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# OPTIMIZATION MODELS OF TRANSPORTATION NETWORK IMPROVEMENT: REVIEW AND FUTURE PROPSECTS

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## Preface

This paper is the second in a series on "Regional Development and Land-Use Models". The purpose of this series is to consider the application of optimizing and behavioural land-use models as tools in the study of regional development. The present paper considers the problem of designing an optimal regional transportation network taking into account the spatial land-use pattern. The paper reviews current models and identifies weaknesses in them. In the context of this series, the main value of this paper, as a complement to (1), is in identifying models which might be combined into integrated land-use and transportation design models.

J.R. Miron
March, 1976

## Papers in the Regional Development and Land-Use Models Series

1. John R. Miron, "Regional Development and Land-Use Models: An Overview of Optimization Methodology", RM-76-27, April, 1976.

2. Ross D. MacKinnon, "Optimization Models of Transportation Network Improvement: Review and Future Prospects", RM-76-28, April, 1976.

Optimization Models of Transportation Network Improvements: Review and Future Prospects

(Abstract)

The paper briefly reviews the alternative approaches to spatial improvements in transportation networks from the early linear programming attempts to the more recent discrete programming approaches; the more analytical geometrical and optimal control methods; narrow cost minimization models and the more comprehensive attempts to incorporate a broad range of economic and social impacts. Finally some personal remarks are made concerning the most promising areas of future research with respect to practical relevance, computational feasibility and theoretical interest.

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# OPTIMIZATION MODELS OF TRANSPORTATION NETWORK IMPROVEMENTS: REVIEW AND FUTURE PROSPECTS*

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Investments in the transportation system represent a major component (frequently the largest share) of a country's public capital expenditures. These investment decisions can, of course, strongly influence how the transportation system will be used for decades to come. In particular, transportation investments may affect subsequent decisions of individuals and firms regarding the type and volume of their activities and ultimately even their locations. Thus, not only the magnitude of capital expenditures in transportation, but also their strategic economic, social and environmental ramifications would seem to make the study of them a practically important and intellectually challenging area of research. It is the purpose of this paper to review the applications of optimization methods to this research question and make some tentative prescriptions regarding the most promising and relevant areas of future research. (A review of research is particularly appropriate in view of the many disciplines and interest groups making contributions to this area of research. They include engineers (primarily civil, electrical and industrial), operations researchers, economists, mathematicians, geographers, regional scientists, urban planners, and perhaps others. Although the volume of research is managable, its far-flung distribution makes the task of keeping informed of current developments a difficult one.)

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The study of transportation is pre-eminantly a spatial problem. The use of a transportation system manifests itself as a set of spatial flows; the most important investment and control decisions are of a spatially specific nature; the consequences of these investments can be highly differentiated spatially; the transportation system is one of the crucial mechanisms which gives rise to locational variations and specialization in economic and social activities. It is these spatial or locational aspects which are emphasized in this review.

Approaches to optimize such investments can be categorized roughly and somewhat arbitrarily into two groups. The first and largest is ultimately concerned with numerical optimization procedures. These approaches frequently attempt to incorporate significant theoretical relationships, but they are devised to provide computationally feasible solution methods for real planning problems. Most of these methods arise directly from the mathematical programming literature. Finally, they deal exclusively with discrete space problem formulations. That is, locations are represented by discrete nodes and the network improvement decision is whether two nodes should be directly connected, and, if so, what the capacity of the link should be.

The second class of optimization methods is more analytical, theoretical and is frequently concerned with space as a continuous variable. It has its roots in analytical geometry and differential calculus. Rather than attempting to solve real planning problems, this approach aims to provide theoretical, general, and essentially qualitative insights on limited aspects of the transportation investment problem. These two approaches, while in principle complementing one another have developed largely independently with minimal influences on one another.

# 1. Numerical Models of Optimal Network Improvement

## 1.1 Linear Programming Models

There are, of course, many formulations of optimal network improvement problems, reflecting differing initial conditions, system behaviour, primary objectives and constraints. Perhaps the most obvious formulation is a simple extension of the capacitated Hitchcock - Koopmans Transportation Problem as presented by Quandt (1960):

$$\text{Minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij} \; X_{ij} \tag{1}$$

$$\text{subject to:} \quad \sum_{j=1}^{J} X_{ij} \leq S_i \qquad i=1,\dots I \qquad \text{(supply)} \tag{2}$$

$$\sum_{i=1}^{I} X_{ij} = D_j \qquad j=1, \dots, J \qquad \text{(demand)} \tag{3}$$

$$X_{ij} - \Delta K_{ij} \leq K_{ij} \qquad \begin{array}{l} i=1,\dots, I \\ j=1,\dots, J \\ \text{(route capacity)} \end{array} \tag{4}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} r_{ij} \cdot \Delta K_{ij} \leq B \qquad \text{(budget)} \tag{5}$$

The objective is to minimize the total cost of shipments of a homogenous product subject to specified supply constraints and demand requirements without violating the capacity of any link [old capacity ($K_{ij}$) + added capacity ($\Delta K_{ij}$)] and not exceeding the budget available for transportation investment expenditures (B).

One of the most interesting aspects of this formulation, as with all linear programming problems, is the dual problem:

$$\text{Maximize} \quad \sum_{j=1}^{J} V_j D_j - \sum_{i=1}^{I} U_i S_i - \sum_{i=1}^{I} \sum_{j=1}^{J} W_{ij} K_{ij} - tB \qquad (6)$$

$$\text{subject to: } V_j - U_i - W_{ij} \leq C_{ij} \qquad (7)$$

$$W_{ij} - r_{ij}\, t \leq 0 \qquad (8)$$

All of the dual variables have the standard interpretation of the change in total costs associated with a unit change in the right hand side of the associated constraint in the primal. Thus, in particular, the dual variable t is the decrease in costs arising from an additional dollar expended on increasing the capacity of the transportation network. This imputed rate of return can be compared to the rate of return for expenditures from other investments in order to assess the wisdom of making more (or less) money available for transportation system improvements.

Quandt presents two alternative formulations of the same problem. One of them minimizes construction cost expenditures, completely ignoring transfer costs. A tolerable level of total transportation costs could be incorporated into this version of the model as a constraint.) The other formulation minimizes the joint costs of operations and investments with the latter costs being amortized on an annual basis using appropriate interest rates.

In a paper which was published earlier but which derives from Quandt's analysis, Garrison and Marble (1958) formulate perhaps the first network generation problem which attempts to incorporate the economic consequences of transportation investment. The objective is to minimize the sum of operating and investment expenditures subject to supply, demand and transportation capacity constraints:

$$\text{Minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} C_{ij} X_{ij} + \sum_{i=1}^{I} \sum_{j=1}^{J} r_{ij} \Delta K_{ij} \tag{9}$$

subject to:

$$\sum_{j=1}^{J} X_{ij} \leq \sum_{k} \sum_{q} a_{ikq} \cdot \Delta K_{kq} + S_i \quad \text{(supply)} \tag{10}$$

$$\sum_{i=1} X_{ij} \geq \sum_{k} \sum_{q} b_{ijq} \cdot \Delta K_{kq} + D_j \quad \text{(demand)} \tag{11}$$

$$X_{ij} \leq K_{ij} + \Delta K_{ij} \quad \text{(link capacity)} \tag{12}$$

In inequalities (10) and (11) $a_{ikq}$ and $b_{jkq}$ are empirically determined coefficients which measure the effects of capacity increments on route kq on the supply capacities and demand requirements at i and j respectively. The difficulties of measuring such coefficients at any one time are rather formidable. More important, perhaps, it should not be expected that they would remain constant as the transportation system, the way it is used, and the distribution of economic activity change over time. It would seem clear that the sensitivity of demands and production to changes in system structure cannot be assumed to remain constant and linear as the system itself undergoes changes. Another apparent flaw in the model is that the "optimal" solution may result in less than optimal transportation investments since low capacities will discourage demands and supplies from increasing and thus total flows will be less than they would be with higher investment levels. Thus, a cost minimizing objective may result in a stifling of economic development, which would of course, be contrary to the objective of most national governments. Having said all this, it must also be emphasized that as one of the first applications of optimization methods to the network generation problem, the Garrison - Marble paper represents an ambitious initial effort to go beyond the more familiar narrowly defined formulations.

Kalaba and Juncosa (1956) have presented another early paper in this research area. Although nominally concerned with a telephone rather than a physical transportation network, many of the principles are similar. In this formulation both nodes (switching centres) and links have capacities which can be increased. Interstation demands are given. The optimal routing problem is defined to be one of maximizing a performance criterion function which is the ratio of satisfied demands to those predicted exogenously. For the design problem, the authors choose to minimize the total cost of adding to existing link and switching capacities subject to constraints on link and switching capacities as well as a constraint on the performance criterion ratio index just mentioned. Thus again we see the flexibility of mathematical programming methods - objectives may appear in the objective function or in the constraints. To a large degree the choice is the modeller's although theoretical principles and practical requirements may provide some guidance.

All of these linear programming formulations can be modified so as to include the possibility of transhipment or can be put into an arc-chain format to accommodate any generalized transportation network. An arc-chain formulation of a network synthesis problem can be expressed in the following way:

$$\text{Minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{q=1}^{Q} C_{ij}^{q} \; X_{ij}^{q} \tag{13}$$

subject to:

$$\sum_{q=1}^{Q} \sum_{j=1}^{J} X_{ij}^{q} \leq S_i \tag{14}$$

$$\sum_{q=1}^{Q} \sum_{i=1}^{I} X_{ij}^{q} = D_j \tag{15}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{q=1}^{Q} a_{ij}^{\ell q} X_{ij}^{q} - \Delta k_{\ell} \leq K_{\ell} \tag{16}$$

$$\sum_{\ell=1}^{L} r_{\ell} \Delta K_{\ell} \leq B \tag{17}$$

The decision variable $X_{ij}^{q}$ is the flow from node i to node j over the $q^{th}$ chain ($q^{th}$ shortest path) from i to j. The decision variable $\Delta k_{\ell}$ is link- or arc-specific - i.e. the increase in capacity of the $\ell^{th}$ link. The coefficient $a_{ij}^{\ell q}$ is defined so that it equals unity if link $\ell$ is on the $q^{th}$ shortest path between nodes i and j and equals zero otherwise.

## 1.2 Discrete Programming Models

One of the major shortcomings of linear programming representations of the network improvement problem is that increases to the capacity of any network link are assumed to be continuously valued variables. More realistically, these capacity improvements might be limited to a set of indivisible entities, such as the addition of entirely new lanes of traffic on the link. Alternatively, the potential changes to a network arc might be restricted to a set of mutually exclusive design possibilities; in this case the variables may be limited to the values zero and one. Integer valued variables, then, emerge very naturally from the typically rather lumpy investment decisions arising in many transportation planning situations. In principle, either cutting-plane methods or tree-searching procedures may be used, but for most problems of interesting size only the latter are feasible.

A typical mixed integer programming model of network improvement could closely resemble the previous linear programming formulation:

$$\text{Minimize} \quad \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{q=1}^{Q} C_{ij}^{q} \; X_{ij}^{q} \tag{18}$$

Subject to:

$$\sum_{q=1}^{Q} \sum_{j=1}^{J} X_{ij}^{q} \leq S_i \tag{19}$$

$$\sum_{q=1}^{Q} \sum_{i=1}^{I} X_{ij}^{q} \geq D_j \tag{20}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{q=1}^{Q} a_{ij}^{\ell s} \; X_{ij}^{s} - \lambda_\ell \; K_\ell \leq 0 \tag{21}$$

$$\sum_{\ell=1}^{L} r_\ell \; \lambda_\ell \leq B \tag{22}$$

The flow - blocking constraints (21) ensure that the total flow on any link $\ell$ is less than or equal to the capacity of the link (the product of the capacity $k_\ell$ of one unit e.g. a lane and the number of units ($\lambda_\ell$) e.g. the number of lanes). The coefficient $r_\ell$ is the cost of constructing one unit of capacity on link $\ell$. The investment decision variables $\lambda_\ell$ may be restricted to the binary set, zero and one, and be interpreted as the decision to add or not to add link $\ell$ to the system. Mixed integer programming models of this type have been applied in a large number of different situations including intercity highway networks (Bergendahl (1969) and, Morlok et al (1969)), intra-urban traffic systems (Ochoa-Rosso (1968), Ochoa-Rosso and Silva (1968), and Hershdorfer (1965)), and national transport systems (Taborga (1968) and Kresge and Roberts (1971)).

One form of network generation problem has received considerable attention - so much that it is frequently referred to as "the optimal network problem". The "optimal network" is that set of arcs linking together a given set of nodes such that the sum of the shortest paths over the network between every pair of nodes is minimized with the restriction that the total length of the network does not exceed some upper bound. The solution to this problem consists of the binary incidence matrix specifying which pairs of network links are to be constructed. The number of different potential solutions quickly becomes enormous as M, the number of potential links, increases. In general, there are $2^M$ potential solution networks. This problem can be solved by any method which is capable of restricting the total number of link combinations that must be examined to some manageable size. Many algorithms have been developed for this purpose - backtrack programming, branch and bound methods and heuristic programming. (See Scott (1971) for a review of these algorithms and some applications.)

All tree searching algorithms are especially flexible solution procedures in that they can be used to solve a variety of mixed integer - real linear or even non-linear programming problems so long as they retain the property of monotonicity i.e. the objective function may be non-linear provided the deletion of a link consistently decreases (or increases) its current value.

In spite of this flexibility, improvements in exclusion operators and bounding procedures and the increasing capabilities of computers, none of these algorithms can be used to obtain solutions to very large problems.* In this event, heuristic methods varying from simple random sampling and trial and error to elaborate search procedures may have to be used.

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*In order to avoid the computationally cumbersome combinational programming methods, Hodgson (1972) develops a linear programming formulation by assuming an extensive two lane highway network already exists; the investment problem consists of determining where the network should be upgraded to a four lane facility. The decision variable in this case is the number of miles of four lane roadway between two adjacent nodes. Partially upgraded links are thus permissible and meaningful. This formulation enables the researcher to take advantage of the extensive software and powerful computational methods of mathematical programming systems to be brought to bear on large network improvement problems with large numbers of constraints.

The models offer the potential of providing feasible solutions with only a minimum of computational effort. Although many of these algorithms converge toward the globally optimal solution, they can terminate with a locally optimal solution which differs from the global optimum by some unknown margin of error. An example of such heuristic algorithm is presented in Scott (1969) and applied to a system of 32 cities in MacKinnon and Hodgson (1970). Various special heuristic algorithms have been developed by Steenbrink (1974) and Barbier (1966) and applied to optimal network problems.

Another category of heuristic procedure consists of methods which take into account the hierarchical structure of transportation networks solving problems first at the most aggregate level, substituting a single link for many which perform similar functions. As the algorithm proceeds a network disaggregation procedure is implemented giving more detailed investment prescriptions. (See,for example, Manheim (1966), Chan (1969) and Chan et al (1968)). More simply, large regions can be decomposed into smaller sub-regions within which network optimization is independently undertaken. The overall solution network is generated as an aggregation of the optimal solutions of the smaller regions.

Finally, interactive programming can be used. With this procedure, the modeller interacts directly with a computer terminal, responding to the interim solutions generated by the program. Links may be added or deleted by the modeller as the computer solves and evaluates each of the proposed networks. Stairs (1968) suggests this method for the network improvement problem and Schneider (1971) uses it to provide quite reasonable results to related public facility location allocation problems.

Interactive programming in its many guises represents the least rigorous of all approaches to network optimization. At its best, it can effectively harness the subjective intuitive understanding that experienced planners can bring to bear on complex, ill-structured problems. At its worst, heuristic programming generates plausible guesses at solutions to specific problems and offers few if any general insights into the problem of transportation network improvement.

### 1.3 Optimal Control Methods

Radically different are the attempts to apply control theoretic methods to the network generation problem.

Wang, Snell, and Funk (1968) assume a given rectangular network geometry, and a given generation, distribution and direction of traffic (all converging on a single CBD). They formulate a problem which jointly minimizes the time costs of flow and investment costs; simultaneously assigning flows and improving the network. Flows can be generated recursively because of the assumed directionality and the well defined simple geometric structure. Travel times are assumed to be a non-linear function of traffic volumes and investment on a link. State variables are defined in terms of flows, investments, and travel time costs. The Hamiltonian function for a typical interior node is generated. Initial conditions for the state variables are of course zero (no flows or antecedent costs beyond the limits of the urban area.) The boundary conditions for the adjoint variables are thus readily determined. An application of a discrete version of the maximum principle then generates a solution of some generality to this network improvement problem. It is suggested that the special case where upper and lower bounds (i.e. inequality conditions) on investment levels in different parts of the network be handled by a search procedure guided by current values of the adjoint and state variables and the partial derivatives of the Hamiltonian function. Two hypothetical numerical examples are presented - the first a pure network generation problem, the second a network improvement problem. The ability of this type of approach to deal with non-linear functions certainly recommends it. Whether it can be extended to model more realistic problems is still unknown.

## 2. Theoretical Models of Optimal Network Improvement

Quite different in style from the models outlined in the previous section are the approaches which attempt to isolate and rigorously analyze a smaller set of factors and where the ultimate aims, stated or not, are theoretical principles and analytical elegance rather than specific numerical solutions to practical planning problems. Although this literature is rather small, a detailed review of all the models is nevertheless impractical. In this section, the problems are enumerated and the general flavour of the approaches are indicated.

The "Steiner problem" is the most widely studied of all these problems. The continuous space analog to the minimal spanning tree problem (Prim, (1957)), the Steiner problem determines the network of minimal length which connects a given set of points to each other. The solution to this problem is of particular relevance in cases where flow costs are insignificant compared to construction costs (eg. communications networks and economies where capital is in very short supply) and where the construction cost surface is uniform and thus Euclidean measures of distance are relevant. The Steiner network provides a lower bound on network length for systems which are to connect a given set of regions or cities.

The solution to the three point Steiner problem has been known for many years - a simple "vee" network if the triangle joining the three points has an angle of more than $120^{\circ}$; otherwise a "wye" network with the three links meeting in the interior of the triangle, forming three $120^{\circ}$ angles. The general n-point case has proven to be more resistant to solution, but methods have been developed to solve n-point cases for any specified network topology. (Gilbert and Pollak, 1968 and Werner, 1969). As the number of topologies increases geometrically with n, it may seem that these methods are severely limited; however, in most situations, many topologies can be discarded as implausible candidates. Thus, even for large values of n, networks which are close to the overall minimal length can be generated.

Instead of ignoring flow costs completely, it would be possible to emphasize them to the exclusion of construction costs. The solution to this problem is the trivial network which connects each pair of points directly. Considerably more interesting are those network models which attempt to include the trade-off between construction and flow costs. One version of this problem is the three point Weber problem. One raw material source $P_1$ is to be connected by a transportation network to two markets $P_2$ and $P_3$ so that the joint costs of construction and flow are minimized. Working in Euclidean space and assuming flow and construction costs are linear, homogeneous functions of distance, the problem reduces to that of finding the location coordinates (X, Y) of an interior point such that

$$c = [c+(f_{12}+f_{13})k]\,[(X_1-X)^2+(Y_1-Y)^2]^{\frac{1}{2}}$$

$$+ \; [c+f_{12}k]\,[(X_2-X)^2+(Y_2-Y)^2]^{\frac{1}{2}}$$

$$+ \; [c+f_{13}k]\,[(X_3-X)^2+(Y_3-Y)^2]^{\frac{1}{2}}$$

is minimized where $(X_i, Y_i)$ are the given location coordinates of point $P_i$, $f_{ij}$ is the flow between i and j, and c and k are the known per mile costs of construction and flow respectively. Werner (1968a) solves this formulation, re-interpreting the problem in trigonometric terms. With more than three points, the problem of specifying the appropriate network topology again arises. Werner develops a method by which a "first design" network is decomposed and a sequence of cost minimizing adjustments are made by applying the methodology developed for the three point case. Only locally optimal solutions can be derived as the order of the adjustments affects the network to which the procedure converges. The specification of optimal network topology can be solved for a few special cases, but Werner states that "...up to now, practically all essential problems concerning topological network design are unsolved."

It is interesting to note that the network which jointly minimizes construction and flow costs is a minimum length network (i.e. a Steiner tree) _if_ we assume that flows linearly decline with distance. This should provide a strong warning against using such models as planning devices. Clearly flows will decline with increased charges, but the primary purpose of a transportation system is _not_ to minimize costs. One must be cautious in defining the objective function and constraints of network improvement models. This comment is closely related to the criticism of the Garrison -

Marble model in the previous section.

Another case where demand is specified at a finite number of discrete locations is a trunk line problem studied by Beckmann (1967) and Hauer (1972). Hauer considers the case of a single origin (eg. place of work) and multiple destinations located at discrete points in Euclidean space (eg. places of residence such as apartment blocks). The problem is to distribute the homeward bound travellers to points close to their destination taking into account the inconvenience that diversions impose on travellers bound for different destinations. More specifically, the objective is to minimize some composite of walking distance, travel time for passengers and travel time of the vehicle. (Although nominally a routing problem, it is, with slight modifications, applicable to the location of fixed facility such as an intercity freeway or a waste disposal facility.) A number of general properties regarding the geometric shape of the optimal route are derived and a graphical method of constructing a route for any set of points is developed. As usual, the order in which the points are to be served, the network topology, must be specified; thus some experimentation may be necessary.

Other approaches relax the assumption regarding the discreteness of the location of transportation demand. For example, one can attempt to derive the optimal transportation network to serve a spatially continuous population distribution. Tanner (1968) develops a methodology to evaluate alternative networks of this type but avoids the more difficult problem of network synthesis.

To make the network generation problem tractable, these formulations focus on a major trip destination such as the CBD to which all trips are assumed to be destined. A homogenous transportation surface is generally assumed, superimposed on which is to be a new arterial network on which transportation costs are to be some specified fraction of "normal" costs. Friedrich (1956) uses a calculus of variations approach to determine the optimum trajectory of a single arterial route servicing a variable trip generating density within a rectangular sector of a city. Werner et al (1968) and Sen (1971) develop extensions and special cases of this approach. Sen generates an arterial network with multiple branches (a herring-bone topology) to serve a uniformly distributed population. For this specified network topology, it is possible to derive general statements about the relationship between the dimensions of the region, the transportation cost differential, the angle of incidence and the spacing of the branch lines with the main stem of the network.

Another continuous space problem is the minimum cost route connecting two points where the costs of construction, because of physical conditions or land acquisition costs, vary with location. The general problem can be formulated as a calculus of variations problem (Werner and Boukidis, n.d.) but in this form can be solved only for special cases.*
Where flow costs can be ignored and the cost surface can be approximated by a set of polygonal homogeneous cost regions, Werner (1968) shows that the problem can be solved as a multivariate extension of the law of refraction.

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*M.J. Hodgson of the University of Alberta is currently working on numerical applications of these methods in cases where the costs can be described by analytical spatial functions.

Newell (1974) comments on why the optimal network problem is under-researched. Not only is it typically at the last stage of transportation planning process, but it is characterized by difficulties which naturally arise out of two characteristics of the mathematical functions relevant to such problems. The first, the fact that many of the decision variables are integer valued has already been discussed at some length. The second is that the relationship between joint flow and construction costs on the one hand and flows is not in general a convex function. This non-convexety arises from the economies of scale achieved by large transportation facilities. Thus, total costs on any link rise in a convex fashion <u>for any fixed facility</u> (as congestion levels are encountered) but as flows rise, larger scale facilities can reduce average flow costs on any route. The relevant cost curve then is the individual cost curves' envelope, which very roughly, can be said to be concave. Concave programming problems are difficult to solve, but in qualitative terms, if the cost curve is concave, then flows between any two destinations will be routed over a single path because of the economies of scale. That multiple path routing arises in reality can be attributed to the fact that the cost curve is not strictly concave, but piecewise convex.

Another observation arising from Newell's discussion is that perfectly symmetrical networks (eg. square grids, non-hierarchical networks, etc.) are almost certain to be suboptimal even if the underlying demands are perfectly symmetrical. This follows directly from the above mentioned economies of scale.

Although Newell is modest in evaluating his contribution, it is possible that these sort of qualitative findings are more significant than the more technical and supposedly operational results of mathematical programming methods. More important than evaluating the relative merits of these approaches, it is apparent that mathematical programming formulations should attempt to incorporate the theoretical findings of Newell and others.

## 3. Perspectives on Future Research Priorities

Having provided a brief but hopefully representative review together with an extensive if not comprehensive bibliography on optimization approaches to the network improvement problem, it is appropriate to ask what are the most interesting remaining research tasks which could be undertaken. The answer to such a question is necessarily subjective and selective. In this section, no attempt is made to provide an objective comprehensive list of research problems. It is nevertheless hoped that participants of the workshop will make suggestions regarding additional focal points for research as well as concrete proposals regarding appropriate research strategies to resolve remaining problems.

The first and perhaps overriding research priority is a rather general one and, in a sense, all others are special cases of it - how to imbed optimal network improvement models into broader transportation planning and even broader urban and regional socio-economic planning contexts?

Transportation planning is typically decomposed into the following sub-problems: (a) prediction of the distribution of population, land use and economic activities; (b) trip generation; (c) trip distribution; (d) modal split; (e) traffic assignment; (f) network analysis, evaluation and modification. Although feedback loops are recognized, in practice the planning process tends to follow this sequence. For simplicity let us ignore the modal split component. Some approaches to network generation assume trip generation is known and the model simultaneously distributes trips, assigns traffic, and generates network improvements. The simple extensions of the capacitated Transportation Problem in either transshipment or arc-chain form are models of this type (eg. Quandt (1960), Carter and Stowers (1963).

More numerous are those which assume internodal demands are specified and independent of network structure. These models assign the known trips to the transportation network while adding links or upgrading the quality of the current network (eg. Ridley (1968), Scott (1969), Boyce, Farhi and Weschiedel (1973), Bushell (1970), Hodgson (1972), Bergendahl (1969b), and many others.) Hutchinson (1972) argues that this is a reasonable strategy for improvements to an already well-developed transportation system. But of course in general the new system configuration will influence future distribution of demands. Even for a given distribution of population and economic activity, better transportation service will induce individuals and firms to increase their utilization of the transportation system. Thus the integration of the network generation problem with trip distribution and generation models is certainly desirable. This has been done very crudely by MacKinnon and Hodgson (1970) by incorporating an unconstrained gravity model into the objective function. More recently Boyce and his associates have developed a method which minimizes the costs of a trip set which is distributed by a doubly constrained interaction model.* This approach, of course, assumes that trip generation is insensitive to transportation service characteristics. Even the same number of people with the same spatial distribution may make more trips as costs decrease and the quality of service improves in other ways. More importantly, perhaps, changes in the transportation system will in the longer run induce households, firms, and other decision making units to change the location of their activities and therefore the spatial structure of their transportation requirements.

The relationships between transportation improvements and the location (and re-location) of other economic and social activities is not very well understood. [See for example Holland (1972), Straszheim (1970) and Putnam (1975)] Models which identify the location of both transportation link improvements and production levels of economic activities such that the joint costs of transportation and production are minimized may be appropriate in some instances. Conceptually and operationally this is a rather simple integration of an interregional input-output model with location-allocation and network improvement models. Given a total schedule of exogenously predicted "final demands", exports, and imports by sector, the cost minimizing location of industry and transportation investments can be determined. Barber (1973, 1975) has applied such a model to interregional systems in Indonesia and Colombia. (In the Indonesian context, see also O'Sullivan et al 1975.) With less than rigidly and centrally planned economies, such formulations may have only heuristic value, setting a lower bound on one criterion of merit, recognizing that firms may make location decisions which do not result in

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*Source: personal communication with D. Boyce, University of Pennsylvania.

cost minimization for the system as a whole. Even in rigidly planned economies, other goals may be important yet difficult to incorporate in the model. Also for any type of economy, input-output coefficients are not stable and more importantly they will be affected by changes in the transportation cost structure. A cost-sensitive interregional input-output submodel could be incorporated into the larger model, but this would give rise to difficult nonlinearities.

Location factors other than transportation costs should be included within the model. Straszheim (1970) and many others would suggest this is particularly true for more advanced economies where the sensitivity of location decisions to transportation costs is rather low because of the relative homogeneity of the transportation cost surface and the typically small proportion of value added accounted for by transportation costs in many of the fastest growing sectors in such economies. Transportation effects on spatial development would appear to be most striking at two extremes of development - on an interregional scale, transportation can be an effective tool to shape development in economically less advanced countries where transportation service is at a low level and the economy is dependent on transportation-intensive activities; on a local scale, transportation can shape the future spatial structure of urban areas.

Dickey (1972) and Dickey and Azola (1972) have provided a valuable first step in broadening the focus of normative urban transportation planning by imbedding an econometric model of land use development (the "Empiric" model) into an optimization framework. The decision variables are interzonal travel times and the goals are stated in terms of desired future social and economic land use patterns. The constraints are composed largely of the equations of the Empiric model. Thus, "future land use changes that take place are constrained to do so in the way the Empiric model says they will."*

Dickey rightly characterizes this approach as exploratory and preliminary Not only is the econometric model subject to some skepticism, but also the decision variables themselves (exponentially transformed travel times) are not in a form which is necessarily directly translatable into action by transportation planners. For one thing, it would not be difficult to generate combinations of "optimal" travel times which were inconceivable since changing times between one zonal pair will change the times between another pair if the first link is on the path between the second pair. Even if this difficulty is not encountered, the costs of changing travel times and the means by which this is to be done are not specified. Given the variety of means and costs (and our imperfect knowledge) this vagueness is perhaps a desirable characteristic. The model suggests directions and approximate magnitudes of actions implied by the goals, not the precise means of implementation.

Lundqvist (1973) presents an even more ambitious model which uses hierarchical decomposition to jointly determine the optimal expansion and use of transportation <u>and</u> other land uses within an urban area. The need for a "recursive dialogue between optimization and simulation techniques" is stressed. The solution to the non-linear combinatorial problem is based on heuristic tree searching methods.

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*One set of non-linearities which naturally arises in the constraints is accommodated by using $e^{-\beta C_{ij}(t+1)}$ as a decision variable rather than the travel times Cij (t+1) themselves ($\beta$ is a distance friction parameter estimated in the econometric model.) Multiplicative non-linearities are handled by assuming one set of variables is constant and searching over their known range of values for their best settings.

Within this general prescription of broadening the scope of optimal network improvement models, two categories of difficulties typically arise - first, data on system inputs, goals, and past behaviour are scarce, costly and/or of poor quality; second, theoretical deficiencies are generally just as severe. There is an apparent need for models which are implementable, relevant, and about which we have some confidence regarding their validity.

Three categories of special problems relating to these models are those aspects related to system dynamics, risk and uncertainty, and specification of objectives. Each of these will be considered in turn.

For the most part, only relatively crude models have thus far been formulated to optimize improvements to transportation systems over time. Most of these approaches consider the network addition problem within a dynamic programming framework. Funk and Tillman (1968) and de Neufville and Mori (1970), for example, explicitly make the simplifying assumption that the benefits and costs arising from each link addition project are independent of each other, and can thus be specified numerically at the outset. More precisely, the interdependencies are only the relatively trivial ones arising from the fact that building a link in an earlier period may preclude the construction of others in subsequent periods - a sequence which, taken as a whole, could result in higher net benefits than one which simply maximizes the net benefits for each period taken separately.

Of considerably more interest (and difficulty) are models which recognize the flow interdependencies between different links. That is, the opening of a new link will divert traffic from other links and may even change the flow of traffic between origin - destination pairs in both a relative and absolute sense. Thus the value of adding a particular link will be dependent on what other links have already been added to the system. For example, the construction of a link in time period t may affect the value of adding a nearby link in period t+1 - positively if traffic generated and/or diverted as a result of the first link causes congestion on the network in this area, and negatively if one of the system objectives is to encourage regionally equitable balance of transportation services and investments.

Behavioural responses to network changes should be an important part of the network link sequencing problem. Changing the network will result in a different assignment distribution, and generation of traffic. Ultimately transportation investments can result in different geographical patterns of population, economic and recreational activities - patterns which will be passed on to subsequent stages of the spatial development process.

Some of these behavioural responses are relatively easy to model. Bergendahl (1969b) assumes that transportation demands between all pairs of points are predictable exogenously and that traffic need only be assigned to the network in each period. He assigns traffic using a linear programming multicommodity flow model. Incorporating link congestion costs, (Bushell (1970) uses a similar approach.) Other network assignment algorithms could be used, but this one provides a measure of the maximum savings in flow costs resulting from transportation investments. The optimal investment sequence is then determined by minimizing a recursive function which is a composite of discounted operating and investment costs:

$$\emptyset^{t+1}(S^{t+1}) = \min_{S^{t+1}} [R' \bar{Z}^{t+1}(S^{t+1}) + C(S^t, S^{t+1}) + R\, \emptyset^t(S^t)]$$

$$\emptyset^{T+1} = \min_{S^T} [R\emptyset^T(S^T) - v(S^T)]$$

$$\emptyset^o(S^o) = 0$$

where $S^t$ is the state of the system at time t (a list of the transportation links and their characteristics)

$\bar{Z}^t(S^t)$ the minimum operating costs associated with the state $S^t$

$c(S^t, S^{t+1})$ the investment costs of changing the system from $S^t$ to $S^{t+1}$

$v(S^T)$ the "scrap value" of the system (eg. the future operating costs associated with the "horizon" network $S^T$.

R and R' appropriate discount factors.

Bergendahl's model is operational and has been applied to highway investment decisions in Sweden. The computational task has been reduced by eliminating many of the implausible link combinations and sequences.

Although more ambitious than other efforts Bergendahl does not attempt to incorporate the traffic re-distribution and generation effects of network improvements. Even though our theoretical knowledge is relatively weak on this subject, it is of interest to note how such relationships could be incorporated. Hodgson (1974) uses a simple gravity model to redistribute and generate intercity traffic estimates over time in response to network changes. Traffic is assigned by the simple shortest path method. Transportation flow estimates are maximized over time subject to investment in each period and the specification of a terminal network which was generated in MacKinnon and Hodgson (1970) using a single stage optimization method. This latter condition is clearly artificial, but it is likely that considerable experimentation with crude approximations is going to be necessary before well-structured solutions to the temporal sequencing problem emerges.

In an important paper Frey and Nemhauser (1972) model the optimal timing of network expansion as a convex programming problem where flows are non-linear functions of service characteristis which in turn are functions in part of flows. The interdependencies between augmenting capacity on different links is fully taken into account and the conditions under which a myopic decision strategy is optimal are delimited. Unfortunately the general nature of these findings are tempered somewhat by the statement that "...although these results may be extendable to large serial networks, it does not seem possible to generalize them to networks with more complex topologies."

Each of the four responses to network change - assignment, distribution, generation, and spatial re-structuring - takes time. The specification of these lags should ultimately be an integral part of a transportation investment model. Although it would be tempting to claim that they are in ascending order of response lags, even this is not clear. Spatial restructuring (e.g. plant re-location) may be initiated in response to <u>anticipated</u> transportation improvements whereas traffic generation and other "operational" responses must await the actual construction of the facility. The characteristics and timing of these responses are not well understood; much research needs to be done, particularly in terms of the spatial restructuring impacts as these often swamp the short term savings in transportation costs which are frequently used to provide the nominal justification for transportation investments.

In view of this current ignorance, an explicit consideration of transportation improvements within the context of risk and uncertainty would appear to be appropriate. Not only is there uncertainty about the interaction effects between the transportation system and its socio-economic environment, but the environment itself is subject to exogenously generated changes, some of them slow trends, others quite abrupt shocks. Is it possible to generate network structures which perform well under a wide variety of future conditions? Extensive sensitivity analyses with networks generated by the optimizing approaches outlined in previous sections of this paper would be the most obvious way of dealing with this problem. Compromise networks could emerge which are best for no <u>single</u> future condition, but are tolerable or good for a wide variety of conditions. Where the probability distributions of exogenous changes are known (e.g. weather conditions, travel demands, etc.), some aspects of these problems may be formulated using stochastic mathematical programming (Kalaba and Juncosa, 1956 and Midler, 1971) and stochastic optimal control approaches.

Measures of system flexibility or adaptability would be useful to include as terms in an objective function or constraints. MacKinnon (1968) and Vuccic (1970) have introduced some issues related to transportation system flexibility, but they have not been formalized. This problem area is closely related to the problems of system resilience and option foreclosure which have received considerable attention at IIASA over the past two years. These problems in turn are closely associated with transportation plan development as a learning process, perhaps capable of being modelled within a Bayesian framework. (See,for example, Hutchinson (1970) and Kahn (1971).

Another weakness in all of the models of network improvement is related to the specification of an appropriate objective function. In the absence of something approximating a social welfare funtion, it is clear that there is a degree of arbitrariness associated with any objective function. Nevertheless, it is equally clear that it may be useful to determine networks which minimize costs or environmental impacts, maximize profits or accessibility, or optimize some other measure - not because these measures represent widely accepted, justifiable and overriding indicators of social merit - but rather because the results would demonstrate the implications of assuming they are such indicators. Thus a number of alternative network configurations may be developed, compared and evaluated in some detail.

Multiple objectives can be handled simply by incorporating some of them in the constraint set and some in the objective function or alternatively by using the recently developed multicriteria methods of mathematical programming. (Kapur, 1970). With considerable experimentation, sensitivity analysis, and interaction with public officials and even citizen groups, optimization approaches may result in a clarification of social objectives, an indication where conflicts are likely to arise, and facilitation of compromise solutions by being able to demonstrate the implications of a variety of objectives.

## 4. Closure

It should be clear by now that the author views optimization methods not as ways of generating well-defined and rigorously justifiable solutions to transportation network improvement problems. Because of our ignorance regarding many of the important system relationships and the potentially far-reaching implications of these decisions, the interpretation of the results of optimization models of network improvement cannot be similar to the interpretation of optimization models of, say, machine replacement in a factory even though the techniques may be exactly the same. In a network improvement context, optimization models are best regarded as heuristic frameworks within which to generate, in rough outline, some alternative investment programs. The ultimate evaluation, specification and adoption of these programs will have to be based on (1) more detailed analyses which capture the multivariate nature of urban and regional transportation systems operations and impacts and/or (2) quite subjective qualitative judgements made by experienced planners, policy makers and perhaps even the general public.

Viewed in this light, optimization models become inputs to the decision making process rather than the decision making process itself. The models, for the reasons already stated, are not credible candidates for the latter role. It is only with these more modest claims that optimization methods will be widely adopted in developing transportation investment programs. They may be useful not only in directly indicating what type of investment programs should be undertaken, but also in clarifying objectives and identifying where additional research would be rewarding.

In the debate regarding the relevance of these methods to transportation system planning, a fundamental question is whether rigorous optimization methods can be applied creatively, honestly and effectively in situations where the systems are relatively poorly understood. It can be argued that an approximate solution to an important problem may be more useful than an exact solution to a relatively unimportant problem. Many operations research models are of the latter type. In one sense, this emphasis is admirable and indicative of a conservative scientific honesty,not claiming to be able to deliver more than is possible. However these exact solutions to narrowly defined, well understood problems, can result in suboptimal solutions in a broader context and an exacerbation of more important problems. Nowhere is this more apparent than in transportation system planning.

## Bibliography

Allen, J.A. 1969. The Optimal Sequencing of Expressway Network Construction. Discussion Paper No. 5, Department of Town Planning, University College, London.

Barber, G.M. 1970. Variational Calculus Approaches to Spatial Design Problems, in Seminar in Quantitative Geography, R.D. MacKinnon and A.J. Scott (eds.). Discussion Paper No. 7, Department of Geography, University of Toronto.

_____. 1973. An Optimization Approach to a Node - Network Development Problem. Ph.D. Thesis, University of Toronto.

_____. 1975. A Mathematical Programming Approach to a Network Development Problem. Economic Geography, 51: 2: 128-141.

Beckmann, M.J. 1967. Principles of Optimum Location for Transportation Networks, in W.L. Garrison and D.F. Marble (eds.) Quantitative Geography: Vol. I. Evanston, Illinois: Northwestern University Studies in Geography No. 13, pp. 95-119.

Beenhakker, H.L. 1973. Approaches to Dynamic Transport Planning. Transportation, 2: 245-280.

_____. and Danskin, J.G. 1973. Economies of Stage Construction for Transport Facilities. Transportation Research, 7:163-178.

Bellheimer, J.W. and Gray, P. 1973. Network Design with Fixed and Variable Cost Elements. Transportation Science, 7: 49-74.

Bergendahl, G. 1968. Capacity Planning Models for Systems of Plant and Road Investments. Swedish Journal of Economics, 94-105.

_____. 1969a. Models for Investments in a Road Network. Stockholm: Bonniers.

_____. 1969b. A Combined Linear and Dynamic Programming Model for Interdependent Road Investment Planning. Transportation Research, 3: 211-228.

Boyce, D.E., Farhi, A. and Weschiedel, R. 1973. Optimal Network Problem: A Branch-and-Bound Algorithm. Environment and Planning, 5: 519-533.

Bos, H.C. and Koyck, L.M. 1961. The Appraisal of Road Construction Projects: A Practical Example. Review of Economics and Statistics, 43: 13-20.

Brotchie, J., Sharpe, R. and Dickey, J. 1974. The Integration of Land Use and Transportation Planning Using Modelling Techniques, in Traffic Control and Transportation Systems. Amsterdam: North-Holland, 311-335.

Burstall, R.M. 1966. Computer Design of Electricity Supply Networks by an Heuristic Method. Computer Journal, 9: 263-274.

_____. 1967. Tree Searching Methods with an Application to a Network Design Problem. Machine Intelligence, 1: 65-85.

Bushell, G. 1970. An Optimization Model of Traffic Assignment and Network Improvement for the Freeway System of Southern Ontario. Research Paper No. 40, Toronto: Centre for Urban and Community Studies, University of Toronto.

Carter, E.C. and Stowers, J.C. 1963. Models for Funds Allocation for Urban Highway System Capacity Improvements. Highway Research Record No. 20, pp. 84-102.

Chan, Y. et al. 1968. Aggregation in Transport Networks: An Application of Hierarchical Structure. Research Report R68-47. MIT Department of Civil Engineering, Cambridge, Mass.

Chapman, L.D. 1973. Investing in Regional Highway Networks. Transportation Engineering Journal, TE2: 353-362.

Cockayne, E.J. and Melzak, Z.A. 1968. Steiner's Problem for Set Terminals. Quarterly of Applied Mathematics, 56: 213-218.

de Neufville, R. and Mori, Y. 1970. Optimal Highway Staging by Dynamic Programming. Transportation Engineering Journal, ASCE, TE1: 11-23.

Dickey, J. 1972. Minimizing Economic Segregation Through Transit System Changes: A Goal Programming Approach, in Traffic Flow and Transportation. Proceedings of the Fifth International Symposium on the Theory of Traffic Flow. New York: American Elsevier, pp. 87-108.

_____. and Azola, M.P. 1972. An Adaptive Programming Technique for Sequencing Highway Improvements to Affect Land Use Growth. Regional Studies, 6: 3: 331-342.

Doulliez, P.J. 1968. Optimizing the Investment Policy on a Transportation Network. Discussion Paper No. 6822, Center for Operations Research and Econometrics. Catholic University of Louvain.

_____. and Rao, R.M. 1969. Removing Arcs from a Network with Increasing Requirements. Discussion Paper No. 6920, Center for Operations Research and Econometrics. Catholic University of Louvain.

Frey, S.C. and Nemhauser, G.L. 1972. Temporal Expansion of a Transportation Network - I. Transportation Science, 6: 3: 306-323. II - 6: 4: 395-406.

Friedrich, P. 1956. Mathematical Description of Life in Settled Areas and Derivation of Plan Layout Thereof as a Problem of Theoretical Town Planning. Second International Symposium in Traffic Flow, pp. 346-368.

Fulkerson, D.R. 1959. Increasing the Capacity of a Network: The Parametric Budget Problem. Management Science, 5: 472-483.

Funk, M.L. and Tillman, F.A. 1968. Optimal Construction Staging by Dynamic Programming. Journal of the Highway Division ASCE, pp. 255-265.

Gardiner, M.T. 1971. The Basic Geometric Approach to Network Optimization Applied to System Design. Proceedings of the Association of American Geographers, 3: 52-56.

Garrison, W.L. and Marble, D.F. 1958. Analysis of Highway Networks: A Linear Programming Formulation. Proceedings of the Highway Research Board, 37: 1-17.

Gilbert, E.N. and Pollak, H.O. 1968. Steiner Minimal Trees. Journal of the Society for Industrial and Applied Mathematics, 16: 1-29.

Goldman, A.J. and Nemhauser, G.L. 1967. A Transport Improvement Problem Transformable to a Best-Path Problem. Transportation Science, 1: 295-307.

Gomory, R.E. and Hu, T.C. 1964. Synthesis of a Communication Network. SIAM Journal of Applied Mathematics, 12: 348-369.

Groboillot, J.L. and Gallas, L. 1967. Optimalisation d'un Projet Routier par Recherche du Plus Court Chemin dans un Graphe à Trois Dimensions. Revue Francaise d'Informatique et Recherche Operationelle, 1: 2: 99-121.

Hanan, M. 1966. On Steiner's Problem with Rectilinear Distance. SIAM Journal. 14: 255-265

Hammer, P.L. 1969. Increasing the Capacity of a Network. Canadian Operations Research Society Journal, 7: 83-91.

Harris, B. 1970. Generating Projects for Urban Research. Environment and Planning, 2: 1-21.

Hauer, E. 1972. Optimal Form of a Class of Collection - Distribution Networks, in G.F. Newell (ed.) Fifth International Symposium on the Theory of Traffic Flow and Transportation. New York: American Elsevier.

Hay, G., Morlok, E.K. and Charnes, A. 1966. Toward Optimal Planning of a Two-Mode Urban Transportation System: A Linear Programming Formulation. Highway Research Record 148, 20-38.

Herschdorfer, A.M. 1968. Optimal Synthesis of a Road Network, in Business Logistics: Policies and Decisions. D. McConaughy and C.J. Clawson (eds.). Los Angeles: University of Southern California Research Institute for Business and Economics, pp. 179-192.

Hoc, H.H. 1973. A Computational Approach to the Selection of an Optimal Network. Management Science, 19: 488-498.

Hopkins, L.D. 1974. Plan, Projection, Policy - Mathematical Programming and Planning Theory. Environment and Planning, 6: 419-430.

Holland, E.P. 1972. Economic System Models for Transport Sector Studies. Economic Staff Working Paper No. 131. International Bank for Reconstruction and Development, Washington, D.C.

Horn, W.A. 1968. Optimal Networks Joining Points in the Plane. Proceedings of the 4th International Symposium of Traffic Flow. Karlsruhe.

Hutchinson, B.G. 1969. Structuring Urban Transportation Planning Decisions: The Use of Statistical Decision Theory. Environment and Planning. 1: 209-220.

Hutchinson, B.G. 1972. Programming of Regional Highway Investments. Transportation Engineering Journal, TE3: 493-508.

Hodgson, M.J. 1972. Highway Network Development and Optimal Accessibility Change in the Toronto-Centered Region. Unpublished Ph.D. Thesis, University of Toronto.

Kadas, A.S. 1975. On Modelling the Interaction of the Transportation Network and Industrial Location. Paper presented at the European Regional Science Meetings, Budapest.

Kahn, A.M. 1971. Transport Policy Analysis: A Decision - Theoretic Framework. Socio-Economic Planning Sciences. S: 159-171.

Kalaba, R.E. and Juncosa, M.L. 1956. Optimum Design and Utilization of Communication Networks. Management Science, 3: 33-43.

Kapur, K.C. 1970. Mathematical Methods of Optimization for Multi-Objective Transportation Systems. Research Publication GMR-965. General Motors Research Laboratories, Warren, Michigan.

Kaufmann, W. 1928. Über eine MinimaleAufgabe der Verkehrstechnik. Der Bauingeniewr. 31: 558-561.

Knight, U.G.W. 1960. The Logical Design of Electrical Networks Using Linear Programming Methods. Proceedings of the Institute of Electrical Engineering. A107: 306-319

Lack, G.N.T., et. al. 1968. A Model for the Economic Evaluation of Rural Road Improvements. Australian Road Research Board, 4: 1: 322-379.

Lundqvist, L. 1973. Integrated Location - Transportation Analysis: A Decomposition Approach. Regional and Urban Economics, 3: 3: 233-262.

MacKinnon, R.D. 1968. The Concept of System Flexibility Within a Transportation Context. Ph.D. Dissertation, Northwestern University.

_____. and Hodgson, M.J. 1970. Optimal Transportation Networks: A Case Study of Highway Systems. Environment and Planning. 2: 267-284.

_____. and Barber, G.M. 1971. A New Approach to Network Generation and Map Representation: The Linear Case of the Location - Allocation Problem. Geographical Analysis, 4: 156-168.

Manheim, M.L. 1966. Hierarchical Structure: A Model of Design and Planning Processes. Cambridge, Mass.: MIT Press.

McLoughlin, G.T. 1973. Transportation Networks. Paper presented at the 44th National Operations Research Society of American Meeting, San Diego.

Melzak, Z.A. 1961. On the Problem of Steiner. Canadian Mathematical Bulletin, 4: 143-148.

Midler, J.L. 1971. Investment in Network Expansion Under Uncertainty. Transportation Research, 5:

Miehle, W. 1958. Link Length Minimization in Networks. Operations Research, 4: 232-243.

Miller, R.E. 1967. An Optimization Model for Transport Planning. Transportation Research, 1: 271-286.

Morlok, E.K., et al. 1969. The Development of a Geographic Transportation Network Generation and Evaluation Model. Northwestern University Transportation Center. Report for Transport Systems Planning Division. Office of High-Speed Ground Transportation.

_____. 1970. A Goal Directed Transportation Planning Model. Transportation Research, 4: 199-213.

Newell, G.F. 1974. Optimal Network Geometry. Sixth International Symposium on Transportation and Traffic Theory, New York: Elseview, pp. 561-580.

Ochoa-Rossa, F. 1968. Applications of Discrete Optimization Techniques to Capital Investment and Network Synthesis Problems. Ph.D. Dissertation, Massachusetts Institute of Technology.

_____. and Silva, A. 1968. Optimum Project Addition in Urban Transportation Networks via Descriptive Traffic Assignment Methods. Research Report No. R68-44. MIT, Department of Civil Engineering, Cambridge, Mass.

Oettli, W. and Prager W. 1971. Optimal and Suboptimal Capacity Allocation in Communication Networks. Journal of Optimization Theory and Applications, 8: 396-411.

O'Sullivan, P., et al. 1975. Regional Road Investment Planning for Developing Countries: An Indonesian Example. Evanston, Illinois: Northwestern University Transportation Center Monograph.

Pearce, C.E.M. 1974. Locating Concentric Ring Roads in a City. Transportation Science, 8: 2: 142-168.

Prager, W. 1961. On the Design of Communication and Transportation Networks. Proceedings of the First International Symposium on the Theory of Traffic Flow. New York: Elsevier.

Prim, R.C. 1957. Shortest Connection Networks and Some Generalizations. Bell System Technical Journal, 36: 1389-1401.

Putman, S.H. 1975. Urban Land Use and Transportation Models: A State-of-the-Art Summary. Transportation Research, 9: 187-202.

Quandt, R.E. 1960. Models of Transportation and Optimal Network Construction. Journal of Regional Science, 2: 27-45.

Ridley, T.M. 1968. An Investment Policy to Reduce the Travel Time in a Transportation Network. Transportation Research, 2: 409-424.

Ripper, M. and Varaiya, P. 1974. An Optimizing Model of Urban Development. Environment and Planning, 6: 149-168.

Roberts, P.O. and Kresge, D.T. 1971. Systems Analysis and Simulation Models. Washington, D.C.: Brookings Institution.

Roe, T. and Shane, M. 1973. Programming Approach to Urban Transit Planning. Transportation Engineering Journal, TE3: 571-584.

Rosenhead, J. 1973. The Optimum Location of Transverse Transport Links. Transportation Research, 7: 107-123.

Schneider, J. B. 1971. Solving Urban Location Problems: Human Intuition Versus the Computer. Journal of the American Institute of Planners. 37: 2: 95-99.

Schwartz, J.G. 1968. Network Flow and Synthesis Models for Transport Planning. Research Report R68-49. MIT. Department of Civil Engineering, Cambridge, Mass.

Scott, A.J. 1967. An Integer Program for the Optimization of a System of Chromatic Graphs. Journal of Regional Science, 7: 291-296.

_____. 1969. The Optimal Network Problem: Some Computational Procedures. Transportation Research, 3: 201-210.

_____. 1971. Combinatorial Programming, Spatial Analysis and Planning. London: Methuen.

Sen, L. 1971. The Geometric Structure of an Optimal Transport Network in a Limited City - Hinterland Case. Geographical Analysis, 3: 1-14.

Stairs, S. 1968. Selecting an Optimal Traffic Network. Journal of Transport Economics and Policy, 2: 218-231.

Steenbrink, P.A. 1974. Optimization of Transport Networks. London: Wiley.

_____. 1974. Transport Network Optimization in the Dutch Integral Transportation Study. Transportation Research, 8: 11-27.

Straszheim, M.R. 1970. Researching the Role of Transportation in Regional Development. Land Economics, 48: 212-219.

Taborga, P. 1966. A Model to Study an Optimal Transportation Policy in Chile. Research Report R66-8, MIT Department of Civil Engineering. Cambridge, Mass.

Tanner, J.C. 1968. A Theoretical Model for the Design of a Motorway System. Transportation Research, 7: 107-123.

Tinbergen, J. 1957. The Appraisal of Road Construction: Two Calculation Schemes. Review of Economics and Statistics, 39: 3: 241-249.

Vickery, W. 1967. Optimization of Traffic and Facilities. Journal of Transport Economics and Policy, 1: 123-136.

Walters, A.A. 1968. A Development Model of Transport. American Economic Review, Papers and Proceedings, 58: 360-377.

Wang, J., Snell, R.R. and Funk, M.L. 1968. Toward a Solution for the Optimal Allocation of Investments in Urban Transportation Networks. Highway Research Record No. 238, 23-45.

Wellington, A.M. 1898. The Economic Theory of the Location of Railways, 6th edition. New York: Wiley.

Werner, C. 1966. Zur Geometrie von Verkehrsnetzen. Die Beziehung zwischen raeumlicher Netzgestaltung und Wirtschaftlichkeit. Free University of Berlin: Publications of the First Department of Geography No. 10.

_____. 1968a. The Role of Topology and Geometry in Optimal Network Design. Papers of the Regional Science Association, 21: 173-190.

_____. 1968b. The Law of Refraction in Transportation Geography: Its Multivariate Extension. Canadian Geographer, 12: 28-40.

_____. et al. 1968. A Research Seminar in Theoretical Transportation Geography, in Geographic Studies of Urban Transportation on Network Analysis. F. Horton (ed.). Northwestern Studies in Geography No. 16, Evanston, Illinois, pp. 128-170.

_____. 1969. Networks of Minimum Length. Canadian Geographer, 13: 47-69.

_____. 1970. Formal Problems of Transportation Impact Research. Annals of Regional Science, 4: 2: 134-150.

_____. and Boukidis, N. n.d. Determination of Optimum Route Connecting Two Locations. Report submitted by Transportation Center, Northwestern University to U.S. Army Transportation Research Centre, Fort Eustes, Virginia.

Wesolowsky, G.O. 1975. Location of the Median Line for Weighted Points. Environment and Planning, 7: 163-170.

Leblanc, L.J. 1975. An Algorithm for the Discrete Network Design Problem. Transportation Science, 9: 3: 183-199

Hodgson, M.J. 1973. Staging Interurban Highway Construction by Dynamic Programming. Annals of Regional Science, 8: 123-136.