

MATHEMATICAL MODELLING AND EVALUATION OF  
BIOLOGICAL RESOURCES OF THE OCEAN

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November 1976

WP-76-29

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## Preface

This paper was prepared by Prof. V.I. Belyaev (Head of the Department of Mathematical Modelling at the Institute of Biology of the South Seas, Sevastopol, USSR), during his visit to IIASA in the framework of the tasks "Regional Environmental Policy Design and Management" and "Hydrophysical and Ecological Models of Water Bodies".

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November 1976

## Abstract

The aspects of mathematical modelling use are examined for the evaluation of biological resources of the ocean. Two types of problems are distinguished: the open sea ecosystem modelling and study of pollution acting on the shelf ecosystem productivity. The first of them is the synthesis of a complex system model by joining partial solutions obtained from the analysis of models of its subsystems. The second approach is the construction of a simple model of informative variables. Both of the approaches are outlined in works of the author which are cited.

## INTRODUCTION

When the resources of the biosphere are evaluated, importance is attached to biological resources of the ocean. The intensity of their use is currently quite great and for some species it approaches its tolerable limits.

Nevertheless the point of view is spreading that ocean resources are almost nonlimited and the degree of their use may considerably increase in the future. The contradiction of these conclusions is caused by the inexactitude of evaluation methods of biological resources of the ocean, by the imperfection of corresponding calculation methods and by errors in the original data. The question is cleared only when some population is found to be on the verge of destruction. Of course this way of using ocean resources cannot be called rational. In most cases the rational use of ocean resources must be based on the continuous recovery of all the ecosystem links that were disturbed by its exploitation.

For this it is necessary to understand well the connections between processes that are going on in the ecosystems. Ocean ecosystems are complex systems and studying them is impossible without mathematical modelling. Finally, there is a need, too, for finding engineering solutions for problems of artificial recovery of ecosystems links whose resources have been rapidly exhausted. Here we are dealing with methods of construction and creation of artificial marine ecosystems on the basis of natural ones. The solving of this problem will lead to the creating of an industrial mariculture. All of this requires the engineering methods of projection based on the mathematical modelling of objects under study.

## A PRIORI MODELLING OF MARINE ECOSYSTEMS

At the moment remarkable progress is being made in the field of mathematical modelling of marine ecosystems. In particular, in the USSR a number of methodological questions are being elaborated corresponding both to this problem as a whole and to particular cases having practical interest (1,2,3).

In cases where carrying out simultaneous observation of marine ecosystem parameters is exceptionally complex, the obtaining of its mathematical models based on experimental data processing is very difficult. In cases where one speaks about forecasting the influence of previously-absent anthropogenic factors on ecosystems, this method is not possible at all. For this reason a priori modelling is very important for solving this problem. Before an experimental study of ecosystems is done, it is necessary to answer these questions: what must be measured, where, and with what accuracy?

In the marine ecosystem an interaction between physical, chemical and biological processes takes place. In a priori modelling, the system of equations describing the ecosystem is derived by

joining of equations that have been obtained earlier for separate processes. In so far as the ocean parameters in their experimental and theoretical study are subjected to smoothing, the joining of equations in the system must be made by taking into account the spectral oscillation characteristics of study parameters.

#### THE EQUATIONS FOR PHYSICAL AND CHEMICAL PROCESSES

The circulation of matter in the ecosystem is carried out by currents, turbulence and self-movement of the organisms. That is why the equation describing the currents and turbulence characteristics must be included in the system of marine ecosystem equations. It must be noted that the form of these equations (in other words, the choice of their coefficients) depends essentially on time-space scales of problems. The majority of works on the theory of currents correspond to steady or very smooth "climatic" currents. The frequency oscillation spectra of such currents does not correspond to the ecosystem behavior in the upper sea levels where significant changes occur during a period of twenty-four hours. For example, at night up to half of the phytoplankton is eaten by zooplankton and in the day-time it is restored by dividing of algae. The attempt to develop the method for the calculation of currents taking into account these circumstances was undertaken in Reference (4).

At the moment a considerable number of studies corresponding to the study of turbulent diffusion have been completed. An important fact was the discovery of the vertical ocean structure that may exert significant influence on its ecosystem state. It must be noted that diffusion of suspended particles, the capacity in which we consider organisms and their exchange products, corresponds to general nonconserving substance diffusion. In this case, the parameters of diffusing particles change continuously or discretely and depend on the environment's parameters. The considerations concerning the descriptions of nonconserving substance diffusion are given in Reference (4).

The study of the transportation of heat and salt and the formation of temperature and salinity fields has reached the same level as the study of diffusion in view of the knowledge of transportation characteristics. But in the general case this problem is included in the problem of the formation of hydro-thermodynamical fields. That is why the temperature and salinity fields are found simultaneously with the velocity field.

At present there are many works giving the possibility of describing the ocean light field in sea water by some mathematical expression. However, the greatest part of the ocean light field belongs to the visible and thermal parts of the spectrum that were investigated in connection with the problems of the formation of temperature fields. The part of the spectrum belonging to the FAR (fotosynthetic active radiation) for one or another algae needs to be investigated further. Nevertheless, in this case some a priori choices are possible for expression parameters for the analogy of investigated parts of the spectrum.

The description of chemical processes is fulfilled by equations for the transportation of concentrations corresponding to chemical combinations. The diffusion equations with source terms caused by the chemical and biological processes are taken as the basis.

#### THE EQUATIONS FOR DESCRIPTIONS OF POPULATIONS AND COMMUNITIES

The main parameters characterizing the biologic components of populations are biomass, size, and presence of any harmful or useful substances in species. Harmful substances may include the accumulation in the species of some substances negatively influencing its development and reproduction. Useful substances include, for example, concentration and combination of fats in the species: it became clear that the fats carry the important information about the ecological fate of species and populations. The totality of species parameters form the phase space of its state. Introducing the functions of species parameters distribution, we are then able to describe the populations and communities. In the phase space, the functions described are similar to the density of multi-dimensional liquids "flowing" through the phase space. The distribution function is given by the expression:

$$\int_{G_i} f_i(q_{1i}, q_{2i}, \dots, q_{ni}) dG_i = N_i \quad (1)$$

Here  $G_i$  is the region of phase space of  $q_{ij}$  parameters corresponding to  $i$ -species, where  $f_i$  differs from zero and  $N_i$  is the total sum of  $i$ -species (in the unit of volume).

Multi-dimensional "liquid" flowing through the phase space may be studied as a usual liquid in the Lagrangian presentation. The moments and coordinates of organisms appearing or coming into the system may be taken as examples of Lagrangian variables. This method is especially convenient for studying the spread of organisms in the environmental medium by the scheme of nonconserving substance diffusion (4).

Generally, for a description of the biological ecosystem components it is necessary to know the regularities of the following processes:

- the absorption or secretion by organisms of substances that take part in the circulation,
- the movement of organisms on their own.

The parameters of the species themselves and of their surrounding medium must be included as variables into formulae describing these regularities. Based on experimental studies up to now, the formulae which described most of the above-mentioned regularities are obtained. Some materials concerned with this question are contained in References (5,6).

In particular, if the movement of the organisms is of a regular character, it may be taken into account by the use of functions with the value of 1 in the elements of space where organisms are

present, and with the value of 0, where they are absent. The chaotic movement of the organisms on their own can be considered as a diffusion theory results.

The introduction of distribution function parameters of organisms that characterized the organisms' state and of the function describing their movement in space, allows one to define the system of equations that describes the circulation of matter in the ecosystem and determine its state.

Summarizing the above, it may be said that up to now methods of mathematical description have been developed for practically any type of ecosystem with the accuracy of empirically determined constants or with the form of functions which describe separate processes in the ecosystem. For a qualitative analysis of ecosystem models for the majority of the parameters mentioned, it is however possible to assign approximate values; for functions the approximated expressions are obtained by common sense.

#### THE METHODS FOR ANALYZING MATHEMATICAL MODELS OF MARINE ECOSYSTEMS

After the construction of a total equation system of ecosystem models, the parameter calculations or predictions problems of interest for us may be solved. In a number of cases we take an interest in the quality conclusions about the fate of ecosystems, as noted by their natural development and as a result of possible anthropogenical influences.

As a rule the total equation system describing marine ecosystems is so complicated that it cannot be solved directly. Also if it were possible, information obtained through this method would be too detailed and superfluous for reaching final conclusions of practical use. Therefore in the analysis of the marine ecosystems there are now two approaches:

- the synthesis of the total solution on the basis of the previously found partial solutions for parts of the ecosystem, and
- the finding of simple models that may describe the behavior of parameters which are of interest to us.

Both of these approaches are used in our investigations. The method taken by us of total solution synthesis, is the following. The complex system is divided into subsystems, which during time step  $\Delta t$  may be considered as isolated with fixed meanings of variables of other subsystems. The continuous values of subsystem parameters are replaced by a number of discrete meanings, for example - 1,2,3,4 - weak, average, strong, very strong. Then the set of equations of subsystems is believed to be solved with regard to its output variables, i.e. the expressions for next operators  $Y_{ij}$  are assumed found:

$$\alpha_{ij}(t + \Delta t) = Y_{ij}(\alpha_{1j}(t), \dots, \alpha_{Nj}(t)) \quad (2)$$



where  $\alpha_{ij}$  is the quality meaning of the  $i$ -parameter of the  $j$ -subsystem.

Then we suppose that the deterministic model (2) describes the passage of parameters from  $t$  to  $t + \Delta t$  with some conditional probability

$$P(\alpha_n/\alpha_i) = \int_{x_i}^{x_{i+1}} \frac{dx^*}{x_{i+1} - x_i} \int_{x_n - x^*}^{x_{n+1} - x^*} f(\varepsilon) d\varepsilon \quad (3)$$

Here  $f(\varepsilon)$  is the probability of error corresponding to the real object description given by model (2);  $x_k$  is the meaning of the parameter corresponding to the left boundary of the parameter interval with degree  $\alpha_k$ .

While passing sequentially from one system/one block to the next one it chooses the meaning  $\alpha_n$  every time instead of  $\alpha_i$  with the probability  $P(\alpha_n/\alpha_i)$ . The entire system is passed many times from input parameters to its output ones, i.e. the method of accidental trajectories is used. Then the obtained meanings of output parameters are statistically processed.

Using the above-mentioned approach to the modelling, one supposes that not all subsystem descriptions are exhausted by the deterministic expression and also requires taking into account the probability distribution  $f(\varepsilon)$ .

The experimental study of any marine ecosystem characteristics is based on the possibility of consideration of almost isolated ecosystem parts. From some time from the total number  $N$  ecosystem parameters one takes into account only the part directly connected with the variable being studied. As a result of the experiment the dependence of the following is established:

$$F_i(q_{1i}, q_{2i}, \dots, q_{Ki}) = 0 \quad .$$

Here  $K$  is the number of the most closely connected variables being studied in the experiment. The other variables with indexes from  $K+1$  to  $N$  are not, as a rule, controlled. The function  $f(\varepsilon)$  in this case determines the accuracy of the real process described by model  $F_i$ . This approach was used in Reference (6) for investigation of a pelagic ecosystem of the upper sea level. In (6) the system of differential equations was derived for description of interacting epy- and bathyplankton ecosystems. The migration of the vertical organisms and selectivity of food absorption was taken into account. The ecosystem investigated has a complicated behavior pattern. In the ecosystem model the inverse influence of the biological components on the temperature and density distribution was taken into account. This influence is caused by the dependence of sun radiation absorption on the suspended plankton concentration. Based on this, the oscillations of parameters are possible in the ecosystem. The approach described was recently used to study the pollution acting on the dynamic North Sea exosystem (7). The second of the approaches discussed consists of a description of complex systems by a simple model based on finding model variables which are the most informative with regards to the studied properties

of the system. A method which is based on the approach mentioned, and has great potential is, for example the method of limiting factors. The principles of this approach are given in Reference (8).

The method of reducing this complex problem to a simple one, is examined in the author's work (9) where the "frontal concentration" concept is proposed for the calculation of pollution zone evolution. The problem of calculating the coastal pollution source influence is reduced to two parts: 1) quality analysis of ecosystem model equations with the aim of finding the "frontal concentration" meanings, and; 2) of solving the problem of fixed substance concentration movement.

### CONCLUSIONS

The evaluation of biological resources of the ocean may be carried out most effectively by the use of mathematical modelling. At the same time, the modelling of open sea ecosystems makes possible the solving of two problems:

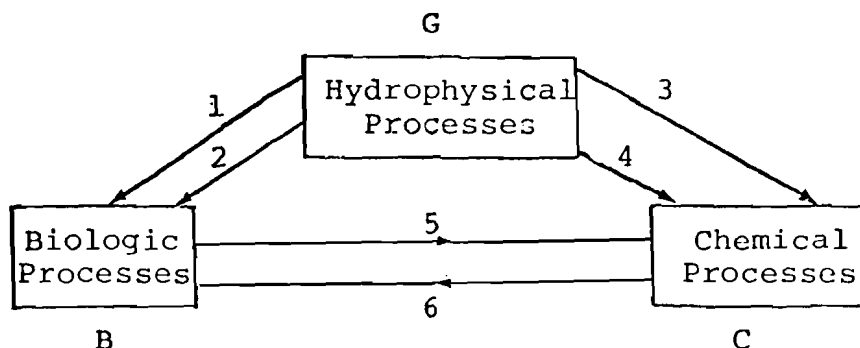
- calculating the meanings of the values for those areas in which data are not available, by using information about other controlled factors.
- establishing a well-grounded bases for processing the primary observed data with the aim of putting the additional data into the single standard.

Finally, the goal of the outer sea ecosystem modelling is the prediction of the ranges of its possible rational use by carrying out some projects where man intervenes and influences the natural ecosystem mechanism.

The second important direction of investigation is the modelling of the shelf ecosystems which have considerable biological productivity. At the same time they are subjected to the most intense pollution effects. To determine potential productivity of these areas, it is necessary to ascertain the boundaries of the zones in which the ecosystems may change through the effect of pollution.

Up to now for all types of problems discussed the solutions were found in a general form. In this paper our works were cited as examples. Other approaches to these problems proposed in the USSR are described in (1,2).

Arbitrarily, any ocean ecosystem may be divided into three subsystems:



In this figure the arrows show the interrelationship between the processes which were taken into account. The comprehensive consideration of all connections is not made here in the analysis of concrete marine ecosystems. For practically solvable problems, the block G is studied without taking into account the connections "2" and "4" although in Reference (6) the tendency of biological processes to influence the hydrophysical ones was noticed. The blocks B and C are studied by taking into account the connections "5" and "6", "1" and "3" although in most cases one of the connections "5" or "6" is examined as a fixed background.

The modelling method described in this paper allows one to combine the results of such separate investigations.

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\* Copies and reprints of these works available in IIASA's library.