A GENERALIZED MATHEMATICAL MODEL OF THE INFLUENCES OF COASTAL POLLUTION SOURCES ON THE MARINE ECOLOGICAL SYSTEM

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Preface

This paper was written by Professor V.I. Belyaev (Head of the Department of Mathematical Modelling at the Institute of Biology of the South Seas, Sevastopol, USSR), during his visit with the Food and Agriculture Program at IIASA in July 1977.

Biological resources of the ocean have an important role in non-agricultural food production. Dr. Belyaev's task was to prepare a suggestion as to whether we should conduct inhouse research in this field or whether we should rely on the findings of other institutions. His paper describes the outlines of a method developed at his home institution to evaluate these special food resources. The method has not yet been applied to the whole scale but certain areas of the investigation show promising results.

Ferenc Rabar
July 1977

Abstract

The concept of "frontal concentration" is proposed. Frontal concentration of pollutant corresponds to disappearance of ecosystem properties that are interesting from our point of view and is determined on the basis of qualitative analysis of a mathematical model of the ecosystem. The position of the pollution zone boundary corresponds to the position of frontal concentration values. The calculation of frontal concentration value movements is carried out by solving the diffusion problem.

Introduction

As a coastal source of pollution, the continuously acting source is considered which may be the output of sewage, estuary, etc. After a certain period of time the pollution zone is formed around the source. In this zone the physical and chemical conditions are changed and some part of animal and vegetable species disappear. At the same time, some other species that existed earlier in this zone in negligible quantities or those that were brought in from outside the zone and could not survive, now begin to develop intensively in this zone. The destruction of part of the primary organisms is caused not only by their direct poisoning by toxic substances, but it is also a result of the ecological equilibrium disturbances--the disappearance of food, the increased number of harmful organisms, the increased concentration of harmful metabolic end products, the change in oxygen concentration, etc.

The number of chemical combinations which comprise pollution is usually very great. The marine ecologic system also contains a great number of components. The direct mathematical formulation of the problem of the coastal pollution source influence on the marine ecologic system leads to the mathematical system containing hundreds of partial derivation equations with nonlinear forcing functions which characterise absorption and secretion of substances.

Thus, the solution of the mathematical problem of destruction of the ecological system of the basin that is caused by the spread of pollutants is very difficult. In this paper, one of the possible methods of reducing it to a more simple problem is considered.

Problem Definition

Let us take into consideration that, after some time in the polluted zone (PZ), the ecological system of the polluted zone (ESPZ) will form. Between the ESPZ and ecological system (ESUZ) located in the unpolluted zone (UZ), a transitional frontal zone (FZ) will exist which has an unsteady ecological system (ESFZ). The disposition of the zones is given on the following diagram:

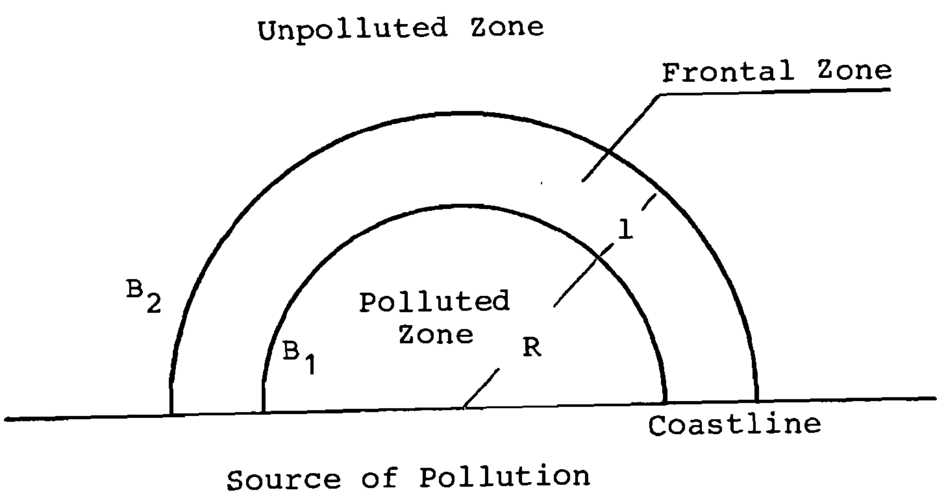

Fig. 1. Scheme of pollution of the basin by coastal pollution source.

Let us designate by q_i the components of the ESUZ which disappear in the FZ. These components could include the concentration of living organisms in the biomass, and the substances formed in the ESUZ which they require for living. By c_i we designate the concentrations of ESPZ components influencing the ESUZ components; c_i includes the harmful substances coming from the pollution source, the organisms developing in the ESPZ and products of their metabolism. The resulting conditions can be expressed by:

$q_i(x,y,z,t) = 0$ at $x,y,z \in PZ$, $c_i(x,y,z,t) = 0$ at $x,y,z \in UZ$.

To solve the question regarding direction of PZ boundary movement, it is necessary to consider the processes going on in the FZ. Let us designate the boundary between PZ and FZ by B_1, and the boundary between FZ and UZ--by B_2.

The ESFZ will be characterized by a set of parameters q_i and c_i which we designate by Q_i and C_i. In general Q_i and C_i will be described by the following equations:

$$\frac{\partial Q_i}{\partial t} = (L_{1i} + F_{1i} - R_{1i})Q_i \quad i = \overline{1,N_1} \quad . \tag{1}$$

$$\frac{\partial C_j}{\partial t} = (L_{2j} + F_{2j} - R_{2j})C_j \quad j = \overline{1,N_2} \quad . \tag{2}$$

$$Q_i = 0 \quad \text{at } x,y,z \in B_1 \ ,$$
$$Q_i = q_i \quad \text{at } x,y,z \in B_2 \quad . \tag{3}$$

$$C_j = 0 \quad \text{at } x,y,z \in B_2 \ ,$$
$$C_j = c_j \quad \text{at } x,y,z \in B_1 \quad . \tag{4}$$

In the equations (1)-(2) the differential operators characterized the transport of components caused by water exchange are designated by L_{lk}; F_{lk} and R_{lk} define the increase and decrease of components respectively $(l = 1,2)$.

We suppose that the processes of conentration profiles formation in the PZ and in the UZ are much slower than those in the FZ, and we think that q_i and c_i are constant in the problem of FZ (i.e. in the conditions (3) and (4)).

The expressions for F_{lk} and R_{lk} may depend on the variables x, y, z, t, Q_i, C_i as well as other components of the ecosystem. In the latter case it is necessary to add to equations (1) and (2) some further equations for closing the system.

In addition, it is necessary to set the boundary conditions for the botton and surface of the basin. These more accurate definitions are required for the concrete calculations. For the purposes of this purely qualitative analysis it is enough to limit oneself to consideration of the equations (1) and (2) with conditions (3) and (4). Interaction of concentration Q_i and C_i in the FZ runs in such a way that while C_i increases from zero point, the R_{1i} increases and F_{1i} decreases; as a result Q_i decreases. The Q_i concentrations cannot be lower than some critical range Q_i* within which corresponding populations perish. From the qualitative analysis of equations (1)-(4) it follows that some values of c_i* on the boundary B_1 of FZ correspond to Q_i^* .

As a result of the absorption and the disintegration of c_i concentrations, their decrease is observed in the direction from the source to the PZ periphery. That is why the PZ boundary or the FZ location will be the curve on which $c_j = c_j$* .

The c_j* concentrations will be called "frontal concentrations". They are determined by analysis of mathematical model ESUZ. In particular, in some cases, the appraisal of c_j* values may be carried out on the basis of qualitative analysis of equations.

In consequence, the movement of the PZ boundary may be considered as the process of a large-scale diffusion of concentration c_j from point source. This process is expressed by the equation (2) where C_j is changed by c_j with boundary condition

$$c_j = 0 \qquad \text{at} \qquad c_j \leq c_j^* \quad . \tag{5}$$

Thus, this problem is reduced to two more simpler problems--to the calculation of diffusion of c_j concentrations and the qualitative analysis of ESUZ model for estimating the values of c_j*. The further simplification of the problem would be possible if, among c_j components, there will be those concentrations acting as leading factors that have an influence on the ESUZ, and if, among parameters q_j, there will be values "limiting" components. This circumstance essentially allows the limitation of the number of variables considered in the problem, taking into account only the leading and limited variables.

Simplified Version of the Problem

When the stationary currents are absent and the horizontal characteristics of the water exchange are isotropic, the disposition of the polluted zone will correspond to the scheme represented in Figure 1. We investigate the version when the width of FZ is much less than the size of PZ (i.e. $l \ll R$).

Using the conditions (3)-(4), we get the approiximate estimations for terms of equation (1) which take into account the diffusion exchange in FZ:

$$L_{1i}Q_i = D_{1i}\frac{\partial^2 Q_i}{\partial r^2} \simeq \frac{D_{1i}}{l^2}(q_i - 2\bar{Q}_i) \quad . \tag{6}$$

The values of diffusion coefficient D_{1i} may be various for different components. In particular, the diffusion model may also be used for description of the organisms spread in their independent chaotic migration.

$\bar{Q}_i$ in expression (6) designates the mean values of Q_i along the FZ-width.

Initial values of parameters Q_i are taken as their undisturbed values, i.e.

$$Q_i = q_i \qquad \text{with} \qquad t = 0 \quad . \tag{7}$$

Taking Q_i in equation (1) as mean value along FZ-width and taking into consideration expression (6), we get:

$$\frac{dQ_i}{dt} + p_i Q_i = \frac{D_{1i}}{l^2} q_i \quad , \tag{8}$$

$$p_i = R_{1i} + \frac{2D_{1i}}{l^2} - F_{1i} \quad . \tag{9}$$

In expressions for p_i the first two items characterize the loss of components due to mortality and their removal from the FZ by means of diffusion.

If the items which express the decrease of components exceed the F_{1i} value in expression (9), which in itself defines the increase of the components, the following inequality will be valid:

$$p_i \geq P_i = \text{const.} \tag{10}$$

We assume that the steady processes of profile concentration formation in P and U zones proceed much more slowly than those in FZ, and q_i in (8) is constant. Then, from (7), (8) and (10) one may write:

$$Q_i \leq q_i \left(\frac{D_{1i}}{l^2 P_i} + \left(1 - \frac{D_{1i}}{l^2 P_i}\right) e^{-P_i t}\right) \quad . \tag{11}$$

Or, with rather great "t":

$$Q_i \leq q_i \frac{D_{1i}}{l^2 P_i} \quad . \tag{12}$$

According to condition (12), some quantities of P* will correspond to the critical values of Q*.

P_i is an increasing function of C_j, and C_j is an increasing function of boundary value c_j that follows from conditions (4). That is the reason for the correlations

$$c_j \geq c_j^* \ , \quad C_j \geq C_j^* \ , \quad C_j^* = C_j(c_j^*) \tag{13}$$

the following inequality will correspond to

$$Q_i \leq Q^* = q_i \frac{D_{1i}}{l^2 P_i^*} \quad , \qquad P_i^* = P_i^*(C_i^*) \quad ,$$

i.e.

$$Q_i \leq Q^* \quad \text{when } c_j \geq c_j^* \quad . \tag{14}$$

When the c_j concentration is spread from the source point, their values increase at a fixed distance from the source starting from zero. When c_j will attain the c_j^* value range, the destruction of populations characterized by parameters Q_i (in FZ) and the shift of PZ in the direction to UZ will occur. It is c_j conentrations that we consider as the "frontal concentrations". They are determined by analysis of mathematical model ESUZ. In particular, in a number of cases c_j* estimation is to be carried out on the basis of qualitative equation analysis as the given example shows.

In accordance with condition (5), the c_j solution, not equal to zero, is within PZ limits. In this case, as it was pointed out earlier, the meanings of R_{2j} and F_{2j} expressions in equation (2) are selected to be suitable for a steady regime. As we have noted above, both processes in the P and U zones are in the steady regimes, i,e. they are not accompanied by the qualitative change of corresponding ecological systems. The unsteady regime of the conversion of one ecosystem type into another occurs in FZ. In this case the process is characterized by the definite time of relaxation t_R. In the example considered, the time of relaxation according to the inequality (11) is estimated by the condition $P_i t_R \simeq 3$. In order to make the described scheme valid, the characteristic diffusion time of c_j (time of the formation of c_j profile) must be much longer than relaxation time t_R of processes in the frontal zone. This condition is not realized at small t. However, in this case the FZ is located directly around the source, surrounded by the UZ. As far as the PZ is concerned, it is not yet formed. In this case, the problem is reduced to analysis of processes in FZ.

The conclusion on the existence of "frontal concentration" may be made in a more general case, when "l" does not satisfy the condition $l \ll R$. After solving diffusion problem (1)-(4) we shall have the following expression for Q_i:

$$Q_i = Q_i\ (q_i,\ c_j,\ x,y,z,t) \quad .$$

The Q_i is a decreasing function of c_j when q_i, x,y,z,t are fixed. From this, the inequality (14) will follow.

The Basic Principles of Method

The question of the PZ boundary shift is reduced to two problems:

1. The estimation of "frontal concentrations" c_j* on the boundary B_1.

   This is the boundary concentration C_j* to which in FZ the critical concentration C_j* destruction of the ecosystem will occur.
   The previous example shows that the problem of c_j* estimation, in a number of cases, may be solved by qualitative analysis of mathematical model equations of UZ ecosystem provided the data on c_j effect their characteristics R_{1j} and F_{1j} is available.

2. The problem of the PZ boundary position estimation where $c_j = c_j$*.
   This problem is solved on the basis of information on distribution of values in the field of water exchange characteristics in the space, i.e. currents and turbulence.

   The c_j* values depend on the viability of ESUZ. If the change of ESUZ characteristics lead to increasing c_j*, then the PZ boundary shifts towards the pollution source, i.e. the attack of UZ and shortening of PZ takes place. The opposite effect will occur if c_j* is decreasing. The expansion or reduction of PZ will occur under the intensification or weakening of the source pollution power.
   The considered scheme allows tracing the influence of water exchange characteristics on the evolution of the PZ. In this case, it is necessary to take into consideration the water exchange in scales of P zone, that make c_j transfer towards FZ.

Analysis of the Simplified Model of the Process

To give the reader a simplified example, let us assume that the limiting component in the ESUZ is concentration q. When q is destroyed the circulation of substances within the ecosystem breaks down and the ecosystem itself ceases to exist. For example, "q" may designate total biomass concentration of the populations of different species that have a similar behaviour in the ecosystem. At the same time "q" may denote nonliving matter (e.g. it may be the oxygen concentration).

Let us suppose further that the "c" concentration plays a role of leading geochemical factor among pollutants. The Q and C concentrations in the FZ may be described by equations:

$$\frac{dQ}{dt} + p_1 Q = \frac{D}{l^2} q_B \quad , \qquad (15)$$

$$\frac{dC}{dt} + p_2 C = \frac{D}{l^2} c_B \quad , \qquad (16)$$

$$C = 0 \quad , \qquad Q = q \qquad \text{at} \quad t = t_0 \quad . \qquad (17)$$

$$p_1 = \frac{2D}{l^2} + \lambda_1 C - F_1 \quad , \qquad p_2 = \frac{2D}{l^2} + \lambda_2 Q \quad ,$$

$$q_B = q(x,y,z,t') \quad \text{at} \quad x,y,z \in B_2 \quad ,$$

$$c_B = C(x,y,z,t') \quad \text{at} \quad x,y,z \in B_1 \quad ,$$

$$t' = t_o + t_R/2 \quad .$$

The decreasing rate of the Q component is taken in proportion to C. The decreasing rate of C, which is determined by the ecosystem ability for selfpurification is taken in proportion to Q; λ_1, λ_2 are appropriate coefficients of proportionality. For real-object calculation it is necessary to consider p_1 and p_2 as being described by empirical or theoretical expressions reflecting their dependence on "C" and "Q" respectively.

The equations (15), (16) describe the oscillation process. The oscillation of C and Q occurs close to the stationary solutions of the system (15), (16). The destruction of the ecosystem will undoubtedly take place if the stationary solution for Q is

The stationary solutions of equations (15), (16) satisfy the following equations:

$$Q = \frac{kq_B}{(2k-F)+\lambda_1 C} \quad , \quad C = \frac{kc_B}{2k+\lambda_2 Q} \quad , \quad k = \frac{D}{1^2} \quad . \tag{18}$$

According to these equations, C increases and Q decreases when c_B grows. That is why the condition on $c_B \geq c_B{}^*$ is in accordance with the condition $Q \leq Q^*$ and, in addition, $c_B{}^*$ is defined by Q^*:

$$c_B{}^* = \frac{1}{\lambda_1} \left(\frac{q_B}{Q^*} - \frac{2k-F}{k}\right) (2k + \lambda_2 Q^*) \quad . \tag{19}$$

Thus, the value of frontal concentration $c_B{}^*$ is estimated by formula (19). The value 1 or width FZ is included into the expression (19). The approximate estimation of 1 is possible from the condition $\left(\frac{\lambda_2 \bar{q}}{D}\right)^{1/2} = 1$, which follows from the approximate solution for distribution of C across the FZ:

$$c(x) \simeq c_B e^{-\sqrt{a}\, x} \quad , \quad \alpha = \frac{\lambda_2 \bar{q}}{D} \quad , \tag{20}$$

where the axis Ox is taken across FZ, and its origin is on B_1, $\bar{q}$-- the characteristic Q value in FZ. For example, one may choose $\bar{q} = q_B/2$.

For defining PZ values, it is necessary to solve the equation (2) with condition (5), where C is changed by c and c* is determined by formula (19). If c* is as small as compared to most values of c in PZ, the condition (5) must be changed by the requirement of c regularity with $r \to \infty$.

For the homogenous one-dimensional diffusion, the solution of the problem on field c(x,y,z,t) is given by the following formula:

$$c(x,y,t) = \frac{I\, e^{\frac{Vx}{2D}}}{\Pi D} \int_0^t \exp\left(- \frac{r^2}{4Dr} - \beta\tau\right) \frac{d\tau}{\tau}, \quad \beta = \frac{V^2}{4D} - \lambda \quad . \tag{21}$$

where

$r = \sqrt{x^2+y^2}$, I -- contamination source power,

λ -- coefficient of c decreasing in PZ conditioned by decay and absorption,

V -- current rate along the coast along the axis Ox.

The stationary solution is defined by formula

$$c(x,y) = \frac{I}{\Pi D}\, e^{\frac{V\kappa}{2D}}\, K_0(x\ r), \quad \kappa^2 = \frac{V^2}{4D^2} + \frac{\lambda}{D} \quad , \tag{22}$$

where K_0 is Macdonald's function.

Placing the values c_B* into left parts of expressions (21) and (22), and solving the inverse problems relative to x,y, we shall estimate, in the first case, the movement of PZ boundary, and in the second one, its stationary position (Fig. 2).

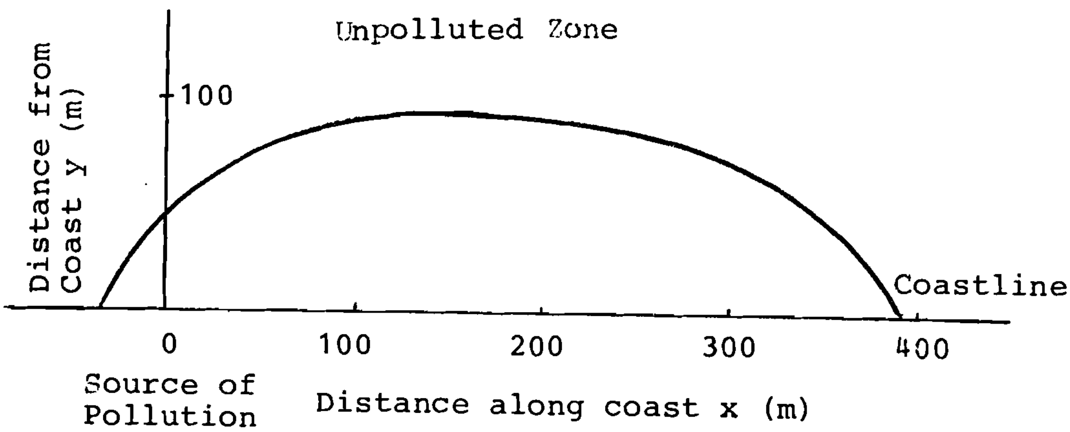

Fig. 2. The boundary position of the frontal zone calculated for a stationary case by the formula (22) with

$c_B* = 12{,}8 \cdot 10^{-7} kg\ cm^{-2}$,

$I = 1\ kg\ sec^{-1}$; $V = 0{,}1\ cm\ sec$;

$D = 1\ m^2 sec^{-1}$; $\lambda = 5{,}25 \cdot 10^{-4} sec^{-1}$, The described procedure for solving the problem, which has just been cross-checked, will be valid for an even more precise solution when a number of factors which characterize conditions of a given basin are taken into account. In this case, the usual hydrological methods and data must be used.

Some Generalization of the Proposed Method

The considered scheme of behaviour of pollution zone boundaries around the coastal point source of pollution is valid in the case of interaction between one leading ecosystem component and a single limiting one. It is possible to generalize the considered scheme for the case when there are a number of such components.

Let us have the C_i $(i = \overline{1,N})$ as the leading pollution components and Q_j $(j = \overline{1,M})$ as the limiting ecosystem components disturbed even by one of the leading pollution components.
On the basis of an ecosystem model that takes into account all interactions between C_i and Q_j, it is necessary to draw up the following table:

i	f_i	Q_1	Q_2	. . . .	Q_M
1	C_1	T_{11}	T_{21}	. . . .	T_{M1}
.	.	.	.	. . . .	
.	.	.	.	. . . .	
.	.	.	.	. . . .	
N	C_N	T_{1N}	T_{2N}	. . . .	T_{MN}
N+1	$C_1\ C_2$	T_{112}	T_{212}	. . . .	T_{M12}
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.
n	$C_i\ C_j\ C_k$	T_{1ijk}	T_{1ijk}	. . . .	T_{Mijk}
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.
m	$C_i \ldots\ldots C_2$	$T_{1,i\ldots 1}$	$T_{2,i\ldots 1}$	. . . .	$T_{Mi\ldots 1}$
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.
.	.	.	.	. . . .	.

Here T_{ij} is time of destruction of the ecosystem by the effect of C_i component on Q_j component; T_{1ij} is time of destruction caused by the effect of $C_i \propto C_j$ combination on Q_1, etc. The components and their combinations shall be considered as f_k factors. The components and their combinations may have a number of values (we may take into account the discrete levels of these values), to which a number of T_{ij}, T_{1ij},... values will correspond. If the interaction between C_k and Q_i, or between combinations of a number of C_i and Q_i are absent, we shall suppose that $T_{(\ldots)} = \infty$.

Naturally, the scheme would be very complicated if it were necessary to make the whole overview of possible combinations for evaluation of all $T_{(\ldots)}$ values.

We suppose that the number of combinations which correspond to limited $T_{(\ldots)}$ values is not very great, and these combinations may be taken from any additional assumptions. Nevertheless, we have to take into account that there is a possibility of any combination in existance in order not to lose the actual acting factors among them. All these possible acting factors must be reflected in the mathematical ecosystem model. The parameters C_i must accordingly exist in the model, in the expressions for its coefficients and terms. $T_{(\ldots)}$, the destruction time of the ecosystem (or the time of its quality changes), depends on C_i concentration (see Fig. 3).

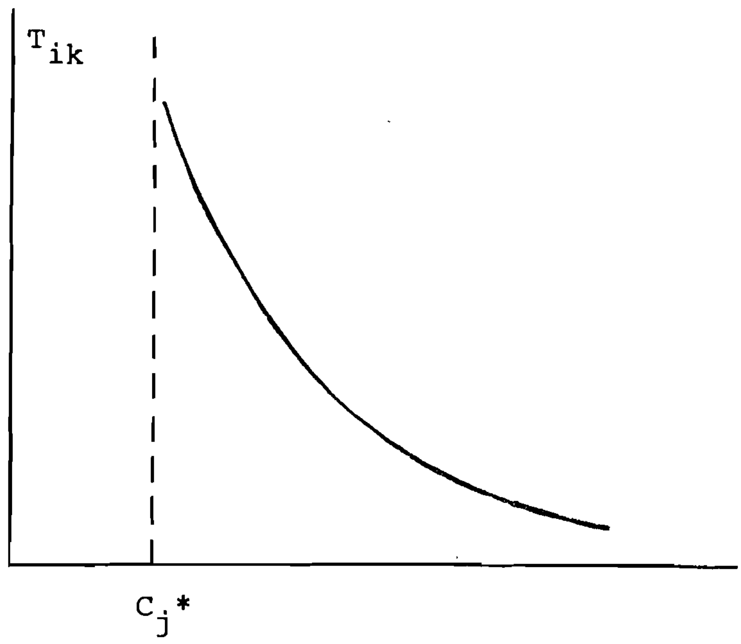

Fig. 3. The dependence of destruction time of the ecosystem on the concentration of leading components.

When the combination of leading pollution components affect the ecosystem, the abscissa axis is substituted by a multi-dimensional plan which has the same number of dimensions as that of leading components. Instead of the point along the abscissa axis that corresponds to C_j*, we shall have a multi-dimensional critical surface that represents a section of the space with C_i coordinates which correspond to value $T_{(\ldots)}$.

As a result of destruction and absorption of C_i components, their values decrease with the distance from pollution source. The factor f_k which, at the fixed levels of C_i concentrations in pollutions, corresponds to the minimal destruction time of ecosystem $T_{(\ldots)}$ will act close to the pollution source. If the other factors were absent, then this factor would form the pollution zone up to such a distance at which, owing to decrease of its C_j concentrations, the following condition $T_{(\ldots)} = \infty$ can be reached.

Under other factors, the following situation is possible: when the rate of action for the first, originally more rapid, factor will decrease more quickly with the increase of distance than the action rate for other originally more slowly acting factors. Starting with the moment when the destruction time of ecosystems of the first and second factors is equal, the second factor will act, and only this factor must be taken into account in the problem.

Further, in a similar way, the third and consecutive factors may begin to act until the distance from the source of pollution will be so large that concentrations of all factors will decrease to values at which it will tend to ∞. The last factors for which this condition will be fulfilled will determine the final size of pollution zone by the value of its frontal concentration. Thus, the pollution zone in the general case will consist of the zones formed by consecutively acting factors (see Fig. 4).

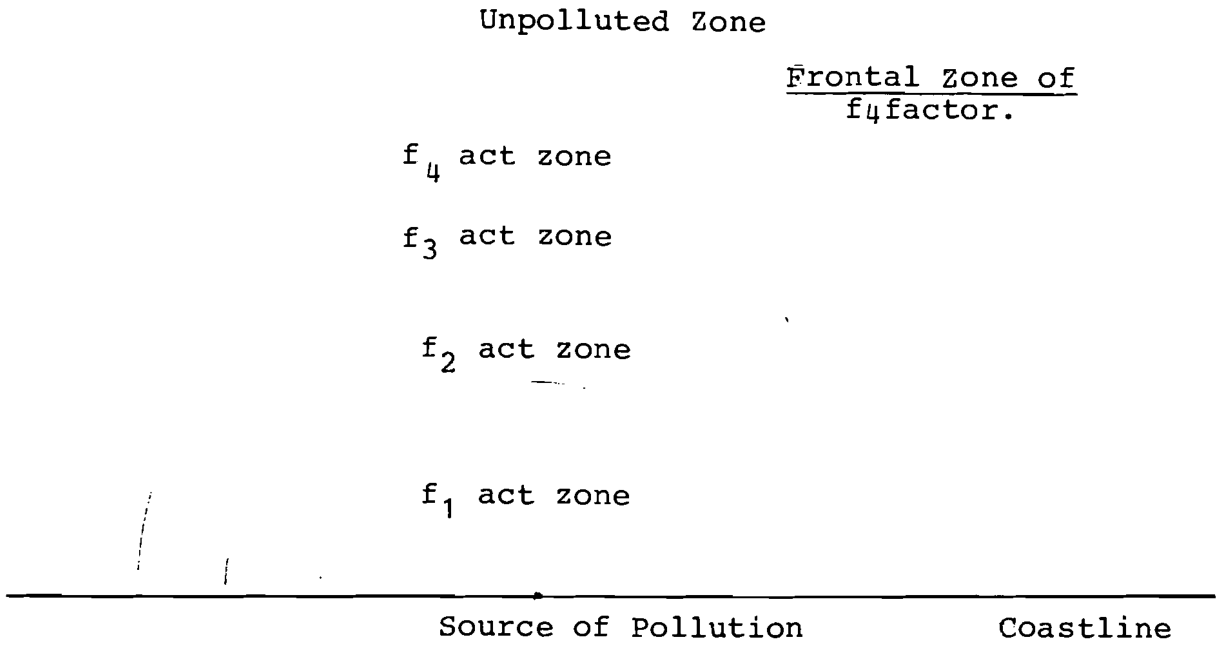

Fig. 4. The pollution zone formed by consecutively acting leading factors: f_1, f_2, f_3 and f_4.

The external boundary for effect of f_i zone is determined by values of C_i* concentrations that comprise f_i factor, by which the destruction time of ecosystem will be larger than that for f_{i+1} factor. Actually it is possible owing to the C_i concentration decrease in the direction towards periphery of polluted zones, the number of consecutively acting factors will not be too large. However, with many factors we shall have the considerable simplification of the problem so far as it is reduced to a set of consecutively solving problems on the movement of fixed values of polluting substance concentrations.

It is also possible that the most rapidly acting polluting components have, as a rule, the most far reaching frontal values. In this case, the frontal concentration movement of a single leading component will completely determine the behaviour of pollution zone boundary.

However, the correct conclusion about such mechanisms of component action will be found when we take into consideration the possibility of the above mentioned combination of many factors affecting the ecosystem.

The other natural generalization of the described problem consists of the substitution of the point source by space distributed one. This generalization is not a simple one and it must be carried out within the framework of the diffusion problems which can be solved by known methods.

Conclusion

1. The described approach to the solution of the problem of pollution zones in water bodies is applicable naturally only in those cases when such zones form and have considerable size. The pollution zone is an area where some of the components of the ecosystem are disturbed by pollution and a new ecosystem appears which is qualitatively different from the previous one--the ecosystem of the pollution zone. Thus, in the water body, three qualitatively different ecosystems are observed (ESPZ, ESFZ, ESUZ), which are described by different mathematical models. The boundaries between these ecosystems may be mobile.

2. The analysis of the frontal zone processes for the determination of the frontal concentration values may be carried out in a number of cases by qualitative methods. In particular, when the width of frontal zone "l" is small its value may be used as a small fixed parameter for the simplifcation of this problem. In the general case, the determination of the fontal concentration values may be carried out by a method for marine ecosystem modelling which was described in a previous paper (V.I. Belyaev, 1976).

3. In those cases when the size of the pollution zone is small and therefore the frontal zone occupies a large part of the affected area, the solution of the problem will be reduced to research on the processes in the frontal zone. In this case we are interested in more precise evaluations of the size of the frontal zone disturbances of the ecosystem components and pollution substances within it, etc. For the solution of these problems the abovementioned method may also be used.

(V.I. Belyaev, 1976)

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