A System Approach to Management of Catastrophic Risks

Yuri M. Ermoliev[1]
Tatiana Y. Ermolieva[1]
Gordon J. MacDonald[1]
Vladimir I. Norkin[2]
Aniello Amendola[1]

[1]*International Institute for Applied Systems Analysis, Laxenburg, Austria*
[2]*Glushkov's Institute of Cybernetics, Kiev, Ukraine*

RR-00-08
March 2000

Reprinted from *European Journal of Operational Research* 122 (2000) 452–460.

International Institute for Applied Systems Analysis • Schlossplatz 1 • A-2361 Laxenburg • Austria
Tel: (+43 2236) 807 • Fax: (+43 2236) 71313 • E-mail: publications@iiasa.ac.at • Web: www.iiasa.ac.at

---

Reprinted from *European Journal of Operational Research* 122 (2000) 452–460.

ELSEVIER

European Journal of Operational Research 122 (2000) 452–460

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/orms

A system approach to management of catastrophic risks

Yuri M. Ermoliev [a,*], Tatiana Y. Ermolieva [a], Gordon J. MacDonald [a], Vladimir I. Norkin [b], Aniello Amendola [a,1]

[a] *International Institute for Applied Systems Analysis, Schlossplatz 1, A-2361, Laxenburg, Austria*
[b] *Glushkov's Institute of Cybernetics, Kiev, Ukraine*

Received 1 October 1998; accepted 1 April 1999

Abstract

There are two main strategies in dealing with rare and dependent catastrophic risks: the use of risk reduction measures (preparedness programs, land use regulations, etc.) and the use of risk spreading mechanisms, such as insurance and financial markets. These strategies are not separable. The risk reduction measures increase the insurability of risks. On the other hand, the insurance policies on premiums may enforce risk reduction measures. The role of system approaches, models and accompanying decision support systems becomes of critical importance for managing catastrophic risks. The paper discusses some methodological challenges concerning the design of such models and decision support systems.

Keywords: Catastrophe modeling; Insurance; Risk; Stochastic optimization

1. Introduction

The increasing vulnerability of modern society to various "failures", accidents, mismanagement, natural and human-made disasters, is an important characteristic of current socio-economic, technological and environmental global changes. Searching for economic efficiency without paying attention to possible risks often leads to "clustering" of individual property, production processes, installations, buildings and other values. George Dantzig has compared modern society to a busy highway [3], where a disruption in one place may cause wide spread traffic jams. Such events as Hurricane Andrew, the Kobe earthquake, the explosion of chemical tanks in Bhopal, the Chernobyl catastrophe and the ecological disaster of the Rhine after an accidental discharge of toxic chemicals at Basel caused large societal losses. Economic losses from Hurricane Andrew and the Northridge earthquake exceeded $45 billion. The Kobe earthquake (Japan) resulted in around $100

* Corresponding author. Tel.: +43-2236-807-208; fax: +43-2236-71313.

E-mail addresses: ermoliev@iiasa.ac.at (Y.M. Ermoliev), ermol@iiasa.ac.at (T.Y. Ermolieva), macdon@iiasa.ac.at (G.J. MacDonald), norkin@dept130.cyber.kiev.ua (V.I. Norkin), amendola@iiasa.ac.at (A. Amendola).

[1] Visiting from EC-JRC, I-21020, Ispra, Italy.

0377-2217/00/$ - see front matter
PII: S 0 3 7 7 - 2 2 1 7 (9 9) 0 0 2 4 6 - 5

billion in property damage. Global climate and socio-economic changes may dramatically increase the severity and frequency of natural hazards in many regions. The key problem is to find ways to improve resilience and to protect society effectively against the increasing risks.

What role can the insurance industry play in encouraging prevention, preparedness and response measures, and providing financial protection against catastrophic risks without exposing itself to the danger of insolvency? Catastrophes represent new challenges for the insurance theory [1,19]. The most significant of them is the ability to cope with dependencies among catastrophic claims. There exist also dependencies among catastrophic events, for example, weather-related natural catastrophes due to the persistence in climate [14]. We must anticipate that large and more frequent future losses would overwhelm the insurance industry as it currently exists [2,12]. The challenge is to evaluate the role of insurance [13] coupled with other policy instruments, such as regulations, standards and new financial instruments as complements to and substitutes for reinsurance. This task requires a system approach.

From a formal point of view the control of insolvency is equivalent to the prevention of certain multidimensional jumping processes to reach critical thresholds, which is a rather general problem in risk management. To deal with dependent catastrophic losses, a geographically explicit dynamic model was developed in [5–7]. The model incorporates information on property values and their vulnerability, generators of catastrophes, risk reduction and risk spreading decisions, and stochastic optimization procedures. The aim of this paper is to discuss specific components of this model and related decision making problems.

Section 2 illustrates the importance of stochastic dynamic models and the discontinuous nature of insurance processes. Sections 3 and 4 show the nonsmooth and implicit character of decision processes. Possible goals and risk functions are discussed in Section 5, which emphasizes their nonlinearity with respect to probabilities and the nested structure of the resulting stochastic optimization problems. Section 6 discusses a decision making problem involving catastrophe bonds and reinsurance contracts. Section 7 outlines the proposed adaptive Monte Carlo optimization techniques, which can also be viewed as an adaptive scenario analysis. It is pointed out that nonsmooth random goal functions may lead to inconsistencies of deterministic sample mean approximations. Section 8 illustrates numerical experiments. Concluding remarks are given in Section 9.

2. Insurability of standard risks

The concept of risk must play the same role in determining economic activities as profits and costs. This notion emphasizes the variability of outcomes, the possibility of gains and, at the same time, the chances of losses. Such "hit-or-miss" situations often lead to nonsmooth and even discontinuous decision models [9], which challenge traditional approaches. Fig. 1 shows a typical trajectory of the risk reserves of an insurance company [4,11].

Claims arrive at random time moments $\tau_1, \tau_2, \ldots$, of sizes $L_1, L_2, \ldots$ The risk reserve at time t is the difference between accumulated premium $P(t)$, initial capital R_0 and aggregated claim $S(t)$: $R(t) = R_0 + P(t) - S(t),\ t > 0$. The premium income $P(t)$ in $[0, t)$ is calculated as $P(t) = \pi t$. As we can see, the timing of claims and their sizes cause ruin at τ_4. If a claim of the same size arrives at time $\tau_4' > \tau_4$, it would not cause insolvency. Insolvency at τ_4 would not occur either if the premium rate is higher after τ_3.

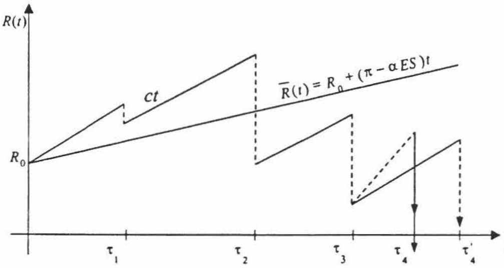

Fig. 1. Stochastic trajectory of the risk reserve.

The insurability of a risk is concerned with the choice of premium rate π, mitigation measures and other decision variables, in a way that the chance of the insolvency drops below an acceptable level and, at the same time, the insurance becomes attractive. In Section 5 we formalize these requirements.

The insurability of standard "frequent-low consequence" risks, such as car accidents, is derived under a set of idealized conditions. These conditions should include a large number of independent exposures. From the law of large numbers it then follows that $\lim_{t\to\infty}[P(t) - S(t)]/t = \pi - \alpha EC$ with probability 1. In other words, the observable profit approaches the expected profit $(\pi - \alpha EC)t$, where α is the intensity of claims and EC is their expected value. Therefore, in the case of positive expected profit, $\pi - \alpha EC > 0$, the expected risk reserves $\overline{R}(t) = R_0 + (\pi - \alpha EC)t$ increase linearly in t, as is shown in Fig. 1. This is a basic actuarial principle: premiums are calculated from the mean value of aggregate claims $S(t)$ increased by a safety loading $\lambda > 0$, $\pi = (1+\lambda)\alpha EC$. As we can see from Fig. 1, although $\overline{R}(t)$ increases, insolvency may occur. It depends on the existence of large claims, which means that λ must be chosen properly. In estimating EC for frequent risks we can look back on large historical databases of past experience. It is also possible to use "trial-and-error" mechanisms for learning about the required λ and its adaptation to changing conditions.

The assessment of insurability for catastrophic risks requires new approaches. The estimation of EC becomes an extremely complicated task in the case of rare catastrophic events with relatively small historical data. The law of large numbers does not operate, since catastrophes produce highly dependent losses and claims. "Learning-by-doing" approaches may be very expensive, dangerous and even simply impossible. Instead, the role of catastrophe modeling [12] and stochastic optimization techniques [8,20] becomes essential for making decisions on premiums, mitigations, etc. Dependent catastrophic losses and claims at different geographical locations and their dependencies on policy variables can be simulated on a computer. Stochastic optimization makes it possible to adjust decision variables to generated catastrophic events and available historical data.

Remark. The deterministic linear function $\overline{R}(t)$ is a very rough approximation of random jumping process $R(t)$. It illustrates the disadvantages of deterministic models, even if complemented by an uncertainty analysis, versus models with explicit treatment of uncertainties. The uncertainty analysis of $\overline{R}(t)$ may indicate only an array of linear functions that do not approach insolvency even with insignificant safety loading, whereas the random process $R(t)$ may often encounter insolvency.

3. Catastrophe modeling

To deal with catastrophic risks from natural, technological and environmental hazards one should characterize patterns of possible disasters, their geographical location, and timing. One should also design a map of regional properties, characteristics of structures, available and implemented mitigation measures, spread of current and possible new coverage, availability of catastrophe securities, etc.

Advances in computers and mathematical modeling then make it possible to simulate a variety of different scenarios of catastrophes using data from historical evidence, scientific facts and experts. Scenarios can then be used to evaluate losses, confidence intervals or histograms of marginal loss distributions for each company and any fixed combination of decisions.

Such straightforward catastrophe modeling [12] facilitates final decision making on a company's solvency, reinsurance requirements, safety loading in premiums, and the effects of mitigation measures, and helps us to understand the fluctuations in space and time of catastrophic risks. Unfortunately, the dependencies of outcomes on decision variables restrict the use of straightforward approaches. They easily run into endless "if–then" analyses without providing a clue to the choice of an optimal and robust solution against possible threats. Simulating rare events to obtain a consistent estimate of their impacts is time consuming. The dynamic aspects of interactions among timing of events, sizes of claims and decisions are crucially important: a 300 year earthquake may happen in two years. All these require fast Monte Carlo simulation and specific analytical analysis of the underlying stochastic processes and decision making tools.

4. Decision variables

In the case of frequent-low consequence risks, the law of large numbers provides a simple [5] "more-risks-are-better" portfolio selection strategy: if the number of independent risks in the portfolio is larger, then the variance of aggregate claims is lower and lower premiums can be chosen. This increases the demand for insurance, the coverage of losses, and, hence, the profits of insurers.

In the case of catastrophic risks the law of large numbers does not operate and the simple more-risks-are-better strategy may increase the probability of ruin for many insurers; for example, if selected risks are positively correlated. To avoid ruin, insurers must deliberately select coverages from different locations with appropriate premiums and support these strategies by investments in catastrophe securities for different "layers" of losses, contingent on different events. This can be modeled by the introduction of appropriate decision variables. It is important to note that increasing number of dependent catastrophic risks may require higher premiums, in contrast to conventional independent risks. The insurance may also encourage individuals to adopt mitigation measures by premiums reflecting the consequent decrease in expected losses. New financial instruments in the form of catastrophe future contracts, call option spreads, or catastrophe bonds (see [10]) assist insurers to spread their risks worldwide. State-mandated insurance pools and governmental catastrophe reinsurance contracts might also provide stability for financing large losses. Catastrophic events affect the whole insurance system through various channels of its business. An insurer cannot evaluate desirable decision variables independent of other participants: insurers, governments and investors. Insurers may deliberately diversify their portfolios; for example, by spreading exposures among themselves (spatial cross-section diversification, swaps), and by promoting mitigation measures (inter-temporal diversification). All these lead to rather rich sets of decisions. Even simple cases illustrate the complexity of arising decision making problems.

Let us assume that a region where insurers operate is subdivided into locations i, $i = \overline{1,N}$. For the simplicity of notations we assume that there is only one insurer. The claim size depends on coverage of the insurer in different locations and patterns of catastrophes. Let us denote by L_i the random losses from possible catastrophes at location i, and $x_i, 0 \leqslant x_i \leqslant 1$, the fraction of L_i covered by an insurance contract. The claim size at time t is defined as $C(x,t) = \sum_i x_i L_i(t)$, where $L_i(t)$ is a random realization of L_i from a catastrophe at time t. Important decision variables are R_0, π and the reinsurer's arrangements; for example, the "excess of loss" reinsurance contract. In this case the insurer retains only a portion, $\min\{y, \sum_i x_i L_i\}$, $y \geqslant 0$, of the claim and passes the remaining portion to the reinsurer. Here y is a decision variable. The reinsurer's contract to protect a "layer" of losses is defined by two decision variables $y = (y_1, y_2)$. In this case the insurer retains $\min\{\max[0, C - y_1],\ y_2 - y_1\}$, $C = \sum_i x_i L_i$, $y_1 \geqslant 0$, $y_2 \geqslant 0$.

A catastrophe bond has the same structure as reinsurance contracts. The bond value is contingent upon catastrophe losses L during the exposure period $[0, T]$. The random pay-out of the investor is zero if losses are less then "trigger" y_1. If losses are in the range between y_1 and y_2, the investor pays $L - y_1$. If losses exceed y_2, the investor pays $y_2 - y_1$. Thus, the investor's payments are defined as $\min\{\max[0, L - y_1],\ y_2 - y_1\}$. Investment decisions, lending, borrowing and swaps modify the risk process $R(t)$ as $R(t) = R_0 + P(t) - S(t) + I(t) - O(t)$, where $I(t)$, $O(t)$ are incomes and outcomes associated with these decisions. The use of mitigation measures "reshapes" the distribution of losses L_i. Thus, in general, $R(t)$ is a complex stochastic jumping process, which depends on various decision variables reshaping its probability distribution. The analysis of cooperation among insurers, governments and other "actors" (investors, reinsurers), for instance, for the feasibility of insurance pools and swaps, requires more state and decision variables (see [5]). In this case $R(t)$ becomes a random vector with mutually dependent components.

The number of possible combinations of decision variables and patterns of catastrophic events exponentially approaches infinity, and straightforward if–then analysis would generate an extremely high number of alternatives. Thus, with only 10 feasible decisions, say the level of coverage for a particular location, and 10 possible scenarios, the number of if–then combinations is 10^{10}. At one second per evaluation, more than 10^2 years are required to carry out the computation. This number would further dramatically increase, if we take into account the spatial dynamic aspect and the continuous nature of various decision variables.

The main idea in dealing with this problem (Section 7) is to avoid an exact evaluation of all possible alternatives and concentrate attention on the most promising directions. From a formal point of view this is equivalent to the design of special search techniques (in the space of decision variables), making use of random outcomes from Monte Carlo simulations of catastrophes. This is the main task of stochastic optimization [8,17,20]. Certain of these search procedures can also be viewed as adaptive Monte Carlo optimization techniques or adaptive scenario analysis. They generate feedback to policy variables and automatically drive them towards desirable combinations without going into exhausting if–then analyses.

5. Goal and risk functions

A sequence of random catastrophes affects different locations $i = 1, \dots, N$ and generates dependent catastrophe losses $L_i(t)$ at different time intervals $t \geqslant 0$. Without insurance and risk reduction decisions, location i faces losses $L_i(t)$. These losses are reduced or compensated after implementing the appropriate decisions. If we denote the vector of decisions by x, then $L_i(t)$ becomes a function $L_i(x,t)$ of x. As we can see from Section 4, this function may have a rather complex nonsmooth structure defined by min, max operators. Hence the corresponding claim process $S(t)$ become a rather general nonsmooth function of x, t. There may be different goals such as expected profits of insurers, risks of their insolvency and losses of individuals. The linearity of the goal functions in underlying probabilities, for example, the standard expected utility, underestimates the impacts of catastrophes. To deal with rare catastrophic events, nonlinear goal functions are required.

Consider the insurer who maximizes expected "wealth" $R(t)$, taking into account the risk of overestimating profits and the risk of insolvency (ruin). Decisions x can be chosen from maximization of the expected value

$$F(x) = E[R(\tau) + \gamma \min\{0, R(\tau) - ER(\tau)\} + \delta \min\{0, R(\tau)\}], \quad (1)$$

where γ, δ are substitution coefficients between the expected wealth $E[R(\tau)]$, the risk of overestimating profits $E[\min\{0, R(\tau) - ER(\tau)\}]$ and the insolvency $E[\min\{0, R(\tau)\}]$; τ is a "stopping" time, for instance, the time of the first catastrophe or the time of insolvency.

The maximization of $F(x)$ requires special techniques. This is a stochastic optimization problem with a number of nonstandard features. First of all, the term $F_1(x) = ER(\tau)$ may be a rather complex nonsmooth function, since losses $L_i(x,t)$ are often defined by min, max operations (Section 4). The essential complexity is associated with the stopping time τ, which may be an implicit random function of x. The risk function $F_2(x) = E[\min\{0, R(\tau) - ER(\tau)\}]$ is nonlinear in the probability measure. This function and $F_3(x) = E[\min\{0, R(\tau)\}]$ create new nonsmooth features of $F(x)$ due to min operations under the expectation. In contrast to the standard stochastic optimization model [8,20], random function $\min\{0, R(\tau) - ER(\tau)\}$ is not directly observable, since it depends on the explicitly unknown expectation $F_1(x) = ER(\tau)$.

Thus, the function $F(x)$ belongs to the family of expectations with the following general structure:

$$F(x) = Ef(x, Eg(x, \omega), x, \omega), \quad (2)$$

where f, g are functions (possibly vector functions), and ω is a random variable. The maximization of such functions requires special stochastic optimization techniques (see [5]). The nonsmooth risk functions F_2, F_3 in (1) correspond to the Markovitz mean–semivariance model [15]. In [16] it was shown that the use of absolute deviations with appropriate choice of risk coefficients is consistent with the stochastic dominance of random outcomes. The applicability of the well-known mean–variance model [15] is usually linked with the normality of the probability distribution, which cannot be assumed for catastrophic risks. The importance of nonsmooth risk functions $F_2(x), F_3(x)$ follows from the following: if risk coefficient δ becomes large enough, then the probability of ruin drops below a given level [5].

6. A catastrophe bond versus insurance

Let us consider an important risk management situation, that illustrates the decision making problems discussed above. Catastrophe securities and bonds have been introduced to assist the insurance industry in spreading risks worldwide. The following shows that catastrophe bonds may be more attractive than similar reinsurance contracts.

Assume that a "client" (insurer, government, firm, etc.) decides to protect a "layer," defined by decision variables (y_1, y_2), of possible catastrophe losses L, either by the bond or similar reinsurance (insurance) contract (see Section 4). The catastrophe may occur at a random time τ. The wealth of the reinsurer at time t is characterized by a random variable W_t, which may assume negative (insolvency) values. The contract changes W_t to $W_t + \pi(1+r)^t - C_t$, where C_t is a random realization of claim $C = \min\{\max[0, L - y_1], y_2 - y_1\}$ at time t, π is a premium to be defined and r is the risk-free rate of returns. The reinsurer and the client are concerned with the risk of overestimating profits $\pi(1+r)^\tau - C_\tau < 0$ for the reinsurance; the risks of underestimating costs $\pi(1+r)^\tau - C_\tau > 0$ for the client, and the risk of insolvency $W_\tau + \pi(1+r)^\tau - C_\tau$, or the credit risk; that is, when the reinsurer is not able to pay the claims. Thus, in a certain sense, for given (y_1, y_2) a "fair" π must "equate" risks of overestimating profits and underestimating costs; in other terms $\pi(1+r)^\tau - C_\tau = 0$ in some probabilistic sense. The credit risk requires an additional "safety loading" of π, which must be chosen by taking into account dependencies between W_τ and C_τ.

A bond with the same random claim C is issued at time 0 with face value $W > y_2 - y_1$ and maturity τ. The wealth of the investor at time τ is $W(1+r)^\tau + \pi(1+r)^\tau - C_\tau$, where π is the premium. The investor and the client are concerned with the risk of overestimating profits (for the investor), $\pi(1+r)^\tau - C_\tau < 0$, and the risk of underestimating costs (for the client), $\pi(1+r)^\tau - C_\tau > 0$. The credit risk does not exist in this case. Thus, for given (y_1, y_2) a fair premium π must be chosen on the same principle, $\pi(1+r)^\tau - C_\tau = 0$, without the safety loading of π, which means that the price of the catastrophe bond in contrast to the same reinsurance contract will be lower. The notion of fairπ can be defined differently for different cases, for example, as the value minimizing $E|\pi(1+r)^\tau - C_\tau|$.

7. Adaptive Monte Carlo optimization

The search for Pareto efficient decisions is achieved through the maximization of weighted sums of different goal functions, such as (2). A principal challenge is that $F(x)$ in (2) is an analytically intractable function. A Monte Carlo simulation of a catastrophe ω produces only random outcomes $g(x, \omega)$ for a given set of decision variables x. Random $g(x, \omega)$ estimates $Eg(x, \omega)$. Unfortunately, $f(x, g(x, \omega), x, \omega)$ cannot be used as an estimate of $E(x) = Ef(x, Eg(x, \omega), \omega)$. Papers [5–7] deal with the design of adaptive Monte Carlo optimization techniques (adaptive scenario analysis), that make it possible to find solutions without time consuming evaluations of $Eg(x, \omega)$. Fig. 2 illustrates the general idea of these techniques.

The process starts with an initial solution x_0. A catastrophe is generated by the catastrophe generator.

The Monte Carlo simulation model produces random outcomes, which the adaptive adjustment procedure then uses to modify some parameters of

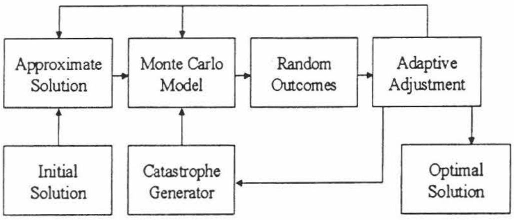

Fig. 2. Adaptive scenario analysis.

the Monte Carlo model, such as estimates of $Eg(x, \omega)$ for a current vector of x, and to update the current approximate solution and, possibly, the parameters of the catastrophe generator, for example, the probability measure, if this depends on some decision variables as it was discussed in [18].

Fig. 3 illustrates the traditional if–then scenario analysis. For each scenario ("if") the model produces the optimal solution ("then"). Different scenarios produce different solutions without indicating optimal and robust final solutions.

The adaptive adjustment is a core module of the adaptive scenario analysis. Random outcomes of the Monte Carlo model depend on simulated catastrophic events and current values of decision variables x. A change of x affects the probabilistic characteristics of outcomes. From these outcomes the adaptive adjustment block estimates a direction of improvement for goal functions and correspondingly adjusts current decision variables (solutions). Special attention is paid to the nonsmooth character of goal functions (2). These functions F have the form of multidimensional integrals or expectations of the nested random goal functions f. Nonsmooth random goal functions f, g may restrict the interchange of the differentiation and integration operation; that is, the direct estimation of gradients and subgradients from observations of functions f, g may not lead to consistent estimates of gradient F_x. The lack of continuous derivatives of the expectation $F(x)$ itself further complicates the problem.

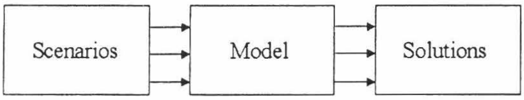

Fig. 3. Standard scenario analysis.

We note that the use of straightforward random search procedures, such as the genetic algorithm, is restricted by the dimensionality and probabilistic nature of the risk function. The probability that purely random search procedures would "hit" even such a set as the positive orthant of n-dimensional Euclidean space is 2^{-n}.

There is an important alternative approach [8,20] to the outlined adaptive Monte Carlo optimization. It proceeds by simulating of a finite number of catastrophic events, which are then used to approximate expectations Eg and $F(x)$ by their sample mean values. The resulting deterministic problem can then be solved by deterministic optimization techniques. This approach requires a well-defined analytical structure of random goal functions f, g that may be problematic for general problems, in particular those with stopping time τ (implicitly) dependent on decision variables. Besides, nonsmooth functions f, g may also cause inconsistencies in using gradients of the sample mean approximations, which may result in solutions having nothing in common with the solutions of the original problem (see discussion in [9]).

8. Numerical experiments

Fig. 4 shows a "landscape" of damaged property values (houses, lands, factories, etc.) for

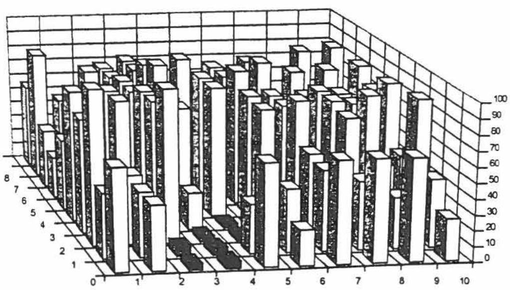

Fig. 4. Landscape of damaged values.

different locations of a region affected by an earthquake (dark part of the landscape).

In general, a catastrophic event, for instance, an earthquake, hurricane, or flood, is simulated as a random field. The distribution of losses at given locations depends on the nature of the catastrophe, the characteristics of the soil, the vulnerability of structures, etc. Trajectories of this field in a particular case may be random lines or trajectories of an asymmetric random walk, of random length and "strength". In our example five insurers operate in a region with 900 locations. Fig. 5 shows the histogram of initial risk reserves for insurer 1, where the horizontal axis shows values $R(\tau)$ and the vertical axis indicates the number of scenarios with values $R(\tau)$ in predefined intervals around 0, -20, 2, and so on. Fig. 6 illustrates improved risk reserves of the same insurer as the result of

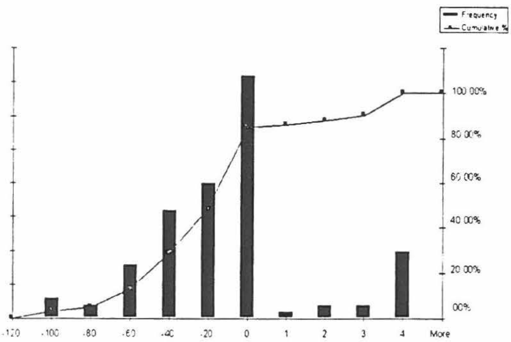

Fig. 5. Histogram of initial risk reserve: Insurer 1.

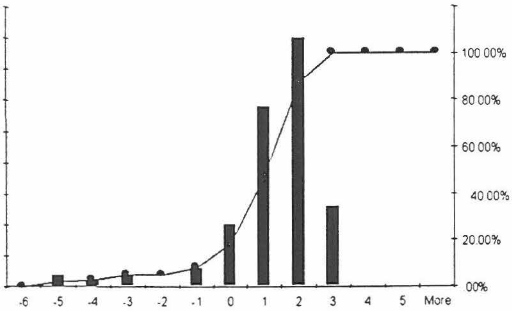

Fig. 6. Histogram of improved risk reserve: Insurer 1.

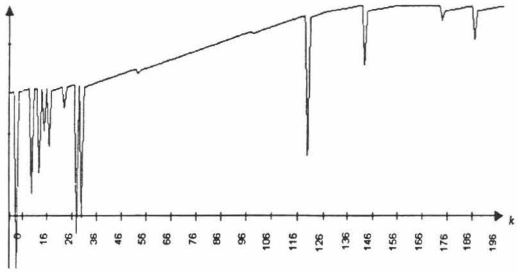

Fig. 7. Improvements of the goal function.

adaptive scenario analysis. As we can see, the possibility of insolvency is reduced from the deficit of risk reserves -120 to -6, with considerably lower frequencies.

A fragment of improvement of a goal function similar to (1) and (2) is shown in Fig. 7. The vertical axis shows $F^K = 1/K \sum_{k=1}^{K} f^k$, where f^k is a specially designed (see [5]) statistical estimate of the goal function $F(x^k)$. It is interesting to note that the value of the goal function is very soon stabilized. But catastrophes still affect insurers, and the last steps further mitigate the influence of rare catastrophes by making adjustments (e.g., redistributing coverages between insurers) towards more robust policies.

9. Concluding remarks

Numerical experiments with different problems show the feasibility of the approaches outlined. The design of optimal risk management decisions can be based on a simulated "history" of catastrophes. Predicting catastrophes from limited historical data is often difficult or simply impossible. The optimization in the presence of uncertainty is, in a sense, a more robust task than such as the prediction: it is much easier to evaluate which one out of the two parcels is heavier than to measure their exact weights. The advantage of the adaptive Monte Carlo optimization methods outlined here stems from the lack of tractable analytical structure of the objective function, which often excludes alternative approaches. The experiments show that the computer time required to search the optimal value has the same order of magnitude as the time required to estimate the value of the objective function at a given initial decision.

References

[1] K.J. Arrow, The theory of risk-bearing: Small and great risks, Journal of Risk and Uncertainty 12 (1996) 103–111.
[2] J.D. Cummins, N. Doherty, Can insurers pay for the "Big One"? Measuring capacity of an insurance market to respond to catastrophic losses, Working Paper, Wharton Risk Management and Decision Processes Center, University of Pennsylvania, Philadelphia, PA, 1996.
[3] G.B. Dantzig, The Role of Models in Determining Policy for Transition to a More Resilient Technological Society, IIASA Distinguished Lecture Series, vol. 1, 1979.
[4] C. Daykin, T. Pentikainen, M. Pesonen, Practical Risk Theory for Actuaries, Monographs on Statistics and Applied Probability, vol. 53, Chapman & Hall, London, 1994.
[5] Y.M. Ermoliev, T.Y. Ermolieva, G.J. MacDonald, V.I. Norkin, On the design of catastrophic risk portfolios, IIASA Interim Report IR-98-056, 1998.
[6] T. Ermolieva, Y. Ermoliev, V. Norkin, Spatial stochastic model for optimization capacity of insurance networks under dependent catastrophic risks: Numerical experiments, IIASA Interim Report IR-97-028, 1997.
[7] T. Ermolieva, The design of optimal insurance decisions in the presence of catastrophic risks, IIASA Interim Report IR-97-068, 1997.
[8] Y. Ermoliev, R. Wets (Eds.), Numerical Techniques of Stochastic Optimization. Computational Mathematics, Springer, Berlin, 1988.
[9] Y. Ermoliev, V. Norkin, On nonsmooth and discontinuous problems of stochastic systems optimization, European Journal of Operations Research, 1997.
[10] K.A. Froot, The Limited Financing of Catastrophe Risk: An Overview, Harvard Business School and National Bureau of Economic Research, Cambridge, MA, 1997.
[11] J. Grandell, Aspects of Risk Theory, Springer Series in Statistics, Springer, Berlin, 1992.
[12] Insurance Service Office, The Impact of Catastrophes on Property Insurance, New York, 1994.
[13] H. Kunrenther, Mitigation Losses and Providing Protection Against Catastrophic Risks, The Role of Insurance and Other Policy Instruments, Working Paper, University of Pennsylvania, Philadelphia, PA, 1996.
[14] G.J. MacDonald, Persistence in Climate, JSR-91-340, The MITRE Cooperation, McLean, VA, 22102-3481.
[15] H.M. Markowitz, Mean Variance Analysis in Portfolio Choice and Capital Markets, Blackwell, Oxford, 1987.
[16] W. Ogryczaki, A. Ruszczynski, From stochastic dominance to mean-risk models: Semideviations as risk measures, IIASA Interim Report 97-027, 1997.
[17] G. Pflug, Optimization of Stochastic Models, Kluwer Academic Publishers, Dordrecht, 1996.
[18] E.L. Pugh, A gradient technique of adaptive Monte Carlo, SIAM Review 8 (3) (1966).
[19] Swiss Re, Sigma, Natural Catastrophes and Major Losses in 1996, (3) 1997.
[20] R.J.-B. Wets, Challenges in stochastic programming, Mathematical Programming 75 (1996) 115–135.