

GAMING MODEL TO STUDY THE PROBLEM
OF SHARING NATURAL RESOURCES

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July 1975

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1. Introduction: General Characterization of the Problem
and the Basic Approach

The problems of sharing natural resources (SNR) are constantly growing in their practical importance. The pattern of the distribution of natural resources is highly irregular and the price for natural resource extraction is high. Moreover, due to intensive exploitation, some resources are becoming scarce and others are approaching exhaustion. These problems affect the pattern of resource exploitation in different parts of the world and eventually the strategies for national development. Thus there is stress upon the necessity for defining reasonable proportions of development with due regard for the trade-offs between restricted natural resources and the requirements for growth over time.

The specifics of the SNR problem are an aggregated representation of the basic variables and controls and the high level of uncertainty which results from the lack of information about the natural pattern of development in different places. This uncertainty is further increased by interacting human interests which although strongly affecting the solution of the problem cannot be represented within a strict analytical framework and are being expressed through subjective, often non-quantifiable, judgments.

There may be several ways to cope with the above mentioned difficulties under different conditions. A very common approach is based on stochastic dynamic models which may change their properties and adjust to reality according to the rules given by a system analyst (SA). Human behavior, if represented at all, is expressed either through a set of simplifying hypotheses reducing the behavior to implementation of strictly given rules, or through a set of human preferences. In both cases human beings are represented with very rough submodels within the general model structure.

There is an alternative approach, however, based on the direct involvement of men in a model. This enriches the properties of a model considerably and moves it closer to reality.

Such an approach to the solution of complex problems is known as gaming or interactive simulation and has been under development for some time.

Gaming simulation has many useful properties, and among them it:

- concentrates a real decision-making experience through involvement of the practical decision makers in the model's operation;
- integrates knowledge with specific mechanisms and processes in complex systems;
- develops an ability to work with incomplete data;
- develops abilities to make decisions in a specified functional environment;
- provides a feedback from the results of decision implementation to a decision maker (DM) who may affect subsequent results;
- stresses the importance of determining critical factors and situations; etc.

Existing experience with gaming simulation shows its high potentiality as an instrument for studying development strategies in systems with multiple conflicting goals.

Unfortunately, to date no theory of building gaming models has existed. This lack is the reason the success or failure of a specific application has depended on the ad hoc procedures developed by model designers. As a result the model under description has a two-fold purpose. It is intended both for use as a testing ground for the development of the gaming theory and also for specific studies of decision making and organizational behavior under the particular conditions of exploitation of restricted vital resources. Both issues stated as broadly as they are cannot be served efficiently with only one model, and thus, a family of models is required.

The first model from this family will deal with the problem of planning and management under conflicting resources requirements. It is noteworthy that similar conceptual approaches to the problem are being developed elsewhere and specifically in [3]. The model has three types of operational units: Elements (E), Nature (N) and Arbitration (A), the functions of which will be specified in detail later on.

The basic philosophy of this model is an aggregate description of all variables with the use of simple relationships between the phase variables within the elements on the one hand, and introduction of the maximum complexity of interaction among the elements and between the elements and nature on the other. An aggregate description of variables and verbal explanation of phenomena within the model may associate it with global models.

To avoid misunderstanding we need to stress the fact that the model as formulated in this presentation is theoretical and not supposed to reflect any reality. Its structure, however, allows for the introduction of real mechanisms if available. Thus we do not believe that the rational patterns of development obtained through the model may be extended to real situations. We hope instead to identify general rules for formulating these patterns in a complex competitive environment which would be applicable to real cases. We also hope to identify efficient ways of dealing with the different types of uncertainty present in real-life situations.

The relationships which represent the basic processes and mechanisms within an element are intentionally oversimplified to make analyses of interactions more tractable and to shift proportions between the "hard" and "soft" decision procedures in the model in favor of the former ones which use rigorous computational principles and do not suffer from the ad hoc nature of human behavior. This allows us to concentrate more attention on studying interactions within the model where human judgments are of critical importance. A strong interactive part of the model and explicit controls make a principal difference between this model and for example the existing global models shown in Table 1.

The goals are not set into the model, but instead are formulated and modified in the course of model operation. The only general assumption is that the elements behave rationally, i.e. do not intentionally approach the extinction point but tend to stay within the domain of active behavior which is known to all of them. Thus at any specific moment there may be several alternative goals within the model and they may vary over time. Of course the goals may be fixed while studying specific development strategy. A goal tree for an element may look like Figure 1. Basic model structure is represented in Figure 2, while Figure 3 exhibits the interaction links and internal features of elements in more detail.

At the operational level the element transforms resources into specific values of phase variables (production capacity, store of capital, population, knowledge, pollution store of paid resources). The transformation is accomplished by means of applying some decision procedures. In a general case resources may be considered as vectors. The domain of activity for an element is determined by the constraints on the phase variables. The values of these variables are used as indicators representing the state of an element and also as a language for expressing short-term, or operational goals. The pattern of development of an element in a phase space is a function of development policy expressed in terms of control variables which effect the phase variables as well as interaction variables.

Nature is the main holder of resources. The total amount

Table 1.

Type of Model	Basic Principle	Principle of Planning	Information about Natural Resources	Interactions among Elements	Availability of Controls
1. Meadows [4]	Rigorously Normative	Extrapolation of Tendencies	Complete data	No	Parametric controls through varying the structure of economy or external investments
2. Forrester [1]
3. Mesarovic and Pestel [5]
4. Fundación Bariloche [3]
5. Gaming Model	Both normative and descriptive	Not fixed, depends on the choice of a planner	Depends on the investments in studying nature	All phase variables except population	Direct control variables

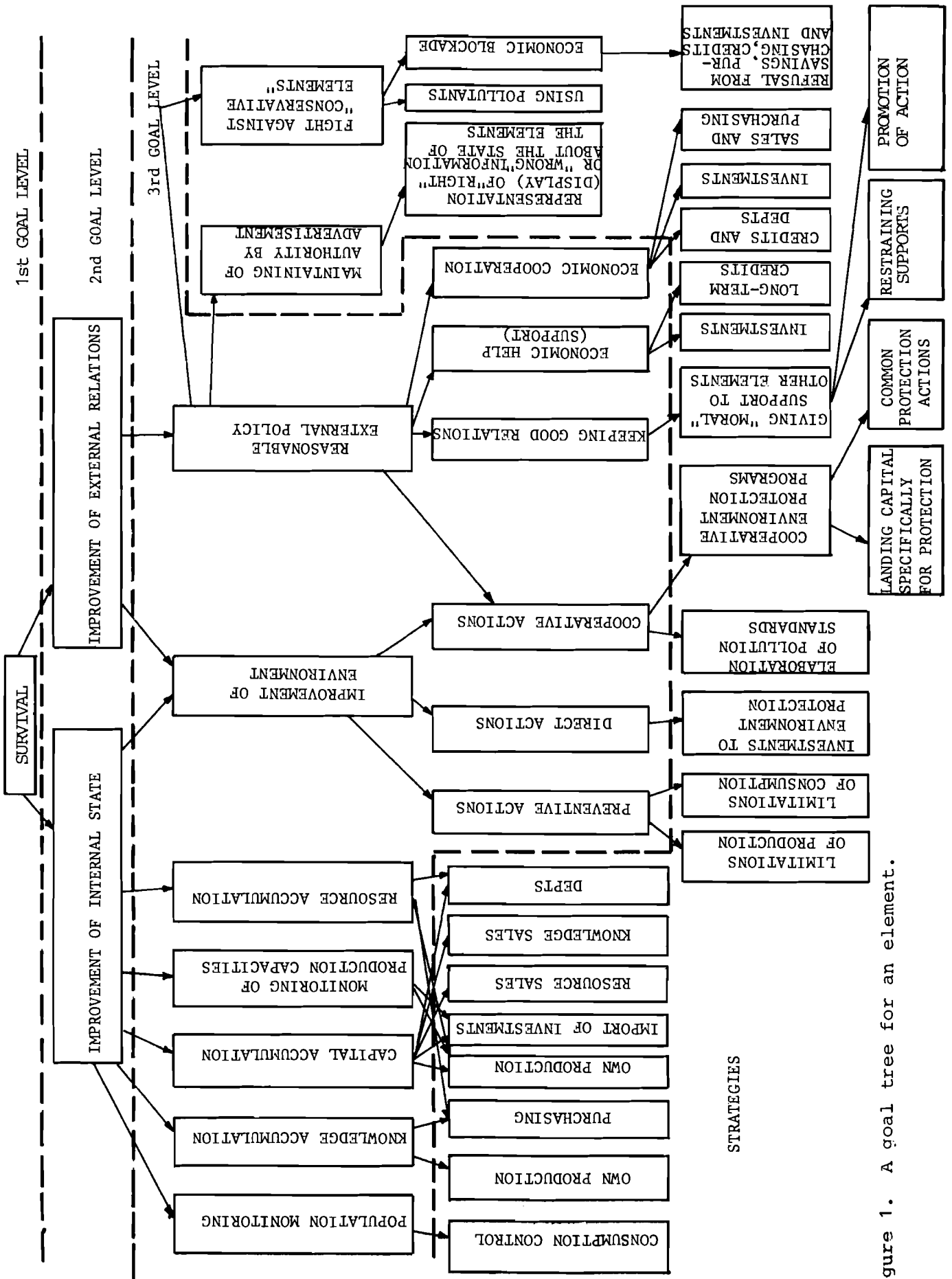


Figure 1. A goal tree for an element.

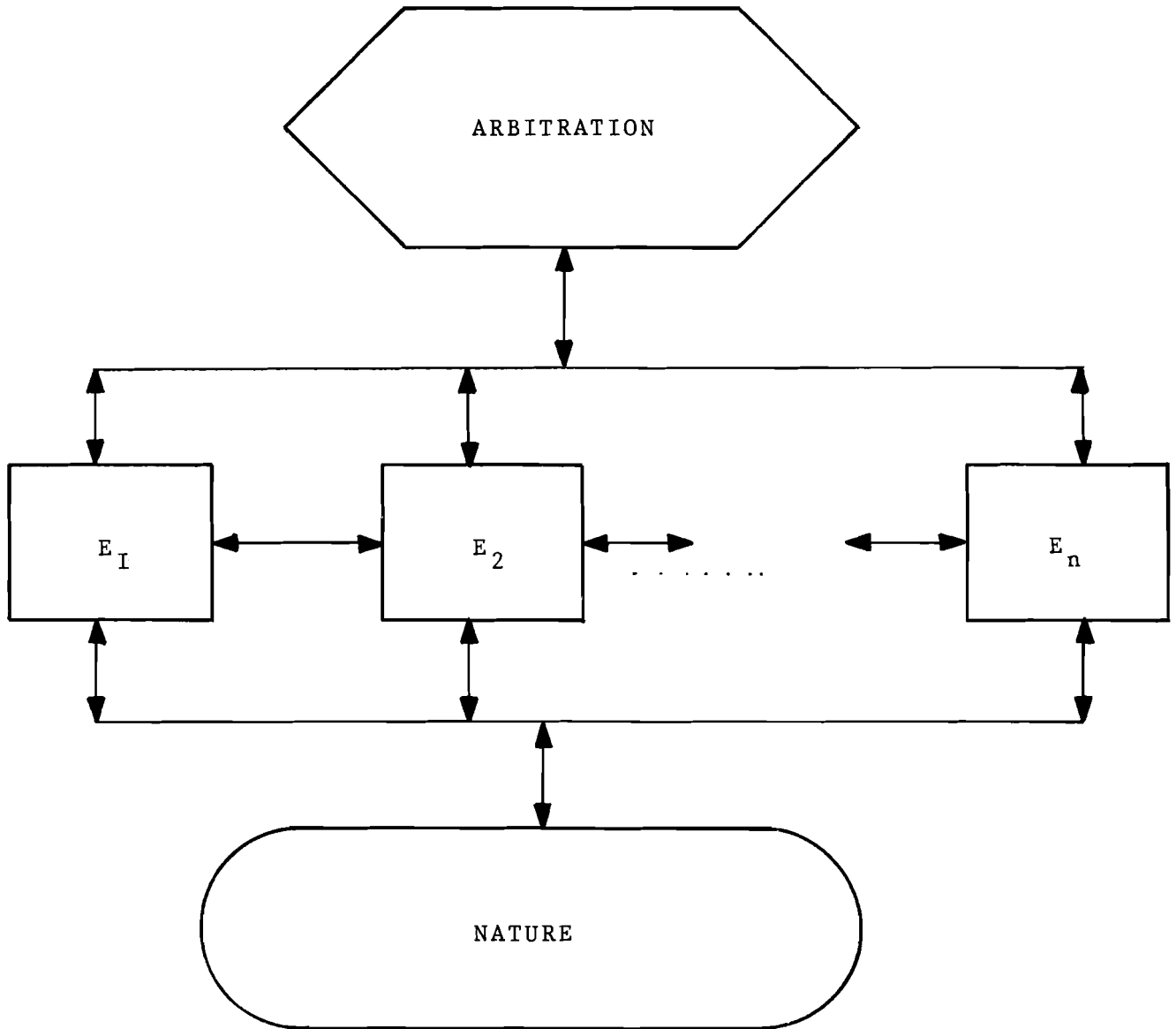


Figure 2. Basic model structure.

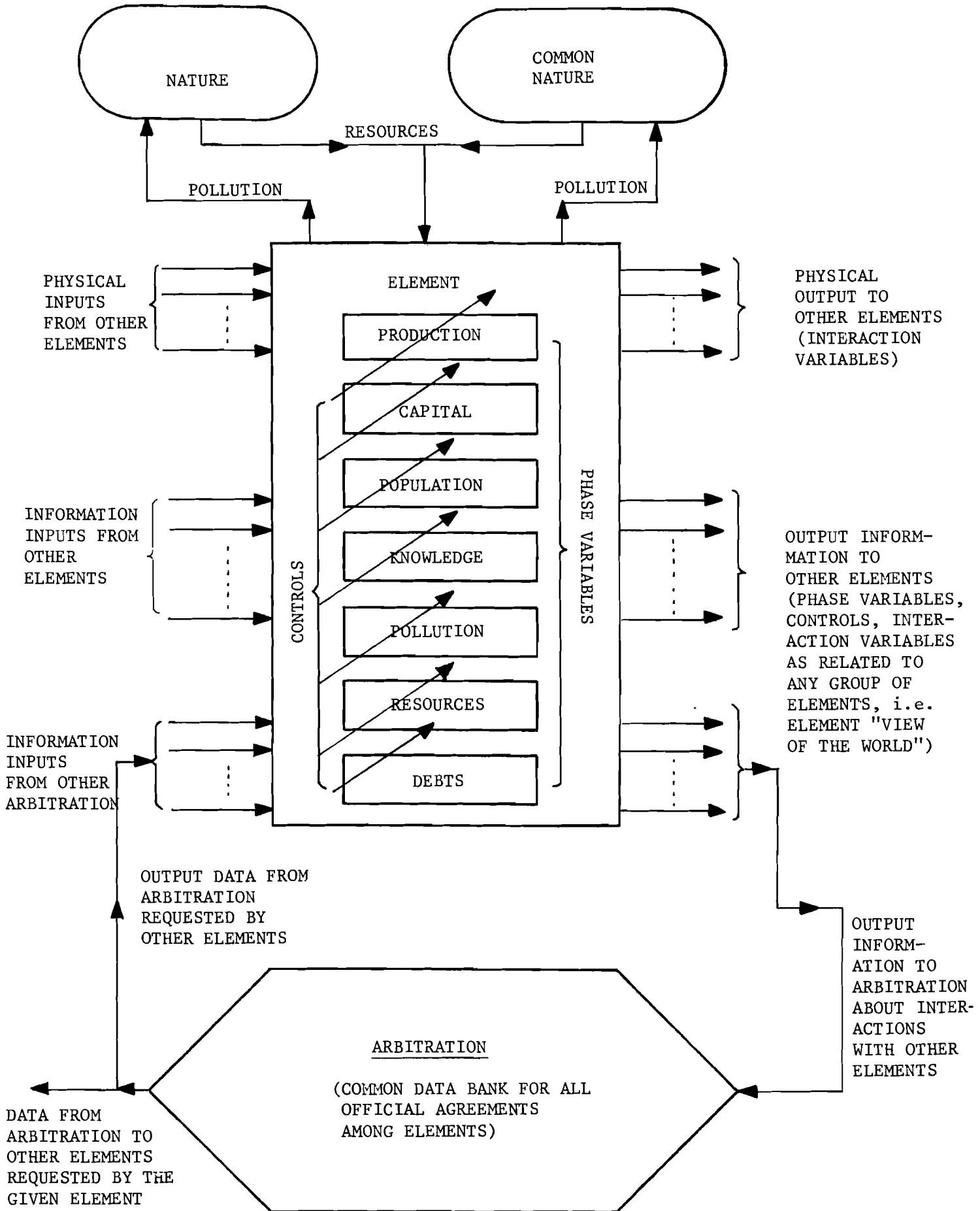


Figure 3. Structure of physical and information interactions in the model.

of resources is assumed unlimited but the elements may simultaneously have an access to only one or several layers where the amount of resources and their prices are fixed and are of common knowledge. The success of the new discoveries depends on the natural barrier met by the element and on the research policy chosen by it. Nature is conventionally divided into two parts. One part is natural resources owned by an element. These may not be used by other elements without permission of the owner. The other part is common resources shared by all elements. These may be exploited by any element depending on its development strategy. Special mechanisms are provided for allocation of scarce resources. Production and consumption within elements pollute nature, i.e. decrease the amount of resources available.

Arbitration is an information unit which keeps data about all official (as opposed to non-registered) agreements among elements concerning exchanges of physical variables (outside investments of capital, buying or selling resources, etc.). An element may be called for an arbitration if for any reason it wants to expose its interactions with other elements. Unless requested by the owner, this information is kept confidential. Thus arbitration is an indirect instrument for affecting "the authority" of elements in cases where their behavior differs from that stated by the respective agreements. The arrows going in two directions in Figure 2 show a bilateral character of all interactions in the model though the nature of feed-forward and feedback links may differ as shown in Figure 3.

There are three types of interactions. The first one represents the physical flows of resources from nature to elements and the flows of pollutants from elements to nature. The second corresponds to purely informational transactions between the elements and arbitration. The third one is a mixed type of interaction.

2. Specification of Model Units

2.1 The Element

The block diagram of the element illustrating the relations among phase variables is shown in Figure 4. To further specify the element we introduce the notation:

R_i	=	store of paid (active) resources within the element;
E_i	=	resultant pollution introduced by the element;
P_i	=	population of the element;
Kn_i	=	level of knowledge within the element;
ϕ_i	=	capital funds of the element's production;

f_i	=	effective production capacity of the element;
μ_i	=	amortization factor of capital funds of the element;
α_i	=	unit man-power capacity of the element production;
β_i	=	unit resource capacity of the element production;
R_i^V	=	resources allocated for production;
V_i	=	production output;
$y^{V,R,KN,E}$	=	investments in production resources, knowledge and antipollution;
α_i	=	factor representing the effect of knowledge on production;
w_i	=	consumption of capital per unit of the population;
w_i^O	=	minimal consumption of capital per unit of population;
P_i^*	=	rated mortality factor due to starvation;
γ_i	=	mortality factor due to pollution;
E_i	=	cumulative pollution;
E_i^O	=	elements sensitivity to pollution;
Q_i	=	a total store of capital;
Q_i^-	=	total external investments of the given element;
Q_i^+	=	total external investments to the given element;
$q_i^{R,KN}$	=	inflow of capital due to selling resources or knowledge, respectively;
$E_i^{V,w}$	=	pollution factor due to production or consumption, respectively;
\bar{E}_i	=	pollution of other elements by a given one;
$\eta^{KN,V,E}$	=	normalizing coefficients corresponding to efficiency of investments into development of knowledge, production and antipollution programs respectively;
λ_p^j	=	price of resources in the j-th layer;
R^j	=	total resources of the j-th layer;
$D_{i\bar{f}}$	=	total debts of the i-th element;
$\lambda_{\bar{a}}^{ij}$	=	the λ -th debt of the i-th element to the j-th one.

Whenever superscripts s and b appear they refer to selling or buying respectively.

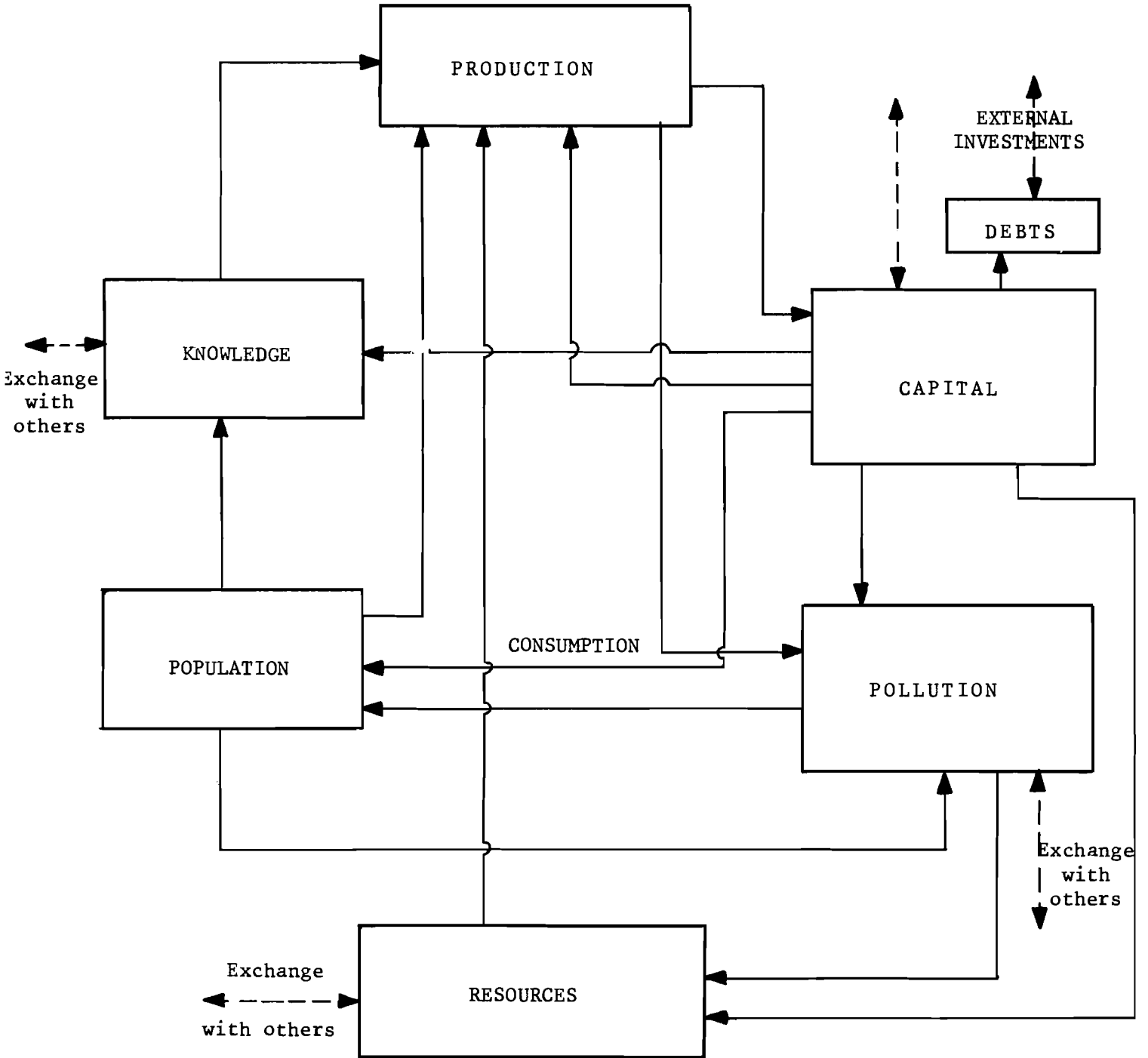


Figure 4. Block diagram of the element.

The following functions will be of help later on:

$$\theta(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1, & \text{if } x \geq 0, \end{cases} \quad (1)$$

$$(x)_+ = \begin{cases} 0, & \text{if } x < 0, \\ x, & \text{if } x \geq 0. \end{cases} \quad (2)$$

Using this notation we may represent all functional relationships for the element. Let us specify first of all the domain of activity for an element.

The domain of definition is

$$\begin{aligned} \phi_i, R_i, E_i, KN_i, P_i &\geq 0, \\ -\infty < Q_i < +\infty \end{aligned} \quad (3)$$

while the domain of activity is

$$\begin{aligned} \phi_i, R_i, E_i, KN_i &\geq 0, \\ P &> 0, \\ -\infty < Q &\infty. \end{aligned} \quad (4)$$

Production within the elements is expressed in terms of capital funds, production function and constraints on the actual capacities. The capital funds at time $(t + 1)$ are

$$\phi(t + 1) = \phi_i(t) - \mu_i \phi_i(t) + y_i(t), \quad (5)$$

while the initial funds are

$$\phi_i(0) = \phi_i^0. \quad (6)$$

The production function is

$$V_i(t) = (\eta_i^V + \alpha_i KN_i) \cdot f(t) \quad . \quad (7)$$

Constraints on the actual capacities are:

$$f_i(t) \leq \phi_i(t) \quad , \quad (8)$$

$$\alpha_i \cdot f_i(t) \leq P_i(t) \quad , \quad (9)$$

$$\beta_i \cdot f_i(t) \leq R_i^V(t) \quad . \quad (10)$$

The second term in (5) represents amortization of capital funds. The third term represents renewals and expansions, and it is a control variable. The production function (7) is expressed in terms of actual capital funds $f(t)$ and accounts for the influence of growing knowledge. The actual capital funds $f(t)$ are determined by the constraints (8)-(10). Although the production as represented above corresponds to a single-sectoral economy it can be easily extended to a multi-sectoral case.

Population at time $(t + 1)$ is

$$P_i(t + 1) = P_i(t) + (w_i - w_i^O)_+ \cdot P_i(t) - (w_i^O - w_i)_+ \cdot P^* - \lambda_i (E_i - E_{iO}) \quad , \quad (11)$$

while the initial population is

$$P_i(0) = P_{i0} \quad . \quad (12)$$

The second term in (11) introduces the growth of population if given the amount of food in excess of the minimum level. The third and the fourth terms represent mortality due to starvation and pollution respectively. The quantity w_i is a control or policy variable selected by the element.

Migration of population among elements is not allowed. Whenever $P_i(t) = 0$ the element is switched off from the model and cannot participate in any form of activity. All its phase variables are conserved and cannot be influenced by others. Of course an option may be introduced which allows for redistribution of stores of capital and resources among active elements.

Knowledge is a mechanism which affects production efficiency and the process of getting resources out of nature. The former effect is represented by (7) while the latter will be described later on. Knowledge is assumed to be a non-decreasing function of time. It can either be sold or bought, but in contrast to resources the selling knowledge does not reduce their total amount.

The equation for knowledge at time $(t + 1)$ is

$$\begin{aligned}
 KN_i(t + 1) = & KN_i(t) + \eta_i^{KN} Y_i^{KN}(t) \cdot P_i(t) \\
 & + \sum_{j=1}^L \{KN_{ij}^b(t) \cdot \theta[KN_j(t) - KN_i(t) - KN_{ij}(t)] \\
 & + [KN_j(t) - KN_i(t)] \cdot \theta[KN_{ij}^b(t) + KN_i(t) - KN_j(t)]\} \\
 & + \theta[KN_j(t) - KN_i(t)] \quad , \quad (13)
 \end{aligned}$$

where L is a total number of active elements and KN_{ij}^b is the amount bought by the i -th element from the j -th element. Initial conditions are represented as

$$KN_i(0) = KN_{i0} \quad . \quad (14)$$

The second term of (13) represents the growth of knowledge as a function of investments and population, while the third one gives the increment of knowledge due to buying it from other elements. Theta (θ) functions are introduced to account for different relationships between the level of knowledge in a given element and the levels in others. Though KN_{ij}^b and Y_i^{KN} are both control variables, the implementation of KN_{ij}^b depends not only on the performance of the i -th element itself but also on the attitude of other elements toward it.

Resources at time $(t + 1)$ are

$$R_i(t + 1) = R_i(t) + \sum_{j=0}^L R_{ji}^b(t) - \sum_{j=1}^L R_{ij}^s(t) - R_i^v \quad , \quad (15)$$

while

$$R_i(0) = R_{i0} \quad . \quad (16)$$

Here $j = 0$ corresponds to buying resources from nature.

$R_i^V, R_{ji}^b, R_{ij}^S$ are control variables.

Pollution at time $(t + 1)$ is

$$E_i(t + 1) = E_i(t) + E_i^V \cdot f_i(t) + E_i^W \cdot w_i(t) + \bar{E}_i \\ - \eta_i^E \cdot y_i^E(t) - \bar{E}_i(t) \quad , \quad (17)$$

while

$$E_i(0) = E_{i0} \quad . \quad (18)$$

Here f_i, w_i, y_i, E_i are controls.

Capital at time $(t + 1)$ is

$$Q_i(t + 1) = Q_i(t) + v_i(t) + \sum_{j=1, j \neq i}^L Q_{ji}^+(t) \\ + \sum_{j=1, j \neq i}^L \left[q_{ij}^R(t) + q_{ij}^{KN}(t) \right] + \sum_j \left[r_{ji}(t) - d_{ij}(t) \right] \\ - \left[y_i^V(t) + w_i(t) \cdot P_i(t) + y_i^E(t) + y^R(t) \right] \\ + \left[y_i^{KN}(t) + \sum_{j=1, j \neq i}^L Q_{ij}^-(t) \right] + \sum_j d_{ij}(t) \quad (19)$$

while

$$Q_i(0) = Q_{i0} \quad . \quad (20)$$

The second through fifth terms in (19) represent the inflow of capital through production, other element investments, selling resources and knowledge, and debt payments respectively.

The last term is the outflow of capital due to different expenditures. The controls in this case are $Q_{ij}^-, q_{ij}^R, q_{ij}^{KN}, y_i^V, w_i, y_i^E, y_i^R, y_i^{KN}, d_{ij}$.

Debts are closely related to interacting among elements. Thus each element may either borrow capital or give it with interest to others without limitations on the number of transactions. All transactions on which agreement has been obtained are assumed to be registered by arbitration. If unregistered, a transaction cannot be officially proved. Under registration, arbitration is given the following information:

$$l_{d_{ij}}, l_{t_{ij}^s}, l_{t_{ij}^f}, l_{\delta'_{ij}}, l_{\delta''_{ij}} .$$

Once registered, the interest rates $l_{\delta'_{ij}}, l_{\delta''_{ij}}$ cannot be changed, unless there is a new bilateral agreement which has a power of its own. Back payments are also assumed to be registered by arbitration. Only in this case they are officially reorganized (as can be shown at the information desk of arbitration). The number of back payments is unlimited, and the fact of each payment should be confirmed by both interested sides who report the amount paid $l_{k_{ij}^r}$ and the time payment, $l_{k_{ij}^t}$ to arbitration. The total amount of debts by time t is

$$\begin{aligned} D_i(t) = & \sum_j \sum_l \{ l_{d_{ij}} [1 + l_{\delta'_{ij}}(t - l_{t_{ij}^s})] + \theta(t - l_{t_{ij}^s}) \\ & - \sum_k l_{k_{ij}^r} \cdot \theta(l_{t_{ij}^f} - l_{k_{ij}^t}) \} \\ & + \{ 1 + l_{\delta''_{ij}}(t - l_{t_{ij}^f}) \cdot \theta(t - l_{t_{ij}^f}) \} \\ & - \sum_k l_{k_{ij}^r} \theta(l_{k_{ij}^t} - l_{t_{ij}^f}) \cdot \theta(t - l_{k_{ij}^t}) \end{aligned} \quad (21)$$

and initial debts are given

$$D_i(0) = D_{i0} . \quad (22)$$

3. Arbitration

As mentioned previously, arbitration is a pure information unit whose functions are to keep and update several data files and also to display the files according to a given rule. The files managed by arbitration are

$$\ell_{d_{ij}}(t), \ell_{t_{ij}^s}, \ell_{t_{ij}^f}, \ell_{\delta_{ij}^'}, \ell_{\delta_{ij}''}, \forall_{i \neq j}, \ell_{k^r_{ji}}(t), \ell_{k^t_{ji}}, \forall_{j=i} \cdot$$

The content of all these files is kept confidential unless otherwise stated by the owners of information who are the lenders of capital by definition. There is no rigid mechanism in the model which guarantees the proper amounts and times of back payments. Thus the "lenders" have only an indirect way of affecting the "borrowers" through displaying their attitude toward debts payments on the information desk of the Arbitration Unit. Arbitration does not have any other goal except data file bookkeeping, and it helps to implement a soft part of management in the model which refers to the unformalizable concept of the element "Authority."

4. Nature

Two types of nature are distinguished within the model: own nature of the element (${}^i n_N$) and common nature (${}^c N$). Both types have a layered structure, each layer being represented by a quadruple

$$\ell^i = \{I^i, C^i, \lambda^i, X^i(t)\},$$

where

- I^i = an information threshold for entering a layer;
- C^i = capital threshold;
- λ^i = the price of a unit of resources;
- $X^i(t)$ = amount of resources in a given layer at time t .

The first three quantities do not change in time. The index i refers to the i -th layer. The superscript ${}^i n$ and c will be introduced to refer to internal and common nature respectively. The amount of layers in both types of nature is assumed unlimited. However, an element knows characteristics of only those layers for which the values of the information threshold are less than its level of knowledge. Thus the whole set of known layers for the k -th element at time t is

$$\bar{L}_k(t) = \text{in}_{L_k}(t) \cup {}^c L_k(t) ,$$

where

$$({}^c) , \text{in}_{L_k}(t) = \{ ({}^c) , \text{in}_{\ell^i} : \text{KN}_k(t) \geq ({}^c) , \text{in}_{I^i} \} .$$

A set of all known layers at time t is

$$L(t) = \cup \bar{L}_k(t) .$$

Not all known layers may be under exploitation, however, so concepts of an active layer for a given element and a set of active layers are introduced.

The layer ℓ^i is active for the k-th element at the time t if $\text{KN}_k(t) \geq I^i$ and $\int_{-\infty}^t y_k^{Ri}(\tau) d\tau \geq C^i$. Thus both information and capital barriers should be passed over.

A set of active layers for the k-th element at time t is

$$\bar{L}_k^*(t) = \text{in}_{L_k}(t) \cup {}^c L_k(t) ,$$

where

$$({}^c) , \text{in}_{L_k}(t) = \{ ({}^c) , \text{in}_{\ell^i} : \text{KN}_k(t) > ({}^c) , \text{in}_{I^i} ,$$

$$\int_{-\infty}^t ({}^c) , \text{in}_{y_k^{Ri}}(t) dt \geq ({}^c) , \text{in}_{C^i} \} .$$

A set of all active layers at time t is

$$L^*(t) = \cup \bar{L}_k^*(t) .$$

Internal nature allocates resources within one layer according to the following rule:

$$in_{R_k^i}(t) = \left\{ \begin{array}{l} \frac{in_{Y_k^{Ri}}(t)}{in_{\lambda^i}} , \quad \text{if } \frac{in_{Y_k^{Ri}}(t)}{in_{\lambda^i}} - a_i E_i \leq in_{X^i}(t) \\ in_{X^i}(t) , \quad \text{if } \frac{in_{Y_k^{Ri}}(t)}{in_{\lambda^i}} - a_i E_i > in_{X^i}(t) \end{array} \right\} ,$$

where

$in_{X^i}(t)$ is the amount of resources in the i -th layer.

There is the following limitation for the exploitation of internal layers

$$\sum_{in_{\ell^i} \in in_{L_k^*}} in_{R_k^i}(t) \leq \sum_{in_{\ell^i} \in in_{L_k}} in_{X^i}(t) .$$

A general internal consumption is

$$in_{R_k}(t) = \sum_{in_{L_k}(t)} in_{R_k^i}(t) .$$

The dynamic equation for the amount of resources in a layer is

$$in_{X^i}(t + 1) = in_{R_k^i}(t) - a_i E_i(t)$$

where the last term represents the effect of pollution.

To characterize exploitation of common resources let us introduce the subset of elements exploiting a given layer as

$$\mathcal{E}^i = \{e_k : KN_k \geq c_I^i , \int_{-\infty}^t c_{Y_k^{Ri}}(t) dt \geq c_C^i\} .$$

Common nature allocates resources of one layer according to the following results:

$$c_{R_k}^i(t) = \begin{cases} \frac{c_{Y_k}^{R_i}(t)}{c_{\lambda}^i} , & \text{if } \sum_{e_s \in \mathcal{E}^i} \frac{c_{Y_s}^{R_i}(t)}{c_{\lambda}^i} \leq c_X^i(t) \\ \frac{c_X^i(t) \cdot c_{Y_k}^{R_i}(t)}{\sum_{l_s \in \mathcal{E}^i} c_{Y_s}^{R_i}(t)} & \text{if } \sum_{e_s \in \mathcal{E}^i} \frac{c_{Y_s}^{R_i}(t)}{c_{\lambda}^i} > c_X^i(t) . \end{cases}$$

The dynamics of resources in a layer are

$$c_X^i(t + 1) = c_X^i(t) - \sum_{e_s \in \mathcal{E}^i} \{c_{R_s}^i(t) - b_s \cdot E_s(t)\} ,$$

the last term again representing the effect of pollution.

5. Information in the Model

Each element formulates its development strategy using four sources of information. Nature provides it with data about all layers whose information threshold is less than the level of knowledge of the element. The number of elements in a specific layer is not displayed, however, and has to be identified by the elements themselves. Arbitration supplies information about those interactions among elements which should become common knowledge according to the wishes of information holders.

Although both sources give objective data they have different mechanisms of data access. While nature has a rigorous mechanism for data access and communication the access to arbitration data depends on the subjective attitudes of elements.

Another source of information is other elements. In contrast to previous sources of specific data they may generate information relating to their own phase variables, phase variables of other elements, forecast of development strategies, state of nature and interactions in the model, etc. Thus this is quite general information but its truthfulness is not guaranteed.

The final source of information is the element which in addition to what has been said is able to generate real data about its own phase variables but their displaying depends on the element policy and the content of data may vary as queries from other elements come.

6. Conclusion

A description of the gaming model is presented which is aimed at studying development planning under the conflicting resources requirements. The model has a rigid part allowing for the calculation of all phase variables and a soft part representing the interaction of elements at the physical and information levels.

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