

THE TRAGEDY OF STRATEGY

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THE TRAGEDY OF STRAGEDY\*

by Jean-Pierre Ponssard, Mark Thompson,  
and Norman Glass

Norman Glass: We intend today to make an introductory description of one important area of systems analysis. That area is game theory.

Mark Thompson: Many people believe that game theory is an analytic activity designed to figure out how best to beat your girlfriend at chess, at backgammon, at go, or at rummy.

N.G.: But they are wrong; game theory is much more. To be sure, an initial success of game theory was development of a strategy which showed how poker should be played. But game theory is important also in analysing a wide variety of situations involving confrontation and cooperation. These include politics, economics, management, warfare and diplomacy.

M.T.: We shall now present a small number of sketches designed to illustrate important concepts in game theory.

N.G.: Perhaps the simplest game of all is one you may have played as children.

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\* This working paper is introduced as a draft for one teaching tool in a series to be developed by IIASA. The skits it comprises were first worked out and given by Ponssard and Thompson in the spring of 1974. In spite of popular demand, Glass and Thompson gave a rerun of the performance in the spring of 1975. The version given by Glass and Thompson is here transcribed.

M.T.: Suppose that we each cover coins on our wrists. Simultaneously, we uncover our coins.

N.G.: If the coins match--either both heads or both tails--then I win Mark's coin.

M.T.: If they do not match, then I win Norman's coin.

N.G.: Why don't we play?

(They play twice. The winner of each game happily pockets the coin of the other; the loser looks very unhappy.)

M.T.: My objective is to turn up the coins to the opposite side of Norman's.

N.G.: My objective is to turn up the coins to match Mark's.

M.T.: Because these objectives are mutually contradictory, there is no simple strategy that would be successful for either one of us.

N.G.: Suppose, for instance, that I have a wonderful rule which tells me what to do. Maybe I should examine carefully the record of the last ten times we played and calculate what to do.

(He thinks hard and sets his coin.)

M.T.: Now if there were such a rule which would tell Norman exactly how to set his coin, all I would have to do is follow exactly the same rule completely to the end of Norman's thought process and then turn up my coin the opposite to the side Norman would turn up his.

(Thompson does this, wins the game, and takes Norman's coin.)

N.G.: The problem of this game is to guess what the opponent is thinking.

M.T.: For instance, I might think that Norman thinks that I will do one thing. But if I think that he thinks that, then I should do the opposite. In this case, I win.

N.G.: On the other hand, if I think that Mark thinks that I will think that, then I can fool him by thinking the opposite. In this case, I win.

M.T.: The point of this is simple: the person who can better guess the mind of the other will win.

N.G.: But if you are afraid that your opponent can guess your mind better than you can read his mind, then there is only one way to make sure that you are winning your fair share of the time.

M.T.: I think I see. I must know that he is thinking that I am thinking that he is thinking that I am thinking that he is thinking and .....

N.G.: Wrong. The only way is to randomize. If I flip a coin to decide how to lay it on my wrist, I will always win my fair share of the time.

M.T.: This is called randomization of strategy. In a game such as this, the better guesser may try to anticipate how his opponent will play. If however, the opponent is the better guesser, then the safest strategy is randomization.

N.G.: The technical term for this situation is a zero-sum game. This name comes from the fact that whatever Mark wins, I lose and vice-versa.

M.T.: This can be seen in diagrammatic form on Diagram 1. Charts of this nature are the identifying badge of game theorists. If you happen to see anyone at IIASA studying such charts, the chances are that he is a game theorist. Now we see on the upper left-hand corner Norman's perspective of the game. If Norman himself plays heads and I play heads, then Norman gains unit. If Norman plays heads and I play tails, we see in the corner at the lower left-hand of the box that Norman loses one coin. My view, we see, is exactly the opposite--the negative--of Norman's view. When both players play heads, for instance, I get a minus one which is the negative of Norman's reward of plus one for this game.

N.G.: Game theorists define the value of the game as the average results if both players play optimally. In this case, optimal randomized play by both players makes each player equally likely to win. The value of this game is therefore zero.

M.T.: Although the coin-flipping game is extremely simple, the structure of this game can be seen in other situations.

N.G.: For instance, a situation that may occur in real life is the following which is depicted in Diagram 2. It happens that Mark has lent me 2000 schillings and I am about to leave IIASA to return to my work in London.

# THE SIMPLEST GAME

## NORMAN'S VIEW

SELF

heads tails

heads	1	-1
tails	-1	1

heads

tails

MARK



## MARK'S VIEW

SELF

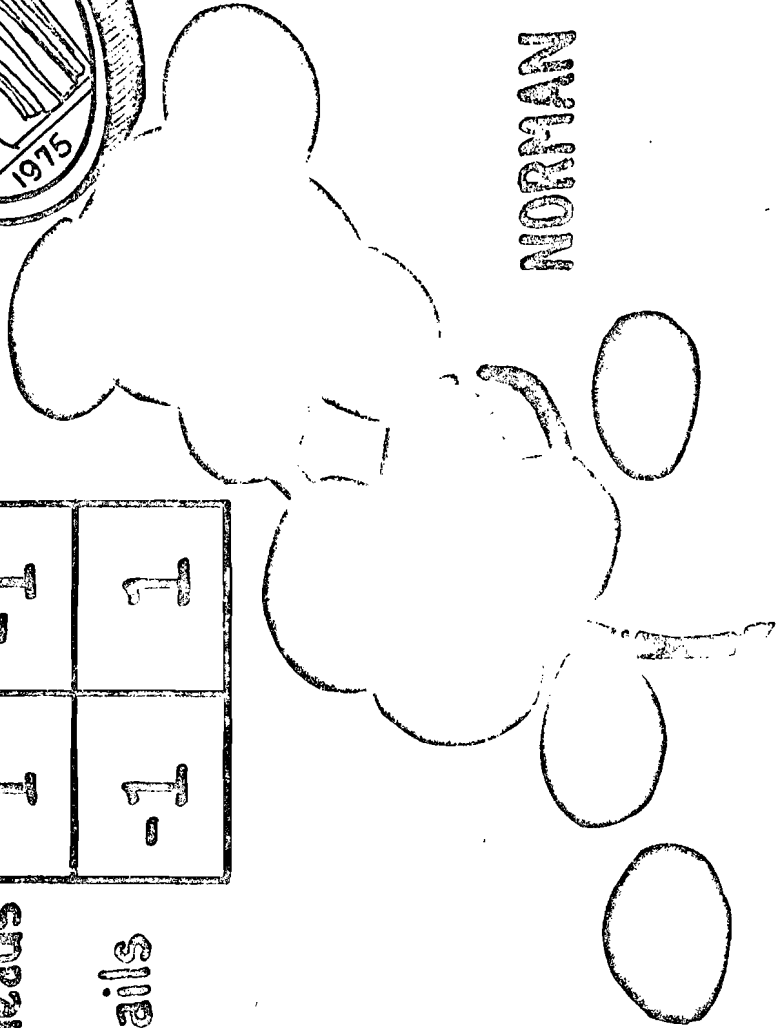
heads tails

heads	-1	1
tails	1	-1

heads

tails

NORMAN



M.T.: I know that Norman is leaving town today. I know also that he will leave either by train or by plane. Unfortunately, both the train and the plane leave at the same time this evening. My problem is to go either to the airport or to the train station and hope to find Norman. If I find him, I will get the 2000 schillings which he owes me.

N.G.: You may be able to see that the structure of this game is exactly the same as the game with the coin flipping. Mark is trying to match me: to go to whichever place I am leaving from. I am trying to do exactly the opposite by avoiding him. What then is the optimal strategy for this game?

M.T.: According to that what we said earlier, I should try to guess where you are going to go. If I think that I have more than half a chance to guess where you are going, then I should simply go where I think you are going.

N.G.: My strategy is the inverse. If I think that I have better than half a chance to guess where you are going, then I should guess and go to the other place--either the train station or the airport.

M.T.: But if I think that you can figure out my thinking better than I can figure out yours, then I should flip a coin to decide where to go.

N.G.: Right. Flipping a coin is the only way that either one of us can be certain of having half a chance of winning the game.



THE  
ZERO-SUM  
GAME



GLASS

airport	station
---------	---------

2000 AS	0
0	2000 AS

THOMPSON

airport
station

Diagram 2.

M.T.: I think I see. The only way I can assure myself half a chance of catching you is by flipping a coin to decide whether to go to the airport or to the train station.

N.G.: Exactly. Now, what is the value of the game?

M.T.: If we both play optimally, I will have a one half chance of catching you and getting from you the 2000 schillings which you owe me. Therefore the value of the game is half of the 2000 schillings, or 3000 schillings.

N.G.: Surprisingly enough, you are right.

M.T.: Thanks a lot.

N.G.: Don't mention it. Now why is this a zero-sum game?

M.T.: Because what I win, you lose and vice-versa.

N.G.: You are doing so well that we can move on to the next game situation which took place when you and I were operating filling stations a few years ago. Perhaps you can explain the situation.

M.T.: Well, each of us had a filling station across the road from the other. The problem was whether or not we should each day set the high price for gasoline or set the low price. If we set the high price for gasoline, we would make a profit of 5 schillings per car.

N.G.: And when we sell gasoline for the lower price we make a profit of 3 schillings per car.

# THE NON-ZERO-SUM GAME

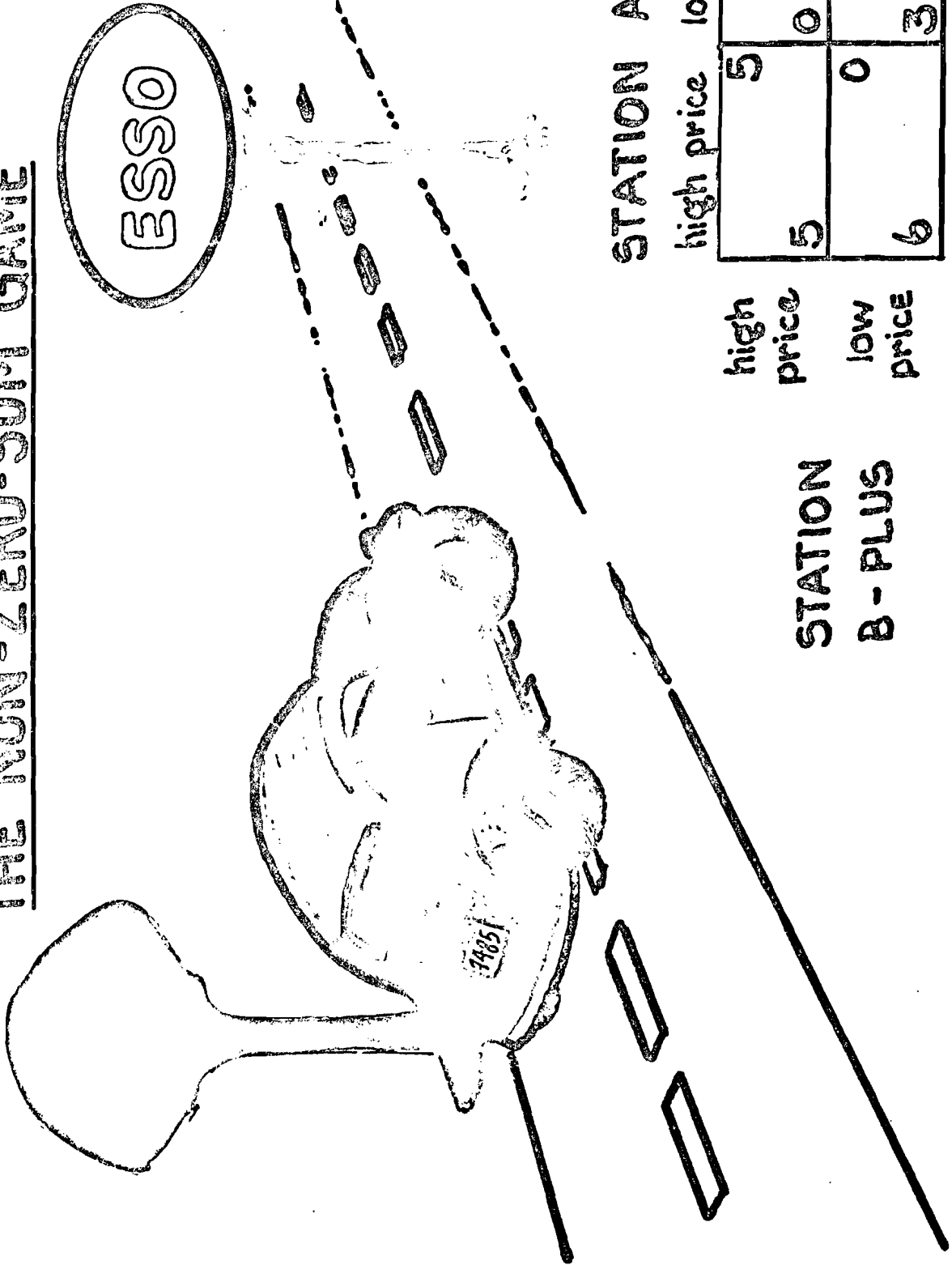


Diagram 3.

M.T.: A daily average of 2000 cars came by the road looking for gasoline. The problem was to make the highest possible daily profit.

N.G.: That's right. We also had to specify simultaneously at midnight whether for the next day we would charge the high price or the low price for our gasoline.

M.T.: If we both charge the high price or if we both charge the low price, then each of us would end up with 1000 cars for the day. At the high price, that would be 5000 schillings which we show here on Diagram 3. If we both charge the low price, we each get 3000 schillings which we see in the lower right hand corner of Diagram 3.

N.G.: But, if one of us is charging the low price and the other one is charging the high price, all the cars will go to the station charging the low price.

M.T.: That is right. In that case, that person will make 6000 schillings that day. His competitor will not make any money at all since no car will come to pay the higher price.

N.G.: I think it is very easy for me to discover the optimal strategy in this game.

M.T.: Really? What is the optimal strategy?

N.G.: Well, look. Here I am at station a-minus. I must choose between charging the high price or charging the low price. Let me assume first that you charge the high price. Now if I also charge the high price, we share the cars and I get 5000 schillings for the

day. But if I charge the low price, then I get all the cars. You didn't get any of the cars and I get 6000 schillings for the day.

M.T.: So in this case, you do better to charge the low price.

N.G.: That is right. Now suppose that you charge the low price. If I also charge the low price, I get 3000 schillings otherwise, I get nothing. Whether you charge the high price or whether you charge the low price, I always do better myself to charge the low price. Therefore I am going to charge the low price. Pretty smart, huh?

M.T.: (Looking at the audience)

Now isn't this ridiculous? Suppose I were to follow the same train of thought that Norman has just followed. You can see that the game is exactly symmetric, so that if I were to follow his process of deduction, I would end up also setting the low price. Just look at the result. If we both charge the low price, we'll both get 3000 schillings each day. If we were to play the game only once, then he would be right. In the long run, however, this is clearly stupid. If Norman was just a little bit smarter, he would see that we would do much better by both charging the high price, each getting 5000 schillings each day, rather than both setting the low price. Somehow I will have to get this through his thick skull.

N.G.: I suppose the only way we are ever going to resolve this argument is by playing the game. Shall we play?

M.T.: By all means.





The last few games, ever since the first, my reward has been 6, 3, 6, 3, 6, 3. He seems to be planning to continue this strategy trying to persuade me to play high. Let me analyze this. When I'm getting 6, 3, 6, 3, 6, 3; as an average I receive  $4 \frac{1}{2}$ . If I agreed with his strategy and decided to play high, then I would get five every time. Since 5 is higher than  $4 \frac{1}{2}$ , I think that I will decide to play high.

M.T.: So finally we achieve the best outcome. Let us play.

Play 8:    NG { plays high }                    MT { plays high }  
              { gets 5000 AS }                    { gets 5000 AS }

M.T.: This then is the long-term stable solution of the game. It may take a shorter or a longer amount of time to arrive at the stable best solution—depending upon the intelligence of the person operating the other gas station.

N.G.: One thing you should notice here is that this situation is similar to price fixing in many industries that are regulated by anti-trust laws. Anti-trust laws usually have the provision that two or more companies may not talk directly to each other in setting prices.

M.T.: That is right. But the problem in this situation we can easily see. It was not necessary for us to talk to each other. All the communication that we needed to signal to one another could be completely transmitted by the prices that we charged each day.

N.G.: That is right. That is probably why anti-trust laws are not very successful in preventing the consumer from being conspired against by large companies. We see also why this is



a non-zero-sum game. When both of us play high instead of both low, we both gain. The only loser is the consumer and he is not in the game.

M.T.: I fear that what you say is true.

N.G.: Well, I'm sorry to say what happened next.

M.T.: Me too.

N.G.: We agreed to have a special meeting to see if we could raise the price even higher.

M.T.: Unfortunately, we were caught as we met secretly trying to raise the price of gasoline still higher and as a result you see us now in our next game's situation which is shown in Diagram 4. Could you explain this, Norman?

N.G.: As the most cruel punishment of all, they have put us in a very special prison with no other prisoners but us two.

M.T.: What is special about this prison?

N.G.: We have been able to show this by reconstructing the prison in this room today. You see here, we have a button which opens the only door to the prison. It is the door over there. (Indicates a door.) Unfortunately, the button operates the door only briefly so that, when one of us pushes the button and then tries to run to the door, the door is shut before he can reach it.

M.T.: The structure of this game is shown in the table at the bottom of the diagram. The person who pushes the button for the

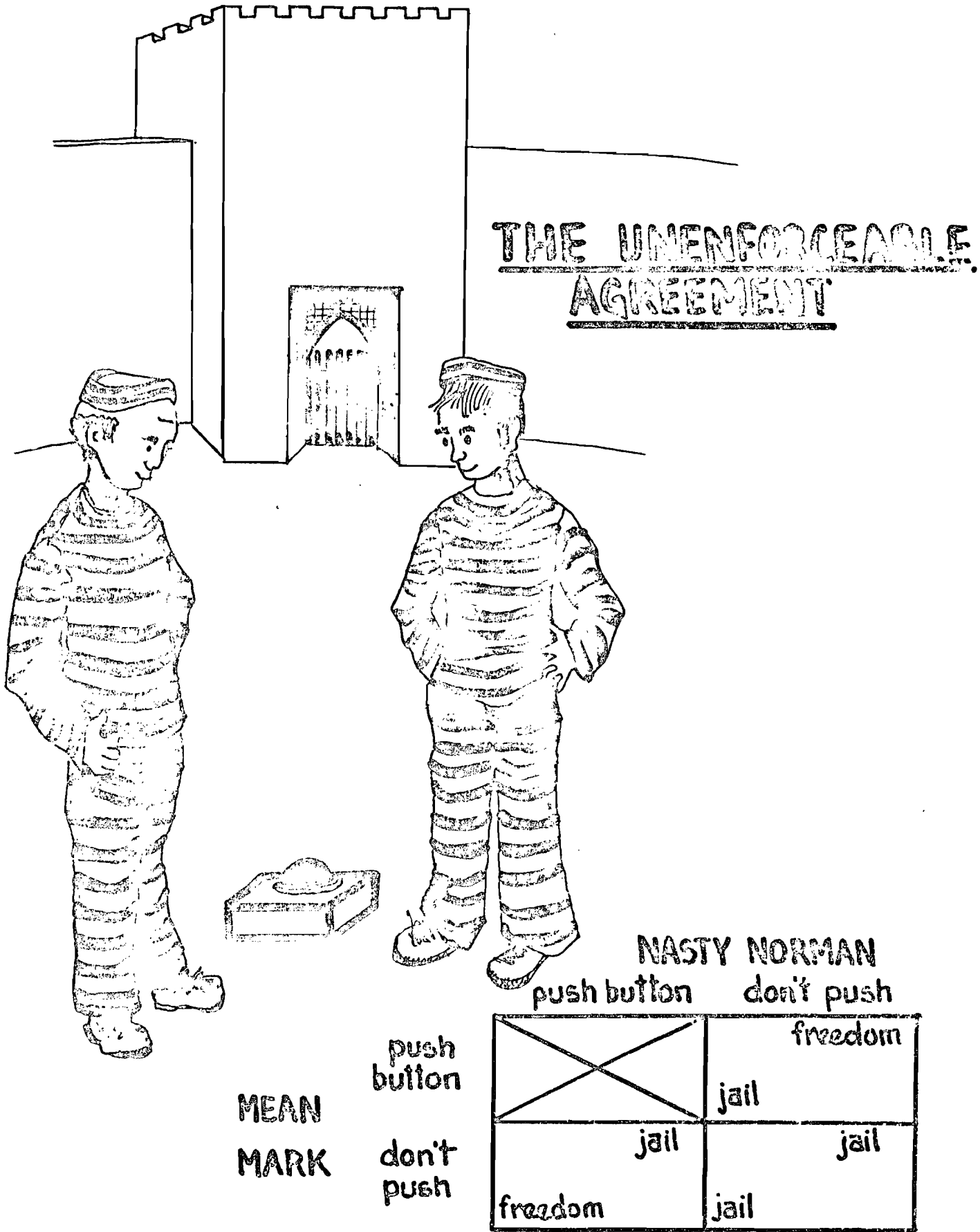


Diagram 4.

other to go free remains in jail. If neither of us pushes the button, then we both remain in jail. In this case I see a clear solution to the game.

N.G.: What is that?

M.T.: It is obvious that you should push the button and that I should go forward to freedom.

N.G.: That solution is not so clear to me.

M.T.: What could you possibly mean?

N.G.: It seems to me that the better solution is for you to push the button and for me to go to freedom.

M.T.: I think that you will find that, when you use completely objective analysis, the solution you propose is a purely selfish one. It is a suboptimal analysis.

N.G.: I could say the same thing about your analysis.

M.T.: Please don't. I can see that we are not going to get anywhere at this rate. Fortunately for us, I am a student of game theory.

N.G.: Why is that fortunate?

M.T.: Well, I have devised the perfect solution that will give each of us a one half chance of escaping to freedom.

N.G.: This isn't another of those randomized strategies, is it?

M.T.: Exactly right. What we must do is form a contract by which we agree to flip a coin. The person who wins the coin

flip gets to leave the prison, the person who loses the coin flip will push the button for the other to leave. Does that make sense?

N.G.: That makes a lot of sense. It gives each of us half a chance to leave. That's better than spending the rest of my life with you. Let's do it.

(They flip the coin, and Glass wins.)

N.G.: Aha, tails it is. Here I go. I can hardly wait. Just, ah, you know, wait till I get to the door, then push the button. (Thompson is looking rather unhappy, Glass gets to the door.)

N.G.: Okay, any time, Mark, any time, open the door.

M.T.: Hm, ha, eh, what are you doing, Norman?

N.G.: Mark, push the button please. Just push the button.

M.T.: Take your time now, we have lots of time. You especially have lots of time. You are going free.

N.G.: Mark, could you please push the button now? I am ready.

M.T.: Hm, Norman, could I ask you one question?

N.G.: What is that?

M.T.: What is in it for me to push the button for you to go to freedom?

N.G.: It's not a question of what's in it for you. We made an agreement and you will now push the button for me so that I can go to freedom. What was in it for you was you had half a chance of my pushing the button for you.

M.T.: Yes, that was what was in it for me, that was a very good agreement until we flipped the coin. But now I don't see any reason for me to push the button for you to go to freedom.

N.G.: I'm sorry Mark, we made an agreement, we must honor our agreement. It's up to you to push the button now.

M.T.: I don't think it was a very good agreement. Why don't we do it again and if you win next time, I promise, really truly, to push the button for you.

N.G.: I don't think that's a very good idea.

M.T.: Well then, I'm not going to push the button.

N.G.: Do you know what we have discovered?

M.T.: What is that?

N.G.: We have here the situation of the unenforceable agreement.

M.T.: I see, the agreement itself is attractive. It is much better than staying in prison with each other, if it could only be enforced. The only problem is that, once we decide how the agreement is to be fulfilled, it is not in the interest of one of us to fulfill it.

N.G.: That's exactly right. That is why this is an unenforceable agreement.

M.T.: So, as you can see, the situation looks very different depending on whether or not we can make enforceable agreements. That is why game theorists specify in the rules if their games are cooperative or non-cooperative. Now, since we play this game non-coopera-

tively (no agreements are enforceable among outlaws), we will both stay in prison.

N.G.: That is true. Fortunately, however, on a special occasion in the nation--the birthday of Professor Afifi--a general amnesty was declared and we both were released from prison.

M.T.: And where did they send us for rehabilitation?

N.G.: Why, where else but the International Institute for Applied Systems Analysis in Laxenburg, Austria.

M.T.: Ah, yes, I remember, it did not take us long at all to learn the ways of the natives.

N.G.: That is right.

M.T.: You see, I had a very good situation worked out at Laxenburg. There was this very nice lady who understood my charms, understood my wit, and her name was Mrs. Norman Glass. Why there she is now. Hello, Mrs. Glass. Did we not have a wonderful situation?

(Glass has put a kerchief on his hair and is acting effeminate.)

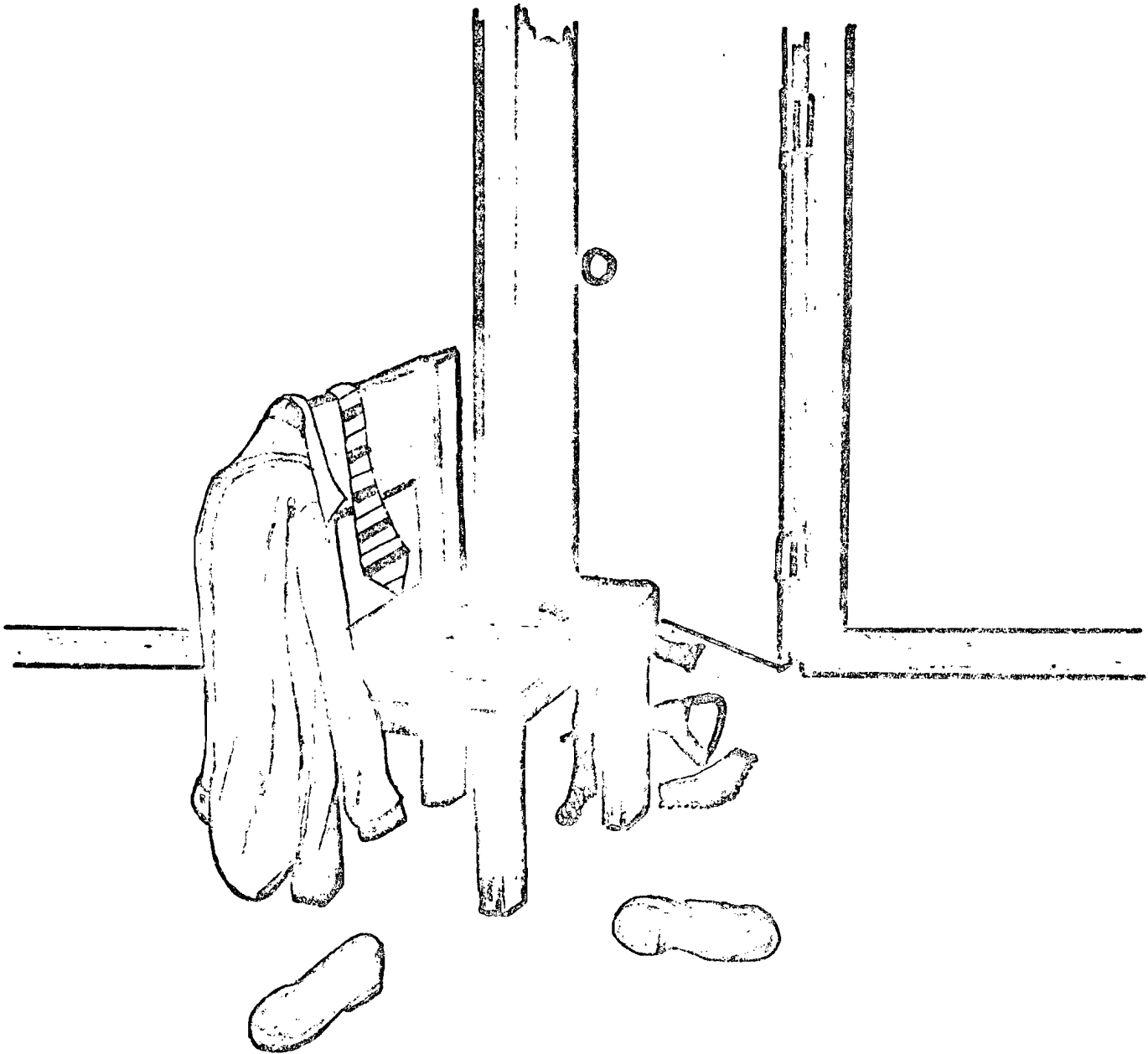
Mrs. N.G.: Well, I would not say it was so wonderful.

M.T.: As I said, it was a perfect situation but then something terrible happened. Mrs. Glass decided to leave me for Les Garner.

Mrs. N.G.: (sighs) I wanted a man, I wanted a real man.

M.T.: But, unfortunately for me, I had studied game theory and was able to come up with the right solution for the situation which we see here drawn in the fifth of our charts. Now,

# THE THREAT



MRS. G.

return to T. not return

HE = HAPPY ENOUGH

MH = MOST HAPPY

UH = UNHAPPY

VUH = VERY UNHAPPY

T.

tell

not  
tell

	VUH	VUH
VUH		VUH
	HE	MH
MH		UH

Diagram 5.

Mrs. Glass, listen to me very carefully.

Mrs. N.G.: I'm listening, what do you want?

M.T.: You have two choices. You can return to me or you can remain with that ridiculous Mr. Garner. Should you choose to return to me, we will both live happily ever after and, as we can see on Diagram 5, I will be most happy and you will be happy enough. Should you decide not to return to me, then I shall tell Norman Glass. He will be very unhappy with you, he will tell my wife and she will be unhappy with me. Both of us will be very unhappy as we see in the upper right-hand corner and the upper left-hand corner of the chart.

Mrs. N.G.: Ah, that would not be very good, would it?

M.T.: No. So you see that, since I have made an irrevocable threat, the game situation is reduced to only two boxes: the lower left-hand corner where you can be happy enough by returning to me; and the upper right-hand corner where, by not returning to me, you must become very unhappy when Norman finds out what you have done. So what is your choice?

Mrs. N.G.: Let me think about this a minute. Yes, I have a better solution.

M.T.: What is that?

Mrs. N.G.: Listen to me very carefully. I have decided absolutely, positively, definitely not to return to you.

M.T.: What?



Mrs. N.G.: Let me explain. You see, by my stating definitely that I will not return to you, the game is reduced only to the two boxes are on the right-hand side of the chart. Since I have made my definite decision not to return to you, you have now only the choice of whether or not you want to tell my husband. If you tell my husband, your wife will find out and you will be very unhappy. But, if you are wiser and do not tell Norman, then I will be most happy and you will be unhappy. Therefore I am certain that you will not tell Norman if I do not return to you.

M.T.: That is completely right.

Mrs. N.G.: I told you so.

M.T.: Except for one thing.

Mrs. N.G.: What is that?

M.T.: What you have forgotten is when I made my threat to you all possibility of your not returning and my not telling Norman of your foolishness was eliminated. The lower right-hand box no longer existed as a possibility for the game. The only two boxes left in the game as result of my threat were the lower left-hand box and the upper right-hand box.

Mrs. N.G.: I think I see. Because you have made your definite threat before I made my definite statement, I no longer had a chance to make my definite statement.

M.T.: That is right. This is the essence of the threat situation: he who makes the first firm threat or definite statement is the winner. You never had a chance to threaten point-blank not to

return to me because, by that time, I had already made my threat. Because I had made my threat, you had only the choice to be very unhappy or happy enough. Since you had only that choice, I now ask you, Dear, which do you choose?

Mrs. N.G.: Ah, it is terrible, but I fear that I have no other choice but to return to you.

M.T.: I can assure you that you'll never regret it.

Mrs. N.G.: I regret it already.

M.T.: But, after a year or two of this game, Norman and I tired of the sport and took up boating.

(Norman removes female costume. Both players put on boating hats.)

N.G.: In the course of our boating, we have learned that there is a very large treasure--one million dollars--on a nearby island. Each of us has the chance to take his boat to this island and to carry away some of the treasure.

M.T.: I have a very large boat that can carry away all of the treasure.

N.G.: My boat is much nicer but, alas, not so large and can carry way only one-half of the treasure.

M.T.: The main problem for me is that every boat must be manned by at least two people.

N.G.: So, it is necessary for us to get together--in technical terms, to form a coalition--if we are to get any of the treasure

at all. And we can see here in the rewards listed on Diagram 6 what the payoffs to various coalitions will be. If either of us tries to go to get the treasure alone, the boat will sink and he will come back with no treasure. Someone who does not have a boat--let us call him Boatless--cannot possibly get the treasure. If, however, we decide to get together, we can have a total reward to be split between the two of us of one million dollars.

M.T.: Yes, and if I--my nickname is Largy--and Boatless get together, we also can have reward of one million dollars as you see here on the fifth line of our reward chart.

N.G.: And if I--my nickname is Smally--and Boatless get together, we can have reward of 500,000 dollars. If the three of us get together, then there will be a reward of 1 million dollars to be split among the three of us.

M.T.: I think I understand the situation. Now, because you are my friend, Smally, I would like to make the following proposition to you. Listen very carefully.

N.G.: Yes, I am listening.

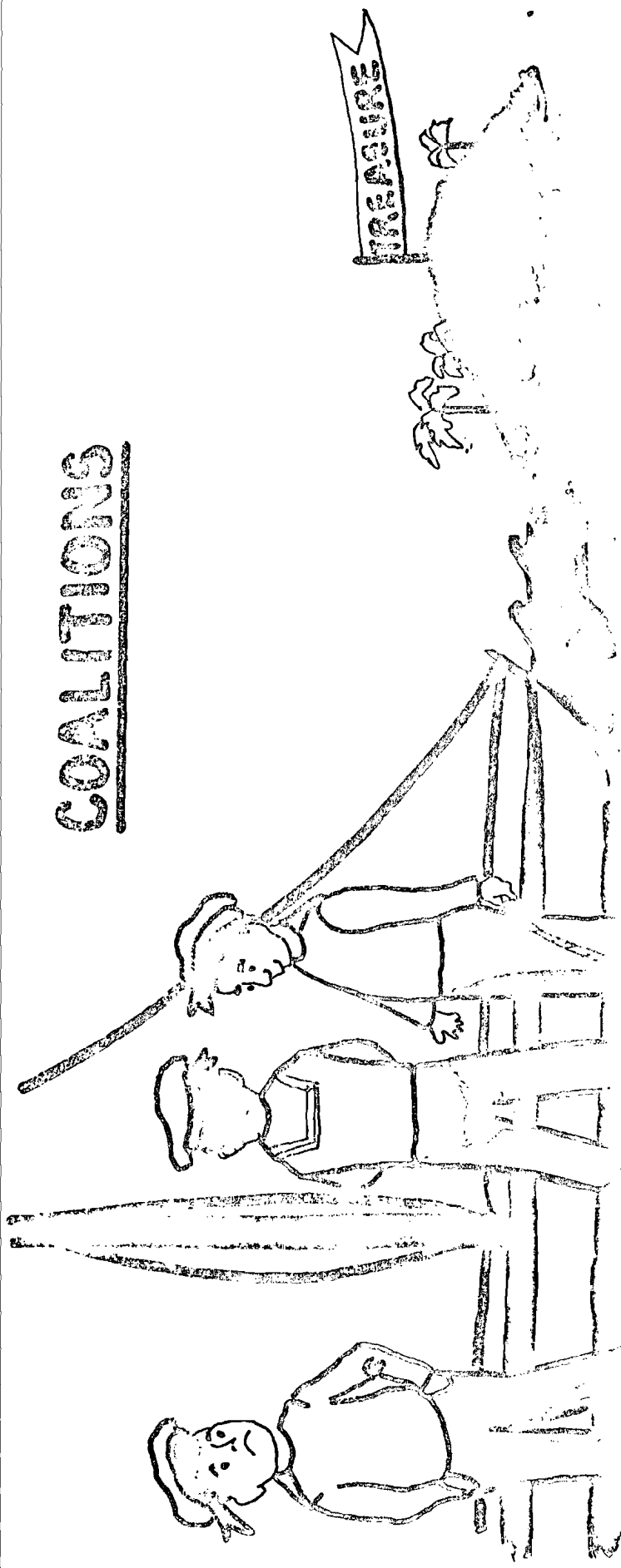
M.T.: We will go, you and I together, to get the treasure. When we have brought it back safely, I shall give you 200,000 dollars.

N.G.: And I suppose you would keep 800,000 dollars.

M.T.: That is correct.

N.G.: I don't think this is a good idea.

# COALITIONS



- REWARD (l, a, g, y) = 0
- REWARD (s, m, a, l, l, y) = 0
- REWARD (b, o, a, t, l, e, s, s) = 0
- REWARD (l, s) = 1
- REWARD (l, b) = 1
- REWARD (s, b) = 1/2
- REWARD (l, - s) = 0

Diagram 6.

M.T.: What do you mean?

N.G.: Let me explain. I can think of a far better arrangement. Suppose, for instance, that I were to go on my boat; it is not so big a boat as your ugly thing. I would go with Boatless to the island. I would gladly give Boatless 50,000 dollars, and I would get 450,000 dollars. Now that is better for Boatless than the proposition that you made since he was not getting any money under it and it is better for me since I will now get 450,000 dollars instead of the paltry 200,000 dollars that you have offered me.

M.T.: It sounds like a weak idea to me.

N.G.: Well, fortunately for you, I have a much better solution for both of us.

M.T.: What is that?

N.G.: I propose we go in your boat and then we will take back the entire treasure and you will give me exactly one half of the treasure, 500,000 dollars.

M.T.: That is ridiculous. Look at this. You are talking about bringing Boatless into the picture. Look, what I could have done for Boatless. I could have offered Boatless 100,000 dollars to come with me in my boat. I would have gotten 900,000 dollars and he would be much better off than under your suggestion of giving him 50,000 dollars. I would be better off in getting 900,000 dollars than with the very generous offer which I made to you of 200,000 dollars to come with me to get the treasure.

But, since you are my friend, Smally, I will make a slightly better offer for you. If you will come with me in my boat to get the treasure, I shall be glad to give you 250,000 dollars and, of course, keep 750,000 dollars for myself.

N.G.: Of course.

M.T.: Well, do you agree?

N.G.: I think so. If I want more than 250,000 dollars for myself then the only way would be to offer less than 250,000 to Boatless. But now, you can always make a more favorable counter offer to Boatless, that is to offer him exactly 250,000 dollars, and keep the 750,000 dollars for yourself. There is no way that I can improve my situation by bringing boatless into the picture. I guess, 750,000 dollars and 250,000 dollars is a fair bargaining agreement.

M.T.: Surprisingly good. We had better stop here. Otherwise your rapidly growing understanding of game theory will outstrip mine.

N.G.: And now, if there are no points on which we have not succeeded in confusing our audience totally, we shall be pleased to answer questions.