WORKING PAPER

Multistate Life Table with Duration-Dependence: An Application to Hungarian Female Marital History

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IIASA
International Institute
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Foreword

A growing body of theoretical research in mathematical demography has shown that heterogeneity in demographic rates can lead to erroneous conclusions about processes that evolve over time and age. One area in which this finding has been particular concern is multistate increment-decrement life table analysis. Prior to the development of the duration-dependent multistate life table (DDMSLT) (D. Wolf, *Mathematical Population Studies*, 1(3):217–245) increment-decrement models of marriage, divorce, fertility, employment, and migration were, of practical necessity, based on either age or duration in status. This represented a major limitation of the multistate approach, given the widely accepted notion that transition rates for demographic events are dependent on both age and time in status. The DDMSLT model retains the simplicity and elegance of the original multistate life table while relaxing one of its most restrictive assumptions.

In this paper, Alain Belanger examines the biases that can be introduced into the marital status life table if duration dependence is ignored. Using data from the 1984 Hungarian microcensus, Belanger shows that the introduction of duration dependence has the greatest impact on the age structure of the stationary population in each marital status, and relatively little impact on the overall time spent in each status. This result is explained by the fact that duration dependence is not relevant until around age 25 when a substantial proportion of women have experienced a first marriage.

From this exercise one can draw the general conclusion that the impact of duration dependence will be a function of age when when the initial status is an age-dependent state. This suggests that duration dependence will be of greater importance for the total time spent in each status when the transition to a duration-dependent state occurs at an earlier age. An example would be the employment-status life table, where transitions from the age-dependent state never-worked to employment are likely to occur at an earlier age than a transition to first marriage. In the case of fertility, the majority of first births follow the transition from never-married to married, which would tend to diminish the impact of introducing duration dependence. On the other hand, fertility transitions are strictly hierarchical and highly duration-dependent, so that the overall impact of introducing duration dependence may be quite substantial. In populations where a substantial amount of movement takes place at the younger ages, one would expect the introduction of duration dependence to have a numerically important effect on the multistate life table for migration.

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Abstract

Multistate life table methods have been recognized as an excellent tool in the analysis of many types of transitions. Yet demographers have never been completely satisfied with the Markovian assumption and have stressed the importance of population heterogeneity. In marital status analysis, for example, the time spent in the current status is thought to be of first importance in determining transitions, but computational problems linked to the introduction of duration prevented any estimation of the bias in life table calculations arising from its omission. Building on recent developments in multistate demography, and using data from the 1984 Hungarian microcensus, this paper analyzes the impact that the introduction of duration-specific transitions has on the results of a multistate life table analysis of marital dissolution. The results show that the inclusion of duration has its greatest impact on the distribution of the stationary population between ages 25 and 35.

Acknowledgements

The author is very much indebted to Douglas Wolf for his help and comments in the early phase of this project, and to Andras Klinger who provided the data. This work was started while the author was at IIASA; he is grateful both to the Canadian Committee for IIASA and to IIASA itself for providing a travel grant and stipend, respectively. Thanks also go to Andrei Rogers and Jennifer Woodward for their comments on an earlier draft.

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Introduction

The first empirical multistate life table using period data was computed by Rogers (1973) and subsequently was applied at IIASA (International Institute for Applied Systems Analysis) in a study of internal migration in its then 17 member countries. The major advantages of such life tables, over the traditional single or multiple decrement life tables, are their ability to permit reentry into previously occupied states as well as exits from one state to another, and their capacity to handle different mortality patterns for different statuses.

These characteristics make them an ideal tool for the analysis of any type of transitions. Examples of their use abound in the recent literature. In addition to migration, multistate life table techniques have been applied to the analysis of working life (Schoen and Woodrow, 1980; Willekens, 1980; U.S. Department of Labor, 1982), fertility (Suchindran *et al.*, 1977; Lutz and Wolf, 1987), active life expectancy (Rogers *et al.*, forthcoming), and marital status changes (Keyfitz, 1988; Espenshade, 1983, 1986; Espenshade and Braun, 1982; Willekens *et al.*, 1982; Schoen and Nelson, 1974).

Based on a time-inhomogeneous, finite-space Markov chain process, multistate methods share the same limiting assumptions of population homogeneity and complete dependence on the current status (i.e., the Markovian assumption). Stated explicitly, all members in a given status and age group have the same transition probabilities, and these probabilities depend solely on the current status and age. The transitions are completely independent of previous occupied statuses and of the duration in the current state.

In some applications, these restrictive assumptions are contradicted by theoretical foundations and/or empirical findings. In the analysis of family formation and dissolution, for example, the time spent in the current status (duration) seems to be of primary importance in the determination of divorce and remarriage probabilities (Land and Schoen, 1982).

Theoretically, marriage dissolution rates are expected to decline with duration of marriage. First, the length of a marriage can be associated with the accumulation of assets that have their greatest value in the union and with the presence of children (Becker *et al.*, 1977). A second explanation emphasizes the selectivity of marriage dissolution. As marriage duration increases, the surviving unions will include fewer and fewer high-risk marriages (Thornton and Rodgers, 1987). However, a minimum of time has to separate divorce from marriage. Divorcing is an important decision and most people will therefore try to make personal adjustments before resorting to it. In any case, a marriage cannot be shorter than the time period necessary to have the divorce legally processed. Remarriage rates also are expected to be negatively correlated with the time spent as divorced or widowed, mainly due to selectivity.

The analysis of correlation coefficients of Californian divorce rates cross-tabulated by age and duration of marriage led Schoen (1977:251) to state that "duration of marriage is the preferred index". However, considering that divorce and remarriage are closely related with other age related events, notably mortality, an age index cannot be discarded. Therefore, he concluded that "if possible, both age and duration should be used as indexes in divorce analysis."

A duration index can be introduced in multistate life tables by expanding the state-space. However, until most recently, such an approach created computational problems related to the size of the matrices to be inverted. Wolf (1987) generalizes Rogers's (1975) linear model by developing a method to introduce duration dependence in multistate analysis that requires inversion of matrices of the same order as those in the case without duration dependence.

The purpose of this paper is to analyze the bias introduced by omitting duration dependence into multistate life tables applied to marital events. First, the model incorporating duration-dependence is briefly described. This is then followed by a description of the data and the methodology used to compute age-specific duration-dependent transitions from a retrospective survey. Finally, the results from duration-dependent and duration-independent models are contrasted.

Multistate Life Table with Duration-Dependence

The duration-independent multistate life table methodology is discussed in several sources (see Espenshade, 1986; Willekens *et al.*, 1982 for application to marital status life tables). A brief review is useful before presenting Wolf's (1987) method to incorporate the duration dimension.

Let $M(x)$ be a n-by-n matrix of transition rates between pairs of states 1 to n, from age x to x+1:

$$M(x) = \begin{bmatrix} m_1(x) & -m_{12}(x) & \dots & -m_{1n}(x) \\ -m_{21}(x) & m_2(x) & \dots & -m_{2n}(x) \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ -m_{n1}(x) & \cdot & \dots & m_n(x) \end{bmatrix} \tag{1}$$

where elements $m_{ij}(x)$ are the transition rates into state i from state j, and the diagonal elements $m_i(x)$ are defined as

$$m_i(x) = m_{di}(x) + \sum m_{ij}(x) \quad , \tag{2}$$

$m_{di}(x)$ being the death rates in state i between ages x and x+1. Also, let $l(x)$ be a vector representing survivors in state 1 to n at exact age x. Then,

$$l(x+1) = (I + \frac{M(x)}{2})^{-1}(I - \frac{M(x)}{2})l(x) \quad . \tag{3}$$

In a manner similar to the single decrement life table, all other functions can be easily obtained from the $l(x)$ arrays (Rogers, 1975).

The first step in incorporating duration is to estimate for each age a duration-specific $M(x)$ matrix of the same form as in (1). These matrices can be labelled $M_d(x)$, and are of the same order as $M(x)$. The marriage duration categories $d = a$, 0, 1, ..., w are defined with respect to the last anniversary reached in a given status at exact age x. "At exact age x, someone in duration category d has been in their current status at least d, but less than d+1, time units" (Wolf, 1987:4). Thus, duration category "a" is the category entered if a transition occurs between age x and $x+1$, and can be conceived as duration category "-1". At exact age $x+1$, those who have experienced a transition between age x and $x+1$ are then in duration category "0". Duration category w is the last duration category and is open-ended. In a fashion similar to the treatment of the last open age group in duration-independent multistate life tables, those who are in duration category w at age x and do not move, will be in the same category at age $x+1$.

These $w+2$ $M_d(x)$ matrices are of course interacting with each other. People who are in duration category d at age x and do not move have to be in duration category $d+1$ at age $x+1$, and those who do move have to be in duration category 0 at age $x+1$. A possible approach would be to simply increase the state-space of the model. However, such an approach can rapidly become unmanageable since a matrix of order $n*(w+2)$ will have to be inverted for each age group. For example, the model of marital transitions used in this paper has 7 states and 15 duration categories. It would thus require one to invert a matrix of order 119 for each age group!

Wolf (1987) ingeniously showed how to rearrange the $Md(x)$ to facilitate such calculations. First, he decomposed the $Md(x)$ matrices[1] into two parts labelled $DM_d(x)$ and $CM_d(x)$. $DM_d(x)$ being the diagonal elements of $M_d(x)$, and $CM_d(x)$ being the original $M_d(x)$ matrix with its diagonal elements replaced by zeros. Then,

$$M_d(x) = DM_d(x) + CM_d(x) \quad . \tag{4}$$

Rearranging to obtain $M^*(x)$, the full matrix of age and duration-dependent transition probabilities, gives:

$$M^*(x) = \begin{array}{c|cccccc} M_a(x) & CM_0(x) & CM_1(x) & CM_2(x) & \cdots & CM_w(x) \\ \hline 0 & DM_0(x) & 0 & 0 & \cdots & 0 \\ 0 & 0 & DM_1(x) & 0 & \cdots & 0 \\ 0 & 0 & 0 & DM_2(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & DM_w(x) \end{array} \tag{5}$$

Finally, the $l(x)$ array must be regrouped by duration category to give the column vector:

$$l^*(x) = [0...0 \,|\, l_{10}(x)...l_{n0}(x) \,|\, ... \,|\, l_{1w}(x)...l_{nw}(x)] \quad . \tag{6}$$

The first n elements of $l^*(x)$ are zeros that correspond to duration category a, which is occupied only temporarily by those who change their marital status between ages x and $x+1$. At exact age x, the survivors are distributed between duration categories 0, 1,..., w only.

Then, $l^*(x+1)$ can be obtained from an operation similar to (3):

$$l^*(x+1) = A(x)(I + \frac{M^*(x)}{2})^{-1}(I - \frac{M^*(x)}{2})l^*(x) \tag{7}$$

where $A(x)$ is a matrix which causes survivors in a given state and duration to advance to the next duration category. Recall that $M^*(x)$ is a matrix of order $n*(w+2)$. Apparently, the computational requirements to solve (7) are the same as if the state-space had been increased without rearranging. That is only apparent. To facilitate the demonstration, let

$$Y = (I + \frac{M^*(x)}{2}) \quad , \tag{8}$$

[1] Except the matrix for duration category a.

and

$$Z = (I - \frac{M^*(x)}{2}) \quad , \tag{9}$$

so that (7) can be rewritten as

$$l^*(x+1) = A(x) Y^{-1} Z \, l^*(x) \quad . \tag{10}$$

Then, by partitioning Y and Z following the dashed lines in (5), and labelling these parts as

$$Y = \begin{bmatrix} R & S \\ 0 & T \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} U & V \\ 0 & W \end{bmatrix} \quad ,$$

Wolf (1987) shows that

$$l^*(x) = \begin{bmatrix} R^{-1}U & R^{-1}(V - ST^{-1}W) \\ 0 & T^{-1}W \end{bmatrix} l(x) \tag{11}$$

T being a diagonal, only R, a matrix of the same order (n-by-n) as in the duration-independent case, needs to be inverted.

A Model of Marital Transitions

The simplest marital status life table would have only two states:[2] married and not married. However, to reduce population heterogeneity, most studies have distinguished four statuses: never married (single), married, divorced, and widowed. Although it is recognized that it would be desirable to provide more information (Keyfitz, 1988; Espenshade, 1986), a distinction between first, second or higher order marriages and dissolutions has been rarely carried out because of data limitations.

Figure 1 sets out our model which separates first marriages and dissolutions from second or higher order ones. However, in this model, it is possible for a woman to be in status divorced 2 (widowed 2) even if she is divorced (widowed) for the first time provided that she was a widow (divorced) before being remarried.

Not all the transitions are duration-dependent. As long as a first event did not occur, the duration in the first state (i.e., single) is equal to the age of the individual and does not bring any additional information. Thus, first marriage rates are only age-specific. In addition, the transition from married (or remarried) to widowed is theoretically more related to the male mortality function and the age difference between spouses than to the marriage's duration. For that reason, transition rates to the widowed statuses are derived from mortality rates of married males two years older. Consequently, among the nine possible transitions, only six are duration-dependent (Figure 1).

In marital status analysis, most of the j to i transitions are impossible. For example, a person cannot move into the divorced status directly from the never married status. Moreover, some transient states, such as single and first married, can never be reentered once left. Therefore, most of the off-diagonal elements of $M_d(x)$ are 0, and each matrix has the following form:

[2] In addition to the absorbing state of death.

FROM

$$
\text{TO}\quad
\begin{array}{cc}
 & \begin{array}{ccccccc} \text{S} & \text{M1} & \text{D1} & \text{W1} & \text{M2} & \text{D2} & \text{W2} \end{array} \\
\begin{array}{c} \text{S} \\ \text{M1} \\ \text{D1} \\ \text{W1} \\ \text{M2} \\ \text{D2} \\ \text{W2} \end{array} &
\begin{bmatrix}
m_s & 0 & 0 & 0 & 0 & 0 & 0 \\
-m_{sm} & m_m & 0 & 0 & 0 & 0 & 0 \\
0 & -m_{m1d1} & m_{d1} & 0 & 0 & 0 & 0 \\
0 & -m_{m1w1} & 0 & m_{w1} & 0 & 0 & 0 \\
0 & 0 & -m_{d1m2} & -m_{w1m2} & m_{m2} & -m_{d2m2} & -m_{w2m2} \\
0 & 0 & 0 & 0 & -m_{m2d2} & m_{d2} & 0 \\
0 & 0 & 0 & 0 & -m_{m2w2} & 0 & m_{w2}
\end{bmatrix}
\end{array}
$$

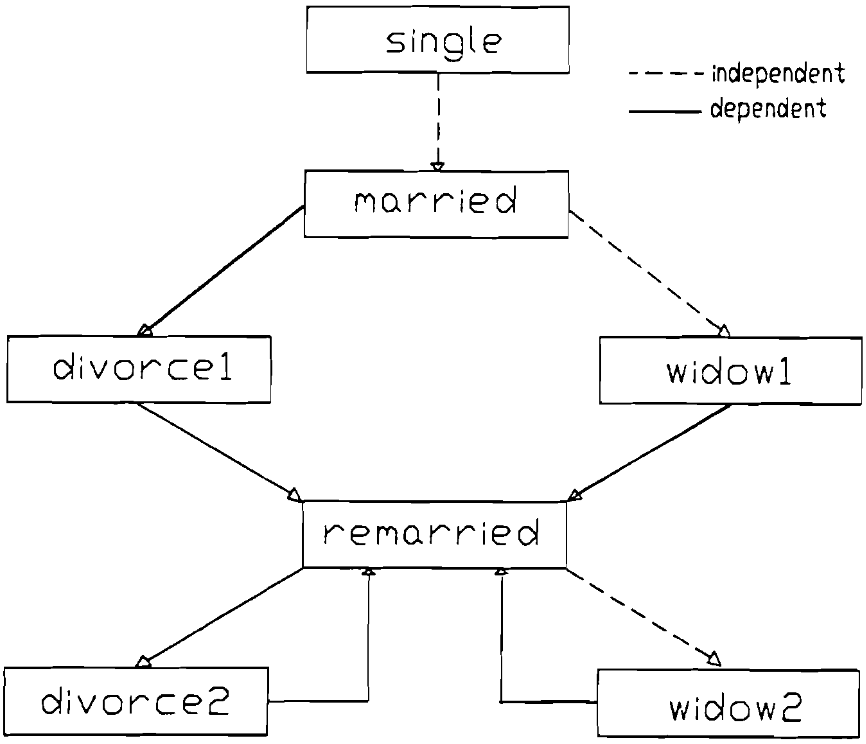

Figure 1. A model of marital transition including duration.

Data

Our data come from retrospective questions asked in the 1984 Hungarian microcensus (2 percent sample) of women aged 15 to 50. In addition to the current marital status and age, the year of ceremony, and that of dissolution (with its motive: divorce or widowhood) up to three marriages are recorded. Mortality rates were extracted from the *Hungarian Demographic Yearbook of 1985* and were available only by age and marital status.

The 2% Hungarian microcensus counted 42,000 women aged 15–50. One common problem that arises in empirical application of multistate models is the estimation of reliable transition probabilities from the available data. Such a problem is greatly increased when single years of age and duration dependence are introduced. We used 15 duration categories[3] to distinguish the duration-dependent transitions. To overcome the problem of small numbers, the sample size was artificially expanded by calculating occurrence-exposure rates retrospectively over the 1980–1984 period. Thus, each cohort aged between 20 and 49 at the time of the census was exposed during five years to various marital status, age, and duration categories.

Two other problems encountered are more specific to the type of data used. It is well established that retrospective surveys tend to underestimate the occurrence of certain events, particularly those bearing a social stigma. Estimated divorce rates are a common example of such underestimation due to misreporting. The Hungarian census is not an exception. Figure 2 contrasts rates of first divorce by age (all duration) estimated from the census data with the corresponding rates officially published (*Hungarian Demographic Yearbook of 1985*). It should also be noted that retrospective data are conditional on survival at the time of the census, and a bias can be introduced from the interaction between mortality and other marital transitions. However, the main concern of this paper is to contrast the results obtained from a duration-independent approach with those obtained when duration-specific transitions are introduced in the life table calculations. The biases caused by misreporting and by the interactions between events are assumed to be similar in both approaches. For that reason, we did not correct the rates estimated from the census data.

Results

Figures 3 to 5 show the effects of duration on Hungarian marital transitions by comparing selected duration categories with the duration-independent rates. Duration-independent first divorce rates (Figure 3) peak for the age group 20–24, and decrease steadily thereafter. Variations between duration categories differ significantly from one age group to the other; while they are minimal for age group 25–29, much larger variations appear after age 30. Generally, first divorce rates are higher for shorter duration categories (categories 1, 3, and 5) than for the duration-independent and longer duration categories.

Duration-independent remarriage rates (Figure 4) also peak for the age group 20–24 and decline rapidly thereafter. The length of time spent since the last marital transition appears to be more important for remarriage than for divorce. Duration categories 1 and 3 rates are consistently higher than the duration-independent rates.

The importance of separating first marriages (and dissolutions) from higher order ones appears clearly when Figure 5 is compared with Figure 3. Looking at the duration-independent rates of first and second marriage dissolution by divorce, we note that the latter are almost twice as high as the former. Also, second divorce rates peak at a different age group. However, duration has a similar effect on second order and on first order divorces.

The age-specific duration-dependent rates are available only for women aged 15 to 50 years in 1984. Thus, the construction of a complete life table, ending at the usual open age group of 85 and over, can only be done by applying the same duration-independent transitions (extracted from published tables) for the remaining age groups. For the purpose of establishing the impact that the introduction of duration-dependent transitions has on multistate life table indices, it appeared misleading and unnecessary to do so. Consequently, a common indicator such as the life expectancy cannot be calculated. In-

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[3]In fact, we used single years of duration up to duration 10 (7 in the case of remarriage and second divorce). An average occurrence/exposure rate has been used for duration categories 10 to 14 (7 to 14 for remarriage and second divorce) and 15 and over. These rates are by single year of age for first divorce, but by five-year age groups for remarriage and second order divorce.

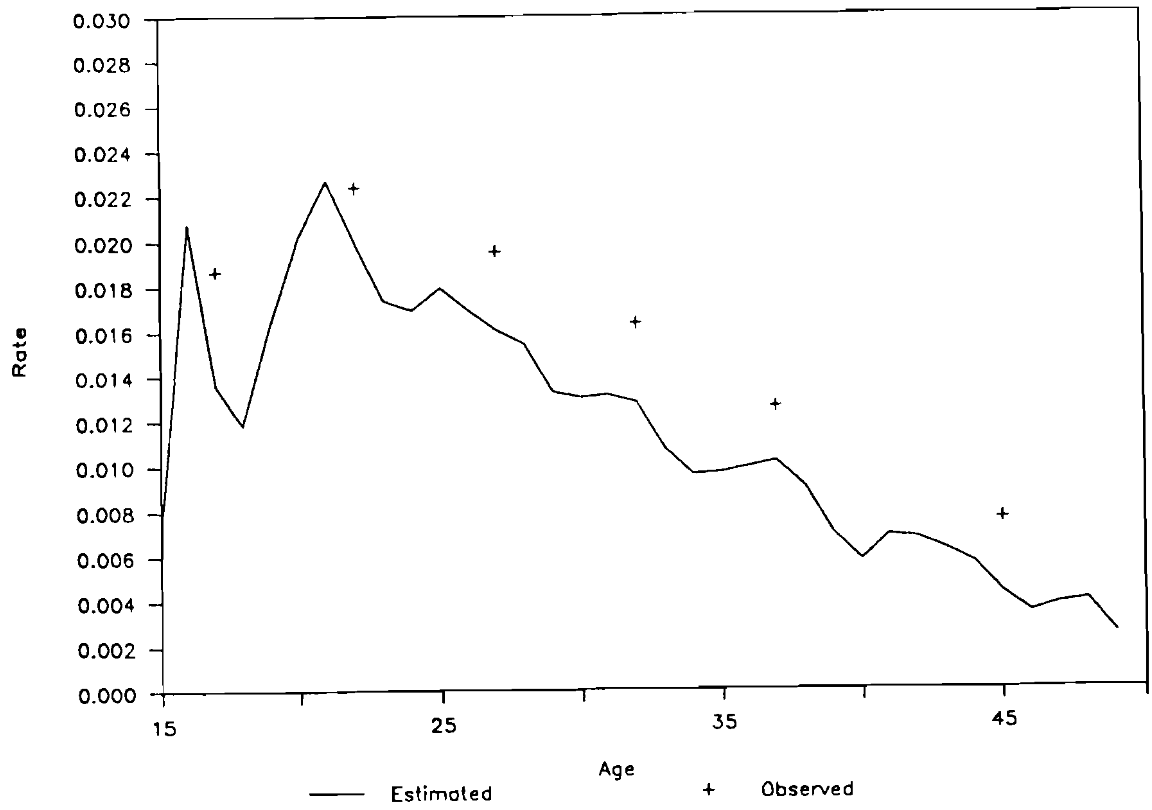


Figure 2. First divorce rates by age: females, Hungary, 1980–1984.

Source: Hungarian Yearbook 1985 and Hungarian 2% microcensus.

stead, it is replaced by the average number of years lived between ages 15 and 50, which is the summation of the L_x matrices between this age interval, and thus can be interpreted in a manner similar to the life expectancy.

The average number of years lived in each marital status for the duration-dependent and independent approaches are compared in Table 1. The largest difference, in terms of years gained or lost, is found for first marriages. The duration-independent life table overestimates the average number of years lived in that category by 0.4 year, or about two percent, when compared to the duration-dependent approach. In relative terms, however, divorces are much more affected by the introduction of duration-dependent rates. The average number of years lived in the first divorce state is 17 percent higher in the duration-dependent life table than in the duration-independent one. Generally, the average number of years lived is larger in the duration-dependent approach for statuses that can be reached only later in the life course. Yet, the absolute difference is never larger than a half year for any status.

Thus, from an examination of the average number of years lived in a given marital status, one can conclude that introducing duration categories does not improve the life table estimates significantly. However, it should be remembered that this indicator, like life expectancy, is not very sensitive to "small" changes. Moreover, just as life expectancy at birth is more sensitive to smaller changes in infant mortality rates relative to changes in older age mortality, the average number of years lived between age 15 and 50 taken out of the marital status life table is more sensitive to variations in early marital transition rates. One has to be married before getting a divorce, and first marriage rates are dura-

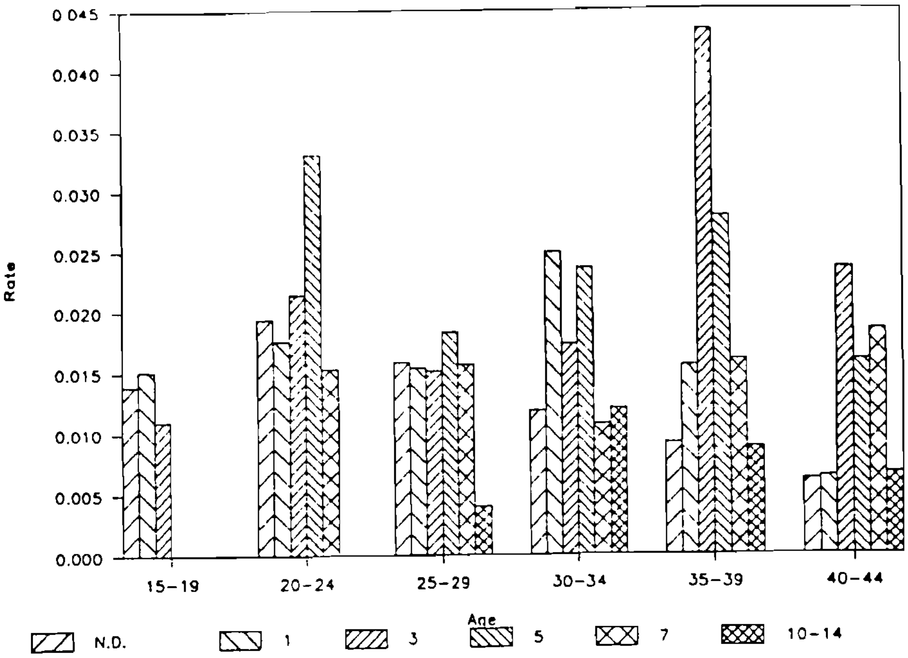


Figure 3. First divorce rates by age groups for selected duration categories: females, Hungary, 1980–1984.

Source: Hungarian 2% microcensus.

tion independent. More than 80 percent of the 35 years that a woman can possibly live, between age 15 and 50, are spent in the never married and first marriage statuses. Consequently, it is important to analyze the effect of duration on a more sensitive indicator, such as the stationary population. Table 2 presents the stationary populations associated with the life table arising from each approach. The single population is the same in both approaches because the same first marriage rates are used. The discrepancies between the two approaches are the largest for first marriages and divorces.

Differences between the duration-independent and duration-dependent stationary populations are presented in Figure 6. Clearly, the effects of duration heterogeneity vary with age. Until age 26, the first married population benefits slightly from the introduction of duration categories, mostly at the expense of the second marriage population. More significant differences in the two stationary populations appear after the population reaches the age when it is first subject to the duration-dependent transitions. It is in the age group 25–29 that duration has the greatest effect. For example, there is no noticeable difference in the divorced populations before age 25. But at age 29, the duration-independent approach underestimates the population in the first divorce category by one third, or 2,223 person-years.

The introduction of duration categories affects mostly the transitions between first divorce and second marriage between ages 30 and 35. The population in the second marriage category was higher in the duration-independent approach before age 31. As the population "at risk" of a second marriage gets larger in the duration-dependent approach, the number of women getting remarried increases. Consequently, the difference between the two approaches in the number of divorced women decreases by one fourth between age 29 and age 35.

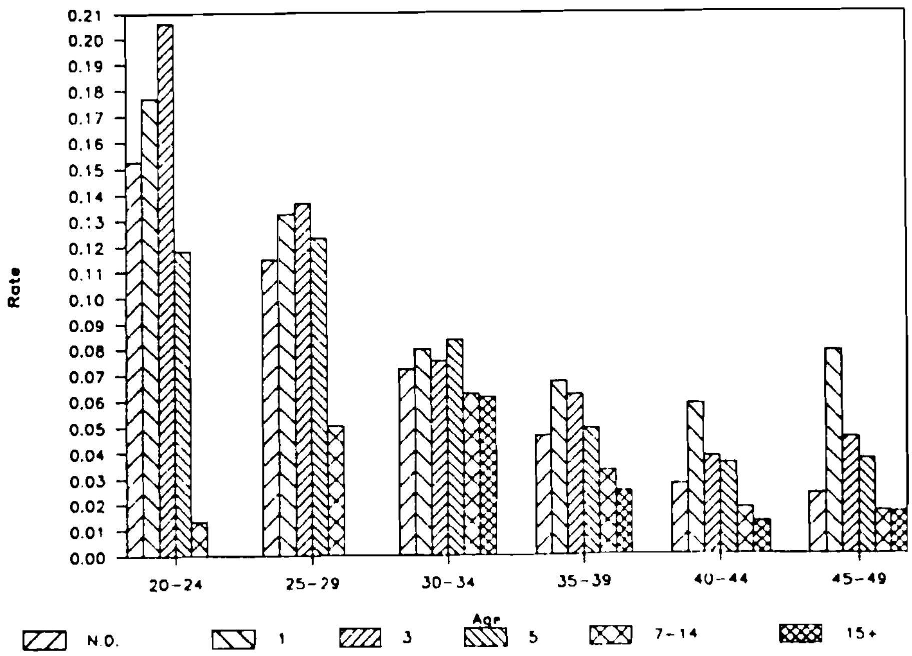


Figure 4. Second marriage rates by age groups for selected duration categories: females, Hungary, 1980–1984.

Source: Hungarian 2% microcensus.

The population is more equally distributed between the different duration categories after age 35, and as a result the differences between the two approaches are almost stabilized. Until age 50, the differences in the number of people in first marriage and first divorce categories continue to decrease slowly, while they slowly increase for the widowed category. However, changes in these differences are negligible compared with those produced between ages 25 and 35. There are now enough women in the older duration categories to compensate for the higher transition rates of the younger ones, and the effects of duration heterogeneity are tempered.

It is also interesting to compare changes in retention probabilities with the time spent in the current status. Mathematically, such probabilities can be defined as:

$$l_{i,d}(x+d)\ l_{i,0}(x)^{-1} \tag{12}$$

where the first subscript refers to the marital status, and the second to the duration category.

Since there is no duration category in the conventional multistate life table, such an indicator doesn't exist for it. However, Willekens *et al.* (1982) define a marital status-based life expectancy. Following the same concept, and using their notation, the probability of surviving in the same status at age $y+t$ for someone entering it at age y can be calculated as:

$$_yl_i(y+t)\ _yl_i(y)^{-1} \quad . \tag{13}$$

Note that both in (12) and in (13), l_i is a scalar. Thus, providing that $d = t$, it is possible to compare retention probabilities between the duration-dependent and duration-independent approaches. Figure 7a shows these retention probabilities for women divorc-

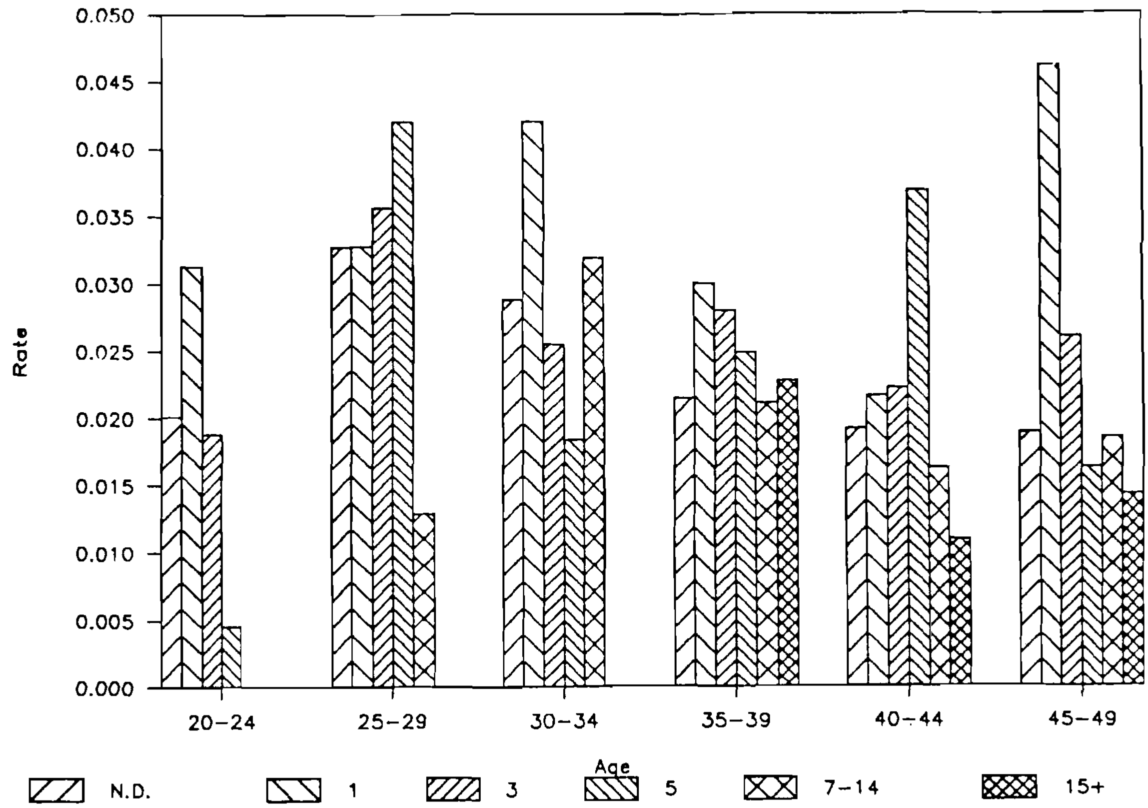


Figure 5. Second order divorce rates by age groups for selected duration categories: females, Hungary, 1980–1984.

Source: Hungarian 2% microcensus.

Table 1. Average number of years lived between age 15 and 50 by marital status, comparison of both approaches.

	Duration-Independent		Duration-Dependent		Difference	
Status	(1) Years	(2) %	(3) Years	(4) %	(5) = (1)-(3)	(6) = (5)/(1)
Single	8.36	24.32	8.36	24.32	0.00	0.00
Married 1	20.39	59.28	19.99	58.13	0.40	1.95
Widowed 1	1.10	3.19	1.05	3.07	0.04	3.80
Divorced 1	2.22	6.46	2.60	7.56	-0.38	-16.93
Married 2	1.86	5.40	1.91	5.55	-0.05	-2.75
Widowed 2	0.13	0.37	0.13	0.39	-0.01	-4.91
Divorced 2	0.34	0.99	0.34	0.99	0.00	0.27
Total	34.39	100.00	34.39	100.00	0.01	0.02

Source: Hungarian 2% microcensus and author's calculations.

Table 2. Life table stationary population by age and marital status, Hungary, 1980–1984.

	A. Duration Independent								B. Duration Dependent							
Age	Single	Mar.1	Widow1	Div.1	Mar.2	Widow2	Div.2	Total	Single	Mar.1	Widow1	Div.1	Mar.2	Widow2	Div.2	Total
15	96403	3563	1	12	1	0	0	99979	96403	3576	1	0	0	0	0	99979
16	89467	10335	3	124	9	0	0	99938	89467	10365	3	102	0	0	0	99938
17	83030	16508	7	318	35	0	0	99898	83030	16554	8	298	9	0	0	99898
18	77057	22186	19	514	84	0	0	99860	77057	22229	21	515	38	0	0	99860
19	71513	27312	39	796	163	0	0	99822	71513	27321	42	843	104	0	0	99822
20	61701	36453	65	1245	308	0	1	99775	61701	36440	71	1340	222	0	0	99774
21	48897	48245	103	1916	549	1	8	99718	48897	48249	112	2045	411	1	4	99718
22	38750	57170	148	2680	897	1	21	99667	38750	57260	161	2790	694	1	11	99666
23	30709	63968	206	3343	1355	3	35	99618	30709	64200	223	3376	1085	2	24	99618
24	24336	69069	274	3926	1893	5	70	99573	24336	69420	297	3904	1558	4	54	99572
25	20086	71941	345	4588	2407	7	138	99512	20086	72344	370	4582	2010	6	115	99512
26	17411	73187	414	5281	2913	10	220	99436	17411	73638	439	5295	2463	8	182	99436
27	15092	74129	481	5850	3498	13	300	99363	15092	73905	504	6602	3008	11	239	99362
28	13082	74784	564	6317	4148	19	377	99291	13082	73464	586	8176	3657	16	308	99289
29	11340	75248	664	6642	4852	26	449	99220	11340	73474	685	8864	4454	22	378	99217
30	10184	75193	777	6984	5419	34	532	99123	10184	73334	787	9135	5199	31	449	99119
31	9513	74644	904	7425	5821	44	649	99000	9513	72726	890	9473	5791	40	560	98993
32	8887	74069	1028	7820	6235	55	784	98878	8887	72090	985	9778	6344	51	734	98868
33	8302	73507	1192	8096	6670	70	920	98756	8302	71512	1113	9929	6900	66	922	98744
34	7755	72990	1393	8233	7135	90	1039	98635	7755	70970	1273	9963	7511	86	1063	98621
35	7371	72332	1591	8418	7521	111	1136	98480	7371	70277	1446	10095	7991	108	1176	98453
36	7135	71499	1786	8698	7791	132	1251	98291	7135	69419	1634	10354	8297	131	1303	98272
37	6907	70655	1975	8973	8066	154	1374	98103	6907	68563	1816	10610	8592	154	1439	98081
38	6686	69378	2626	9194	8328	233	1470	97914	6686	67304	2446	10809	8855	237	1553	97890
39	6472	67764	3720	9284	8554	369	1563	97726	6472	65726	3502	10879	9095	381	1645	97699
40	6312	66215	4756	9331	8687	505	1681	97486	6312	64211	4508	10902	9258	525	1742	97457
41	6203	64626	5731	9444	8751	638	1803	97195	6203	62652	5469	10993	9339	668	1840	97163
42	6096	63044	6659	9575	8838	769	1922	96902	6096	61117	6385	11096	9428	810	1936	96867
43	5990	61464	7597	9671	8949	907	2029	96606	5990	59589	7311	11172	9513	960	2034	96570
44	5887	59901	8544	9721	9089	1053	2115	96310	5887	58080	8248	11212	9600	1116	2127	96270
45	5779	58392	9451	9713	9201	1199	2205	95939	5779	56632	9133	11168	9704	1271	2210	95897
46	5668	56941	10321	9645	9272	1344	2305	95496	5668	55231	9968	11055	9812	1423	2295	95451
47	5559	55540	11147	9560	9347	1488	2414	95053	5559	53868	10766	10945	9897	1575	2396	95005
48	5452	54034	12055	9486	9398	1653	2533	94611	5452	52423	11648	10835	9934	1748	2521	94561
49	5347	52480	13039	9373	9456	1836	2638	94169	5347	50943	12609	10681	9963	1942	2632	94116

Source: Hungarian 2% microcensus and author's calculations.

ing for the first time at ages 20, 25, and 30. It is clear that for a given amount of time spent since the divorce, retention probabilities are higher for women divorcing at higher ages, reflecting the decrease in remarriage rates as age increases. More interesting is the cross-over effect in the retention probabilities according to the two approaches. Because remarriage rates are higher for recently divorced women, the duration-dependent approach shows lower retention probabilities at lower duration categories than its duration-independent counterpart. That occurs before duration category 8 for women getting di-

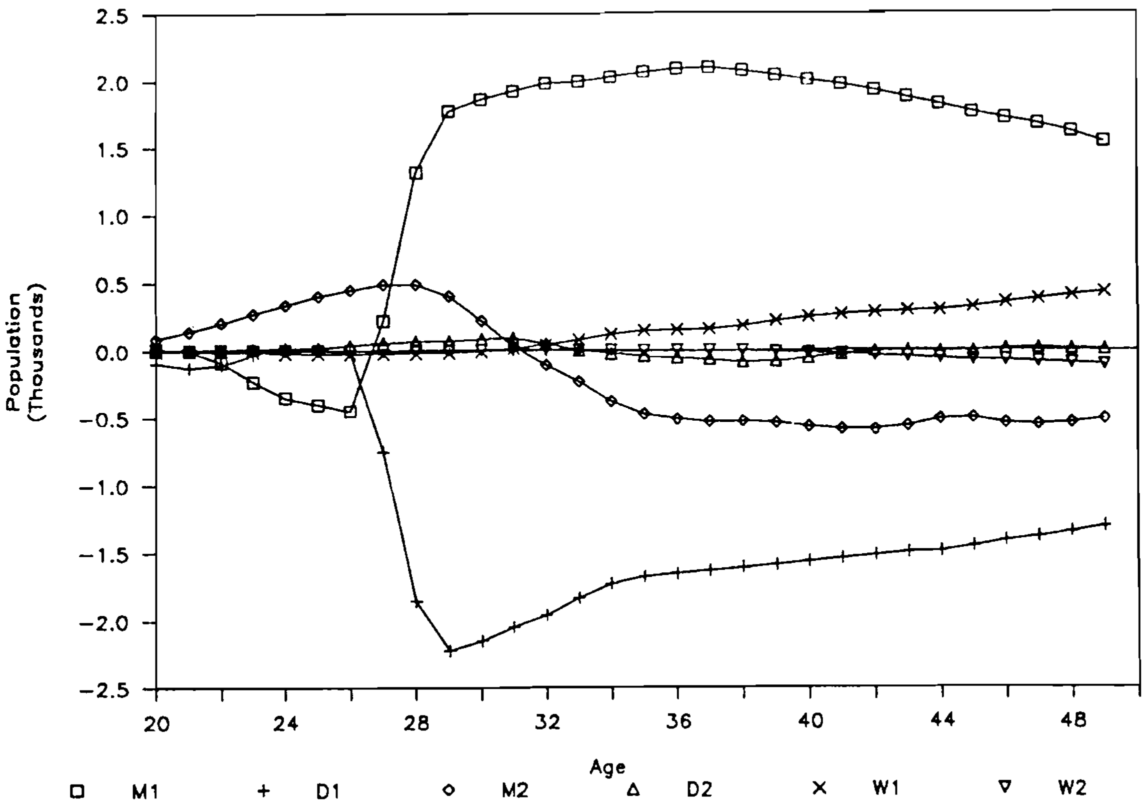


Figure 6. Differences between the duration-independent and duration-dependent stationary populations: females, Hungary, 1980–1984.

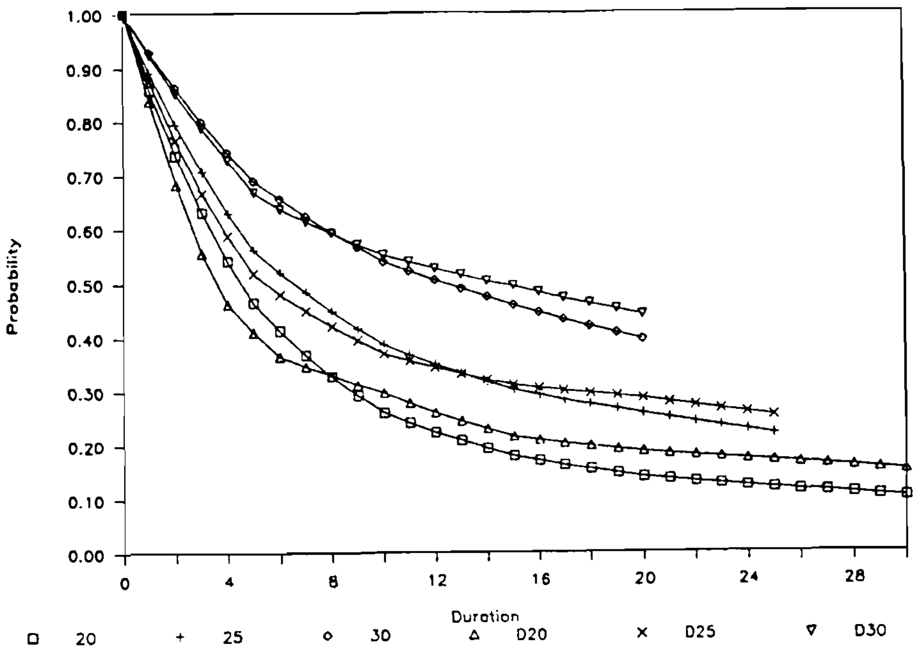
Source: Hungarian 2% microcensus and author's calculations.

vorced at age 20 and 30, and duration category 12 for those divorcing at age 25. The duration-independent approach ultimately overestimates the number of transitions out of the divorced status for less recent divorcees and creates the cross-over effect.

Retention probabilities for women getting remarried at age 25 and 30 are presented in Figure 7b. Notice that these retention probabilities do not exhibit any cross-over effect, but they do exhibit relatively larger differences between the two approaches. Notice also that in the duration-independent approach, retention probabilities of women getting remarried at age 30 are much higher than those of women getting remarried at age 25, while in the duration-dependent approach they are almost identical.

Although most marital status-based measures of retention of the duration-independent approach can be compared with the corresponding duration-dependent retention measures, there are some indicators specific to the duration-dependent approach that have no equivalencies in the conventional multistate life table. For example, the effect of the time spent in the current status on the retention probabilities can be only analyzed from a duration-dependent perspective. Table 3 presents the proportion of women surviving in the status occupied at age 30, according to the time already spent in that status at this initial age. For example, the proportion of women surviving in their first marriage at age 40, who got married at age 30 ($d = 0$) is 80.6 percent. This proportion rises to 83.6 percent for those who at age 30 were married since five years ($d = 5$), and 84.6 percent for those who were in this status since 10 years ($d = 10$).

a) Marital Status: First Divorce



b) Marital Status: Second Marriage

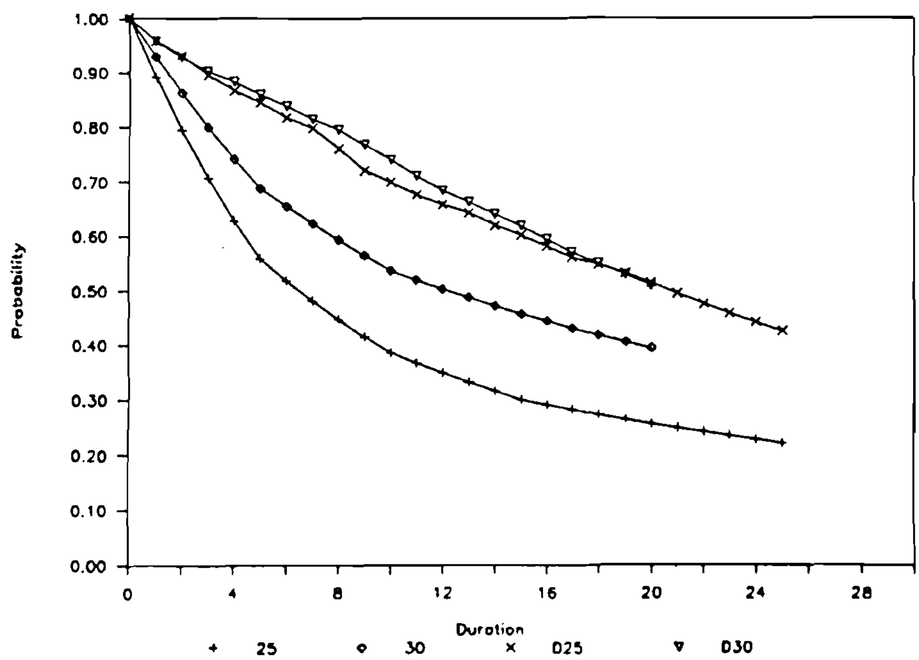


Figure 7. Comparison of retention probabilities by duration categories: females, Hungary, 1980–1984.

Source: Hungarian 2% microcensus and author's calculations.

Table 3. Retention proportions between age 35 and 50 for different duration in the initial status (age = 30), females, Hungary, 1980–1984.

	Initial Status								
	Married			Divorced			Remarried		
Age	d=0	d=5	d=10	d=0	d=5	d=10	d=0	d=5	d=10
35	92.37	91.86	92.59	66.73	69.93	71.94	85.25	82.40	84.06
40	80.61	83.56	84.55	54.99	58.25	62.59	73.33	70.60	71.01
45	70.61	73.42	74.30	49.32	53.44	57.55	61.21	60.51	60.87
50	61.54	64.02	64.77	43.35	47.59	51.08	50.50	49.78	50.72

Source: Hungarian 2% microcensus and author's calculations.

For women in their first marriage, the difference in the retention proportions over time (i.e., at ages 35, 40, 45, and 50) between those in duration category 5 and those in duration category 10 is relatively small compared to the difference between women in duration category 5 and those just married ($d = 0$). The proportion surviving in the status divorced also increases with the time already spent in the status at age 30, but the differences between duration category 0 and 5 are almost the same as those between duration 5 and duration 10. Finally, from the last three columns, we learn that for second marriage, the time spent in the current status at age 30 has a much smaller effect on the retention proportions at older ages.

Another indicator specific to duration-dependent multistate life tables is the average duration in the current status. Assuming linearity, women in duration category d have spent on average $d + 0.5$ years in their current status. Thus, average duration in the current status can be calculated as:

$$\bar{d}_i(x) = 1/2\ l_{i,0}(x) + 3/2\ l_{i,1}(x) + \cdots + (2x-1)/2\ l_{i,x-1}(x) + x l_{i,x}(x) \quad . \tag{14}$$

Figure 8 presents the average duration in the current status for both first and second order marriages, and divorces. The general pattern for all statuses is that average duration increases slowly at younger ages and faster later in the life course. Looking at first marriage for example, for each single year of age the average duration increases by less than half a year before age 30 and by close to a full year around age 50. The average durations of other statuses behave in a similar fashion, except that the turning point occurs later in life.

Conclusion

Using Wolf's derivation (1987), and given the year of birth and the year of last event, it is possible to calculate a multistate life table with duration dependence. This has been done for marital dissolutions of Hungarian women, using data from the 1984 census. The results were compared with a duration-independent approach. The calculation of occurrence/exposure transition probabilities by duration categories is not much more complicated than calculating their duration-independent counterparts, but it necessitates a larger sample size in order to avoid estimations based on small numbers.

The average number of years lived between ages 15 and 50 by marital status, a measure comparable to the life expectancy, was found to be not very sensitive to the introduction of duration categories. However, the analysis of the age structure of the stationary population showed that introducing duration does have an important impact on the transitions from married to divorced statuses for those between ages 25 and 29, and on the transitions from divorce to remarriage for the next age group.

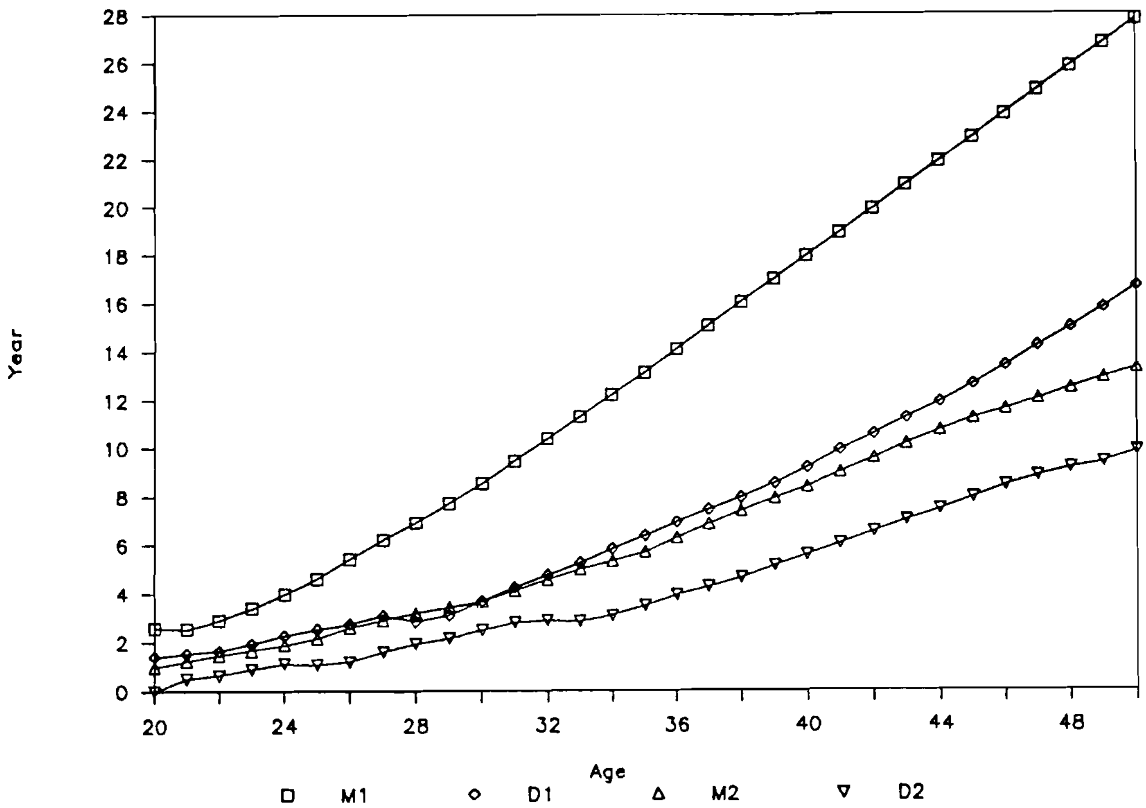


Figure 8. Average duration in the current status by age: females, Hungary, 1980–1984.
Source: Hungarian 2% microcensus and author's calculations.

Moreover, some indicators are specific to the duration approach. In addition to the average duration in the current status, only duration-dependent multistate life tables allow one to contrast retention probabilities between age x and age $x+t$ for different duration categories at the initial age.

Following Henry's seminal study (1952) of divorce, the French (classical) approach to marital dissolution has favored the duration index over the age index. One foreseeable consequence of the introduction of duration-dependence in multistate life tables, is the possible reconciliation between the French classical approach to marital dissolution and the multistate model more widely accepted in America.

Finally, given that the required data are available, duration-dependent life tables can be easily applied to the analysis of all other types of transitions where duration is known to have a significant impact. An analysis similar to the one presented here, but applied to migration, would have the additional advantage of testing the validity of the Markovian assumption in the original field where multistate analysis was first developed.

Appendix

Allocation of occurrences and exposures in an age-duration time-space

Introducing duration complicates the allocation of time between statuses now characterized by age and duration. This allocation has to be seen in a three-dimensional space where time, age, and duration are related. Figure A.1 shows a representation of it based on the juxtaposition of two Lexis diagrams. The first one represents the familiar relation between time and age, and the second one shows the relation between age and duration.

The interaction of time, age, and duration results in a three dimensional space where age as well as duration in the current status both change. For individuals who don't experience any change in their current status during a given year, this age-duration space can be divided into four "sub-spaces":

- region ABCa + region ACDa where their age is x years and their status is in duration category d;
- region BCab where their age is still x, but their status is now in duration category $d+1$;
- region DCad where their age is now $x+1$, but their status is in duration category d;
- region DaCc + region Cabc where their age is $x+1$ and their status is in duration category $d+1$.

From geometry we see that the volume of any part of the space ABCDabcd, characterized by four points, is equal to the determinant of a 4 by 4 matrix where each line has the coordinates of each point along the three axes in the first three columns. The fourth column is a vector of ones. For example, the volume of ABCa can be calculated as:

$$\text{Volume of ABCa} = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{a} \end{array} \begin{array}{c} \begin{array}{cccc} \text{X} & \text{Y} & \text{Z} & \end{array} \\ \begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} \end{array} = 1/6\ .$$

Assuming a linear distribution of events over age and duration categories, occurrences and time exposures can be allocated as shown in Table A.1.

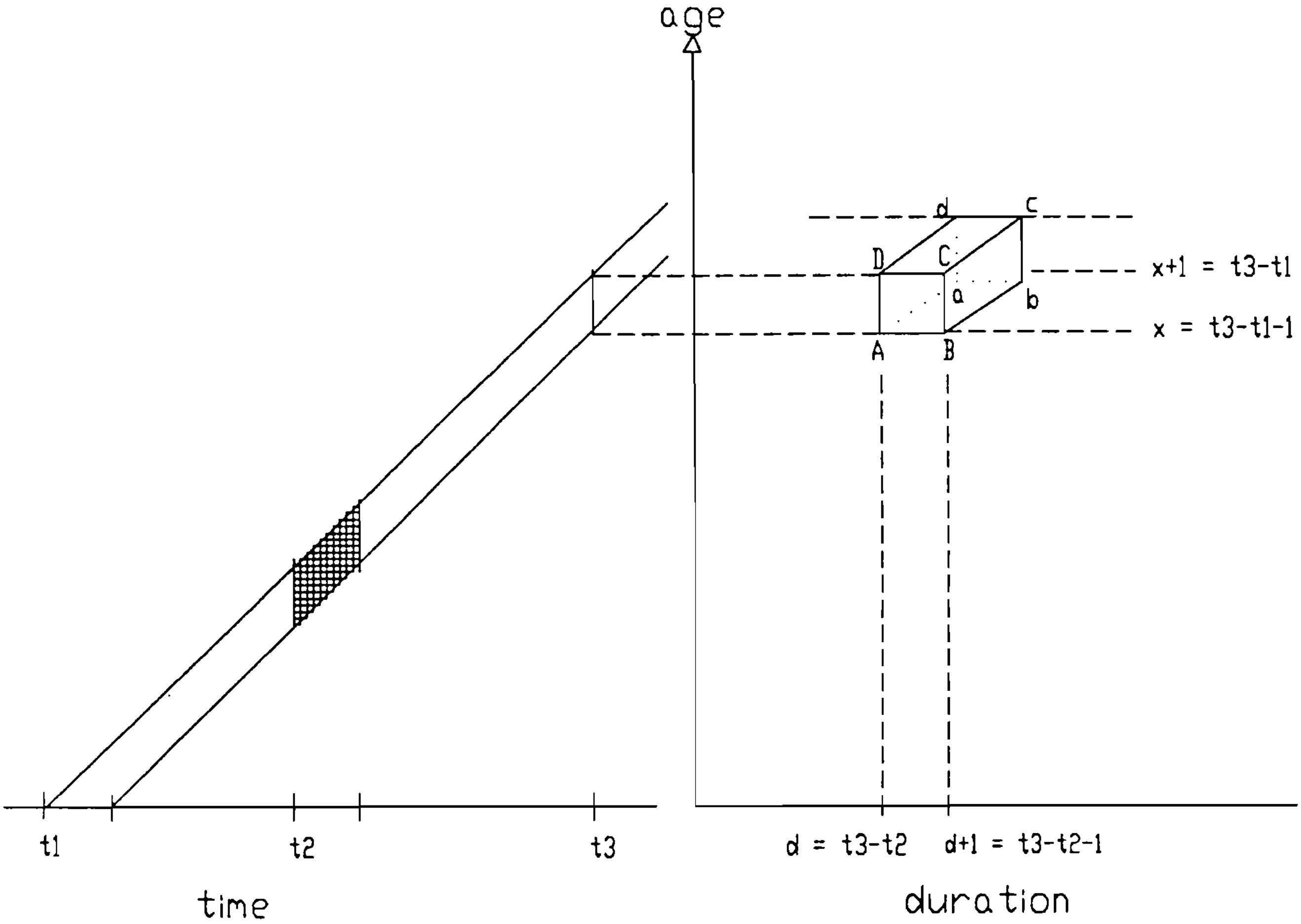


Figure A.1 A Lexis diagram representation of the age-duration space through time.

Table A.1 Allocation of occurrence and exposure by age and duration categories.

Number of events occurring during the year	a,d	(a+1),d	a,(d+1)	(a+1),(d+1)	Total
A. Occurrence Allocation					
1 Event	0.333	0.167	0.167	0.333	1
2 Events					
First Occurrence	0.584	0.166	0.166	0.084	1
Second Occurrence*	0.250	0.750	0.000	0.000	1
B. Exposures Allocation					
No Event	0.333	0.167	0.167	0.333	1
1 Event					
First Status	0.292	0.083	0.083	0.042	0.5
Second Status*	0.125	0.375	0.000	0.000	0.5
2 Events					
First Status	0.235	0.043	0.043	0.012	0.333
Second Status*	0.167	0.167	0.000	0.000	0.333
Third Status*	0.056	0.277	0.000	0.000	0.333

Notes: "a" stands for age, "d" stands for duration.
* duration = 0.

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