COMBINED MIGRATION-DIFFUSION MODELS: ANALOGIES FOR REGIONAL DEVELOPMENT

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Preface

Stimulated by Ross MacKinnon's WP-75-22, "A First Attempt to Combine Interregional Migration and Spatial Diffusion Models," this working paper briefly notes some striking analogies between one view of regional development problems and an extensive literature of models in epidemiology and chemical engineering. It is intended as a stage in continuing discussion and exploration of modelling approaches, and is, therefore, deliberately open-ended and somewhat speculative.

Combined Migration-Diffusion Models: Analogies for Regional Development

In WP-75-22, Ross MacKinnon has outlined a "first attempt to combine interregional migration and spatial diffusion models." It may be of interest to note that the combination of physical migration (movement) and "diffusion" of attributes has interested workers in several other fields--particularly epidemiology (where the interest is primarily in migration of disease carriers or vectors such as insects and the "diffusion" of contagious diseases by infective contact) and chemical engineering (where the movement is convective transport, locally and over long distances, of substances that react, diffuse, and radiate energy). In both these fields, this interest has led to a substantial body of mathematical analysis, some of which appears to shed light on the basic regional development problems that motivate the work in WP-75-22.

Epidemiology:

For example, in epidemiology, the model presented in WP-75-22 is known as a multi-location logistic process. This basic logistic model is one of the simplest used; it considers only two of the major transport and diffusion contributions, omitting several others. The analogy is as follows: the mechanism described in regional development as "telling" corresponds to infection; the non-knowers correspond to the susceptible population not yet infected; physical migration is much the same in both cases. (The main difference is that, in epidemiology, infection is usually assumed to require spatial proximity, so that the matrix $\underline{Q}$ presented in WP-75-22 would usually be diagonal.)

Does the analogy tell us anything new? In this instance, it seems to provide considerable insight. For example, the standard epidemiological models show that the logistic model (and hence the regional development model in WP-75-22) omits the following contributions:

a) Non-susceptibles in the population. MacKinnon has assumed that the entire population not yet informed is susceptible to being informed. In some cases, that may be so; in other cases, the information or attribute being disseminated may be sufficiently complex that only some people are capable of receiving it effectively and perhaps fewer are capable of transmitting it further.

   Then the susceptible population, per se, becomes of great interest: do they migrate differentially faster or slower to or from particular places? What control policies (e.g., education, training programs, bonuses for skilled personnel in particular locations, targeted transmission techniques, etc.) can affect their numbers in particular places? Are there differences between receivers and transmitters? Etc.

In epidemiology, of course, the aim is to increase the <u>non</u>-susceptibles, mainly by immunization programs, which have been among the major contributions to over-all public health in this century. In epidemiology, diffusion-type considerations also occur nested within migration; for these the susceptible population can also be an important control variable. Here, the mass sterilization and release of male screw-worm flies (to minimize the fraction of all females able to have fertile matings) is a classic example.

b) <u>Disappearance of attributed population</u>. MacKinnon has assumed that information (or whatever other attribute is being transmitted) sticks with the people who receive it for the entire period under examination. That may well not be so: if the people with the knowledge or skills are mainly older (e.g., skilled artisans), they may retire or die; no matter what age people are, they can (and will) forget the information or skill or lose the attribute (e.g., through underemployment); or they may migrate out of the system being considered (e.g., the "brain drain" of so much concern a few years ago).

In epidemiology, the disappearance of infected cases is often an important mechanism: people become immune to some diseases after having been sick; others die; still others simply recover from the disease and become susceptible again.

c) Exogenous supply of susceptibles. MacKinnon has assumed that the susceptible population is essentially fixed at the beginning, and then simply reshuffled through time by migration. When one takes the total population as being susceptible, new supply is not likely to become important unless there is substantial net population growth. When one considers a more restricted susceptible population, however, exogenous supply may become quite important. New college or technical school graduates, educated women entering or re-entering the labor force, etc., may be significant contributions, especially in less developed regions and countries (about which concern is presumably the greatest).

In epidemiology, the classic cases of exogenously supplied susceptibles are children beginning school and transients (tourists, etc.), who may also be or become exogenous sources of infection.

The epidemiological literature is now sizeable. Two references (chosen simply because they happened to be at hand) that provide some entree into that literature, and give some of the relevant equations (though by no means all of them) are:

- o M.S. Bartlett, An Introduction to Stochastic Processes, Cambridge University Press, 2nd edition, 1966. See especially Chapter 4.4, "Epidemic Models", particularly p. 143, which gives a general equation including spatially distributed effects.
- o J. Gani, "Point Processes in Epidemiology", pp. 757-774 in P.A.W. Lewis (Editor), Stochastic Point Processes, Wiley-Interscience, 1972. See especially the references.

Solution Insights:

As these references show, for a single region, MacKinnon's model reduces to the difference form of the logistic differential equation:

$$Y(t + 1) = Y(t)[1 + q(1 - Y(t)/X)] \quad \text{, or}$$

$$Y(t + 1) - Y(t) = (q/X)[X - Y(t)]\ Y(t) \quad .$$

There appears to be no simple closed-form solution to this difference equation, though the corresponding differential equation (representing the deterministic form of the stochastic model)

$$\frac{dY(t)}{dt} = A[X - Y(t)]\ Y(t) = A\left[XY - Y^2\right]$$

has the simple solution

$$Y(t) = \frac{X}{1 + \left[(X-Y_o)/Y_o\right] \exp(-AXt)}$$

which is the well-known logistic function (yielding the characteristic S-curve). For "large" X and moderately large t, the solution to the difference equation is essentially indistinguishable from this function. The stochastic formulation matters primarily when Y is small.

Since no closed-form solutions seem able to be obtained even for the single region case, it is apparent that solutions to the multi-region case will need to be computed even for the very simple form the model now assumes. Though there may be surprises in store from extreme choices of coefficients, my experience with similar models in chemical engineering (representing flows between process units) indicates that the solutions in each region will resemble mixtures of logistic functions, with appropriate changes in slope added by the inter-regional transport and communication. One may be able to get staircase-type solutions in some instances, where first one region, then another and another exert a dominant impact on some otherwise isolated region.

Future Directions:

The logistic model is a special case of the so-called "pure birth process"; in that the numbers of informed people can only grow. In general, the solutions to these, and the controls on these, are not considered very interesting. There is not enough room to maneuver effectively and they are not considered very realistic except in highly specialized situations.

Very interesting phenomena can be observed, however, as soon as one adds differentially susceptible populations and/or disappearances ("deaths" in the stochastic process terminology). Then one can get critical threshholds, fluctuations, and a variety of interesting control possibilities. Since everything is likely to have to be computed anyway, one does not add a great deal of complexity by adding one or more of these features, and one may gain a great deal of realism and interesting solution characteristics.

Chemical Engineering:

As a postscript, and to provide food for further thought, it may be interesting to note analogies also to chemical engineering problems. In these, the formulation is usually changed into the form of a generalized diffusion equation (a second-order, parabolic partial differential equation) either by applying the theory of transport processes directly or, perhaps more pleasing esthetically, by developing the forward or backward Kolmogorov equations describing the underlying Markov process. In some instances, these may be easier to treat numerically (though that seems doubtful for such simple models). In most instances, these will yield considerable intuitive insight into the nature of the phenomena being described, since such a very great deal of work has been done exploring the nature of the solutions to diffusion equations of all types.