$. Smaller (a+b) delays the diffusion, but the speed of diffusion is faster after the diffusion has substantially set in.

5. Numerical Examples

In order to visualize the implications described in the previous chapter, some numerical examples are shown in this chapter.

The parameters are set for a basic case as follows:

$t = 0$ (1980)

$\lambda = 0.001$

$\alpha+\beta = 0.05$

$a+b = 0.2$

$P_0 = 30$

$W_0 = 1.5$

$l_0 = 1$

$n = 3$

The curves of diffusion rate R(t) are shown in Figure 10. The impacts of parameters λ, $(\alpha+\beta)$ and (a+b) are also illustrated in a visual way.

Supposing that unit prices for three applications, A, B, and C, are 30, 40 and 50 at t = 0, respectively, the total diffusion curve for A + B + C can be obtained by summing up the three curves as shown in Figure 11.

6. Conclusions

A new approach to formulate the penetration mechanism of high technological equipment into industry has been proposed in this paper. This approach aims at the integration of company-level studies and macro-economic level studies in the penetration mechanism. In other words, the outline of our model might be summarized as follows:

a) The basic mechanism of penetration is a decision-making rule to introduce the advanced equipment at the company level.

b) Cost/benefit at the company level depends upon the size of the company, the technological progress and the wage increase.

c) The diffusion rate at the macro-level is obtained by applying the decision-making rule to the whole companies through the distribution function of the company size.

d) Total diffusion of advanced equipment in the national economy is calculated by applying the above procedures to all industrial sectors and applications.

The formulation from a) to c) leads us finally to a simple mathematical model which is a well-known Gompertz curve, although we introduce the various factors as variables and parameters in formulating the "empirical laws".

It might be concluded that the theoretical reasoning for applying a Gompertz curve to forecast the diffusion of technologies into industry is reinforced in this process.[5]

In addition, new sophisticated implications can be added to the parameters of the Gompertz curve through our model as

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[5]Logistic curves are usually applied, as, e.g., in [Tani, 1978a; Tchijov, 1987b].

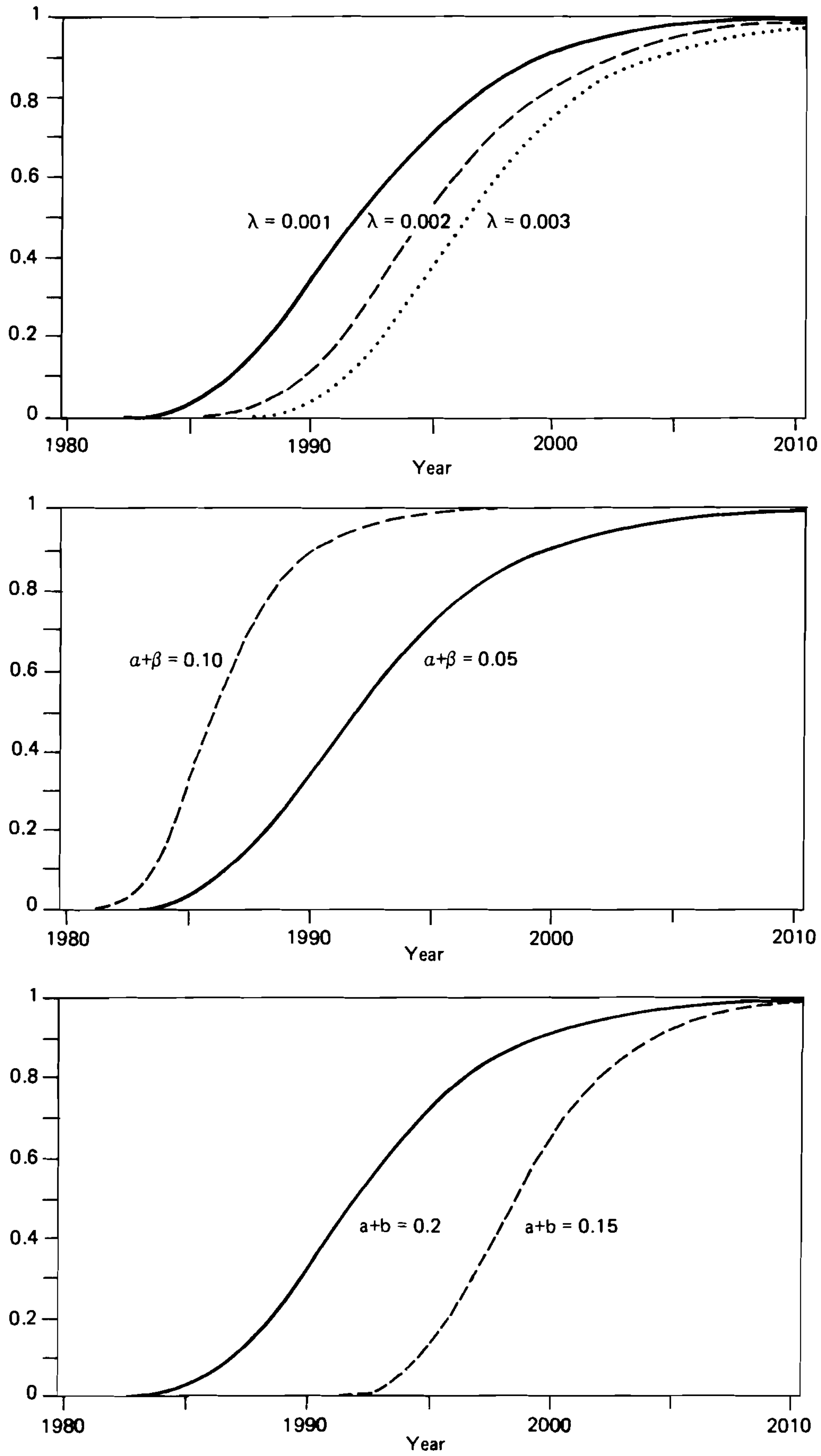

Figure 10. Numerical examples.

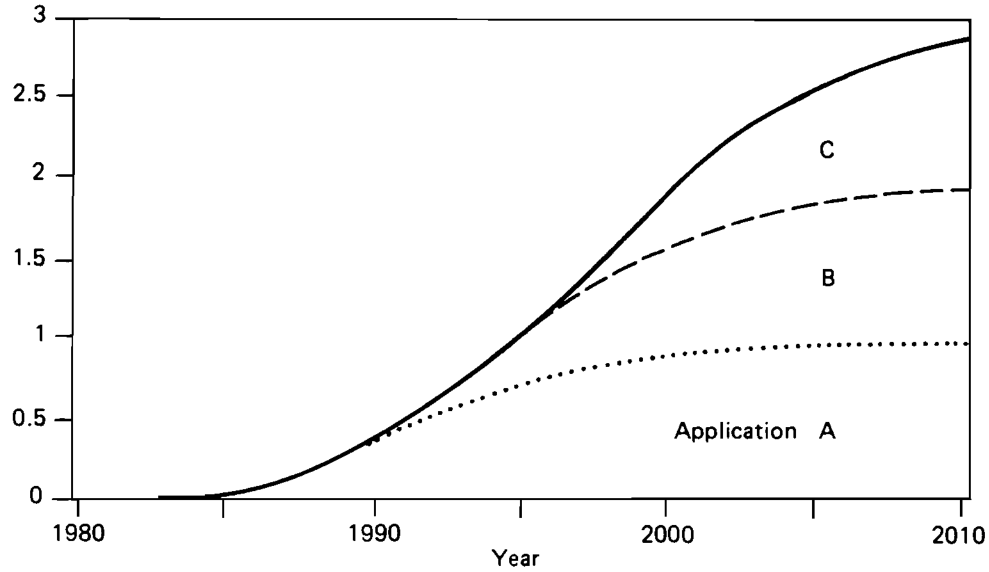

Figure 11. Typical diffusion curve for various applications.

described in Chapter 4. However, it should be noted that the Gompertz curve model is an example of the results by the formulation of penetration mechanism proposed by this paper.

To be more adequate for actual cases, we could adopt other kinds of functions for sub-model equations, which would mean that we could derive different shapes of diffusion curves from the same formulation. In other words, the principal aim of this paper is not to specify the shape of diffusion curves, but to formulate the mechanism of penetration in a mathematical form.

Although we focused on the benefits of labor-saving in our model, our formulation itself is also applicable to other cases, in which other kinds of benefits dominate, if we can express the benefits as a function of company size x.

Generally speaking, there are several kinds of benefits generated by introducing advanced technologies. In the case of CIM, the benefits from flexibility and quality of production are considered to be important as well as labor-saving ones.

Our formulation in this paper assumes that the benefit at company level is proportional to the wage rate and company size. It should be noted that direct labor-saving is an example of such benefits.

For instance, it might require additional labor to achieve high flexibility and quality without advanced technologies. The above benefits might be expressed as the saving of such additional labor.

Finally, any model has its own limitations of applications. The model proposed in this paper depends greatly upon the several assumptions listed. If some assumptions are not adequate in cases of applications to other sectors and countries, different models could be derived from a similar formulation method.

In order to apply this model to real-world cases, some modifications are necessary. The general form of our model is explained in Appendix B. For example, the exponential function of company size distribution shows a deviation in the range of very small companies. As a decision-making criterion, a probabilistic function should be introduced. The experience curve or economy of scale in production should be incorporated instead of the simple trend function.

However, it may be concluded that the mathematical model proposed here can be regarded as a useful step towards further investigations on the diffusion mechanism of new technologies such as CIM.

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APPENDIX A: Notation of Variables and Parameters

x:	company size (number of employees in company)
L:	total number of employees in industry
$f(x)$:	labor distribution density function with respect to company size x
λ:	parameter of labor distribution $[f(x) \propto e^{-\lambda \cdot x}]$
$u(x)$:	number of units introduced in company of size x
u_0:	density of advanced equipment (units per employee)
$l(x)$:	labor saved by introducing advanced equipment in company of size x
l_0:	labor saved per unit of equipment
$C(x,t)$:	investment cost for introducing equipment in company of size x at time t
$p(x,t)$:	unit cost of equipment in company of size x at time t
a:	parameter showing the effect of "economy of scale" in user cost (cost $\propto x^{-a}$)
α:	annual rate of price decrease (price $\propto e^{-\alpha t}$)
P_0:	constant coefficient which denotes the unit cost of equipment in company size $x=1$ at time $t=0$
$B(x,t)$:	benefit of labor saving in company of size x at time t
$w(x,t)$:	annual wage in company size x at time t
b:	parameter showing the effect of the wage gap between large and small companies (wage $\propto x^{b}$)
β:	annual wage increase rate (wage $\propto e^{\beta t}$)
W_0:	constant coefficient which denotes the annual wage in company size $x=1$ at time $t=0$
n:	decision criterion for investment (years)
$X(t)$:	size of companies which decide to introduce advanced equipment at time t
$U(t)$:	population of advanced equipment in industry at time to
$U\infty$:	upper limit of the population
$R(t)$:	diffusion rate of advanced equipment $[R(t) = U(t)/U\infty)$
$t*$:	a point of inflection in $R(t)$
t_y:	turning point of impact of parameter y

$$q \equiv \lambda \left[\frac{P_{\infty}}{W_{\infty} \cdot l_{\infty} \cdot n} \right]^{\frac{1}{a+b}}$$

$$r \equiv \left[\frac{\alpha + \beta}{a + b} \right]$$

$y = 1/(a+b)$

$V = P_{\infty}/(W_{\infty} \cdot l_{\infty} \cdot n)$

$h = (\alpha+\beta) \cdot t$

APPENDIX B: General Form of the Model

In Chapter 3 we specified the functions of $p(x,t)$, $w(x,t)$ and $f(x)$. In addition, we assumed the extreme criterion of decision-making as shown in equation (11). These assumptions might be too simple to apply the model to real cases.

Therefore, the general form of our model is explained as follows:

$$U(t) = \int_0^\infty (u(x)/x) \cdot f(x) \cdot g(z)dx \qquad \text{(B-1)}$$

where $g(z)$ denotes a probabilistic decision-making function, and z is defined below.

$$z = \frac{p(x,t) \cdot u(x)}{n \cdot w(x,t) \cdot l(x)} \qquad \text{(B-2)}$$

The function $g(z)$ satisfies the following conditions:

$$\begin{cases} g(0) = 1 \\ g(1) = 0.5 \\ g(+\infty) = 0 \end{cases} \qquad \text{(B-3)}$$

As a function of $g(z)$, a cumulative logarithmic normal distribution would be adequate.

$$g(z) = 1 - N(\log z) \qquad \text{(B-4)}$$

where N( ) means a cumulative normal distribution function.

The shape of $g(z)$ is illustrated in Figure B. Finally, in case of a wider generalization, $f(x)$ can also be treated as a time-dependent function.

The experience curve could be incorporated into our model as described below.

Instead of $p(x,t)$, the following equation is introduced.

$$p(x,t) * H(M_t) \qquad \text{(B-5)}$$

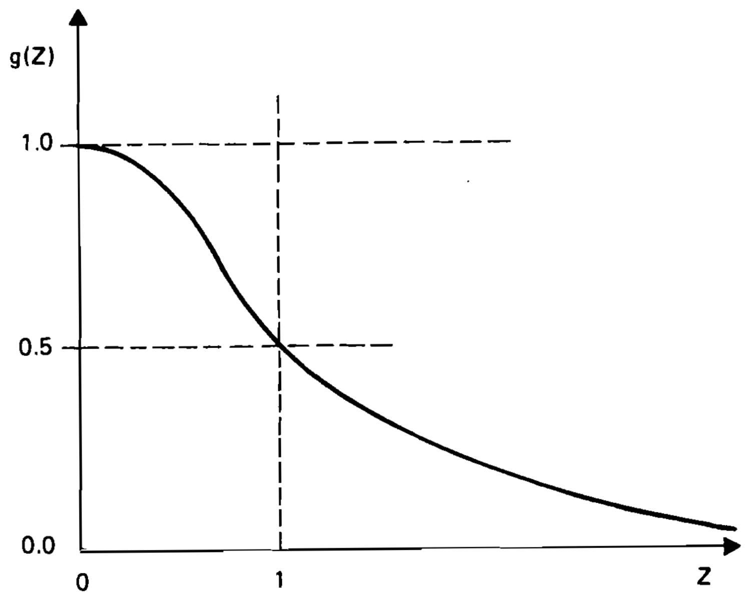

Figure B. Probabilistic decision-making function g(Z).

where H( ) and M_t denote a function of the experience curve and cumulative units of production up to time t, respectively.

M_t can be expressed as below if there are no imports and exports:

$$M_t = \sum_{i=0}^{\infty} U(t-i*m) \qquad \text{(B-6)}$$

where m denotes a replacement period.

Although it is impossible to solve the above equations analytically with respect to U(t), we could apply the above-stated general model by numerical methods.