International Institute for Applied Systems Analysis • A-2361 Laxenburg • Austria
Tel: +43 2236 807 • Fax: +43 2236 71313 • E-mail: info@iiasa.ac.at • Web: www.iiasa.ac.at

INTERIM REPORT IR-97-77/October

Exploring the Unknown On Entrepreneurship, Coordination and Innovation Driven Growth

Giovanni Dosi (dosi@iiasa.ac.at)*
*Giorgio Fagiolo** (fagiolo@datacomm.iue.it)*

Approved by
Gordon J. MacDonald (macdon@iiasa.ac.at)
Director, IIASA

Contents

Abstract

Notwithstanding the revival of attention recently displayed by the economic discipline about self-sustained processes of economic growth fueled by technological advances, an enormous gap still remains between what we historically know about technical change and its economic exploitation, one the one hand, and the ways we represent them in formal growth models, on the other. Building on some general properties of the empirical patterns of innovation and diffusion that seem to be neglected in a good deal of contemporary growth literature, we present a stylized computer-simulated model in which self-sustained growth appears as the outcome of a coordination process among heterogeneous agents locally interacting in a decentralized economy characterized by: (i) notionally endless opportunities of endogenously introducing innovations; (ii) path-dependency in learning achievements; (iii) dynamic increasing returns grounded upon collectively shared 'learning paradigms'.

By means of extensive Montecarlo-like studies, we show that the model is able to generate GNP time-series exhibiting the statistical properties displayed by empirically observable data. Finally, we show simple but quite general settings in which collective economic growth finds its necessary condition in the presence of a number of 'irrationally' entrepreneurial agents.

Keywords: Innovation, Endogenous Growth, Evolution, Exploitation vs. Exploration

JEL Classification: O30, O31, C15, C22

Acknowledgments

Support to this research by the Italian National Research Council (CNR), the Italian Ministry of Research (MURST) and IIASA, Laxenburg, Austria is gratefully acknowledged. An early sketch of some ideas behind the model which follows is in Dosi and Orsenigo (1994). Comments by several participants to the Conference of CNAM, Paris, October 1996 helped a lot in sharpening the arguments. Usual caveats apply.

About the Authors

*	Dept. of Economics, University of Rome “La Sapienza”
and IIASA, Laxenburg, Austria

**	European University Institute, Florence, Italy

Exploring the Unknown On Entrepreneurship, Coordination and Innovation Driven Growth

Giovanni Dosi and Giorgio Fagiolo

1 Introduction

The determinants of economic growth in general, and the possibility of a self-sustained process fueled by technological advances, have recently brought back the attention of the economic discipline, with respect to both formal theorizing and historical analysis. Concerning the former, 'Endogeneous Growth' models (broadly in the spirit of Romer (1986, 1990) and Grossman and Helpman (1991a, 1991b)) and 'Schumpeterian' and 'Evolutionary' models (mainly building on Nelson and Winter (1982)) have been all trying - in different perspectives - to tell stories where per-capita incomes grow (also) as the outcome of positive feed-backs in knowledge accumulation. At the same time, a rapidly expanding empirically grounded literature on the economics of technological

change has been exploring the drivers of innovation and diffusion; the mechanism through which they occur; and their effects - at the levels of firms, sectors and whole countries[1].

Notwithstanding all that, we largely share the assessment spelled out in much more detail by Nelson (1997) of an enormous gap still remaining between what we historically know about technical change and its economic exploitation, one

[1] See, among others, Freeman (1982) and (1994), Rosenberg (1982) and (1994), David (1975), Dosi (1988), Nelson (1993), Lundvall (1993), Grandstrand (1994), Stoneman (1995), and fair parts of Dosi et al. (1988) and Foray and Freeman (1992).

the one hand, and the ways we represent them in formal growth models, on the other[2].

While some tensions between 'appreciative' (empirically drawn) generalizations and much more 'reduced forms' models is likely to always appear, our departing diagnostics is somewhat more pessimistic than that. In fact, a few general properties of the empirical patterns of innovation and diffusion seem to be neglected in a good deal of contemporary (formal) growth literature. Among them, and strictly related to the model presented here, in our view, there are the following.

First, aggregate formal accounts (in most of both the 'old' and 'new' growth models) tend to neglect the systematic heterogeneity in microeconomic technological competencies highlighted in the empirical literature. Relatedly, note that any 'representative agent' reduction might be highly misleading whenever the aggregate dynamics depends not only on the mean characteristics of any population but also on the distributions themselves and on the details of the interaction mechanisms among microentities[3].

Second, there appear to be a striking conflict between the incredibly sophisticated forward-looking rationality one typically imputes to agents in aggregate formal stories and the messy experimentation which empirical students of innovation and business history usually find - full of stubborn mistakes, 'animal spirits' and unexpected discoveries[4].

Third, partly as a consequence, it seems quite hard to interpret macrodynamics as equilibrium paths isomorphic to some underlying 'representative' behavioral pattern.

Fourth, economic change appears to be driven at least as much by time-consuming diffusion as from innovation[5].

Here, we shall present a model of growth that builds on the foregoing properties, together with few other 'stylized facts' stemming from empirical analyses of technological change but often neglected in formal aggregate endeavors. In Section 2, we shall outline the building blocks and theoretical conjectures supporting the model presented in Section 3. Next, we discuss some simulations results (Section 4), and, finally, flag some research developments ahead (Section 5).

[2] Cf. also Dosi, Freeman and Fabiani (1994), where one tries to outline a series of historical 'stylized facts' which the theory should ideally account for.

[3] For highly pertinent considerations on this point cf. Kirman (1989) and (1992) and Allen (1988).

[4] For example, on entry dynamics of new firms cf. the evidence discussed in Dosi and Lovallo (1997).

[5] This point has indeed been emphasized within otherwise rather orthodox models by Jovanovic and Rob (1989), Jovanovic (1995), and, of course, is near the concerns of evolutionary modelers (cf. Nelson and Winter (1982), Silverberg et al. (1988), Metcalfe (1988) and (1996)).

2 Decentralized Knowledge Accumulation and Collective Outcomes: Some Preliminaries

Technological advances, to a significant extent, are generated, *endogenously*, through resource-expensive search undertaken by a multiplicity of profit-motivated agents. Search itself is generally uncertain and innovative entrepreneurs (or, for that matter, incumbent firms undertaking innovative activities) are driven by the beliefs that "there might be something profitable out there", but are generally unable to form probability distributions on the outcomes of their search efforts.

Innovations are not entirely appropriable: knowledge progressively diffuses to other agents who might well catch-up by investing in imitation - most likely, with a lag proportional to some measure of the distance between the knowledge which they master and that which they want to acquire.

Knowledge accumulation generally entails dynamic increasing returns both at the levels of individual agents (typically, business firms) and collection of them (i.e. industries), grounded upon collectively shared 'learning paradigms'. However, radically new technologies involve, to different degrees, ruptures and 'mismatchings', so that only part of the old knowledge might be useful to the exploitation of future technologies[6].

On the grounds of these basic building blocks, the model that follows addresses three major issues.

First, under what circumstances processes of innovation and diffusion with the above characteristics can *self-organize* and yield aggregate outcomes with the properties corresponding to the empirically observed patterns of growth ? Since the model does not rest on any *a priori* commitment to individual rationality and collective equilibria, the question involves an issue which could be called of Schumpeterian coordination, namely: can 'boundedly rational' agents, heterogeneous in their beliefs and technological competences, (imperfectly) coordinate their efforts of search for novel opportunities and of exploitation of

[6] On these points, see in particular Rosenberg (1982) and Freeman (1982) regarding technologies uncertainty; Freeman (1982), Levin et al. (1987), Nelson and Winter (1982) and the remarks in Dosi (1997) and Nelson (1997) on appropriability; Arrow (1962a), Arthur et al. (1987), David (1975) and (1988), Romer (1990); Atkinson and Stigliz (1969), Nelson and Winter (1982), Dosi (1988), Malerba and Orsenigo (1993) on different - theoretical and empirical - appreciations of dynamic increasing returns; Nelson and Winter (1977), Dosi (1982), Freeman and Perez (1988) on somewhat complementary notions of 'technological paradigms' and relatively ordered 'trajectories' in learning patterns.

what they already know such as to yield relatively ordered patterns of self-sustained aggregate growth ?[7]

Second, we shall undertake some experiments of comparative dynamics and map different conditions of generation and diffusion of knowledge into the resulting growth patterns. For example, what happens to the mean (and higher moments) of the distribution of growth rates across independent sample paths as technological parameters change (including the richness of innovative opportunities, the easiness of imitation/diffusion, and the degree of path-dependence in learning processes) ?

Third, the model highlights a few sources of potential conflict between individual and collective rationality. It is an established result that in presence of externalities and dynamic increasing returns of some kind, one should not in general expect the dynamics generated by self-seeking agents to correspond with the socially optimal one. Abandoning 'representative agents' compression of the microeconomics of innovation makes the point even more vividly clear: there is no reason to expect that a decentralized economy would handle the dilemma between 'exploration' of novelty and 'exploitation' of incumbent knowledge the same way as an omniscient (and benign) planner would[8]. Moreover, by relaxing the assumption of hyper-rational agents with correct *technological* expectations, one is also able to consider those circumstances where collective growth finds its *necessary* condition in the presence of a number of 'irrational' entrepreneurs; that is, the vindication of innovative 'animal spirits' as public virtue, even when 'irrational acts' of private *hubris* ...

In this work, we explicitly take on board four out of the five 'facts' that Paul Romer (1994) identifies as underlying New Growth Theories, namely: (i) multiplicity of agents; (ii) non-rivalry in the use of knowledge; (iii) replicability of physical production activities; (iv) endogeneity of discovery efforts. The fifth one - i.e. the rents associated with successful discoveries - is implicitly there but plays no role. On purpose, we mean to partly de-link the expectations on these rents from their actual average values (which is implied by the abandonment of any rational technological expectation hypothesis). Hence, while acknowledging that agents search for innovations because they can *sometimes* earn a rent on them, we don not assume any monotonic relation between the 'true' expected value of those

[7] For a thorough discussion on the exploitation-exploration trade-off arising in adaptive systems see March (1991), Schumpeter (1934), Holland (1975), Allen and McGlade (1986) and Kuran (1988). See also Levinthal and March (1981) and Levitt and March (1988) on the trade-off between the refinement of an existing technology and invention of a new one.

[8] But any actual planner, too, would fall well short of that standard, being equally ignorant of long-run learning opportunities.

rents and the propensity to innovate. As a first approximation, we prefer to study the ways the patterns of knowledge accumulation, together with institutionally nested 'animal spirits', affect growth - with rent-related incentives just as permissive conditions, above a minimum threshold [9].

Moreover, well in the spirit of an evolutionary perspective, we assume: (i) heterogeneity among agents in their technological and behavioral features - e.g. their problem-solving knowledge and their propensity to search and to quickly imitate[10] - ; (ii) diversity in the knowledge-bases upon which agents are able to draw; (iii) path-dependency in learning achievements; (iv) bounded rationality in both decisions to allocate resources to search and choices on the directions of search efforts (hence, unlike stochastic New Growth models of 'creative destruction' - such as Cheng and Dinopoulos (1991) and Aghion and Howitt (1992) - or 'hybrids' between 'old' and 'new' ones - such as Jovanovic and Rob (1990) and Jones and Newman (1994) - we shall not confine the analysis to those rather special cases whereby decentralized agents on average 'get it right'...[11]); (v) 'open-ended' dynamics in the technology space (so that learning opportunities are notionally unlimited, but what each agent can achieve at any one time is constrained by what one has learned in the past).

However, unlike full-fledged evolutionary models[12], we do not account for any selection dynamics through which individual agents (*in primis*, firms) grow, shrink on die according to their revealed technological and market success. Hence, the following could be regarded as a reduced form 'toy model' of evolutionary growth, focusing upon the collective outcomes of decentralized patterns of knowledge accumulation, while suppressing - alike most traditional growth models - any explicit competitive interaction.

[9] In fact, this is quite in tune with the empirical evidence. While it is obviously true that with zero appropriability of innovation no private actor has any incentive to undertake expensive search (e.g. for a long time agricultural research on new varieties of seeds, etc.), on the other side, to our knowledge, there is non convincing evidence, either cross-country or over time, that innovative efforts respond smoothly to the fine tuning of appropriability conditions.

[10] Parts of the overwhelming evidence on this point are surveyed in Nelson (1981), Freeman (1982), Dosi (1988).

[11] On this point, the empirical evidence indeed matches quite solid theoretical reasons on the impossibility of forming unbiased expectations on future technological advances. After all, innovation is about solving problems that one has been unable to solve so far. But if one could know, even in probability, how to solve them, that would mean that the solution algorithm has already been found ! The issue bears on prolem-solving complexity and, more generally, on the predictability of discovery. More on this is in Dosi and Egidi (1991) and Dosi, Marengo and Fagiolo (1996), within a vast literature.

[12] See, among others, Nelson and Winter (1982), Winter (1984), Chiaromonte and Dosi (1993), Silverberg and Verspagen (1994), Dosi et al. (1994), Silverberg and Lehnert (1993), Conlisk (1989) and Metcalfe (1988).

3 The model

Think of a *knowledge base* (i.e. a technological paradigm) as a metaphorical 'island' on a stochastic n-dimensional lattice (in the following 2-dimensional for simplicity). Each island is characterized by dynamic increasing returns, associated to knowledge-accumulation, which drive the exploitation of any knowledge base.

However, notionally unlimited opportunities exist - so that, as time goes to infinity, whatever economic performance measure may go to infinity, too.

Relatedly, conflicts between 'exploration' of known technologies and 'exploration' of potentially superior ones might emerge (cf. March (1991) for an illuminating illustration of the dilemma)[13]. Moreover, we assume that individual efforts of 'exploration' slowly yield a collective externality, via, first, diffusion of knowledge, and, second, incremental improvements upon specific knowledge bases[14].

Search (i.e. exploration of new islands), as well as imitation, require a resource investment, which we assume to be proportional to the average current per capita output of the economy. Labor is the only formally accounted input - although one can easily think of a much higher dimensionality of the actual search and production input spaces as ultimately projected into labor productivity dynamics.

In this spirit, the economy is represented as a set of production activities, 'spatially' distributed on the 2-dimensional integer lattice $\aleph^2$ and it is composed of a fixed population of agents $I=\{1,2,..,N\}$, $N<<\infty$, and a countable infinite number of islands, indexed by $j\in\aleph$. There is only one good, which can be 'extracted' from every island. Time is discrete and the generic time-period is denoted by $t\in\aleph\cup\{0\}$.

The lattice, i.e. the sea, is endowed by the 'Manhattan' metric d_1. Each node $(x,y)\in\aleph^2$ can be either an island or not, while each island has a size of one node.

[13] The distinction between 'incremental' and 'radical' technical progress (i.e. between paradigm changes and within-paradigm improvements) is increasingly accepted also in other modeling perspectives: cf. for example Cheng and Dinopoulos (1992), Jovanovic and Rob (1990), Jovanovic and McDonald (1994), Amable (1995).

[14] Again, the issue of a time-consuming (and/or resource-consuming) adaptation and diffusion is beginning to make inroads also into equilibrium growth models: cf. Jovanovic and McDonald (1994), Jovanovic (1995) and Jones and Newman (1994). In the model below we especially emphasize 'creative destruction' aspects of technological discontinuities, with relatively lower attention to the possible complementarities among them (on this point, in the formal growth literature, cf. A.Young (1993)). However, note that the complementarity aspect is implicit in the possibility that we allow in our model for agents to 'carry over', so to speak, part of their previous production skills to new knowledge bases.

Let $\pi(x,y)$ be the probability that the node $(x,y)\in\aleph^2$ is an island. We will assume throughout that $\pi(x,y)=\pi$, all $(x,y)\in\aleph^2$, where $\pi\in(0,1)$ [15].

Each island $j\in\aleph$ is completely characterized by its coordinates (x_j,y_j) in the lattice[16] together with an initial (or intrinsic) 'productivity' coefficient $s_j=s(x_j,y_j)\in\Re_+$.

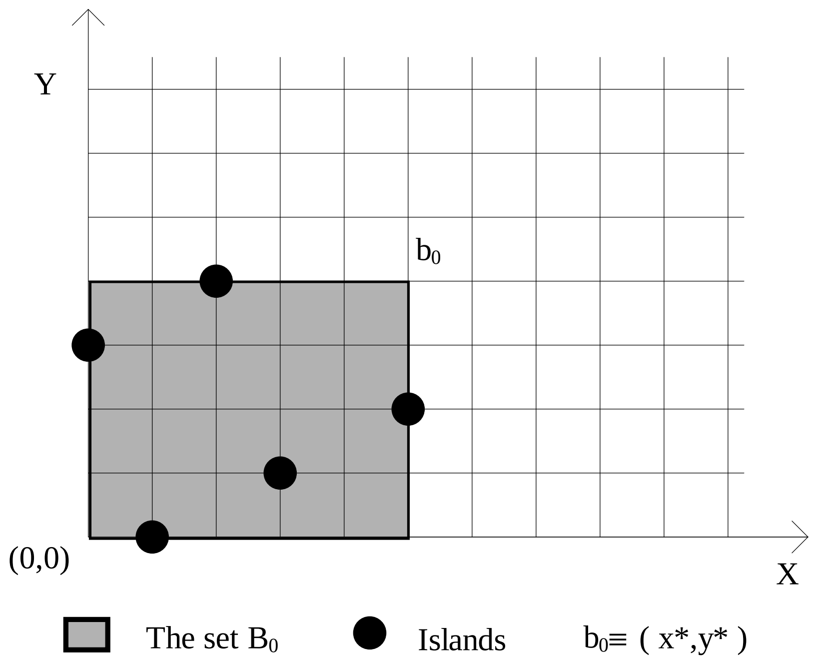


Fig. 1: A simple example of initial distribution of islands (ℓ_0=5)

Without loss of generality, we suppose that, at time t=0, the population is randomly distributed on a (small) set of islands $L_0=\{1,2,...,\ell_0\}\subset\aleph$. More precisely, assume that $d_1[(x_j, y_j)]\le d_1[(x_{\ell_0},y_{\ell_0})]$, all $j\in L_0$, and that each agent $i\in I$ has an initial location $(x_{i,0}, y_{i,0})$, such that, for all $i\in I$, there exists a $j\in L_0$: $(x_{i,0}, y_{i,0})=(x_j, y_j)$. Furthermore, let initial productivity coefficients s_j to be uniformly distributed with mean $d_1[j] = d_1[(x_j,y_j)] = x_j + y_j$, $j\in L_0$, and variance σ_s, so that, on average, the performance of a 'mine' increases with its distance from the origin of the lattice.

All agents are thus initially mining inside the smallest box containing islands in L_0, i.e. $B_0=\{(x,y)\in\aleph^2: x\le x_0^* \text{ and } y\le y_0^*\}$, where $x_0^*=\max\{x_j, j\in L_0\}$ and

[15] As Silverberg and Verspagen (1995) point out, following Nelson and Winter (1982), 'innovation should be modeled stochastically, to reflect the uncertainty in the link between effort and outcome'.

[16] Notice that there is a one-to-one mapping between the index $j\in\aleph$ and the pair $(x_j,y_j)\in N^2$, $j\in\aleph$.

$y_0^*=\max\{y_j,\ j\in L_0\}$[17]. In Fig.1 a very simple example of a conceivable initial configuration of the economy is depicted in order to make clearer the above assumptions.

Finally, assume that each agent $i\in I$ has an exogenously determined *willingness to explore* defined by the number $\varepsilon_i\in[0,1]$[18].

Dynamics and Endogenous Novelty

Let us turn now to the description of how the economy evolves. At time t=1,2,.. each agent can be in one of three different states, namely be a 'miner', an 'explorer' or an 'imitator'. Let $a_{i,t}$ the state of agent $i\in I$ at time t, where $a_{i,t}\in\{$'mi', 'ex', 'im'$\}$, and denote by $j\in\aleph$ the island currently occupied by the 'miner' $i\in I$, i.e. the agent $i\in I$ such that $a_{i,t}$='mi'.

Agents are allowed (with a certain probability) to leave the island they are working on, gradually explore the lattice around and, possibly, discover previously unexploited (and possibly more productive) islands. In order to illustrate how this is formalized, we need some additional notation.

Denote by $n_t(x_j, y_j)$ the number of miners working on island $j\in\aleph$ at time t. Then, define an island $j\in\aleph$ to be 'known' at time t if $n_\tau(x_j, y_j)>0$ for at least a $0\leq\tau\leq t$, i.e. if it currently has some people on it or if it was so at least some finite time in the past. Accordingly, let the set of currently 'known' islands be given by:

$$L_t=\{j\in N:\ \exists\ 0\leq\tau\leq t:\ n_\tau(x_j, y_j)>0\}. \qquad (1)$$

Among all known islands, let us call 'colonized' those currently exploited, i.e. all $j\in L_t : n_t(x_j, y_j)>0$. Conversely, all islands in $\aleph\backslash L_t$ will be called 'unknown', since no agent has previously exploited them. Furthermore, denote the cardinality of L_t by ℓ_t and the current location in the lattice of agent $i\in I$ by the pair $(x_{i,t},\ y_{i,t})$[19]. Finally, as we did for L_0, consider the smallest box containing all islands in L_t, i.e. let

[17] This does not mean, however, that islands j=1,2,.. (both in L_0 and in $\aleph\backslash L_0$) are sorted (in some way) by their distance from the origin.

[18] As we will see below, in each time period the 'miner' $i\in I$ decides to leave its island and explore around it with probability ε_i.

[19] Notice that the location of an agent at time t will correspond to that of an island, say j, if only if currently there is at least one 'miner' on j, i.e.: for t>1, $(x_{i,t},\ y_{i,t})=(x_j,\ y_j)$, some $j\in L_t$ if and only if $a_{i,t}$='mi'.

$$B_t = \{ (x,y)\in \aleph^2 : \; x\leq x_t^* \quad \text{and} \quad y\leq y_t^* \}, \tag{2}$$

where $x_t^*=\max\{x_j, j\in L_t\}$ and $y_t^*=\max\{y_j, j\in L_t\}$. Since the node $b_t^*\equiv(x_t^*, y_t^*)$ will only coincide by chance with a 'known' island, we can think of b_t^* just as a 'proxy' of the most efficient island (i.e. the best practice) currently exploited by the agents[20].

The model allows for an endogenous dynamics on the set L_t and, consequently, on the box B_t, in the sense that the set L_t changes in time because of the actions of agents in I. A crucial distinction has to be made here between what we will call the 'currently realized' economy and the economy *tout court*. As the box B_t contains all exploited technologies up to time t, it therefore represents a proxy of what is actually at disposal of the economy, i.e. the current set of 'fundamentals' or the 'realized economy'.

However, outside B_t there is a whole - eventually better - world waiting to be discovered. The model depicts precisely the process of the gradual endogenous discover of the economy by economic agents themselves.

Hence, given the endogenous nature of innovation/imitation activities, it is crucial to account for the process by which agents in different states make 'crucial decisions', i.e. irreversible choices that change forever the economic environment[21]. Let us consider the 'mining' process, first.

Mining

A 'miner' $i\in I$ currently located on island $j\in L_t$ with co-ordinates (x_j,y_j), will necessarily get, at no cost, a gross output $q_{i,t}$ according to the simple production function:

$$q_{i,t} = s(x_j,y_j)\,[n_t(x_j,y_j)]^{\alpha-1}, \tag{3}$$

where $s(x_j,y_j)$ is the initial 'productivity' coefficient defined above, $n_t(x_j,y_j)$ is the number of 'miners' currently working on island j and $\alpha > 1$. Returns to scale are thus increasing at the islands' level, since the current total gross output of island $j\in L_t$ is:

[20] Unlike most neoclassical models, generally based on technical change embodied in different vintages of equipment (Solow, 1960; Kaldor and Mirrlees, 1962), at any given moment in time there is no a single best-practice technique, but many competing technologies located near the frontier of the box B_t (see also Silverberg et al., 1988).

[21] See also Shackle (1955) and Davidson (1996) for some hints in a similar spirit.

$$Q_t(x_j,y_j) = s(x_j,y_j)\,[n_t(x_j,y_j)]^{\alpha}. \qquad (4)$$

The economy total gross output (GNP) will then be:

$$Q_t = \sum_{j\in L_t} Q_t(x_j, y_j)\,. \qquad (5)$$

As all agents are 'miners' at t=0, then, all $i\in I$:

$$q_{i,0} = s(x_{i,0},y_{i,0})\,[n_0(x_{i,0},y_{i,0})]^{\alpha-1} \qquad (6)$$

so that, aggregating, one obtains: $Q_0(x_j,y_j)=s(x_j,y_j)\,[n_0(x_j,y_j)]^{\alpha}$ for all $j\in L_0$ and $Q_0 = \sum_{j=1}^{\ell_0} Q_0(x_j, y_j)$.

Exploring

At time t, each miner has the opportunity to become 'explorer'. For sake of simplicity, we will assume here that this happens with probability $\varepsilon_i=\varepsilon$, for all $i\in I$ which are in the state of 'miner'. As soon as a 'miner' currently working on island $j\in L_t$ decides to become 'explorer' (i.e. $a_{i,t+1}$='ex'), it leaves its island, 'sailing' until it finds another one - possibly not known. Notice that up to now we have not endowed agents with any 'forecasting' skill. However, when a 'miner' leaves its island at time t, we let it to carry the memory of the last output which the agent was able to get in the state of 'miner' (i.e. its past knowledge and skills). We denote the memory of explorer $i\in I$ leaving island j at time τ by $q_{i,\tau}$. During the search, it does not extract any output but rather it pays a per-period 'transportation' cost equal to a given share $\beta\in[0,1)$ of the last-period per-capita GNP, raised to $\delta\geq 0$[22], i.e. if $a_{i,t}$='ex' then individual transportation cost in period t will be: $c_{i,t}=\beta\cdot[Q_{t-1}/N]^{\delta}$. From time t+1 on, it moves on the lattice following the 'naïve' stochastic rule:

[22] This form of cost has been assumed for sake of normalization. However, since in this version of the model the willingness of explore is assumed to be independent of transportation costs, the latter have no effects on the dynamics of the model, but only on the magnitude of the total net output: see also Section 4.4.

$$\text{Prob}\{ (x_{i,t+1}, y_{i,t+1}) = (x,y) \} = \begin{cases} \frac{1}{4} & \text{if } |x - x_{i,t}|+|y - y_{i,t}| = 1 \\ 0 & \text{otherwise} \end{cases}, \text{ all } (x,y)\in \aleph^2. \quad (7)$$

That is, at each time period the 'explorer' moves from its current node $(x_{i,t}, y_{i,t})$ by randomly selecting one out of the four adjacent nodes. Notice that we are assuming that agents are not aware of the fact that islands are (on average) more and more productive the further away one goes from the origin of the lattice.

The new location of the explorer $(x_{i,t+1}, y_{i,t+1})$ might obviously be: (i) the 'sea'; (ii) a 'known' island $j\in L_t$; (iii) a 'new' island $j\in \aleph\backslash L_t$. In the first case, i.e. $(x_{i,t+1}, y_{i,t+1})\neq(x_j,y_j)$ for all $j\in \aleph$, we still have $a_{i,t+1}$='ex' and the exploration goes on. In the second case, there will be a $j\in L_t$ such that $(x_{i,t+1}, y_{i,t+1})=(x_j,y_j)$ and hence the explorer $i\in I$ becomes miner on $j\in L_t$, i.e. $a_{i,t+1}$='ex'. The third case is the most important. Suppose, for simplicity, that at time t each explorer is allowed to find new islands only outside the box B_t[23]. As stated above, the node occupied by the 'explorer' $i\in I$ at time t+1could be a 'new' island with probability π. In case of discovery, the new island j* with co-ordinates $(x_{j*},y_{j*})=(x_{i,t+1}, y_{i,t+1})$ is added to the set of 'known' islands, i.e. $L_{t+1}=L_t\cup\{j^*\}$ and $\ell_{t+1}=\ell_t+1$. Moreover, both the set $B_{(\cdot)}$ and the 'best practice' proxy $(x_{(\cdot)}^*, y_{(\cdot)}^*)$ are accordingly updated.

Path-Dependence and 'Ordinary' vs. 'Extraordinary' Discoveries

In the model we allow discoveries to be either 'ordinary' or, to different extents, 'extraordinary'. In order to capture the distinction from the innovation literature between innovations within existing knowledge bases and the introduction of radically new 'technological paradigms' (Dosi, 1982), the 'initial' productivity coefficient of a 'new' island j* discovered by the 'explorer' $i\in I$ carrying the output memory $q_{i,\tau}$, will be given by:

$$s_{j*} = s(x_{j*},y_{j*}) = (1+W) \cdot \{ d_1[(x_{j*},y_{j*})] + \varphi\, q_{i,\tau} + \xi \}, \quad (8)$$

[23] This is not a necessary assumption, however. As we will see above, the economy is naturally driven, although only on average, toward more efficient islands by the process of diffusion of information, so that the event of finding a new island inside L_t is in fact irrelevant in our description.

where $d_1[(x_{j*},y_{j*})] = x_{j*} + y_{j*}$ is, as usual, the distance of j* from (0,0); W is a random variable distributed as a Poisson with mean $\lambda>0$; ξ is a uniformly distributed random variable, independent of W, with mean zero and variance σ_ξ and, finally, $\varphi\in[0,1]$. The interpretation of Eq. (8) is straightforward. The initial productivity of a 'new' island depends on four factors, namely: (i) its distance from the origin (as for initial islands); (ii) a cumulative learning effect directly linked to the past 'skills' of the discoverer, i.e. $\varphi\, q_{i,\tau}$; (iii) a random variable W which allows low probability 'jumps', that is, changes in technological paradigms[24]; (iv) a stochastic i.i.d. zero-mean noise ξ.

Two considerations are in order. First, the mechanism through which innovations are introduced in the economy is both path-dependent (Arthur, 1988 and 1994) and influenced by random (small) events (Arthur, 1989; David, 1992). On one hand, a large φ implies that more skilled 'explorers' (i.e. more efficient past 'miners') are likely to discover more productive islands and to produce more in the future, thanks to a sort of 'learning-to-learn' mechanism (Stiglitz, 1987). Moreover, the stochastic nature of innovation, together with increasing returns associated with learning by doing (as in Arrow (1962b) and Parente (1994)), allow even 'ordinary' discoveries to drive the process of growth. Second, notice that, as by independence:

$$\mathbf{E}s(x_{j*},y_{j*}) = (1+\lambda)\,[(x_{j*}+y_{j*}) + \varphi\, q_{i,\tau}], \qquad (9)$$

then, on average, a larger λ lets 'extraordinary' discoveries to be more likely in the economy. The parameter λ, together with π, are measures of the degree of notional 'opportunities'. Indeed, a large λ lets, in expectation, the productivity of a newly discovered island to be sensibly larger than those associated to the currently 'known' islands; likewise, a larger π implies a larger average number of per-period discoveries.

Diffusion of Knowledge and Imitation

Due to the uncertainty of the exploration process and to within-island dynamic increasing returns, there is an incentive for both 'miners' and 'explorers' to imitate the most productive islands existing in the 'currently realized' economy. In the model we formalize a process of diffusion of knowledge which tries to capture some basic features of empirically observed patterns of imitation and diffusion

[24] As happens in Nelson and Winter (1982) or Silverberg and Verspagen (1994), innovation is a local process.

(Nelson and Winter, 1982; David, 1975; Dosi, 1988 and 1992; Freeman, 1994; see also Jovanovic and Rob, 1989; Jovanovic and McDonald, 1994).

Let m_t be the number of 'miners' currently present in the economy. At time t, from each 'colonized' island $j \in L_t$ a signal is delivered and instantaneously[25] spread all around. Signals are characterized by an *intrinsic intensity* proportional to the share of miners present on $j \in L_t$ - i.e. $n_t(x_j,y_j) / m_t$ - and a *content* given by the actual productivity of the island - i.e. $Q_t(x_j,y_j)/ n_t(x_j,y_j)$. Moreover, they decay exponentially with the distance from the source, so that the *actual intensity* with which a signal delivered from (x_j,y_j) reaches an agent currently located at (x,y) is given by:

$$w_t(x_j, y_j; x, y) = \frac{n_t(x_j, y_j)}{m_t} \, exp\{-\rho[\, |x-x_j| + |y-y_j| \,]\}, \rho \geq 0. \qquad (10)$$

Agent i will then collect the 'contents' of all *received* signals (i.e. those coming from islands $j_{h_1}, \ldots, j_{h_M}$, where $M \leq \ell_t$ is a random variable) and contrast them with *its own performance*. The latter is simply agent i's current productivity if it is a 'miner' (say on island j), or the 'memory' on the productivity of its island of origin (say, j), if it is an 'explorer'. Hence, it will choose among the M+1 available options by drawing from the set $\{j, j_{h_1}, \ldots, j_{h_M}\} \subseteq L_t$, with probabilities proportional to the associate productivities. If the choice is j, then it will decide not to imitate any island but rather to remain in the current state. Otherwise, it will become an 'imitator' - i.e. $a_{i,t+1}$= 'im' - and it will move toward the imitated island, say (x',y'), reaching it after $k=d_1[(x',y');(x,y)]=|x-x'|+|y-y'|$ time periods - i.e. making one step at each period and following the shortest path. During this lapse of time, an 'imitator' behaves as an 'explorer' for what concerns both production and transportation costs[26]. Finally, once the imitated island is reached, it will turn again its state into 'miner', i.e. $a_{i,t+k+1}$= 'mi'.

Interactions

Interactions in our economy are basically 'local'[27]. Indeed, agents locally interact both deterministically through increasing returns in the mining process and

[25] In an alternative version of the model, not discussed here, to every signal is also associated a 'speed' which measures how quickly the signal is spread around the economy.

[26] For the sake of simplicity, notice that an imitator cannot be reached by any other signal while committed to a particular destination.

[27] A more detailed discussion of local interaction models in Fagiolo (1997a).

stochastically through the process of knowledge diffusion. In the latter, the parameter $\rho \geq 0$ tunes the 'degree of locality' of the interactions: the larger ρ, the more the process of diffusion of knowledge is local, in the sense that signals tend to reach, in probability, only the nearest neighbors. Two extreme cases are: (i) $\rho=0$, i.e. interactions are global, in that they do not depend on the distance between source and receiver; and (ii) $\rho=\infty$, i.e. no signal is spread, i.e. there is no diffusion of information.

Micro and Macro System Variables

At each time period t=0,1,2,... , the economy will be completely characterized by the following *micro variables*. Concerning islands: (a) the set of 'known' islands L_t; (b) the co-ordinates set: $Z_t=\{(x_j,y_j)\ ,\ j\in L_t\}$; (c) the initial productivity coefficients $S_t=\{s_j\ ,\ j\in L_t\}$. Concerning agents, one might consider the mappings A_t: I $\rightarrow$ {'mi', 'ex', 'im'}, C_t: I $\rightarrow \aleph^2$ and Θ_t: I $\rightarrow \Re$, recording current states, coordinates and individual gross outputs.

The *macro variables* of interest are: (i) the triple $(m_t, e_t, i_t) \in \aleph^3$, $m_t+e_t+i_t=N$, i.e. the current number of 'miners', 'explorers' and 'imitators' in the economy; (ii) the pair $(\ell_t, \ell^C_t) \in \aleph^2$ (where ℓ_t is the number of currently *known* islands and $\ell^C_t \leq \ell_t$ is the number of the *colonized* ones), together with their coordinates and their initial productivity; (iii) the log of GNP, namely $q_t=\log Q_t$; (v) the growth rate of GNP, denoted by g_t.

4 Some results

Let us start with some qualitative results focusing on the different patterns of growth the model is able to generate. To begin, note that the model is an example of 'artificial economies', which one is bound to study mainly via computer simulations. Analytical solutions - at least as long as one looks at the model in its full-fledged form - are indeed not achievable because of the underlying complexity of the stochastic processes which update micro - and accordingly macro - variables[28].

Some other considerations are in order. First, we will mainly focus on the aggregate properties of simulated time series of (log of) GNP and growth rates. The main goal is to analyze how the model behaves in some 'benchmark'

[28] For a thorough discussion of 'artificial economies' models, see Lane (1993).

parametrizations, in order to assess the roles played by knowledge-specific increasing returns, imitation and exploration in the dynamics of the economy. In particular, we will address the question whether the model is able to display self-organizing patterns of persistent growth[29] and - if so - under which behavioral and system parametrizations (especially concerning innovation and diffusion rates). Second, let us emphasize the preliminary nature of the results which follow. In order to get a deeper understanding of the behavior of the model, one should actually perform even more systematic searches of the parameter space and try to accurately map different regions of that space into (statistically) different behaviors of the variables of interest[30].

4.1 A closed economy *without* exploration

Let us analyze a very simple 'stationary' case. Assume exploration is not allowed, i.e. let $\varepsilon_i=\varepsilon=0$, $\forall i\in I$. In this set-up, the economy is 'closed', since agents can only exchange information about the initial set of islands and exploit them (i.e. act on the ground of *given* fundamentals), but are not supposed to endogenously introduce innovations. Without loss of generality, we can assume $\ell_0=2$ and $s_1\leq s_2$. In this case, given the initial productivities, the system is completely characterized by a stochastic process on $\underline{\mathbf{m}}=(m_1, m_2)$, with $m_1+m_2=N$ (i.e. on the number of miners on island j=1,2), which is a Markov chain with two absorbing states, namely $\underline{\mathbf{m}}^1*=(N,0)$ and $\underline{\mathbf{m}}^2*=(0,N)$. Accordingly, the GNP will converge with probability one to the attractor set $\Theta=\{s_1N^\alpha, s_2N^\alpha\}$. However, the process on $\underline{\mathbf{m}}$ is not *ergodic*, implying also potential inefficiency of the economy[31]. Indeed, path-dependency entailed by increasing returns will tend to drive all agents, through waves of imitation, toward the island with the *actual* (not *initial*) best productivity. Hence both initial conditions $\{s_1, s_2$ and $(m_{10}, m_{20})\}$ - i.e. productivity coefficients and the initial distribution of miners on islands j=1,2 - and 'small stochastic events' - i.e. stochastic imitation decisions - could lead agents to converge on the inefficient island j=1[32].

However, the probability that the system will be absorbed by the 'efficient' limit state, i.e. $p^*=\text{Prob}\{\lim_{t\to\infty} Q_t = s_2N^\alpha\}$, will be increasing in both $\Delta s_0=s_2-s_1$ and

[29] More on the notion of self-organization is in Lesourne (1991). See also Silverberg (1988) and the remarks in Coriat and Dosi (1995).

[30] As shown below for the full-fledged model, we did indeed begin this type of analysis.

[31] For a more detailed discussion of these properties of path-dependency cf. Arthur (1994) and David (1988).

[32] The behavior of the model in this simple set-up is close to those obtained in different frameworks by David (1992), Arthur et al. (1987).

$\Delta m_0 = m_{20} - m_{10}$ [33]. For what concerns GNP and growth rates one usually observes simulated time series as that in Fig.2. Hence, in this simple setting, growth is a transitory phenomenon because, once the lock-in on an island is achieved, no further dynamics is allowed in the system and no fluctuations will arise thereafter.

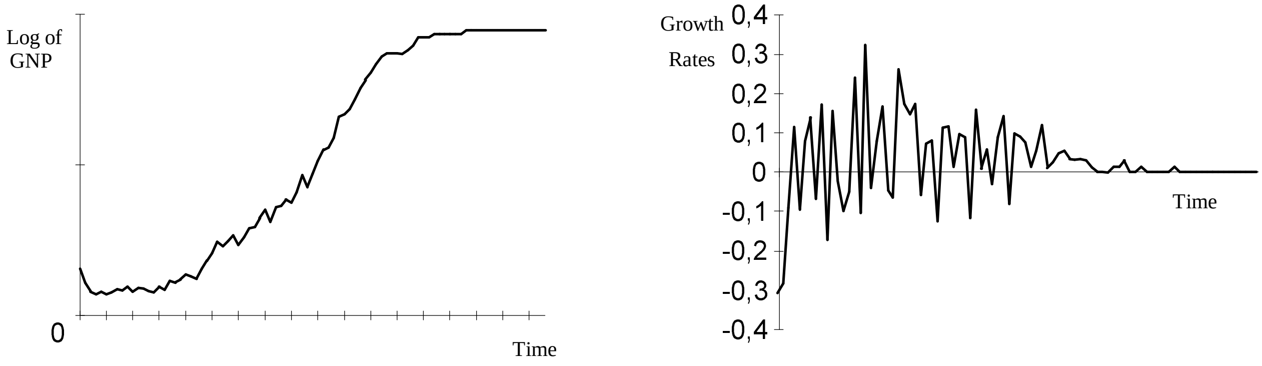


Fig.2: GNP (left) and Growth Rates in a Closed Economy without Exploration

($N=100$, $\pi=0.1$, $\sigma_s=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0.1$)

4.2 A closed economy *with* exploration

Suppose now that exploration is allowed, i.e. let $\varepsilon_i=\varepsilon>0$, $\forall i \in I$, but only *inside* the initial 'realized economy', i.e. inside an unchanged set of 'knowledge bases'. This means that 'miners' can become 'explorers' but they can 'sail' only inside the box B_0. Hence, they are still not able to 'innovate' (i.e. to discover islands other than the already 'known' ones) and must necessarily exploit the existing technologies. However, unlike the previous case, they can always decide to leave the island they are working on, even though all agents are mining on it. All that introduces a potential source of 'exploration', or, more extremely, of 'irrationality' and 'idiosyncrasy' in individual behaviors. Although the decision to become explorer is not linked - in this version of the model - to any system variable, we are tempted to define this behavior as a 'nonconformist' one, as in a few models of 'social interaction' and 'herd behavior'[34]. Indeed, when exploration is allowed, the lock-in of the system will not generally, since there is always a positive probability that 'non conformist' decisions will induce phase transitions in the system.

[33] A Montecarlo study of the frequencies of absorption as functions of Δs_0 and Δm_0, not reported here, gives quantitative supports to intuition.

[34] See Brock and Durlauf (1995), Hirshleifer (1993), Bikchandani et al. (1992), Scharfstein and Stein (1990), Kirman (1993).

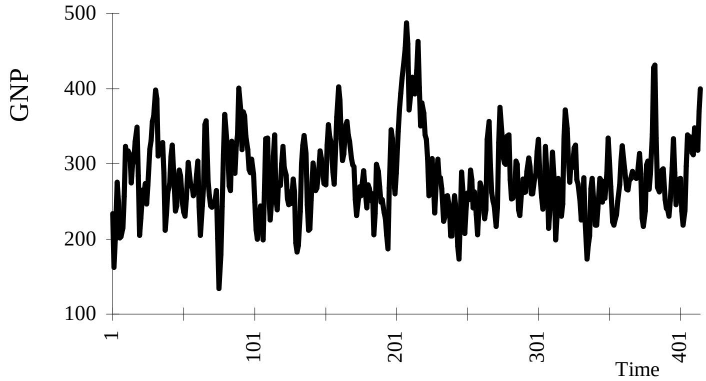


Fig. 3: GNP in a Closed Economy with Exploration

($N=100$, $\pi=0.1$, $\sigma_s=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0$, $\varepsilon=0.1$)

In this setting, the economy can be described as before by a Markov process over the **m** states which the system can attain. However, unlike the previous case, the transition probabilities are not only influenced by the propensities to imitate technologies with a higher (revealed) efficiencies, but also involve a certain probability of 'exploring'. Islands represent here 'basins of attraction' among which the system persistently oscillates exhibiting the mentioned phenomena of phase transitions[35]. The stochastic process of exploration/imitation yields persistent output fluctuations. Indeed, as depicted in Fig.3, the simulated time-series of GNP display an autoregressive stationary pattern - as econometric analyses (not reported) usually show. Note that, in this setting, over finite time periods, the number of miners working on each island obviously depends on earlier states of the system, as Figures 4(a) and 4(b) show for the two cases $s_1=s_2$ and $s_1<s_2$. Increasing returns and knowledge diffusion induces agents - on average - to move toward currently more efficient islands. However, exploration allows with positive probability 'de-locking' bursts, also toward notionally less efficient islands. In a sense, persistent fluctuations are in this case generated by a problem of *imperfect Schumpeterian coordination* in presence of dynamic increasing returns to learning[36].

[35] These properties are quite similar to those displayed by models based on Fokker-Planck equations. Cf. also Kirman (1993) and Orléan (1992).

[36] Notice here the loose analogy with the coordination-related dynamics treated by Cooper and John (1988) and Durlauf (1994).

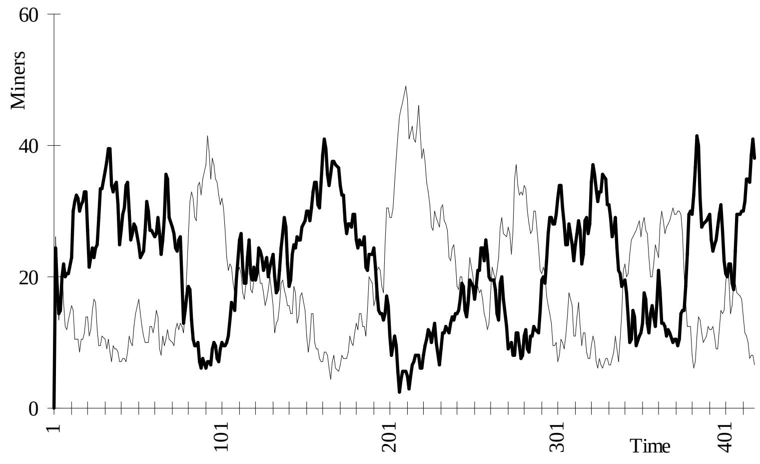


Fig. 4(a): Number of miners on islands j=1,2 when $s_1=s_2$ (thick line: Island 2)

($N=100$, $\pi=0.1$, $\sigma_s=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0$)

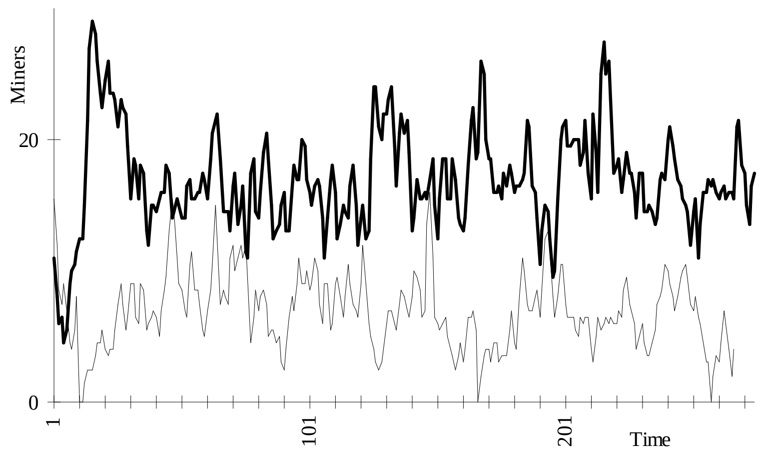


Fig. 4(b): Number of miners on islands j=1,2 when $s_1<s_2$ (thick line: Island 2)

($N=100$, $\pi=0.1$, $\sigma_s=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0$, $\varepsilon=0.1$)

Moreover, here - as in the case closed-economy / no exploration case - as long as one does not allow for the possibility of endogenous novelty, self-sustaining growth could emerge only if one superimposes an exogenous Solow-like drift on the best-practice production function.

4.3 Exploring in an open-ended economy: the emergence of self-sustaining growth

Consider now the more general case of exploration in an *open-ended* economy. In this set-up the economy displays, for a wide range of parameters, patterns of self-sustaining growth[37].

Typically, the simulated time-series of GNP are exponentially shaped (so that its logarithm displays a linear trend, as in Fig.5). More precisely, what one usually find in the case of self-sustaining growth is that the time-series of the (log of) GNP seem to be 'difference stationary', according to standard ADF tests[38] (see Table 1). Indeed, irrespective of whether the constant and/or the trend terms are included in the ADF regression, one is unable to reject at 5% the null of a unit root, which is on the contrary not accepted for both first differences Δq_t and growth rates $g_t=(q_t-q_{t-1})/q_{t-1}$[39].

[37] In the following, a Montecarlo analysis giving a more precise meaning to this statement is presented.

[38] The lag order k=5 in the standard ADF regression $\Delta q_t=\mu+\gamma t+\theta_0 q_{t-1}+\theta_1\Delta q_{t-1}+ \ldots + \theta_{k-1}\Delta q_{t-k+1} + \zeta_t$ has been suggested by both Akaike and Schwarz criteria. All econometric analyses reported here refer, as an example, to a single time-series (i.e. that plotted in Fig.5). Nevertheless, the same conclusions appear to hold in all simulations displaying self-sustaining growth. However, in order to give more rigorous bases to the above outcomes, a Montecarlo study of the percentage of rejection of the null of a unit-root (for different parametrizations) has been undertaken.

[39] The above results seem to match those obtained for GNP time-series for the U.S. by Nelson and Plosser (1982) and Stock and Watson (1986). However, it is a well-known result that standard ADF tests for 'stochastic trend' (against 'deterministic trend' alternatives) suffer from very low power. In particular, many authors have recently shown that unit-root tests are unlikely to discriminate between difference- and trend-stationarity, (see Christiano and Eichenbaum (1989) and Rudebusch (1993)), giving birth to the so-called 'we don't know' literature. Conversely, many other contributions have recently appeared suggesting that unit-root tests can be nonetheless informative, at least over long spans (DeJong and Whiteman, 1991 and 1994). In this connection, Cochrane (1988) has pointed out that the use of longer GNP samples (as in our case) may produce sharper unit-root inference. Yet, evidence stemming from this strand of literature seems to conclude that U.S. aggregate output is *not* likely to be difference stationary (Diebold and Senhadji, 1996; Bernd, 1994). Hence, the question of deterministic vs. stochastic trend in real economic aggregates remains open.Notice also that whenever the permanent component is interpreted as the outcome of productivity shocks -- as thoroughly argued by Lippi and Reichliu (1994) -- ``the random walk identification assumption is not appropriate because it does not take into account well-known features of the way in which technological change is absorbed by different firms throughout the economy. In fact, the random walk carries several implausible implicit assumptions, about the technical change process; e.g. it excludes any learning at the firm-level; it implies simultaneous adoption of technical innovation by all firms, so that even the co-existence of different capital vintages is ruled out'' (Lippi and Reichliu (1994), p. 19). This is indeed the case of our model, where -- absent capital vintages -- there is however a time-consuming process of diffusion of heterogenous pieces of knowledge.

Table 1

ADF Tests on simulated series of log of GNP [q_t],

first differences of log of GNP [Δq_t] and growth rates [$g_t=(q_t-q_{t-1})/q_{t-1}$]*

(a) 1500 Obs., Critical values: 5%=-2.864, 1%=-3.438; Constant included

Variable	Lag	ADF t-Test	σ	t Lag	t-Probability
	5	0.1169	0.072001	1.3317	0.1832
	4	0.14585	0.072021	2.8253	0.0048
Log of GNP	3	0.20439	0.072199	1.1937	0.2328
q_t	2	0.22725	0.07221	-1.2462	0.2129
	1	0.20157	0.072225	0.49482	0.6208
	0	0.21226	0.072205		
	5	-13.397**	0.071939	-1.707	0.088
	4	-15.086**	0.071988	-1.2748	0.2026
First Diff.	3	-17.190**	0.072004	-2.7825	0.0055
Δq_t	2	-21.328**	0.072176	-1.1552	0.2482
	1	-27.255**	0.072185	1.2672	0.2053
	0	-37.052**	0.072201		
	5	-15.046**	0.010963	-2.7796	0.0055
	4	-17.329**	0.010989	1.1188	0.2634
Growth Rates	3	-18.595**	0.01099	-0.81087	0.4176
g_t	2	-21.829**	0.010989	-4.8012	0
	1	-32.801**	0.011075	7.3257	0
	0	-38.851**	0.01128		

(b) 1500 Obs., Critical values: 5%=-3.415; 1%=-3.97; Constant and Trend included

Variable	Lag	ADF t-Test	σ	t Lag	t-Probability
	5	-2.4513	0.071869	1.4905	0.1363
	4	-2.3567	0.0719	2.9716	0.003

Log of GNP	3	-2.1664	0.0721	1.3261	0.185
q_t	2	-2.0876	0.072119	-1.1085	0.2678
	1	-2.1655	0.072125	0.6366	0.5245
	0	-2.1283	0.07211		
	5	-13.398**	0.071961	-1.7014	0.0891
	4	-15.086**	0.07201	-1.2704	0.2042
First Diff.	3	-17.190**	0.072025	-2.7771	0.0056
Δq_t	2	-21.326**	0.072197	-1.1506	0.2501
	1	-27.251**	0.072206	1.2718	0.2036
	0	-37.044**	0.072221		
	5	-15.313**	0.010939	-2.6601	0.0079
	4	-17.595**	0.010962	1.2743	0.2028
Growth Rates	3	-18.837**	0.010965	-0.6385	0.5233
g_t	2	-22.057**	0.010963	-4.5807	0
	1	-33.052**	0.01104	7.5376	0
	0	-39.016**	0.011257		

* Econometric analyses refer to the following parametrization: N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5.

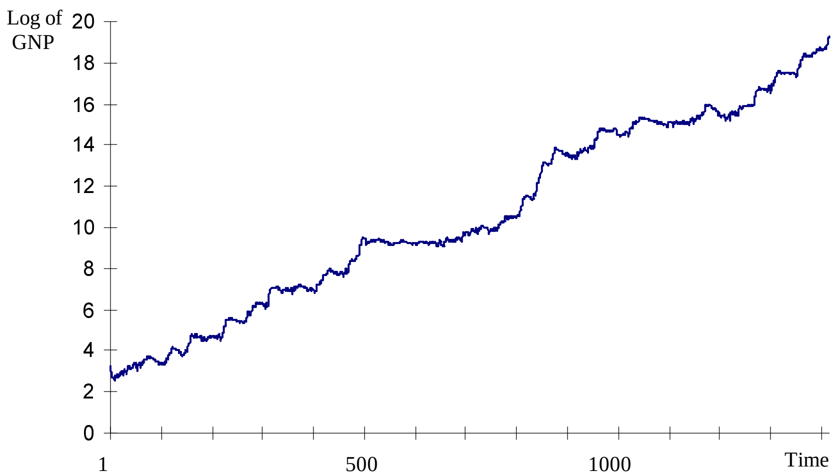
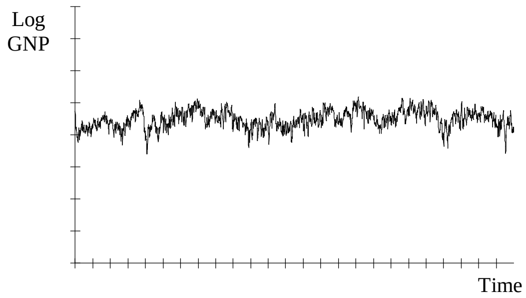


Fig. 5: Exponential Growth in an Open-Ended Economy with Exploration (Log of GNP)

(N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

In the Appendix, some further results about persistence of output fluctuations are reported. In analogy with Campbell and Mankiw (1987, 1989), we address the question of whether fluctuations in GNP are characterized by a permanent component and how big such a component might be. They consider two different measures of persistence[40] based on sample estimates of auto-correlations of changes in log of GNP, finding that in "six out of seven countries a 1% shock to output should change the long-run univariate forecast of output by well over 1%". We computed the same statistics for both time-series of change in log of GNP (i.e. Δq_t) and growth rates (i.e. $g_t=(q_t - q_{t-1})/ q_{t-1}$), getting similar results. As table 4 in the Appendix shows, all estimated measures of persistence generally exceed unity, suggesting that our simulated GNP is characterized by non transitory fluctuations[41].



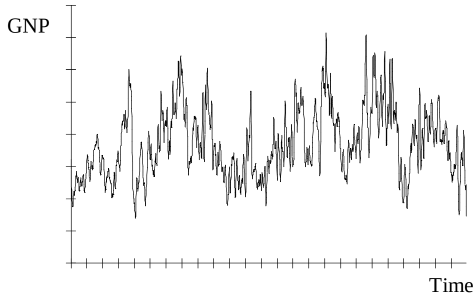


Fig.6(a): No Growth in an Open-Ended Economy with Exploration

($N=100$, $\pi=0.1$, $\sigma_s=\sigma_\xi=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0$, $\varepsilon=0.1$, $\lambda=1$, $\varphi=0.1$)

However, exponential growth is not the sole regularity one can get from the simulated time-series of GNP. Indeed, for different parametrizations, the model is able to generate 'no growth' economies as in Par.4.2 - see Fig.6(a) - or 'low growth' ones[42], as depicted in Fig.6(b).

[40] See the Appendix. For details, cf. also Cochrane (1988).

[41] Notice, incidentally, that our estimates are very close to those of the U.S. (log of) real GNP obtained by Campbell and Mankiw (1989). Again, there is no consensus in the literature about the size of the long-run response of actual real GNPs to an innovation. Christiano and Eichenbaum (1989), for instance, show that Campbell and Mankiw's results are very sensitive to the choice of the ARMA representation of the data.

[42] By a 'low growth' economy we mean a situation where the GNP time-series fluctuates around a linear (stochastic) trend, while its log follows a 's-shaped' pattern, so that in the long run growth rates tend to become stationary around zero.

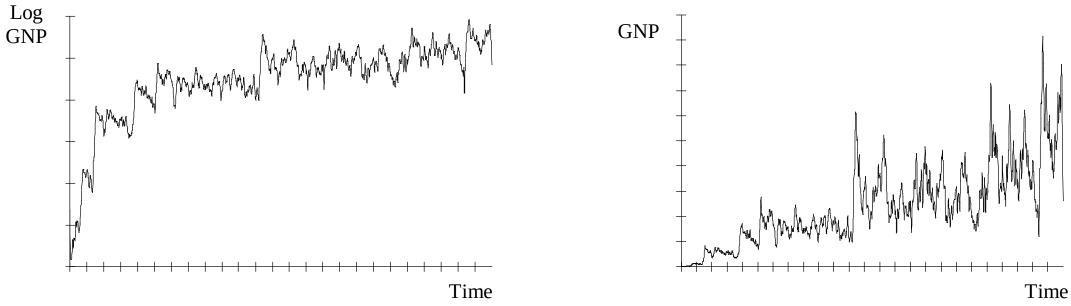


Fig.6(b): 'Linear' Growth in an Open-Ended Economy with Exploration

(N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.2)

Our conjecture is that necessary conditions for the model to exhibit exponential growth are, of course, the presence of increasing returns, but, moreover, the following ones - or a suitable mix of them - ought to apply, namely: (i) both the level of opportunities and the average number of current 'explorers' have to be sufficiently large; (ii) knowledge diffusion is not too 'local'; (iii) there is some path-dependency in innovation. Putting in another way, one should expect self-sustaining growth to emerge for large values of φ, π and λ and for small values of ρ.

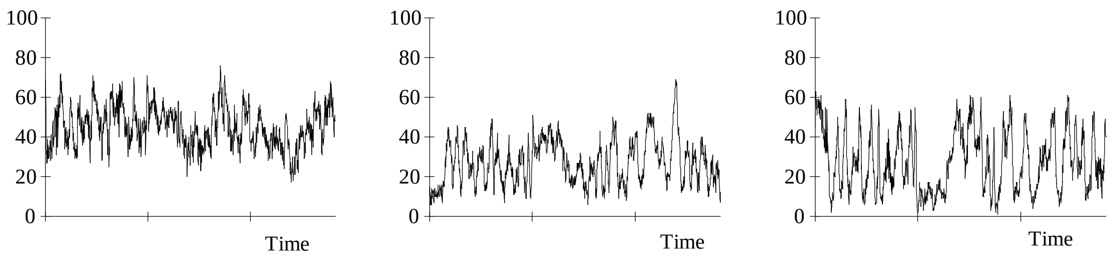
In the following, some support to this conjecture will be shown.

The sources of self-sustaining growth: Some 'Qualitative' Evidence.

A basic insight stemming from a qualitative analysis of the behavior of the model is that self-sustaining growth seems to be generated in the system - above certain thresholds - by non-linear interactions among innovation, path-dependency, increasing returns and diffusion of knowledge and *not* by any of these forces taken in isolation. In order to illustrate this point, assume to start from a fairly uniform distribution of the N agents on the initial 'known' islands L_0. On the one hand, diffusion of knowledge is likely to drive agents to concentrate on a relatively small cluster of 'known' islands - generally close to the frontier of the 'realized economy'- which, by dynamic increasing returns, might be, often but not always, the most efficient ones. On the other hand, some 'lucky' explorers - which have decided not to imitate one out of the cluster of colonized islands - will sometimes find intrinsically superior islands outside the 'realized economy'. Although they might not be able to adequately exploit the opportunities of the 'new' island by themselves, the 'extraordinary' character of their discovery might nevertheless induce other agents to move there in the future and, consequently, increase its actual productivities. Hence, a ‘rare event’ (i.e. the exceptional discovery), feeding path-dependently upon diffusion and incremental innovations thereafter, might be able to trigger a self-reinforcing process whose ultimate outcome might be a pattern of exponential growth.

The above conjecture can be further supported by looking at some other pieces of qualitative evidence on the dynamics of the model. Indeed, given a set-up yielding exponential growth[43], the story that simulated time-series tell us might be rephrased as follows.

First, time series of the number of ‘miners’, ‘explorers’ and ‘imitators’ typically follows a stationary pattern, see Fig.7.



(a) Miners (b) Explorers (c) Imitators

Fig.7: Number of Miners, Explorers and Imitators in an Open-Ended Economy displaying self-sustaining growth ($N=100$, $\pi=0.1$, $\sigma_s=\sigma_\xi=0.1$, $\rho=0.1$, $\alpha=1.5$, $\beta=0$, $\varepsilon=0.1$, $\lambda=1$, $\varphi=0.5$)

Second, although the number of currently ‘known’ islands (at any τ) displays a linear trend, both the ratio ‘colonized’/ ‘known’ islands and the number of ‘colonized’ ones - Fig. 8(a) and 8(b) respectively - fall quickly and then follow a stationary process. Hence, imitation leads agents to exploit (i.e. to ‘colonize’) a small subset of islands (out of the ‘known’ ones).

[43] Unless differently stated, we refer throughout, as an example, to the basic parametrization: $N=100$, $\sigma_s=\sigma_\xi=0.1$, $\beta=0$, $\alpha=1.5$. All results reported in this sub-section refer to: $\pi=0.1$, $\rho=0.1$, $\varepsilon=0.1$, $\lambda=1$, $\varphi=0.5$.

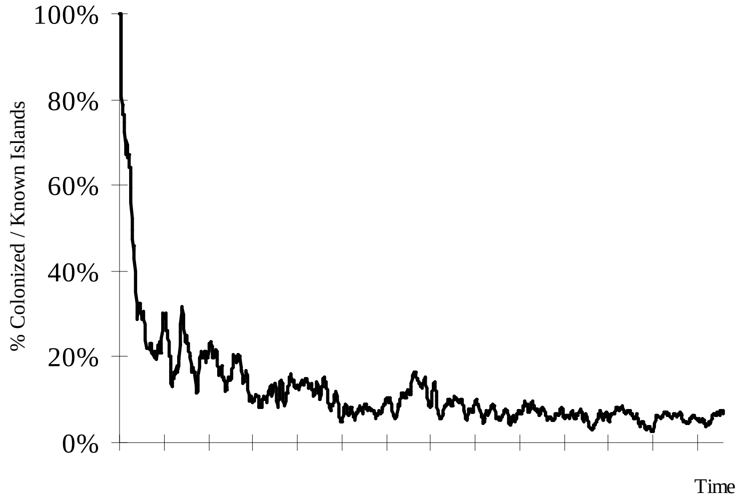


Fig.8(a): % of Colonized Islands in an Open-Ended Economy displaying self-sustaining growth (N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

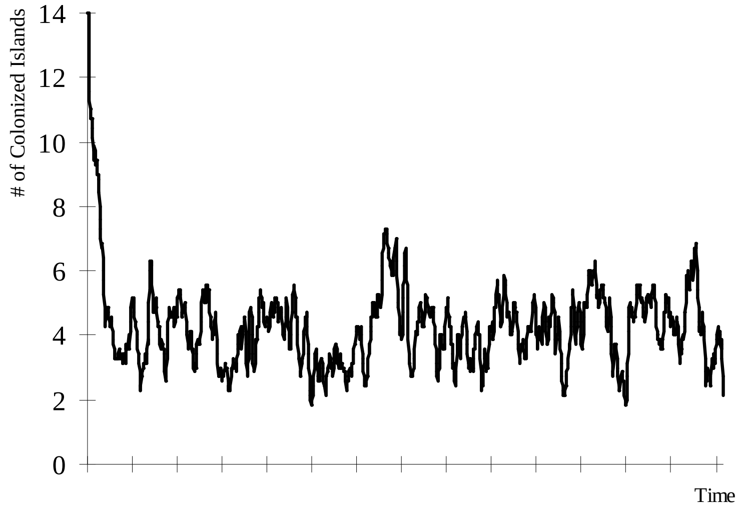
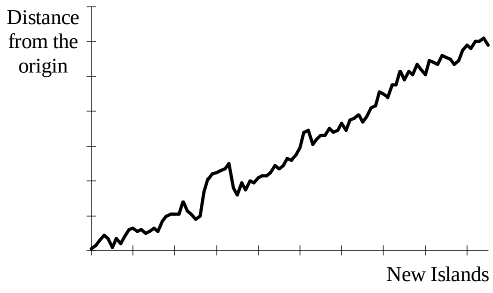


Fig.8(b): Number of Colonized Islands in an Open-Ended Economy displaying self-sustaining growth (N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

Third, since the number of 'explorers' is a stationary process, the average per-period number of 'discoveries' keeps constant. Moreover, as the uniform nature of the 'exploration' rule should suggest - cf. Eq. (7) - the distance from the origin of a new island increases linearly with the number of discovered islands (see Fig.9(a)). However, the path-dependent nature of innovation implies that the



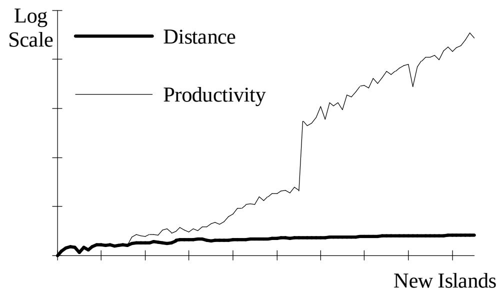


Fig.9: Distance from the origin and actual productivities of new islands (Number of new islands on x-axis) in an economy displaying self-sustaining growth

(N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

initial productivity of a new island (i.e. the coefficient s_{j*}) is generally greater than the average current productivity over all 'known' islands (see Fig.10) while the one-time push irregularly caused by the introduction of 'new paradigms' keeps the order of magnitude of initial productivity of new islands constantly above their distance from the origin (see Fig.9(b)).

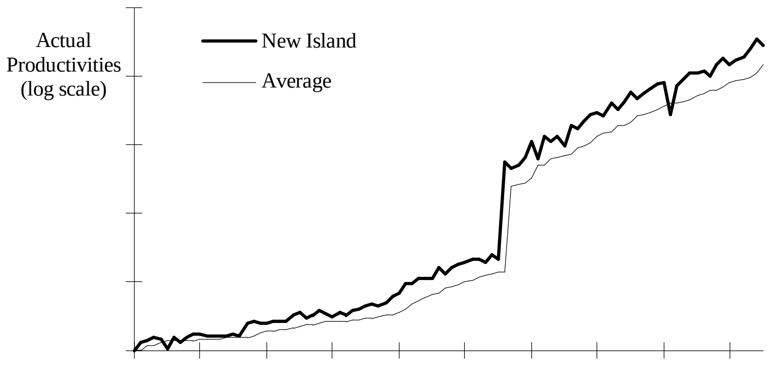


Fig.10: Actual productivity of new islands vs. average current productivity of 'known' islands in an economy displaying self-sustaining growth

(N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

Finally, relatively ordered spatial patterns of colonized islands are likely to emerge, due to the local nature of both the exploration and imitation processes. In Fig.11 the path of expansion of the 'best practice' proxy b_t* is plotted together with four 'snapshots' showing the locations of currently 'colonized' islands for different time periods t= 0, 500, 1000, 1500. While in the early time periods of the simulation small (stochastic) events select the region of the lattice where the exploration is going to take place, the path-dependent nature of the overall process tends to keep the economy inside that region. At each time period, only few islands are exploited and the economy is seldom producing under the notionally most efficient conditions.

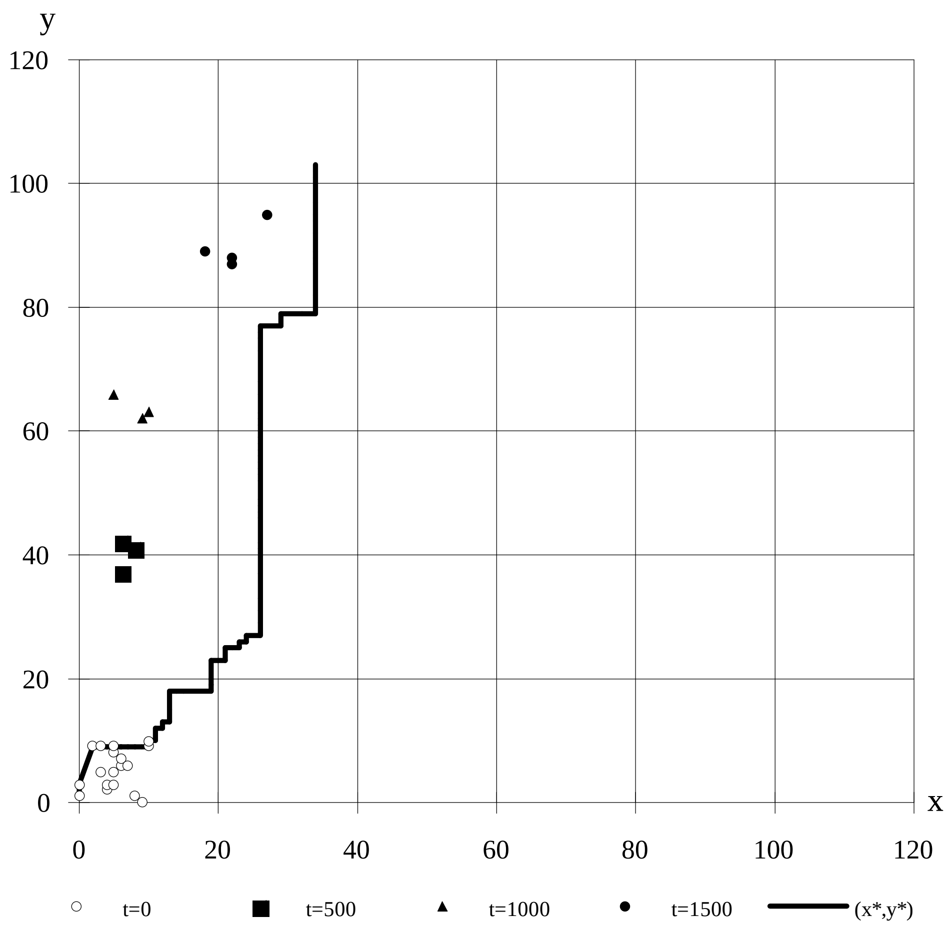


Fig.11: Spatial Diffusion Patterns of Colonized Islands and 'Best Practice' proxy b_t*=(x_t*,y_t*) in an economy displaying self-sustaining growth

(N=100, π=0.1, σ_s=σ_ξ=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5)

A Montecarlo Analysis

In order to give strength to the above interpretation, we have performed some Montecarlo (MC) studies with the goal of investigating (i) how behavioral and system parameters affect average growth rates (AGRs); and (ii) the robustness of the results across different sample paths, holding the parametrization constant[44].

Table 2

Means of Montecarlo Estimates of Frequency Distributions of Growth Rates within a Simulation

(100 Simulations; N=100, $\sigma_s=\sigma_\xi=0.1$, $\beta=0$, $\varepsilon=0.1$, $\alpha=1.5$)

Path-Dependency and Globality of Diffusion		Means of Distributions of Growth Rates	
		Low Opportunities ($\lambda=1$; $\pi=0.1$)	High Opportunities ($\lambda=5$; $\pi=0.4$)
φ	ρ		
∞	0	0.4678	0.4618
0.7	0.1	0.4779	0.4771
0.6	0.2	0.4838	0.5085
0.5	0.3	0.4961	0.5518
0.4	0.4	0.5157	0.5946
0.3	0.5	0.5440	0.6801
0.2	0.6	0.6230	1.3825
0.1	0.7	0.7124	1.5167
0	0.8	0.7905	1.8653

First, we have considered the role played by opportunities, path-dependency and locality of knowledge diffusion in the emergence of ‘self-sustained’ growth.

For a given level of ‘willingness to explore’ ($\varepsilon=0.1$), two benchmark cases, namely a ‘low opportunities’ set-up (i.e. $\pi=0.1$ and $\lambda=1$) and a ‘high opportunities’ one (i.e. $\pi=0.4$ and $\lambda=5$), have been analyzed. For different

[44] For a given parametrization, let $\{g_m, m=1,2,..,M\}$ be the Montecarlo sample of average growth rates, where, for a given simulated time series $\{q_t=log Q_t, t=0,..,T\}_m$, we simply define $g_m=100\cdot[(q_T/q_0)^{1/T}-1]$. In the following, T=2500 and M=1000.

combinations of ‘path dependency’ and ‘locality of knowledge diffusion’[45], a sufficiently large number of independent simulations have been run, yielding correspondent distributions of AGRs[46]. In Fig.12a (low opportunities setup) and 12b (high opportunities), MC mean values and variances of the distributions of AGRs are plotted. The histograms for mean values seem to confirm the above intuition. Mean values of AGRs are increasing in both path-dependency (φ) and globality of knowledge diffusion (ρ)[47] for a given level of opportunities, while high-opportunity AGRs are larger than low-opportunity ones for a given combination of path-dependency and globality of knowledge diffusion. Moreover, histograms of MC variances suggest an interesting emergent property of the model. Indeed, as a general result, one observes a strong positive correlation between high AGRs and larger variances in the MC distributions (see also below)[48]. Finally, a recursive analysis of the first four moments of AGRs MC distributions (not reported here) has been undertaken. For each combination in the above parameter grid, moments of MC distribution over the first M* simulations - where $M^* = M_0, M_0+1, \ldots, M$ - have been computed and plotted against M*. In all cases one can observe convergence of the first four moments after a number of simulations well below M=1000.

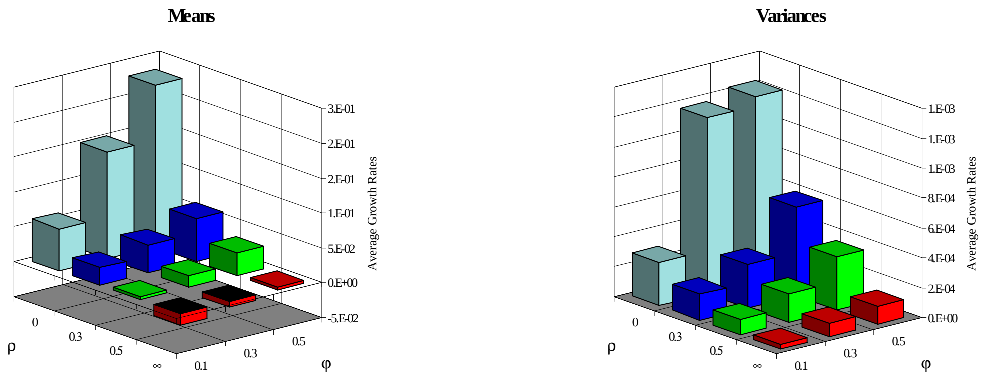


Fig. 12 (a) : Low Opportunities (π=0.1; λ=1)

[45] In each case, a grid for ρ and φ has been prepared, namely: $\rho \in \{0, 0.1, 0.5, \infty\}$ and $\varphi \in \{0.1, 0.2, 0.3, 0.5\}$. Notice that if ρ=0 the knowledge diffusion is ‘global’, while if $\rho=\infty$ it is absent.

[46] The null of normality is accepted at 5% for all AGR Montecarlo distributions (χ^2 test).

[47] Notice that, as a ‘rule of thumb’, only mean values of AGR above 0.06 imply ‘self-sustained growth’, or, put it differently, a I(1) process for the log of GNP.

[48] For a similar property of actual time series in a cross-section of countries, cf. Fatas (1995).

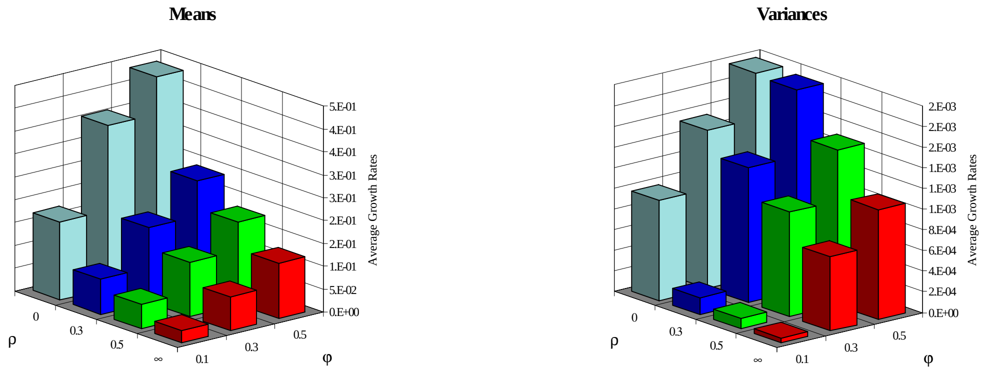


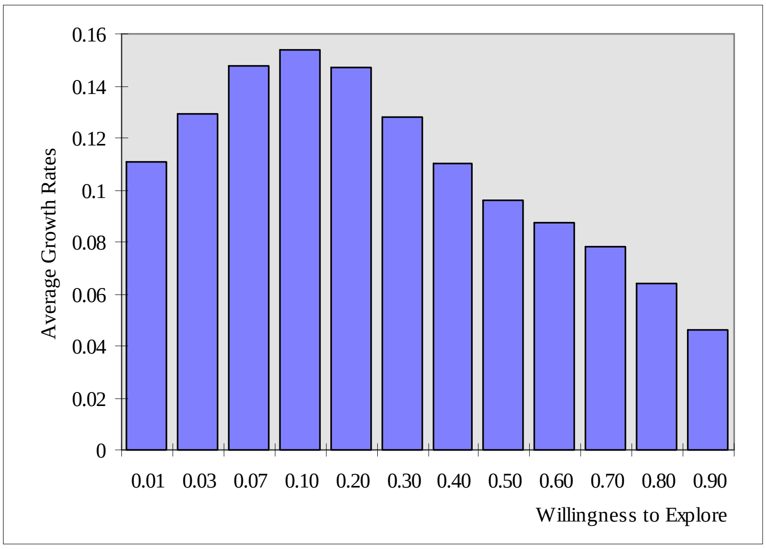
Fig. 12 (b) : High Opportunities (π=0.4 ; λ=5)

Figure 12 : Montecarlo Means and Variances of the Distributions of Average f Rates Growth (1000 Sim., N=100, $\sigma_s=\sigma_\xi=0.1$, $\alpha=1.5$, $\beta=0$, $\varepsilon=0.1$)

Second, the net effect of ‘willingness to explore’ on AGRs (i.e. the effect of a change in ε, everything else being constant) has been investigated. For a given parametrization yielding as a usual outcome a pattern of self-sustaining growth[49], we have performed several simulations for varying ε, under the two above opportunities setups. An interesting emergent property is that MC means of AGRs seem to be small whenever the ‘willingness to explore’ is either very low or very large - see Fig.13(a) and 13(b). Furthermore, the system appears to be characterized - in both opportunities setups - by ‘optimal’ levels of ‘willingness to explore’, somehow increasing in the notional level of opportunities. The intuition here corresponds to that suggested in March (1991, p.71). As he points out, systems that engage in exploration to the exclusion of exploitation “exhibit too many undeveloped new ideas and too little distinctive competences”, while, conversely, at the opposite extreme, they “are likely to find themselves trapped in sub-optimal stable equilibria”. Hence, in our model the losses stemming from the exploration-exploitation trade-off seem to be minimized by an appropriate balance between the two forces (March, 1991; Allen and McGlade, 1986), which, however, agents are generally unable to correctly evaluate ex-ante.

[49] The parametrization is ρ=0.1 and φ=0.5. For each value of $\varepsilon \in \{0.01, 0.03, 0.07, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90\}$, M=1000 simulations have been run.

(a) Low Opportunities (λ=1; π=0.1)



(b) High Opportunities (λ=5; π=0.4)

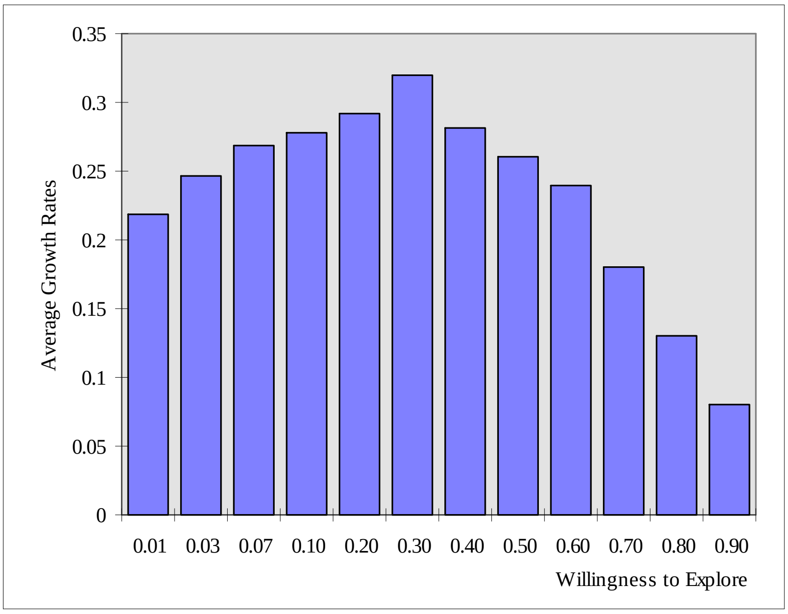


Figure 13: Montecarlo Means of Average Growth Rates vs. Willingness to Explore (ε)

(1000 Simulations; N=100, σ_s=σ_ξ=0.1, β=0, α=1.5, ρ=0.1, φ=0.5)

Third, in order to further investigate the emergence of some positive correlation between higher AGRs and larger variances in growth rates, we have computed, for different parametrizations, MC estimates of the frequency distribution of the simulated time-series of growth rates. This has been done by averaging, over M=100 simulations, the frequency distributions of the time-series $\{h_t=(Q_t-Q_{t-1})/Q_{t-1}$, t=1,..,2500$\}$. The results about the mean of those distributions (Table 2), together with those obtained before, suggest that ‘self-sustaining’ growth seems to be strongly related to a larger variability in the distributions of growth rates both *across* independent simulations and *within* a single sample path. The interpretation of this emergent property is strongly related to both the non-linear and self-reinforcing nature of the mechanisms involved. Indeed, what one usually get by gradually increasing the strength of the sources of growth in the model is that the self-reinforcing mechanisms of exploration, innovation and production become somewhat explosive.

Self-sustaining growth appears to *imply* the co-existence of periods of moderate growth intertwined by ‘jumps’ caused by radical innovations (i.e. the arrival of new ‘paradigms’) which however diffuse through the economy thanks to a time-consuming process of adjustment of all agents to the new knowledge base. Hence, the model, despite it simplicity, is able to account for some of those ‘retardation factors’ emphasized by Abramovitz (1989, 1993) and David (1991), and, relatedly, for the appearance over finite time periods of distinct patterns (or ‘phases’) of development.

Moreover, higher average rates of growth entail *higher within-simulation* variability in the rates themselves and *also a higher cross-simulations variability* of AGRs[50]. The latter property seems to suggest a sort of path-dependency in growth patterns which becomes more marked the more one ‘fuels’ the economy with learning opportunities.

Size of the economy and growth

A well-known drawback of many models of endogenous growth based on some forms of increasing returns - involving dependence of a *flow* variable upon a *stock* variable, e.g. arrivals of technological ‘blueprints’ as a function of their levels - is that sheer size effects influence growth rates[51]. For instance, many one-factor

[50] For similar findings, see Aghion and Howitt (1992).

[51] We refer here to R&D-based models of endogenous growth, such as Aghion and Howitt (1990, 1992), Romer (1986, 1990), Grossman and Helpman, (1991a, 1991b). In these models, size-effects stem from three related assumptions, namely (i) technology is non rival, so that increases in the scale of the economy entail larger profits for all innovators; (ii) there are strong inter-temporal spillovers, i.e. each innovator can improve existing technology at any time; and (iii) new technologies are substitute for the old ones, so that returns to innovation are decreasing in the rate of innovation. Conversely, in many models in which growth is endogenously generated by the accumulation of human and physical *rival* capital, any increase in the scale of the economy has no impact on growth rates (cf. Lucas (1988), Jones and Manuelli (1990) and Rebelo (1991)). Furthermore, cf. Young (1995) and Jones (1995a) for recent examples of R&D-based models of endogenous growth *without* scale-effects.

models, such as Aghion and Howitt (1990) and Romer (1986), predict that growth rates are increasing, other things being equal, in the size of the population. Furthermore, when one considers extensions of these basic models - such as multi-factors models (Aghion and Howitt, 1992; Grossman and Helpman, 1991a; Romer, 1990) and with international trade (Grossman and Helpman, 1991b) - the standard result is that growth rates are increasing in the factor used intensively in the 'innovative' activity (e.g. skilled labor)[52].

Table 3

Montecarlo Mean Values of Average Growth Rates (AGRs*)

as a function of the Size of the Economy (N) and the Econometric Sample Size (T)

(100 Simulations, π=0.4, λ=5, $\sigma_s=\sigma_\xi$=0.1, β=0, α=1.5, ρ=0.1, ε=0.1, and φ=0.5)

	Size of the Economy				
Sample Size	N=50	N=100	N=200	N=500	N=1000
T=250	0.2526	0.2402	0.2454	0.2196	0.1275
T=500	0.2879	0.2104	0.2278	0.1602	0.1563
T=1000	0.2300	0.2262	0.1901	0.1889	0.1485
T=1500	0.2448	0.2536	0.2287	0.2044	0.1895
T=2500	0.2529	0.2048	0.2102	0.1707	0.1912
T=5000	0.2347	0.2141	0.2163	0.2267	0.2156

The present model, notwithstanding increasing returns to learning, *does not* display that unreasonable property. To see this, we have computed MC mean values of AGRs across M=100 simulations holding all parameters constant[53] but just increasing the size of the economy N, i.e. the number of agents. Moreover, in order to ascertain whether the time-length of observed histories affects our results, we have reported MC mean values of AGRs computed at different time-periods (i.e. for different econometric sample periods T). As Table 3 shows, there is a weak evidence on falling AGRs the larger the economy is *for a given time-length*,

[52] Taken literally, they would predict India growing faster than, say, Singapore. Cf. Jones (1995b) for a detailed discussion on empirical evidence on these points.

[53] In what follows (cf. Table 3), we report as an example the results obtained considering a 'high-opportunity' set-up yielding 'exponential growth', i.e. we set π=0.4, λ=5, $\sigma_s=\sigma_\xi$=0.1, β=0, α=1.5, ρ=0.1, ε=0.1, and φ=0.5. However, the same pattern holds also for other opportunity setups and different parametrizations of knowledge diffusion, path-dependency and dynamic increasing returns.

while AGRs do not display any monotone pattern even if one compares AGRs for N and T both increasing.

The intuition behind this is that, while *ceteris paribus* larger economies face potentially higher returns to knowledge exploitation, it is also true that they must cope, in probability, with higher 'adjustment lags' to new knowledge bases (as proxied in our model by the time it takes to move a certain fraction of the N agents to the notionally superior islands). Hence, larger economies which are potentially able to fuller exploit increasing returns to any one knowledge base need also a relative longer time to achieve persistently higher growth rates.

4.4 Individual vs. Collective rationality: A Simple Example

As conjectured above, the model highlights a few sources of potential conflict between individual and collective rationality. In order to illustrate this point, consider the following simple example. Assume an economy characterized by: (i) constant returns to scale (i.e. α=1); (ii) no knowledge diffusion (i.e. $\rho=\infty$); (iii) no path-dependency in innovation (i.e. φ=0); (iv) all N agents working at time t=0 on a single island (ℓ_0=1) with co-ordinates (x*, y*) and initial productivity s*=x*+y*[54]; (v) a constant positive transportation cost β (i.e. δ=0, $\beta\in[0,1)$, see Section 3.2).

Given the above parametrization, we will consider two different settings for what concerns behavioral assumptions.

In the first one, the population is composed of N agents behaving according to the behavioral rules defined in Section 3.

In the second one, we will introduce a 'representative individual' (RI) endowed by 'rational expectations'. More precisely, assume that the latter has unbounded computational skills and complete information, so that it knows: (i) the co-ordinates (x*, y*); (ii) the system parameters; (iii) the model of the economy. Although it knows that, on average, the initial productivity of a new island is increasing in its distance from the origin, he does not know where new islands are actually located. Hence, starting from the node (x,y), it will make use of an exploration rule which gives equal probability to the nodes (x+1,y) and (x,y+1). Finally, assume for simplicity that the intertemporal discount rate is zero[55].

At time t=1, the problem for the RI is to decide whether to continue to extract the good at time t=2 or start to explore. In the first case, it will get a per-period net

[54] Notice that with constant returns to scale the output of the agent working on island (x*,y*) is equal to its initial productivity s*, irrespective of the number of agents are working on the island.

[55] Our conjecture is that the following results will hold *a fortiori* for a strictly positive discount rate.

output from mining equal to θ_M=s*. In the second case, the expected per-period net output from exploration will be: $\theta_E=[(1+\lambda)(s^*+\tau) - \beta\tau]/\tau$, where $\tau=1/\pi$ is the expected length of exploration[56]. Then, the RI will decide to remain on island (x*,y*) if and only if $\theta_M>\theta_E$, i.e. iff:

$$\pi < \frac{1}{1+\lambda} - \frac{1}{s^*} + \frac{\beta}{(1+\lambda)\cdot s^*} = \pi^*(\beta,\lambda,s^*) \quad (11)$$

As one can easily check, $\pi^*(\beta,\lambda,s^*)$ is decreasing in λ and increasing in s* and β, as expected[57]. More generally, one could single out - for given values of s* - a correspondent region in the space spanned by feasible values of (β,λ) satisfying (7) for some $\pi\in(0,1)$.

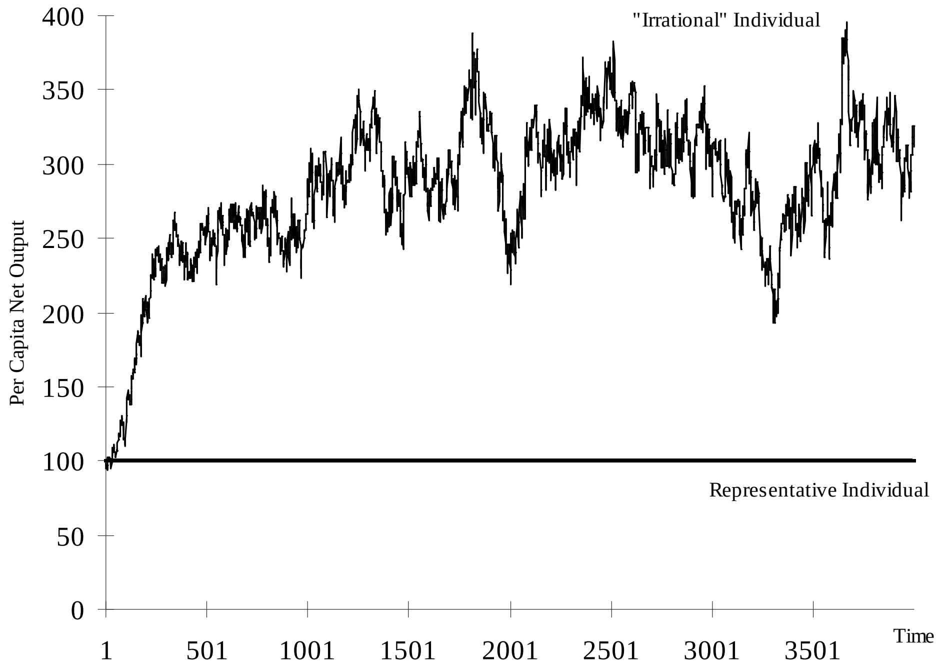


Figure 14: Individual vs. Collective Rationality: A simple example

($s^*=100$, $N=100$, $\varepsilon=0.05$, $\beta=0$, $\delta=0$, $\varphi=0$, $\lambda=5$, $\pi=0.15$, $\rho=\infty$, $\sigma_s=\sigma_\xi=0.1$, $\alpha=1$)

[56] Notice that τ is also the expected distance between (x*,y*) and a new island.

[57] If we allow β to be greater than the unity, then $\pi^*(\beta,\lambda,s^*)$ is increasing in s* only if $\lambda>\beta-1$, i.e. if opportunities are large enough. Notice that if $s^*\to\infty$ the RI will always stay on (x*,y*), while if $\lambda^*\to\infty$ it will always leave.

For instance, assume for simplicity β=0. Then, the pair π=0.15 and λ=5 satisfies Eq. 11 for s*=100. In this setup, the RI will decide to continue to work as a 'miner'. Hence, such an economy will get a net per-capita output θ*=100. On the contrary, consider an economy characterized by the same parametrization[58], composed of N=100 agents, all starting as 'miners' on the island (x*,y*), x*+y*=100, and behaving as described in Section 3. Notice that agents live here in a rather 'poor' environment, in which there is *neither* knowledge diffusion, *nor* path-dependency in innovation, *nor* increasing returns to scale. Furthermore, assume that agents are characterized by a very low 'willingness to explore' (i.e. ε=0.05). Notwithstanding all that, simulations show (Fig.15) that the economy is able to get, as a general outcome, a per-capita net output persistently greater than θ*=100.

Thus, even in this very simple setting, collective growth finds its necessary condition in the presence of a number of 'irrational' individuals.

Even more so, this potential conflict between individual rationality and collective welfare emerges in the general setting with unlimited notional opportunities of exploration and transportation costs born up front by the 'explorers' themselves.

Note that as mentioned earlier this property significantly expands upon the common result from e.g. New Growth literature that in presence of externalities or dynamic increasing returns a systematic divergence between endogenously generated growth rates and socially optimal ones (whatever the latter means...) is likely to emerge. Here, one may require indeed the presence of straightforwardly *irrational* agents in order to have endogenous growth at all.

5 Conclusions

The foregoing model presents a rather simple dynamics through which 'incremental' knowledge accumulation, diffusion and random discoveries of new technologies interact as to yield persistent - and persistently - fluctuating growth.

As mentioned, it could be considered as a sort of 'reduced form' evolutionary model, with an almost exclusive emphasis upon the learning/diffusion aspects of economic evolution, while repressing the competition/selection features of market interactions.

While the limitations of this reduced form are quite obvious (for example, the 'microeconomics' is bound to be rather poor), on the upside, it still allows predictions on the dynamics of aggregate variables (and first of all growth rates of

[58] That is β=0, δ=0, φ=0, λ=5, π=0.15, ρ=∞, σ_s=σ_ξ=0.1, α=1.

the economy), mapping them into system- and behavioral parameters capturing the conditions of generations and diffusion of knowledge.

In particular, the model is able to study the effects upon the patterns of growth of: a) *technological opportunities* (as captured by both the density of 'islands' and the probability of Poisson jumps to radically new paradigms); b) *cumulativeness of learning* and *path-dependency* (i.e. the increasing return coefficient α, for each island, and the fraction of idiosyncratic knowledge, φ, that agents are able to carry over to newly discovered technologies); c) *locality of learning* (i.e. an indirect inverse proxy for appropriability), captured by the diffusion parameter ρ; and, finally, on the behavioral side, d) the *propensity to explore*, ε.

Note also, that, in principle, the above variables and parameters can find empirical (although inevitably rough) proxies. Therefore, one might not dispair to test the qualitative properties generated by the model against actual data.

As simple as it is, the model is comparable with New Growth ones, with some overlappings and some major differences. It is similar to the former in that it identifies in knowledge diffusion *cum* dynamic increasing returns the primary sources of self-sustained growth. However, it departs from them in a few important respects.

First, knowledge is neither treated as entirely appropriable or a pure externality: rather, its benefits partly accrue to those who embody it and partly leak out as a sort of spill-over.

Second, dynamic increasing returns to learning are, at least to some extent, technology-specific.

Third, diffusion takes time rather than being instantaneous (and indeed is a major source of growth).

Fourth, problems of 'Schumpeterian coordination' always emerge out of microeconomic heterogeneity in both technical knowledge and innovative decisions.

Finally, the radical uncertainty intrinsic in the innovation process involves the possibility that agents make *systematic* mistakes in innovative search and adoption.

Among other properties, our model shows how a decentralized economy with heterogeneous interacting agents, under certain technological and behavioral conditions, can *self-organize* into exponential growth[59], without appealing to the forecasting powers of any far-sighted 'representative agent'. In fact the result is stronger than that, since the economy might require *non-average* (and individually irrational) behaviors in order to achieve such a self-sustained path[60]. Hence the

[59] Cf. Lane (1993), Krugman (1996) and Fagiolo (1997b).

[60] A similar point on non-average behaviors inducing symmetry breaks in the distribution of particular features or performances of a population of agents is in Allen (1988).

permanent dilemma between *exploitation* of what one knows and *exploration* of the unknown (March, 1991) and, consequently, also the crucial collective role of entrepreneurial ‘animal spirits’, even when ill-grounded in the ‘true’ probability distributions of gains and losses stemming from innovative search.

As it stands, the model seems quite well suited to account for some generic properties of knowledge-driven growth. Nevertheless, further developments come easily to mind.

First, one could try to see how this basic story about growth is modified by the introduction also of a ‘Keynesian’ coordination problem affecting interdependent demand generation mechanisms.

Second, one might likewise study the relevance of adding explicit selection processes affecting the frequency in the population (i.e. the size) of different agents which are ‘carriers’ of different technologies.

And, on a methodological side, together with computer simulation, it might not be out of reach to study some analytical properties, at least in some special cases, of the Markovian process plausibly underlying the model presented here.

However, even before all that come, it seems to us that the foregoing work might contribute to the understanding of how endogenous learning processes, with imperfect collective adaptation and heterogeneous agents, drive growth notwithstanding (or rather *because of*) the absence of fantastically rational agents and equilibria fulfilled throughout.

Parameters of the Model

N = Number of agents

ε = Willingness to Explore

π = Probability that a node is an island

λ = Expected Value of Jumps in Innovation

ρ = Globality of Knowledge Diffusion

φ = Path Dependency in Innovation

α = Returns to Scale

β = Transportation Cost (NB. $c_{i,t}=\beta\cdot[Q_{t-1}/N]^{\delta}$)

δ = Transportation Cost (NB. $c_{i,t}=\beta\cdot[Q_{t-1}/N]^{\delta}$)

σ_s = Variance of the distribution of initial productivity coefficients for islands inside L_0

σ_ξ = Variance of the noise in the initial productivity coefficients for islands outside L_0

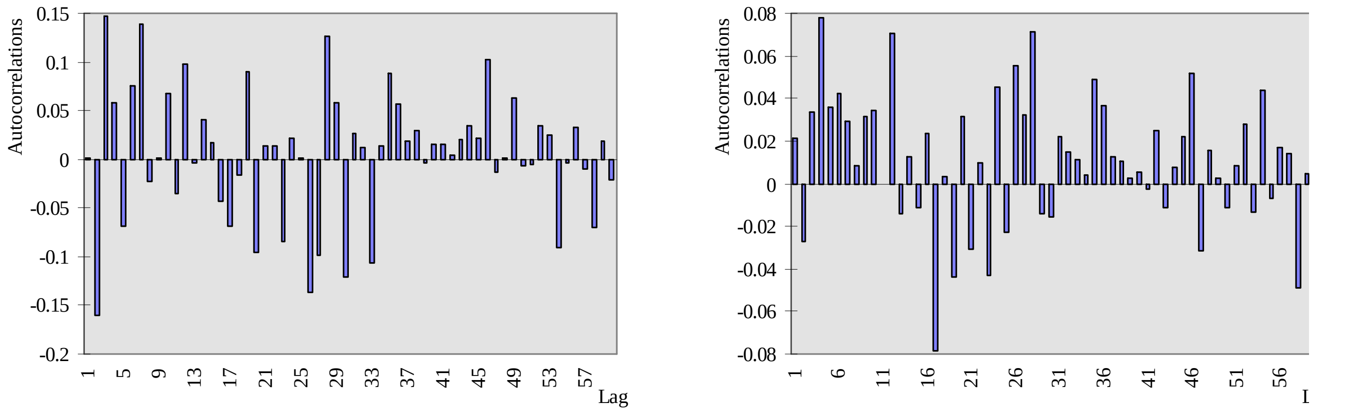
Appendix

Some results on persistence of output fluctuations

Assume that the change in log of GNP follows a stationary process with moving average representation: $\Delta q_t = A(L)\nu_t$, where $A(L)=\sum_{j=0}^{\infty} A_j L^j$, $A_0=1$ and ν_t is white noise. Following Campbell and Mankiw (1987, 1989) and Cochrane (1988), we computed estimates of the following persistence measures: (i) $V \equiv \lim_{k\to\infty} V^k$, where $V^k=[\,1 + 2\sum_{j=1}^{k}(1-\frac{j}{k+1})\rho_j\,]$ and ρ_j is the *j*th autocorrelation coefficient of Δq_t; (ii) $A(1)=\sum_{j=0}^{\infty} A_j$. An estimate of V^k (which consistently estimate V for large k) is found simply by replacing population auto-correlations with sample counterparts, while A(1) must be estimated non-parametrically (for large k) by $\hat{A}^k(1)=\sqrt{\frac{\hat{V}^k}{1-\hat{\rho}_1^2}}$. Since both $\hat{V}^k$ and $\hat{A}^k(1)$ are downward biased, they have been multiplied by the correction factor T/(T–k). For a random walk A(1) and V^k equal one, while for any series stationary around a deterministic trend A(1) is zero and V^k approaches zero for large k. Thus, if both measures are above unity the output exhibits fluctuations with high persistence. Campbell and Mankiw (1987, 1989) and Cochrane (1988) provide Montecarlo studies on 90% critical values of $\hat{V}^k$ and $\hat{A}^k(1)$ for different data generation processes and k=20,40,60.

Sample autocorrelation functions for the change in log of GNP [Δq_t] and for growth rates [$g_t=(q_t - q_{t-1})/q_{t-1}$] are reported in Fig.15 [A61]. In Table 4 both statistics $\hat{V}^k$ and $\hat{A}^k(1)$ are computed for for Δq_t and g_t and k=20, 40, 60. Autocorrelation coefficients are quite small (in particular for Δq_t) but similar to those obtained in reality (see Campbell and Mankiw, 1989). Moreover, all estimates of persistence are greater than unity and quite similar to those obtained for empirical data. Comparing them with the corresponding 90% percentiles, one is able to reject all stationary processes with larger root less or equal to 0.9. In particular, the values of $\hat{V}^k$ for Δq_t fit quite well the case where q_t is generated by an AR(2) process with roots (1, 0.25).

[A61] As done in Table 1, econometric analyses refer to a simulation generated by the following parametrization: N=100, π=0.1, $\sigma_s=\sigma_\xi$=0.1, ρ=0.1, α=1.5, β=0, ε=0.1, λ=1, φ=0.5.



(a) Growth Rates [$g_t=(q_t - q_{t-1})/q_{t-1}$]

(b) Change in log of GNP [Δq_t]

Figure 15: Sample Auto-correlations

Table 4: Estimates of persistence in simulated series of log GNP

k	g_t	Δq_t
	Bias Corrected $\hat{V}^k$	
20	1.35	1.46
40	1.30	1.65
60	1.46	1.89
	Bias Corrected $\hat{A}^k(1)$	
20	1.18	1.22
40	1.17	1.32
60	1.26	1.43

References

Abramovitz, M. (1989), *Thinking about Growth*, Cambridge, Cambridge University Press

Abramovitz, M. (1993), "The search for the sources of growth", *Journal of Economic History*, 53, 2

Aghion, P., Howitt, P. (1990), "A model of growth through creative destruction", National Bureau of Economic Research, Working Paper #3223

Aghion, P., Howitt, P. (1992), "A model of growth through creative destruction", *Econometrica*, 60, 2

Allen, P.M. (1988), "Evolution, innovation and economics", in Dosi, G. et al. (1988)

Allen, P.M., McGlade, J.M. (1986), "Dynamics of discovery and exploitation : the case of the scotian shelf groundfish fisheries", *Canadian Journal of Fishery and Aquatic Sciences,* 43

Amable, B. (1995), "Endogenous growth and cycles through radical incremental innovation", CEPREMAP Discussion Paper, 9504

Arrow, K. (1962a), "Economic welfare and the allocation of resources to invention", in Nelson, R. (Ed.), *The Rate and Direction of Inventive Activity*, National Bureau of Economic Research, Princeton

Arrow, K. (1962b), "The economic implications of learning by doing", *Review of Economic Studies*, 29, 155-173

Arthur, W.B. (1988), "Self-reinforcing mechanisms in economics", in Anderson P.W. et al. (Eds.), *The economy as an evolving complex system*, Addison-Wesley

Arthur, W.B. (1989), "Competing technologies, increasing returns and lock-in by historical small events: the dynamics of allocation under increasing returns to scale", *Economic Journal*, 99

Arthur, W.B. (1994), *Increasing returns and path-dependency in economics*, Ann Arbor, Univ. of Michigan Press

Arthur, W.B., Ermoliev, Yu.M., Kaniovski, Yu.M. (1987), "Path-dependent processes and the emergence of macro-structure", *European Journal of Operational Research*, 30

Atkinson, A., Stigliz, J. (1969), "A new view of technical change", *Economic Journal*, 79, 573-578

Bernd, L. (1994), "Testing for unit-roots with income distribution data", *Empirical Economics*, 19, 4, 555-573

Bikhchandani, S. et al. (1992), “A theory of fads, fashion, custom and cultural change as informational cascade”, *Journal of Political Economy*, 100, 5

Brock, W.A., Durlauf, S.N. (1995), “Discrete choice with social interactions: (I) Theory”, Working Paper, University of Wisconsin at Madison

Campbell, J.Y., Mankiw, N.G. (1987), “Are output fluctuations transitory ?”, *Quarterly Journal of Economics*, 102, 857-880

Campbell, J.Y., Mankiw, N.G. (1989), “International evidences on the persistence of economic fluctuations”, *Journal of Monetary Economics*, 23, 319-333

Cheng, L.K. and Dinopoulos, E. (1991), “Stochastic Schumpeterian economic fluctuations”, mimeo, University of Florida

Cheng, L.K. and Dinopoulos, E. (1992), “Schumpeterian growth and international business cycles”, *American Economic Review Papers and Proceedings*, 82, 2, 409-414

Chiaromonte F., Dosi, G. (1993), “Heterogeneity, competition and macroeconomic dynamics”, *Structural Change and Economic Dynamics*, 4, 39-63

Christiano, L.J., Eichenbaum, M. (1989), “Unit roots in real GNP: do we know and do we care ?”, Natioanl Bureau of Economic Research, Working Paper #3130

Cochrane, J.H. (1988), “How big is the random walk in GNP?”, *Journal of Political Economy*, 96, 893-920

Conlisk, J. (1989), “An aggregate model of technical change”, *Quarterly Journal of Economics*, 104, 787-821

Cooper, R., John, A. (1988), “Coordinating coordination failures in Keynesian models”, *Quarterly Journal of Economics*, 103

David, P.A. (1975), *Technical choice, Innovation and Economic Growth*, Cambridge, Cambridge Univ. Press

David, P.A. (1988), “Path-dependence: putting the past into the future of economics”, Stanford, Institute for Mathematical Studies in Social Sciences, Technical Report 533

David, P.A. (1991), “Computer and dynamo: the modern productivity paradox in a not-too-distant mirror”, in OECD, *Technological and Productivity*, Paris, OECD

David, P.A. (1992), “Path-dependence and predictability in dynamic systems with local externalities: a paradigm for historical economics”, in Foray D., Freeman C. (1992)

Davidson, P. (1996), “Reality and economic theory”, *Journal of Post-Keynesian Economics*, 18, 4

DeJong, D.N., Whiteman, C.H. (1991), “The case for trend-stationarity is stronger than we thought”, *Journal of Applied Econometrics*, 6, 4, 413-421

DeJong, D.N., Whiteman, C.H. (1994), “The forecasting attributes of trend- and difference-stationary representations for macroeconomic time-series”, *Journal of Forecasting*, 13, 279-297

Diebold, F.X., Senhadji, A.S. (1996), “The uncertain unit root in real GNP: Comment”, *American Economic Review*, 86, 5, 1291-1298

Dosi, G. (1982), “Technological paradigms and technological trajectories”, *Research Policy*, 11(2), 147-162

Dosi, G. (1988), “Sources, procedures and microeconomic effects of innovation”, *Journal of Economic Literature*, 26, 126-171

Dosi, G. (1992), “Research on innovation diffusion: an assessment”, in Grubler, A., Nakicenovic, N. (Eds.), *Innovation Diffusion and Social Behaviors*, Heidelberg-Berlin-New York, Springer Verlag

Dosi, G. (1997), “Opportunities, incentives and the collectives patterns of economic change”, *Economic Journal*, forthcoming

Dosi, G., Egidi M. (1991), Substantive and procedural uncertainty, *Journal of Evolutionary Economics*, 1

Dosi, G., Fabiani, S., Aversi, R., Meacci, M. (1994), “The dynamics of international differentiation: a multi-country evolutionary model”, *Industrial and Corporate Change*, 3, 1, 225-242

Dosi, G., Freeman C., Fabiani S. (1994), **“**The process of economic development”, *Industrial and Corporate Change*, 3, 1.

Dosi, G., Freeman, C., Nelson R., Silverberg, G., Soete, L. (Eds.) (1988), *Technical change and economic theory*, London, Pinter

Dosi, G., Lovallo D. (1997), “Rational entrepreneurs or optimistic martyrs ? Some considerations on technological regimes”, in Garud, R., Nayyar P., Shapiro, Z. (Eds), *Foresights and Oversights in Technological Change*, Cambridge University Press

Dosi, G., Orsenigo, L. (1994), “Macrodynamics and microfoundations: an evolutionary perspective”, in Granstrand, O. (Ed.), *The economics of technology*, North Holland, Amsterdam

Durlauf, S.N. (1994), “Path-dependence in aggregate output”, *Industrial and Corporate Change*, 3,1

Fagiolo G. (1997a), “Spatial interactions in dynamic decentralized economies: a review”, European University Institute, mimeo

Fagiolo G. (1997b), “An artificial model of coordination and innovation-driven growth in decentralized economies with heterogeneous interacting agents”, European University Institute, mimeo

Fatas, A. (1995), “Endogenous growth and stochastic trends”, INSEAD, Working Papers 95/85/EPS

Foray, D., Freeman, C. (1992), “Technology and the wealth of nations”, Paris, OECD

Freeman, C. (1982), “The economics of industrial innovation”, London, Pinter

Freeman, C. (1994), “The economics of technical change”, *Cambridge Journal of Economics*, 18, 463-514

Freeman, C., Perez, C. (1988), “Structural crisis of adjustment: business cycles and investment behavior”, in Dosi, G. et al. (1988)

Grandstrand, O. (1994) (Ed.), *The economics of technology*, North Holland, Amsterdam

Grossman, G.M., Helpman, E. (1991a), *Innovation and growth in the global economy*, Cambridge: MIT Press

Grossman, G.M., Helpman, E. (1991b), “Quality ladders and product cycles”, *Review of Economic Studies*, 106, 557-586

Hirshleifer, D., (1993), “The blind leading the blind: social influence, fads and informational cascades”, The J.Anderson Graduate School of Management, UCLA, Working Paper 24-93

Holland, J.H. (1975), *Adaptation in natural and artificial systems*, Ann Arbor, Univ. of Michigan Press

Jones, C.I. (1995a), “R&D-based models of economic growth”, *Journal of Political Economy*, 103, 4, 759-784

Jones, C.I. (1995b), “Time series tests of endogenous growth models”, *Quarterly Journal of Economics*, 110, 495-525

Jones, L.E., Manuelli, R. (1990), “A convex model of equilibrium growth”, *Journal of Political Economy*, 98, 1008-1038

Jones, R., Newman, G. (1994), “Economic growth as a coordination problem”, Working Paper, University of British Columbia, 94/11

Jovanovic, B. (1995), “Learning and growth”, National Bureau of Economic Research, Working Paper #5383

Jovanovic, B., McDonald, G.M. (1994), “Competitive diffusion”, *Journal of Political Economy*, 102, 1

Jovanovic, B., Rob, R. (1989), **“**The growth and diffusion of knowledge”, *Review of Economic Studies*, 56

Jovanovic, B., Rob, R. (1990), **“**Long waves and short waves: growth through intensive and extensive search”, *Econometrica*, 58, 6

Kaldor, N., Mirrlees, J.A. (1962), “A new model of economic growth”, *Review of Economic Studies*, 29, 174-192

Kirman, A.P. (1989), “The intrinsic limits of modern economic theory: the emperor has no clothes”, *Economic Journal*, 99 (Conference 1989), 126-139

Kirman, A.P. (1992), “Whom or what does the representative individual represent ?”, *Journal of Economic Perspectives*, 6, 2.

Kirman, A.P. (1993), “Ants, rationality and recruitment”, *Quarterly Journal of Economics*, 108

Kuran, T. (1988), “The tenacious past: theories of personal and collective conservatism”, *Journal of Economic Behavior and Organization*, 10, 143-171

Lane, D.A. (1993), “Artificial worlds and economics, parts I and II”, *Journal of Evolutionary Economics*, 3 (2 and 3), 89-108 and 177-197

Lesourne, J. (1991), “Economie de l’ordre et du désordre”, Economica, Paris

Levin, R.C., Klerovich A., Nelson R., Winter, S. (1987), “Appropriating the returns from industrial research and development”, *Brookings Paper on Economic Activity*, Special Issue in Microeconomics, 783-820

Levinthal, D.A., March J.G. (1981), “A model of adaptive organizational search”, *Journal of Economic Behavior and Organization*, 2, 307-333

Levitt, B., March J.G. (1988), “Organizational learning”, *Annual Review of Sociology*, 14, 319-340

Lippi, M., Reichlin, L. (1994), “Diffusion of technical change and the decomposition of output into trend and cycle”, *Review of Economic Studies*, 61, 19-30

Lucas, R.E. (1988), “On the mechanics of economic development”, *Journal of Monetary Economics*, 22, 3-42

Lundvall, B.A. (1993), *National systems of innovation*, London, Francis Pinter

Malerba, F., Orsenigo, L. (1993), “Technological regimes and firm behaviors”, *Industrial and Corporate Change*, 2 (1), 54-71

March, J.G. (1991), “Exploration and exploitation in organizational learning”, *Organization Science*, 2, 1

Metcalfe, J.S. (1988), “Trade, technology and evolutionary change”, University of Manchester, Mimeo

Metcalfe, J.S. (1996), *Lectures on evolutionary theory*, Manchester, Manchester University, Mimeo

Nelson, C.R., Plosser, C.I. (1982), “Trends and random walks in macroeconomic time series”, *Journal of Monetary Economics*, 10, 139-162

Nelson, R. (1981), “Research on productivity growth and differences”, *Journal of Economic Literature*, 19, 1029-1064

Nelson, R. (1994), “What has been the matter with Neoclassical Growth Theory ?”, in Silverberg, G., Soete, L. (Eds.), *The economics of growth and technical change*, Edward Elgar

Nelson, R. (1997), “The agenda for growth theory: a different point of view”, forthcoming in *Cambridge Journal of Economics*

Nelson, R. (Ed.) (1993), *National innovation systems: a comparative analysis*, New York, Oxford University Press

Nelson, R., Winter, S. (1977), “Simulation of Schumpeterian competition”, *American Economic Review*, 67, 1, 271-276

Nelson, R., Winter, S. (1982), *An evolutionary theory of economic change*, Belknap Press, Harvard Univ. Press

Orléan, A. (1992), “Contagion des opinions et fonctionnement des marchés financiers”, *Revue Économique*, 4

Parente, S.L. (1994), **“**Technology adoption, learning by doing and economic growth”, *Journal of Economic Theory*, 63, 2

Rebelo, S. (1991), “Long run policy analysis and long run growth”, *Journal of Political Economy*, 99, 500-521

Romer, P.M. (1986), **“**Increasing returns and long-run growth”, *Journal of Political Economy*, 94

Romer, P.M. (1990), **“**Endogenous technological change”, *Journal of Political economy*, 98

Romer, P.M. (1994), “The origins of endogenous growth”, *Journal of Economic Perspectives*, 8, 1, 3-22

Rosenberg, N. (1982), *Inside the black-box*, Cambridge, Cambridge Univ. Press

Rosenberg, N. (1994), “

Rudebusch, G.D. (1993), “The uncertain unit root in real GNP”, *American Economic Review*, 83, 1, 264-272

Scharfstein, D.S., Stein, J.C. (1990), “Herd behaviour and investment”, *American Economic Review*, 80, 3

Schumpeter, J.A. (1934), *The theory of economic development*, Cambridge MA, Harvard Univ. Press

Shackle, G.L.S. (1955), *Uncertainty in economics*, Cambridge, Cambridge Univ. Press

Silverberg, G., Dosi, G., Orsenigo L. (1988), "Innovation, diversity and diffusion: a self-organization model", *Economic Journal*, 98, 1032-1054

Silverberg, G., Lehnert, D. (1993), "Long waves and evolutionary chaos in a simple Schumpeterian model of embodied technical change", *Structural Change and Economic Dynamics*, 4, 9-37

Silverberg, G., Verspagen, B. (1994), "Collective learning, innovation and growth in a boundedly rational, evolutionary world", *Journal of Evolutionary Economics*, 4

Silverberg, G., Verspagen, B. (1995), "Evolutionary theorizing on economic growth", IIASA Working Paper, WP-95-78

Solow, R. (1960), "Investment and technical progress", in Arrow K. et al. (Eds.), *Mathematical methods in social sciences*, Stanford, Stanford Univ. Press

Stiglitz, J.E. (1987), "Learning to learn, localized learning and technological progress", in Dagupta, P., Stoneman, P. (Eds.), *Economic policy and technological performance*, Centre for Economic Policy Research, Cambridge University Press

Stock, J.H., Watson, M.W. (1986), "Does GNP have a unit root", *Economic Letters*, 22, 147-151

Stoneman, P. (Ed.) (1995), *Handbook of the economics of innovation*, Oxford, Basic Blackwell

Winter, S. (1984), "Schumpeterian competition in alternative technological regimes", *Journal of Economic Behavior and Organization*, 5, 287-320

Young, A. (1993), "Substitution and complementarity in endogenous innovation", *Quarterky Journal of Economics*, 108, 3, 775-807

Young, A. (1995), "Growth without scale effects", National Bureau of Economic Research, Working Paper #5211