RM-75-76

ENERGY POLICY AND VALUE TRADEOFFS

Ralph L. Keeney

December 1975

Preface

The work described in this report is a part of an ongoing IIASA research effort on Integrated Management of Energy/Environment Systems. The primary goal of that effort is to develop the methods and means by which we embed energy systems within regional environments. The outputs of the program include concepts, applied methodologies, and the evaluation of case studies. During 1975 the case studies were emphasized; they focussed on three greatly differing regions, namely the German Democratic Republic, the Rhône-Alpes Region in southern France, and the state of Wisconsin in the U.S.A. The IIASA research was conducted within a network of collaborating institutions composed of the Institut für Energetik, Leipzig; the Institut Economique et Juridique de l'Energie, Grenoble; and the University of Wisconsin, Madison.

This report by Ralph Keeney is concerned with one of the most important components of the research--the development of an appropriate framework for evaluating the tradeoffs associated with alternative energy/environment strategies. The work is a first step in linking a large energy/environment simulation model (developed at the University of Wisconsin) with a formal methodology for assessing preferences and values. The approach presented in this paper has been extended to the three-region study mentioned above; this work will be described in a later report.

W.K. Foell
Leader, Ecology/Environment
Project

Abstract

This report has two main purposes. The first is to indicate the appropriateness of multiattribute utility theory for addressing the tradeoff issues invariably present in selecting energy policy. The second is to illustrate by an example how one puts the theory into practice. Specifically, an eleven-attribute utility function over attributes including deaths, SO_2 pollution, radioactive waste, health effects, and electrical energy generated is assessed. A dialogue indicating the procedure used, with comments on why various questions were asked, is presented in detail. The resulting utility function is being used to examine energy policies differing in terms of main fuel (fossil or nuclear) and degree of conservation.

Table of Contents

1. Introduction

The selection of an optimal energy policy, or even the choice of the better policy between two alternatives, depends very much on the preference structure of the decision making body. For instance, the choice between generating 70% of the electricity produced in a region using nuclear power and the other 30% with fossil-fuel plants, or an alternative of generating the same total capacity from 70% fossil facilities and 30% nuclear, would, among other factors, depend heavily on one's preferences for (i.e., willingness to accept) various levels of sulfur-dioxide and particulate pollution compared to the possible levels of radioactive release. As another example, whether it is desirable to double the electricity production in the next ten years would depend on the perceived advantages of the additional electricity and the perceived disadvantages of the additional pollution and other unwanted impacts. For both of these prototypical problems, there is no objective solution. No decision procedure, formal or informal, can get around the fact that preferences are a critical aspect in such problems, and that they are inherently subjective.

Most experiences to date in analyzing energy policy formalize many technological and economic relationships in trying to present an accurate picture of the "total impact" of the various options. Thus, for instance, the decision unit will eventually receive a report indicating the impact--sometimes probabilistically specified--of alternative A in terms of various pollution levels at different locations, the acres flooded because of dams, the number of people who must be dislocated, the likelihoods of minor and major disasters at plants, the forecasted impacts on electricity rates, the overall effect on system reliability, perhaps the estimated health impacts on the population, and so on. Similar information is provided for the other alternatives. But this is where it ends. The decision making group must then somehow integrate all this information in their minds and come up with an (optimal) policy. Selecting this policy is very difficult. Three major aspects creating this difficulty are

1. The large uncertainties about what the impact of any alternative will eventually be, and the difficulty in separating this from one's preferences concerning "possible" consequences;

2. The multiple objective aspects of the problems and the necessity to make value tradeoffs among various levels of different indicators;

3. The large differences among the preference structures of the individual members of the decision making unit and the lack of systematic procedures to articulate these differences and provide a mechanism for constructive compromise.

In this paper, we suggest and try to support the contention that multiattribute utility analysis can be a considerable help in dealing with the three difficulties mentioned above. Our vehicle for doing this involves describing in detail the assessment of a multiattribute utility function over eleven measures of effectiveness used to indicate the environmental impact of alternate energy development scenarios in the State of Wisconsin. The Wisconsin effort, directed by Professor Wes Foell, has specified possible consequences of several energy alternatives. To date they have not attempted to quantify the preference structure for the decision makers involved. This paper is a first step in such an effort.

Section 2 summarizes aspects of the problem which concerns us. Section 3 states the technical terms and the main theoretical result to be used. This result says that subject to certain assumptions, the utility function must have a particular form. In Section 4, the assessment of Bill Buehring's utility function is illustrated in detail. The discussion essentially presents the dialogue between Buehring and myself the first time the assessment was done. Section 5 gives some follow-up assessments conducted a week later. In the interim, Buehring had assessed Wes Foell's utility function over the same eleven measures and had several discussions about preferences over these attributes. In Section 6, we calculate Buehring's utility function.

The final section suggests uses of the utility function for addressing the three difficulties outlined earlier.

2. Impacts of Alternatives for Electrical Energy Production in Wisconsin

Over the past several years, Buehring and Foell and others (see Buehring and Foell [2], and Buehring, Foell, and Dennis [3]) have tried to assess the impact of various alternatives for producing electrical energy in Wisconsin from now until the year 2000. Rather than go into any detail, let me briefly mention aspects of their work relevant to our discussion here. The primary policies that Buehring and Foell are examining differ in terms of two main characteristics: the total electrical power generated and the percentages generated from nuclear and fossil sources.

At the beginning of their work, a set of desired energy policy objectives were generated. The process was essentially creative. Alterations were made after discussions to arrive at a reasonably comprehensive set of objectives. The next step involved specifying attributes (i.e., measures of effectiveness) to measure the degree to which these several objectives were met. These attributes, indicated by $X_1, \ldots, X_{11}$ are listed in Table 1. Also in the table, we list the unit used to measure each attribute as well as the range for the possible impacts of any of the alternatives. It was simple to check that in fact, for all attributes except electricity generated, less of an attribute was preferred to more. Hence for later purposes, Table 1 lists best and worst levels rather than maximum and minimum levels.

Buehring [1] has specified in great detail exactly what impacts each attribute is meant to capture. Let us simply try to clarify a few aspects here. Fatalities include deaths due to working in the coal mines, transporting the fuel, nuclear power plant disasters, and prolonged pollution, for example. Permanently unusable land may result from radioactive waste being stored at a location. Attribute X_2 measures the impact due to the loss of usable land, whereas X_8 is meant to indicate

Table 1. Attributes for Evaluating Energy Policy.

Attribute	Measure	Level Worst	Level Best
X_1 = fatalities	deaths	700	100
X_2 = permanent land use	acres	2000	0
X_3 = temporary land use	10^3 acres	200	10
X_4 = water evaporated	10^{12} gallons	1.5	0.5
X_5 = SO_2 pollution	10^6 tons	80	5
X_6 = particulate pollution	10^6 tons	10	0.2
X_7 = thermal energy needed	10^{12} kwh (thermal)	6	3
X_8 = radioactive waste	metric tons	200	0
X_9 = nuclear safeguards	tons of plutonium produced	50	0
X_{10} = chronic effects	tons of lead	2000	0
X_{11} = electricity generated	10^{12} kwh (electric)	0.5	3

the implications (e.g., genetic impact) resulting from the waste itself. Attributes X_3 and X_4 measure the land and water resources not available because of electrical power generation. Attributes X_5 and X_6 are intended to capture all the undesirable effects of air pollution other than health impacts--measured by X_{10}--and deaths from acute SO_2 exposure, measured by X_1. The attribute X_7 indicates the thermal power needed to generate the electrical power measured by X_{11}. Together, these provide an indication of the waste and efficiency of the system. In addition, X_{11} is a proxy indicator of the desirable impacts on quality of life due to more energy. Attribute X_9 is used to indicate the vulnerability of the system to theft of nuclear material.

The next and major part of Buehring and Foell's work was to estimate the impact of the various alternatives being investigated in terms of the eleven attributes of Table 1, by trying to trace backwards from the generation of electrical power to all the impacts produced on the way. This included obtaining, transporting, using, and disposing of the fuel. After this was done for each option, one was at the situation described in the introduction. The question is, how does one aggregate all the data in a responsible fashion to select a reasonable, if not the best, policy? After a diversion to introduce the terminology, we return to this in Section 4.

3. Terminology and Main Result

This section summarizes the technical terms and the theoretical result used in this paper. Let $X \equiv X_i \times X_2 \times \cdots \times X_n$ be a consequence space, where X_i is the i^{th} attribute. A specific consequence will be designated by $\underline{x}$ or $(x_1,x_2,\ldots,x_n)$. We are interested in assessing the utility function over X, denoted by $u(x_1,x_2,\ldots,x_n)$ or $u(\underline{x})$, which is valid in the von Neumann - Morgenstern sense. (See von Neumann and Morgenstern [7].) Such a utility function has the important property that in choice situations involving uncertainty, one should choose the option leading to the highest expected utility. An implication of this when there is no uncertainty is that $u(\underline{x}) = u(\underline{y})$ if and only if $\underline{x}$ is indifferent to $\underline{y}$. Hence, the preference structure and all the tradeoffs among attributes are specified once u is known.

Let us define $\bar{X}_{ij}$ to mean $X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_{j-1} \times X_{j+1} \times \cdots \times X_n$ and $\bar{x}_{ij}$ to be a specific level of $\bar{X}_{ij}$. Similarly, the notation $\bar{X}_i$ is defined as $X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$, and $\bar{x}_i$ is a level of $\bar{X}_i$.

The main assumptions used in the paper concern the concepts preferential independence and utility independence. We will say $\{X_i,X_j\}$ is <u>preferentially independent</u> of $\bar{X}_{ij}$ if one's preference order for consequences $(x_i,x_j,\bar{x}_{ij})$, with $\bar{X}_{ij}$ held fixed, does not depend on the fixed amount $\bar{x}_{ij}$. This is equivalent to assuming that tradeoffs under certainty among various amounts of X_i and X_j do not depend on $\overline{X}_{ij}$. The preferential independence assumption implies that the indifference curves over $X_i \times X_j$ are the same regardless of the value of $\overline{X}_{ij}$.

In a similar fashion, we say X_i is utility independent of $\overline{X}_i$ if one's preference order over lotteries on X_i, written $(\tilde{x}_i, \overline{x}_i)$, with $\overline{X}_i$ held fixed does not depend on the fixed amount $\overline{x}_i$. This implies that the conditional utility function over X_i, given $\overline{X}_i$ is fixed at any value, will be a positive linear transformation of the conditional utility function over X_i, given $\overline{X}_i$ is fixed at any other value.

The main result used in this paper is the following THEOREM. Let $X \equiv X_1 \times X_2 \times \cdots \times X_n$, $n \geq 3$. If for some X_1, $\{X_1, X_j\}$ is preferentially independent of $\overline{X}_{1j}$ for all $j \neq 1$ and X_2 is utility independent of $\overline{X}_2$, then either

$$u(\underline{x}) = \sum_{i=1}^{n} k_i u_i(x_i) \quad , \quad \text{if } \sum k_i = 1 \quad , \tag{1}$$

or

$$1 + ku(\underline{x}) = \prod_{i=1}^{n} [1 + kk_i u_i(x_i)] \quad , \quad \text{if } \sum k_i \neq 1 \quad , \tag{2}$$

where u and the u_i are utility functions scaled from zero to one, the k_i are scaling constants with $0 < k_i < 1$, and $k > -1$ is a scaling constant.

Equation (1) is the additive utility function and (2) is the multiplicative utility function. More details about these, including suggestions for assessment, are found in Keeney [5].

4. The First Assessment of Buehring's Utility Function

Assessing a utility function is a process in which you, the assessor, ask the decision maker a series of questions about his or her preferences. From the responses, you construct his or her utility function. First one asks some questions to determine the general shape of the utility function. Then one asks more specific questions to quantify a specific utility function. Finally, there should be consistency checks and modification. My experience has been that almost invariably in multiattribute contexts, the decision maker will make some modifications to his preferences as first articulated. This should not be disturbing, as a major purpose of the assessment is to force the decision maker to understand the implications of his preferences in these very complex situations. Since the problem is complicated, it is unlikely that one can immediately articulate consistent preferences that correctly represent the individual's feelings.

The assessment process is dynamic. What you, the assessor, do next depends on how the person whose preferences are being assessed responds to the current question. It depends not solely on the answer itself, but on other factors such as your perception of the ease the decision maker had in responding, on his understanding of the question, and on the desirability of going into more detail. At a point where you feel your previous question was misunderstood, and hence wrongly answered, you can ask a similar question to verify your intuition. If there was a misunderstanding, questions should be repeated.

In spite of these dynamic aspects, one can more or less follow a pattern in assessing utility functions. Once the attributes are specified, the assessment process might be broken into five parts:

1. Familiarization with the terminology of motivation for the assessment,
2. Verification of independence assumptions concerning preferences,
3. Assessment of the tradeoffs among attributes,
4. Assessment of the individual attribute utility functions,
5. Checking for consistency and modification.

In the dialogue which follows, we essentially cover parts 2 through 5. Before this assessment, Buehring read several sources to familiarize himself with the terminology and procedures used in such assessments. When we met for the utility function assessment, Buehring and I first went over the meanings of all the attributes. Then we discussed the point of view that Buehring should take in the assessments; that is, whether he should articulate his own preferences, or those he feels the government has or should have, or what. Clearly, the different perspectives would lead to different responses. We concluded that for this assessment, the preferences should be his. With this, we were ready.

4.1 Verification of Preferential Independence Assumptions

Keeney: So, let's begin. First we can examine the preference structure by looking at some preferential independence conditions. As a start, consider attribute X_1, fatalities, versus X_2, the permanently unusable land. We will use Figure 1 to help in questioning. Note that the range of fatalities, 100 to 700, is plotted on the abscissa and the range of land permanently unusable, 0 to 2000 acres, is on the ordinate. So I guess the best point is (x_1 = 100, x_2 = 0); is that right?

Buehring: Yes.

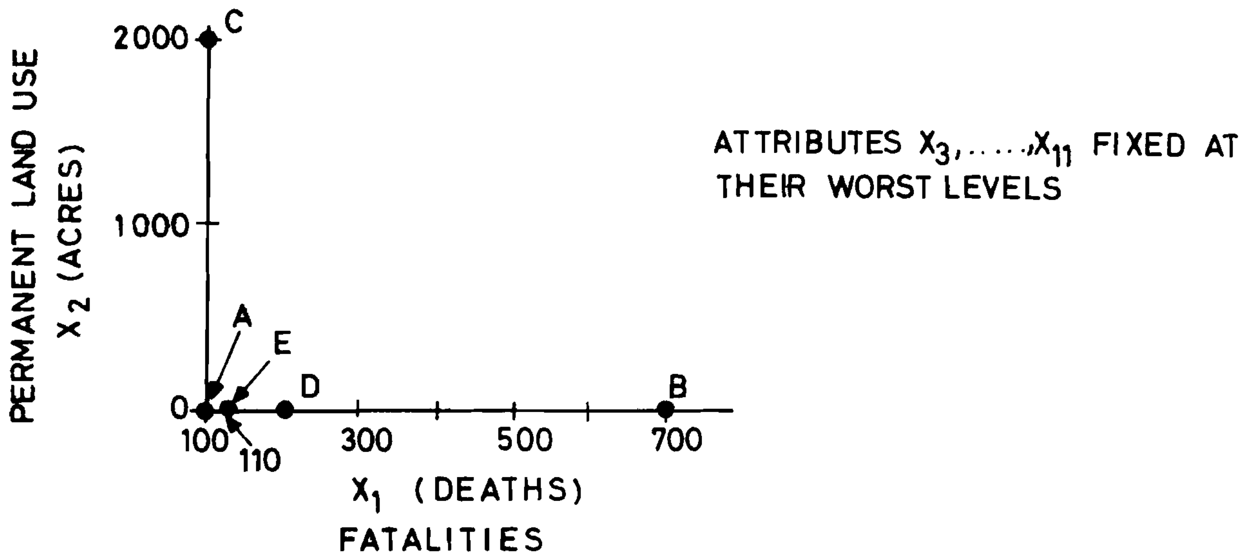


Figure 1. Fatalities versus permanent land use: other attributes at worst level.

K: And for now, let us assume that all other attributes X_3, X_4,...through X_{11} are at their worst levels as defined in Table 1. Now suppose you are at the best point A (Figure 1); would you rather lose 600 people and move to point B or lose 2000 acres and go to C?

B: Lose the 2000 acres.

K: [That question seemed to be easy to answer since it came quickly. Thus I lowered the fatalities greatly.]
Okay, would you rather lose 100 additional people (point D) or the 2000 acres?

B: I'd still rather lose the acres.

K: Not that this number needs to be precise--you can certainly change any numbers anywhere in the process--how many people on a first guess would you be willing to give up to be indifferent to these 2000 acres?

B: That's pretty tough--that's permanent commitment for land. But, relative to fatalities, it just doesn't seem that important to me.

K: How about 110 people (point E)?

B: It's probably in that neighborhood. It's very small. I'm just trying to think whether it's bigger than 101...I guess it is bigger than 101. Maybe 105, how does that sound?

K: That's fine for now. One thing that comes to my mind is, what is included in attribute X_2? Is it concerned only with the loss of use of the land and not with the psychological worry that an individual who lives near a radioactive waste facility may feel, for example?

B: No, it isn't supposed to include that. The problem of waste is captured by attribute X_8 which includes both the high-level and low-level waste.

K: Fine, let's proceed. We want to move over to Figure 2 now and ask essentially the same question with all the other attributes, X_3 through X_{11}, at their best level.

Which would you rather do, go from ($x_1 = 100$, $x_2 = 0$), point A', up to 700 on X_1 (point B') or up to 2000 on X_2 (point C')?

B: 2000.

K: And what if I made this 400 on X_1 (point D') or 2000 on X_2?

B: Still 2000.

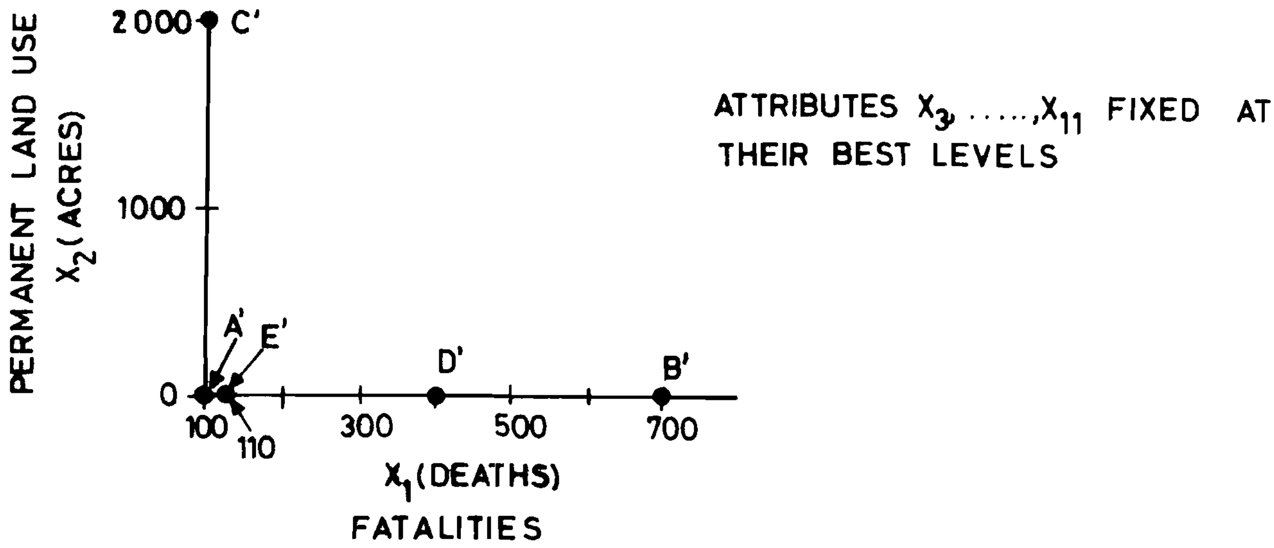


Figure 2. Fatalities versus permanent land use: other attributes at best level.

K: [The responses seemed easy so I jumped ahead.] Is there any reason why the indifference point in this case should be any different from that in Figure 1, that is the 105 for X_1?

B: It's essentially the same, I think.

K: This says one thing, that the consequences ($x_1 = 105$, $x_2 = 0$) and ($x_1 = 100$, $x_2 = 2000$) must be on the same indifference curve for the two levels of the other attributes specified. Of course, it says nothing about any of the rest of the preference structure. But let me save the

general questions until after we try another specific case.
Let us look at the preference tradeoffs between X_2 and X_3, temporarily unusable land. What is included in X_3? [Such questions are asked partly to help me understand the attribute. Then I can use my understanding of the situation as an indicator of whether we are communicating. If some response seems odd to me, I can press the decision maker for his reasoning. Sometimes one should also do this when the response seems reasonable. The spirit is to force serious consideration of the consequences and reconciliation of the discrepancies. The other reason for such questions is to get the decision maker to consider all the factors in making tradeoffs.]

B: This includes temporary disruptions (e.g., surface mining) of land that can be recovered. Another example is land use at a power station. Usually a large area surrounds a facility, which is called an exclusion area at a nuclear plant and something else at a coal plant. In any case more land is bought than is actually used.

K: One thing you'll note is that although these attributes have a relatively clear meaning to you, since you have developed them and worked with them, they are probably not quite so clear to someone else. Thus, in interpreting your results, not only here with the preferences but also with the impacts, it would be nice to have a clear statement of what is and what is not to be included in each attribute.

B: Right.

K: So, let's return to the tradeoffs between X_2 and X_3, fixing the other nine attributes at their worst levels. The ranges are indicated here in Figure 3. For X_2, they are the same as before, and for X_3, the best is 10,000 and the worst 200,000 acres. The best point is A, no land permanently unusable and 10,000 acres of land temporarily unusable. If you must move up to either 2000 permanent, point C, or 200,000 temporary, point B, which would you do?

B: Let's see. I guess the 200,000 recoverable acres would be preferred.

K: That means you'd rather have (x_2 = 0, x_3 = 200,000) than (x_2 = 2000, x_3 = 10,000).

B: Yes.

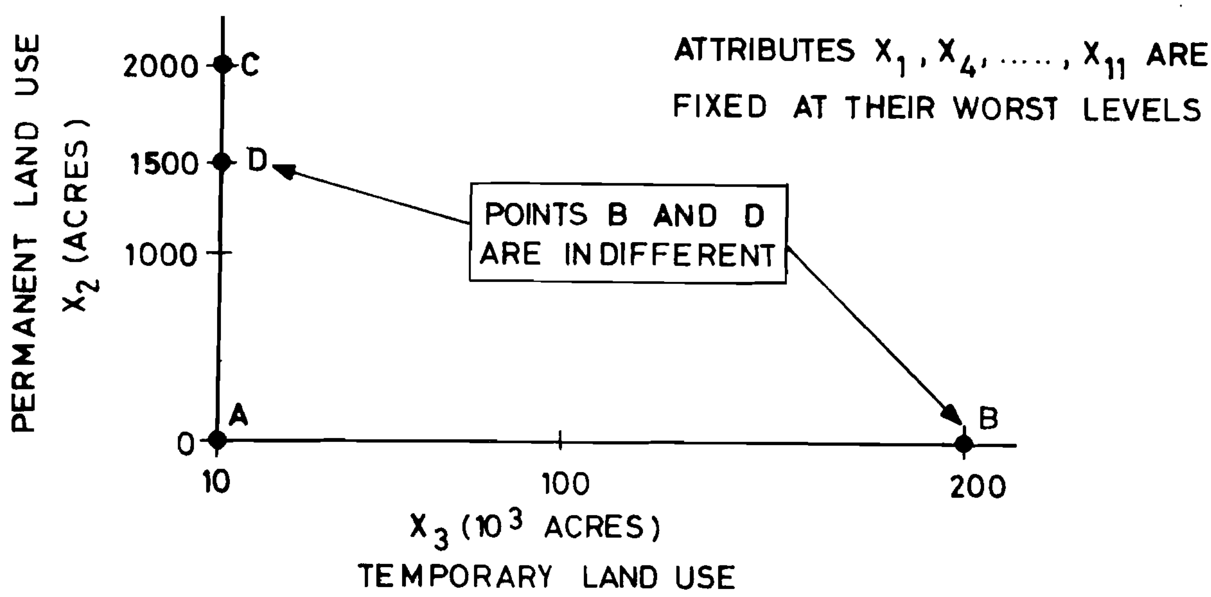


Figure 3. Temporary versus permanent land use: other attributes at worst level.

K: I might question that. If X_2 is just supposed to measure the land use as opposed to why the land is actually unusable, here is a ratio of 200 to 2 (land temporarily unusable to land permanently unusable in the last question), and you prefer to give up the temporary usage.

B: That's true. It's the problem of quantifying the permanency.

K: Of course there is no right or wrong answer here.

B: Right, I understand that. I don't feel too much difference between the two (points B and C in Figure 3), to be perfectly honest. If you changed the question to go to either 1000 permanent or 200,000 recoverable, I'd switch and choose the 1000. [This means that (x_2 = 1000, x_3 = 10,000) is preferred to (x_2 = 0, x_3 = 200,000).]

K: Let's take those as two answers, and now choose x_2 = 1500. You either lose 1500 permanently (point D) or 200,000 temporarily (point B). I am actually looking for some level of X_2 between 1000 and 2000 where you are indifferent.

B: 1500, that is about it, I think.

K: Now when you consider this tradeoff, did you think at all about where the other attributes were fixed?

B: No, I didn't.

K: Well, I think we can speed up a bit now. Consider Figure 4 where attributes other than X_2 and X_3 are at their best levels. Do you prefer point B' or C'?

B: Point B', I guess.

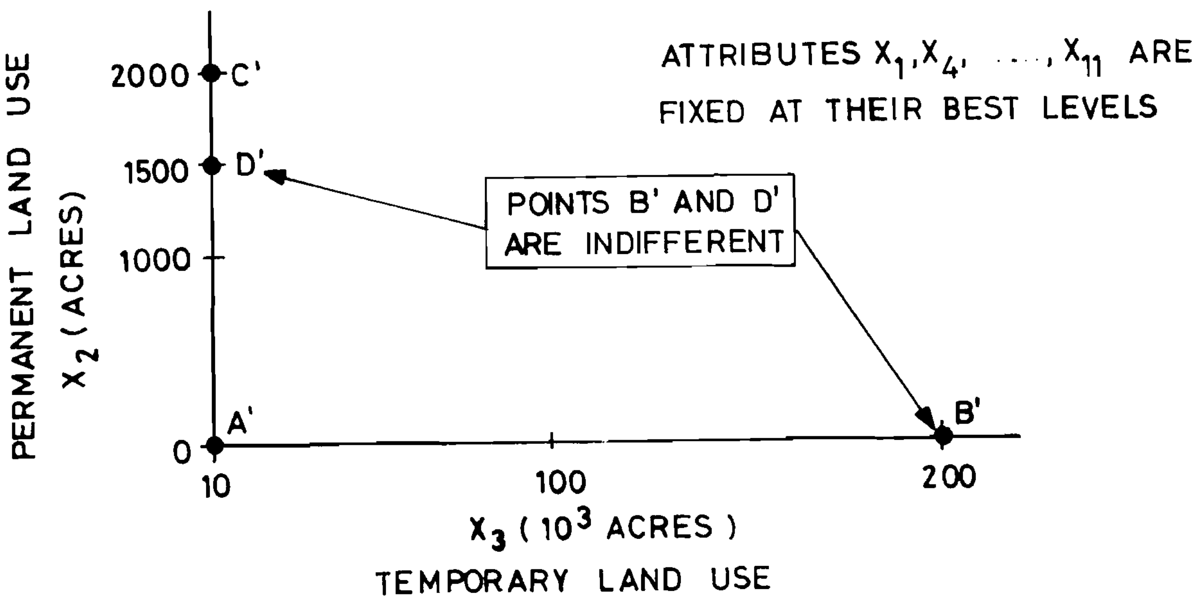


Figure 4. Temporary versus permanent land use: other attributes at best level.

K: Would the indifference point be the same? That is, are you indifferent to D' and B'?

B: Yes, I think so; it's the same. I don't feel any difference.

K: This is an indication that the indifference curve over X_2 and X_3 is the same regardless of where the other attributes are. This is the preferential independence condition. Do you think that as a general rule, the tradeoffs between X_2 and X_3 don't depend on the other attributes?

B: Yes, that's true...as long as the other attributes are fixed.

K: This indicates that the pair $\{X_2,X_3\}$ is preferentially independent of the other attributes.
So let's go on. Which one of these first three attributes is easiest for you to think about? I will then use that to examine additional tradeoffs.

B: Oh, I see. Frankly, with X_1, I feel the indifference levels would be very low; I think one of the other two would be better.

K: Then I'll take X_3, and we'll examine the tradeoffs between X_3 and X_4. Let me begin with a naive question. Is it better to have less water evaporated than more, always?

B: Indeed.

K: So the best consequence in Figure 5 is point A. Here we have fixed all attributes but X_3 and X_4 at the worst levels. Which would you prefer, point B or point C?

B: I would say the water loss is preferred; I'd rather lose the additional trillion gallons than the 200,000 acres.

K: How about if you compare point B to point D where X_3 is at 100,000 acres? This is actually a change of 90,000 acres...from 10,000 at point A.

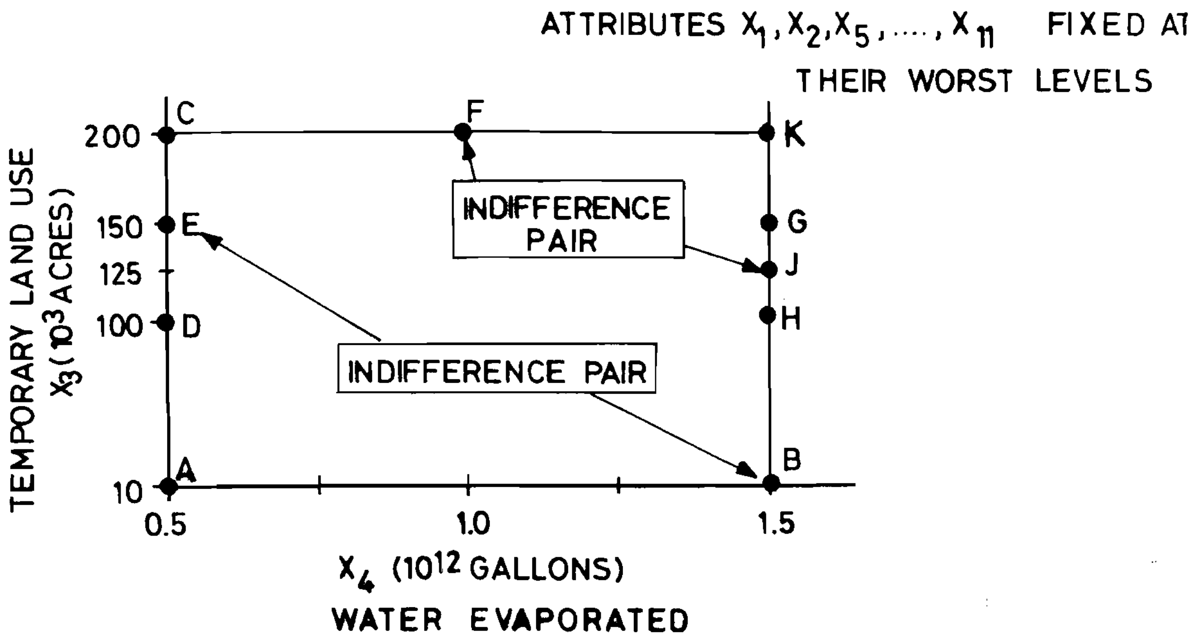


Figure 5: Water evaporated versus temporary land use: other attributes at worst level.

B: A 100,000 acres versus a trillion gallons...that's a lot of water. I guess I'd go with the land loss. I'd rather lose the land than a trillion gallons.

K: Since D is preferred to B which is preferred to C, there must be a point between C and D where you are indifferent. What I usually do then is halve the difference. I say usually because, for instance, if you easily answered that B was preferred to C, but had a hard time deciding D was preferred to B, this would imply that the indifference level was near 100,000. Well then, I might give you 170,000 to make the answer a little easier and help you to converge. [Discussion like this is meant to take the mystery out of the assessment process, to help develop rapport and to give one a break now and then.]

B: I see, that's a good policy.

K: So, how about x_3 = 150,000 (i.e., point E) or the 1.5 trillion gallons?

B: I'd say that's about as closely as I could define it. I'm indifferent.

K: Moving to Figure 6, if we change all the attributes other than X_3 and X_4 to their best level, do you see any difference?

B: No, I don't.

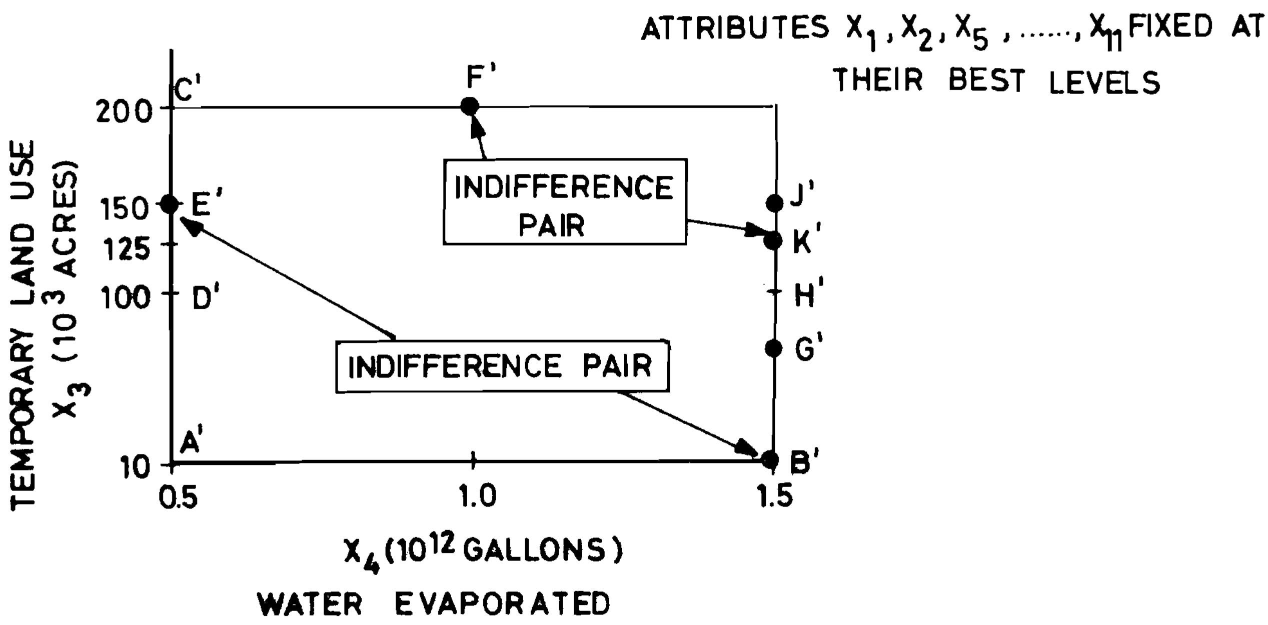


Figure 6. Water evaporated versus temporary land use: other attributes at best level.

K: Fine. Now let's back up to Figure 5 and consider point F versus point G. [Strictly speaking, one should ask questions in different ranges of the $\{X_3,X_4\}$ consequence space to verify preferential independence conditions. Assumptions that are reasonable for part of the $\{X_3,X_4\}$ space cannot necessarily be extrapolated to all the $\{X_3,X_4\}$ space. The analyst's judgment must be used to decide exactly how much can be implied by specific responses.]

B: Okay, I have a choice of 200,000 acres and one trillion gallons versus 150,000 acres and 1.5 trillion gallons.

In this case...I'm not sure if I'm confused but I think I'd take F. Maybe I should sit down with my pencil and think about this a little bit.

K: One way to look at this is as follows. You are at F. Are you willing to give up further 0.5 trillion gallons, in addition to the one trillion, in order to reduce land temporarily unusable from 200,000 to 150,000 acres?

B: Would I go that way...no, I don't think I would.

K: You'd stay at F.

B: I think I would.

K: How about if you could go from F to H?

B: Yes, I think I would do that.

K: And where might you be indifferent to F between G and H? How about 125,000 acres?

B: Yes, that's about as close as I can come.

K: Then let's jump back to Figure 6 and consider F' versus G' with X_3 at 75,000 acres. Which would you prefer?

B: I would prefer the 75,000 acres and the 1.5 trillion gallons.

K: And how about 140,000 acres?

B: That's very close again.

K: Do you see any reason why it should be different than before?

B: I don't see any differences.

[To try to promote independent thinking each time, the order used to converge to indifference is varied. To see this, compare the sequence G,H,J in Figure 5 to the sequence G',H',J',K' in Figure 6. With the given responses, we can reasonably assume that $\{X_3, X_4\}$ is preferentially independent of the other attributes.]

K: So now let's try the tradeoffs between X_3 and X_5, the sulfur dioxide emission. My understanding is that the sulfur dioxide is here for effects other than on health. Is that correct?

B: Yes; both X_5 and X_6 are meant to include aspects other than health effects. The health effects of chronic air pollution are considered as part of attribute X_{10}. The fatalities from acute SO_2 exposure are in X_1.

K: So what effects do you wish to pick up here?

B: Visual effects, damage to buildings, odors, more frequent washing of clothes, damage to property, reducing land values, crop damage, etc...things like that.

K: Okay, consider the tradeoffs between X_3 and X_5 illustrated in Figure 7. Note that one advantage of using the same attribute in the tradeoffs is that you get used to thinking in terms of that attribute. Suppose you are at the best point A and must move to either B or C.

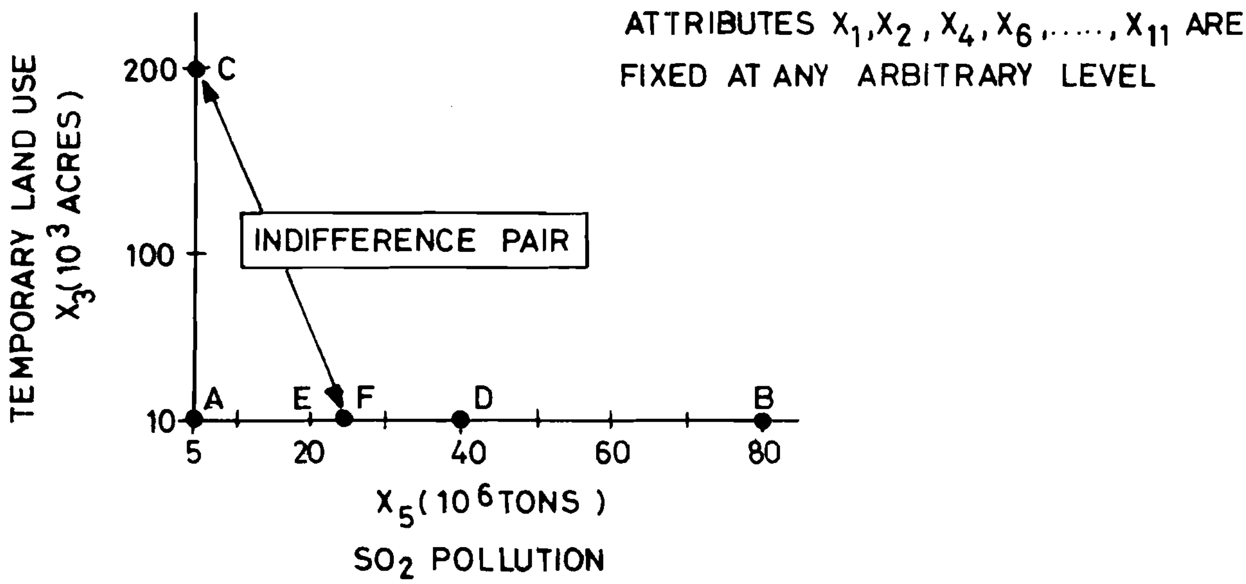


Figure 7. SO_2 pollution versus temporary land use: other attributes at arbitrary levels.

B: I would prefer to lose the 200,000 acres.

K: And what if we reduce X_5 to 40 million tons? Do you prefer point C or D?

B: I think I would still prefer to go to the 200,000 acres (point C).

K: And if X_5 is 20 (i.e., point E)?

B: I guess at 20, I'd take the sulfur dioxide to the 200,000 acres.

K: Where will this break?

B: Oh, 25 million tons of SO_2.

K: [If one has a hard time answering this last question, I'll usually offer a specific level of X_5. In the first case, the decision maker must select an indifference level from an entire range, and in the second case, he must only decide on which side of the specific given level is indifference. The second question is easier to answer.]
Is it reasonable to assume that your answer above does not depend on where the levels of the other attributes are fixed, since I didn't specify them and you didn't ask me?

B: That's right. I really don't feel any difference. As long as they are held fixed, they are not involved in the tradeoff between X_3 and X_5.

K: Let me push a bit farther, because I would personally find it easier to argue against the assumption in this case than in many others. In particular, let's look at the impact due to knowing the particulate pollution level measured by X_6. Suppose I fix the particulate pollution at its worst level--10 million tons--and I again ask whether you would be indifferent between C and F in Figure 7.

B: But I am stuck with 10 million tons of particulate in both cases?

K: Yes, you're stuck with the 10 million tons.

B: Then I'd still be indifferent between C and F.

K: Now suppose you have just 0.2 million tons of particulate in all cases, would your answer change?

B: No, I'd still be indifferent.

K: Let me suggest a rationale that would imply that there should be a difference, depending on whether particulate pollution was low or high. Of course, remember that there is no right or wrong. My rationale is that people would view air pollution as a whole. If there is a lot of particulate, a little increase in SO_2 may have serious effects, whereas if particulate pollution were low, the same increase in SO_2 would be relatively unimportant. In such a case, one might for example give up more of usable land to reduce SO_2 from 40 to 30 million tons if particulate pollution were high than if it were low. Such preferences would violate the preferential independence condition.

B: As a matter of fact, there have been some studies which indicate that SO_2 and particulate together cause more health effects than equivalent amounts do separately. However, this is health effects, and these are excluded from attributes X_5 and X_6. In terms of damage costs, it's the acid more than anything from the SO_2, whereas it is the sooting from the particulate. There doesn't seem to be much synergism in this context, so I would remain with my previous responses.

K: Fine, then we can assume that $\{X_3, X_5\}$ is preferentially independent of the other attributes. I'm not sure that I ought to belabor the point. Is there...

B: I don't think so. If I considered each of these other attributes compared to say land use, I don't see why the tradeoffs would depend on the levels of the additional attributes as long as they are held fixed.

K: Thus we will assume that each of the pairs $\{X_3, X_i\}$ for $i = 1,2,4...,11$ is preferentially independent of the other nine attributes. This satisfies the preferential independence conditions necessary to invoke either an additive or a multiplicative utility function. [The formal result is given in the theorem of Section 3. Strictly speaking, we did not check to see whether $\{X_1, X_3\}$ was preferentially independent. However, since $\{X_1,X_2\}$ and $\{X_2,X_3\}$ were each P.I., it follows from a result of Gorman [4] that $\{X_1,X_3\}$ is P.I.]

4.2 Verification of Utility Independence Conditions

K: Now let's check the utility independence assumption. We will begin by looking at your preferences for different numbers of fatalities, indicated by attribute X_1, with all the other attributes fixed. Consider the lottery illustrated in Figure 8. This lottery gives you a one-half chance of 100 fatalities with all other attributes at their best level, and a one-half chance of 700 fatalities with all other attributes fixed at their best level. The question is, would you prefer the lottery or an option which gives you 600 fatalities for sure with all other attributes at their best level?

Figure 8. A lottery involving fatalities; other attributes at best levels (before assessing certainty equivalent).

B: I'd take the lottery.

K: Okay, how about if fatalities are 150, with other attributes at the best level?

B: I'd take the 150.

K: How about 200?

B: I'd take the 200.

K: 450?

B: That's pretty close. At 450, I'd take the lottery I guess.

K: 375? The average of the lottery as you know is 400.

B: It would take something slightly under 400 for me to choose it. I'd take 375.

K: And what if it were 400 versus the lottery?

B: At 400 I'd take the lottery, but if it were slightly under 400, I'd be very tempted to take the sure consequence.

K: At 390?

B: Yes, I'd choose around 390 as an indifference point.

K: Now why is this slightly under 400 as opposed to right on?

B: Well, I feel that as long as the expected value is the same, I'd prefer to accept the risk for the chance that it might come out right. But if there is a little bonus in there, I think it not worth the risk that 700 people may die.

K: With that reasoning, should you perhaps prefer 399 to the lottery? That's a bonus of one expected life.

B: That's right. Maybe that's it. I'd be indifferent at 399. For all practical purposes, I guess it could be 400. [After this process, Figure 8 ends up as Figure 9].

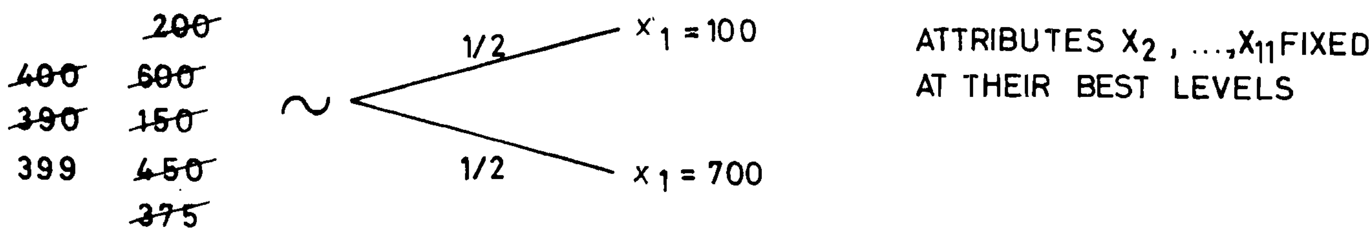


Figure 9. A lottery involving fatalities; other attributes at best levels (after assessing certainty equivalent).

K: Going on, let's ask a similar set of questions concerning levels of fatalities with all other attributes fixed at the worst level. Refer to Figure 10, a lottery with a one-half chance of 100 fatalities and a one-half chance of 700 fatalities. Would you prefer the lottery or 600 fatalities, with all other attributes again fixed at their worst levels?

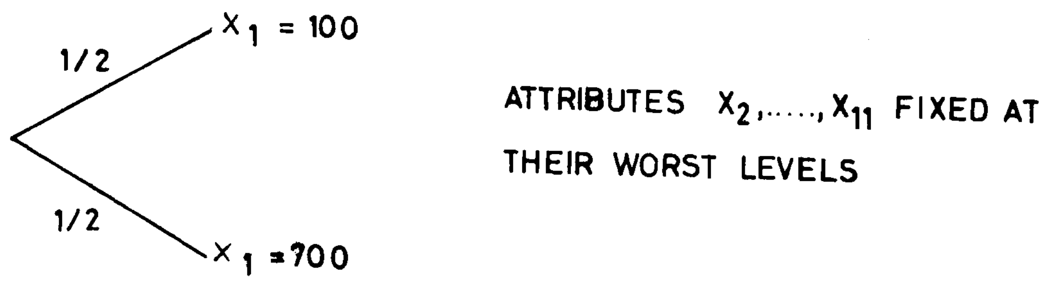


Figure 10. A lottery involving fatalities; other attributes at worst levels.

B: I'd take the lottery.

K: How about the lottery versus 250?

B: I'd take 250.

K: How about 300?

B: I'd take 300.

K: 500?

B: I'd take the lottery.

K: [All the previous four questions seemed easy to answer so I asked a general question.]
And where would you be indifferent?

B: Essentially the same point, just a shade under 400.

K: The thing to note here is that it appears that your indifference point does not depend on the levels of the other attributes. The relative preferences that you attach to different levels of fatalities seem to be independent of the other attributes as long as their levels are fixed. Is this true in the general case?

B: That's right.

K: This implies that X_1 is utility independent of the other attributes. This assumption, together with the preferential independence assumptions which we already verified, implies that your utility function must be either additive or multiplicative. [The conditions for the theorem in Section 3 have been verified. These conditions also imply that each attribute must be utility independent of all the others.] But let's just try one more attribute as a check. How about taking X_8, radioactive waste storage, since we haven't said much about that. Let's fix all attributes at their worst levels and examine the lottery in Figure 11. Here you get either 200 or 0 metric tons, each with a probability of one-half. Would you prefer the lottery or 40 metric tons with all other attributes at their worst amounts?

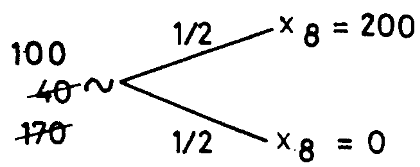


Figure 11. A lottery over radioactive waste with certainty equivalent shown.

B: I'd take the 40.

K: How about 170 metric tons?

B: I'd take the lottery. In this case I'd go right to the expected value of 100. Yes, 100 metric tons would by my indifference point.

K: Does your answer to this depend on the other attributes?

B: No.

K: [I felt that plutonium produced might have some effect on the previous response. Although Buehring's general response implied that this was not so, a specific check to see whether some aspect has been overlooked is sometimes prudent.]

For instance, suppose I told you that X_9 was high--that many tons of plutonium were produced. This could lead to a high theft level of such material. If I told you theft was high, would it change your 100 indifference level in Figure 11?

B: No.

K: Fine. Now let's check X_8 between 0 and 100 metric tons produced. If you had a fifty-fifty lottery yielding 0 or 100 metric tons, again with other attributes at their worst levels, where are you indifferent? [Figure 12 was used as an illustration of the lottery.]

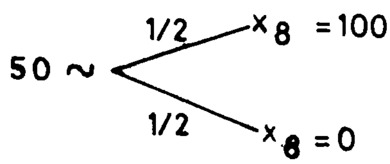


Figure 12. A lottery over radioactive waste with certainty equivalent shown.

B: Right at 50.

K: Does this answer depend on the other attributes?

B: No.

K: To cover the range of X_8, are you always indifferent to the choice between 150 and a lottery yielding 100 or 200 with equal probabilities, as long as other attributes do not vary?

B: Yes.

K: And now the general question. For any such lottery questions involving X_8, regardless of where the other attributes are fixed, are you linear? Would you always be indifferent at the expected value?

B: Yes. Of course there is one little complication. Since both X_8 and X_9 are nuclear effects, if terrorist activity related to plutonium theft were very high, I suppose there would be some extra resentment of radioactive waste. Is that okay?

K: I'll let you answer that. Suppose there is that high resentment, what is your indifference point to the lottery in Figure 11 yielding 0 or 200 metric tons of waste?

B: I'm not sure I feel this myself, but I could see how someone might say: "If a lot of plutonium blackmailing is going on, I am going to feel worse about radioactive waste storage. Therefore I'd demand something lower than 100 before I'd be indifferent."

K: I think you may be mixing up two things. Suppose you said you were indifferent to 80 metric tons and the lottery.

B: Okay.

K: Well, then I offer you 90 versus the lottery, and you say no, since there is so much resentment. Well, there would be a lot more resentment to the 200 metric tons that you're apparently willing to risk.

B: That's true.

K: From my viewpoint, let me try to state what I think is your concern. Let's return to where you said you were indifferent to 100 and the lottery in Figure 11 regardless of where other attributes are fixed. Take the case where there is no theft and no resentment. Then, sloppily speaking, you might say that the jump from 0 to 100 isn't too important; but then neither is the jump from 100 to 200 too important. They are equally important, but neither one is critical. However, if the theft is high and there is much resentment, then the jump from 0 to 100 is very important because of all the concern about waste; but the jump from

100 to 200 is also very important. Again they are equally important. What we are concerned with in finding your indifference levels to the lottery of Figure 11 is whether the jumps from 0 to 100 and 100 to 200 are equally important <u>given</u> the other attributes, and not, for instance, whether it is more important to go from 100 to 200 if theft and resentment are high or low. The qualitative feeling that you're giving to me is that you would be a lot more concerned about high levels of radioactive waste storage if there were theft than if there were no theft.

B: Yes.

K: This does not imply that your relative preferences for various storage levels change depending on where theft (as represented by production available) is fixed.

B: Yes, I agree.

K: Now what such an attitude may do is affect your tradeoffs between say X_3 and X_8, given levels of X_9. (That is, it may affect the preferential independence condition.) For instance, if theft is high, you may be willing to give up more usable land temporarily to reduce radioactive waste from 100 to 50 metric tons, than you would be if theft were low. This is the type of preferences indicated by your comments. So it has to do with the evaluation of radioactive waste versus other attributes as a function of tons of plutonium produced, rather than with the relative preferences for various levels of waste as a function of plutonium produced.

B: I believe I was thinking of simultaneous changes in the level of theft as I was changing radioactive storage levels. I can see how the argument says that 100 should be my indifference level for the fifty-fifty lottery of 0 or 200 metric tons of waste.

K: Okay, then we'll assume that X_8 is utility independent of the other attributes.
[Next we went back and explicitly checked whether Buehring did feel that tradeoffs among X_3, land temporarily unusable,

and X_8, radioactive waste, depended on the tons of plutonium produced. It turned out that he did not. Hence we continued to assume that $\{X_3,X_8\}$ was preferentially independent of the other attributes.]

4.3 Ordering the Scaling Constants

K: Now we come to the assessment part. The conditions we have just verified imply that either the additive form or the multiplicative form of the utility function discussed in Section 3 must hold. To assess either of these, we need to get the k_i's and the u_i's. From the theorem in Section 3, k is calculated from the k_i's if the multiplicative form holds. The tough part is probably assessing the k_i's.
As a first step, let's try to order the k_i's. To do this, refer to Table 1 and assume that all attributes are at their worst levels. To get the rankings, we need to know the order in which you would push these attributes up from their worst to their best levels if you had the choice. First, if you could push just one of them from the worst to the best level, which attribute would you choose? To help you think about this, let me go through some of them pairwise.
Take attributes X_1 and X_2. Consider an option leading to 700 fatalities and 2000 acres of land permanently unusable; both attributes are at their worst level. Would you rather move up to 100 fatalities or to 0 acres of land?

B: 100 fatalities.

K: This answer, which implies that k_1 is greater than k_2, seemed clear from the beginning of our discussion. [Had Buehring responded 0 acres, I would have pushed for the reasoning.]

B: Right.

K: So now I'll take the better of these two and compare it with temporary land unusable. Would you rather go from 700 to 100 fatalities or 200,000 to 10,000 acres of land?

B: Change the fatalities. (This implies that k_1 is greater than k_3.)

K: How about water evaporated, 1.5 to 0.5 trillion gallons, or...

B: Fatalities; the 600 additional fatalities are going to be the most important I think.

K: Well, let's try radioactive waste: 200 to 0 metric tons stored or 700 to 100 fatalities. Now presumably some of the thoughts here concern possible genetic impacts of the radioactive wastes.

B: Yes, that's true.

K: So is that worth the 600 people between now and the year 2000?

B: No, it isn't; the 600 is still worth more. (Thus k_1 is greater than k_8.)

K: How about nuclear safeguards: is it better to go from 50 to 0 tons produced or from 700 to 100 fatalities?

B: I'd still prefer to save the people.

K: And the lead produced measured by X_{10}?

B: Chronic health effects--that's a mysterious one. That could be worth more than 600 actually, but I don't think it is. [An analysis of preferences often indicates questions like this which are important in determining policy, but for which the decision maker needs more information. Often such information is available. Once the question is clearly articulated, one can begin to look for the answer.]

K: So you'd take the 600?

B: Yes.

K: And how about electricity generated, 0.5 to 3?

B: That's an interesting one; preferences go the other way.

K: I think the way to think about this involves what happens to Wisconsin if only 0.5 trillion kilowatt-hours are produced.

B: It's hard for me to think about X_{11}, electricity generated. The level of 0.5 trillion kilowatts might not cause that much suffering. [Then Buehring checked some electricity consumption table for Wisconsin.] At our current consumption rate, we will use 0.9 trillion kilowatts between now and 2000. That is a cut of almost half. But I think I'd still make the choice of saving the 600 people.

K: Of course, some of this electricity may run kidney dialysis machines, for example.

B: Yes, that's true; but I'm assuming that cuts would be selective and such things as hospitals and schools would stay in operation.

K: However, with a fifty percent cut in electricity, you would certainly affect life style. But anyway, you choose the 600 fatalities to be the most important.

B: That's right.

K: This means the largest k_i is k_1. [A common error made in many studies is to ask which of several attributes is most important, independent of their ranges. If the range of fatalities were changed from 700 to 690, changes from best to worst on several other attributes would have been more important than 700 to 690 fatalities. See Chapter 5 of Keeney and Raiffa [6] for details.] Now we need to look for the next-most important change after fatalities. To be quick--based on your previous answers--let's start with radioactive waste, chronic health effects, and electricity generated, all at their worst levels. Which of these would you rather move up to its best?

B: Chronic health effects, I'd have to say. [This implies that k_{10} is greater than k_8 or k_{11}.]

K: How about chronic effects relative to nuclear safeguards?

B: That's very close, but I think chronic effects.

K: Just glance over the other attributes now: energy needed, X_7, for example.

B: That one doesn't bother me so much.

K: So chronic health effects would be No. 2.

B: Yes. [This implies that k_{10} is second-largest next to k_1].

K: What would be No. 3?

B: It's difficult for me to grasp all the implications of energy generated. If electrical energy production is really 0.5 trillion kilowatts, it will be pretty tough.

K: So would you like to go back and give up those 600 people?

B: No, no, I wouldn't, but electricity generated is important.

K: And chronic health effects is still No. 2?

B: Yes, but I think electricity generated comes in here now; then I think the nuclear safeguards.

K: You mean the change in tons of plutonium produced from 50 to 0 is more important than the change in waste from 200 to 0 metric tons.

B: Yes, sure.

K: Okay, and now assume that all the attributes X_2 through X_8 are fixed at their worst levels.

B: I would first pick SO_2.

K: It's more important than the waste problem?

B: Yes, that's my bias; I'm not too worried about radioactive waste. I do worry about nuclear safeguards, but I don't think waste is that big a problem. However, I would put radioactive waste next.

K: Okay, now you have attributes X_2, X_3, X_4, X_6, and X_7 left. [We are continually using Table 1 in the discussion.]

B: Next is permanently unusable land. Of the alternatives, I'd prefer to move it from 2000 to 0 acres.

K: Okay, now there is X_3, X_4, X_6, and X_7.

B: Energy needed doesn't bother me. That one is going to be last. I think temporarily unusable land, then water, then particulates, and then thermal energy needed. So it goes X_3, X_4, X_6, X_7.

K: Good, let's check the order then. We started out with all attributes in their worst case. And you preferred moving fatalities from 700 to 100 to eliminating the chronic effects due to 2000 tons of lead pollution.

B: Yes, that's right.

K: You preferred avoiding 2000 tons of lead pollution to raising electricity generated from 0.5 to 3.0 trillion kilowatt hours.

B: Yes.

K: And that electricity increase you preferred to reducing the plutonium produced from 50 to 0 tons. [We continued in this manner and found no changes in the order. Thus, we have $k_1 > k_{10} > k_{11} > k_9 > k_5 > k_8 > k_2 > k_3 > k_4 > k_6 > k_7$.]

K: Many of your above responses could have been inferred from earlier choices when we were checking for preferential independence. For example, look at the tradeoffs between temporarily unusable land and water evaporated in Figure 5 (see page 15).

B: Okay.

K: There you said you preferred consequence B to consequence C. Thus if you began at point K in that figure and had to go to either B or C, you would prefer to go to B. This says that you would rather move from 200,000 to 10,000 acres used than move from 1.5 to 0.5 trillion gallons of water evaporated, which is exactly what you said in evaluating k_3 versus k_4. [Other information given in checking for preferential independence conditions was also consistent with the ordering of the k_i's.]

4.4 <u>Assessing the Scaling Constants: Tradeoffs Among Attributes</u>

K: Now that we have the order of the k_i's, let's assess the tradeoffs to get their relative values. Let's start with X_1 and X_{10}. Refer to Figure 13. You said previously that you prefer consequence B to consequence A. Thus, if you were at A, 700 fatalities and 0 tons of lead, you would be willing to increase lead to 2000 tons in order to decrease fatalities to 100. Is that right?

B: Yes.

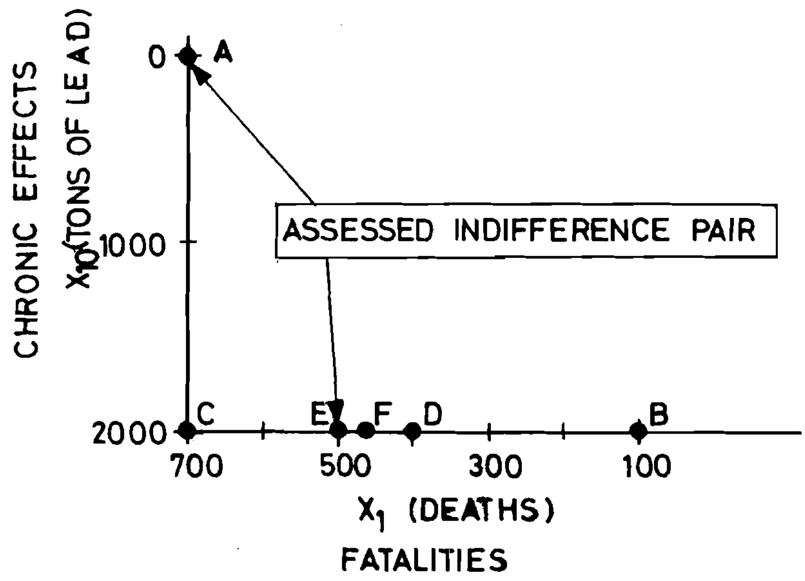


Figure 13. Fatalities versus chronic health effects.

K: What if you only got to move to 400 fatalities? That is, would you be willing to move from A to D?

B: I would still rather save the 300 people [i.e., the 700 -400].

K: What if you can only go to E--500 fatalities--and you are saving 200?

B: I'd say that's pretty close to what I feel is equivalent to the chronic effects, so at that point I might switch.

K: You would switch or be indifferent?

B: Be indifferent.

K: Let's look at what this says. Because of the preferential independence conditions, we can assume that all other attributes are at their worst level--so $u_i = 0$ for $i \neq 1$, 10--and equate the utilities of points A and E since you are indifferent between them. Using either the additive or multiplicative utility function, we find that the utility of A is k_{10} and the utility of E is $k_1 u_1(500)$.

B: Okay.

K: Hence the relationship between k_1 and k_{10} is $k_{10} = k_1 u_1(500)$, where the utility function u_1 is measured on a zero-to-one scale. Based on what you told me in checking for utility independence, your utility function for fatalities is essentially linear. Since $u_1(100) = 1$ and $u_1(700) = 0$, $u_1(500)$ must be about 0.333. Thus we would have $k_{10} = 0.333k_1$. We'll refine this later, but for now let's go on.

Look at the tradeoffs between X_{11} and X_1 in Figure 14. We want to find a point on the X_1 axis, with $X_{11} = 0.5$, that is indifferent to point A. The question is, how many fatalities must you save in order to accept the decrease in electricity from 3.0 to 0.5 trillion kilowatt hours? That's tough I know, but I'll ask it anyway.

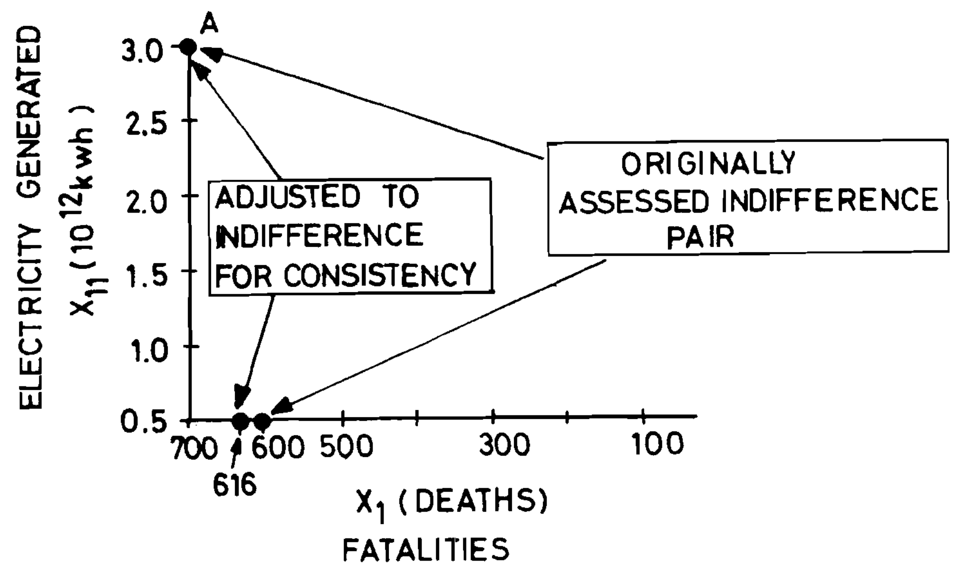


Figure 14. Fatalities versus electricity generated.

B: That is tough. It is certainly less than the last question, less than the 200 [i.e., the point $x_1 = 500$].

K: That follows in order to have $k_{10} > k_{11}$.

B: All right; well, I'm still confident of that. About 100 at the most.

K: Let's try 50. Suppose you had 650 deaths and 0.5 trillion kilowatt-hours, or 700 deaths and 3.0. Which would you prefer?

B: I might take the 700 and 3.

K: And what if it's 550 and 0.5 or 700 and 3?

B: Okay, I'd take the 550.

K: And where would you be indifferent; how about at 600?

B: That's about it. I'd say that's pretty close.

K: What this implies is that $k_{11} = k_1 u_1(600)$ or $k_1(.167)$, because that's...

B: One-sixth.

K: Now we can run checks on this. Refer to Figure 15 and presume you are at point A, 2000 tons of lead and 0.5 trillion kilowatts. Would you rather eliminate the lead (point B) or increase electricity production to 3.0 (point C)?

B: Lose the lead.

K: That's consistent with your previous responses, since with other attributes at their worst levels, both the additive and multiplicative utility functions imply that the utility of B is k_{10} and the utility of C is k_{11}. And you have said k_{10} is greater than k_{11}.
Now back up X_{10}. Suppose you could only go to 1500 lead (point D) or 3.0 million kilowatt hours (point C). Which do you prefer? Another way to think of this is, you're at point D with 0.5 electricity and 1500 lead, and you are told you can increase electricity to 3.0 if you are willing

to accept 500 more tons of lead. Would you do it?

B: Yes, I guess I would.

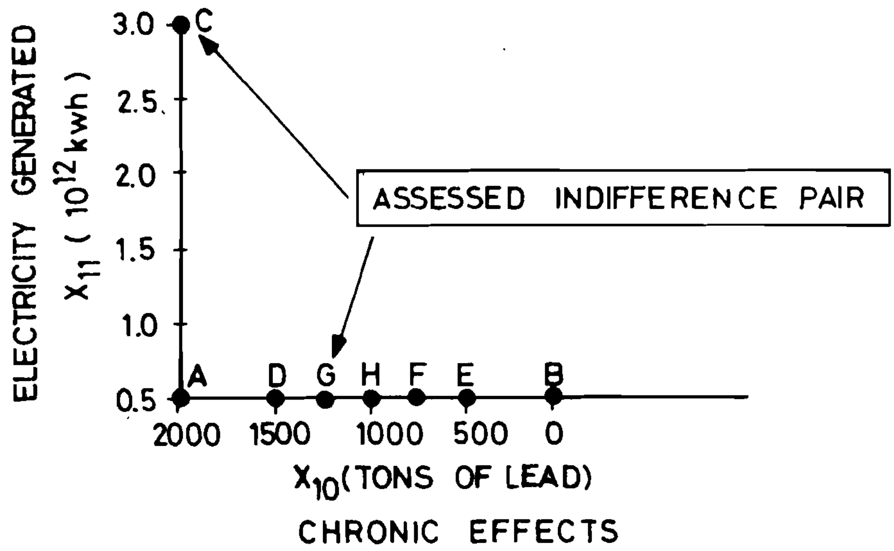


Figure 15. Chronic health effects versus electricity generated.

K: How about if you started at 500 tons of lead (point E); would you accept the additional 1500 to jump up to 3?

B: No.

K: If you started from 750 tons (point F)?

B: No.

K: How about at 1250 (point G)?

B: Okay. That's pretty close. That sounds about where I'd be indifferent.

K: How about if you are at 1000 (point H) and someone says, "For an additional 1000 tons of lead, I can move you to 3". Would you accept that or not?

B: No, I don't think so.

K: You had to tell me that because, if you are indifferent to accepting 750 more tons (i.e., x_{10} = 1250), you'd better not accept 1000 more. What this says is...

B: ...that I'm probably confused. This probably isn't consistent.

K: Since C and G are indifferent, we set their utilities equal and find $k_{11} = k_{10}u_{10}(1250)$. I don't know what $u_{10}(1250)$ is, but we can do a quick assessment of u_{10}. Refer to Figure 16. We've got a range of X_{10} from the worst point, 2000 tons to the best point, 0 tons. Because of our scaling convention, we set $u_{10}(2000) = 0$ and $u_{10}(0) = 1$. Now consider the fifty-fifty lottery of 0 or 2000 tons shown in Figure 17, and suppose you have this option or $x_{10} = 500$ for sure. Which would you take?

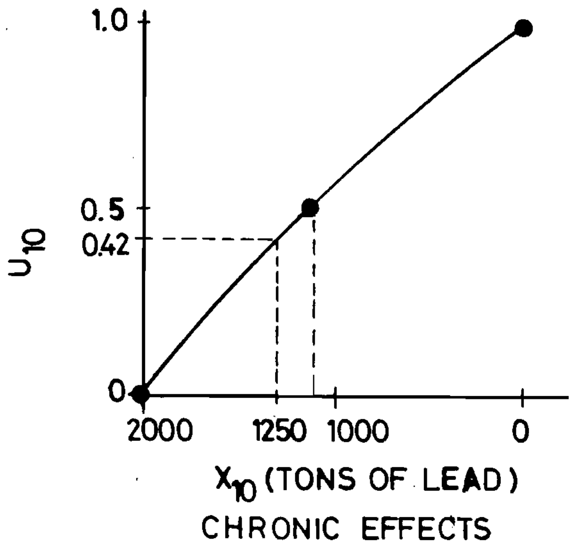


Figure 16. Utility function for chronic health effects.

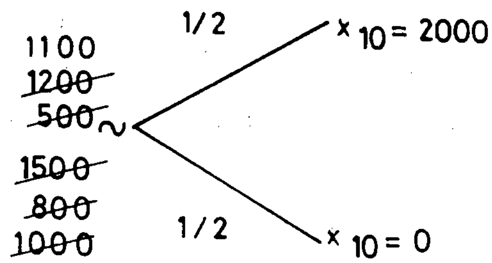


Figure 17. Finding the point of indifference to a lottery.

B: 500.

K: The lottery or 1500?

B: I'd take the lottery.

K: How about at 1200?

B: I'd still probably take the lottery.

K: Then we go to 800?

B: I'd take the 800 for sure.

K: 1000?

B: I'd take the 1000, I think.

K: The average in the lottery is 1000, as you know. So, 1100?

B: That's pretty close--you can probably say 1100 is indifferent. I'd take 1050.

K: You'd take 1050 over the lottery?

B: Yes. I'd take 1050 over the lottery.

K: And not 1100?

B: 1100, I don't know; that's pretty close.

K: How about 1200?

B: At 1200, I'd take the lottery.

K: Okay. I'll take 1100 as indifferent.

B: All right.

K: This says that the utility assigned to 1100 must equal to the utility of the lottery. It's assigned that way so we can use expected utilities in evaluating alternatives. Hence we assign $u_{10}(1100) = 0.5$ and plot it on Figure 16. Would you prefer the fifty-fifty lottery yielding 0 or 1100 tons of lead or an option of the average 550 tons for sure?

B: I'd take the 550.

K: And which would you choose between the fifty-fifty lottery yielding 1100 or 2000, and 1550 for sure?

B: Again, I'd take the sure consequence, the 1550.

K: These last answers imply that you are risk averse in the attribute chronic health effects, so, as a first approximation, we can sketch in the concave utility function u_{10} in Figure 16. [Later a constantly risk averse function will be fit. This degree of precision on the single-attribute utility functions in a multiattribute problem is probably sufficient in most cases. Subtle differences in risk attitudes on the individual attributes are likely to have little effect relative to variations in the k_i values and the general shape of the u_i functions.]

Now we can return to the equation $k_{11} = k_{10}u_{10}(1250)$. Eyeballing it from Figure 16, I'd say $u_{10}(1250) = .42$, implying $k_{11} = .42\ k_{10}$. With this, we have assessed three equations relating k_1, k_{10}, and k_{11}: namely $k_{10} = .333\ k_1$, $k_{11} = .167\ k_1$, and $k_{11} = .42\ k_{10}$. They are reasonably consistent, but a slight alteration is required. If just one of them is changed, we find that the parameter in the first one must be .4, so $k_{10} = .4\ k_1$, or that the second becomes $k_{11} = .14\ k_1$, or that the third becomes $k_{11} = .5k_{10}$. Let's see how much your answers leading to the original three equations would have to change in order to get the new consistent equations.

Assuming linear preferences for fatalities, $u_1(460) = .4$, so you would have to be indifferent to points A and F in Figure 13 to adjust the first equation. Alternatively, in Figure 14, since $u_1(616) = .14$, the indifference point should be at 616 fatalities instead of 600 to change the second equation. To adjust the third equation, you could either be indifferent between ($x_{10} = 1100$, $x_{11} = 0.5$) and ($x_{10} = 2000$, $x_{11} = 3$) in Figure 15 or adjust from 1100 to 1250 the value of X_{10} for which you are indifferent to the lottery in Figure 17. Of course, there are options of adjusting each of these by a small amount. However, it is easier just to move one to be consistent.

B: Yes; the one I feel the least strongly about is the one in Figure 14. You said you could move the 600 to 616 fatalities, is that right?

K: Right.

B: I think I would notice the 40 additional fatalities in Figure 13; 1100 seems low on the tradeoff in Figure 15, and 1250 seems high as an indifference amount for the lottery. Yes, I think I'd be happy to change the 600 to 616 in Figure 14.

K: Fine, then for now we are consistent in our tradeoffs among attributes X_1,X_{10}, and X_{11}. What we should do now is go through the same procedure for each of the other eight attributes. We can look at the tradeoffs relative to deaths, attribute X_1, or to attribute X_{10} or X_{11}. Let's choose something other than deaths just to indicate how to do that. Since we already have a rough utility function u_{10}, consider attribute X_{10}, tons of lead, versus attribute X_9, nuclear safeguards. Note that X_9 was the attribute whose scaling factor was the fourth-largest. In Figure 18, the worst point G of the possible X_9,X_{10} combinations is (x_9 = 50, x_{10} = 2000). If you were at that point, would you rather move to point A or point B, saving respectively 50 tons of plutonium produced or 2000 tons of lead produced?

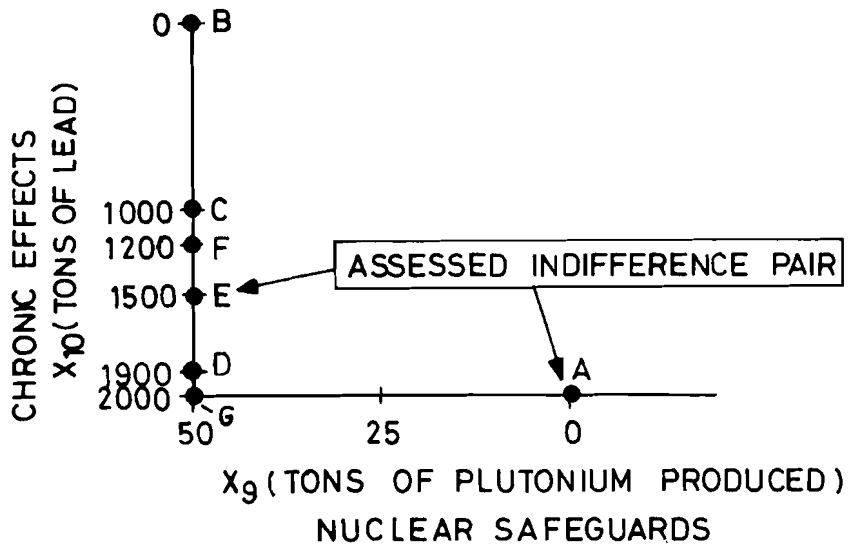


Figure 18. Nuclear safeguards versus chronic health effects.

B: I'd prefer to save the 2000 tons of lead.

K: That you have already answered a couple of times before. I just asked it again for a check.

B: Each time it helps me to organize my thoughts.

K: This means that the utility of point B is greater than the utility of point A, and we know the utility of point B is k_{10} and that of point A is k_9. What we want to do now is come down the X_{10} scale toward 2000 and find a point that is indifferent to point A. Then we equate the utilities and we have one equation relating the scaling constants k_9 and k_{10}. So, if you are offered point C, 50 tons of plutonium produced and 1000 tons of lead, or point A, which would you prefer?

B: That's very difficult. There are so many things to think about.
[Since the question seemed difficult, I changed to one that I thought would be easier to answer.]

K: Well then, let's consider point D, 1900 tons of lead and 50 tons of plutonium, versus point A. Which would you prefer?

B: Oh, then I'd certainly prefer point A.

K: Would you prefer point E or point A?

B: Let me take an easier one now that I've got my thinking straightened out. Returning to point C versus point A, I would rather have point C.

K: Okay. How about if X_{10} is 1200, point F, versus point A?

B: Here I'd still take point F. Thinking about it, I guess at X_{10} equal to about 1500 I'd be indifferent.

K: That is, at point E you'd be indifferent.

B: Yes.

K: That's fine for now. This implies that k_9, the utility of A, equals $k_{10}u_{10}(1500)$, the utility of E. From Figure 16, it appears as if the utility $u_{10}(1500)$ is approximately 0.3, so $k_9 = 0.3\ k_{10}$.

[We continued in this fashion successively evaluating tradeoffs between two attributes at a time. The next pairs were $\{X_5, X_{10}\}$ and $\{X_8, X_{10}\}$. Then because k_{10} was clearly much larger than the scaling constants k_2, k_3, k_4, k_6, and k_7 of the remaining five attributes, we chose X_8 for the basis of comparison with them. That is, we considered tradeoffs between $\{X_8, X_2\}$, $\{X_8, X_3\}$, etc. As a final result, we had ten equations with eleven unknowns: $k_1, k_2, \ldots, k_{11}$. These are displayed in Table 2.

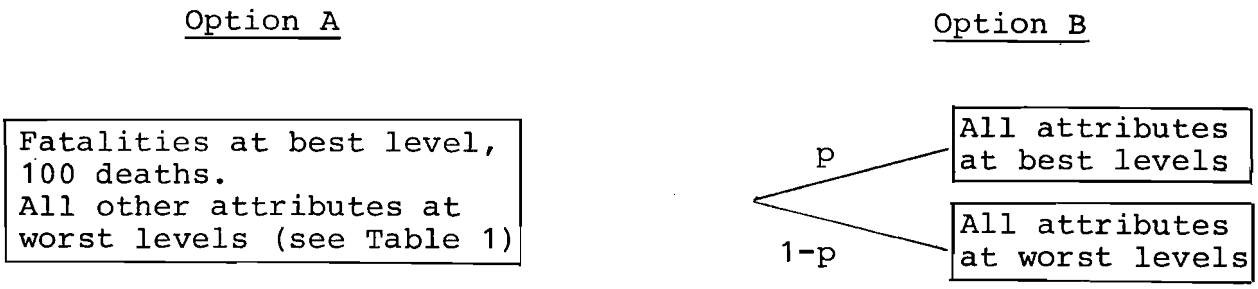
[Next we wanted to check whether u was additive or multiplicative. Two separate methods were used for this.]

Table 2.

$k_{10} = k_1 u_1(500)$	x_1 in deaths,
$k_{11} = k_1 u_1(616)$	- " -
$k_9 = k_{10} u_{10}(1500)$	x_{10} in tons,
$k_5 = k_{10} u_{10}(1600)$	- " -
$k_8 = k_{10} u_{10}(1700)$	- " -
$k_2 = k_8 u_8(50)$	x_8 in metric tons,
$k_3 = k_8 u_8(75)$	- " -
$k_4 = k_8 u_8(100)$	- " -
$k_6 = k_8 u_8(150)$	- " -
$k_7 = k_8 u_8(180)$	- " -

K: Now our ten equations specify the relative values of the k_i's. To get their absolute values, I need to ask you one very tough question. It is not necessary to ask such a difficult question; however, it does simplify the calculations that are needed to determine the k_i's. It is also illustrative of another method to determine scaling constants, so let's try.

Consider the two options in Figure 19. Option A is a consequence with fatalities at its best level, that is 100 fatalities, and all other attributes at their worst level as shown in Table 1. Option B is a lottery which gives you all eleven attributes at their best levels with probability p or otherwise all attributes at their worst levels with probability 1 - p. The question is, what is p such that you are indifferent between options A and B? Let's try out some numbers. Suppose p is 0.8 and 1 - p is 0.2, which would you prefer?



For the assessed probability p = 0.6, Options A and B were indifferent.

Figure 19. Assessing the indifference probability: at p = 0.6 .

B: With p = 0.8, I'd have to go with the lottery, I think.

K: One way to look at this is as follows. Suppose you have the consequence in option A and decide to switch it for the lottery with p = 0.8. Then if you are unlucky and move to the worst case, the difference from option A is 600 additional deaths, and this occurs with probability 0.2. If you are lucky, of which there is a 0.8 chance, you maintain the lowest level of 100 fatalities and improve on all other ten attributes. Does that seem reasonable?

B: Yes.

K: How about your preference between options A and B when p = 0.7?

B: I think I'd still take the lottery at 0.7.

K: How about $p = 0.2$?

B: I'd take option A in that case.

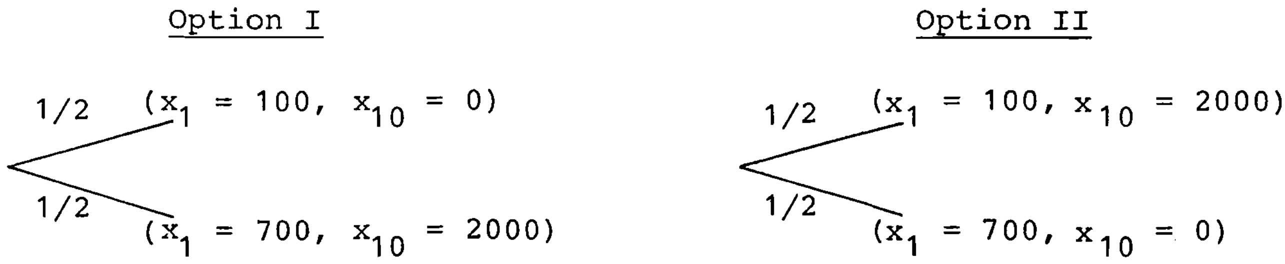
K: And if $p = 0.4$?

B: I'd take option A.

K: How about 0.6?

B: At 0.6, that's pretty close, I think. At $p = 0.5$, I'd still take option A. Yes, I think 0.6 is about the indifference point.

K: What this implies is that the utility of option A must equal the utility of option B when $p = 0.6$. The utility of A is simply k_1 using either the additive or multiplicative utility function, and the utility of B is p times 1, the utility of all attributes at their best levels, plus $1 - p$ times 0, the utility of all attributes at their worst levels. Hence $k_1 = p = 0.6$. Now we can combine this equation with the previous ten to calculate values for all eleven k_i's. [This was done roughly and quickly by hand in a couple of minutes. The sum of the k_i's, that is $k_1 + k_2 + \ldots + k_{11}$ equaled 1.14. As indicated by equations (1) and (2), if $\sum k_i = 1$, the utility function is additive, and if $\sum k_i \neq 1$, it is multiplicative. Because the sum of the k_i's is quite near to 1.0, an additive utility function may be appropriate. We will now try to find out whether this is so.]
Now let's try to get a qualitative feeling for your preferences in situations involving more than one attribute being varied. Consider the two options in Figure 20. Option I gives you a one-half chance at 100 deaths with 0 lead, and a one-half chance at 700 deaths with 2000 tons of lead. Option II is similar, but it gives you one-half chances at either 100 deaths with 2000 tons of lead or 700 deaths with 0 lead. You can consider all attributes other than fatalities X_1 and lead X_{10} to be fixed at any levels, but the same fixed levels for each option. Which option, I or II, do you prefer?



Options I and II were found to be indifferent.
(x_1 measured in deaths and x_{10} in tons of lead.)

Figure 20. Preferences for combinations of fatalities and tons of lead.

Before answering, let me point out that with both options, you have an identical chance at 100 or 700 deaths and an identical chance at 0 or 2000 tons of lead. So considering one attribute at a time, the consequences are the same. However, with Option I you get either the best or worst of both attributes, whereas with Option II you will get the best of one but the worst of the other attribute. Do you have a preference or are you indifferent?

B: I think I am indifferent. Yes, I think I am indifferent.

K: Let me suggest very rough arguments for preferring one or the other. You may say that with either consequence in II, the situation will be "very bad," whereas at least with Option I, there is a one-half chance to come out okay. This implies that I is preferred to II. Alternatively, you may say I can handle either case resulting from II, but the second possibility in Option I is simply untenable; therefore I'd prefer II. Or these two effects may balance each other and you would be indifferent.

B: I understand the two positions and I like the idea of having a shot at both at their best, but it is very close to indifferent.

K: What this implies is that the k_i's should sum to 1.0. If you had preferred II to I, the $\sum k_i$ should be greater than 1.0, and if you preferred I to II, the $\sum k_i$ should be less than 1.0.

B: That's interesting, because if I had selected one, I would have taken II.
[I now repeated the same test for additivity with a pair of attributes that I felt might indicate non-additivity. In assessing utility functions, the assessor should play the devil's advocate.]

K: Consider one more similar question involving the attributes X_5, SO_2 pollution, and X_6, particulate pollution. In Figure 21, with Option III, you get either 5 million tons of SO_2 pollution with 0.2 million tons of particulate or 80 million tons of SO_2 with 10 million particulate. And I think Option IV is clear. Which do you prefer, or are you indifferent? Here again the implications are identical taking one attribute at a time. The difference is in how the attribute levels are combined.

Option III

1/2 $(x_5 = 5,\ x_6 = 0.2)$

1/2 $(x_5 = 80,\ x_6 = 10)$

Option IV

1/2 $(x_5 = 5,\ x_6 = 10)$

1/2 $(x_5 = 80,\ x_6 = 0.2)$

Options III and IV were found to be indifferent.
(x_5 measured in 10^6 tons of SO_2 and x_6 in 10^6 tons of particulate.)

Figure 21. Preferences for combinations of SO_2 and particulate pollution.

B: Again I'm reasonably close to indifferent. Although there is perhaps a little synergistic effect with these two attributes, I would still be very close to indifferent.

K: Then we will assume that your utility function is additive, implying again that the k_i's sum to 1.0. This relationship together with the ten equations relating the relative values of the k_i's implies (after a little calculation) that $k_1 = 0.526$. Let's return to Figure 19 and examine this implication. It means that you should be indifferent between Option A and Option B when $p = 0.526$. Does this seem reasonable?

B: Yes, it does. I don't think that distorts my feelings.

4.5 Assessing the Single-Attribute Utility Functions

K: Good, then the only assessments that remain are the individual utility functions, the u_i. Actually we have already assessed u_1 and u_{10}, so let's try u_2. Refer to Figure 22 where we have scaled X_2, the permanent land use, from 2000 acres, the worst point, to 0 acres, the best. Thus we assign $u_2(2000) = 0$ and $u_2(0) = 1$ as illustrated in the figure. Now consider a choice between a fifty-fifty lottery yielding either $x_2 = 2000$ or $x_2 = 0$, and an option giving you 800 acres used for sure. Which would you prefer?

B: 800.

K: How about 1400 versus the lottery?

B: The lottery.

K: And 900 versus the lottery?

B: At 900, I would take it.

K: A thousand?

B: That's going to be the point of indifference.

K: So then the utility function is probably very close to linear as shown in Figure 22.

B: In this case I think so.

K: Good. Then let's go on to u_3. Temporary land use goes 200,000 to 10,000 acres. If you had a fifty-fifty lottery

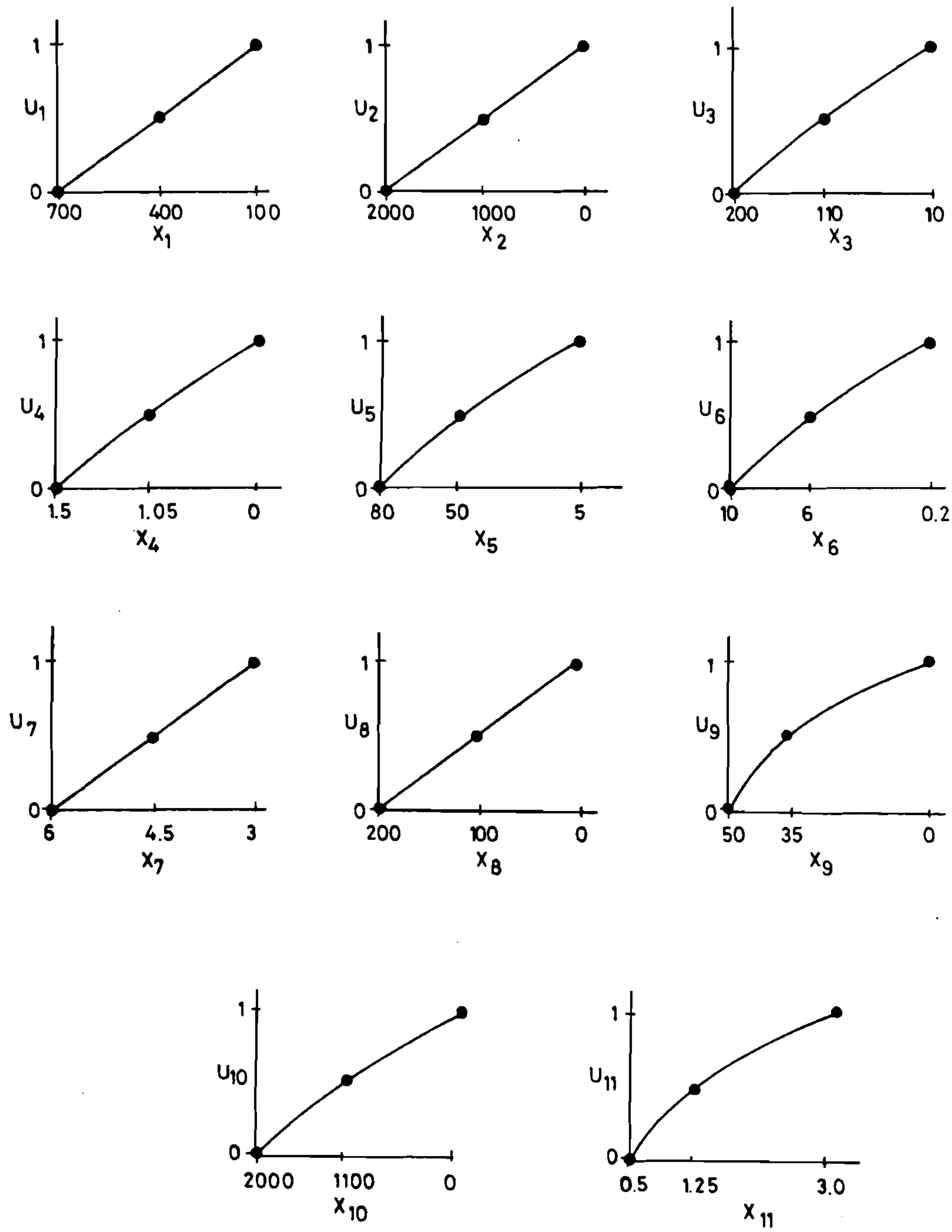


Figure 22. First-cut utility functions.

yielding either of these or 80,000 for sure, which would you take?

B: The 80,000.

K: How about 130,000 acres versus the lottery?

B: I'd take the lottery.

K: How about 110,000 acres?

B: I'd stick with the lottery. Again I think in this case I would be indifferent at the mean for such lotteries.

K: Then u_3 is also linear.

[We assessed utility functions for the other attributes in a similar fashion. The results are shown in Figure 22. For public problems, it seems to be especially true that several utility functions are linear in their respective attributes. This is largely a result of the range of possible consequences. Let me illustrate this with an excerpt from the assessment of the utility function for radioactive waste.]

K: Let us now assess u_9. You can probably figure out what the question will be.

B: Yes. This one I've thought about; it's going to be linear. The maximum is only 200 metric tons. Now if that were 2000 metric tons, my answer would be much different. My indifference point to a fifty-fifty lottery of 0 or 2000 metric tons would be quite a bit over the mean.

K: [We also will illustrate part of the assessment of the utility for electricity generated.]

Electricity generated goes from 0.5 to 3.0 trillion kilowatt hours. Because this is a proxy variable, you've got to think about what you would do with the various energy amounts if you had them. Consider a fifty-fifty lottery of 0.5 or 3.0 versus 2.0 for sure.

B: If I could have 2.0 for sure, I'd take it.

K: How about 1.75?

B: Let's see, the mean of this lottery is 1.75. I'd take 1.75

for sure rather than the lottery.

K: How about 1.0 versus the lottery?

B: Now 1.0 is about the current level of electricity. I'd come close to taking 1.0, but I guess I would take the lottery.

K: How about 1.5?

B: I'd take the 1.5.

K: And 1.25?

B: That's about it, I think. That's where I'm indifferent.

K: That seems reasonable to me, too. This completes a first-cut assessment of the u_i's, and we now have all the information needed to specify your utility function. We have found out that for you,

$$u(x_1,x_2,\ldots,x_{11}) = \sum_{i=1}^{11} k_i u_i(x_i) \quad ,$$

where the k_i's are found by solving the equations in Table 2 plus $\sum k_i = 1$, and the u_i's are shown in Figure 22. Let us examine an implication of your utility function. In particular, refer to Figure 23 where we have pictured the $\{X_1,X_8\}$ consequence space. Now since $k_1 > k_8$, there must be some consequence, call it C, between points A and B which is indifferent to point D. If, for C, the level of X_1 is designated by x_1', then equating the utilities of C and D, we find $k_1u_1(x_1') = k_8$. Given the values of k_1 and k_8 which we have calculated, this implies $u_1(x_1') = .0667$. From Figure 22, it follows that $x_1' = 660$. Put together, this implies that you should be indifferent between C: ($x_1 = 660$, $x_8 = 200$) and D: ($x_1 = 700$, $x_8 = 0$). Does this seem reasonable or out of the question or a little high or...?

B: It seems quite reasonable.

5. The Second Assessment of Buehring's Utility Function

I didn't do all the curve fitting and calculations necessary to specify the overall utility function given the assessed information. The reason was that we planned to

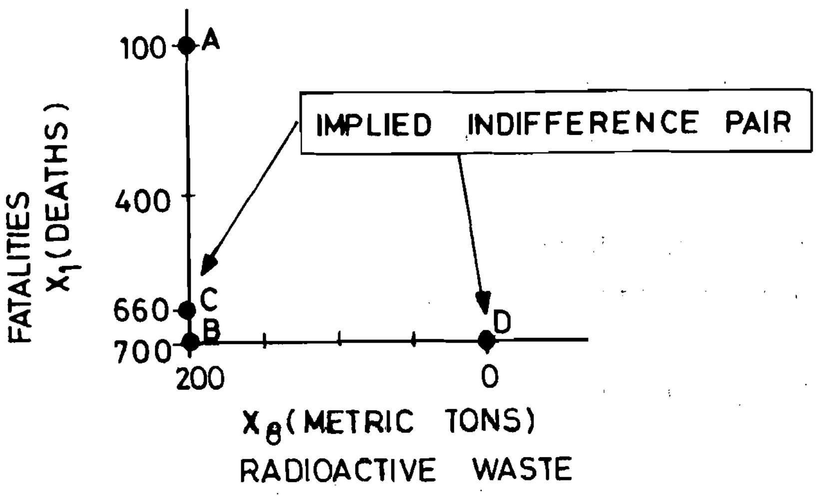


Figure 23. Implied tradeoff between radioactive waste and fatalities.

reassess aspects of Dr. Buehring's utility function in a few days after enough time for reflection. In the meantime, Buehring assessed Wes Foell's utility function over the same attributes. Dr. Foell is the head of a project on Integrated Energy Systems at IIASA, and as mentioned, is also the leader of the Wisconsin research team that developed the Wisconsin model. This interaction allowed Bill Buehring to "get some feedback on his preferences." For instance, if Foell's preferences were radically different from Buehring's, then Buehring could ask for the reasons and incorporate the answer (i.e., the new information) modifying his own preference structure. What follows is our second-cut assessment of Dr. Buehring's utility function. Because of the work behind us, it is obviously much more streamlined.

K: Could you give me the ordering of the k_i's, that is, the order in which you would like to move attributes from their worst to best levels in Table 1? If we run into inconsistencies later, we can simply revise the list.

B: All right: 10, 1, 9, 5, 8, 11, 3, 2, 4, 7, 6. I think that's it.

K: This means you would prefer going from 2000 to 0 tons of lead rather than 700 to 100 fatalities?

B: Yes.

K: And you'd prefer going from 700 to 100 fatalities to going from 50 to 0 tons of plutonium produced?

B: Yes.
[We continued down the list this way as a simple check and found no changes necessary. This implies

$$k_{10} > k_1 > k_9 > k_5 > k_8 > k_{11} > k_3 > k_2 > k_4 > k_7 > k_6] \ .$$

K: In Table 3 we have the attribute list with the old and new ranking of the k_i scaling constants. If we compare these lists, nothing moved more than one position except k_{11}, which moved three positions. Why do you feel this happened?

Table 3. Ranking of the scaling constants.

Attribute	Ranking	
	Old	New
k_1	1	2
k_{10}	2	1
k_{11}	3	6
k_9	4	3
k_5	5	4
k_8	6	5
k_2	7	8
k_3	8	7
k_4	9	9
k_6	10	11
k_7	11	10

B: I think that I overvalued the increase of energy from 1.5 to 3.0 trillion kilowatt hours in my previous answers. The shape of the utility function over the last part of the curve for energy generated will be very close to flat. It is only slightly better to have 3.0 than 1.5 trillion kilowatt hours.

K: There are three places where there are simple position interchanges among the k_i's. The first is between k_{10} and k_1. How did this come about?

B: I've always felt that the trace elements are very important. After interviewing Wes, I decided that the health impact of 2000 tons of lead could be much larger than 600 quantified fatalities. Furthermore there are the esthetic impacts due to the lead pollution. The more I thought about it, the worse it became.

K: Another reversal had to do with permanent land use and temporary land use.

B: I think before I was concentrating on the permanency question and ranked $k_2 > k_3$. After thinking more about the magnitudes of land involved, $k_3 > k_2$.

K: And finally, you reversed particulate pollution and energy needed. Now $k_7 > k_6$.

B: After more thinking about the implications of the worst level of particulate pollution, I decided it wasn't so bad. It isn't black soot coming out of the stack.

K: Okay, let's go ahead and get your relative k_i values. Consider Figure 24. You have said point A is preferred to point B.

B: Yes.

K: How about point C versus B? Would you prefer C ($x_1 = 700$, $x_{10} = 1000$) or B ($x_1 = 100$, $x_{10} = 2000$)?

B: I'd take B.

K: Point D versus B?

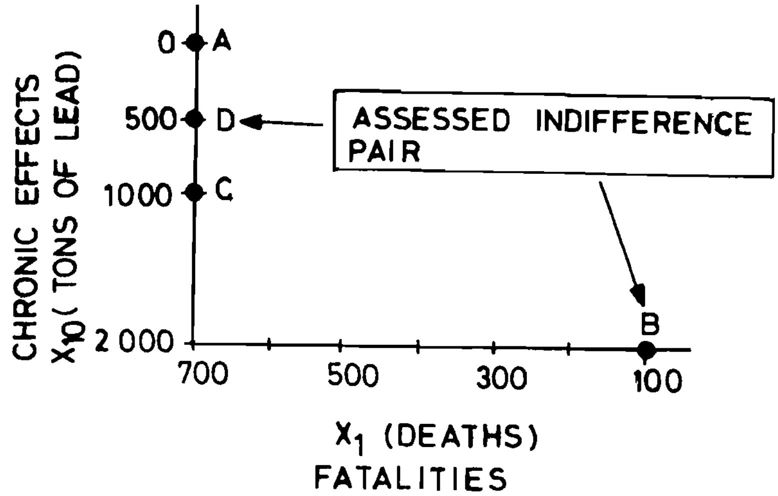


Figure 24. Reassessed tradeoff between fatalities and chronic health effects (cf. Figure 13).

B: Here I'd be indifferent. Yes, that's about it.

K: Okay, this means $k_1 = k_{10}u_{10}(500)$ since the utility of points B and D must be equal.
[We continued in this manner as illustrated before and generated the ten equations in Table 4.]
Let me now ask you a question on additivity. Maybe since you assessed Wes' utility function you've already thought more about it.

B: Yes, I have and I am additive.

K: Well, let's try one check. Refer to Figure 25 where there are two lotteries involving X_1 and X_{10}, the two attributes whose ranges are most heavily weighted. Do you have a preference between them?

B: No, I am indifferent.
[This implies again that $\sum k_1 = 1$, which together with the equations in Table 4 gives us eleven equations with eleven unknowns: $k_1, k_2, \ldots, k_{11}$. Later on we will solve for these.]

Table 4.

$k_1 = k_{10}u_{10}(500)$	x_{10} in tons,
$k_9 = k_{10}u_{10}(1200)$	x_{10} in tons,
$k_5 = k_{10}u_{10}(1700)$	x_{10} in tons,
$k_8 = k_5u_5(10)$	x_5 in million tons,
$k_{11} = k_5u_5(20)$	x_5 in million tons,
$k_3 = k_5u_5(60)$	x_5 in million tons,
$k_2 = k_3u_3(50)$	x_3 in thousands of acres
$k_4 = k_3u_3(75)$	x_3 in thousands of acres,
$k_7 = k_3u_3(125)$	x_3 in thousands of acres,
$k_6 = k_3u_3(150)$	x_3 in thousands of acres.

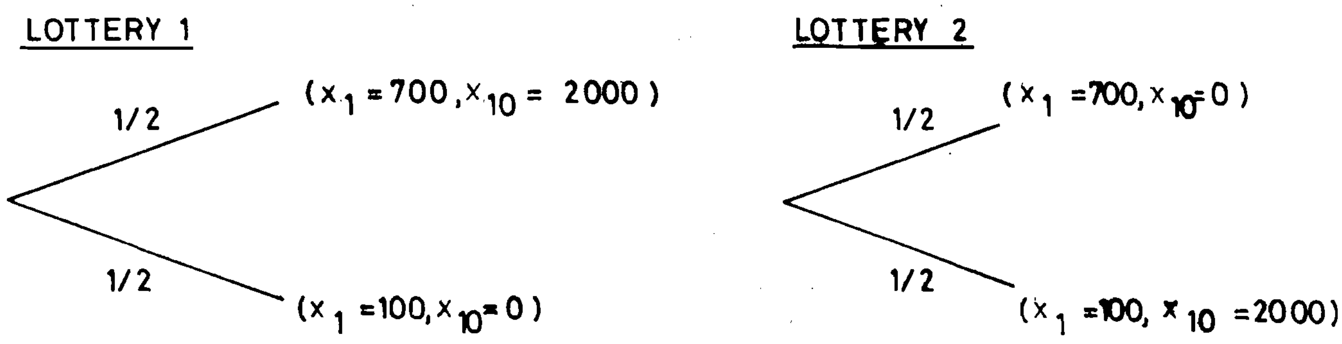


Figure 25. Reassessment of preferred combinations of consequence levels (cf. Figure 20): Buehring remained indifferent to lotteries 1 and 2.

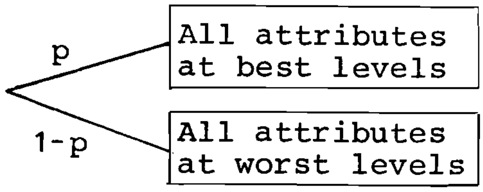
K: Going on, refer to Figure 26. Option I gives you fatalities of 100 for sure and Option II gives you a p chance at all of the attributes at their best or a 1 - p chance of all at their worst. Which would you choose if p = 0.5?

B: That's tough.

Option I

Fatalities at best level,
100 deaths.
All other attributes at
worst levels (see Table 1)

Option II

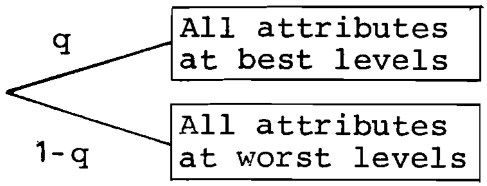


Options I and II were assessed to be indifferent for p = 0.35. This was adjusted to p = 0.3 to be consistent with the response for Options V and VI below.

Option III

Chronic effects at best level,
0 tons of lead.
All other attributes at worst
levels (see Table 1).

Option IV

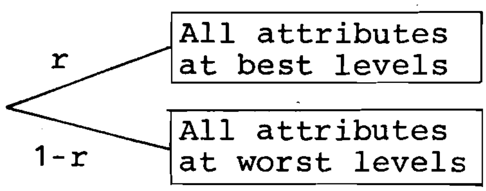


Options III and IV were assessed to be indifferent for q = 0.45. This was adjusted to q = 0.4 to be consistent with the response for Options V and VI below.

Option V

Fatalities at best level,
100 deaths.
Chronic effects at best
level, 0 tons of lead.
All other attributes at
worst levels (see Table 1)

Option VI



Options V and VI were assessed to be indifferent for r = 0.7. For consistency with the additive utility function, this indifference r must equal the indifference p plus the indifference q above.

Figure 26. Assessment of indifference probabilities.

K: Well, suppose you have Option I. Would you risk a 0.5 chance at fatalities increasing to 700 for a 0.5 chance at all other attributes raised to their best level?

B: At 0.5 I'd take the lottery.

K: What if $p = 0.4$?

B: I'd still take the lottery, but just barely.

K: At $p = 0.3$?

B: At $p = 0.3$...at $p = 0.35$ I'd essentially be indifferent.

K: Now consider the same type of question only between Options III and IV in Figure 26. If $q = 0.5$ which would you choose?

B: At 0.5 I think I'd take the lottery, but that is close.

K: How about $q = 0.3$?

B: At 0.3, I'd take Option III.

K: What if $q = 0.4$?

B: At 0.4, I'm almost indifferent. That's a little low; how about 0.45?

K: This is interesting and quite consistent with earlier responses in this session. Which do you prefer between the two sure Options I and III?

B: Well, III as I've already said.

K: Sure, so if Option I is indifferent to II for some value $\bar{p}$, and III is indifferent to IV for some value $\bar{q}$, which should be bigger, $\bar{p}$ or $\bar{q}$?

B: I guess $\bar{q}$ since IV must be preferred to II given the respective indifference options.

K: Yes, and in fact an in-the-head calculation implies that the ratio of 0.35 to 0.45 is very consistent also. [These numbers should equal k_1 and k_{10} respectively. A later calculation indicates that the implied ratio is in fact very consistent.] Now consider Option V with lead and fatalities both at their best levels and all other attributes at their worst levels, or Option VI with probability $r = 0.5$.

B: At 0.5, I'd take the sure thing, Option V.

K: Suppose $r = 0.9$.

B: At 0.9, I'd take it.

K: You'd take the lottery?

B: Yes, I'd take it.

K: At $r = 0.6$?

B: At 0.6, I don't think I'd take the lottery. I'd go back to Option V. What I'm saying is that the other attributes don't mean much here, aren't I?

K: What you are implying is that you are not willing to change from V and take a 0.4 chance at 2000 tons of lead and 600 additional deaths in order to get a 0.6 chance at pushing all the other attributes up to their best level.

B: Well, I guess I do feel this way. It's not that those others are meaningless though; this bothers me a little bit. If this were a consistent answer with the two previous choices, would the indifference probability $\bar{r}$ be 0.8?

K: Yes, for consistency with the additive utility function, $\bar{r}$ must equal $\bar{p} + \bar{p}$.

B: At 0.8, it seems too high. Maybe the other indifference probabilities should be a little lower.

K: That is exactly the type of thinking we want to promote with utility assessments.

B: That is a good check.

K: Let's return to $r = 0.6$.

B: I'd still take Option V at $r = 0.6$, but maybe at $r = 0.7$, I'd be indifferent. Yes, I can't believe it's as high as 0.8, so there must be something wrong with the other indifference probabilities.

K: Okay, then the sum of the first two indifference probabilities must be 0.7. A simple way to do this is make $\bar{p} = 0.3$

and $\bar{q}$ = 0.4. [I marked these on the respective lotteries for Options II and IV.] How does this seem?

B: I'll buy that. Yes, at 0.3 and 0.4, they seem very close.

K: We can do a quick sloppy check to see whether these numbers are at all reasonable. We'll assume that all the utility functions are linear, an approximation that is probably okay for present purposes. Referring to Table 4, we see that k_9 would be about 8/20 of k_{10} or 0.16, since the decrease from 2000 to 1200 tons of lead is equivalent to the entire range of nuclear safeguards, 50 tons to zero. Similarly, k_5 would be 3/20 or 0.06. Now k_8 would be almost the same as k_5. Specifically, it would be 14/15 times 0.06, but we'll assume that it is 0.06. And k_{11} would be 12/15 of k_8, or about 0.05. Anyway, summed up, we see that the eleven k_i's would equal approximately 1.1. For additivity, as you know, they should sum to 1.0. However, given the roughness of our calculations, the numbers seem to check out reasonably well.

B: That's not bad; it's amazing.

K: The last assessments we'll do here will be to get the single-attribute utility functions. I only need one certainty equivalent on each of these. Let's take the lead first and consider a fifty-fifty lottery yielding either 0 or 2000 tons. For what sure level are you indifferent to this lottery?

B: It's a little over 1000, 1100 maybe.

K: Okay. The next assessment is for X_1.

B: That's going to be linear, that's fatalities.

K: In our last session, you felt it was very close to linear. Now do you think it is linear, or do you have the same feeling as before?

B: Fatalities? I think it is linear, perfectly linear.

K: The third one is X_9, nuclear safeguards. Consider a fifty-fifty lottery yielding 0 or 50 tons of plutonium produced or 25 tons for sure. Which would you choose?

B: I'd take the sure 25.

K: How about 40 for sure versus the lottery?

B: At 40, I'd take the lottery.

K: 35?

B: I think that's about it.

[This procedure was continued for all the other attributes. The results are shown in Figure 27. The three points marked by dots in the figure were those used in the assessment. The middle one was the certainty equivalent for a fifty-fifty lottery of the outer two.]

K: We can compare these responses to those you gave before if you are interested.

B: I am, actually.

K: Okay, let's look at Figures 22 and 27. On attribute X_{10}, before you gave 1100 and this time 1100 for the certainty equivalent of a fifty-fifty lottery yielding 0 or 2000, which seems rather consistent. On attribute X_1, you were linear both times. For X_9, you gave 35 as your certainty equivalent both times. Actually, the only three that seem to be much different at all are X_{11}, X_3, and X_6. On X_{11}, you gave 1.0 this time and 1.25 last time. You can reflect on this and change your mind if you want.

B: I think I should stay with the 1.0, because I feel the utility of 2.0 trillion kwh is very close to 1. There is simply not much difference between 2.0 and 3.0 trillion kwh.

K: Suppose it is 0.9, what does that mean to you?

B: It means that over the range 0.5 to 3.0, I'll have received 90 percent of utility possible by reaching 2.0. Said another way, if I have a fifty-fifty lottery yielding 0.5 or 2.0, or a fifty-fifty lottery yielding 0.5 or 3.0, my certainty equivalents probably would not be that different.

K: That's right.

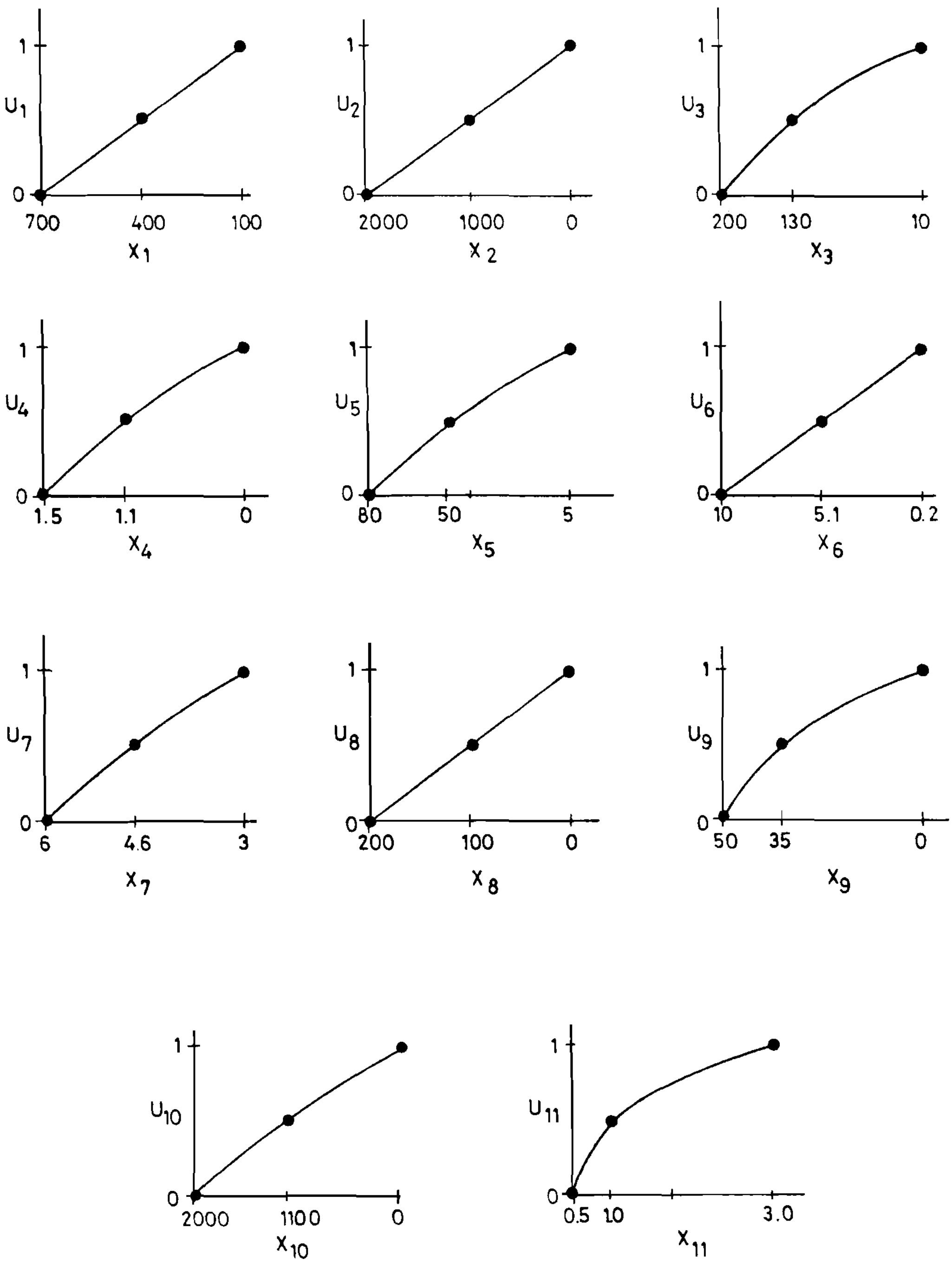


Figure 27. Second-cut utility functions.

B: And so I want to stick with 1.0 as the certainty equivalent.

K: Good. Let's go on with X_3: you are now indifferent to the fifty-fifty lottery yielding 10,000 or 200,000 acres and 130,000 acres for sure, whereas before you were linear. Do you know anything you were thinking about that seemed different in the two assessments?

B: I'm thinking now of some sort of cumulative effect. By the time we get to 200,000 acres, it's getting to be a very noticeable impact.

K: Finally, now you are linear in attribute X_6, particulate pollution, whereas before you were a little off linear. Any reflection on that?

B: As I think about it, the worst case for particulates is not very noticeable. Therefore there are no real cumulative effects in this range.

K: Well, now I guess all the information is here to calculate the utility function.

6. Calculating the Utility Function

From the theorem presented in Section 2 of this report and our assessments, we know the utility function is

$$u(x_1,x_2,\ldots,x_{11}) = \sum_{i=1}^{11} k_i u_i(x_i) \quad , \tag{3}$$

where u and the u_i's are scaled zero to one, the k_i's are positive, and $\sum k_i = 1$. To specify the utility function u, we need to calculate the u_i's and k_i's.

First exponential curves were fit to the nonlinear single-attribute utility functions using the data in Figure 27. The final results are given in Table 5.

Next, given the utility functions in Table 5, we could solve the set of ten equations in Table 4 and the equation $\sum k_i = 1$ for the eleven unknown k_i's. The eleven equations used for the solution are given in Table 6, as well as the solution itself in

Table 5. The single-attribute utility functions.

Attribute	ui	x_i, measured in	Range Worst	Range Best
X_1 = fatalities	$u_1(x_1) = (700 - x_1)/600$	deaths	700	100
X_2 = permanent land use	$u_2(x_2) = (2000 - x_2)/2000$	acres	2000	0
X_3 = temporary land use	$u_3(x_3) = 1.496 - 0.466e^{0.00581x_3}$	10^3 acres	200	10
X_4 = water evaporated	$u_4(x_4) = 1.784 - .520e^{0.822x_4}$	10^{12} gallons	1.5	0.5
X_5 = SO_2 pollution	$u_5(x_5) = 1.784 - .742e^{0.011x_5}$	10^6 tons	80	5
X_6 = particulate pollution	$u_6(x_6) = (10 - x_6)/9.8$	10^6 tons	10	0.2
X_7 = thermal energy needed	$u_7(x_7) = 4.260 - 2.495e^{0.0892x_7}$	10^{12} kwh (thermal)	6	3
X_8 = radioactive waste	$u_8(x_8) = (200 - x_8)/200$	metric tons	200	0
X_9 = nuclear safeguards	$u_9(x_9) = 1.198 - 0.198e^{0.036x_9}$	tons of plutonium	50	0
X_{10} = chronic effects	$u_{10}(x_{10}) = 3.017 - 2.017e^{.0002013x_{10}}$	tons of lead	2000	0
X_{11} = electricity generated	$u_{11}(x_{11}) = 1.039 - 2.003e^{-1.313x_{11}}$	10^{12} kwh (electric)	0.5	3

Table 6. Solving for the Scaling Constants.*

k_i	Relationship to k_{10}	k_i equals	Value of k_i
k_{10}		k_{10}	.339
k_1	$k_1 = k_{10}u_{10}(500)$	$0.786k_{10}$	.226
k_9	$k_9 = k_{10}u_{10}(1200)$	$0.449k_{10}$	.152
k_5	$k_5 = k_{10}u_{10}(1700)$	$0.177k_{10}$	.060
k_8	$k_8 = k_5u_5(10) = k_{10}u_{10}(1700)u_5(10)$	$0.169k_{10}$	.057
k_{11}	$k_{11} = k_5u_5(20) = k_{10}u_{10}(1700)u_5(20)$	$0.152k_{10}$	.051
k_3	$k_3 = k_5u_5(60) = k_{10}u_{10}(1700)u_5(60)$	$0.063k_{10}$	.021
k_2	$k_2 = k_3u_3(50) = k_{10}u_{10}(1700)u_5(60)u_3(50)$	$0.054k_{10}$	.018
k_4	$k_4 = k_3u_3(75) = k_{10}u_{10}(1700)u_5(60)u_3(75)$	$0.048k_{10}$	.016
k_7	$k_7 = k_3u_3(125) = k_{10}u_{10}(1700)u_5(60)u_3(125)$	$0.033k_{10}$	.011
k_6	$k_6 = k_3u_3(150) = k_{10}u_{10}(1700)u_5(60)u_3(150)$	$0.023k_{10}$	.008
		$\sum = 2.953k_{10}$	$\sum = 1.0$

*Solving $\sum k_i = 2.953k_{10} = 1$ yields $k_{10} = 0.339$ from which the other k_i's are evaluated.

the final column.

The final utility function is given by equation (3) above plus Tables 5 and 6.

7. Discussion

Let me briefly comment on two topics: the assessment procedure itself and the uses for the resulting utility function.

First, for convenience in calculation, some of the questions asked of Dr. Buehring in specifying his utility function were difficult to consider. These are not necessary, especially if one has some computer support. For instance, one never needs to ask for indifference probabilities directly as we did with the options in Figure 26. In this case, we did not actually use them in specifying the utility function.

A second point is that, had there been a preference between the lotteries in Figure 25, the overall utility function u would have been multiplicative. As is seen from equation (2), this would mean that an additional scaling constant, the k in (2), would need to be specified. It is evaluated directly from the values of the eleven k_i's. The point is that the multiplicative utility function is only slightly more difficult than the additive to specify and use. See Keeney and Raiffa [6] for several applications.

The utility function u should be carefully scrutinized to make sure it does capture Mr. Buehring's preferences. For instance, it would now be easy to draw sets of indifference curves given u. By examining these, one may find aspects of the utility function which are not appropriate. When this is the case, the "errors" should be corrected.

As an example, the values of k_1 and k_{10} in Table 6 imply that $\overline{p}$ and $\overline{q}$ in Figure 25 should be .266 and .339 rather than .3 and .4 respectively, if the tradeoffs among the other nine attributes and X_1 and X_{10} remain the same and if the additive utility function is to be used. Alternatively, if the assessed values .3 and .4 seem more reasonable and if the above-mentioned tradeoffs remain fixed, a multiplicative utility function must be employed. Such discrepancies need to be reconciled before using the utility function to evaluate policy.

The uses of the utility function might be categorized as either formal evaluation or informal structuring. The first is rather clear. One combines this preference model with the output of an impact model (probabilistic or deterministic) and simply calculates the expected utility of alternatives.*

---

*See Buehring [1] for an evaluation of six policy options using such a utility function.

Of course, a sophisticated analysis should include sensitivity analyses, etc., but at least in theory, this is relatively straightforward. In this way, the utility function directly addresses the first two complexities (uncertainties and multiple objectives) raised at the beginning of the paper.

What I mean by "informal structuring" includes thinking about one attribute at a time, deciding whether the measure is good, and seeing where critical information is lacking (e.g., relationships between pollution levels and fundamental health effects.) It also includes helping to focus discussion with others to sort out where differences and agreements are, to define attributes precisely, and to indicate weaknesses of the impact model because it fails to include aspects which clearly affect preferences. In the context of a team working on a problem, the intent is to improve the quality of the interaction to lead to a better overall model in the end.

Used as an aid to focus discussion, utility analysis can be helpful in addressing the third complexity mentioned at the beginning of this paper. The procedures can lead to a much better understanding of the points of agreement and the points of disagreement among individuals in a decision making unit. It may indicate reasons for these disagreements and suggest directions of research that would tend to minimize them. Once such differences are as small as possible, analysis may prove that the same policy options are preferred using any of the candidate utility functions of the different individuals; or at least there may be uniform agreement to eliminate some alternatives from consideration. Finally, utility analysis may serve as a mechanism for creative and constructive compromise among individual members of the decision making unit.

The set of assessments discussed here took about eight hours of Mr. Buehring's time. Consequently, one fairly common comment about such assessments is, "This all seems fine, but when is anybody who is a real decision maker going to take all the time necessary to do this? We need simpler, even though oversimplifying, procedures to get the sense of the decision maker's preferences quickly." I agree that this often is a problem.

On the other hand, I feel that with the man-years of effort and millions of dollars being spent to model such crucial problems as those concerning energy policy, we should be able to 'free' a real decision maker (decision makers), who has a comprehensive knowledge of the problem area, for a week or so--at least long enough to reasonably structure his or her preferences. It may be prudent even to have a team of policy makers and analysts work together in a several-man-months effort to construct a good preference model. This would then be coupled with the impact model for evaluating policy. The default, of course, is that our decision makers are expected to simultaneously consider and balance all the multidimensional consequences of the impact model, as well as their implications, in their heads and then arrive at a responsible decision.

8. Acknowledgment and Postscript

I appreciate very much Bill Buehring's comments on this paper as well as his willingness to have his preferences appear in print. The dialogue is clearly altered; however, the changes were very minor--mainly correcting grammar, deleting uh's and huh's, removing interruptions, and the like. The complete sense of the discussion is preserved.

Three months after the assessments took place, Bill and I went over this paper and informally discussed his preferences. In the interim, he had evaluated selected policies using his utility function, assessed some other individuals' preferences, and learned more about some of the consequences of various levels of the proxy attributes (e.g., SO_2 emissions) used in this paper. This has led to some minor changes in his preferences, as you might expect. In light of this plus the fact that the assessments were done under some time pressure, it is inappropriate to interpret the utility function specified by equation (3) with Tables 5 and 6 as "the final utility function of Bill Buehring." However, had he been required to make a policy decision three months ago, the expressed utility function could have been of considerable help in examining and choosing among the alternatives.

References

[1] Buehring, W.A. "A Model of Environmental Impacts from Electrical Generation in Wisconsin." Unpublished Ph.D. dissertation, Department of Nuclear Engineering and Institute of Environmental Studies, University of Wisconsin, Madison, 1975.

[2] Buehring, W.A., and Foell, W.K. "A Model of Environmental Impact of Wisconsin Electricity Use." IES Report 32. Madison, Wisconsin, University of Wisconsin, November 1974.

[3] Buehring, W.A., Foell, W.K., and Dennis, R.L. "Environmental Impact of Regional Energy Use: A Unified Systems Approach." French-American Conference on Energy Systems Forecasting, Planning, and Pricing. Madison, Wisconsin, September 1974.

[4] Gorman, W.M. "The Structure of Utility Functions." Review of Economic Studies, 35, 1968, p. 367-390.

[5] Keeney, R.L. "Multiplicative Utility Functions." Operations Research, 22, 1974, p. 22-34.

[6] Keeney, R.L. and Raiffa, H. Decisions with Multiple Objectives. New York, Wiley, to be published (1976).

[7] von Neumann, J. and Morgenstern, O. Theory of Games and Economic Behavior, 2nd edition, Princeton, New Jersey, Princeton University Press, 1947.