Working Paper

Sensitivity of Water Balance to Climate Change and Variability

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WP-91-047
November 1991

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Foreword

The IIASA Water Resources Project addresses the development and application of methods and procedures needed to identify policy strategies for water resources planning and operation. Due to population growth, industrial and agricultural development, increased pollution and the impact of global climatic change, the reliability of water supply may substantially decrease in various parts of the world, causing serious social and economic problems. There is a need for studies on possible policy actions, aimed at the development of more resilient and more robust water systems, based on a sound understanding of geophysical processes which regulate the hydrological cycle in a changing environment.

Professor Zdzisław Kaczmarek's paper concerns methodological tools for the sensitivity analysis of the water balance components to changing climatic forcings. It presents a new meso-scale hydrological model based on the stochastic storage theory, and its application to the sensitivity analysis and to water balance impact studies. The model allows to calculate runoff characteristics, evaporation and catchment storage on the basis of standard climatological data, and eventually on the basis of alternative climate scenarios. It was tested for a number of river catchments in Europe and Africa.

The possible effects of the expected changes in air temperature and precipitation will give rise to various problems in many fields of water resource management. For this reason, the paper may be of interest not only to hydrologists but also to decision makers in water industry.

Professor Bo Döös
Leader
Environment Program

Sensitivity of Water Balance to Climate Change and Variability

*Zdzisław Kaczmarek**
*Dariusz Krasuski***

1 Introduction

Scientists and politicians are faced with an unusual problem, in that mankind is going to change the global environment due to increased population stress, industrial development and often unwise resource management. Serious disturbances in climatic processes may be expected during the XXI century due to increased concentration of greenhouse gases in the Earth's atmosphere and related changes in the radiation balance. In spite of all the uncertainties associated with the climate issue, the world scientific community is expected to evaluate possible consequences of atmospheric processes on vegetation, hydrology, sea level rise, economic activities and global security. This paper is concerned with the possible impact of climate variations on water resources, in particular on water balance components.

The concept of a decisive role of climate in shaping hydrological processes was formulated more than a hundred years ago by the Russian climatologist A.I. Voejkov by saying that "rivers are the product of climate" (Voejkov, 1886). In the pioneering paper on runoff processes Langbein says sixty years later that "runoff, like soil moisture, evaporation, and other components of the hydrological cycle, may be regarded as a manifestation of climate" (Langbein 1949). In a recent Statement of the Second World Climate Conference we read: "Among the most important impacts of climate change will be its effects on the hydrological cycle and water management systems, and through these, on socioeconomic systems" (Climate Change, Science, Impacts and Policy, 1991). It is clear that the problem of interrelations between atmospheric and hydrological processes is of great concern both from a theoretical and a practical point of view.

There are at least five main areas of research related to the climate/water resources interface:

1. Studies aimed at detecting changes in atmospheric processes by means of hydrological indicators, including paleohydrological investigations;
2. Analysis of the sensitivity of water balance components to changes in climate characteristics;
3. Assessing the possible implications of climate fluctuations on water supply and demand, and consequently on water management;
4. Studies on the impact of climate change on physical, chemical and biological processes in rivers, lakes and reservoirs;
5. Research aimed at sound and more accurate parameterization of land surface processes in global and meso-scale atmospheric models.

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The progress in all these directions during the last ten years is evident, but most of the relevant theories, methods and models are open to improvements.

This paper describes a methodological framework for the climate/water balance sensitivity analysis. In the next section we present a simplified approach to a quantitative assessment of possible changes in average annual runoff due to future variations in climate forcing, and some critical comments on earlier works in this field. It should be added that a rough estimate of possible increase or decrease of mean annual runoff may be informative for those who are interested in a global-scale picture of the influence of possible climate shifts on hydrology. It is usually of limited value for the regional water resources assessment as well as for analyzing socioeconomic implications of changes in the catchment-scale availability of water.

The main part of the paper is devoted to a description of a IIASA modeling scheme aimed at the simulation of monthly or seasonal responses in main water balance components - runoff, water storage and evapotranspiration - to future changes in climatic forcing. This model, based on the stochastic storage theory, allows to calculate mean values, standard deviations and the correlation structure of hydrological variables. Combined with simulation techniques it could be used for generating time series of monthly or seasonal flows. It was required that input data should be based on standard climatic and hydrological observations and on typical outputs from the Atmospheric General Circulation Models. The model has to be calibrated and validated for historical data sets, and then could be run for alternative equilibrium climate scenarios in order to provide differences between water balance elements calculated on the basis of various meteorological inputs. The model was intended to serve as a methodological tool for studying river catchments' reaction to changes in climatic forcing, but it can be applied for grid cells inside the catchment if necessary data and model parameters can be determined for each cell.

The model has been documented in F77L for the IBM compatible PC-AT microcomputers and in RPN for the Hewlett-Packard 48-SX calculator. The report includes a number of numerical examples in order to illustrate the way in which the described methodology may be applied.

2 Sensitivity of Annual Runoff to Climate

We shall now try to quantify the sensitivity of annual catchment runoff to changes in annual climatic characteristics, such as annual precipitation or mean annual air temperature. In this study sensitivity will be defined as the ratio of change in runoff to the change of a given climatic value. Lets assume that the annual runoff R is a known function of T (temperature) and P (precipitation)

$$R = R(T, P) \quad . \tag{1}$$

In an approximate way the differential

$$dR = \left(\frac{\delta R}{\delta P}\right)_o dP + \left(\frac{\delta R}{\delta T}\right)_o dT \tag{2}$$

describes the relative importance of changes of P and T on dR. In the hydrological literature one can find a number of formulae relating annual runoff to the mean annual climatic parameters — from simple empirical equations to more physically based models. Common examples are the Langbein diagrams (Langbein, 1949) where annual runoff is a function of the catchment mean annual precipitation and of the weighted mean annual temperature, the Budyko method (Budyko, 1948, 1984), and the Turc formula (Turc, 1954). If we assume that a given relationship is valid for future climatic conditions (this assumption may, however, be questioned), then it can be applied for the runoff/climate sensitivity analysis by calculating numerical values of derivatives in the equation (2). Lets first investigate the Turc formula

$$R_a = P_a \left[1 - \frac{L_a}{\sqrt{cL_a^2 + P_a^2}}\right] \tag{3}$$

where $L_a = 300 + 25T_a + 0.05T_a^3$. Parameter c may be estimated for each river catchment on the basis of known values of annual runoff R_a, the annual sum of precipitation P_a, and the mean annual temperature T_a. A generalized value $c = 0.9$ has been suggested by Turc if there is no possibility for calibration. The formula (3) is valid only if $P_a > (1-c)^{0.5}L_a$.

Assuming that for the present ($1 \times CO_2$) conditions $P_a = P_{ao}$ and $T_a = T_{ao}$, we may calculate sensitivity indicators

$$\left(\frac{\delta R_a}{\delta P_a}\right)_{P_a=P_{ao},T_a=T_{ao}} \text{ and } \left(\frac{\delta R_a}{\delta T_a}\right)_{P_a=P_{ao},T_a=T_{ao}} \qquad (4)$$

in equation (2). In Table 1 numerical values of these indicators for four European river basins, with highly differentiated climatic conditions, are presented. We may find for example, that if for a $2 \times CO_2$ equilibrium climate precipitation in the Vistula catchment will increase by 10%, and at the same time the mean annual temperature increases by 3.0°C, then

$$R = 0.80 \times 0.1 \times 604 + (-20.40) \times 3.0 = -12.9 \text{ mm} \quad ,$$

which means the runoff decrease of 7.5%.

Table 1: Sensitivity of runoff to precipitation and temperature (Turc formula)

River Basin		Júcar	Vistula	Seine (Paris)	Volga (Upper)
area	(km^2)	42,900	194,900	427,000	161,700
P_{ao}	(mm)	519	604	715	520
T_{ao}	(°C)	13.7	7.5	10.0	2.8
c		0.88	0.55	0.76	1.16
R_{ao}	(mm)	69	173	231	226
$\left(\frac{\delta R}{\delta P}\right)_o$		0.43	0.80	0.76	0.79
$\left(\frac{\delta R}{\delta T}\right)_o$		-10.53	-20.40	-21.03	-13.05

In the case of the Budyko method the annual runoff is a function of precipitation and of the net radiation balance, given in the form

$$R_a = P_a - \sqrt{r_a P_a \left[1 - \exp\left(-r_a/P_a\right)\right] \tanh\left(P_a/r_a\right)} \qquad (5)$$

where r_a is the water depth equivalent of the mean annual net radiation balance

$$r_a = 12.61 \left[Q_{sr}(1 - \text{alb}) + Q_{lr}\right] \qquad (6)$$

expressed in mm, why the incoming short-wave radiation Q_{sr} and the long-wave radiation balance Q_{lr} are given in (W/m^2). Because r_a is a function of a number of meteorological factors (including air temperature), a method for its numerical evaluation has to be decided before the runoff (5) can be calculated. After consideration of a number of methods, an algorithm proposed by (Morton, 1983) has been selected for the purpose of this study. It allows to calculate monthly values of Q_{sr}, Q_{lr}, and albedo provided that monthly data on air temperature, relative air humidity and sunshine duration are available. Assuming that the relative humidity and sunshine duration can be considered as constant from the present climate to $2 \times CO_2$ conditions, we may transform equation (2) to be presented in the form

$$dR = \left(\frac{\delta R}{\delta P}\right)_o dP + \left(\frac{\delta R}{\delta r}\right)_o \left(\frac{dr}{dT}\right)_o dT \qquad (7)$$

where r is defined by (6) and an approximate value of its derivative is done by

$$\left(\frac{dr}{dT}\right)_o = 12.61\frac{Q_{lr}(T_o+\varepsilon) - Q_{lr}(T_o-\varepsilon)}{2\varepsilon} \tag{8}$$

(index $_a$ has been omitted for simplicity). As an example, numerical values of sensitivity parameters for the Vistula basin ($r_a = 640mm$) calculated by means of the Budyko formula are

$$\left(\frac{\delta R_a}{\delta P_a}\right)_o = 0.66, \text{ and } \left(\frac{\delta R_a}{\delta r_a}\right)_o\left(\frac{dr_a}{dT}\right)_o = -3.11 \quad .$$

Comparing the results of the Turc and Budyko methods for this and other catchments, we may come to the conclusion that the sensitivity of the Budyko formula to changes in air temperature is generally lower than in the case of the Turc formula. Numerical results should, however, be taken with necessary caution because of the empirical character of relations involved. It should also be remembered that some important features are disregarded in this analysis, as for example the possible direct effect of CO_2 increase on evapotranspiration. The results obtained by the Budyko method seem to be in good agreement with the conclusions of a sensitivity study done for the United States (Karl and Riebsame, 1989). Analyzing runoff changes associated with the recent climate fluctuations on the U.S. territory, the authors conclude that "The effects of recent temperature fluctuations on streamflow are minimal, but the impact of relatively small fluctuations in precipitation (about 10%) are often amplified by a factor of two or more, depending on basin and climate characteristics".

The relative effect of precipitation change on runoff may be better illustrated by dividing both sides of equation (2) by the mean annual runoff

$$\begin{aligned}\frac{dR_a}{R_{ao}} &= \left(\frac{\delta R_a}{\delta P_a}\right)_o\frac{dP_a}{R_{ao}} + \left(\frac{\delta R_a}{\delta T_a}\right)_o\frac{dT_a}{R_{ao}} = \\ &= \frac{1}{\varphi_o}\left(\frac{\delta R_a}{\delta P_a}\right)_o\frac{dP_a}{P_{ao}} + \left(\frac{\delta R_a}{\delta T_a}\right)_o\frac{dT_a}{R_{ao}}\end{aligned} \tag{9}$$

where φ_o is the mean annual runoff coefficient calculated for the present climatic conditions. The parameter

$$\Phi = \frac{1}{\varphi_o}\left(\frac{\delta R_a}{\delta P_a}\right)_o \tag{10}$$

is equivalent to the elasticity measure introduced by Schaake (Schaake, 1990), but numerical values of (10) differ from Schaake's annual model for which Φ is always equal to one. The elasticity parameter (10), calculated for the Turc formula on the basis of data in Table 1 is

Júcar catchment	$\Phi = 3.23$
Vistula catchment	$\Phi = 2.79$
Seine catchment	$\Phi = 2.35$
Volga catchment	$\Phi = 1.82$.

In general, values of the elasticity parameter (10) will be higher in arid and semi-arid regions, where the coefficient of runoff is usually low.

A somewhat different approach to the sensitivity analysis of runoff has been proposed by Wigley and Jones (1985). They define sensitivity of runoff *change* to changes in annual precipitation and annual evapotranspiration as

$$S_\alpha = \frac{\delta\zeta}{\delta\alpha}, \text{ and } S_\beta = \frac{\delta\zeta}{\delta\beta} \tag{11}$$

where

$$\alpha = \frac{P_1}{P_0}, \quad \beta = \frac{Ev_1}{Ev_0}, \text{ and } \zeta = \frac{R_1}{R_0}$$

are ratio values of water balance components for future (index 1) and present (index 0) climatic conditions. It can easily be shown that

$$\zeta = \frac{\alpha - \beta(1 - \varphi_o)}{\varphi_o} \tag{12}$$

where φ_o is the runoff coefficient. Assuming that α and β are independent variables, a relative sensitivity measure was defined as

$$S_\zeta = \frac{\frac{\delta\zeta}{\delta\alpha}}{\frac{\delta\zeta}{\delta\beta}} = \frac{\frac{1}{\varphi_o}}{\frac{1-\varphi_o}{\varphi_o}} = \frac{1}{1 - \varphi_o} \tag{13}$$

Because the coefficient of runoff is always less than one, the authors conclude that "...runoff is always more sensitive to precipitation changes than to evapotranspiration changes, particularly for higher values of φ_o".

In this connection two comments can be made. First, that the parameter S_ζ represents the sensitivity of runoff change ζ, but not of runoff itself. Second, the above argumentation would be valid if changes in precipitation and evapotranspiration were mutually independent. This may be true in the case of potential evapotranspiration, but is obviously incorrect for actual evapotranspiration, which highly depends on moisture conditions, i.e. on precipitation. Taking into account that β depends on α we will get

$$\frac{\delta\zeta}{\delta\zeta} = \frac{1}{\varphi_o} - \frac{1 - \varphi_o}{\varphi_o}\frac{\delta\beta}{\delta\alpha} \tag{14}$$

$$\frac{\delta\zeta}{\delta\beta} = \frac{1}{\varphi_o}\frac{\delta\alpha}{\delta\beta} - \frac{1 - \varphi_o}{\varphi_o} \tag{15}$$

and consequently

$$S_\zeta = \frac{1 - (1 - \varphi_o)\frac{\delta\beta}{\delta\alpha}}{\frac{\delta\alpha}{\delta\beta} - (1 - \varphi_o)} = \frac{\delta\beta}{\delta\alpha} \tag{16}$$

It is obvious that numerical values of (16) depend on the form in which annual evapotranspiration is dependent on annual precipitation. It can be shown that if we apply the Turc formula (3), then after some transformations

$$S_\zeta = c\frac{(1 - \varphi_1)^3}{1 - \varphi_o} \tag{17}$$

where φ_1 is the coefficient of runoff for future climatic conditions. For almost all the world's river catchments, the parameter S_ζ defined by (17) will be less than one, what evidently contradicts conclusions of the Wigley and Jones paper.

It should be stressed again that the above considerations are based on simplified transfer functions, linking the annual runoff with annual climatic variables. Their results should be understood rather as a qualitative description of the climate/runoff sensitivity problem, than as a quantitative estimation of sensitivity parameters. In the next sections more sophisticated methods will be presented, taking into account the intraannual distribution of hydrological elements and, in particular, the decisive role of catchment storage in the formation of runoff and actual evapotranspiration.

3 A Seasonal Stochastic Water Balance Model

In this section we will describe a stochastic hydrological model aimed at simulation of seasonal characteristics of catchment storage, runoff and evapotranspiration. It will be called "stochastic" because both input and output contain probability distributions and/or their parameters of water balance components. The term "season" means usually a month but it may refer to any other time interval shorter than one year. Essential requirements for the model are as follows:

1. Input data should be based on standard observations of hydrological and meteorological elements;

2. Averaged (lumped) characteristics of climatic elements and land surface processes should be used;

3. The number of calibrated parameters should be kept as small as possible;

4. By replacing "historical" input data by data for alternative climate scenarios, the model should be able to evaluate the sensitivity of water balance to climate.

Spatially lumped, deterministic water balance models were applied for climate/water resources impact studies in a number of regional investigations (see e.g. Cohen, 1986; Gleick, 1987; Schaake, 1990; Vörösmarty, 1991). Our model differs substantially from previous methods for two main reasons: first, it takes into account random properties of input and output variables, and second, that in contrary to most of the earlier studies it allows to evaluate not only the sensitivity of runoff, but also the possible impact of climate perturbations on the catchment storage and on evapotranspiration.

Figure 1 provides an overall structure of the seasonal stochastic water balance model. The first step is to establish and to solve the fundamental water balance equation, and to select methods for calculating its components, e.g. the rate of potential evapotranspiration. During the second phase, a number of stochastic matrices should be calculated in order to define the probabilistic structure of catchment storage levels. Once this structure is determined we may move to the third step and calculate statistical characteristics (mean values, standard deviations, etc.) of the seasonal water balance components. The calibration of model parameters will be done by minimizing the mean quadratic difference between observed and computed runoff values. Finally, in the last step the input data are disturbed by assuming alternative climate scenarios in order to evaluate the impact of climate change on the water balance components.

3.1 Water Balance Equation

Since the end of the XIX century hydrologists were concerned with the problem of connections between precipitation, evapotranspiration, the catchment storage, and runoff. In the year 1896, the water balance equation was formulated by Penck (1896), and since then this technique has been applied to many hydrological problems. Recently it was suggested as an efficient tool for the climate/water resources impact studies.

For the purpose of our model the water balance equation will be used in a form of a differential formula

$$S_{\max}\frac{dz}{dt} = P(t) - R_s(z,P,t) - R_g(z,t) - Ev(z,PET,t) \tag{18}$$

where:

$S_{\max}$ *(mm)* is the catchment water-holding capacity,

$z = S(t)/S_{\max}$ is the relative storage level, defined as ratio of actual storage to the catchment capacity,

$P(t)\,(mm\ day^{-1})$ is the effective catchment inflow based on rainfall measurements and snow budget estimates,

$R_s(z,P,t)\,(mm\ day^{-1})$ represents surface runoff,

$R_g(z,t)\,(mm\ day^{-1})$ represents subsurface runoff,

$Ev(z,\mathrm{PET},t)\,(mm\ day^{-1})$ is the actual evapotranspiration, and PET is the potential evapotranspiration rate.

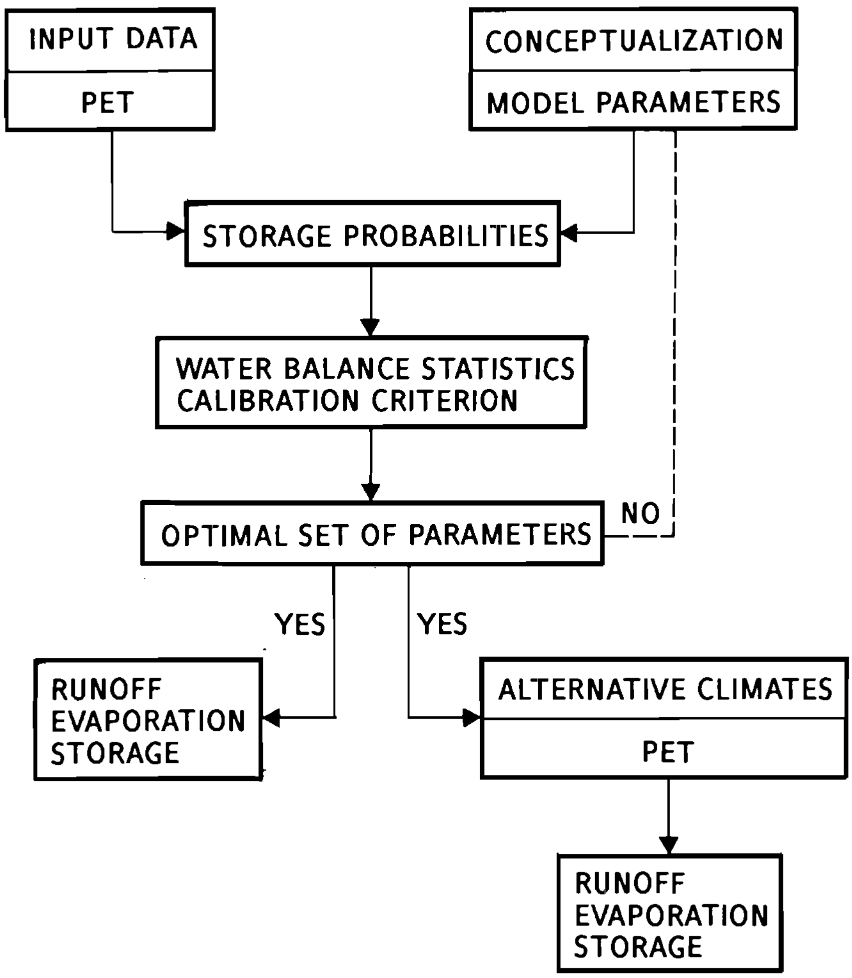

Figure 1: Structure of stochastic water balance model

Lets shortly comment on some of the above water balance elements.

The effective catchment precipitation for a given time interval can be calculated as

$$
\begin{aligned}
P(t) = & \text{ measured precipitation} \times \text{correcting factor} - \text{interception} + \\
- & \text{ snow accumulation} + \text{snow melting.}
\end{aligned}
$$

It is generally recognized that the existing precipitation gauges underestimate precipitation due to the wind effect, wetting losses and evaporation from the gauge itself. Unfortunately, there is no universally accepted methodology for correcting the measured values of rainfall, although in some countries such adjustments are made. The correcting factor largely depends on the type of the precipitation gauge and on the geographical location of the catchment. There is little hope that a standardized methodology for adjusting precipitation data will be developed in the near future.

The amount of water intercepted by the canopy is usually small, except for some specific regions as, for example, the tropical forests. In addition, there is no satisfactory theory of interception and existing field experiments show widely variable results. For all these reasons we provisionally assume that no correction in measured values of precipitation will be made, and that the interception loss can be neglected. The resulting errors should be partly eliminated by the calibration procedure.

The role of snow accumulation and snow melting in hydrological processes is significant in some regions of the world and cannot be neglected. Several methods for incorporating the effects of snow budget into the water balance models were proposed (see e.g. Yevjevich, 1989; Vehviläinen, 1989). In some countries routine snow measurement data are available and may be used for developing regional relationships for snow balance estimates. The user of the model should decide which of the existing methods suits best his needs for including snow processes into the computational scheme.

A very significant role in modeling water balance components has a proper estimation of potential evapotranspiration. PET has a considerable seasonal variability, but for a given month the interannual variability is usually rather low. We decided therefore to use mean monthly (seasonal) values of potential evapotranspiration as constant model parameters. A number of methods for calculating PET values have been developed and documented, among them the best known are:

- Budyko–Zubenok method (World Water Balance and Water Resources of the World, 1978),
- Priestley and Taylor formula (1972), and
- Thornthwaite formula (1948).

Our model may be run in conjunction with each of these methods, but the Budyko–Zubenok submodel has been used for our case studies because of its sound physical basis. It will be described below.

According to the Budyko–Zubenok method, the mean monthly rate of potential evapotranspiration is calculated by means of the formula

$$PET = 86400\frac{0.622\rho}{p\rho_w}D[e_s(Tw) - e] \quad (mm\ day^{-1}) \tag{19}$$

where the air density $\rho = 1.293$ $(kg\ m^{-3})$, density of water $\rho_w = 1000$ $(kg\ m^{-3})$, p is the air pressure in (hPa), and e (hPa) means the mean monthly vapor pressure. The integral coefficient of diffusion was assumed by the authors to have two numerical values, namely $D = 3.0$ $(mm\ s^{-1})$ during the cold part of the year, and $D = 6.0$ $(mm\ s^{-1})$ for months with positive values of mean air temperature. In addition, Zubenok observed (World Water Balance, 1978) that in arid regions D may increase up to 10.0 $(mm\ s^{-1})$. In order to get a continuous relation between

D and T, and taking into account the dependence of D on the level of catchment aridity, the following heuristic rules are proposed:

$$\text{If } T < 0, \quad \text{then} \quad D = 6.0 + 0.3T,$$
$$\text{If } T > 0, \quad \text{then} \quad D = 5.2 + (349 + 70P_a)/T_a$$

where T_a and P_a are annual values of the mean air temperature and the catchment precipitation.

The saturated vapor pressure $e_s(Tw)$ may be calculated by means of an approximate formula

$$e_s(Tw) = 6.11 \exp\left(\frac{17.27Tw}{237.2 + Tw}\right) + 0.09(T_{\max} - T_{\min})^2 \frac{d^2 e_s(Tw)}{d^2 Tw} \tag{20}$$

where $T_{\max}$ and $T_{\min}$ are the mean monthly extreme temperature values. The second component on the right side of the equation (20) presents a correcting factor aimed on eliminating an error caused by calculating e_s (Tw) based on mean monthly values of Tw. The second order derivative in (20) may be replaced by an approximate relation

$$\frac{d^2 e_s(Tw)}{d^2 Tw} = 0.029 + 0.0025Tw \quad (0 \leq Tw \leq 25) \tag{21}$$

The parameter Tw in the Budyko–Zubenok method would be an apparent land surface temperature, would the catchment be supplied with an unlimited amount of water. To find its numerical value, the energy balance equation

$$\begin{aligned} Q_{sr}(1 - alb) \quad &+ \quad Q_{lr} - G - \frac{0.622\rho L}{1000p} D[e_s(Tw) - e] + \\ &- \quad \frac{\rho c_p}{1000} D(Tw - T) = 0 \end{aligned} \tag{22}$$

has to be solved for Tw. L is the latent heat of vaporization equal $2,470,000$ $(J\ kg^{-1})$, c_p = 1005 $(J\ kg^{-1}\ deg^{-1})$ is the specific heat of dry air, G $(W\ m^{-2})$ is the energy flux between surface and soil, and other elements were defined earlier.

For long-wave radiation balance in (22), the slightly modified Brunt formula (Brutsaert, 1982) may be used

$$Q_{lr} = 5.5 \times 10^{-8}\left[0.552e^{1/7}(T + 273.2)^4 - (Tw + 273.2)^4\right](0.2 + 0.8n_s) \tag{23}$$

where n_s is the monthly relative sunshine duration. For calculating G we use the Albrecht formula (Henning, 1989)

$$G = 0.0017\varphi_L [Q_{sr}(1 - alb) + Q_{lr}]_{\max} \sin\left(\frac{\pi}{6}(Mo - Mo_{\max} + 4)\right) \tag{24}$$

where φ_L is the average catchment latitude (negative in the Southern Hemisphere), Mo is a number of a current month, and $Mo_{\max}$ is the number of the month for which the net radiation balance reaches maximum.

After defining $P(t)$ and PET we shall now return to the water balance equation and integrate it for z and t. In our model runoff and actual evapotranspiration depend on the catchment storage. Various expressions were discussed in the literature to conceptualize these relationships. For example, in the case of evapotranspiration, some authors assume linear approximation

$$Ev = PET \cdot z$$

(see e.g. Schaake, 1990, World Water Balance, 1978). In the $GFDL - GCM$ parameterization (Delworth and Manabe, 1988)

$$Ev = PET \frac{z}{0.75}, \text{ if } z < 0.75, \text{ and}$$
$$Ev = PET, \text{ if } 0.75 \leq z \leq 1.00 \quad .$$

In another paper (Wood et al., 1991) this relation has a non-linear form

$$Ev = PET\left[1 - (1 - z)^{5/3}\right] \quad .$$

Similar differences may be found in the case of runoff conceptualization. At present there is no way to prove in advance that one approach is better than another and we have to agree with Gburek (1971) that "...a model system is merely a researcher's idea of how a physical system interacts and behaves, and in the case of watershed research, watershed models are usually extremely simplified mathematical descriptions of a complex situation ...".

In our water balance model we apply the following conceptualization of relations between runoff, evaporation and storage

$$R_s(z, P, t) = \frac{\varepsilon}{1 + \varepsilon - z} P \quad , \tag{25}$$

$$R_g(z, t) = \alpha z^2 \quad , \tag{26}$$

$$Ev(z, PET, t) = PET \cdot z \tag{27}$$

where ε and α are parameters with constant values during the year. Their values as well as $S_{\max}$ should be estimated by means of model calibration. Substituting (25) - (27) into (18) we obtain

$$S_{\max}\frac{dz}{dt} = P\left[1 - \frac{\varepsilon}{1 + \varepsilon - z}\right] - \alpha z^2 - PET \cdot z \quad . \tag{28}$$

Before integration some additional assumptions have to be established. We have examined two approaches leading to somehow different solutions:

- Alternative (A): catchment storage changes linearly in a given month from the initial level z_i to the final level z_j;
- Alternative (B): P is assumed to be constant during the month, and equal to the mean monthly effective precipitation.

It can experimentally be shown that if the time interval is not longer than one month, the resulting probabilistic structure of catchment storage will be similar in both alternatives. The reason for this result is that integration of equation (28) under alternative (B) leads to the relation $z = z(t)$ not far from a linear form. Computer programs are available for both alternatives and it is a user's decision to select the one he prefers.

In the case of alternative (A) we may write

$$z = a + bt \quad , \quad \frac{dz}{dt} = b \quad ,$$

where

$$a = z_i, \quad b = \frac{z_j - z_i}{\tau} \quad ,$$

and τ is the length of a month expressed in days. Substitution to (28) leads to an expression

$$P = \left(1 + \frac{\varepsilon}{1 - a - bt}\right)\left(bS_{\max} + \alpha(a + bt)^2 + PET(a + bt)\right) \tag{29}$$

and the mean monthly value is

$$P_{ij} = \frac{1}{\tau}\int_0^\tau P dt \quad . \tag{30}$$

Algebraic manipulation of (29) and (30) leads to the following relation between mean monthly catchment inflow and the initial and final storage levels:

$$
\begin{aligned}
P_{ij} &= \frac{S_{\max}}{\tau}\left[z_j - z_i + \ln\frac{(1-z_i)}{(1-z_j)}\right] + \\
&+ \alpha\left[\frac{z_i^2 + z_i z_j + z_j^2}{3} + \varepsilon\left(\frac{1}{z_j - z_i}\ln\frac{(1-z_i)}{(1-z_j)} - \frac{(z_i+z_j)}{2} - 1\right)\right] + \\
&+ \left[\frac{z_i + z_j}{2} + \varepsilon\left(\frac{1}{z_j - z_i}\ln\frac{(1-z_i)}{(1-z_j)} - 1\right)\right] PET
\end{aligned}
\tag{31}
$$

Equation (31) is valid if $z_j \neq z_i$. In the opposite case

$$
P_{ij} = \left(\alpha z_i^2 + PET \cdot z_i\right)\left(1 + \frac{\varepsilon}{1 - z_i}\right) \quad . \tag{32}
$$

In the case of approach (B), the effective precipitation rate is assumed to be constant for each month, and consequently the integrated water balance equation may be written as

$$
\int_{z_i}^{z_j} \frac{dz}{P_{ij}\left(1 - \frac{\varepsilon}{1+\varepsilon-z}\right) - \alpha z^2 - PET \cdot z} = \frac{\tau}{S_{\max}} \quad . \tag{33}
$$

In order to obtain P_{ij} as a function of storage levels z_i and z_j, the equation (33) is solved in our model by means of iterative numerical integration. The procedure requires few seconds at the IBM-386 microcomputer.

Because $P_{ij} \geq 0$, for its lower limit a minimum value $z_j = z_{j,\min}$ may be found by means of equation (31) or (33). For alternative (A), P_{ij} should be set equal to zero in (31) and then $z_j = z_{j,\min}$ may be found by solving the resulting equation. In the case of alternative (B), by integration of (33) for $P_{ij} = 0$ we get

$$
z_{j,\min} = \frac{PET z_i}{(\alpha z_i + PET)\exp\frac{\tau \cdot PET}{S_{\max}} - \alpha z_i} \quad . \tag{34}
$$

It can easily be shown that for the conceptualization (28) the following inequalities are always valid

$$
z_{j,\min} < z_j < 1 \quad , \quad z_{j,\min} > 0 \quad . \tag{35}
$$

Expressions (31) - (34) will be used in the next section to calculate the stochastic matrices of storage levels.

3.2 Stochastic properties of Storage Dynamics

To properly model the water balance components, the dynamics of catchment storage described by equations (28) to (33) become a crucial factor. Because the effective precipitation $P(t)$ is a stochastic process, random properties of $z(t)$ and as a result of runoff are investigated in our model. To this end we will utilize concepts and techniques developed some time ago in the framework of stochastic storage theory originally intended to solve problems connected with design and operation of water reservoirs. Although some papers were published in Russian literature in the forties (see e.g. Kritskij and Menkel, 1940), the main stream of work started with Moran's paper on the stationary probability distribution of storage levels (Moran, 1954). Since then, a number of mathematicians and engineers were developing an elegant theory that on the one hand became a part of "pure" mathematics, but on the other hand also provided a suitable basis for solving practical water resources problems. An excellent state-of-the-art summary of stochastic storage models may be found in (Klemeŝ, 1981) and (Phatarfod, 1989).

Up till now, the stochastic storage theory has not been applied for developing hydrological catchment models. There is however a clear similarity between the conceptualization of the water balance model done by (28), and the dynamics of a water reservoir described by a mass conservation equation

$$S_{\max}\frac{dz}{dt} = inflow(t) - outflow(t) \quad .$$

It can be proved (Kaczmarek, 1963; Abramišvili and Čitašvili, 1969) that if:

1. inflow is a m-order Markov process,
2. storage level is always kept between 0 and 1, and
3. a unique value of mean inflow P_{ij} can be found for each pair (z_i, z_j),

then storage levels will form a $(m + 1)$-order Markov process. It means, in particular, that for white-noise monthly (or seasonal) precipitations, the resulting storage levels $z(1)$, $z(2)$, $\ldots z(n)$ form the first-order Markov process, where $z(n)$ is the storage level at the beginning of nth month.

The stochastic storage theory in a form presented e.g. in (Kaczmarek, 1974) may be, in principle, applied for any type of inflow processes. Unfortunately, the computational burden rises dramatically with the increase of the order of lags, and consequently only the white noise process and first-order Markov process were applied in practical applications. In the case of catchment modeling it is generally accepted that for periods long enough (e.g. one month) precipitation values may be approximately assumed to be independent for consecutive time intervals. For example, in (Delworth and Manabe, 1989) the authors observe that except for a few small regions, the time series of monthly runoff give lag-one auto-correlations near zero. The white-noise assumption was therefore employed in our water balance model. It can of course be verified for each particular region by means of standard statistical testing procedures. We shall further assume that the probability distributions of monthly precipitation may be approximated by log-normal density function with two parameters estimated by means of sample moments.

The first step in analyzing random properties of storage dynamics requires calculation of conditional probabilities (see Figure 2)

$$\begin{aligned} p_{ij,(n)} &= p\left(z_j \le z_{(n+1)} < z_{j+1} \mid z_{(n)} = z_i\right) = \\ &= p\left(P_{ij} \le P_{(n)} < P_{i,j+1}\right) \end{aligned} \tag{36}$$

for each month (or season), and $i, j = 1, 2, \ldots, K$. The accuracy of the model depends on the number K of intervals into which the storage space has been divided. Our experience shows that $K = 20$ should in most cases be the sufficient number of storage states. For each time period we need K^2 probability values (35).

The sequence of storage states for months 1 to $S = 12$ forms a discrete-state Markov chain over the space {1,2, ..., K} with transition probability matrices

$$MCP_{(n)} = \{p_{ij,(n)}\}, \quad n = 1\ldots S, \quad i,j = 1\ldots K \tag{37}$$

It should be remembered that effective precipitation rates P_{ij} and $P_{i,j+1}$ should be obtained from equations (31) or (33), and that the probability values are calculated using standard procedures for the log-normal distribution function.

The second step in our analysis comprises steady-state or ergodic probabilities of storage levels. In this connection we employ a widely accepted concept of stationarity of water balance components, storage levels included, for time moments separated by an interval of one year. It means, for example, that the probability distribution of storage levels by 1st January will be the same for consecutive years. Denoting steady-state probabilities by

$q_{i,(n)}$ = ergodic probability of z_i at the beginning of nth month,

$q_{ij,(n)}$ = ergodic probability of a conjunction (z_i, z_j) of initial and final storage levels in month n,

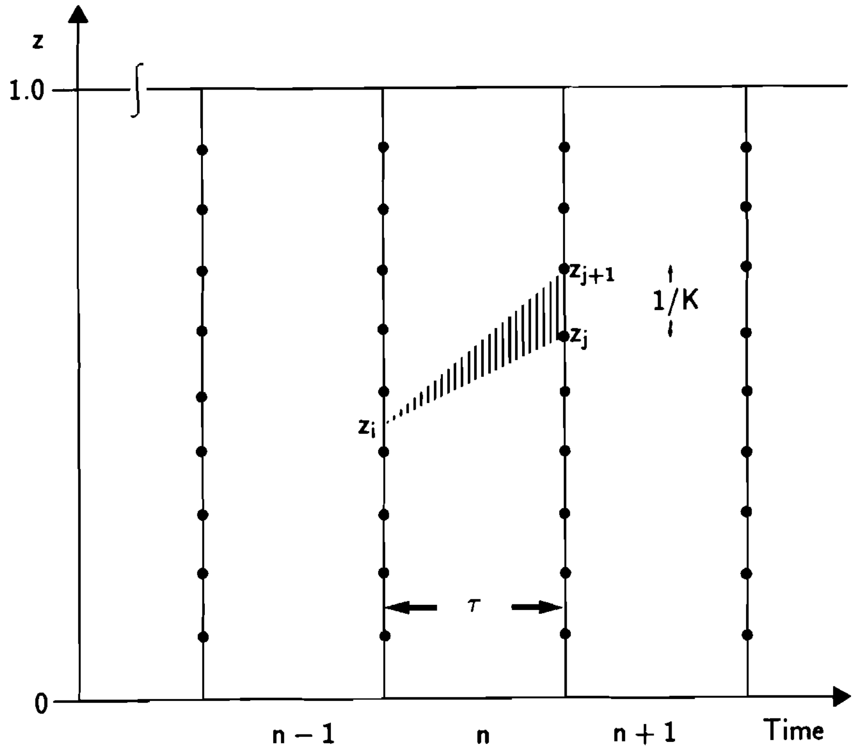

Figure 2: Schematic overview of storage levels

and the respective matrices

$$MQI_{(n)} = \{q_{i,(n)}\}$$
$$MQIJ_{(n)} = \{q_{ij,(n)}\} \quad ,$$

and assuming that $MQI_{(1)} = MQI_{(1+S)}$, we may find $q_{i,(n)}$ by solving a system of linear equations

$$MQI_{(1)} \cdot \prod_{n=1}^{S} MCP_{(n)} - MQI_{(1)} = 0 \tag{38}$$

in which the last equation should be replaced by

$$\sum_{i=1}^{K} q_{i,(1)} - 1 = 0 \quad . \tag{39}$$

Note that $MQI_{(n)}$ are matrices of one row and K columns. Once $q_{i,(1)}$ are evaluated, all other matrices $MQI_{(n)}$ are obtained by successive multiplications

$$MQI_{(n+1)} = MQI_{(n)} \cdot MCP_{(n)} \quad , \quad n = 1, \ldots, S \tag{40}$$

In order to calculate the joint ergodic probabilities $q_{ij,(n)}$, the matrix $MQI_{(1)}$ should be transformed into a diagonal matrix $MD_{(1)}$, where $q_{i(1)}$ are its diagonal elements. Then

$$MQIJ_{(1)} = MD_{(1)} \cdot MCP_{(1)} \tag{41}$$

and

$$MQIJ_{(n+1)} = MQIJ_{(n)} \cdot MCP_{(n)} \quad , \quad n = 1, \ldots, (S-1) \tag{42}$$

This completes the evaluation of the storage level probabilities for the water balance model, conceptualization of which is done by (28). It should be remembered that these probabilities depend not only on input data, but also on the form of functions (25) – (27), linking various components of water balance with the level of storage. This implicates certain subjectivity in our results, a typical phenomenon associated with conceptual hydrological models.

In the third and final step, in the examination of storage dynamics, we shall calculate for each month (or season) a number of numerical characteristics, demonstrating in a concise way the influence of climatic input on temporal variability of storage. The most important and commonly used measures are mean values, standard deviations and lag-one autocorrelation coefficients. The way in which climate processes influence these measures may be helpful in overall explanation of linkages between atmospheric and hydrological processes.

The long-term mean values of storage for the beginning of nth month can be calculated as

$$E(z)_{(n)} = \sum_{i=1}^{K} z_i q_{i,(n)} \quad , \quad n = 1, \ldots, S \tag{43}$$

Similarly, the standard deviation and lag-one coefficient of correlation are

$$SDEV(z)_{(n)} = \sqrt{\sum_{i=1}^{K} \left(z_i - E(z)_{(n)}\right)^2 q_{i,(n)}} \tag{44}$$

$$CORR(z_{(n)}, z_{(n+1)}) = \frac{1}{SDEV(z)_{(n)} SDEV(z)_{(n+1)}} \cdot \sum_{i=1}^{K} \sum_{j=1}^{K} (z_i - E(z)_{(n)})(z_j - E(z)_{(n+1)}) q_{ij,(n)} \tag{45}$$

where $q_{i,(n)}$ and $q_{ij,(n)}$ were defined earlier. It should again be stressed that the accuracy of formulae (43) - (45) depends on the number K of intervals into which the storage space is divided. Our calculations done for a number of river catchments show that particularly sensitive to K is the coefficient of correlation. It should also be noticed that (43) and (44) measure mean value and standard deviation for the relative storage, but by multiplying by $S_{\max}$ we may easily get respective values for the storage itself.

3.3 Runoff and Actual Evapotranspiration

From the practical point of view, the most important impact of climate change is on runoff and actual evapotranspiration. In most of the World's regions the catchment runoff is the main source of water used for meeting requirements of population, agriculture, industry and the energy sector. There are still a lot of controversies concerning the sensitivity of runoff to changes in climatic and land-use processes. Our model is intended to serve as a technical tool in the climate/water resources sensitivity studies.

The way in which runoff and evapotranspiration may be calculated depends on assumptions laid down before the integration of equation (28). We shall first consider alternative (A), according to which the catchment storage is for a given month a linear function of time. It is obvious that because precipitation and storage are random variables, also runoff and evapotranspiration have random properties. Lets denote by C any of the water balance components under investigation, and by

$$\overline{C}(z_i, z)_{(n)} = \frac{1}{T} \int_o^T C(z_i, z)_{(n)} dt \quad , \quad z = a + bt \tag{46}$$

its *partial* mean value, calculated under the condition that the catchment storage changes from z_i to z during a given month. The overall expected value of C in the nth month will therefore be

$$E(C)_{(n)} = \sum_{i=1}^{K} q_{i,(n)} \int_{o}^{\infty} \overline{C}(z_i, z)_{(n)} f(P) dP \tag{47}$$

where $f(P)$ is the log-normal density function of the effective precipitation. Substituting relations (25) - (27) into (46) we will get, after necessary algebraic manipulations, the following expressions for partial mean values:

For surface runoff:

$$\begin{aligned}
\overline{C}(z_i, z)_{(n)} &= \overline{R_s}(z_i, z)_{(n)} \cong \varepsilon \left[\left(\frac{S_{\max}}{\tau} + \frac{\alpha + PET_{(n)}}{z - z_i} \right) \left(\ln \frac{1 + \varepsilon - z_i}{1 + \varepsilon - z} + \right. \right. \\
&- \left. 2arth \frac{\varepsilon(z - z_i)}{\varepsilon(z_i + z - 2) - 2(z_i - 1)(z - 1)} \right) + \\
&- \left. (\alpha + PET_{(n)}) - \frac{\alpha}{2}(z_i + z) \right]
\end{aligned} \tag{48}$$

For subsurface runoff:

$$\overline{C}(z_i, z)_{(n)} = \overline{R_g}(z_i, z)_{(n)} = \frac{\alpha}{3}(z_i^2 + z_i z + z^2) \tag{49}$$

For evapotranspiration:

$$\overline{C}(z_i, z)_{(n)} = \overline{Ev}(z_i, z)_{(n)} = \frac{PET}{2}(z_i + z) \quad . \tag{50}$$

Expressions (48) - (50) should then be substituted into (46). Before numerical integration will be possible, an approximate relation

$$z = z(z_i, P) \tag{51}$$

has to be established for each month. Skipping technical details it should be mentioned that (51) may be obtained on the basis of relation (31) and then incorporated into (47). Finally, integration of (47) gives for each month the expected values of runoff and evapotranspiration. In a similar way one may calculate the second moment of C, and consequently standard deviations

$$SDEV(C)_{(n)} = \sqrt{E(C^2)_{(n)} - \left(E(C)_{(n)}\right)^2} \tag{52}$$

The algorithm for calculating the lag-one coefficient of correlation is more complicated and will not be discussed here.

In case of alternative (B), when $P_{ij} = P$ is assumed to be constant in a given month, the algebraic procedures leading to expected values and standard deviations of water balance components are similar, with the only difference that the partial mean values $\overline{C}(z_i, z)_{(n)}$ are obtained by means of numerical integration. Consequently

$$E(C)_{(n)} = \sum_{i=1}^{K} q_{i,(n)} \int_{o}^{\infty} \frac{1}{\tau} \int_{zi}^{z(P,zi)} \frac{S_{\max} w(z) dz}{P\left(1 - \frac{\varepsilon}{1+\varepsilon-z}\right) - \alpha z^2 - PET \cdot Z} f(P) dP \tag{53}$$

where

$$\begin{aligned}
w(z) &= \tfrac{\varepsilon P}{1+\varepsilon-z} && \text{for surface runoff,} \\
w(z) &= \alpha z^2 && \text{for subsurface runoff, and} \\
w(z) &= PET \cdot z && \text{for actual evapotranspiration.}
\end{aligned}$$

Similarly to the alternative (A), numerical integration of equation (53) requires that the relation $z(z_i, P)$ should previously be established.

It can be seen from the above considerations that characteristic values of water balance components depend on climatological input data presented in the form of:

- probability distribution functions of monthly effective precipitation, with parameters estimated for each month,
- mean monthly values of potential evapotranspiration, usually calculated on the basis of standard climatological data.

They also depend on three parameters: $S_{\max}, \alpha$ and ε, which at the moment have to be calibrated by comparing computed and observed values of monthly runoff. The calibration procedure is a standard one and will not be discussed here. One of the future tasks may be a kind of generalization of the model's parameters by relating them to catchment characteristics, such as physical properties of soil, types of vegetation, and catchments topography. Such generalization would allow broader application of the water balance model in regions where runoff data are not easily available, or if the calculations should be done on the basis of grid squares climatological information.

By altering input data, the model may be used as a tool for climate impact studies and sensitivity analysis. The weak point of this reasoning is that parameters of this and similar hydrological models are obtained for past climatic conditions. As long as their independence on climate change has not been proved, the comparative results for alternative climate scenarios should be taken with caution. This dilemma, raised at the time of the First World Climate Conference (Schaake and Kaczmarek, 1979) is still unresolved and open for future investigations.

4 Climate Scenarios For Sensitivity Assessment

It is widely acknowledged that climatic and hydrological phenomena are stochastic processes with a significant seasonal component. Their properties may be described by means of multivariate probability distributions, or by a set of parameters if the form of PFD is assumed to be known in advance. If any process remains in equilibrium about a constant set of parameters it is called to be stationary. Up to now, in practical water resources applications the hydrological and climatic processes are treated as periodically stationary. They are usually considered as being discrete in relation to time.

We shall define a climate scenario as a complete description of stochastic processes representing climate elements under investigation. If, for example, the monthly average air temperatures at a given meteorological station are assumed to be the white-noise Gaussian process with mean values and standard deviations estimated on the basis of 30-year long observational records, it means that we have created a temperature scenario. This scenario may differ from the real geophysical process because in reality the distribution function is not known, and the parameters are estimated with some error. Nevertheless, such temperature scenario may be used for solving some practical problems. By assuming changes in the form of probability distributions of climate elements and/or in their parameters, we are creating alternative climate scenarios.

The concept of alternative climate scenarios has in the last 10 years been closely linked with the global warming issue. In reality we do not know whether there will be any future equilibrium climate and what will be the changes in stochastic climatic processes, but for policy decisions it may be useful to create some visions of what may eventually happen in our environment. In most cases alternative climates caused by increased concentration of greenhouse gases in the atmosphere are created in a simplified way by assuming changes only in some of the parameters, e.g. in long-term mean values. It means implicitly that other parameters, such as e.g. coefficients of variation or the autocorrelation functions remain unchanged.

Alternative climate scenarios are often used for water resources impact studies. In this connection we may observe that the terms "sensitivity analysis" and "impact studies" are frequently used in an interchangeable way, which may lead to some confusions. To avoid this, we shall clearly distinguish between these two concepts. In this paper we define sensitivity as a reaction of a physical system to *changes* in external influences. If

$$y = f(x_1, x_2, \ldots, x_n) \tag{54}$$

then sensitivity of y to changes in x_i may be found as

$$\left(\frac{\delta y}{\delta x_i}\right)_{x_1=x_{10},\ldots,x_n=x_{n0}} \qquad i=1,\ldots,n \quad . \tag{55}$$

where index $_o$ denotes the base set of factors $x_j (j = 1, \ldots, n)$.

To find numerical characteristics of water balance sensitivity to climate change there is no need to assume alternative climate scenarios. What is really needed is the knowledge of a transfer function (54), where y means a water balance component and x_i are climatic factors under investigation. In the case of simple annual models the sensitivity characteristics based on (55) can be obtained by methods described in chapter 2. For more complicated seasonal models an approximate value of the sensitivity parameter may be calculated by means of relation

$$\frac{\Delta y}{\Delta x_i} = \frac{y(x_{io}+\lambda,\Omega_o) - y(x_{io}-\lambda,\Omega_o)}{2\lambda} \tag{56}$$

where Ω_o is the base set of climatic factors other than x_i, and λ is a small disturbance of x_i.

On the other hand, an impact study will be understood as an assessment of water balance components on the basis of a set of alternative climatic factors which in some way differ from the historical data. It means that to perform such an impact study one has to specify one or more climatic scenarios, and apply them to a model which serves as a transfer function between climatic forcing and hydrological variables. A number of approaches have been suggested for creating regional climate scenarios (see e.g. Giorgi and Mearns, 1991). They may be classified into the following groups of methods:

1. Purely hypothetical scenarios designed for a particular impact study, assuming for example that the catchment precipitation will decrease by 10% and the air temperature will increase by 2.0° C;

2. Scenarios constructed by using instrumental data records, on the basis of which warm or particularly dry periods are identified;

3. Paleoclimatic and paleohydrological analogues;

4. Various types of GCM-based scenarios, when large-scale GCM information is in a certain way translated into regional statistics of climatic variables of interest for hydrological modeling.

None of these methods is perfect, and the resulting climatic characteristics should be considered as feasible scenarios, not as climate forecasts. It is generally acknowledged that General Circulation Models give the best large-scale information on the reaction of climate to increased CO_2 concentration, and that scenarios based on GCMs are internally consistent. But unfortunately the results can vary significantly from one GCM to another, in particular for precipitation patterns. In addition, their spatial distribution is too coarse to provide good input data for catchment water balance models.

Consequently, an analyst has to take his own subjective decision how to create climatic scenarios for the water balance impact study, which adds a new kind of uncertainty to the issue. May be that in the context of limited knowledge of the regional climate change, the sensitivity analysis, as defined above, is more justified than impact studies based on questionable climate scenarios. If, however, such a study is undertaken, the resulting water balance statistics should again be treated as hydrological scenarios, not forecasts.

5 An Example: Warta River Catchment

The Warta river is the largest tributary of the Odra (Oder) river and is located in Western Poland (15.0° - 19.5° E and 50.5° - 54.0° N). The key basin characteristics are:

Table 2: Basic Data for Warta River Catchment

Month	Historical Statistics						$2 \times CO_2$ (GFDL data)	
	P	Cv_P	T	e	n_s	R	T	P
XI	1.49	0.45	3.5	7.2	0.16	0.256	6.4	1.34
XII	1.31	0.66	-0.8	5.4	0.14	0.319	6.2	1.81
I	0.91	0.55	-2.7	4.6	0.17	0.336	2.4	1.28
II	1.13	0.77	-2.3	4.7	0.22	0.392	4.7	1.59
III	1.30	0.40	1.3	5.6	0.32	0.496	8.9	1.41
IV	1.83	0.55	7.5	7.8	0.41	0.511	13.4	1.44
V	1.85	0.42	12.4	10.5	0.39	0.391	16.0	2.35
VI	2.05	0.39	16.8	13.5	0.43	0.275	20.8	1.64
VII	2.80	0.51	17.8	15.1	0.40	0.226	23.3	2.33
VIII	2.20	0.31	17.0	14.7	0.42	0.235	22.7	2.57
IX	1.53	0.58	13.3	12.3	0.41	0.216	18.7	1.88
X	1.28	0.64	8.7	9.6	0.32	0.233	12.7	1.15

P and R in $(mm\ day^{-1})$; T in $(°C)$; e in (hPa).

- catchment area: 54.529 km^2,
- average altitude above sea level: 148 m,
- mean annual precipitation: 598 mm,
- mean annual runoff: 118 mm,
- runoff coefficient: 0.197.

Mean monthly statistics of various climatic elements (precipitation, air temperature, vapor pressure and relative sunshine duration) were calculated on the basis of records at nine climatological stations located in the basin. Precipitation values were corrected to avoid instrumental bias and adjusted for snow accumulation and snow melting. Climatic values based on historical records are given in Table 2. Mean monthly rates of potential evapotranspiration were calculated by means of the Budyko method described in section 3.1.

The stochastic water balance model has been calibrated to minimize mean square deviations between observed and modeled monthly runoff values. The resulting values of model parameters are: $S_{\max} = 629\ mm, \alpha = 0.286, and\ \varepsilon = 0.0090$. The simulated and observed mean monthly values of catchment runoff are presented in Figure 3, which generally shows a good agreement between the two patterns. It should be added that all calculations were done for $K = 40$ storage levels, but very similar results were obtained for $K = 20$. After calibration the model was used for:

1. Assessment of runoff sensitivity to precipitation change;
2. Construction of water balance scenario for the $2 \times CO_2$ climate, based on the Geophysical Fluid Dynamic Laboratory model.

The sensitivity analysis was done by running the water balance model for precipitation values increased and decreased by 1.0% in relation to historical values given in Table 2. The resulting

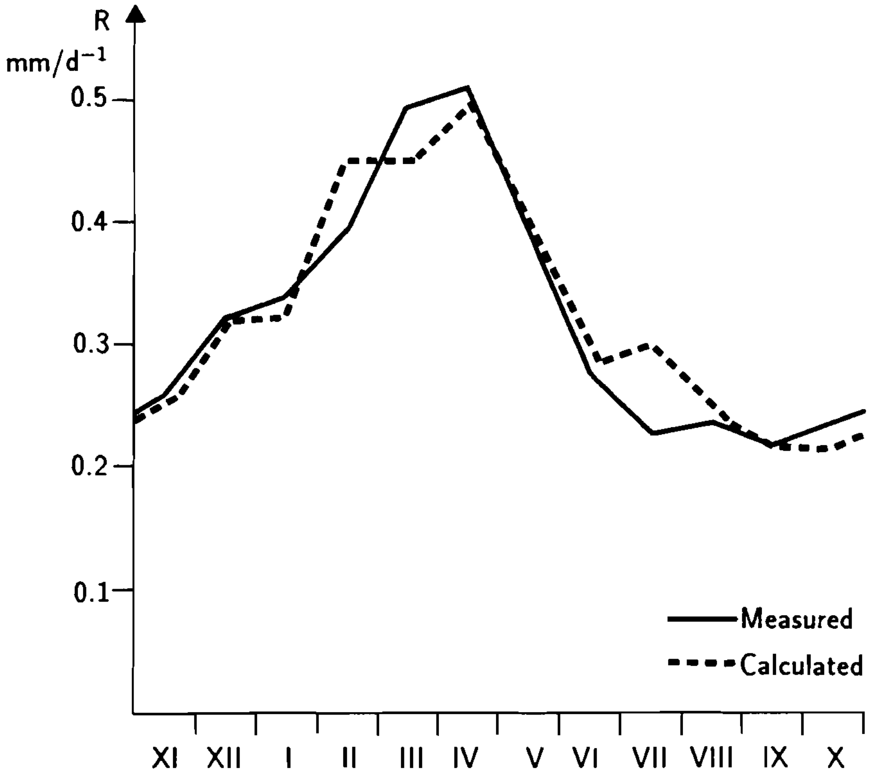

Figure 3: Measured and calculated 1 $\times$ CO_2 runoff values for Warta river

annual runoff is:

$$R = 122.06\, mm\; for\; P = 1.01\, P, and$$
$$R = 115.40\, mm\; for\; P = 0.99\, P,$$

all other climatic characteristics being assumed invariant. The sensitivity parameter (56) is therefore equal to

$$\frac{122.06 - 115.40}{2 \cdot 5.98} = 0.577.$$

Dividing the above value by the catchment runoff coefficient we get the elasticity measure (10)

$$\Phi = \frac{0.577}{0.199} = 2.80 \quad .$$

It is worthy to note that similar calculations done for a number of river basins in Europe and Africa give the values of elasticity measure of runoff to precipitation in the range from 2.0 to 4.0 which indicates an amplification similar to that obtained for the United States (Schaake, 1990).

The next step was to construct the water balance scenario for $2 \times CO_2$ climate based on the mean monthly temperature increments and monthly ratios of $2 \times CO_2$ and $1 \times CO_2$ precipitation, obtained from the GFDL Global Circulation Model. The numerical values of these characteristics, extracted from the IIASA data base, are given in Table 2. For further calculations it was assumed that the values of coefficient of variation Cv_P, relative humidity and relative sunshine duration will not change from the present to the $2 \times CO_2$ climate. The results are given in Table 3, and the differences of runoff characteristics for the present and alternative climates are shown in Figure 4. It can be seen that for this particular case:

Table 3: Model Results for Warta River Catchment

Month	E(z)		R_s		R_g		R		Ev	
	$1 \times CO_2$	$2 \times CO_2$	$1 \times CO_2$	$2 \times CO_2$	$1 \times CO_2$	$2 \times CO_2$	$1 \times CO_2$	$2 \times CO_2$	$1 \times CO_2$	$2 \times CO_2$
XI	0.748	0.728	0.075	0.065	0.173	0.163	0.249	0.227	0.190	0.261
XII	0.799	0.769	0.119	0.169	0.196	0.186	0.315	0.355	–	0.154
I	0.847	0.831	0.108	0.169	0.214	0.210	0.322	0.379	–	0.089
II	0.874	0.870	0.225	0.358	0.229	0.228	0.454	0.586	–	0.258
III	0.906	0.905	0.210	0.214	0.237	0.234	0.448	0.449	0.739	1.076
IV	0.911	0.900	0.256	0.138	0.233	0.219	0.489	0.357	1.786	2.143
V	0.889	0.848	0.149	0.148	0.214	0.198	0.363	0.346	2.528	2.724
VI	0.839	0.813	0.103	0.068	0.186	0.169	0.289	0.237	3.125	3.291
VII	0.773	0.722	0.131	0.076	0.169	0.142	0.300	0.217	2.810	2.941
VIII	0.758	0.681	0.087	0.075	0.162	0.133	0.248	0.208	2.347	2.469
IX	0.739	0.676	0.059	0.058	0.156	0.133	0.214	0.191	1.494	1.639
X	0.730	0.679	0.053	0.085	0.158	0.144	0.211	0.228	0.695	0.800

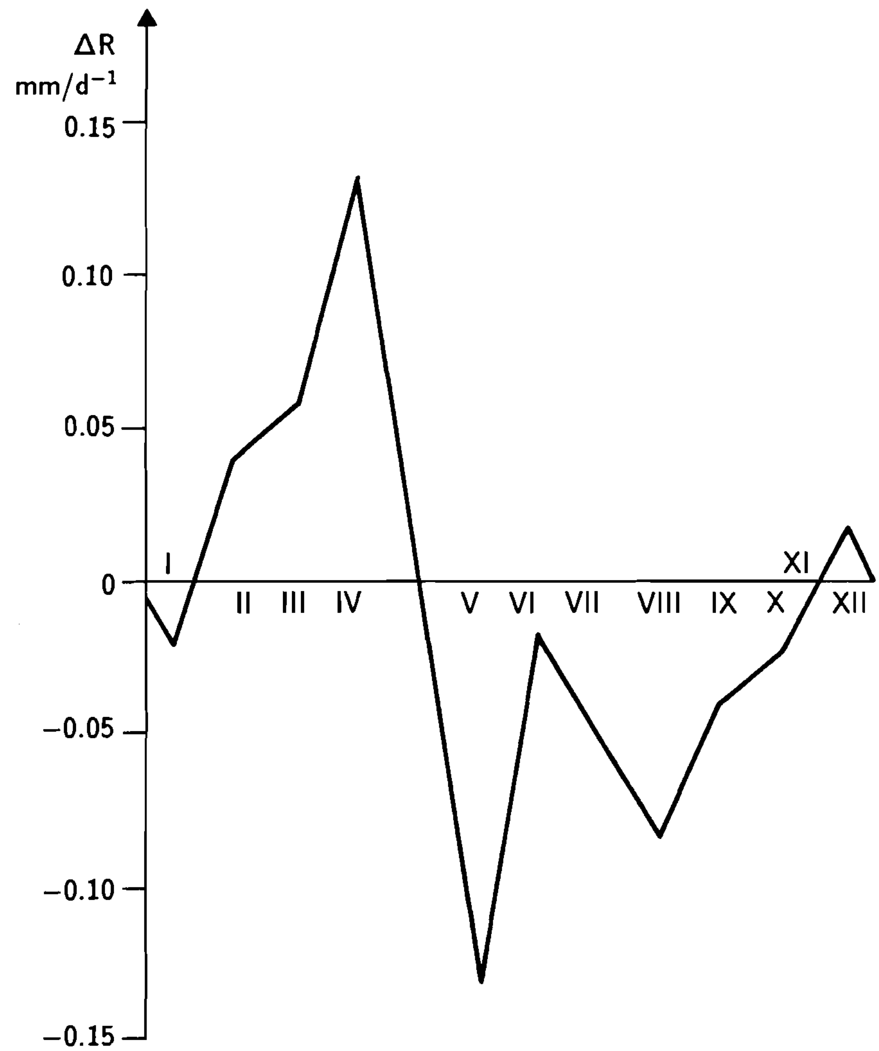

Figure 4: Runoff differences for 2 × CO_2 (GFDL) and 1 × CO_2 climates for Warta river

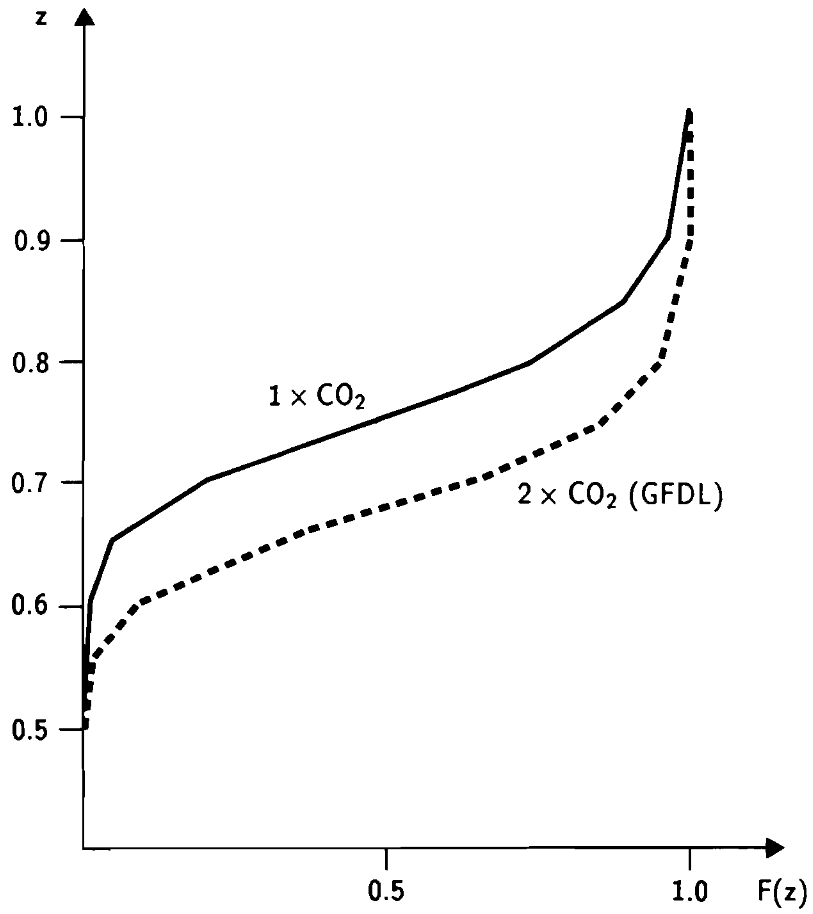

Figure 5: Probability distribution of relative storage for 1 $\times$ CO_2 and 2 $\times$ CO_2 (GFDL) climates

- relative storage level $z = S/S_{\max}$ decreases during the whole year, and particularly in summer,
- the annual runoff remains quite unchanged, with the winter runoff (December to February) increased by 21.0% and the summer runoff (June to August) decreased by 24.0%,
- actual evaporation rates are higher for all months.

It should be stressed again that this is *not* a forecast of future hydrological regime in the Warta river catchment, but only a scenario dependent on the assumed changes in climatic forcing. Similar calculations implemented for other GCM outputs give of course different results.

In addition to mean statistics of the water balance components, the model allows to calculate some other characteristics, such as probability distribution of storage levels, standard deviations and coefficients of correlation. An example of storage distributions for the $1 \times CO_2$ and $2 \times CO_2$ climates in one of the summer months for the Warta basin is presented in Figure 5, which shows a significant shift towards higher probability values for lower moisture levels in the case of the GFDL climate scenario. Such additional information may help to better understand the hydrological processes which lead to changes in runoff characteristics. It may also be useful in other impact studies, as e.g. in analyzing possible consequences of climate change on the agricultural production. In conclusion, the stochastic water balance model may be offered as effective and relatively simple tool for sensitivity analysis and climate impact studies. Its flexibility allows to expect that for most of the World's regions the model will give satisfactory results. The to expect that for most of the World's regions the model will give satisfactory results. The main underlying idea of applying stochastic storage theory to the catchment processes may be combined with various forms of conceptualization of the water balance equation. The model can be expanded for Markow-type precipitation processes, i.e. for time intervals shorter than one month, but this will significantly increase computational requirements.

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