MENTAL MAPS: WITHOUT SPAGHETTI THEY ARE BALONEY

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Gould and White [1974] show how "mental maps" of countries can be constructed. These maps can represent many facets of individual attitudes (preference, ignorance, prejudice, etc.). Once an individual preference structure has been constructed, Gould and White describe a technique for aggregating the preferences of individuals into "national" or "regional" preferences. Thus they illustrate the mental map of school leavers at Bristol, Aberystwyth, Liverpool, and Inverness over the preference for location in Great Britain (p.71 to 80), as well as maps for the United States for different regions. These maps are aggregations of individual maps, or preference structures. The technique used for constructing maps is to start with an ordinal preference map for individuals, then by the technique of factor analysis to combine or weight the individual preferences into an "aggregate preference structure".

One might start off by quibbling with the use of factor analysis to construct the aggregate preference structure. Why not use simply averages of the preferences rather than averages where the weights are determined by the factor loadings? There is, however, a deeper reason to question the technique, and that is simply that under the normal definition of a map, or a preference surface, aggregate mental maps cannot be constructed.

To see why, we need to ask, what are the normal qualities which we associate with a map of a preference surface? We call them the AXIOMS OF MENTAL CARTOGRAPHY. First, it must be a complete map of the region. Second, it must be a transitive map, indicating that if region A is higher than region B and region B is higher than region C, then A must be higher than C. Third, when comparing two regions we assume that the height of a town is independent of the height of other towns; thus to compare the height of Baden and Vienna, we need not know the height of Laxenburg. Fourth, these maps must be meaningful aggregates in the sense that they reflect the tastes of more than one or two persons; thus, let us say that if at least half the people think A is higher than B, then the aggregate must also show A to be higher than B.

Under these innocuous conditions, it is easy to show that aggregate mental maps cannot be generally constructed. A simple example will make the process clear: let us have three individuals (A, B, C) and three locations (Vienna, Baden, and Laxenburg). The ranking of the three towns is as follows:

Person	Baden	Laxenburg	<u>Vienna</u>
A	1	3	2
В	2	1	3
С	3	2	1

where a higher number indicates less preferred locations. Thus
A prefers Baden over other locations.

In this example, by rule 4, we have Laxenburg is higher (more preferred) than Baden, Baden higher than Vienna, and Vienna higher than Laxenburg. Thus in this example any map

satisfying rule 4 will violate rule 2.

Perhaps the example is very special and nasty. When in general can we construct a map consistent with the axioms of mental cartography? We can where two conditions are met: first, we must be able to cut up a map so that it is, topologically, a piece of spaghetti. Second, we must then be able to represent each individual's mental map so that the altitude on the piece of spaghetti rises monotonically from the ocean to a continental divide, then falls monotonically to the ocean. Not everyone need have the same continent divide, but everyone must use the same piece of spaghetti. When the Spaghetti and Continental Divide conditions are met, then an aggregate mental map can be constructed which satisfy the Axioms of Mental Cartography. For a proof we can apply the reasoning of Arrow [1951].

References;

Gould, Peter and Rodney White, Mental Maps, Penguin, London, 1974.

Arrow, K.J. [1951], <u>Individual Choice and Social Welfare</u>, Wiley,

New York, 1951.