

NOTES ON RESILIENCE MEASURES

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## B. PERTURBATIONS AND RESILIENCE

Steele suggested that R is defined usefully only by the system and the perturbation. He guessed that the really difficult thing was going to be specifying the nature of the perturbations in a meaningful way. Several related notions emerge:

- 1) Perturbation Structure: Since in most biological systems  $X \propto C_x^2$ , the perturbation structure  $\Delta X \propto aX$  with (a) a positive constant might be most realistic. But  $\Delta X \propto 1/bX$  with (b) a positive constant might be more dangerous because of control pathologies, etc. (Perturbations get proportionally greater as X gets closer to zero.)
- 2) Holling's approach takes the relevance of the perturbation into his R measures via his log - transform arguments; Häfele does not.
- 3) Relation to components and hazard typology work: Steele's point that R is a function of both system and perturbation is of course an idea we have encountered previously. We (and the Energy Group) rejected it largely in the hope of developing a R concept explicitly free of assumptions about the (unknown, surprising) perturbations which would occur.

Note however that in my empirical components work I almost always end up trying to characterize the character of the perturbation in addition to - or even instead of - that of the system. This approach is carried to its extreme in the Kate-White-Slovic

Typology of Hazards notions. Furthermore, my components theory note of January 1975 discusses Class I phenomena in a context explicitly defined by the scale of the relevant perturbation.

Altogether it seems that we might do well to reconsider explicit integration of perturbation structure into the R concept and R-measures. This has a certain attractiveness to it in that it lets you begin shifting your concern from the detailed nature of the system (which by premise is unknown) to the general character of the perturbations. And if the Kates-White-Slovic typology arguments stand up under examination, and can be cast somewhat more generally, then "the general character of perturbations" may be something which we can in fact get a handle on.

Points to pursue in this context would include viewing the components as structures which could be applied to particular hazard types, and trying to cast R measures as a function of perturbation and components alone, with little or no reliance on detailed character of the system (the latter being again, by premise, unknown).

C. DIMENSIONAL ANALYSIS AND R-MEASURES

Steele also raised the possibility of using dimensionality arguments in designing R-measures. This has been touched on in your note of the 15 July working session, but the following notions might also be noted:

We have been criticized by many for the dependence of some of the proposed R-measures on the scales of the state-space. For generality, we must strive for dimensionless measures. In practice, this may consist of nothing more than the approach implied in my January 1975 Resilience note: i.e. defining "the system" with respect to the perturbation.

In general, both state space behavior and perturbation effect can be expressed as some combination of "length" (L) and "time" (T). The dimensions should almost certainly be included in any R-measure (Instantaneous perturbations and point locations can be dealt with as special cases "in the limit").

The following table began to emerge during the discussions. I would hope it could be recast and completed perhaps including your 15 July table (attached).

Assumptions	Most specific R - measure available	Dimensions
1) No expectation of location in state space	area of stable basin	$L^n$
2) probability expectation of location over some time period.	expected distance to edge over same period	$\frac{E(L)}{T}$
3) given trajectory over same period	average distance to edge over same period	$\frac{L}{T}$
4) given location at a specific time (time in the limit?)	distance to edge	$L$ (? T )

I think the dimensions may be wrong here, and maybe we should follow earlier comments in revising it explicitly incorporating a column on "type of perturbation" giving the latter dimensions as well.

Finally it was suggested, and I shall record without understanding, that any time a R-measure was cast as a dimensionless number (say, as a perturbation in  $LT^{-1}$  over a trajectory in  $LT^{-1}$ ), we were left with a probability ( a dimensionless number). This feels quite wrong, but surely we have often viewed R-measures precisely in the "probability-of-persistence" context, (see my January 1975 note, and Neil Gilbert's critique of my 1974 NRC-Status Report write-up). The question remains as to how, the probability-of-persistence and dimensionless-number R-measures are related.

D) R-MEASURE "CLASSES" (in a system, of a system, for a system..)

As you know, I feel that we have been unjustifiably sloppy in our use of terminology here. While I concur with your "Good Afternoon Wolf" reply to Casti's jargon criticism, the conventions adopted in the 15 July "Summary" table are bound to lead to great confusion. One difficulty hangs on the uncritical use of the word "system". Which system? That defined by the perturbation? That including Restorative Components? That encompassing Contingency Component elements? I don't insist that we address these other aspects of Resilience in a measurement sense now, but we should make use of what we know about them in our choice of terms.

CLASS	PERTURBATION			TRANSFORM	EQUATION	DIMENSIONS	REMARKS
	Direction	Intensity					
Resilience within a system	State Variable	any	Multiplicative (I = cx)	log.	$R = \frac{\int_A \int \rho(x,y) p(x,y) dx dy}{\int_A \int dx dy}$	none	(ρ measured to boundary in state variable space only)
Resilience of a system	Parameter	-"-	-"-	log.	-"-	none	(ρ measured to boundary in parameter space only)
Trajectory Resilience	State Variable	-"-	Constant (I = c)	none	$R = \frac{1}{\int_{s_0}^{s_1} \frac{ds}{dt} \rho(s)}$	LT <sup>-1</sup>	*
	Time fixed	-"-	Multiplicative (I = cx)	log.	$R = \frac{1}{t \int_0^t \frac{dt}{\rho(t)}} \quad \text{or} \quad R = \frac{1}{t} \int_0^t \rho(t) \cdot dt$	none	

NOTE 1: We have left out a class which is essential, i.e., policy resilience