

AN EXPERT-BASED FRAMEWORK FOR  
PROBABILISTIC NATIONAL POPULATION  
PROJECTIONS: THE EXAMPLE OF AUSTRIA

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# An Expert-Based Framework for Probabilistic National Population Projections: The Example of Austria

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Lutz, W. and Scherbov, S., 1998, An Expert-Based Framework for Probabilistic National Population Projections: The Example of Australia. *European Journal of Population* 14: 1–17.

**Abstract.** The traditional way of dealing with uncertainty in population projections through high and low variants is unsatisfactory because it remains unclear what range of uncertainty these alternative paths are assumed to cover. But probabilistic approaches have not yet found their way into official population projections. This paper proposes an expert-based probabilistic approach that seems to meet important criteria for successful application to national and international projections: 1) it provides significant advantages to current practice, 2) it presents an evolution of current practice rather than a discontinuity, 3) it is scientifically sound, and 4) it is applicable to all countries.

In a recent *Nature* article (Lutz et al., 1997) this method was applied to 13 world regions. This paper discusses the applicability to national projections by directly taking the alternative assumptions defined by the Austrian Statistical Office. Sensitivity analyses that resolve some methodological questions about the approach are also presented.

Lutz, W. et Scherbov, S., 1998, Une approche sous contrôle d'experts pour des projections probabilistes de populations nationales: l'exemple de l'Autriche. *Revue Européenne de Démographie* 14: 1–17.

**Résumé.** La façon traditionnelle de traiter de l'incertitude dans les projections de population, en introduisant des variantes hautes et basses, est peu satisfaisante, car ces variantes ne couvrent pas un domaine clairement défini. Les approches probabilistes n'ont pas encore trouvé leur voie dans les projections officielles de population. Cet article propose une approche probabiliste basée sur une expertise, qui semble réunir des critères importants pour une bonne application à des projections, tant nationales, qu'internationales: 1) elle fournit des avantages significatifs par rapport aux pratiques courantes; 2) elle présente une évolution de ces pratiques plutôt qu'une discontinuité; 3) elle a des bases scientifiques claires; et 4) elle est applicable à tous les pays.

Dans un article récent de *Nature* (Lutz et al., 1997) cette méthode a été appliquée à 13 régions du monde. Cet article discute de son application à des projections nationales, en prenant directement en compte les hypothèses alternatives faites par l'Office statistique autrichien. Il présente aussi des analyses de sensibilité qui permettent de résoudre certains problèmes méthodologiques liés à cette approche.

## 1. Introduction and approach

One major unresolved issue in population projection is how to deal with uncertainty. This concerns both the handling of uncertainty in the calculations and the presentation of probabilistic results to the users.

There is little doubt among users and producers alike that it is meaningful to produce at least one ‘medium’, or ‘central’, projection that is somehow considered the best projection at the time of production. For many users such a best guess will suffice. They typically take it as an exogenous input into their own models for school planning, social security considerations, energy outlook, etc. These projections may turn out to be wrong due to unforeseen circumstances, but given our knowledge today they reflect the best assumptions we can make. Hence a medium projection should be an indispensable part of any set of published projections.<sup>1</sup>

It has become practice by the UN and many national statistical agencies to publish, in addition to the medium projection, ‘variants’ that are generally based on higher and lower fertility paths. But in the publications themselves or elsewhere in the demographic literature, it is difficult to find exact definitions of what such ‘variants’ actually stand for. Are they just sample paths, or do they demarcate certain ranges? The only thing that is sometimes explicitly stated is that they should *not* be interpreted as giving any sort of confidence intervals in a probabilistic sense. But this is exactly what most users take them to be, and we cannot blame them because an uncertainty distribution is the only logical interpretation of any set of ‘high’, ‘medium’, and ‘low’ lines published. To an informed non-demographer, e.g. a scholar from another scientific discipline, who is unfamiliar with the traditional practice of demographers, an immediate question will be, whether the range given by the variants is assumed to cover 100 per cent, 90 per cent, or any other proportion of all possible future paths. But the demographic producers generally refuse to be precise about their subjective probability distribution, and do not give the user a satisfactory answer to this crucial question. But what should the user do with the variants if he is not told how to interpret them?

In addition to this lack of precision in what one is actually doing, there seem to be two other serious problems with the traditional ‘variants’:

- 1) They are in many cases still based only on variations in fertility assumptions (as spearheaded by the published UN projections) and disregard uncertainty about future mortality and migration trends, which also impact on population size and even more so age distributional aspects such as on the old-age dependency ratio. For such ratios the uncertainty range due to mortality may be more significant than that due to fertility.
- 2) The high/low variants presented for the total world population or groups of countries are based on the assumption that in all countries fertility trends simultaneously follow the maxima/minima defined for each country.<sup>2</sup> This is a very unreasonable assumption. In reality, in some countries fertility will

be above the assumed average, and in others below. For the global total population size these diverging trends will partly cancel out. Because of these compensations the global population size is by orders of magnitude less likely to hit the value given under the 'high' variant than it is in any particular country, no matter what likelihood is assumed for each country.

What are the possible ways around such serious problems? One can go in either of two directions: The first approach is explicitly to call the alternative projections sample paths or scenarios designed to demonstrate the consequences of certain specified conditions. A constant fertility scenario is an example of this, where there is no need to specify a probability because it is only for illustrative purposes. Some of IIASA's (Lutz, 1994) scenarios (such as the 'African Food Crisis Scenario') followed this direction. For the UN it would be an interesting attempt to specify a scenario that would demonstrate the long-term impacts of a successful implementation of the quantitative goals of the 'Cairo Programme of Action' in the fields of health and unmet need for family planning. The current 'low' variant certainly does not reflect such a scenario (since it does not assume extra efforts in health), although one sometimes hears this association.

The other direction is to systematically consider possible deviations from the most likely path for all three components. This can, on the one hand, be done by applying errors from past population projections or making assumptions about variance derived from past time series, or on the other hand, by having experts define *ex ante* probability distributions. While most of the literature on probabilistic population projections so far follows the first approach (Lee and Tuljapurkar, 1994; Lee, 1993; Keilman and Cruijsen, 1992; Lee and Carter, 1992; Alho, 1990; Alho and Spencer, 1985) this contribution chooses to go the other way.

A recent article in *Nature* (Lutz et al., 1997) presents probabilistic world and regional population projections that make use of expert opinion on both the trends in fertility, mortality, and migration, and on the uncertainty range of those trends. Using simulation techniques the authors have derived distributions of population sizes and age structures from those expert judgements. The range of uncertainty was defined in terms of three values (central, low, and high) for each component for a given year (2030) where the area between 'low' and 'high' should cover 90 per cent of all possible cases. Due to limited space, two important methodological issues of this approach could not be discussed in that article:

- 1) It needs to be studied to what degree results depend on the precision of the expert statement about the 90 per cent range; in other words, does it make much difference if the low-high interval is alternatively taken to cover 85 per cent or 95 per cent?
- 2) Caution has been expressed (see Lee, 1998) that the assumption of piece-wise linear random paths, e.g. in total fertility rates, from the starting point to the end point underestimates the variance of the resulting population age distrib-

ution in comparison to a presumably more realistic random path of short-term fluctuations with some degree of autocorrelation.

These two methodological issues will be discussed using Austrian data and will, in our view, receive satisfactory answers that suggest the method for broader applications also in the field of national population projections. More importantly, using the alternative assumptions of the most recent official Austrian population projections produced by the Central Statistical Office, this paper also illustrates how these 'conventional' projections can directly be converted into a probabilistic framework that provides a more meaningful way of stating uncertainty than the traditional way of publishing variants. By directly taking the assumptions as already defined by national experts for the official statistics, and simply assuming a standard normal distribution over those fertility, mortality, and migration assumptions (which happen to be symmetric),<sup>3</sup> we believe that this approach is a more likely candidate for implementation by statistical offices than complex time-series based approaches that require a number of structural and parameter choices which are not intuitively clear, in addition to detailed time series data which are not available for many countries in the world.

## 2. Probabilistic population projections for Austria

In 1996 the Austrian Central Statistical Office published a new population projection to 2050. Assumptions were defined in the usual way by discussing in an inter-agency meeting proposals prepared by the projections unit. Because international migration has been playing a very important role in Austria recently, the committee decided to implement three alternative migration assumptions, namely, annual net migration gains of 10,000, 17,000, and 24,000 to be effective in the first projection year and stay constant during the whole projection period. In contrast to migration, fertility has been very stable in Austria over the past 15 years, with a total fertility rate of between 1.5 and 1.4. At present, it is at 1.4 and is assumed to increase to 1.5 by 2010 under the central assumption. In the high and low cases it is assumed to reach 1.8 and 1.2, respectively.<sup>4</sup> Values up to 2010 are derived by linear interpolation. Beyond that fertility is kept constant.<sup>5</sup> Life expectancy for men is assumed to increase from presently 74 years to 79 years in 2030 under the central assumption (76 and 82 years under low and high). For women over the same period it is assumed to rise from 80 to 85 years in the central, 83 in the low and 87 in the high case.<sup>6</sup>

The combination of the three central assumptions forms the basis for the official medium variant. Results of this projection indicate that after 2001 the balance of births and deaths will turn negative and remain so at an ever-increasing magnitude over the whole projection period. In 2015 the deficit will reach 1.2/1000, in 2030 3.0/1000, and in 2050 8.0/1000, which means an absolute deficit of more than 60,000. It is only due to assumed net migration gains that the total population size

will continue to grow from presently 8.0 million to 8.4 million in 2025. This is projected to be the turning point after which population will start to decline rather rapidly due to the increasing birth deficit that then will outweigh immigration. Throughout the period population ageing will rapidly advance with the proportion above age 60 increasing from presently 19.7 per cent to way above 30 per cent, and the mean age of the population increasing from presently 38.5 years to close to 50 years.

Hence, there is no doubt that Austria will experience very significant population ageing. But the extent and speed of ageing crucially depends on future fertility, mortality, and migration trends. There is significant uncertainty about the future paths of these demographic components, as already described above, through the alternative assumptions made. Instead of discussing here the results of alternative projections combining various assumptions, we will immediately present the results of 1000 simulation runs that randomly combine different fertility, mortality, and migration paths from the above described normal distributions for each component.<sup>7</sup> This is done by drawing a value for the target year of each component and using the linearly interpolated values for the intermediate years (in the same way the original variants were defined), see also Hanika et al., 1997. Each simulation run is therefore based on three random draws, one for each component, an approach that has been labelled the 'random scenario approach' by Lee (1998).

Figure 1 shows selected fractiles of the resulting distribution in total population size. The inner 20 per cent (dark shaded area) follow the path described above for the main variant: an increase to 2025 followed by a decline in total population size. The inner 60 per cent of the resulting distribution still covers a relatively narrow range over the next two decades, but then starts to open up markedly. The corresponding data are given in Table 1. In 2025 the range covers approximately half a million potential Austrians; in 2050 it is already far above 1 million. The 95 per cent interval shows a similar trumpet shape with a difference of around 1 million in 2025 and close to 3 million in 2050. In other words, the specified expert-based model implies that with a probability of 95 per cent the Austrian population in 2025 will lie between 7.8 and 8.9 million, and in 2050 between 6.5 and 9.2 million. The results also show that with a probability of around 60 per cent the population in 2050 will be lower than today, while in roughly 40 per cent of all simulations population size turns out to be greater than today.

To characterise the results with respect to population ageing, in Figure 2 the uncertainty distribution for the old-age dependency ratio is depicted. Unlike with population size above there is no doubt about the direction of change. Even the lower bound of the 95 per cent interval shows significant increases in old-age dependency. The inner 60 per cent range from a doubling of the ratio to an increase by a factor of 2.4 by the year 2050. This is a much smaller range of uncertainty than with population size partly because the migration factor is less important.

The social implications of alternative degrees of population ageing are likely to be significant, at least in terms of financial difficulties of the pension system.

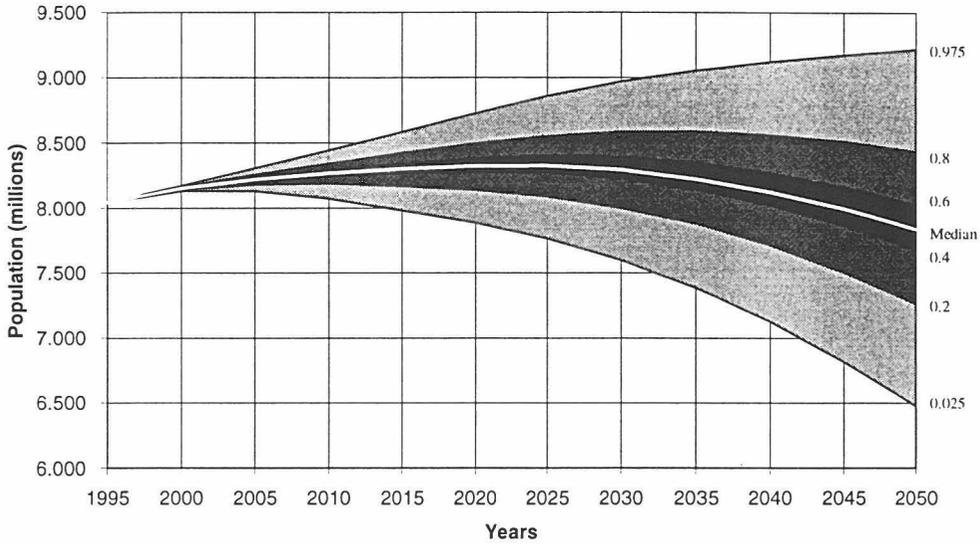


Figure 1. Fractiles of resulting distribution for total population size in Austria 1995 to 2050.

Table 1. Resulting uncertainty distributions for population size and the old-age dependency ratio for Austria 1995 to 2050

Percentiles	1995	2000	2005	2010	2015	2020	2025	2030	2035	2040	2045	2050
<b>A. Total population size</b>												
2.5%	8.040	8.127	8.127	8.069	7.977	7.883	7.765	7.600	7.385	7.126	6.816	6.475
20.0%	8.040	8.141	8.181	8.189	8.166	8.140	8.088	8.000	7.880	7.713	7.506	7.264
40.0%	8.040	8.148	8.209	8.245	8.267	8.279	8.272	8.228	8.141	8.020	7.867	7.682
60.0%	8.040	8.154	8.233	8.291	8.339	8.381	8.404	8.394	8.349	8.271	8.164	8.026
80.0%	8.040	8.161	8.258	8.341	8.424	8.495	8.550	8.584	8.586	8.556	8.509	8.433
97.5%	8.040	8.173	8.306	8.443	8.583	8.726	8.859	8.970	9.055	9.121	9.174	9.219
Mean	8.040	8.151	8.220	8.265	8.299	8.322	8.327	8.300	8.236	8.139	8.013	7.855
Median	8.040	8.151	8.220	8.267	8.301	8.323	8.326	8.295	8.223	8.123	7.996	7.835
St. dev.	0.000	0.011	0.045	0.092	0.149	0.208	0.271	0.340	0.417	0.502	0.591	0.682
<b>B. Old-age dependency ratio</b>												
2.5%	0.315	0.321	0.348	0.360	0.378	0.416	0.483	0.550	0.578	0.569	0.566	0.554
20.0%	0.315	0.321	0.350	0.365	0.386	0.429	0.504	0.583	0.621	0.622	0.631	0.631
40.0%	0.315	0.321	0.351	0.367	0.390	0.436	0.516	0.599	0.642	0.647	0.662	0.671
60.0%	0.315	0.322	0.353	0.370	0.394	0.442	0.524	0.613	0.661	0.672	0.694	0.708
80.0%	0.315	0.322	0.354	0.372	0.398	0.448	0.536	0.632	0.689	0.706	0.736	0.763
97.5%	0.315	0.323	0.356	0.378	0.406	0.462	0.558	0.666	0.738	0.774	0.828	0.883
Mean	0.315	0.322	0.352	0.369	0.392	0.439	0.520	0.607	0.654	0.664	0.684	0.697
Median	0.315	0.322	0.352	0.369	0.392	0.439	0.519	0.606	0.651	0.659	0.677	0.688
St. dev.	0.000	0.001	0.002	0.004	0.007	0.012	0.019	0.030	0.041	0.052	0.066	0.082

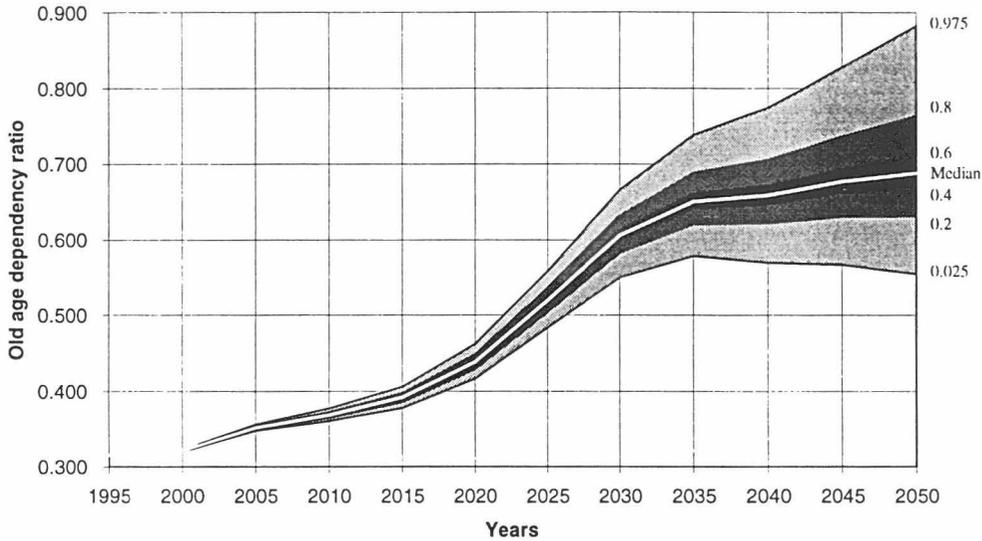


Figure 2. Fractiles of resulting distribution for the old-age dependency ratio in Austria, 1995 to 2050.

Put in a nutshell, the Austrian pay-as-you-go system already has serious coverage problems, and receives one-fourth of its funds from the general budget, although there are still only 31 elderly (above age 60) for 100 adults (15 to 60). The median projections show an increase to 52/100 by 2025 and 69/100 in 2050. In other words, in 2050 there are likely to be 7 elderly for 10 adults in working age. But these demographic dependency ratios are still more favourable than the actual economic dependencies, because young men and women do not start to contribute to the system at age 15 (and education is increasing rather than declining), and a certain proportion of that age group will stay out of the labour force or be unemployed. Hence, the actual ratio of contributors to beneficiaries of the pension system is likely to deteriorate even more strongly, unless there is a radical increase in the mean age of retirement, which in Austria is now as low as 57 years even for men.

Every percentage point in the old-age dependency ratio means that billions of Austrian schillings are available or not available in the Austrian pension system. Hence, for the planning of a reformed pension system, it will make some difference whether in 2040 the ratio is 64 per cent or 67 per cent (the inner 20 per cent of the distribution). It will make a very significant difference whether it is at 57 per cent or 77 per cent (the 95 per cent interval). Seen in another way, these probabilistic projections can help the designers of the new pension system to construct it in a way that it will have a certain probability of not crashing. If the system should be viable with a probability of 80 per cent, then it should be able to handle an old-age dependency ratio of 71/100 in 2040. If politicians feel more confident with a system that will not crash in 97.5 per cent of all cases, they must make it still more efficient to handle even a dependency ratio of 77/100. Or put in still another way: If a ratio of 60/100 (which is about twice the dependency burden of today) is the

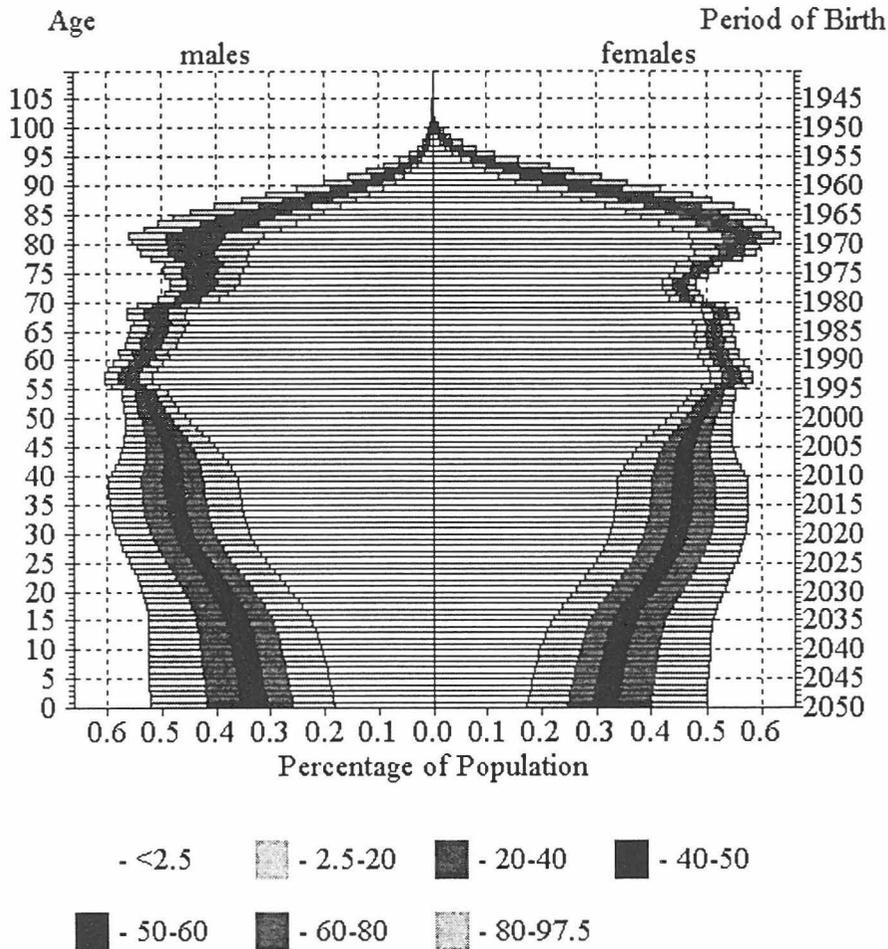


Figure 3. Fractiles of resulting distributions of the Austrian age pyramid in 2050.

point when a given system will crash, we can derive from our model that with a probability of 60 per cent, it will have crashed by 2030 and that there is only a probability of about 5 per cent that it will not crash before 2050.

Figure 3 shows the fractiles of the uncertainty distributions in 2050 for the full age pyramid. It clearly indicates that the distribution is widest at the younger ages due to the uncertainty about future fertility. Especially under age 25, the fertility uncertainty enters twice because we are talking about the children of mothers still to be born. Uncertainty is lowest between ages 55 and 70 in 2050 because these cohorts are already born and have not yet entered the ages of highest mortality. During those higher ages the uncertainty about future life expectancy is clearly reflected in increasing dispersion. For many planning issues related to specific age groups this kind of representation of future uncertainty may be more relevant than that of aggregate dependency ratios.

### 3. Sensitivity analysis

In this section we will address two issues that are potential points of criticism of the above-described method of probabilistic population projections based on expert opinion. The first issue concerns the 90 per cent confidence intervals that are defined by experts in order to specify the magnitude of fertility, mortality, and migration variation. Since some experts may not be able to provide such specific intervals we will test the sensitivity of the results with respect to the alternative assumptions of 85 per cent and 95 per cent intervals.

The second issue deals with the algorithm of scenario generation and the impact of short-term fluctuations. As has been mentioned above, we apply a random scenario approach introduced in Lutz et al. (1996, 1997). Lee (1998), however, conjectures with respect to this approach that it 'could not possibly correctly represent the probability distribution for the age structure (dependency ratios, for example) or for any other measure that depends on the shapes of vital rates trajectories. . . . [T]he evidence is not yet in.' In order to shed more light on this issue, we tried to conduct a systematic analysis of this question by means of simulation since the problem is difficult to solve analytically due to the complexity of the Leslie matrix. Essentially, we compare the results of the random scenario approach with an alternative approach based on adding an auto-regressive random component with a given autocorrelation to the process, as will be described in detail below.

There are several other issues worth testing. The selection of a certain probability distribution (normal, uniform or some asymmetric distribution) also impacts on the results of the simulation. But since earlier sensitivity analysis (Lutz et al., 1996) has shown that the normal and uniform distributions yield very similar results especially in the inner 60 to 80 per cent of the resulting distributions, the major open question is that of asymmetric distributions. Since the assumptions defined by the Austrian Central Statistical Office happened to be symmetric (as are the assumptions for the different world regions in Lutz (1996)) it was decided to leave an in-depth analysis of that issue until we encounter a well justified candidate for a clearly asymmetric distribution. But in principle the method works as well with any specific asymmetric probability distribution that the experts might choose.

Another general issue that will not be discussed here because it relates to any population projection and not just to the proposed probabilistic approach (although the question is more apparent here) is that of a possible correlation between future fertility and mortality. Especially for countries that are in the midst of a demographic transition, there is strong evidence for such a correlation. And as demonstrated in Lutz et al. (1996) projection results look very different in the case of assumed correlation. In a country like Austria that is well advanced in the transition, however, there is little basis for assuming a non-zero correlation between fertility and mortality trends.

Let us now address the first issue, namely the effect of the width of the confidence intervals defined by experts with respect to possible future fertility, mortality,

Table 2. Results of the sensitivity analysis with respect to different assumed uncertainty intervals (Random Scenario Model)

Year	Assumed proportion of all cases between high and low values					
	85%			95%		
	Standard deviation	0.2 Fractile	0.8 Fractile	Standard deviation	0.2 Fractile	0.8 Fractile
A. Total population size						
2010	0.114	8.16	8.36	0.089	8.19	8.35
2030	0.415	7.94	8.67	0.326	8.03	8.60
2050	0.821	7.14	8.58	0.649	7.31	8.42
B. Old-age dependency ratio						
2010	0.005	0.364	0.373	0.004	0.365	0.372
2030	0.038	0.575	0.639	0.028	0.582	0.632
2050	0.102	0.610	0.785	0.075	0.632	0.762

and migration levels. Since the analysis presented above was based on the assumption that the interval between the low and high values covers 90 per cent of all future cases, we chose for the sensitivity analysis the two alternative distributions in which the same intervals are supposed to cover 85 per cent and 95 per cent.<sup>8</sup> As can be expected, the standard deviations of the randomly drawn vital rates are greater in the case of 85 per cent intervals and lower in the case of 95 per cent intervals.

Table 2 shows the results of this sensitivity analysis in terms of two major output parameters, namely, total population size and the old-age dependency ratio. For three selected years the table compares the standard deviations resulting from the three alternative models. Because output parameters can have any kind of distribution and standard deviations do not sufficiently describe these distributions, the table also lists the 0.2 and 0.8 fractiles that encompass the inner 60 per cent of the distribution.

As can be expected for all points in time, the standard deviations are largest and the differences between the fractiles greatest in the case of the assumption that only 85 per cent of all possible cases lie between the high and low values for each component. The 90 per cent interval shows intermediate results, while the 95 per cent assumptions result in the smallest uncertainty range. It is interesting, however, that the difference between the 85 per cent case and the 90 per cent case is generally much larger than that between the 90 per cent and 95 per cent case. This holds with respect to standard deviations and fractiles for total population size and for the old-age dependency ratio. The reason for this lies in the fact that in the case of 85 per cent, there are not only more cases outside the given high-low range, but also the tails of the normal distribution are disproportionately longer.

Also, it has to be kept in mind that the whole process is highly non-linear due to the non-linearity of mortality. As a result the differences between the 90 per cent and 95 per cent assumptions are insignificant by any standard. Formal t-tests on the Null-Hypothesis of equal means show that this hypothesis cannot be rejected at any period. This is even true for the difference between the 90 per cent and the 85 per cent case. In terms of concrete population number, e.g. in 2030, the range of the inner 60 per cent of the uncertainty distribution of population size decreases from 0.73 million in case of the 85 per cent assumption, to 0.58 million for the 90 per cent case, and 0.57 million for the 95 per cent case. In terms of the old-age dependency ratio these ranges are 0.064 (85 per cent), 0.050 (90 per cent), and 0.050 (95 per cent); here the uncertainty ranges even turn out to be identical (to 3 decimal places) for the 90 per cent and 95 per cent cases.

Summing up the results of this sensitivity analysis, one can say that under the conditions of the Austrian demographic regime (which is probably not very different from other European countries), it makes practically no difference whether the defined range between high and low projection assumptions is assumed to cover 90 per cent or 95 per cent of all possible cases. In case of the 85 per cent assumption, the range of uncertainty increases visibly but still not very significantly.

Let us now address the issue of the sensitivity of projection results with respect to different approaches in scenario generation. We introduce an auto-regressive component to our scenario in the following way. Let  $y(t)$  be a function that passes through the mean values of corresponding demographic indicators as defined by experts. Suppose also that  $\sigma^2(T)$  is the variance of the scenario variable defined from the 90 per cent range given by experts for year T. For comparative purposes we created scenario  $z(t)$  in the following way:

$$z(t) = y(t) + x(t)$$

where  $x(t)$  is described by a first order autoregressive process:

$$x(t) = \alpha x(t-1) + \varepsilon(t) \quad \varepsilon(t) \approx N(0, \sigma_\varepsilon^2)$$

For a given autocorrelation  $\alpha$  (in our case, when the scenario was set in 5-year time steps we selected  $\alpha = 0.8$ ) we computed  $\sigma_\varepsilon^2$  in such a way that  $\sigma_{z(T)}^2 = \sigma^2(T)$ , for time point T for which expert data are defined (e.g. 2030), using the following equation:

$$\sigma_{z(T)}^2 = \sigma_\varepsilon^2 (1 - \alpha^{2T}) / (1 - \alpha^2)$$

Table 3 presents the results of this alternative model that includes fluctuation in vital rates at a given autocorrelation of 0.8. This seems to be an appropriate value for 5-year steps (in the case of single-year steps the corresponding values would be in the order of 0.96) as suggested by empirical analysis of past trends in the U.S. (see Lee, 1998). For comparison the results of the standard random scenario approach are given in parentheses.

*Table 3.* Results of the alternative model with random fluctuations (based on 90 per cent, assumed to be between high and low values). Results of the standard random scenario approach (see Table 1) are given in parentheses

Year	Standard deviation	0.2 Fractile	0.8 Fractile
A. Total population size			
2010	0.102 (0.092)	8.18 (8.19)	8.35 (8.34)
2030	0.267 (0.340)	8.07 (8.00)	8.53 (8.58)
2050	0.473 (0.682)	7.44 (7.26)	8.22 (8.43)
B. Old-age dependency ratio			
2010	0.008 (0.004)	0.362 (0.365)	0.376 (0.372)
2030	0.027 (0.030)	0.583 (0.583)	0.629 (0.632)
2050	0.070 (0.082)	0.635 (0.631)	0.752 (0.763)

Because the assumptions, as defined by the Austrian Central Statistical Office, are based on a linear interpolation between the current vital rates and their target values, the implied range of uncertainty for the first projection years is extremely low. The alternatively-defined random process does not have this restriction and therefore produces greater standard deviations for approximately the first 20 years, as seen from the data for 2010 in Tables 1 and 3. This underestimation of near-term variability in the case of linear interpolation has no impact for the longer-term results, but can be embarrassing to the publishing institutions if someone points out 2 or 3 years after publication that the current fertility rate is already outside the high-low range, which at this point is still extremely narrow. An easy fix for this is the definition of a piece-wise linear interpolation that opens up very quickly and then moves linearly towards the target value (as has been done by Lutz et al., 1996).

Beyond 2020 however, the dispersion of the alternative model with random fluctuations is consistently lower than with the standard random scenario model. The standard deviations are clearly lower for both population size and dependency ratio in 2030 and 2050. Looking again at the inner 60 per cent (the distance between the 0.2 and 0.8 fractiles) for population size the 0.58 million under the random scenario model compared to 0.46 million under the alternative model. For the old-age dependency ratio the difference of 0.050 under the random scenario model compares to 0.046 under the alternative model. Hence, it is evident that in the medium to long run the random scenario model presented and recommended here for Austria has a consistently higher variance in the two key output parameters studied than the alternative model that assumes random fluctuations. This can be explained intuitively by the fact that the random scenario model has more persistent

deviations from the mean, whereas the shorter term fluctuations in the alternative model tend to cancel out their effects over time.<sup>9</sup>

In summing up this sensitivity analysis exercise, one can say that the expressed suspicion that the random scenario approach will systematically underestimate the variation of output parameters does not hold in the medium and long term (and even not in the short term, if scenario assumptions are opened up quickly at the beginning). This has been demonstrated here using only Austrian data, but there is no reason to assume that it is not a general property. We can therefore conclude that the random scenario approach is clearly on the safe side in the sense that it does not underestimate variance in either population size or age dependency.

#### **4. Discussion: Can the expert-based probabilistic scenario approach be generally recommended to national statistical institutes?**

When recommending the change of a long-established tradition, the burden of proof tends to be with those suggesting the reform. With respect to population projections this is probably not any different. Generally, it is possible for such efforts to be successful if four criteria are met:

1. The new practice must have clear advantages as compared to the current one.
2. It should be consistent with other work done by the producing institution, and present an evolution along established lines rather than a discontinuity.
3. The proposed approach should be internally consistent and based on accepted scientific work.
4. It should be practical for both the users and producers, and not cost too much.

In the following we will briefly discuss a possible application of the expert-based random scenario approach for official national population projections in light of these four criteria. The same arguments can also be applied to international agencies producing population projections, i.e., primarily the UN Population Division and Eurostat.

##### **4.1. FIRST CRITERION**

The major advantage of probabilistic population projections is that they provide the user with more information. This information about the likely range of uncertainty may not be needed by all users; as discussed above, many may be satisfied with just being given one best guess. But for users who are interested in the question of uncertainty of future trends, a probabilistic projection clearly gives more useful information than the usual high and low variants that do not have a clear interpretation as either a sample path or as giving the bounds of possible trends. As indicated, such more precise information on the degree of uncertainty is particularly relevant for questions for which deviations from the main variant are associated with costs as is the case, for example, in the social security system. It is also relevant to see that

different demographic indicators (such as dependency ratios) have a much more narrow range of uncertainty than others. For these reasons several national statistical institutes have been considering the production of probabilistic projections, but have had difficulties settling on the appropriate methodology to do so.

#### 4.2. SECOND CRITERION

There are essentially three approaches to probabilistic population projections that are proposed in the scientific literature: One based on the time-series analysis of past vital rates; one based on the analysis of past projection errors; and the probabilistic scenario approach based on expert opinion. It is argued here that institutionally, the third approach is the most easy to adopt for statistical offices because it is essentially isomorphic to their current practice. It can utilise the established mechanisms of expert committees that define the alternative assumptions, and it does not require difficult choices associated with the first two methods as to the length of time series on which the assumed future variance should be based, or specific past projections that should be assumed to have the same error as the new projections. These are very difficult questions to find a consensus answer because there are no clear criteria for choice. The random scenario approach, on the other hand, only requires the additional assumption that the values already defined cover approximately a range of 90 to 95 per cent of all future cases. This seems to correspond to the intuition of most experts who say that in the specified range between high and low assumptions, they did not include very unlikely extreme events.

#### 4.3. THIRD CRITERION

Better institutional acceptability of a method does not necessarily imply that it is the better method from a scientific point of view. All three approaches for assuming future variation in demographic trends (time series analysis, past errors, and expert opinion) are based on sound scientific work, have been published in refereed journals, and none could be rejected on the basis of scientific scrutiny. As to the question of internal consistency, the first two approaches typically only derive the future variance from past trends, while for the assumption of the average level (and sometimes even the bounds), they also refer to expert opinion. This seems to be based on the assumption that experts are better in giving an average value than in giving a range of uncertainty. A verification of this assumption is difficult and requires future research that goes deeply into psychology and cognitive science. The extensive literature on Delphi methods (as summarised, for example, in Linstone and Turoff (1975) or Adler and Ziglio (1996)) does not seem to address this question explicitly, and therefore cannot resolve the issue. The third approach presented in this paper is based on expert opinion for both the mean and the range of uncertainty, and may therefore in a way be considered a more consistent source of assumptions. But as long as there is no additional research on this issue, it remains

a matter of taste rather than of scientific arguments. The one condition where the fully expert-based approach is clearly the only viable possibility, is the case of countries without adequate time series data. For a large number of less developed countries, as well as for global-level projections, the fully expert-based approach is therefore the only way to go.

#### 4.4. FOURTH CRITERION

The practical feasibility and cost of publishing probabilistic projections have two aspects: production and dissemination. Once the methodological approach is chosen, the additional production costs are virtually zero. The only thing required is a piece of software that can perform such calculations. Since proponents of the different approaches have functioning programs available (and presumably are willing to share them) this is more a question of communication than of financial resources. As to the presentation and publication of the results, the description of certain fractiles of the distribution of relevant output parameters at different points in time is a viable solution as demonstrated in this paper.<sup>10</sup> Clearly the median or main variant should be described in detail in the same way as it is currently being done. The fractiles could then replace the different variants in the tabulations and graphs. For purely educational purposes and the demonstration of the possible consequences of certain policies, it can sometimes be instructive to describe certain sample paths or specific scenarios such as 'constant fertility' or describe the impact of a certain trend in one component on the total outcome. The full distribution of simulation runs should also be kept at the producing institution, in case of specific questions for which the published fractiles may be too wide.

We can conclude that for national statistical institutes as well as for international agencies producing population projections, the transition from the current practice of variants to expert-based probabilistic projections is more a question of mental and institutional transition than of additional funds required or extensive publication to present the results. It is the view of the authors that the randomized scenario approach presented in this paper for the case of Austria is a good candidate to facilitate this mental and institutional transition because it is a direct extension of current practice.

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## Notes

<sup>1</sup> In this respect the projections produced by Eurostat in 1991, which only present two scenarios without telling the users which one to use, has presumably made a number of users unhappy, since their most recent projections do include a medium projection ('Benchmark Scenario').

<sup>2</sup> The same is often true if national projections are considered to result from aggregating provincial projections.

<sup>3</sup> For lack of convincing alternatives, experts tend to choose symmetric distributions as the simplest case. In case of the assumption of non-symmetry, other probability distributions can be chosen.

<sup>4</sup> Of course, the specific age patterns of fertility, mortality, and migration also present additional sources of uncertainty. But since they tend to be of only secondary importance in European countries, the Austrian projections assume constant age patterns.

<sup>5</sup> This implies the assumption that the anticipated uncertainty range of fertility does not further increase over time, the validity of which can certainly be discussed. While under a time-series approach one might expect further increasing variance, this need not necessarily be the case for expert views on random scenarios.

<sup>6</sup> Actually, life expectancy was the only variable where the committee initially did not define three values but only the central and high values, because a low scenario was not intended for calculation. Hence for the purpose of this probabilistic projection, the low values were assumed as being symmetrical to the high values.

<sup>7</sup> An expert group consisting of demographers from the Austrian Statistical Office and the authors defined the additional assumptions that for fertility and mortality, 90 per cent of all paths should lie between the high and low values, but only 67 per cent for migration due to a higher perceived uncertainty.

<sup>8</sup> Alternatively, this could also be expressed by different intervals still covering 90 per cent.

<sup>9</sup> A new attempt to prove this analytically at least in the asymptotic case is presently under work.

<sup>10</sup> If users are interested in certain specific parameters or indicators that are not published, they should be able to derive them easily in electronic form from the data bank (storing all simulation results) of the institution producing the projection.

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