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Endogenous Growth, Absorptive Capacities and International R&D Spillovers

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This paper is the outcome of a joint research effort on endogenous growth theory and alternative approaches between Gernot Hutschenreiter from the Austrian Institute of Economic Research (WIFO) during his stay at IIASA in the summer of 1995 as a guest scholar with the Systems Analysis of Technological and Economic Dynamics Project (TED), Yuri Kaniovski (TED) and Arkadii Kryazhimskii (Dynamic Systems Project). An accompanying paper by A.Kryazhimskii (WP-95-93) contains a comprehensive mathematical analysis of the model developed in this paper, providing a valuable reference for researchers working on the family of models related to the one presented here.

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Preface

The research project on *Systems Analysis of Technological and Economic Dynamics* at IIASA is concerned with modeling technological and organisational change; the broader economic developments that are associated with technological change, both as cause and effect; the processes by which economic agents – first of all, business firms – acquire and develop the capabilities to generate, imitate and adopt technological and organisational innovations; and the aggregate dynamics – at the levels of single industries and whole economies – engendered by the interactions among agents which are heterogeneous in their innovative abilities, behavioural rules and expectations. The central purpose is to develop stronger theory and better modeling techniques. However, the basic philosophy is that such theoretical and modeling work is most fruitful when attention is paid to the known empirical details of the phenomena the work aims to address: therefore, a considerable effort is put into a better understanding of the 'stylized facts' concerning corporate organisation routines and strategy; industrial evolution and the 'demography' of firms; patterns of macroeconomic growth and trade.

From a modeling perspective, over the last decade considerable progress has been made on various techniques of dynamic modeling. Some of this work has employed ordinary differential and difference equations, and some of it stochastic equations. A number of efforts have taken advantage of the growing power of simulation techniques. Others have employed more traditional mathematics. As a result of this theoretical work, the toolkit for modeling technological and economic dynamics is significantly richer than it was a decade ago.

During the same period, there have been major advances in the empirical understanding. There are now many more detailed technological histories available. Much more is known about the similarities and differences of technical advance in different fields and industries and there is some understanding of the key variables that lie behind those differences. A number of studies have provided rich information about how industry structure co-evolves with technology. In addition to empirical work at the technology or sector level, the last decade has also seen a great deal of empirical research on productivity growth and measured technical advance at the level of whole economies. A considerable body of empirical research now exists on the facts that seem associated with different rates of productivity growth across the range of nations, with the dynamics of convergence and divergence in the levels and rates of growth of income, with the diverse national institutional arrangements in which technological change is embedded.

As a result of this recent empirical work, the questions that successful theory and useful modeling techniques ought to address now are much more clearly defined. The theoretical work has often been undertaken in appreciation of certain stylized facts that needed to be explained. The list of these 'facts' is indeed very long, ranging from the microeconomic evidence concerning for example dynamic increasing returns in learning activities or the persistence of particular sets of problem-solving routines within business firms; the industry-level evidence on entry, exit and size-distributions – approximately log-normal – all the way to the evidence regarding the time-series properties of major economic aggregates. However, the connection between the theoretical work and the empirical phenomena has so far not been very close. The philosophy of this project is that the chances of developing powerful new theory and useful new analytical techniques can be greatly enhanced by performing the work in an environment where scholars who understand the empirical phenomena provide questions and challenges for the theorists and their work.

In particular, the project is meant to pursue an 'evolutionary' interpretation of technological and economic dynamics modeling, first, the processes by which individual agents and organisations learn, search, adapt; second, the economic analogues of 'natural selection' by which interactive environments – often markets – winnow out a population whose members have different attributes and behavioural traits; and, third, the collective emergence of statistical patterns, regularities and higher-level structures as the aggregate outcomes of the two former processes.

Together with a group of researchers located permanently at IIASA, the project coordinates multiple research efforts undertaken in several institutions around the world, organises workshops and provides a venue of scientific discussion among scholars working on evolutionary modeling, computer simulation and non-linear dynamical systems.

The research focuses upon the following three major areas:

1. Learning Processes and Organisational Competence.
2. Technological and Industrial Dynamics
3. Innovation, Competition and Macrodynamics

Abstract

Interaction between own research and externally produced knowledge has been observed both at the firm and the macro level. This paper intends to fill a gap between endogenous growth models which treat knowledge capital as either a purely national or global public good. As a generalization, we introduce "absorptive capacities" of a smaller country "tapping" the knowledge stock of a larger autarkic country (the technological leader, evolving along its steady state) by means of own investment in R&D into an endogenous growth model with brand proliferation due to Grossman and Helpman. The asymptotic behavior of the ensuing non-linear dynamic model is analyzed in detail. Long-run solutions for the variables representing the evolution of the follower country are compared with the perfect-autarky steady states for both countries. Along a trajectory consistent with perfect foresight in the valuation of the firm, the follower's long-run rate of innovation approaches that of the leader which implies an improvement in the long-run innovative and growth performance of the follower as measured against the perfect autarky benchmark. The limit ratio of the knowledge stocks originating in the two countries is calculated explicitly. The present model is richer in its dynamic behavior than the basic model of brand proliferation. Unlike the basic model, an originally stagnant, non-innovating follower economy might still be able to embark on an equilibrium trajectory with ongoing innovation, in particular when it is linked informationally to a rapidly innovating leader. The model accounts for a number of observed facts about international economic growth. Perfect-foresight trajectories resulting in catching up and secular changes in leadership by taking over in terms of knowledge stocks and total factor productivity are generated by the present model under well-defined conditions. At the same time, the opportunities of catching up and taking over are shown to be limited in the sense that the backward and forward time-lags vis-à-vis the leader tend to finite values. While long-run rates of innovation and growth are equalized, implicit transitional growth differentials give rise to statements about convergence (or divergence) across countries depending on the set of conditions prevailing in the two countries initially.

Keywords: R&D, spillovers, absorptive capacities, endogenous growth theory

JEL Classification: O31, O40

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1. Introduction

There is a current of economic thought tracing its origins to Joseph A. Schumpeter, which views innovation as the key driving force of contemporary economic growth[1]). In progressively integrated economies linked by international trade, foreign direct investment and transborder flows of information, economic performance, and productivity in particular, can be expected to depend not only on domestic R&D but also on the research efforts abroad. The ability or failure to exploit scientific and technological progress accomplished in other countries has become a public concern in many nations. Opportunities for gaining from international R&D spillovers have been perceived as vital by small and lagging countries. More recently, international R&D spillovers have emerged as an issue for large countries operating at the technological frontier. Post-war development of industrialized countries was marked by convergence with both a favorable, sometimes impressive economic performance of a number of latecomers and a concomitant erosion of prevailing technological leadership. In particular, based on high technological competence and complementary assets, some followers have emerged as serious competitors of technological "first movers". On the other hand, trends towards globalization of innovative activities as reflected in the allocation of R&D resources across technological centers

[1]) As highlighted by Grossman - Helpman (1991, 1994A), the role this line of thought assigns to innovation goes far beyond its contribution to growth as calculated in traditional growth accounting which does not provide an answer to the question what would have happened *without* innovation. See also the critical assessment of four decades of growth accounting by one of its pioneers, Abramovitz (1993).

and in the formation of strategic alliances between multinational corporations substantiate the increased significance of transborder R&D spillovers among the large, advanced economies[2]).

Much of the recent theoretical work on international knowledge spillovers has been carried out in the framework of endogenous growth theory (see Rivera-Batiz - Romer, 1991 and the research effort summarized in Grossman - Helpman, 1991)[3]). So far, this literature typically treats knowledge either as a purely "local" (national) or "global" public good[4]), thus leaving aside the issue of interaction between knowledge capital stocks of the actors in the world economy which has a number of policy implications. In this paper it is attempted to model explicitly a mechanism behind international R&D spillovers where a country's capacity to absorb knowledge produced in another country depends on its own, domestic R&D effort. The basic ideas underlying the notion of "absorptive capacities" and some of their sources are outlined in Section 2. Endogenous growth theory provides a consistent framework to assess the impact of transborder knowledge spillovers on the evolution of key macroeconomic variables such as the rate of innovation, the growth rate of output and total factor productivity, or, briefly TFP (R&D spillovers proper). Section 3 reviews some major findings of endogenous growth theory in this context. In the main body of the paper, the idea of absorptive capacities is incorporated into an endogenous growth model. Section 4 sets the stage by recalling as a reference the main elements of the basic model of brand proliferation due to Grossman and Helpman, representing a world economy composed of perfectly autarkic economies with Romer-type local public knowledge capital. In Section 5, we bring together two strands of economic reasoning by introducing absorptive capacities into a model of that kind. The case of a smaller country (the technological follower) "tapping" the knowledge stock of a larger, autarkic country (the technological leader) by means of its own R&D activities is explored. Using asymptotic analysis, the long-run model solutions are analyzed in detail and the limit ratios of the knowledge stocks originating in the two countries are calculated explicitly. Long-run solutions for the follower country are compared with the steady state of the leader

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[2]) There seems to be little dispute that "the penalties for failure to keep abreast of innovations *in other countries* and to imitate them where appropriate have grown" (Baumol, 1986, p. 1077). Taking the argument one step further, there is one interpretation of the evidence produced by the literature on convergence arguing "that just as markets and business have become more global, the network of individuals and organizations generating and improving new science-based technologies have become less national and more transnational, so that convergence reflects a diminution of the saliency of nation-states as technological and economic entities" (Nelson - Wright, 1992, p. 1933).

[3]) Theoretical research is presently being complemented by empirical studies on international R&D spillovers.. See, e.g., Lichtenberg (1992), Mohnen (1992A), Verspagen (1994), Bernstein - Mohnen (1994), Irwin - Klenow (1994), and - explicitly referring to endogenous growth theory - Coe - Helpman (1995), Coe - Helpman - Hoffmaister (1995). Surveys were prepared by Mohnen (1992B, 1994).

[4]) An exception is Grossman - Helpman (1990) and, in a different setting, the analysis of imitation in Grossman - Helpman (1991).

(evolving in its perfect-autarky steady state) and its own perfect autarky outcomes. In Section 6, conditions for catching up and taking over in terms of the stock of knowledge and TFP along a perfect-foresight trajectory are given. Section 7 summarizes the major findings of the paper. A proof of the main Proposition is outlined in the Appendix. A thorough mathematical analysis of the model developed in Section 5 is contained in Kryazhimskii (1995).

2. R&D Spillovers and Absorptive Capacities

Economic studies of different provenance have investigated the issue whether investment in own R&D may facilitate the absorption of knowledge produced elsewhere. Empirical observations indicate that these assertions may in fact be valid[5]). *Firm level* studies of technology diffusion[6]) have asserted that firms with own R&D activities may have enhanced capabilities to monitor and assess technological developments accomplished elsewhere. In the Yale Survey "independent R&D was rated as the most effective means of learning about rival technology" (Levin et al., 1987, p. 806). It was noted that in-house R&D increases the ability for reverse engineering and reduces the costs of imitation (see David, 1986, p. 383) as well as the costs of technology transfer (Teece, 1977). Mowery - Rosenberg (1989, p. 6) note that "transferring and exploiting the technical and scientific information that is necessary for innovation constitute a costly process that itself is knowledge intensive"[7]). At the *macro level*, the issue of interaction has emerged from the literature on convergence. Simple catch-up models designed for testing the convergence hypothesis usually include the distance of countries to the technological frontier as an explanatory variable for productivity growth. However, the quasi automatic, one-way flow of "technology" from the technological leader to initially lagging countries implied in these models has been a subject of criticism. Simple catch-up models do not explain a number of important

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[5]) Of course, what is defined as "own" and what as "external" depends on the choice of the unit of analysis and thus on the level of aggregation. The significance of the latter for the analysis of R&D spillovers was highlighted by Griliches (1979, 1992).

[6]) For a survey of the literature on the firm level see Stoneman (1983), Baldwin - Scott (1987), and Geroski (1995).

[7]) Thus, Mowery - Rosenberg (1989, p. 6) conclude that traditional "market failure analysis must be supplemented by an analysis of the conditions affecting the utilization of the results of R&D. Utilization of the results of research is heavily influenced by the structure and organization of the research system within an economy, a topic on which the neoclassical theory is either silent or incorrect".

phenomena including developments at the technological frontier (such as changes in leadership[8])) and changes in the rank order of or persistent technology gaps[9]) between countries or groups of countries; there are no disadvantages of backwardness of any degree. These shortcomings have led Abramovitz (1986, 1991) to draw his well-known distinction between *potential* and *realization.* Technologically lagging countries have a potential for faster growth. The realization of this potential, however, depends on the presence of "*social capabilities*", which enable them, among others, to successfully make use of technologies applied in the more developed countries[10]). Own technological efforts and appropriate diffusion are seen as important factors for the realization of growth potentials[11]). A lack of own R&D may reduce the capability for effective and rapid imitation and adoption, in particular when technologies involved are advanced[12]).

Although the literature on technical change abounds in empirical observations of and allusions to the importance of interaction between own and external R&D efforts (at the firm as well as the macro level) formal models of these interactions are scarce. A notable exception, though formulated in a different context, is the model presented by Cohen - Levinthal (1989)[13]). Using a game-theoretical framework, Cohen and Levinthal examine how the equilibrium level of R&D investment of firms is affected by the presence of interaction between R&D performed within the firm and a pool of external knowledge produced by the universe of firms. R&D performed by the innovators feed a universal pool of existing knowledge. This knowledge stock may be thought of being protected by various barriers to knowledge dissemination. The pool of accessible knowledge comprises of the fraction of universally existing knowledge that is not effectively protected and thus is, in principle, available for adoption by all innovators in the system. However, the appropriation of this knowledge is not costless, but depends once more on own R&D performed by potential adopters. R&D thus plays a dual role in this model. On

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[8]) See, e.g., Nelson - Wright (1992) on the erosion of American leadership.

[9]) See Dosi - Pavitt - Soete (1990), and Fagerberg (1994) for a survey of the literature.

[10]) The notion of "social capabilities" is not easily made operational and is often used as a chiffre. The present inclusion of absorptive capacities into an endogenous growth model may be seen as an attempt to operationally specify at least one facet of "social capabilities".

[11]) One of the important conclusions derived from the catch-up literature is that even in the case of the more developed economies it cannot be assumed that international technology transfer (and thus the elimination of productivity gaps) are "inevitable" in the long run. *De Long* (1988, p. 1148) concludes that "the capability to assimilate industrial technology appears to be surprisingly hard to acquire, and it may be distressingly easy to loose".

[12]) *Steindl* (1982, p. 7) raises a similar argument in his criticism of Maddison's version of the catch-up hypothesis.

[13]) Jaffe (1986, 1989A) also addresses interaction between own R&D and a spillover variable empirically: Productivity of own R&D depends on the pool and the effect of the pool depends on the amount of R&D performed by the individual actors.

the one hand, an actor's R&D effort contributes to the pool of universal knowledge, on the other hand it serves as a means to appropriate existing knowledge. Cohen and Levinthal explain the apparent neglect of this kind of interaction in formal modeling by the characterization of knowledge as a (pure) public good (or, more precisely, to the related connotation of costless appropriability). This treatment of knowledge can be traced to the "modern classics" of the literature such as Nelson (1959) and Arrow (1962)[14]. However, unlike textbook examples of public goods, technological knowledge may not be appropriable unless the potential user has created adequate means to do so. Cohen and Levinthal call this capability to appropriate externally produced knowledge the "learning" or "absorptive capacity". In this paper, this basic idea of the dual role of knowledge production is incorporated into an endogenous growth model.

3. International Knowledge Spillovers and Endogenous Growth Theory

Endogenous growth theory provides a consistent framework to assess the impact of transborder knowledge spillovers on key macroeconomic variables such as the rate of innovation and the growth rates of output and TFP (international R&D spillovers proper). Grossman and Helpman compare the results derived for complete autarky of the economies involved with those for complete, instantaneous and, for that matter, costless international knowledge diffusion. In those extreme cases, knowledge is either a local or a global public good. Using their basic model of brand proliferation, Grossman - Helpman (1991, Chapter 9) show that perfect international dissemination of knowledge accelerates innovation and growth in both countries comprising their stylized world economy. Furthermore, in this case, opening up to international trade has a positive impact on the steady-state growth rate by eliminating duplicative research of the two countries. In the absence of perfect cross-border knowledge spillovers the integration of product markets has the effect that the larger country will eventually dominate the market for innovative goods. While the large country retains the same long-run rate of innovation, the small country will innovate less rapidly than in autarky. On the other hand, the presence of local public goods generally speaking prepares the ground for hysteresis. Thus, "history" (as reflected in initial conditions) not only matters, but is decisive. There is a long tradition in

[14]) It must be noted that both authors have developed their views substantially since then.

economic theory, particularly prominent in Alfred Marshall's analysis of "localized industries", to ascribe local externalities to the production of knowledge[15]). Modern means of communication tend to erode this local exclusiveness, as Marshall (1920, p. 284f) already had indicated. However, there is evidence that localization still plays a role[16]). Though "pedagogically useful" (Grossman - Helpman, 1994B, p. 39), these two polar cases - knowledge as purely local or global public good - are admittedly unrealistic[17]). One major purpose of this paper is to contribute to closing this gap.

Closely related to the R&D spillovers theme and the notion of absorptive capacities is the issue of imitation[18]). The treatment of imitation in Grossman - Helpman (1991, Chapter 11) is formally analogous to innovation in the sense that the cumulative number of imitated products enhances the productivity of imitative activities in the follower country. However, there remain important differences. First, insofar as imitators do *not* contribute to the global stock of new "blueprints" they *do not innovate*. In practice, the borderline between imitative and innovative activities is not easily drawn. In many cases, innovation will comprise elements of imitation, and vice versa. Furthermore, imitation and innovative activities based on cross-border interaction of knowledge capital may well be conducted simultaneously. The model of Grossman and Helpman abstracts from this kind of complication and restricts itself to "pure" imitation. In that model - and this is the second major difference - the presence of a manufacturing cost differential between innovators and imitators (conveniently termed "Northern" and "Southern" firms) is essential for the viability of imitation. Unit costs in the Southern firm (which equal the wage rate prevailing in the South) must be below the price charged by its Northern rival in order to render market entry profitable.

The restriction to pure imitation seems useful in analyzing economic development strategies and the role of R&D spillovers therein. While technological laggards may initially be more or less "passive" importers of foreign technology in the sense that no substantial own R&D activities (either imitative or

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[15]) See Marshall (1920, Book IV, Chapter X). In his discussion of Marshall, however, Krugman (1991, p. 53f) in fact dismisses knowledge spillovers as a legitimate subject of economic inquiry: "Knowledge flows ... are invisible; they leave no paper trail by which they may be measured and tracked, and there is nothing to prevent the theorist from assuming anything about them she likes". In view of recent empirical research in the field, primarily based on patent information (see, e.g., Jaffe, 1989B, Jaffe - Trajtenberg - Henderson, 1992), this view appears overly restrictive.

[16]) Even some phenomena of globalization may be due to the presence of local externalities. Recent empirical research on foreign direct investment, for example, has revealed the establishment of "listening posts" in high-tech areas to be an important motivation for the location of R&D subsidiaries (see Pearce - Singh, 1992).

[17]) An intermediate case between those extremes is set out in Grossman - Helpman (1990) by means of introducing lags allowing for asymmetries in the velocity of knowledge dissemination within and between countries.

[18]) See Grossman - Helpman (1991) and Helpman (1993).

innovative) of the recipient country are involved, more advanced countries are likely to exhibit a much more complex pattern of own research and assimilation of externally produced knowledge. High-performing newly industrialized countries do not seem to have followed a purely "passive" strategy[19]), possibly orchestrated by technologically advanced corporations from developed countries. Rather, deviating from the Vernonian product cycle model, imitative activities combined with manufacturing cost advantages seem to have played a key role in their moving up-stream technologically[20]). However, pure *imitation* is not a viable strategy for countries with unit manufacturing costs equal to those of the technological leader. On the other hand, interaction of knowledge stocks based on absorptive capacities may be hypothesized to play an important role for the most advanced countries. Furthermore, there may be critical levels for the domestic R&D capital stock in order to interact effectively with external R&D capital stocks[21]). The importance of interactions between domestic and external knowledge capital stocks may thus be expected to increase with a country's level of economic development[22]). The model we present in this paper is concerned with innovative activities enhanced by cross-border interaction of knowledge capital stocks and, unlike models of pure imitation, is not confined to the case where manufacturing cost differentials persist. It may be thought of being of particular interest for economies beyond the pure "imitation stage" of economic development.

4. Autarky

Before we introduce absorptive capacities, we recall as a reference the basic endogenous growth model with brand proliferation due to Grossman and Helpman. We will start by assuming a world

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[19]) Fagerberg (1988, p. 451) concludes that "to catch up with the developed countries ... semi-industrialized countries cannot rely only on a combination of technology import and investments, but have to increase their national technological activities as well".

[20]) This was stressed by Grossman - Helpman (1991, p. 310). Furthermore, Helpman (1993, p. 1250) points out that "most technological imitation ... takes place in newly industrialized countries (first and second generation), while the majority of LDCs engage in this activity only marginally ...".

[21]) This is reminiscent of the statement made by Baumol - Blackman - Wolff (1989, p. 204) that "a bit of backwardness may contribute to a higher growth rate, but beyond some point it seems clearly to become pure handicap".

[22]) See Fagerberg (1994, p.1161) for a similar assessment.

comprised of self-contained autarkic economies which are neither linked by trade, investment nor knowledge flows. In order to avoid country indices, we first outline the model for a single autarkic economy. For brevity, we do not intend to restate the model in all its aspects here, but restrict our attention to those parts with particular relevance in the present context[23]. In the model of Grossman and Helpman, the representative household maximizes utility over an infinite time horizon. Intertemporal preferences are assumed to be of the form

$$U(t) = \int_t^{\infty} \exp(-\rho(\tau - t)) \log D(\tau) d\tau \quad .$$

Following Dixit - Stiglitz (1977) an index of consumption $D(.)$ reflecting "love of variety"[24] is defined by

$$D(.) = \left[\int_0^{n(t)} x(j)^{\alpha} dj \right]^{1/\alpha} ,$$

with $n(t)$ the measure of products invented before time t and $x(j)$ representing consumption of brand j. The parameter $0 < \alpha < 1$ is related to the elasticity of substitution, ε , by $\varepsilon = 1/(1-\alpha)$. Following Ethier (1982), $D(.)$ can also be interpreted as a production function for final output produced by a set of differentiated inputs. Intertemporal maximization of utility requires aggregate spending $E(t)$ to evolve according to

$$\frac{\dot{E}(t)}{E(t)} = i(t) - \rho ,$$

where $i(t)$ stands for the nominal interest rate and ρ for the time-invariant discount rate. Aggregate nominal household spending, $E(t)$, is normalized so that

$$E(t) = 1 \qquad \text{for all } t .$$

Thus, the nominal interest rate, $i(t)$, equals the discount rate ρ , i.e.

$$i(t) = \rho .$$

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[23]) For a complete statement of the model the reader is referred to Grossman - Helpman (1991, Chapter 3).

[24]) For a discussion see Helpman - Krugman (1985, Chapter 6).

Each brand of intermediates is produced by a single, representative firm by means of a common constant-returns-to-scale technology involving a single primary factor of production ("labor"). With an appropriate choice of units, one unit of labor is required to turn out one unit of manufacturing output. Brand producers maximize operating profits at time t by pricing the unique differentiated input they manufacture according to the mark-up pricing rule

$$p(t) = \frac{w(t)}{\alpha},$$

where $w(t)$ represents the wage rate in the country at time t which in turn equals the marginal and average cost of a unit of output. In an equilibrium, operating per-brand profits $\pi(t)$ at time t are given by

$$\pi(t) = \frac{1-\alpha}{n(t)}.$$

The production function for "blueprints" is Romer-type[25]). It includes public knowledge capital as an input. The productivity of resources devoted to the development of new intermediates - in our case labor employed in R&D, L_n - is enhanced by the stock of knowledge capital, $K(n)$. Thus, we have

$$\text{(4.1)} \qquad \dot{n} = \frac{1}{a} L_n K(n)$$

where $1/a$ stands for the amount of labor involved in R&D needed to develop a unit of new product. Moreover, for the time being, $K(.)$ is assumed to be proportionate to the number of intermediates invented in the economy so far. Thus, with an appropriate choice of units, we have

$$\text{(4.2)} \qquad K(n) = n.$$

R&D has several aspects in this model. First, every research project resulting in a new intermediate good gives rise to a stream of monopoly profits appropriated by the "innovating" firm. Second, every research project adds to the existing stock of knowledge and thus, finally, increases the productivity of resources devoted to R&D. The stock of knowledge capital is a public input into the production of blueprints. The absorptive capacities of R&D establishments operating within the national context are not explicitly addressed in this model. They are implicitly assumed to be at their maximum attainable

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[25]) See Romer (1990) and Grossman - Helpman (1991, Section 3.2).

level. The value $v(t)$ at time t of the representative firm manufacturing a differentiated input is assumed to be equal to the cost of developing a blueprint. Since the cost of a blueprint is $w(t)a / n(t)$ in an equilibrium with continuing innovation (i.e. $\dot{n} > 0$), we arrive at the "free-entry condition"

$$v(t) = \frac{w(t)a}{n(t)} .$$

Equilibrium in the capital market requires that total return on equity claims is equal to the interest rate. This gives the "no-arbitrage condition"

$$\frac{1-\alpha}{n(t)v(t)} + \frac{\dot{v}(t)}{v(t)} = \rho ,$$

where the left hand side comprises the ratio of operating profits per brand to the value of the representative firm and the rate of change of the latter. Finally, labor market clearing requires

$$\frac{a}{n(t)}\dot{n}(t) + \frac{1}{p(t)} = L ,$$

i.e. the sum of the labor demands of R&D and manufacturing enterprises equals the exogenous labor supply L which is assumed to be constant over time. Grossman and Helpman examined the long-run properties of a model combining these conditions with the pricing equation. Most importantly, the presence of public knowledge capital has the implication that stylized economies of that kind continue to grow endogenously. Let us pass to a stylized world economy composed of two autarkic countries A and B, both being of the kind described in the present section. Let us define the rate of innovation by $g(t) \equiv \dot{n}(t) / n(t)$. Assuming that the parameters α and a as well as the diescount rate ρ do not vary across countries, their steady-state growth rates are given by

$$\text{(4.5)} \quad \bar{g}^i = (1-\alpha)\frac{L^i}{a} - \alpha\rho \quad i = (A,B) .$$

The parameter restrictions $L^i / a > \alpha\rho / (1-\alpha)$ $(i = A,B)$ ensure that the steady-state equilibrium values of the stock market and the rates of innovation are positive. This disjoint world consisting of two economies is, in general, not a world of equals since the rate of innovation is positively related to country size as measured by the size of the total labor force L^i $(i = A, B)$[26]). Identical rates of innovation will prevail only if the countries are of equal size[27]).

5. Absorptive Capacities in an Endogenous Growth Model

The following analysis dealing with a large country A and a smaller country B (with the size measured in terms of their time-invariant supply of homogeneous labor L^A and L^B) modifies the basic brand proliferation model in one important aspect. While we leave the representation of the evolution of economy A unchanged, we introduce an interactive way of blueprint production in country B. Departing from the model of the last section, we now assume that country B has become a "smart follower" by developing the ability to "tap" the knowledge stock accumulated in country A by means of its own investment in R&D, using the ensuing addition to the available stock of knowledge to enhance its productivity in developing new blueprints of its own. The appropriation of knowledge stocks created externally, thus, is not assumed to be costless. While the knowledge stock of country B is purely local, due to the absorptive capacities of country B, the knowledge stock of country A may be described as being of mixed local-global character. To put it differently, the knowledge stocks in country A and country B are public inputs entering the production function of manufacturers operating within the national borders of the respective country. Against this pure public goods property of "national" knowledge capital we introduce a kind of friction in the international dissemination of knowledge which constitutes a costly process. Obviously, this model remains restrictive in a number of respects. Knowledge flows, for example, are assumed to be purely unidirectional: Knowledge is transmitted from A to B, but not from B to A. Moreover, transmission of knowledge (where it occurs) is instantaneous and markets remain segmented (no trade). For simplicity, the autarkic country A is assumed to have reached its steady state. Thus, as before (see (4.8)), its rate of innovation is given by

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[26]) This, of course, may be viewed as a rather undesirable (since counterfactual) property of the brand proliferation model.

[27]) Or - in case productivity in transforming knowledge into innovative intermediates (as reflected in the size of the parameter a) were allowed to vary across countries - if higher/lower productivity in product development just matches the impact of smaller/larger country size.

$$\bar{g}^A = (1-\alpha)\frac{L^A}{a} - \alpha\rho$$

and its knowledge stock is growing exponentially:

(5.1) $\quad n^A(t) = n^A(0)\exp(\bar{g}^A t)$.

Hence, instead of considering $n^A(t) = n^A(0)\exp(\bar{g}^A t)[1+o_t(t)]$ with $o_t \to 0$ as $t \to \infty$, we confine ourselves to the main term of this representation. For notational convenience we assume that everything starts at $t = 0$.

Let us next turn to the interactive mode of product development in country B which constitutes our major modification of the original model. The knowledge stock $K^B(t)$ available in country B at t is assumed to consist of the sum of the knowledge originating in country B which is represented by the number of differentiated inputs developed domestically, $n^B(t)$, and a term comprising externally produced knowledge appropriated by country B:

(5.2) $\quad K^B(t) = n^B(t) + \gamma(n^B(t))\delta n^A(t)$.

A fraction $\delta n^A(t)$ of the knowledge stock produced in country A enters a pool of knowledge which is accessible for adoption in country B, in principle. The value of δ may be thought of being determined, among others, by the stringency of property rights protection, the effectiveness of secrecy and international communication, but may also reflect the extent of duplicative research which will not be targeted by the follower. The restriction $0 \le \delta \le 1$ may be introduced to confine the degree of permeability between the poles of perfect protection and perfect openness regarding knowledge flows[28]). Obviously, the special case with $\delta = 0$ or $\gamma(.) \equiv 0$ results in the perfect autarky model, whereas the case with $\delta = 1$ and $\gamma(.) \equiv 1$ implies complete, instantaneous and costless knowledge spillovers from country A to country B, reminiscent of the traditional neoclassical ubiquity of technology. Our analysis, however, is concerned with the more general case that absorptive capacity is created by own R&D investment. The values of the function $\gamma(.)$ are assumed to be in the range $0 \le \gamma(.) \le 1$. Furthermore, the restrictions $\gamma(0) = 0$ and $\gamma'(.) > 0$ are imposed. The first condition means that in the absence of domestic innovation absorptive capacity is zero. The second just implies that absorptive capacity increases as the stock of knowledge produced in the country grows. Dividing both sides of equation (5.2) through $n^B(t)$ we obtain

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[28]) In the mathematical analysis we will treat δ as a positive constant.

(5.3) $$k^B(t) = 1 + \delta\gamma(n^B(t))\frac{n^A(t)}{n^B(t)},$$

where $k^B(t) \equiv K^B(t)/n^B(t)$ represents the ratio of the knowledge stock *available* in country B to cumulated knowledge *produced* in country B. Thus, the rate of innovation in country B, $g^B(t) \equiv \dot{n}^B(t)/n^B(t)$, is given by

(5.4) $$g^B(t) = \frac{1}{a}k^B(t)L_n^B.$$

The redefinition of the knowledge capital stock in country B by (5.2) calls for modifications of the original perfect-autarky model. Now, for country B, the free-entry condition becomes

(5.5) $$\frac{w^B(t)a}{K^B(t)} \geq v^B(t),$$

where equality holds whenever $\dot{n}^B(t) > 0$. The labor market clearing condition is reformulated to yield

$$\frac{a\dot{n}^B(t)}{K^B(t)} + \frac{1}{p^B(t)} = L^B.$$

Combining these two conditions with the unchanged mark-up pricing equation $p^B(t) = w^B(t)/\alpha$ and the no-arbitrage condition and substituting our specification of $K^B(t)$ (see (5.2)) yields

(5.6) $$\dot{n}^B(t) = \begin{cases} \left[n^B(t) + \gamma(n^B(t))\delta n^A(t)\right]\frac{L^B}{a} - \frac{\alpha}{v^B(t)} & \text{for } v^B(t) > \frac{\alpha a}{\left[n^B(t) + \gamma(n^B(t))\delta n^A(t)\right]L^B} \\ 0 & \text{for } v^B(t) \leq \frac{\alpha a}{\left[n^B(t) + \gamma(n^B(t))\delta n^A(t)\right]L^B} \end{cases}$$

(5.7) $$\dot{v}^B(t) = \rho v^B(t) - \frac{(1-\alpha)}{n^B(t)}.$$

This completes the description of the model. To sum up, our stylized world economy is given by equation (5.1) for the dynamics of growth of the stock of knowledge for the autarkic country A (the technological leader), the system of differential equations (5.6) and (5.7) representing the evolution of the follower country B and associated restrictions on the parameters and the functional form of $\gamma(.)$[29]). Also, the initial values are restricted to be $n^B(0) > 0$ and $v^B(0) > 0$.

Since we concentrate on the long-run relative innovative and growth performance of the follower country with respect to the leader, the main task of our analysis is to examine the asymptotics of the ratio of innovative products introduced in the two countries up to time t (which is interpreted as the ratio of the knowledge stocks produced in the two countries),

$$r(t) = n^B(t) / n^A(t)$$

as t goes to infinity, i.e.

$$\lim_{t\to\infty} r(t) = r_\infty .$$

Given the steady-state rate of innovation of the technological leader A, $\bar{g}^A$, inference can be made to the long-run rate of innovation of the follower country B relative to A, or, introducing the time lag $\tau(t)$, it can be shown how many time units ago the stock of knowledge of country A was the same as the knowledge stock of country B at time t,

$$n^B(t) = n^A(t - \tau(t)) = \exp \bar{g}^A((t - \tau(t)))n^A(0) .$$

It is also clear that

$$\lim_{t\to\infty} \exp(-\bar{g}^A \tau(t)) = r_\infty .$$

Furthermore, let

$$\gamma_\infty = \lim_{n^B\to\infty} \gamma(n^B) .$$

In the Appendix we outline the proof of the fact that three asymptotics are admissible for a solution of the system of differential equations (5.6)-(5.7). For these asymptotics, the limit ratios of cumulatively introduced new products ("knowledge stocks") are computed. The main statement of the mathematical analysis is contained in the following Proposition 1.

[29]) It is straightforward that equations (5.6) - (5.7) can be transformed to a system of differential equations which bears resemblance to that addressed by Grossman - Helpman (1991, Appendix A3.1) in a general way in their analysis of non-linear accumulation of knowledge capital in a closed economy. Our two-country model is more complicated because $k^B(.)$ is specified as a non-linear function in both $n^B(t)$ and $n^A(t)$.

Proposition 1. *The set of all potentially admissible (i.e. positive) initial values* $n^A(0)$, $n^B(0)$, $v^B(0)$ *is split at most into three subsets. Starting from any point of the first, the second, and the third subset, the trajectory* $n^B(.)$, $v^B(.)$ *has, respectively,* lower, upper, *and* intermediate *asymptotics.*

The lower *asymptotics is characterized by* $n^B(t) = const$ *for large* t, *and*

(5.8) $r_\infty = 0$;

this asymptotics is feasible, i.e. is realized by a certain trajectory. The upper *asymptotics is characterized by* $n^B(t)$ *and* $v^B(t)$ *growing to infinity as* t *goes to infinity and*

(5.9) $$r_\infty = \begin{cases} \gamma_\infty \delta \dfrac{\overline{g}^A}{\overline{g}^A - L^B / a}, & \overline{g}^A > L^B / a \\ \infty, & \overline{g}^A \le L^B / a \end{cases} ;$$

this asymptotics is feasible. The intermediate *asymptotics is characterized by* $n^B(t)$ *growing to infinity,* $n^B(t)v^B(t)$ *approaching the limit* $1/(L^A / a + \rho)$ *as* t *goes to infinity, and*

(5.10) $$r_\infty = \gamma_\infty \delta \frac{1}{(L^A / L^B) - 1}.$$

Trajectories with the upper and lower asymptotics always exist, while for the existence of a trajectory with the intermediate asymptotics the condition

(5.11) $L^B < L^A$

is necessary, and condition (5.11) together with condition

(5.12) $\alpha(L^A / a + \rho)L^B / a < 1$

are sufficient.

A sketch of the proof of this Proposition is given in the Appendix.

Let us now turn to the three admissible asymptotics we have identified and discuss their economic implications. *First*, in the case of *lower asymptotics* the knowledge stock of the follower does not grow. In this case, the ratio of the stock of knowledge produced in country B to the stock of knowledge originating in country A approaches zero (see (5.8)). Equivalently, it can be stated that the time lag of the follower B vis-à-vis the technological leader A grows to infinity, i.e. $\tau_\infty = \infty$. In this case, innovation ceases or never gets started in country B while the value of the representative firm in that country, $v^B(t)$, declines over time and eventually turns negative. In that instant, of course, the value of the stock market $v^B(t)n^B(t)$ becomes negative as well. Lower asymptotics occurs, if the initial value $v^B(0)$ is small enough.

Second, with a trajectory characterized by *upper asymptotics* the long-run ratio of knowledge stocks either goes to infinity or approaches a constant value. One case ($r_\infty = \infty$) occurs when $a\overline{g}^A \leq L^B$ holds true. Since - by (4.1) and (4.2) - $a\overline{g}^A$ equals country As steady-state employment in R&D, $\overline{L}_n^A$, this inequality has the straightforward interpretation that R&D employment in country A does not exceed the total labor force of country B. This case implies that the "follower" country B grows faster and will eventually command a higher stock of knowledge than the "leading" country A in the long run. As we will see below, in this case, country B realizes higher long-run growth rates of final output and TFP compared to country A. The other case occurs, if the inequality $a\overline{g}^A > L^B$ holds, i.e. if R&D employment in country A is larger than total employment in country B. In this case, $r(t)$ approaches a constant $\gamma_\infty \delta \overline{g}^A / (\overline{g}^A - L^B / a) = \gamma_\infty \delta / (1 - L^B / \overline{L}_n^A)$. Since $r(t)$ approaches a constant, country B exhibits exactly the same long-run rate of innovation as the leader A, which is given by $\overline{g}^A$. Along a trajectory having the upper asymptotics, both the number of innovative products $n^B(t)$ and the value of the representative firm $v^B(t)$ and hence the aggregate value of the stock market $v^B(t)n^B(t)$ grow without bound. The upper asymptotics occurs, if the initial value $v^B(0)$ is large enough.

Third, *intermediate asymptotics* may be considered the most interesting case. With intermediate asymptotics, the ratio of the knowledge stocks produced in the two countries approaches a positive constant which is given by (5.10). In order to render intermediate asymptotics feasible it is necessary that country B is smaller than country A, and sufficient that country B is not too large given the size of country A in the specified way. Since $r(t)$ approaches a constant, the rate of innovation of the follower B equals that of the technological leader A, $\overline{g}^A$, as t goes to infinity. Since (see (4.5)) the autarky steady-state growth rate is positively related to country size measured in terms of total labor force, the autarky steady-state rate of innovation of the smaller country B, $\overline{g}^B$, is lower than that of country A. This impact of country size on the rate of innovation (and thus, as we shall see, on the growth rates of final output and TFP) is offset here[30]). Along an intermediate trajectory the knowledge stocks of both the larger and the smaller country grow at the same rate (i.e. exponentially) in the long

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[30]) We shall see, however, that relative country size remains a crucial parameter in other respects.

run. Thus, we clearly observe an improvement in the dynamic innovative performance of the relatively smaller follower as compared to the perfect autarky outcome.

A trajectory having intermediate asymptotics has one outstanding property of economic interest. Following Grossman and Helpman let us assume[31]) that the stock market sets the value of the firm at time t equal to the present value of its stream of profits subsequent to t, i.e.

$$(5.13) \quad v^B(t) = \int_t^\infty \exp(-\rho(\tau - \tau)\pi^B(\tau)d\tau \, .$$

A perfect-foresight trajectory, i.e. a trajectory along which expectations are met is realized when the actual value of $v^B(t)$ determined by our system of differential equations (5.6)-(5.7) coincides with (5.13) at any moment of time[32]). The outcomes of applying the above rule for the valuation of the firm to the present model are analogous to those derived by Grossman - Helpman (1991, Chapter 3) for the basic model with brand proliferation. It is intuitively clear that a trajectory having lower asymptotics does not qualify as a perfect-foresight equilibrium. With a constant number of innovative products, per-brand profits remain constant as well (see (4.2)). However, a solution with lower asymptotics implies that the value of the representative firm declines steadily and eventually becomes negative. Thus trajectories having lower asymptotics are inconsistent with perfect foresight in the valuation of the firm. Trajectories characterized by upper asymptotics are also inconsistent with perfect foresight. As the number of new intermediates increases, profits per brand are declining. Investors cannot rationally expect the value of the representative firm to rise continually. Only a trajectory characterized by intermediate asymptotics qualifies as a perfect-foresight equilibrium where expectations are met along the trajectory. In fact, the following Proposition 2 is proved.

Proposition 2. *Every trajectory $n^B(.), v^B(.)$ having the* intermediate *asymptotics is a perfect-foresight trajectory, i.e. satisfies (5.13), where*

$$\pi^B(t) = \frac{1-\alpha}{n^B(t)}.$$

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[31]) For a more subtle discussion of the role of this assumption see Grossman - Helpman (1991, p. 50).

[32]) In this context, note that differentiation of $v^B(t)$ in (5.13) with respect to time yields the no-arbitrage condition $\pi^B(t) + \dot{v}^B(t) = \rho v^B(t)$ which - see (5.7) - entered the system of differential equations under examination.

Every trajectory $n^B(.), v^B(.)$ *that does not have the* intermediate *asymptotics is not a perfect-foresight trajectory.*

Proof of Proposition 2. Consider a trajectory $n^B(.), v^B(.)$. From the model equation (5.7) we have

$$v^B(t) = -\int_0^t \exp(\rho(t-\tau)\frac{1-\alpha}{n^B(\tau)}d\tau + \exp(\rho t)v(0).$$

Hence for every $T > t$

(5.14) $$v^B(t) = \exp((\rho(t-T))v^B(T) + \int_t^T \exp(\rho(t-\tau))\frac{1-\alpha}{n^B(\tau)}d\tau .$$

If $n^B(.), v^B(.)$ follow the intermediate asymptotics, then by Proposition 1, $n^B(T)$ goes to infinity as T goes to infinity and $n^B(T)v^B(T)$ remains bounded. Hence, $v^B(T)$ goes to zero as T goes to infinity. Now, passing to the limit in (5.14) as T goes to infinity, we get (5.13).

Let $n^B(.), v^B(.)$ not follow the intermediate asymptotics. Then, by Proposition 1, we have either the upper asymptotics, or the lower one. Let the upper asymptotics take place, assume that (5.13) holds, and show that this leads to a contradiction. By (5.13) we have

(5.15) $$v^B(t)\exp(-\rho t) = \int_t^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau$$

and, also, for $T > t$

(5.16) $$v^B(t) = \exp(\rho t)\left[\exp(-\rho T)\int_t^T \exp(-\rho\tau)\pi^B(\tau)d\tau + \exp(-\rho T)v^B(T)\right].$$

The integral on the right hand side of (5.15) exists for every t, which implies

(5.17) $$\lim_{s\to\infty}\int_s^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau = 0 .$$

Since $\exp(-\rho T) < \exp(-\rho t)$ for $T > t$, we obtain from (5.16)

$$v^B(t) \le \exp(\rho t)\left[\exp(-\rho t)\int_t^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau + \exp(-\rho T)v^B(T)\right] =$$

$$= \int_t^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau + \exp(\rho t)\exp(-\rho T)v^B(T).$$

Hence, by (5.15),

$$v^B(t) \le \int_t^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau + \exp(\rho t)\int_T^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau.$$

Passing to the limit as $T \to \infty$, by (5.17) we get from this estimate

$$v^B(t) \le \int_t^\infty \exp(-\rho\tau)\pi^B(\tau)d\tau,$$

which by (5.17) implies that

$$\lim_{t\to\infty} v^B(t) = 0.$$

The latter equality is not possible since, by Proposition 1, for the upper asymptotics $v^B(t) \to \infty$ as $t \to \infty$. The contradiction shows that a trajectory with upper asymptotics does not satisfy (5.13), i.e. such a trajectory is not a perfect-foresight trajectory.

Now let the lower asymptotics take place. From Proposition 1, we know that there is a finite time instant t_0 such that $n^B(t) = c$ for $t > t_0$, where c is a positive constant. The latter implies that $\dot{n}^B(t) = 0$ for $t > t_0$. By (5.6) we conclude from this that

$$v^B(t) \le \frac{\alpha a}{[c + \gamma(c)\delta n^A(t)]L^B}, \qquad t > t_0$$

Since $n^A(t) \to \infty$ as $t \to \infty$, the latter inequality implies that

(5.18) $\limsup_{t\to\infty} v^B(t) \le 0.$.

From (5.7) we get

$$(5.19)\quad v^B(t) = \exp(\rho t)\left[v^B(0) - \int_0^t \frac{1-\alpha}{n^B(t)}\exp(-\rho\tau)d\tau\right] =$$

$$= \exp(\rho t)\left\{v^B(0) - c_1 + \frac{1-\alpha}{c\rho}\left[\exp(-\rho t) - \exp(-\rho t_0)\right]\right\},$$

where

$$c_1 = \int_0^{t_0} \frac{1-\alpha}{n^B(\tau)}\exp(-\rho\tau)d\tau > 0.$$

Since $\exp(-\rho t) \to 0$ as $t \to \infty$, to ensure (5.18) we must have

$$v^B(0) < c_1 + \frac{1-\alpha}{c\rho}\exp(-\rho t_0).$$

The latter inequality implies by (5.19) that $v^B(t) < 0$ for all sufficiently large t, but by (5.13) $v^B(t) \geq 0$ for all $t \geq 0$. The contradiction shows that a trajectory having the lower asymptotics cannot be a perfect-foresight trajectory.

This Proposition is proved.

Along an intermediate trajectory, the value of the stock market $v^B(t)n^B(t)$ approaches a positive constant $(1-\alpha)/(\overline{g}^A + \rho)$ (which equals the steady-state value of the stock market in country A) as time goes to infinity, while the introduction of new intermediates continues (see Proposition 1). This implies that the rise in the number of intermediates is balanced by a corresponding decline of the value of the representative firm, i.e. $\dot{v}^B(t)/v^B(t)$ approaches $-\overline{g}^A$ in the long run (this obtains if we substitute the limit for $v^B(t)n^B(t)$ into (5.7)).

Concentrating on intermediate asymptotics we can see from the definition of $k^B(.)$ (see (5.3)) and the expression for the limit ratio r_∞ in (5.10) that in the long run *the ratio of the knowledge stock available in country B to cumulated knowledge originating in that country approaches*

$$k_\infty^B = \frac{L^A}{L^B},$$

i.e. the size of the leading country relative to that of the follower country. For the case of intermediate asymptotics we have from (5.6) that

$$\frac{\dot{n}^B(t)}{n^B(t)} = \left[1 + \gamma\,(n^B(t)\delta\,\frac{n^A(t)}{n^B(t)}\right]\frac{L^B}{a} - \frac{\alpha}{v^B(t)n^B(t)}.$$

Since we set $g^B(t) = \dot{n}^B(t)/n^B(t)$, then $g^B_\infty = \bar{g}^A$, where $g^B_\infty = \lim_{t\to\infty} g^B(t)$. Together with (5.4), this implies that long-run R&D employment in country B approaches

$$L^B_{n\infty} = \frac{L^B}{L^A} a g_\infty{}^B.$$

Since $g^B(t)$ approaches $\bar{g}^A$ in the long run, and - by (4.1) and (4.2) - R&D employment in country A equals $a\bar{g}^A$ in the steady state, we can derive two conclusions regarding the long-run allocation of resources in our stylized world economy. First, *the allocation of labor to R&D and manufacturing remains in the long run constant in country B (and is constant in country A).* Second, *the shares of R&D employment in the total labor force L^i_n / L^i $(i = A, B)$ tend to be the same in both countries in the long run, namely $a\bar{g}^A / L^A$.*

Since the allocation of labor to manufacturing and R&D remains approximately constant, aggregate manufacturing output of intermediates $X^B(t) = n^B(t)x^B(t)$ (where $x^B = 1/n^B(t)p^B(t)$ denotes the output of each brand) also approaches a constant in the long run. Given the production function $D(.)$, final output at time t is given by $D^B(t) = (n^B(t))^{1/\alpha} x^B(t) = X^B(n^B(t))^{(1-\alpha)/\alpha}$ and, consequently, TFP at time t by $D^B(t)/X^B(t) = (n^B(t))^{(1-\alpha)/\alpha}$. The growth rate of final output and TFP is identically $g^B(t)(1-\alpha)/\alpha$. As we have shown, in the case with absorptive capacities the rate of innovation of the follower, $g^B(t)$, approaches the autarky rate of innovation of the larger country, $\bar{g}^A$ in the long run. Since the steady-state autarky rates of innovation are positively related to country size (see (4.5)), *both output and TFP of the follower grow at a higher rate than in autarky.*

In the remainder of this section we take a broader look at the dynamic behavior of the present model in order to illustrate how the introduction of absorptive capacities influences the asymptotics that the follower country realizes. In an explanatory manner, we discuss the dynamics starting from too low an initial value of the representative firm to sustain product development. We shall see that in this case the dynamics is strikingly different from the basic perfect autarky model of Grossman and Helpman. Moreover this case is interesting for expositional purposes since any of the three admissible asymptotics may occur starting from certain low enough initial values of the firm.

Let us denote the curves where the right hand sides of equations (5.6) and (5.7) vanish the n^B-barrier and the v^B-barrier, respectively. Their equations are, respectively,

$$v^B = \mu_n(n^B, n^A), \quad v^B = \mu_v(n^B),$$

where

$$\mu_n(n^A, n^B) = \frac{\alpha a}{[n^B + \gamma(n^B)\delta n^A]L^B}$$

$$\mu_v(n^B) = \frac{1-\alpha}{\rho n^B}.$$

In the following we will use notations

$$\mu_n[t] = \mu_n(n^B(t), n^A(t)),$$

$$\mu_v[t] = \mu_v(n^B(t)).$$

By the model equation (5.6)

$$\dot{n}^B(t) = 0 \qquad \text{if} \qquad v^B(t) \le \mu_n[t],$$

i.e. no innovation takes place at time t if the value of the representative firm, $v^B(t)$, is at or below the n^B-barrier.

In order to show the difference to the original model, let us first consider the case where country B is a perfectly autarkic economy of the type described in Section 4. In this case, the equation for the n^B-barrier is

$$v^B = \tilde{\mu}_n(n^B)$$

where

$$\tilde{\mu}_n(n^B) = \frac{\alpha a}{n^B L^B}.$$

Thus, using a notation similar to the one introduced above,

$\tilde{\mu}_n[t] = \tilde{\mu}_n(n^B(t))$.

No innovation takes place if the value of the representative firm in the economy is below the n^B-barrier. Since $\tilde{\mu}_n(.)$ is a decreasing function of n^B, a technologically "backward" country which has inherited a small stock of knowledge requires a higher threshold value of the representative firm than a more developed country with a higher initial stock of knowledge in order to get innovation started in the first place. In this sense, a backward country runs a higher risk of being trapped into a no-innovation/no-growth trajectory (which, as we have seen, is inconsistent with perfect foresight in the valuation of the firm). Furthermore, in the perfect autarky case we have

$$\dot{\tilde{\mu}}[t] = 0 \qquad \text{if} \qquad v^B(t) \leq \tilde{\mu}_n[t].$$

Thus, there is no chance for a country with an initial value of the firm below the n^B-barrier to ever escape the no-innovation trap.

In the case with absorptive capacities, the dynamics at or below the n^B-barrier is considerably more complex. In this case the derivative of $\mu_n[.]$ is given by

$$\dot{\mu}_n[t] = -\sigma \frac{\dot{K}^B(t)}{[K^B(t)]^2},$$

where $\sigma = \alpha a / L^B$ and $K^B(t) = n^B(t) + \gamma(n^B(t))\delta n^A(t)$ designates the knowledge stock available in country B. Since $\dot{n}^B(t) = 0$ if $v^B(t)$ is at or below the n^B-barrier, the change in the knowledge stock of country B, $\dot{K}^B(t)$, reduces to

$$\dot{K}^B(t) = \gamma(n^B(t))\delta\dot{n}^A(t) = \gamma(n^B(t))\delta\bar{g}^A n^A(t),$$

where $\gamma(n^B(.)) = const$. At any moment of time, country B adds a constant fraction of the incremental knowledge stock of the leader A to its own stock of knowledge, thereby lowering the cost of developing a blueprint of its own. Also, $\mu_n[.]$ is strictly decreasing and thus the threshold value of $v^B(.)$ - above which product development in country B is viable - is declining. For given initial values $n^A(0)$ and $n^B(0)$, parameters determining σ as well as δ, the leader's rate of innovation, $\bar{g}^A$, determines the velocity of $\mu_n[.]$. The higher the rate of innovation in the leading country, the faster the decline of $\mu_n[.]$. *In this sense it appears promising for a technological laggard to link up informationally with a rapidly innovating country.* However, in the relevant area, the value of the representative firm, $v^B(.)$, is declining as well. Thus the final outcome regarding the kind of asymptotics realized by country B depends on the velocity of $\mu_n[.]$ relative to that of $v^B(.)$.

If the velocity of $v^B(.)$ at some large enough time instant t_0 is no greater than the derivative $\dot{\mu}_n[.]$, i.e.

$$\dot{v}^B(t_0) \leq \dot{\mu}_n[t_0],$$

$v^B(.)$ is falling behind $\mu_n[.]$. We can show (see for details Kryazhimskii (1995)) that this implies that innovation will never get started in country B. In this case we arrive at the lower asymptotics. If, however, the derivative of $v^B(.)$ exceeds that of $\mu_n[.]$ at some large enough time instant $t = t_0$, i.e.

$$\dot{v}^B(t_0) > \dot{\mu}[t_0],$$

then $v^B(.)$ catches up with $\mu_n[.]$. We can show that the economy B will eventually find itself between the n^B- and v^B-barriers. In this case, however, all three asymptotics - lower, intermediate, and upper - may occur depending on the precise constellation. It follows that in the presence of absorptive capacities the follower country may be able to avoid the no-innovation trap although the initial value of the representative firm is too low to sustain product development initially. In particular, country B may be able to embark on an equilibrium trajectory characterized by intermediate asymptotics. So far, we have touched upon the dynamics at or below the n^B-barrier as well as between the two barriers. For completeness, let us add that starting from a position at or above the v^B-barrier we arrive at upper asymptotics. The dynamic behavior of the model is schematically summarized in the phase diagram given in *Figure 1*.

6. Catching Up and Taking Over

Let us thus turn to the question whether there exists a trajectory having the intermediate asymptotics along which a process of catching up or taking over takes place. We call a trajectory $(n^B(.), v^B(.))$ *catching up* if $r(t)$ is strictly increasing, and *overtaking* if it is catching up and

$$r(0) < 1 < r_\infty.$$

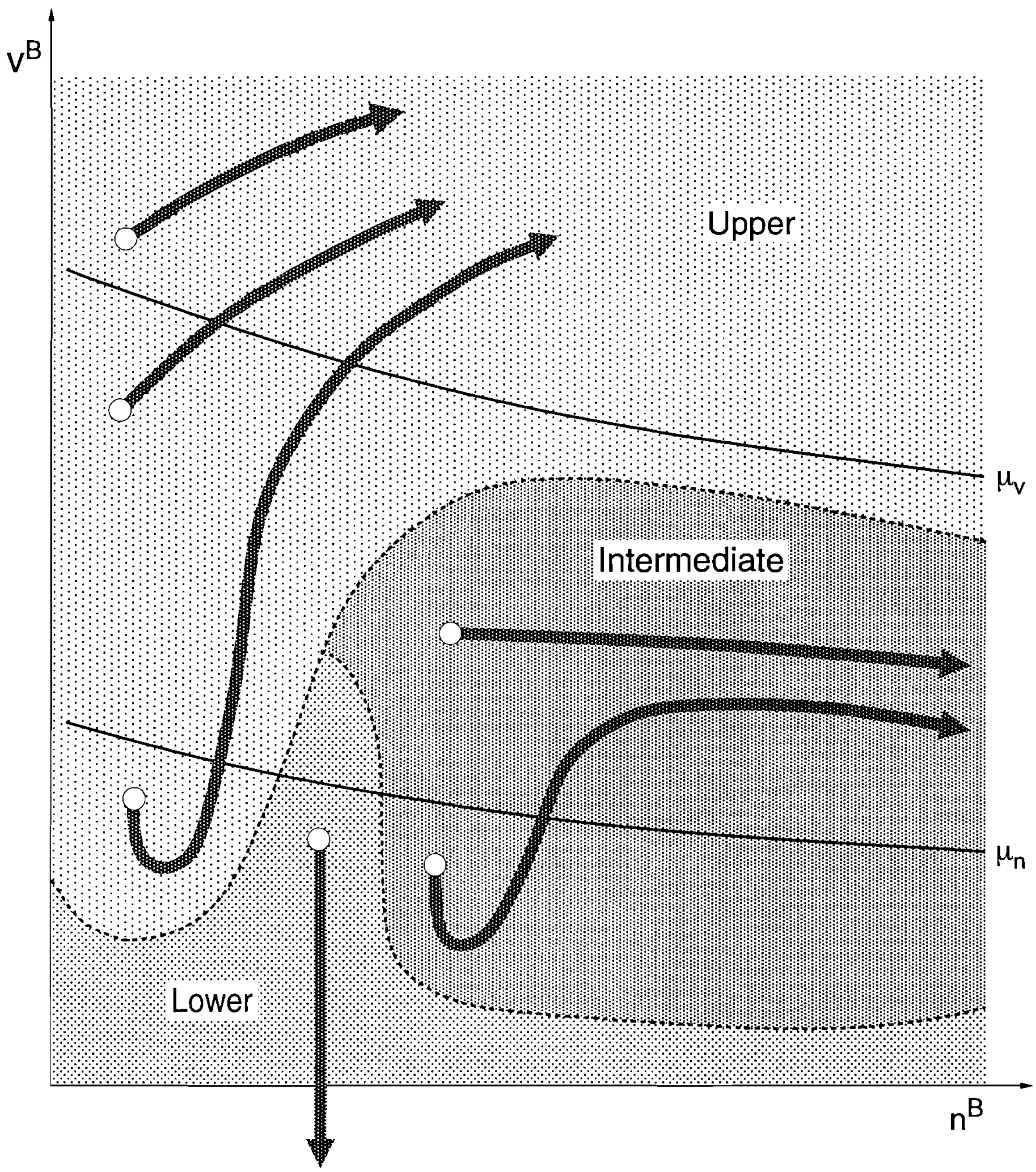

Figure 1: A schematic representation of the n^B- and v^B-barriers and the separation of the n^B-, v^B-plane into domains which lead to the lower, intermediate and upper asymptotics.

In the latter case the initially lagging follower ends up with a larger stock of knowledge than the initially leading country. Due to the properties of the production function $D(.)$ catching up or taking over in terms of the knowledge stock implies catching up or taking over in terms of TFP.

Introduce the following *Non-Polinomiality Condition*: for every interval $[p_1, p_2]$ of nonzero length with $p_1 \geq 0$ there do not exist positive a, β such that

$$\gamma'(p) = ap^{-\beta}$$

for all $p \in [p_1, p_2]$. This condition is of a purely technical nature. Conceptually, it is needed to prevent $n^B(.)v^B(.)$ from being constant over an interval, in particular, to ensure that this product cannot equal its limit value over an interval. As such, this condition is very natural, however. Two examples of $\gamma(.)$ satisfying the Non-Polynomiality Condition are $\gamma(p) = 1 - \exp(-p)$ and $\gamma(p) = \frac{2}{\pi} arctg(p)$.

Proposition 3. *Let the Non-Polinomiality Condition and inequalities* (5.11), (5.12) *be satisfied, and*

$$\bar{g}^A < L^B / a.$$

Then

(i) there exists a catching-up trajectory with the intermediate asymptotics;

(ii) if (see 5.10*))*

$$r_\infty = \gamma_\infty \delta \frac{1}{L^A / L^B - 1} > 1,$$

then there exists an overtaking trajectory with the intermediate asymptotics.

This Proposition can be proven in a similar way as Proposition 1. For details see Kryazhimskii (1995).

We have thus shown that in our model with absorptive capacities there, in fact, exists a perfect-foresight trajectory along which the follower country catches up with or takes over the leading country under certain well-defined conditions. To establish any of these two results we employ $L^B > a\bar{g}^A$ as a sufficient condition. As we have noted above, this inequality states that total employment in the smaller follower country is larger than R&D employment in the larger country A. To establish overtaking we applied the additional condition $r_\infty = \gamma_\infty \delta / (L^A / L^B - 1) > 1$ (see 5.10) implying that there are combinations of L^A / L^B and $\gamma_\infty \delta$ (with $\gamma_\infty \delta > L^A / L^B - 1$) for which the knowledge stock (and thus TFP in manufacturing) in the follower country exceeds that of the leader in the long run. In this case with $r_\infty > 1$ (or, equivalently, $\tau_\infty < 0$), the follower's knowledge stock "runs ahead" that of the leader for time $-\tau(t)$ close to a finite value $-\tau_\infty$, where

$$\tau_\infty = -\frac{1}{\bar{g}^A} \ln r_\infty$$

with r_∞ given by (5.10). Note that only the value of the limit γ_∞ of $\gamma(.)$ is relevant for the size of the long-run ratio of knowledge stocks[33]). The limit ratio r_∞ is inversely related to the size of country A relative to country B. However, long-run effectiveness in "tapping" externally produced knowledge is mitigating this disadvantage of small size. Taking over the leader in terms of the knowledge stock produced in the country is impossible when the difference in the size of the two countries is too large. In particular, since both $\gamma(.)$ and δ are restricted to values not exceeding unity, taking over is precluded if the follower country B is no larger than half the size (in terms of its total labor force) of the leading country A. If, however, $\tau_\infty \geq 0$ (or $r_\infty \leq 1$), then, for large t, the follower's knowledge stock "tracks" that of the leader with time lag $\tau(t)$ approaching a finite value τ_∞. In this sense, the opportunities of the follower for "catching up" and "forging ahead" (to allude to Abramovitz, 1986) are limited. In establishing the existence of a perfect-foresight trajectory and the feasibility of the follower country's catching up and taking over along such a trajectory, we have introduced a variety of conditions. To summarize, *Figure 2* provides a schematical representation of these conditions in terms of the country sizes L^A and L^B for fixed parameter values. A summary interpretation of the emerging patterns is given, among others, in the following conclusions.

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[33]) However, the function $\gamma(n^B(t))$ has a bearing on which of the admissible asymptotics is realized.

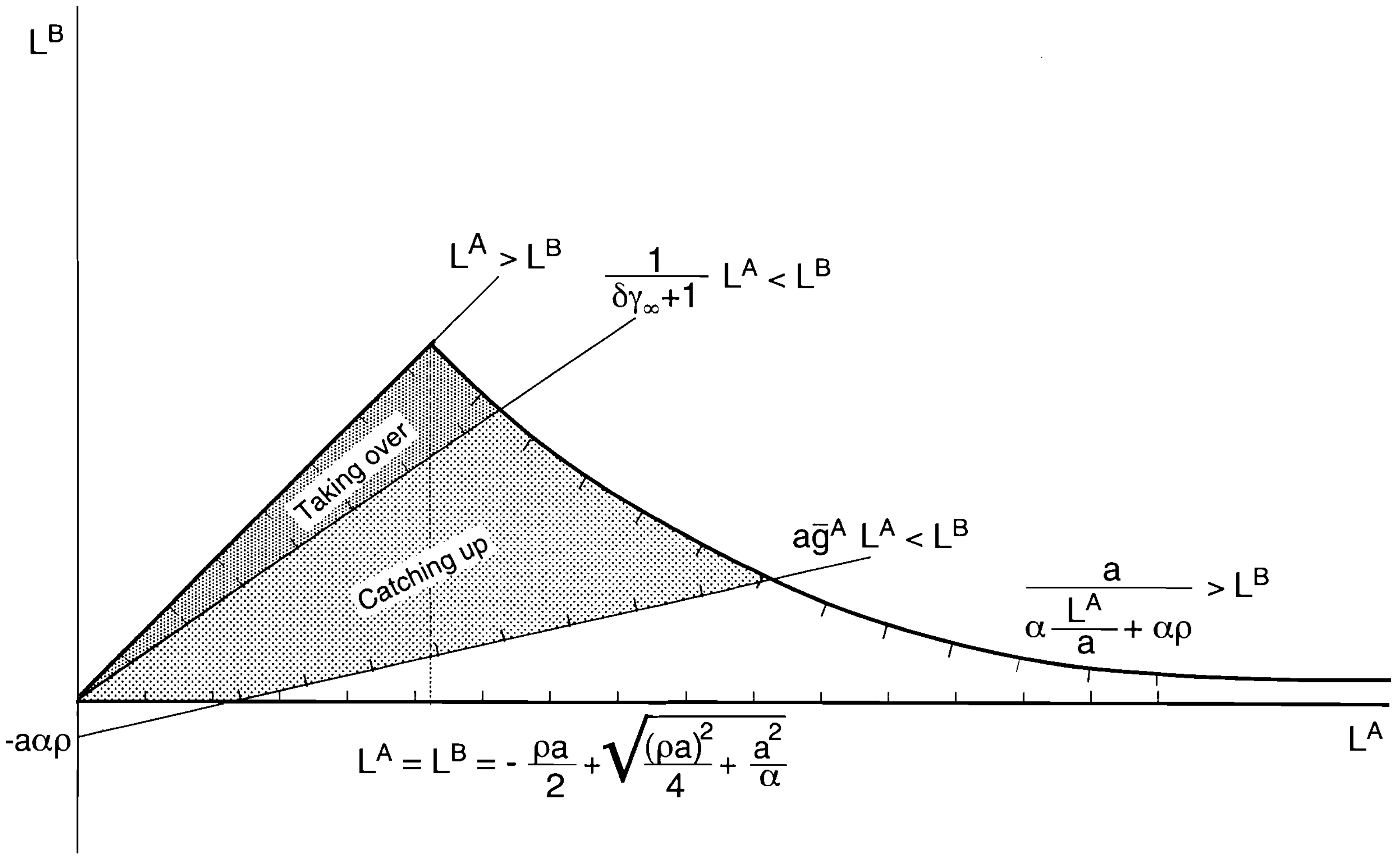

Figure 2: A feasibility domain for a perfect-foresight trajectory in the L^A/L^B-space.

7. Conclusions

This paper is concerned with the impact of absorptive capacities, i.e. the ability of a country to appropriate externally produced knowledge by means of its own R&D investment on its long-run innovative and growth performance. We have limited our analysis to the case of a smaller country (the technological follower) with absorptive capacities and a larger, autarkic country (the technological leader), the latter having reached a steady state characterized by an exponential growth of its knowledge stock. For the model presented here, three asymptotics are shown to be admissible. We have shown that there exists a trajectory consistent with perfect foresight in the valuation of the firm characterized by ongoing innovation and endogenous growth. Under these conditions the follower's rate of innovation - and thus final output and total factor productivity (TFP) growth - approaches that of the leader, and the long-run shares of R&D employment are equalized across countries. The ratio of the knowledge stock produced by the follower to that of the leader approaches a positive constant which is inversely related to the size of the leading country relative to that of the follower country. The disadvantage of the small size of the follower country is mitigated by its long-run absorptive capacities. Country size no longer determines the follower's long-run rate of innovation as it does in perfect autarky, implying that the innovative and growth performance of the follower shows a clear improvement as compared to the perfect autarky benchmark. For the existence of a perfect-foresight trajectory, the condition that the follower country B is strictly smaller than the leading country is necessary, and the same condition combined with the constraint $a/\alpha(L^A/a+\rho) > L^B$ is sufficient. Taking a closer look at these necessary and sufficient conditions (see *Figure 2*), we notice that there is a maximum size for the follower country B consistent with a perfect-foresight trajectory determined by the intersection of the two constraints. This maximum size of the follower country is inversely related to the shared R&D productivity parameter $1/a$. Furthermore, for consistency with perfect foresight, the follower country has to be smaller, the larger the size of the technological leader. A perfect-foresight trajectory is feasible for an arbitrarily small follower, irrespective of the size of the leader.

This latter statement, however, does not hold true for the feasibility of a catch-up process. Catching up by a lagging country along a perfect-foresight trajectory - which is defined by a strictly increasing ratio of its knowledge stock relative to that of the leader - is feasible when R&D employment in the leading country is smaller than the total labor force of the follower. Empirically, this constraint is not particularly restrictive given the fact that R&D employment still constitutes a very small fraction of total employment even in the technologically most advanced countries. Catching-up implies transitional positive differential TFP and output growth of the follower. For taking over, the "follower" must catch up *and* end up with a higher stock of knowledge than the initial "leader" in the long run. In order to be able to take over in this sense, the follower country must be relatively close to the leader in terms of the size of the total labor force. In particular, taking over along a perfect-foresight trajectory is not feasible if the follower is no larger than half the size of the leader. It has to be noted, however, that the opportunities of catching up and taking over have their limits in the sense that the time-lags in "tracking" or "running ahead" of the leader tend to finite values. The latter part of the statement implies that the follower cannot "forge ahead" without bound.

As a first attempt, the present model is deliberately kept simple and extremely stylized. In particular, it is confined to the case of two countries characterized by purely unidirectional knowledge flows. Nevertheless, it appears richer in its dynamic behavior and its explanatory scope than the basic model of brand proliferation. Unlike the basic model of Grossman and Helpman, for example, an originally stagnant follower economy (with too low a value of the representative firm to fund product development) might still be able to embark on an equilibrium trajectory with ongoing innovation, in particular when it is linked informationally to a rapidly innovating leader. The model accounts for a number of observed facts about international economic growth. As we have seen, processes of catching up and taking over in terms of countries' knowledge stocks and TFP can be generated by the present model under well-defined conditions. Since we concentrated on perfect-foresight trajectories, variations of the rates of innovation or growth across countries cannot occur in the long-run. Knowledge stocks, TFP and output levels then neither converge nor diverge over time, but move in parallel. During the transition to the long-run solution, however, differential growth will generally occur, thus leading to convergence or divergence of countries and occasional changes of technological and economic leadership (taking over) in the course of time. Whether convergence or divergence, e.g. of TFP, takes place, depends on the relative sizes of the initial knowledge stocks in the two countries. In analyzing the process of taking over, we naturally assumed that the initial knowledge stock of the technological leader is larger than that of the follower. In this case, TFP converges over a certain interval and then diverges. If we retain the assumption that the initial knowledge stock of the leader exceeds that of the relatively smaller follower, then TFP transitionally converges in the case of catching up and diverges in the remaining case. Moreover, it should be borne in mind that the case with absorptive capacities set to zero (perfect autarky) which generates long-term rates of innovation which differ depending on country size can be treated as a special case of the present model.

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Appendix

Propositions 1 and 3 can be proved by similar arguments, which can be qualified as monotonicity analysis of appropriate functions and trajectories. Here we outline the method; for details see Kryazhimskii (1995).

Proof of Proposition 1. The argument consists of two parts. In the first part, existence of the upper and the lower asymptotics is established. Also, it is shown that if a trajectory has neither the upper nor the lower asymptotics, then it has the intermediate one. In the second part, the existence of a trajectory with the intermediate asymptotics is proven under conditions (5.11) and (5.12).

Let us introduce two curves $v^B = \mu_n(n^B, n^A)$ and $v^B = \mu_v(n^B)$, where the right hand sides of equations (5.6) and (5.7) vanish. Their equations are

$$\mu_n(n^B, n^A) = \frac{\alpha}{c^B[n^B + \gamma(n^B)\delta n^A]},$$

$$\mu_v(n^B) = \frac{1-\alpha}{\rho n^B},$$

where $c^B = L^B / a$. Given a trajectory $(n^B(.), v^B(.))$, we set

$$\mu_n[t] = \mu_n(n^B(t), n^A(t)), \qquad \mu_v[t] = \mu_v(n^B(t)).$$

Fix a trajectory and a sufficiently large time instant t_0. There are only three possibilities:

(1) $\quad v^B(t_0) \geq \mu_v[t_0]$,

(2) $\quad v^B(t_0) < \mu_v[t_0], \qquad v^B(t_0) \leq \mu_n[t_0]$,

(3) $\quad \mu_n[t_0] < v^B(t_0) < \mu_v[t_0]$.

Consider inequality (1). By (5.6), $n^B(.)$ is a nondecreasing function, hence the negative term in (5.7) is nonincreasing in absolute value. Since by (5.7) inequality (1) we have that $\dot{v}^B(t_0) \geq 0$, this implies that $v^B(.)$ remains nonnegative after t_0, and, hence, $v^B(.)$ is nondecreasing after t_0. Thus, the negative term in (5.6) does not increase in absolute value. On the other hand, the positive term in (5.6) grows to infinity since $n^A(.)$ does. To see this, note that $\delta > 0$, $n^B(.)$ is a nondecreasing function with $n^B(0) > 0$ and $\gamma(.)$ is an increasing function with $\gamma(0) = 0$. Consequently, sooner or later, the right hand side of (5.6) becomes positive and, hence, $n^B(.)$ starts growing. Since $\dot{n}^B(.)$ becomes larger than any positive constant, $n^B(.)$ grows to infinity. Due to the argument given above, this implies that the negative term in (5.7) goes to zero and, hence, $\dot{v}^B(.)$ becomes positive, larger than any given constant. Consequently, $v^B(.)$ also grows to infinity. Thus, to accomplish the proof that (1) generates a trajectory with upper asymptotics, we have to show that $r_\infty = \lim_{t\to\infty} n^B(t)/n^A(t)$ exists and is given by (5.9).

Since $n^A(.)$ is exponentially growing, we can introduce the universe function $t = f(n^A)$ which is increasing to infinity. Now, n^B can be thought of as a function of n^A, in particular, $n^B[n^A] = n^B(f(n^A))$. Viewing this function as parametrically given, we have for its derivative

(4) $$\frac{dn^B[n^A]}{dn^A} = \frac{\dot{n}^B(t)}{\dot{n}^A(t)} = \frac{1}{\bar{g}^A n^A(t)}\left\{c^B\left[n^B(t) + \gamma(n^B))\delta n^A(t)\right] - \frac{\alpha}{v^B(t)}\right\} =$$

$$= \frac{c^B}{\bar{g}^A}\left[\frac{n^B(t)}{n^A(t)} + \gamma(n^B(t))\delta\right] - \frac{\alpha}{\bar{g}^A n^A(t) v^B(t)} = \frac{c^B}{\bar{g}^A}\left[\frac{n^B[n^A]}{n^A} + \gamma_\infty \delta\right] + \vartheta[n^A],$$

where $\vartheta[n^A] = \frac{c^B}{\bar{g}^A}\left[\gamma(n^B[n^A]) - \gamma_\infty\right]\delta - \frac{\alpha}{\bar{g}^A n^A v(f(n^A))}$. Since $f(.)$ increases to infinity as n^A grows, we have that

$$\gamma(n^B[n^A]) \to \gamma_\infty \qquad \text{as} \qquad n^A \to \infty$$

and, since we have shown that under (1) $v^B(t) \to \infty$ as $t \to \infty$,

$$\frac{\alpha}{\bar{g}^A n^A v^B(f(n^A))} \to 0 \qquad \text{as} \qquad n^A \to \infty.$$

These two relations give

(5) $$\vartheta[n^A] \to 0 \qquad \text{as} \qquad n^A \to \infty.$$

Integrating the linear differential equation (4) for $n^B[n^A]$, we get an explicit expression for $n^B[.]$ through $\vartheta[.]$. Dividing this expression by n^A, we obtain by (5) that

$$\lim_{t\to\infty} r(t) = \lim_{n^A\to\infty} \frac{n^B[n^A]}{n^A} = r_\infty$$

with r_∞ defined by (5.9). Thus, we have shown that (1) implies the existence of a trajectory with the upper asymptotics with r_∞ given by (5.9).

Inequalities (2) imply by (5.6) and (5.7) that $\dot{v}^B(t_0) < 0$ and $\dot{n}^B(t_0) = 0$. If we show that $\dot{n}^B(t) = 0$ for $t \geq t_0$, then the trajectory $(n^B(.), v^B(.))$ has the lower asymptotics. By (5.6) this equality holds if for $t \geq t_0$

(6) $\quad v^B(t) \leq \mu_n(t)$.

Thus, let us show that (6) takes place if

(7) $\quad \dot{v}^B(t_0) \leq \dot{\mu}_n[t_0]$.

By the second of inequalities (2), (6) holds at least on $t \in [t_0, t_0 + \varepsilon]$ for some $\varepsilon > 0$. Hence, $n^B(.)$ is constant on this interval. By (5.7) this implies that $\dot{v}^B(.)$ decreases (remaining negative) on $[t_0, t_0 + \varepsilon]$. If $\dot{\mu}_n(.)$ is nondecreasing after t_0, then we get (2) and (7) with t_0 replaced by $t_0 + \varepsilon$. Repeating the argument over and over again, we can show that (6) holds for $t \geq t_0$ if $\dot{\mu}_n(.)$ is nondecreasing on $[t_0, \infty)$. On the other hand, it can be proven that $\dot{\mu}_n(.)$ increases on $[t^A, \infty)$, where $t^A = \max[0, \quad -\frac{1}{g^A}\ln(4\gamma\,(n^B(0))\delta n^A(0))]$.

Hence, if $t_0 \geq t^A$, then (6) holds for $t \geq t_0$. Consequently, we have shown that inequalities (2) and (7) for $t_0 > t^A$ imply the existence of a trajectory $(n^B(.), v^B(.))$ with the lower asymptotics.

Consider inequalities (3). Without loss of generality, we can assume that $t_0 \geq t^A$. There are three possibilities:

1) there is $t_1 > t_0$ such that $\mu_n[t] < v^B(t) < \mu_v[t]$ for $t_0 \leq t < t_1$ and $v^B(t_1) = \mu_v[t_1]$;

2) there is $t_1 > t_0$ such that $\mu_n[t] < v^B(t) < \mu_v[t]$ for $t_0 \leq t < t_1$ and $v^B(t_1) = \mu_n[t_1]$;

3) $\mu_n[t] < v^B(t) < \mu_v[t]$ for $t \geq t_0$.

The first case reduces to (1) with t_0 substituted by t_1. Hence, by the argument given above, $(n^B(.), v^B(.))$ has the upper asymptotics. Similarly, the second case reduces to (2) with t_0 replaced by t_1. Consequently, $(n^B(.), v^B(.))$ has the lower asymptotics in this case. A trajectory which corresponds to the third case we call an *intermediate trajectory*. Thus, we have shown that inequalities (3) imply the following three possibilities:

i) $(n^B(.), v^B(.))$ has the lower asymptotics;

ii) $(n^B(.), v^B(.))$ has the upper asymptotics;

iii) $(n^B(.), v^B(.))$ is an intermediate trajectory;

A similar logic leads us to the conclusion that the same three options are the only possibilities for the situation when inequalities (2) hold. Since we have also shown that (1) implies the upper asymptotics, and for a given trajectory only one of cases (1)-(3) may occur, we conclude that one and only one of the situations i)-iii) may occur. Thus, to complete the first part of the proof of Proposition 1, it is sufficient to show that if a trajectory is intermediate, i.e. has neither lower nor upper asymptotics, then it has an intermediate asymptotics. We shall do this now.

Let $(n^B(.), v^B(.))$ be an intermediate trajectory. Introduce new variables

$\chi(.) = n^B(.)v^B(.)$ and $\xi(.) = n^A(.)v^B(.)$.

From (5.6) and (5.7) we get

(8) $$\dot{\chi}(t) = (c^B + \rho)\chi(t) + c^B\gamma(n^B(t))\delta\xi(t) - 1,$$

(9) $$\dot{\xi}(t) = \left[\bar{g}^A + \rho - \frac{1-\alpha}{\chi(t)}\right]\xi(t).$$

The inequalities given by 3) are equivalent to

(10) $$\chi_n(t) < \chi(t) < \chi_v \quad \text{for} \quad t \ge t_0,$$

where

$$\chi_v = \frac{1-\alpha}{\rho}, \quad \chi_n(t) = \frac{\alpha c^B n^B(t)}{n^B(t) + \gamma(n^B(t))\delta n^A(t)}.$$

Set

$$\chi_\xi = \frac{1-\alpha}{\bar{g}^A + \rho}, \qquad r_\infty = \gamma_\infty \delta \frac{1}{L^A / L^B - 1}.$$

The intermediate asymptotics is characterized by the following three relations:

(11) $\lim_{t\to\infty} n^B(t) = \infty$,

(12) $\lim_{t\to\infty} \chi(t) = \chi_\xi$,

(13) $\lim_{t\to\infty} r(t) = r_\infty$.

Let us prove that these three equalities take place.

Consider (11). Assume to the contrary that $\sup_{t\geq 0} n^B(t) = n_*^B < \infty$. Then,

$$\mu_\nu[t] \geq \frac{1-\alpha}{\rho n_*^B} = \mu_\nu^*.$$

There are only two possibilities:

a) there is $t_1 \geq t_0$ such that $v^B(t_1) < \mu_\nu^*$;

b) $v^B(t) \geq \mu_\nu^*$ for $t \geq t_0$.

In case a) by (5.7) we get

$$\dot{v}^B(t) = \rho[v^B(t) - \mu_\nu^*] + \kappa(t),$$

where $\kappa(t) \leq 0$. Integrating this equation on $[t_0, \infty)$, we see that since $v^B(.)$ gets below μ_ν^* at t_1, it must converge to $-\infty$ as t increases. Thus, $v^B(t) \to -\infty$ as $t \to \infty$. This is not possible, since $v^B(.)$ must be positive by (3). Consequently, a) leads to a contradiction.

Let b) take place. Then, taking into account inequalities 3), we have

(14) $\mu_\nu^* \leq v^B(t) < \mu_\nu[t]$ for $t \geq t_0$.

Since $n^B(.)$ is a nondecreasing function, $\mu_\nu[.]$ is a nondecreasing one, too. Hence, $\mu_\nu[t] \to \mu_\nu^*$ as $t \to \infty$ and by (14) we conclude that

$$\lim_{t\to\infty} \nu^B(t) = \mu_\nu^*.$$

This equality implies

$$(15) \qquad \lim_{t\to\infty} \chi(t) = n_*^B \mu_\nu^*,$$

and, taking into account that $n^A(t) \to \infty$ as $t \to \infty$,

$$(16) \qquad \lim_{t\to\infty} \xi(t) = \infty.$$

Since $\gamma(.)$ is an increasing function and $n^B(.)$ is a nondecreasing one, $\gamma(n^B(t)) \geq \gamma(n^B(t_0)) > 0$ for $t \geq t_0$. Thus, by (8), (15) and (16), $\dot{\chi}(t)$ becomes arbitrarily large as $t \to \infty$. This contradicts (15). Consequently, b) leads to a contradiction.

Thus, we have shown that the assumption that $n^B(t)$ remains bounded as $t \to \infty$ leads to a contradiction. Thus, (11) must take place.

Consider (12). Observe that

$$(17) \qquad \dot{\xi}(t) > 0 \qquad \text{if} \qquad \chi(t) > \chi_\xi,$$

$$(18) \qquad \dot{\xi}(t) < 0 \qquad \text{if} \qquad \chi(t) < \chi_\xi.$$

By (8) and (9) we conclude that if for some $t_1 > t_0$ we have $\chi(t_1) = \chi_\xi$ and $\dot{\chi}(t_1) > 0$, then $\chi(.)$ increases after t_1; moreover $\chi(t)$ tends to infinity as $t \to \infty$. The latter contradicts (10). Hence, the above situation is not possible: $\chi(.)$ cannot pass through χ_ξ increasing. There are only three remaining alternatives for the behavior of $\chi(.)$ with respect to χ_ξ:

j) $\quad \chi(t) > \chi_\xi$ for $t \geq t_0$,

jj) $\quad$ there is $t_1 > t_0$ such that $\chi(t) \geq \chi_\xi$ for $t_0 \leq t \leq t_1$ and $\chi(t) < \chi_\xi$ for $t > t_0$,

jjj) $\quad \chi(t) < \chi_\xi$ for $t \geq t_0$.

Focus only on case j). The other possibilities can be considered similarly. Suppose to the contrary that (12) does not hold. Then j) implies that there is $\beta > 0$ and a sequence $\{t_i\}$, $t_i \to \infty$ as $i \to \infty$ such that $\chi(t_i) > \chi_\xi + \beta$, $i \geq 1$. Also, j) implies, by (8) and positiveness of $\xi(.)$, that $\dot{\chi}(.)$ is uniformly bounded from below on $[t_0, \infty)$. Hence there is $\varepsilon > 0$ such that

$$\chi(t) \geq \chi_\xi + \beta / 2 \qquad \text{for} \qquad t_i < t < t_i + \varepsilon .$$

Consequently, by (3),

$$\dot{\xi}(t) = \left[\overline{g}^A + \rho - \frac{1-\alpha}{\chi(t)} \right] \xi(t) > l\xi(t),$$

where $t_i \leq t \leq t_i + \varepsilon$ and $l = \overline{g}^A + \rho - \dfrac{1-\alpha}{\chi_\xi + \beta / 2} > 0$. By (9) and (17) $\xi(.)$ is increasing. Also, $\xi(.)$ is positive and $\dot{\xi}(t) \geq l\xi(t)$ on $[t_i, t_i + \varepsilon]$, $i \geq 1$, which implies that on each of these intervals $\xi(.)$ increases at least $\exp(\rho\varepsilon)$ times. Consequently, $\xi(t) \to \infty$ as $t \to \infty$. The latter, as we can deduce from (8), implies that $\chi(t)$ goes to infinity as $t \to \infty$ which contradicts (10). Hence, assuming that (12) does not hold, we arrived at a contradiction. Thus, (12) must take place.

Consider (13). Note that $r(t) = \chi(t) / \xi(t)$. Consequently, by (12), to show (17) it is enough to prove that

$$\text{(19)} \qquad \lim_{t \to \infty} \xi(t) = \xi_\infty = \frac{1 - (c^B + g)\chi_\xi}{c^B \delta \gamma_\infty} .$$

By (17) and (18) we conclude that in each of the cases j)-jjj) the function $\xi(.)$ is monotone for all sufficiently large t. This implies the existence, finite or not, of $\overline{\xi} = \lim_{t \to \infty} \xi(t)$. If $\overline{\xi} = \infty$, then, by (8), $\chi(t) \to \infty$ as $t \to \infty$. This contradicts (10). Consequently, $\overline{\xi}$ is a nonnegative finite number. By (8) and (12) we have that

$$\Delta = (c^B + \rho)\chi_\xi + c^B \gamma_\infty \delta \overline{\xi} - 1,$$

where $\Delta = \lim_{t \to \infty} \dot{\chi}(t)$. If $\overline{\xi} \neq \xi_\infty$, then $\Delta \neq 0$. The latter implies that as $t \to \infty$ either $\chi(t) \to -\infty$ (if $\Delta < 0$), or $\chi(t) \to \infty$ (if $\Delta > 0$). Both outcomes are not possible (since $\chi(t) \geq 0$ for all t or by (10)). Hence, we must have $\Delta = 0$, or $\overline{\xi} = \xi_\infty$. Thus we have established (19), which implies (13).

Consequently, we have shown that an intermediate trajectory has the intermediate asymptotics (but we have not proved that such a trajectory exists). This accomplishes the first part of the proof of Proposition 1. Let us now show the existence of an intermediate trajectory which has, from the above, the intermediate asymptotics.

Assume that (5.11) and (5.12) hold true. Fix positive $\xi(0)$ and $v^B(0)$. For every $\chi(0) \in [\chi_\xi, \chi_v]$ there exists a single $n^B(0) \in [\chi_\xi / v^B(0), \chi_v / v^B(0)]$ such that $\chi(0) = n^B(0)v^B(0)$. A trajectory $(n^B(.), v^B(.))$ such that

$$n^A(0) = \xi(0) / v^B(0) \qquad \text{and} \qquad n^B(0) = \chi(0) / v^B(0)$$

will be called trajectory corresponding to $\chi(0)$. To make explicit that $\chi(.)$ starts at $\chi(0)$, let us use $\chi(.|\chi(0))$ from now on. Introduce Ξ the set of $\chi(0) \in [\chi_\xi, \chi_v]$ such that $\chi(.|\chi(0))$ satisfies the following conditions:

e) $\chi(.|\chi(0))$ crosses χ_ξ and $\chi_n(.)$,

ee) $\chi(.|\chi(0)) \le \chi_\xi$ after crossing χ_ξ,

eee) $\chi(.|\chi(0))$ is strictly decreasing before crossing χ_ξ.

We show that if $\xi(0)$ is sufficiently large and $v^B(0)$ is sufficiently small, then $\chi(.|\chi^*)$ satisfies (10). Here, $\chi^* = \sup\limits_{\chi(0)\in\Xi} \chi(0)$. As we have noted, this implies that trajectory $(n^B(.), v^B(.))$ corresponding to χ^* satisfies the inequalities 3), that is, represents an intermediate trajectory.

To prove that $\chi(.|\chi^*)$ satisfies (10), we need several auxiliary statements, First, we verify that $\chi_\xi \in \Xi$ and $\chi_v \notin \Xi$. Hence, $\chi^* \in [\chi_\xi, \chi_v)$. Then we show that $\chi(.|\chi^*)$ decreases before crossing χ_ξ, in particular, this function is everywhere decreasing if there is no crossing. Since $\chi_\xi < \chi_v$, the latter implies that $\chi(.|\chi^*)$ cannot break (10) by crossing the upper bound χ_v. Further, if $\chi(.|\chi^*)$ does not cross χ_ξ, then, since by (5.7) we have $\chi_n(.) < \chi_\xi$, it does not cross $\chi_n(.)$ either. Hence, inequalities (10) hold. Assuming that $\chi(.|\chi^*)$ crosses χ_ξ decreasing (it cannot cross this level increasing due to the argument given after (17) and (18)), we prove that after the crossing $\chi(.|\chi^*)$ remains forever below χ_ξ. Now, if $\chi(.|\chi^*)$ never crosses $\chi_n(.)$, then (10) holds true and, consequently, the corresponding trajectory is an intermediate one. Otherwise, if $\chi(.|\chi^*)$ breaks (10) by crossing $\chi_n(.)$, we obtain a contradiction, which shows that this outcome is not possible. Indeed, let $\chi(.|\chi^*)$ cross $\chi_n(.)$ in a finite time. Then, referring to the fact that solutions to ordinary differential equations depend continuously on initial data, we can show that for a sufficiently small $\varepsilon > 0$, $\chi(.|\chi^* + \varepsilon)$ has the properties e)-eee). But this contradicts the fact that χ^* is defined as the supremum for all $\chi(0)$ such that $\chi(.|\chi(0))$ satisfies e)-eee). Thus, we have shown that an intermediate trajectory exists.

This accomplishes the proof of Proposition 1.