RM-77-57

MODEL MIGRATION SCHEDULES AND THEIR APPLICATIONS

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Preface

Interest in human settlement systems and policies has been a critical part of urban-related work at IIASA since its inception. Recently this interest has given rise to a concentrated research effort focusing on migration dynamics and settlement patterns. Four sub-tasks form the core of this research effort:

I. the study of spatial population dynamics;

II. the definition and elaboration of a new research area called demometrics and its application to migration analysis and spatial population forecasting;

III. the analysis and design of migration and settlement policy;

IV. a comparative study of national migration and settlement patterns and policies.

This paper, the thirteenth in the dynamics series, is a summary review of IIASA's work on model migration schedules. It combines the major findings of two earlier publications (RM-75-57 and RR-76-09) with more recent, as yet, unpublished work.

Related papers in the dynamics series, and other publications of the migration and settlement study, are listed on the back page of this report.

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Abstract

This paper considers and contrasts two alternative approaches for capturing the regularities exhibited by age patterns in observed migration rates. The mortality approach is considered first and it is shown how such an approach may be used to infer migration flows from two consecutive place-of-residence-by-place-of-birth census age distributions. The fertility approach is considered next, and techniques for graduating migration age profiles are described. The advantages and disadvantages of both approaches are then briefly assessed.

Acknowledgements

The authors gratefully acknowledge the computational assistance of Frans Willekens. In carrying out this research we also have benefited from discussions with Michael Stoto, Ansley Coale, and Frans Willekens. This study could not have been carried out without the generous provision of detailed Swedish population data by Arne Arvidsson of the Swedish Central Bureau of Statistics. Our thanks also go to Larry Long of the U.S. Census Bureau, and Kazimierz Dziewonski and Piotr Korcelli of the Geographical Institute of the Polish Academy of Sciences for providing us with U.S. and Polish migration data, respectively.

Model Migration Schedules and their Applications

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Model Migration Schedules and their Applications

1. INTRODUCTION

The evolution of a human population undisturbed by emigration or immigration is determined by the fertility and mortality schedules it has been subject to. If such a "closed" population system is disaggregated by region of residence, then its spatial evolution is largely determined by the prevailing schedules of internal migration.

The age-specific fertility, mortality, and migration schedules of most human multiregional populations exhibit remarkably persistent regularities. The age profiles of these schedules seem to be repeated, with only minor differences, in virtually all developed and developing nations of the globe. Consequently, demographers have found it possible to summarize and codify such regularities by means of hypothetical schedules called *model* schedules.

Model schedules have two important applications: 1) they may be used to infer (or "smooth") empirical schedules of populations for which the requisite data are lacking (or inaccurate), and 2) they can be applied in analytical mathematical examinations of population dynamics.

The development of model fertility and model mortality schedules and their use in studies of the evolution of human populations have received considerable attention (Arriaga, 1968; Coale and Demeny, 1966; Coale, 1972; Coale and Trussell, 1974; Rele, 1967); however, the construction of model *migration* schedules and their application to studies of the *spatial* evolution of human populations disaggregated by region of residence, have not. This paper addresses the latter problem and shows how techniques that have been successfully applied to treat the former problem can readily be extended to deal with the latter. We begin, in

Section 2, by considering the regularities exhibited by observed migration schedules. We then follow this description of observed regularities with an examination, in Sections 3 and 4, respectively, of two alternative approaches for summarizing such regularities: the mortality approach and the fertility approach. Section 5 offers concluding remarks and points to future directions for research.

2. REGULARITIES IN OBSERVED MIGRATION SCHEDULES

Demographers have long recognized that persisting regularities appear in empirical age-specific migration schedules (e.g., Lowry, 1966; Long, 1973). Migration viewed as an event, is highly selective with regard to age, with young adults generally being the most mobile group in any population. Levels of migration are also high among children, varying from a peak during the first year of age (the *initial peak*) to a *low point* around age 16. The migration age profile then turns sharply upward until it reaches a second peak (the *high peak*) in the neighborhood of 22 years, after which it declines regularly with age, except for a slight hump (the *retirement peak*) around ages 62 through 65.

The regularities in observed migration schedules are not surprising:

> Young adults exhibit the highest migration rates because they are less constrained by ties to their community. Their children generally are not in school, they are more likely to be renters rather than home owners, and job seniority is not yet an important consideration. Since children move only as members of a family, their migration pattern mirrors that of their parents. Consequently, because younger children generally have younger parents, the geographical mobility of infants is higher than that of adolescents. Finally, the small hump in the age profile between ages 62 to 65 describes migration after retirement... (Rogers, 1975, pp. 146-147)

2.1 Migration Profiles

The shape, or *profile*, of an age-specific schedule of migration rates is a feature that may be usefully studied independently of its intensity, or *level*. This is because there is considerable empirical evidence that although the latter tends to vary significantly from place to place, the former is remarkably similar in various localities. Illustrations of this property appear in Figures 1A and 1B, which set out migration rates for the U.S.A. and Sweden, respectively.

Figures 1A and 1B repeat the fundamental age pattern of migration described above, with peaks occuring at infancy, during the young adult ages and, in one instance, at retirement. Variations in the location of the high peak and in the levels of migration at retirement ages indicate that as, in the case of mortality, age profiles of migration may be usefully disaggregated into families that are distinguished by the location and relative height of their peaks. Alternatively, such a disaggregation may be carried out, in the manner of fertility schedules, by means of the *mean age* of the migration schedule M(x),

$$\bar{n} = \sum_{x=0}^{z} (x + 2.5)\, M(x) \Bigg/ \sum_{x=0}^{z} M(x) \quad ,$$

which readily may be used to classify migration schedules into "young" and "old" categories, perhaps with suitable gradations in between.

Figure 1A indicates that the age profile of migration tends to be remarkably similar for residential movers, intra- and inter- county migrants, and migrants between states. What does vary is the level of migration, the level being higher for smaller territorial units.

Figure 1B shows that important age-specific variations exist between the migration rates of males and females. The high peak for males follows that of the female schedule by a few years, and in the Swedish case it is also lower in height.

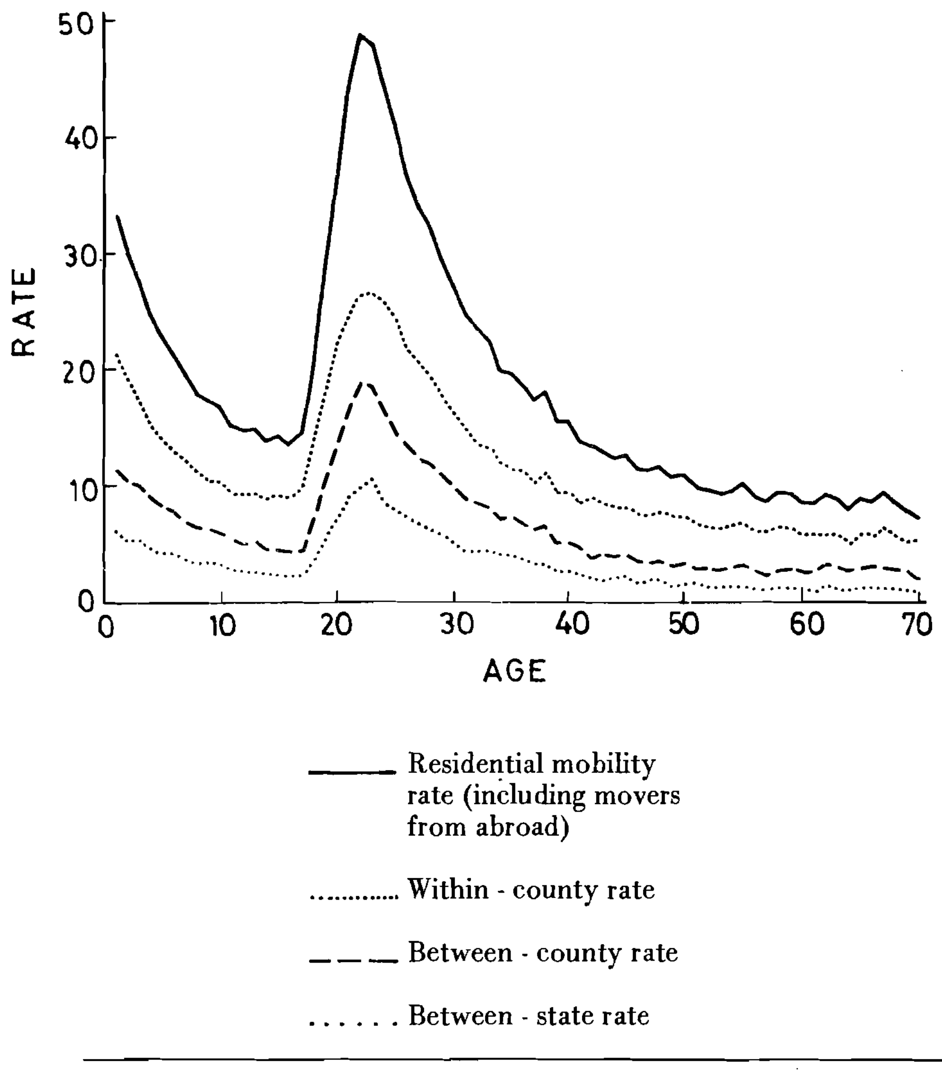

Figure 1.A: Age-Specific Annual Migration Rates of the Total United States Population by Category of Move: Average of 1966-1971.

Source: Long (1973), p. 38.

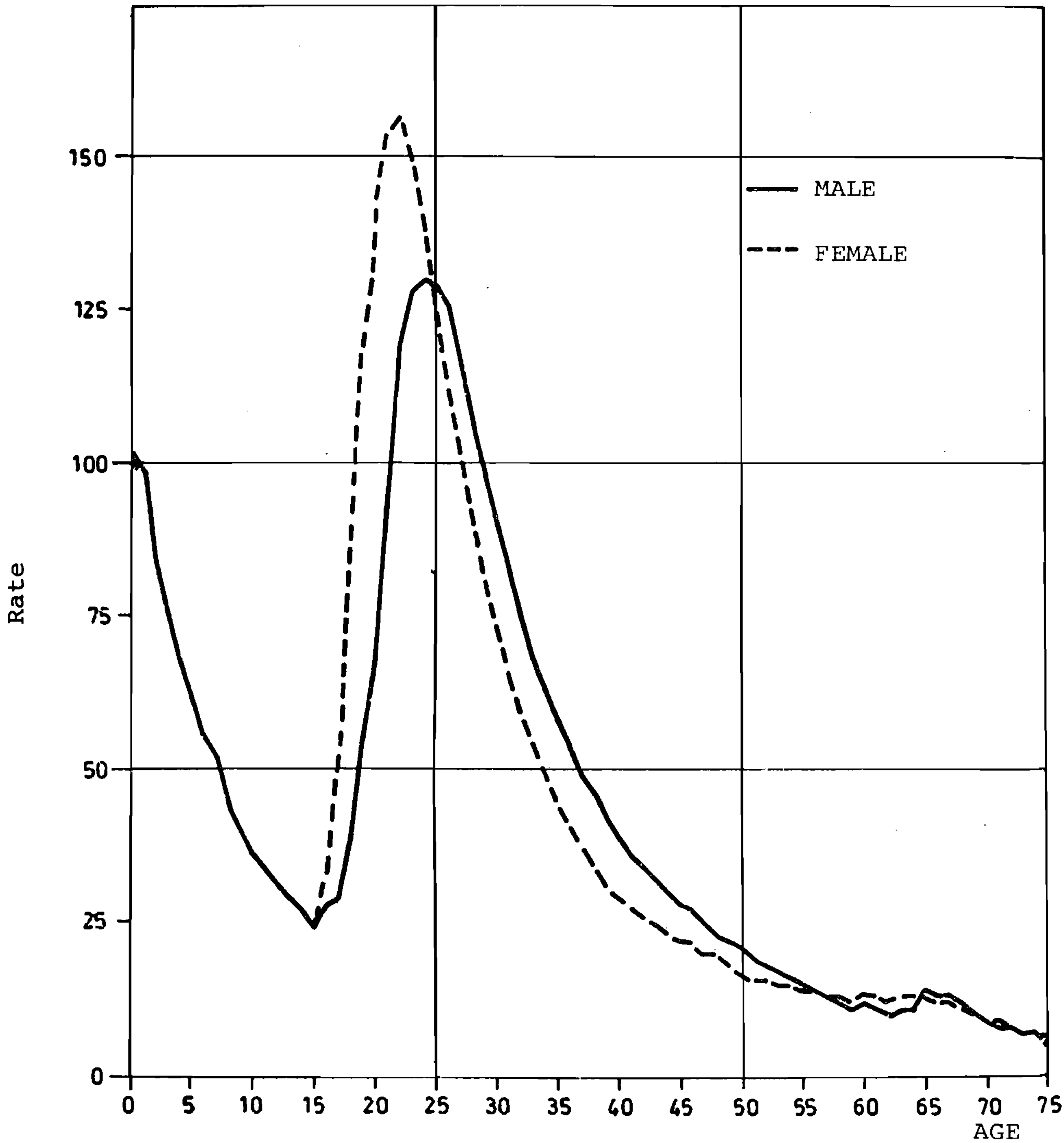

Figure 1.B: Age-Specific Annual Migration Rates of the Swedish Population by Sex: Average of 1968-1973.

Source: Internal Migration in Sweden 1968-1973, 1974, p. 10.

2.2 Migration Levels

The level of migration, like that of *mortality*, can be measured in terms of an expected duration time, for example, the fraction of a lifetime that is expected to be lived at a particular location. However, like *fertility*, migration is a potentially repetitive event, and its level therefore can be expressed in terms of an expected number of moves per person. Summarizing observed regularities within the context of the former perspective leads to the development of a regression approach similar to the one used by Coale and Demeny (1966) to summarize regularities in mortality schedules; the latter perspective suggests an alternative procedure--one analogous to that used by Coale and Trussell (1974) to summarize fertility schedules.

The most common demographic measure of level is the notion of *expectancy*. Demographers often refer to life expectancies, for example, when speaking about mortality, and to reproduction expectancies when discussing fertility. They have calculated for instance that 73 is the average number of years a female could expect to live under the mortality schedule of the U.S. in 1958, and 1.71 is the average number of baby girls she could expect to bear during her lifetime under the then prevailing fertility schedule. The former measure is known as the *expectation of life at birth*, e(0); the latter index is called the *net reproduction rate*, NRR.

A related index is the *gross reproduction rate*, GRR. This measure totally ignores the effects of mortality on reproduction and may be viewed as the net reproduction rate that would arise among a cohort if all of them survived to the end of their child-bearing ages. For this reason, the GRR of a population is, of course, always larger than the corresponding NRR.

Expectancies also have been used in migration studies by Wilber, (1963) and Long (1973). However, their definitions are *nonspatial* inasmuch as they view migration as an *event* in a national population rather than as a *flow* between regional populations. The study of *spatial* population dynamics can be considerably enriched by explicitly identifying the *locations* of events and flows. This permits one to define spatial expectancies such as the expectation of life at birth or the net reproduction rate of individuals born in region i (respectively, ${}_{i}e(0)$ and ${}_{i}NRR$, say), and the expected allocation of this lifetime or rate among the various constituent regions of a multiregional population system (${}_{i}e_{j}(0)$ and ${}_{i}NRR_{j}$, respectively, $j = 1,2,\ldots,m$). For example, it has been estimated (Rogers, 1975) that the expectation of life at birth of a California-born woman exposed to the 1958 U.S. schedules of mortality and migration would be 73.86 years, out of which 24.90 years would be lived outside of California. The net reproduction rate of such a woman, on 1958 fertility rates, would be 1.69, with 0.50 of that total being born outside of California.

Adopting the second perspective, Wilber developed a set of migration expectancies describing the average number of moves experienced by an individual during his remaining lifetime. The application of his formula for calculating migration expectancies for individuals just born produces the direct analog of the conventional formula for the net reproduction rate. The formula, with x set equal to zero, may be expressed as

$$\sum_{x=0}^{z} L(x)M(x) \quad , \tag{1}$$

where L(x) is the stationary life table population aged x to x + 5 years at last birthday, M(x) is the annual rate of migration among individuals in that age group, and z is the starting age of the last interval of life. The corresponding formula for the net reproduction rate is

$$NRR = \sum_{x=0}^{z} L(x)F(x) \quad , \tag{2}$$

where F(x) is the age-specific fertility rate. The similarity between (1) and (2) suggests the designation of (1) as the *net* *migraproduction rate*, a quantity we shall denote by NMR. Thus NRR denotes the average number of *babies* per person, and NMR denotes the average number of *moves* per person, both taken over that person's entire lifetime. Observe that both measures depict the average number of occurrences of a recurrent event over an individual's lifetime. Only the latter, however, is influenced by the spatial extent of the territorial unit.

Earlier we proposed a spatial migration expectancy based on *duration* times, specifically, the expected number of years lived in region j by individuals born in region i. The correspondence between the net migraproduction and net reproduction rates suggests an *alternative* definition of spatial migration expectancy--one reflecting a view of migration as a recurrent event. Just as NRR was apportioned among the constituent regions of a multiregional system, so too can NMR be similarly disaggregated by place of birth and residence. Thus the formula for the spatial net reproduction rate:

$$ {}_{i}NRR_{j} = \sum_{x=0}^{z} {}_{i}L_{j}(x) F_{j}(x) \quad , \qquad (3) $$

suggests the following definition for the *spatial net migraproduction rate:*

$$ {}_{i}NMR_{j} = \sum_{x=0}^{z} {}_{i}L_{j}(x) M_{j}(x) \quad , \qquad (4) $$

where ${}_{i}L_{j}(x)$ denotes the stationary life table population of region j aged x to x + 5 years at last birthday and born in region i, and $M_{j}(x)$ is the age-specific outmigration rate in region j.

The spatial net migraproduction rate ${}_{i}NMR_{j}$ describes the average lifetime number of moves made out of region j by an individual born in region i. The summation of ${}_{i}NMR_{j}$ over all regions of destination ($j \neq i$) gives ${}_{i}NMR$, the net migraproduction rate of individuals born in region i, i.e., the average number of moves an i-born person is expected to make during his (or her) lifetime.

Associated with the concept of the *net* reproduction rate (NRR) in fertility analysis is the notion of the *gross* reproduction rate.

$$GRR = 5 \sum_{x=0}^{z} F(x) \quad .$$

The notion of a *gross migraproduction rate*

$$GMR = 5 \sum_{x=0}^{z} M(x)$$

has a similarly useful interpretation in migration analysis. It measures the intensity of migration between two regions at a particular point in time. The measure, therefore, has basically a cross-sectional character, in contrast to the NMR which measures the intensity of migration over a lifetime. Consequently, the *gross* migraproduction rate often may prove to be a more useful measure than the net rate in that it is a "purer" indicator of migration, in the same sense as the gross reproduction rate. However, the gross rate measures the intensity of migration at a given moment and not over a lifetime. Hence, in instances where return migration is an important factor, the gross rate and the net rate may give differing indications of geographical mobility.

Table 1 presents net and gross migraproduction rates for the total U.S. population in 1958, disaggregated into four regions. The corresponding mean ages of migration are set out in parentheses. Figure 2 plots the gross migraproduction rate against the mean age for the migration schedules of the four-region U.S. population system in 1958 and in 1968, respectively. We find evidence of a division into four groups:

high GMR - high $\bar{n}$;
high GMR - low $\bar{n}$;

Table 1. Net and gross migraproduction rates and mean ages of migration: total United Stated population, 1958.

A. Net migraproduction rates: ${}_iNMR_j$

Region of Birth	Region of Residence 1	2	3	4	Total
1. Northeast	0.4122	0.0366	0.0589	0.0331	0.5408
2. North Central	0.0204	0.4923	0.0604	0.0600	0.6331
3. South	0.0300	0.0629	0.4397	0.0479	0.5805
4. West	0.0203	0.0540	0.0602	0.4181	0.5526

B. Gross migraproduction rates and mean ages of migration: GMR_{ij} and $\bar{n}_{ij}$

Region of Origin	Region of Destination 1	2	3	4	Total
1. Northeast	-	0.1202 (26.99)	0.3168 (33.46)	0.1532 (29.43)	0.5902
2. North Central	0.0891 (28.15)	-	0.3201 (32.16)	0.3289 (30.54)	0.7381
3. South	0.1504 (28.59)	0.2511 (27.77)	-	0.2299 (27.27)	0.6314
4. West	0.0887 (27.73)	0.2167 (30.03)	0.2819 (27.61)	-	0.5873

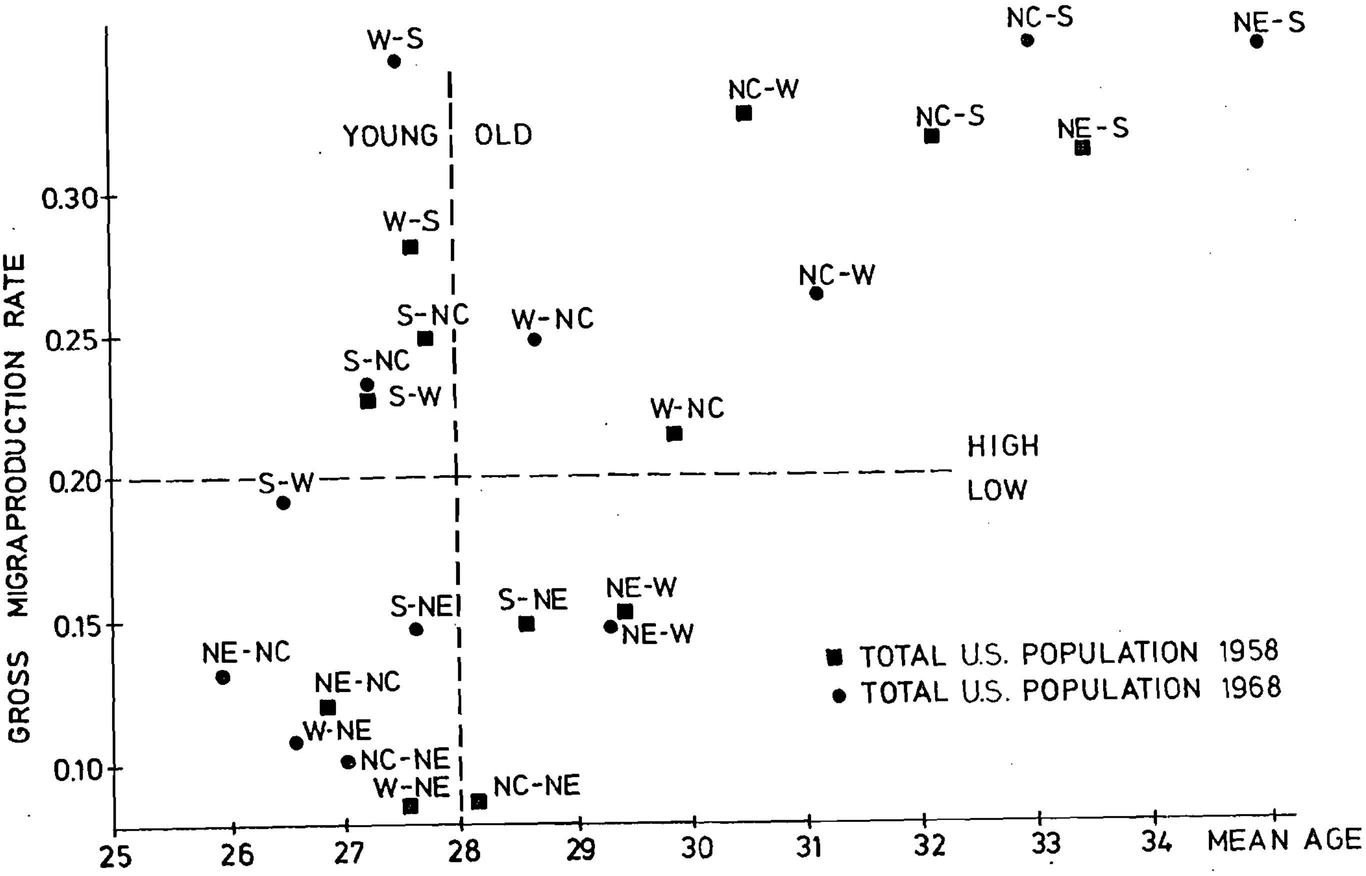

Figure 2: Relation between observed gross migraproduction rate and mean age of migration schedule: total U.S. populations, 1958 and 1968.

low GMR - high $\bar{n}$;

low GMR - low $\bar{n}$.

Migration flows from the North Central Region to the South, for example, exhibit an "old" profile and a mean age of about 32.5 years. The reverse migration flows, on the other hand, take on the shape of a "young" profile and show a mean age that is about five years younger. This suggests that it may be useful to develop a *family* of basic model migration schedules so that the various age profiles exhibited by empirical migration schedules can be more accurately captured and summarized.

3. MODEL MIGRATION SCHEDULES: THE MORTALITY APPROACH

Regularities in the age patterns of observed death rates have fascinated demographers almost since published records of mortality first became generally available. The search for universal "laws of mortality" gave birth to the well known Gompertz-curve graduation of the mortality schedule and, more recently, to two sets of "model" life tables published by the United Nations (United Nations, 1955 and 1967).

The model life table, or mortality, approach for capturing regularities in observed rates may be applied in the study of migration regularities. Such an application, however, first requires the concept of the *multiregional* life table.

3.1 The Multiregional Life Table

Conventional life tables describe the evolution of a hypothetical cohort of babies born at a given moment and exposed to an unchanging age-specific schedule of mortality. For this cohort of babies, they exhibit a number of probabilities of dying and surviving and develop the corresponding expectations of life at various ages.

Life table calculations normally are initiated by estimating a set of age-specific probabilities of dying within each interval of age, q(x) say, from observed data on age-specific death rates, M(x) say. The conventional calculation that is made for an age interval five years wide is (Rogers, 1975, p.12)

$$q(x) = \frac{5M(x)}{1 + \frac{5}{2} M(x)} ,$$

or alternatively,

$$p(x) = 1 - q(x) = [1 + \frac{5}{2} M(x)]^{-1} [1 - \frac{5}{2} M(x)] , \qquad (5)$$

where p(x) is the age-specific probability of surviving from *exact age x* to *exact age x + 5*. The latter probabilities, in turn, may be used to define the corresponding probabilities of survival from one *age group* to the next (Rogers, 1975, pp.16 and 85):

$$s(x) = [1 + p(x + 5)]p(x) \; [1 + p(x)]^{-1} \quad . \tag{6}$$

To avoid any possible confusion between the two sets of probabilities, we shall hereafter refer to s(x) as a survivorship proportion, i.e., the proportion of individuals surviving from *age group x to x + 5* to *age group x + 5 to x + 10*.

One of the most useful statistics provided by a life table is the average expectation of life at age x, e(x) say, calculated by applying the probabilities of survival p(x) to a hypothetical cohort of babies and then observing their average length of life beyond each age. Expectations of life at birth [e(0)] are particularly useful as indicators of the level of mortality in various regions and countries of the world.

Conventional life tables deal with mortality, focus on a single regional population, and ignore the effects of migration. To incorporate the latter, and at the same time to extend the life table concept to a spatial population comprised of several regions, requires the notion of a multiregional life table (Rogers, 1973). Such life tables describe the evolution of several regional cohorts of babies, all born at a given moment and exposed to an unchanging *multiregional* age-specific schedule of mortality and migration. For each regional birth cohort, they provide various probabilities of dying, surviving, and migrating, while simultaneously deriving regional expectations of life at various ages. These expectations of life are disaggregated both by place of birth and by place of residence; they will be denoted by ${}_{i}e_{j}(x)$, where i is the region of birth and j is the region of residence.

Multiregional life table calculations are greatly facilitated by the adoption of matrix algebra. This leads to a compact notation and an efficient computational procedure; it also very clearly demonstrates a simple correspondence between the single region and the multiregional formulas. For example, equations 5 and 6 may be shown to have the following multiregional counterparts (Rogers and Ledent, 1976; Rogers, 1975,p.85):

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}[\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \tag{7}$$

and

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \underset{\sim}{P}(x + 5)]\ \underset{\sim}{P}(x)\ [\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} \tag{8}$$

The diagonal elements of $\underset{\sim}{P}(x)$ and $\underset{\sim}{S}(x)$ are probabilities of survival and survivorship proportions, respectively; the off-diagonal elements will be called probabilities of migrating and migration proportions, respectively.

Expectations of life in the multiregional life table reflect the influences of mortality and migration. Thus they may be used as indicators of levels of internal migration, in addition to carrying out their traditional function as indicators of levels of mortality. For example, consider the regional expectations of life at birth that are set out in Table 2 for the U.S. population with both sexes combined. A baby born in the West, and exposed to the multiregional schedule of mortality and migration that prevailed in 1958, could expect to live an average of 69.94 years, out of which total an average of 8.95 years would be lived in the South. Taking the latter as a fraction of the former, we have in 0.1279 a useful indicator of the (lifetime) migration level from the West to the South that is implied by the 1958 multiregional schedule. (Compare these migration levels with those set out earlier in Table 1).

Table 2. Expectations of life at birth and migration levels by region of residence and region of birth: total United States population, 1958.

Region of Birth	Region of Residence 1	2	3	4	Total
1. Northeast	50.90 (0.7295)	4.49 (0.0643)	8.88 (0.1273)	5.50 (0.0788)	69.76 (1.00)
2. North Central	3.18 (0.0452)	48.45 (0.6889)	9.10 (0.1294)	9.60 (0.1365)	70.32 (1.00)
3. South	4.58 (0.0664)	7.52 (0.1091)	49.21 (0.7134)	7.67 (0.1111)	68.98 (1.00)
4. West	3.18 (0.0454)	6.60 (0.0944)	8.95 (0.1279)	51.22 (0.7322)	69.94 (1.00)

Age-specific probabilities of migrating, $p_{ij}(x)$, in empirical multiregional life tables mirror the fundamental regularities exhibited by observed migration rates. Some of these regularities are illustrated in Figures 3,4, and 5, respectively. (We focus only on the total population but consider data for all four Census Regions and for two points in time: 1958 and 1968.) Figure 3 shows that a strong and positive association exists between the height of the initial peak, $p_{ij}(0)$, and the level of migration as measured by, for example, ${}_i\theta_j$, the fraction of the expected lifetime of an individual born in region i that is expected to be lived in region j. Figure 4 indicates that a similarly strong and positive relationship exists between the height of the low point and the height of the initial peak. Finally, Figure 5 describes the positive association between the heights of the high peak and the low point. Thus a direct line of correlation appears to connect the general migration level between two regions to the values assumed by the corresponding age-specific probabilities of migrating. This suggests that a simple linear regression equation may be used to associate a set of probabilities of migration at each age x, $p_{ij}(x)$, with a single indicator of migration level, say ${}_i\theta_j$. We explore this possibility next.

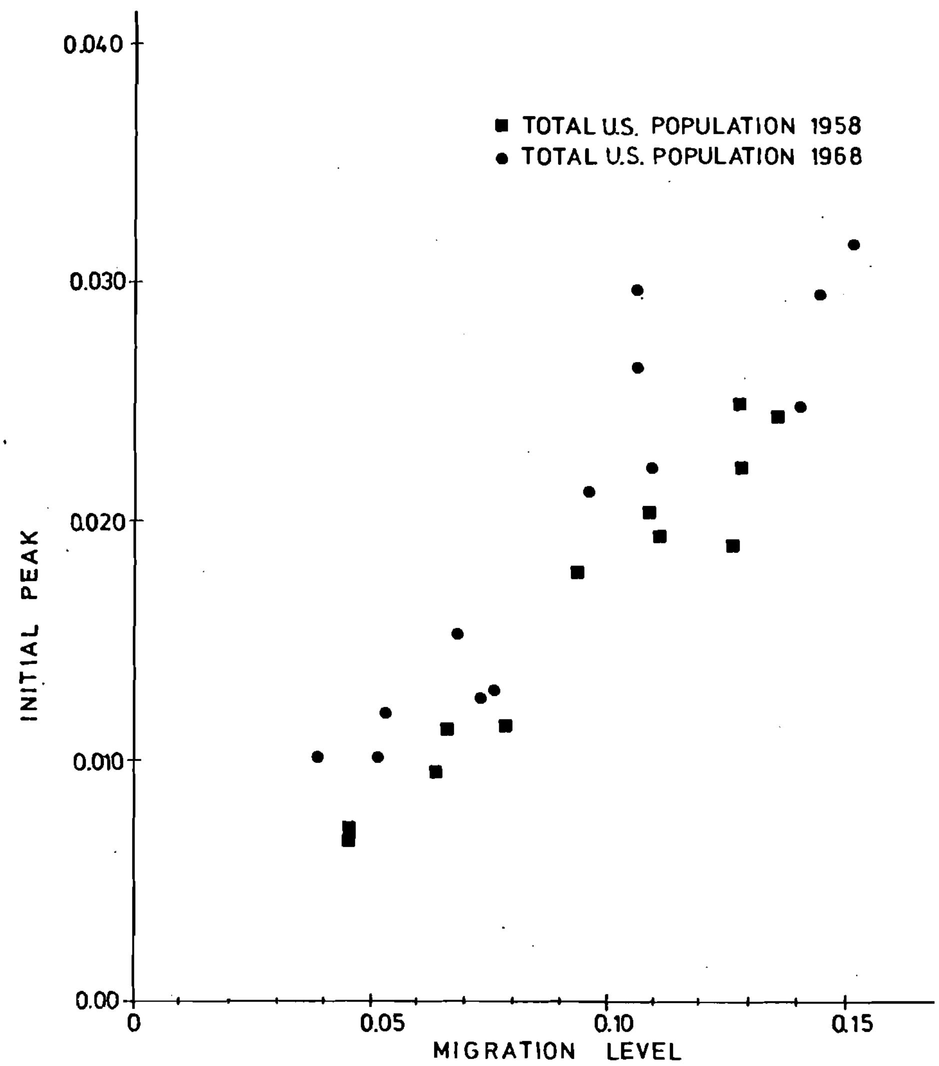

Figure 3. Relation between initial peaks and migration levels in two observed migration schedules.

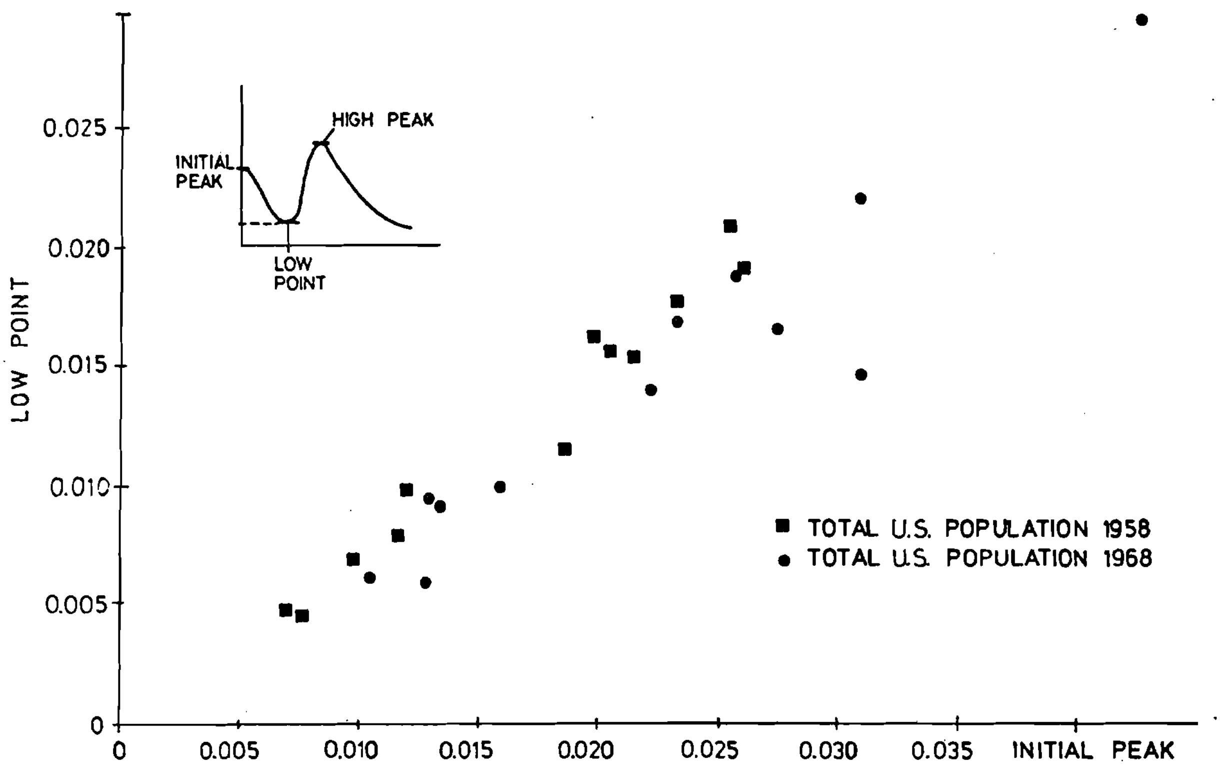

Figure 4. Relation between peaks in migration schedules.

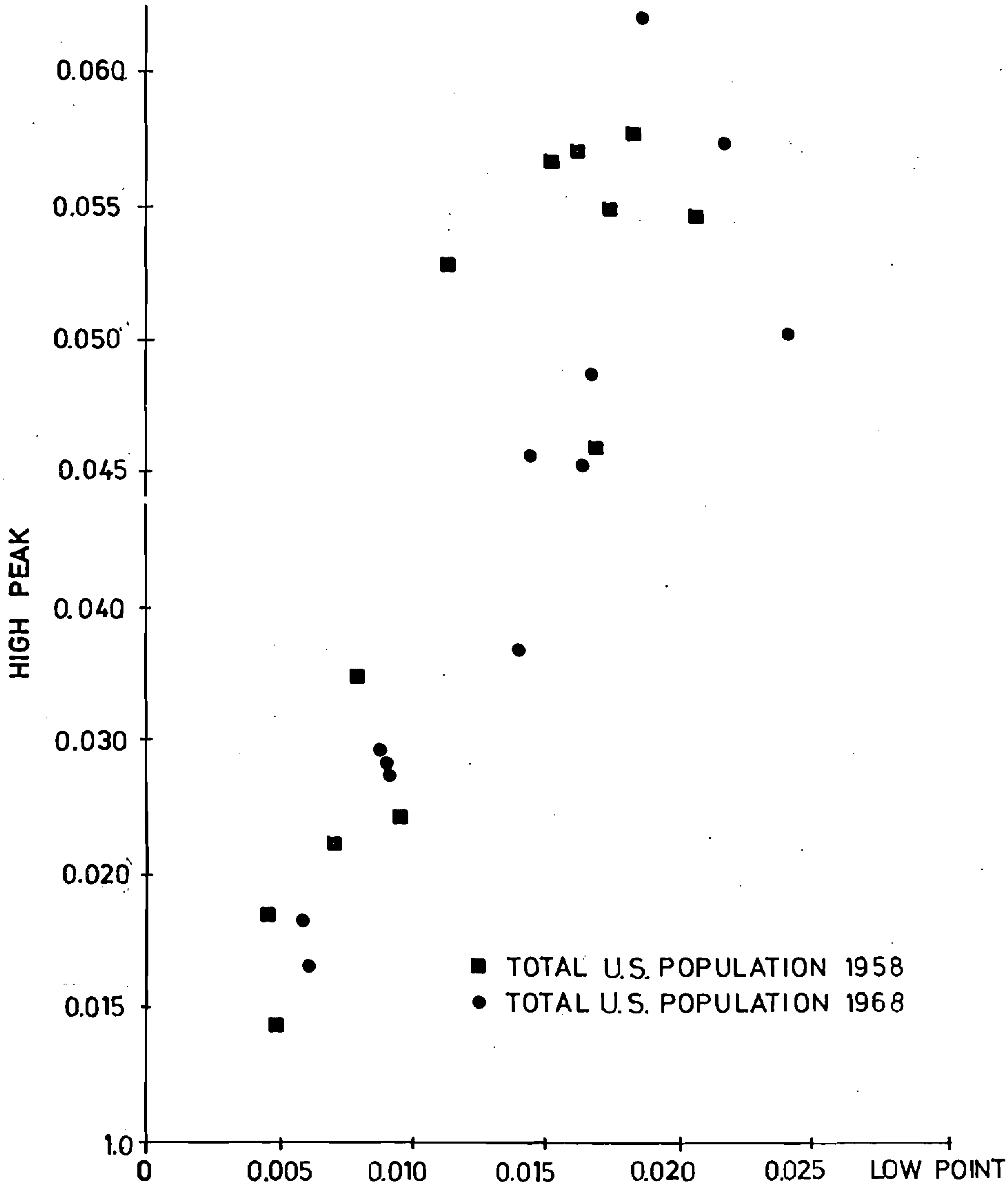

Figure 5. Relation between high peaks and low points in migration schedules.

3.2 Summarizing the Regularities: Regression.

The migration risks experienced by different age and sex groups of a given population are strongly interrelated, and higher (or lower) than average migration rates among one segment of a particular population normally imply higher (or lower) than average migration rates for other segments of the same population. This association stems in part from the fact that if socioeconomic conditions at a location are good or poor for one group in the population, they are also likely to be good or poor for other groups in the same population. Since migration is widely held to be a response to spatial variations in socioeconomic conditions, these high intercorrelations between age-specific migration risks are not surprising.

Figures 3,4, and 5 support the above conjecture and, moreover suggest a way of summarizing the observed regularities in migration probabilities. They indicate that a relatively accurate accounting of the variation in the height of the initial peak (and through it in the height of the rest of the migration schedule) may be obtained by means of a straight line fitted to the scatter of points in Figure 3. Thus a linear regression of the form

$$p_{ij}(0) = \gamma + \beta \; {}_i\theta_j$$

would seem to be appropriate.[1] But $p_{ij}(0)$ cannot take on negative values; a convenient way of ensuring that this possibility never arises is to force the line through the origin by adopting the zero-intercept simple linear regression model

$$p_{ij}(0) = \beta \; {}_i\theta_j \quad . \tag{9}$$

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[1] Since changes in fertility also affect the height of the initial peak, a possible further refinement of the model would be to include a variable describing the level of fertility, for example, the reproduction rate.

The least-squares fit of such an equation to the data illustrated in Figure 3 gives

$$p_{ij}(0) = 0.17392 \ {}_i\theta_j$$

for the 1958 observations, and

$$p_{ij}(0) = 0.22002 \ {}_i\theta_j$$

for the 1968 data points. The fit in each instance is quite satisfactory, yielding coefficients of determination (r^2) of 0.94 and 0.84, respectively.

Given estimates of β and ${}_i\theta_j$ we can obtain an estimate of $p_{ij}(0)$. Figures 4 and 5 suggest that with the value of $p_{ij}(0)$ fixed, we can find the corresponding value of the low point and use that, in turn, to estimate the value of the high point. Generalizing this argument to all age groups beyond the first, we may adopt the simple model

$$p_{ij}(x + 5) = \gamma(x) \ p_{ij}(x) \quad , \tag{10}$$

where $p_{ij}(0)$ is estimated by equation 9. Thus

$$p_{ij}(5) = \gamma(0) \ p_{ij}(0) = \gamma(0)\beta_i\theta_j = \beta(5)_i\theta_j \quad ,$$

$$p_{ij}(10) = \gamma(5) \ p_{ij}(5) = \gamma(5)\beta(5)_i\theta_j = \beta(10)_i\theta_j \quad ,$$

and in general,

$$p_{ij}(x) = \beta(x) \ {}_i\theta_j \quad , \tag{11}$$

in which the β in (9) now is designated by $\beta(0)$. Note that as a consequence of our definitions

$$\gamma(x) = \frac{\beta(x + 5)}{\beta(x)} \quad ,$$

and

$$p_{ij}(x + 5) = \gamma(x) \frac{\beta(x)}{\beta(0)} p_{ij}(0) = \frac{\beta(x + 5)}{\beta(0)} p_{ij}(0) \quad , \quad (12)$$

Equation 11 may be treated as a simple (zero-intercept) linear regression equation, and its coefficient $\beta(x)$ may be estimated using the conventional least-squares procedure. Table 3 presents two sets of such coefficients for the U.S. total population. The first set was obtained using 1958 data, the second set was estimated on the basis of 1968 data. In both instances the observed migration flows were those between the four U.S. Census Regions.

The regression coefficients in Table 3 may be used in the following way. First, starting with a complete set of multiregional migration levels ${}_i\theta_j$ one calculates the matrix of migration probabilities $\underset{\sim}{P}(x)$ for every age, using equation 11 and one of the two sets of regression coefficients in Table 3. (Figure 6 illustrates a range of such probabilities by way of example.) With $\underset{\sim}{P}(x)$ established, one then may compute the usual life table statistics, such as the survivorship proportions defined in equation 8 and the various region-specific expectations of life at each age. The collective results of these computations constitute a *model multiregional life table.*

3.3 Families of Model Migration Schedules

In this section we consider the effects on the migration age profile of various disaggregations of our data on the U.S. population system. Specifically, we examine how the regression coefficients set out earlier in Table 3, and illustrated below in Figure 7, respond to various disaggregations of the empirical population on the basis of which they were estimated. First, we disaggregate the total population by sex. Next, we introduce a disaggregation according to mean age. Then we consider a spatial disaggregation of the four Census Regions into their

Table 3. Regression coefficients for obtaining model probabilities of migration.

Age	Total (1958)		Total (1968)	
	β	r^2	β	r^2
0	0.17392	0.94	0.22002	0.84
5	0.13460	0.95	0.15553	0.89
10	0.15736	0.86	0.15040	0.94
15	0.30757	0.93	0.29195	0.85
20	0.32271	0.72	0.26370	0.72
25	0.23251	0.96	0.20037	0.90
30	0.17897	0.95	0.17907	0.94
35	0.12912	0.95	0.14392	0.96
40	0.09790	0.93	0.10397	0.95
45	0.07522	0.86	0.07378	0.91
50	0.06838	0.73	0.06352	0.76
55	0.07347	0.63	0.07362	0.54
60	0.08254	0.47	0.08320	0.43
65	0.06086	0.50	0.06425	0.47
70	0.04488	0.58	0.04919	0.64
75	0.03019	0.67	0.03951	0.64
80	0.01342	0.18	0.02058	0.63

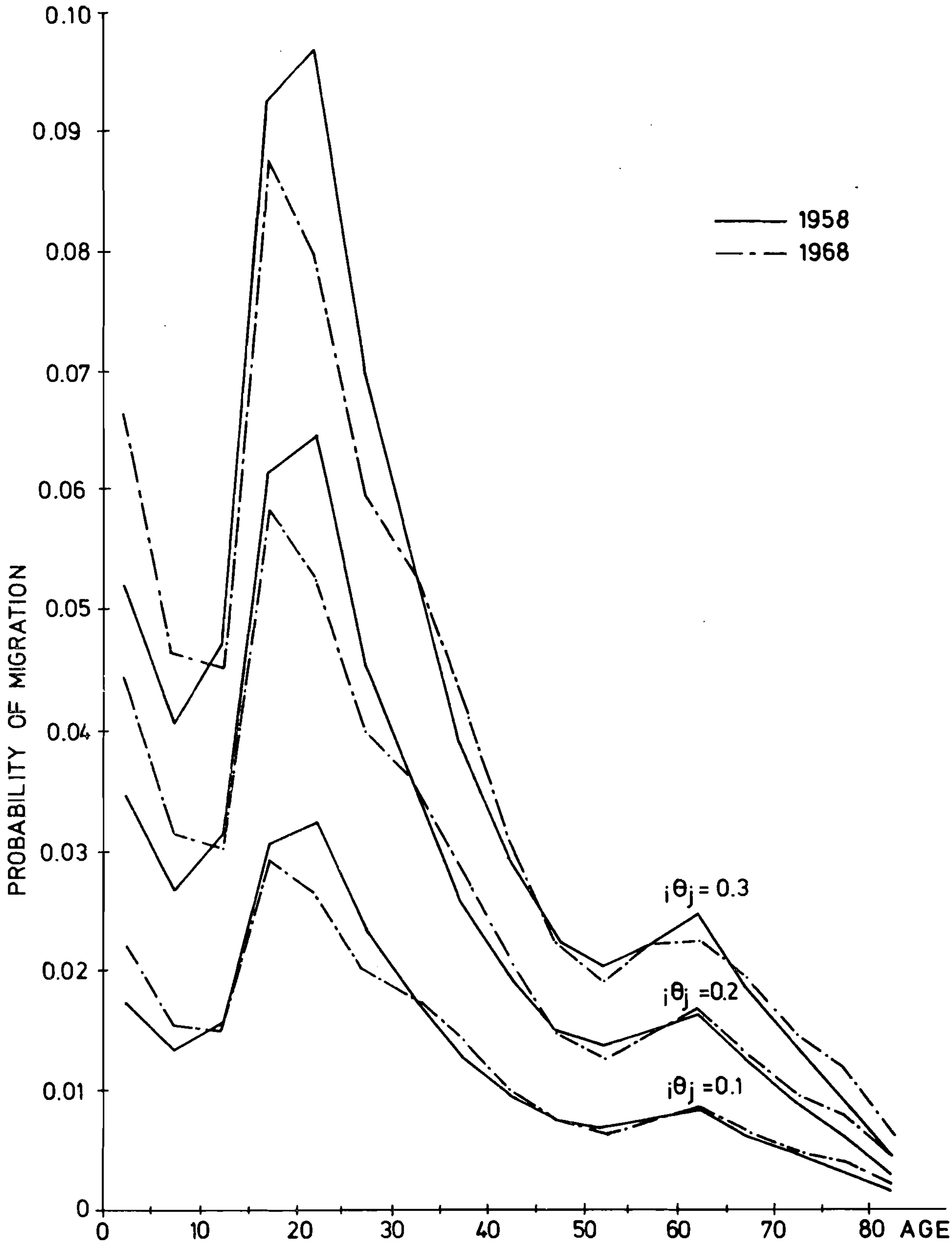


Figure 6. Age-specific model probabilities of migration at various levels of migration.

constituent nine Census Divisions. Finally, we explore the impact of an even finer deconsolidation by mean age.

The two regression coefficient profiles in Figure 7 mirror the fundamental age profile of migrants that was analyzed earlier in this paper. The principal differences between the two coefficient profiles are the higher and older high peak in the 1958 migration schedule, and the higher and older low point of the corresponding 1968 schedule. Beyond the mid-thirties the two profiles are quite similar, with both showing a retirement peak in the 60-64 year-old age group.

Profile Differences by Sex

A disaggregation of the 1968 regression coefficient profile introduces important variations by sex, according to Figure 8. The male coefficients are higher from the very early teens to the mid-forties and are lower at all other ages. The locations of the high peak and the retirement peak are the same in both profiles, but the low point among males comes at a younger age than in females. Also, the retirement peak among females is broader and starts at an earlier age.

Profile Differences by Mean Age

Figure 9 indicates that a division of migration schedules into "young" and "old" categories might be a useful way of disaggregating the regression coefficients illustrated in Figures 7 and 8. It shows two basic age profiles which are distinguishable by the presence of a high retirement peak in one profile and its virtual absence in the other. We designate the former profile as a retirement profile and the latter as a labor force profile. An alternative designation is old and young profile, respectively.

Profile Differences by Size of Areal Unit

Because migration normally is defined as a crossing of a regional boundary, it is clear that reducing the size of a spatial unit should increase the level of outmigration from that unit, since some of the moves that previously did not cross over the old borders now will be recorded as migrations over the new borders. But what of the age profile in each case? Should not this feature of the observed migration flows remain essentially unchanged, at least for the relatively large areal units? Figure 10 (like Figure 1A before it) gives some evidence that this conjecture is valid. The two regression coefficient profiles that it illustrates were estimated on the basis of the same data set, using first a nine and then a four-region spatial delineation of the total 1958 U.S. population. The fact that the former is always higher than the latter is perhaps a consequence of some confounding of profile and level introduced by aggregation bias.

Profile Differences by Several Mean Age Classes

The spatial disaggregation of our data from four to nine areal units increases the number of observations from 12 to 72 and thereby affords us an opportunity to examine the impact of a finer classification by mean age. Specifically, we now consider the disaggregation of the 1958 regression coefficient profile into four instead of two mean age categories: "very young" ($\bar{n}_{ij} \leq 26$); "young" ($26 < \bar{n}_{ij} \leq 28$); "old" ($28 < \bar{n}_{ij} \leq 30$); and "very old" ($\bar{n}_{ij} > 30$).

Except for variations with respect to the retirement peak, the principal impact of the finer disaggregation by mean age appears not so much in the age profile as in the relative height of that profile for a given value of the migration level ${}_i\theta_j$. Thus, for example, the age curve of the "very old" profile in Figure 11 is almost everywhere higher than the corresponding curve of the "very young" profile, *for the same level of migration*. The reason for this is not immediately apparent and merits further study. A possible explanation may lie in the fact that ${}_i\theta_j$ is an index which combines an age-specific migration pattern with a specific (life-table) age composition. This particular confounding of schedule and composition could perhaps generate the variations in profile heights that appear in Figure 11,

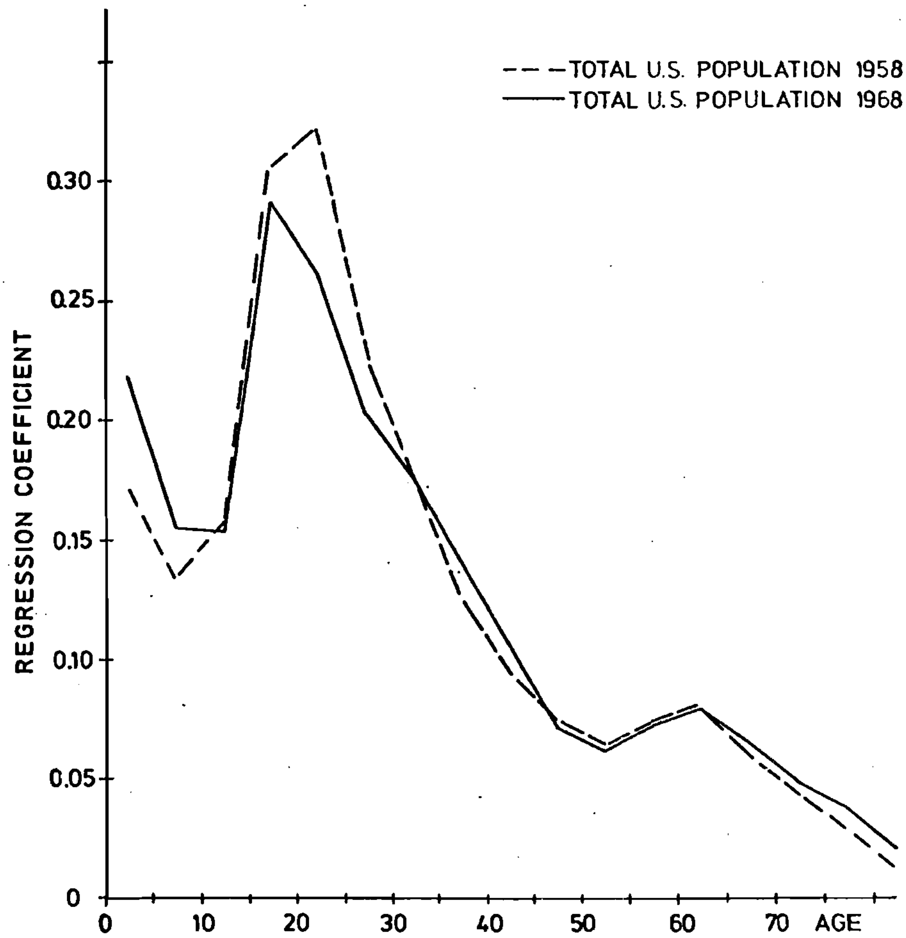


Figure 7. Regression coefficients for model migration schedules: total U.S. populations, 1958 and 1968.

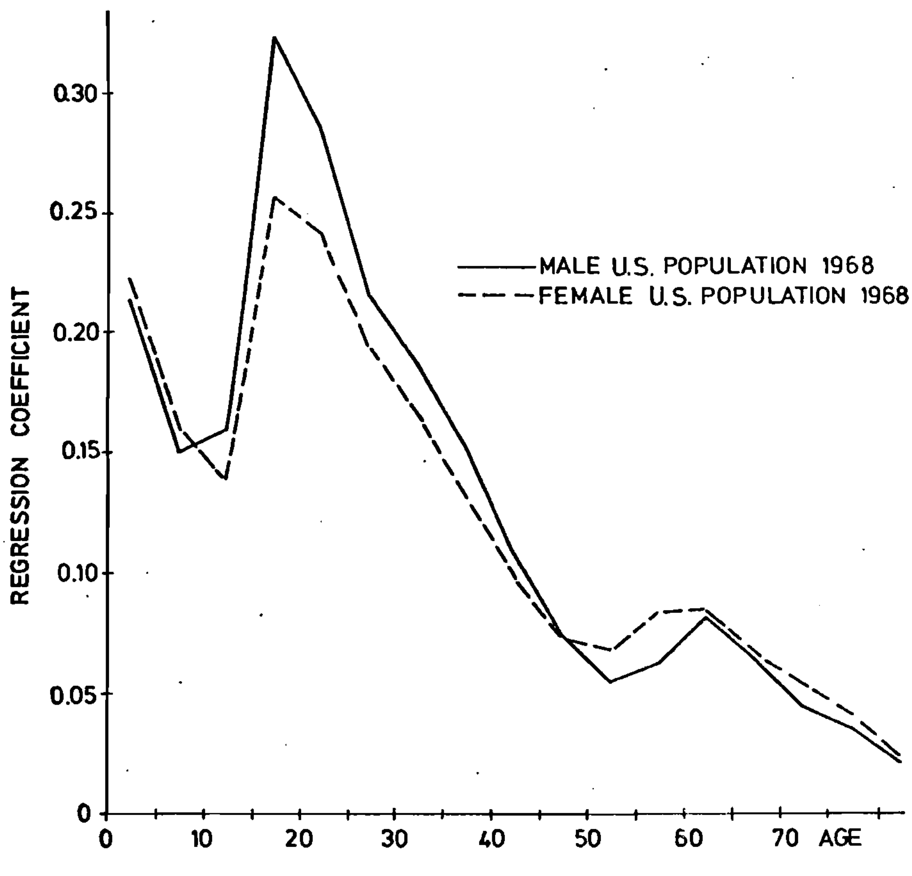


Figure 8. Regression coefficients for model migration schedules: male and female U.S. populations, 1968.

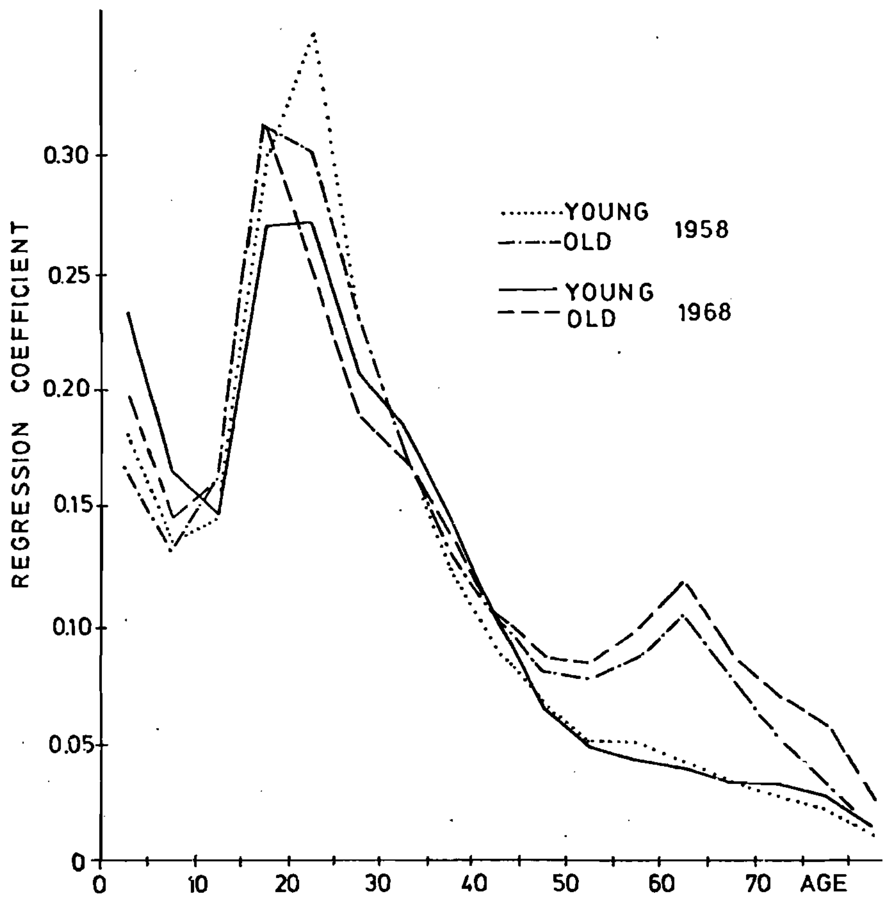


Figure 9. Regression coefficients by "young" and "old" classification ($\bar{n}_{ij} \leq 28$ and $\bar{n}_{ij} > 28$): total U.S. populations, 1958 and 1968.

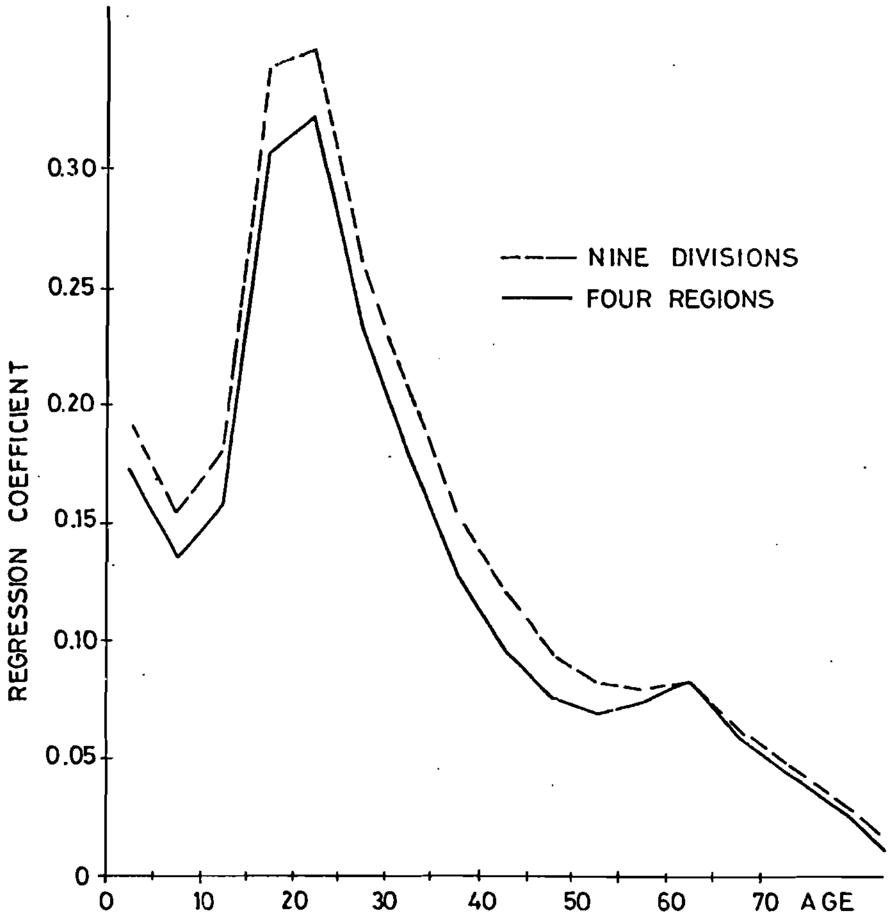


Figure 10. Regression coefficients for model migration schedules: total U.S. population, 1958, by region and division.

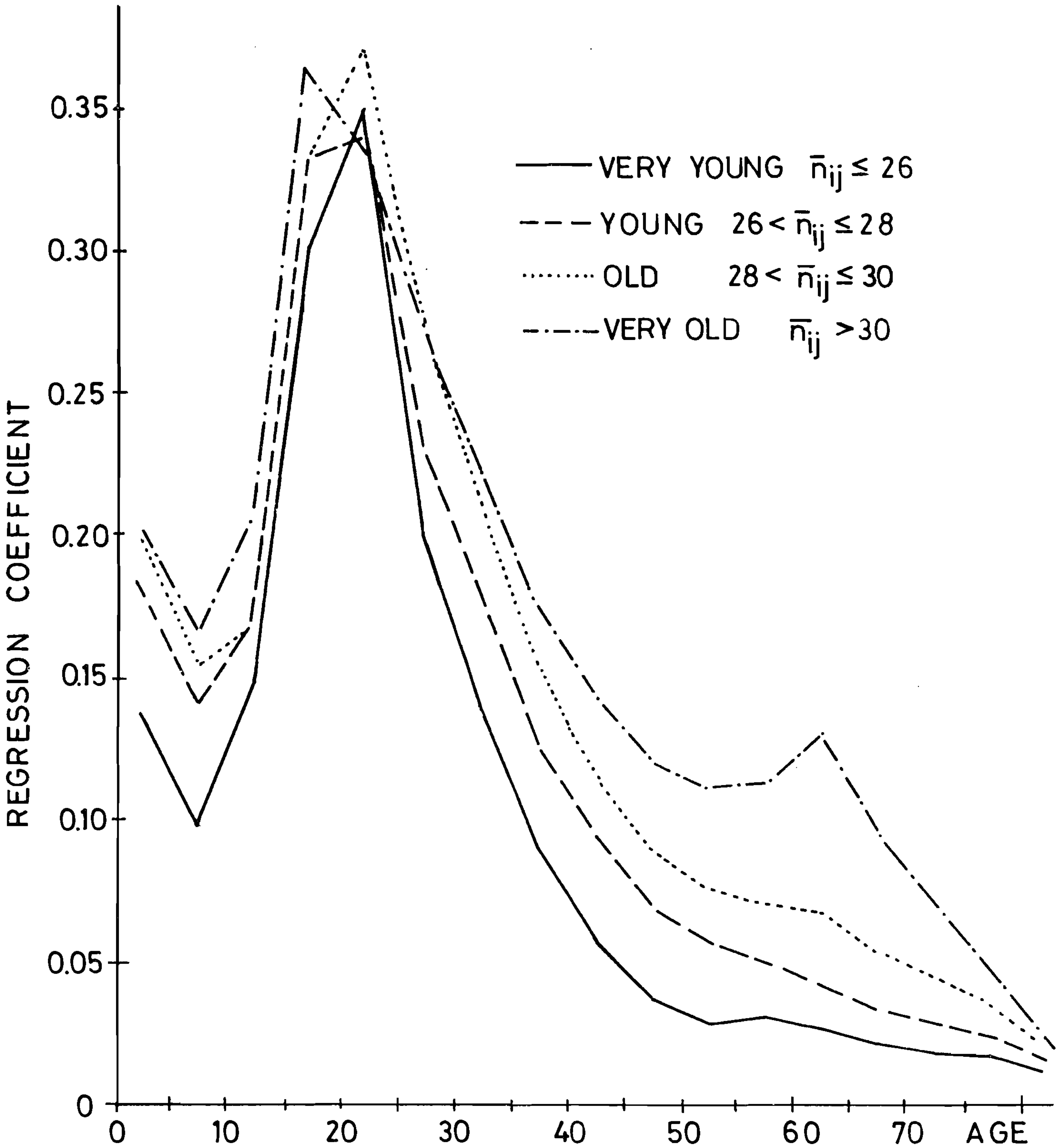


Figure 11. Regression coefficients for model migration schedules: total U.S. population, 1958, by several mean age classes.

although the underlying dynamics of this are by no means self-evident. Consequently, it may well be the case that the "fertility approach" with its focus on the GMR as an index of migration level has a built-in advantage over the "mortality approach" that we have been following in this section. This possibility is considered later in this paper.

The regression coefficients illustrated above in Figures 7 through 11, may be said to form *families* of model migration probabilities or schedules. Those associated with different categories of mean age give "young" and "old" profiles; those that do not consider mean age as an index give "average" profiles. We next illustrate an application of the female "average" profile by constructing a specimen model multiregional life table.

3.4 Application: Inference

We have noted earlier that single-region life tables normally are computed using observed data on age-specific death rates. In countries lacking reliable data on death rates, however, recourse is often made to inferential methods that rely on *model life tables* such as those published by the United Nations (United Nations, 1967). These tables are entered with empirically determined survivorship proportions to obtain the particular expectation of life at birth (and corresponding life table) that best matches the levels of mortality implied by the observed proportions.

The inferential procedures of the single-region model (*the U.N. method,* say) maybe extended to the multiregional case (Rogers, 1975, Ch.6). Such an extension begins with the notion of *model multiregional life tables* and uses a set of initial estimates of survivorship and migration proportions to identify the particular combination of regional expectations of life, disaggregated by region of birth and region of residence, that best matches the levels of mortality and migration implied by these observed proportions.

Model Multiregional Life Tables

Model multiregional life tables approximate the mortality and migration schedules of a multiregional population system by drawing on the regularities observed in the mortality and migration experiences of comparable populations. That is, regularities exhibited by mortality and migration data collected in regions where these data are available and accurate are used to systematically approximate the mortality and migration patterns of populations lacking such data. Table 4 gives the four regional expectations of life at birth and the dozen migration levels that together characterize the patterns of regional mortality and interregional mobility of U.S. females in 1968. Interpolating in the "WEST" family of model life tables developed by Coale and Demeny (1966), we first obtain the appropriate set of model probabilities of dying at each age for each of our four Census Regions. Inserting, in turn, each of the dozen values of ${}_i\theta_j$ into equation 11, with $\beta(x)$ taking on the column of "average" values illustrated for females in Figure 8, we may derive initial approximations for $p_{ij}(x)$. These probabilities of migration may then be used in conjunction with the associated interpolated model probabilities of dying to obtain the matrix of survivorship proportions defined in equation 8. By appropriately manipulating equation 7, we also can find the associated model migration rates. And then, following the normal computational procedures of multiregional life table construction (Rogers, 1975, Ch.3), we may derive, for example, the corresponding matrix of expectations of life at birth, appropriately disaggregated by region of birth and region of residence. Unfortunately the latter matrix usually will not yield the same migration levels that were used to generate the $\underset{\sim}{P}(x)$ matrix. Such inconsistencies occur in the model life tables of Coale and Demeny (1966). To eliminate them one must resort to iteration. Only in this way can one obtain a model multiregional life table whose statistics and parameters are internally consistent.

Figure 12 illustrates four sets of model migration rates that were generated in the course of constructing our specimen model multiregional life table for U.S. females. Adjoining the four model schedules are the corresponding empirical schedules observed in 1965-1970. A comparison of the two sets of schedules suggests that, although the degree of correspondence is fairly close, further improvement would be highly desirable.

The U.N. Method Generalized

The U.N. method of obtaining initial age-specific estimates of 10-year survivorship proportions from two consecutive decennial census-enumerated age distributions may be generalized to multiregional population systems if age-specific place of residence by place of birth (PRPB) data are available for both census years. This easily may be demonstrated by expressing the single region procedure in algebraic form and reverting to matrix algebra to define the corresponding multiregional method. First, we observe that the single-region procedure for estimating s(x) may be expressed as follows:

$$_{10}\hat{s}(x) = \frac{K^{(t+1)}(x+10)}{K^{(t)}(x)} = K^{(t+1)}(x+10)K^{(t)}(x)^{-1} \quad , \quad (13)$$

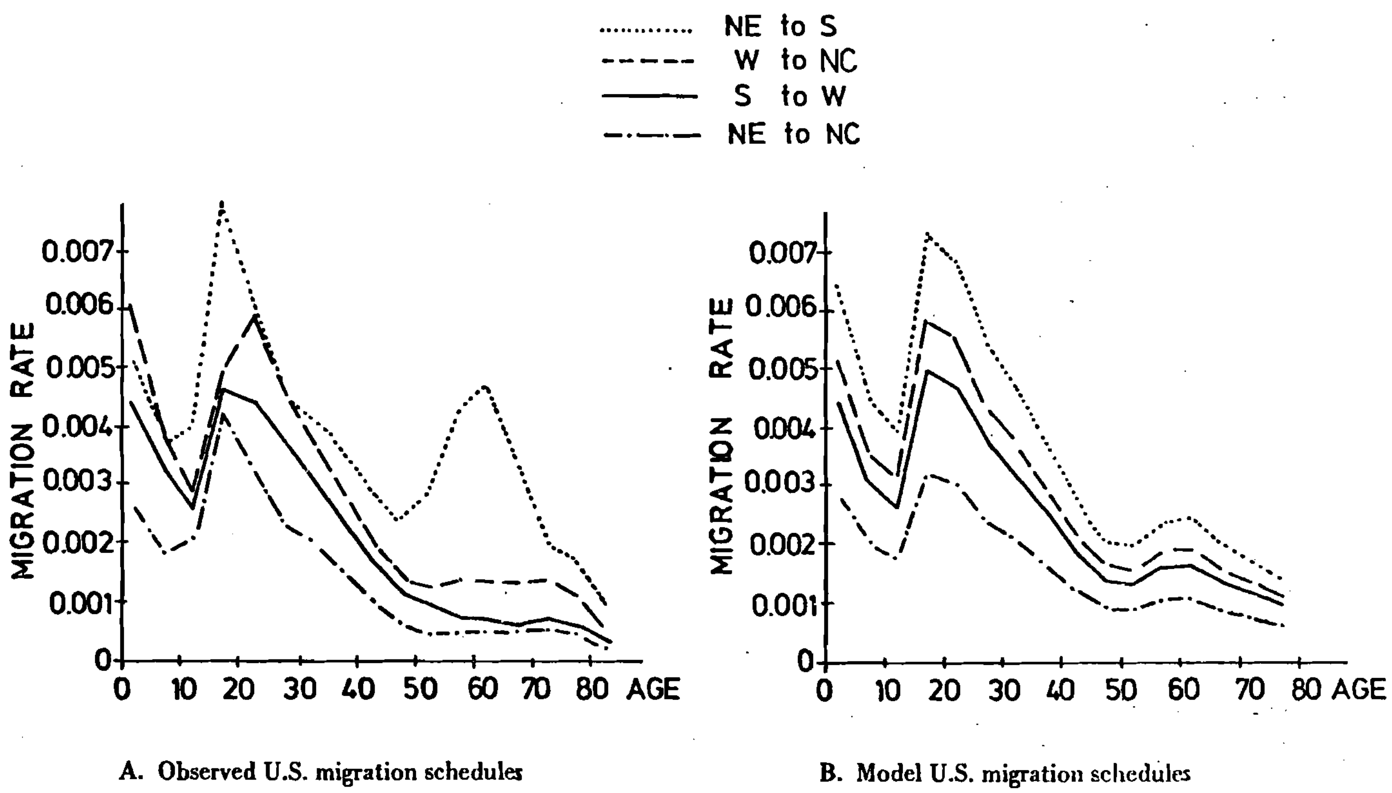


Figure 12. Observed and model female migration schedules.

Table 4. Expectations of life at birth and migration levels by region of residence and region of birth: female United States population, 1968.

Region of Birth	Region of Residence 1	2	3	4	Total
1. Northeast	54.13 (0.7260)	5.08 (0.0681)	10.11 (0.1356)	5.25 (0.0704)	74.56 (1.00)
2. North Central	3.76 (0.0506)	52.14 (0.7005)	10.48 (0.1408)	8.05 (0.1081)	74.44 (1.00)
3. South	5.06 (0.0680)	7.88 (0.1060)	54.53 (0.7328)	6.93 (0.0931)	74.40 (1.00)
4. West	3.90 (0.0516)	7.94 (0.1051)	11.32 (0.1497)	52.41 (0.6936)	75.57 (1.00)

where $K^{(t)}(x)$ denotes the number of persons aged x to $x + 9$ years at time t. Next we recall the multiregional demographic model that projects populations disaggregated by place of residence and place of birth (Rogers, 1975, p.172):

$$_0\underset{\sim}{K}(x + 5) = \underset{\sim}{S}(x)\,_0\underset{\sim}{K}(x) \quad . \qquad (14)$$

Normally our interest in this model centers on the determination of $_0\underset{\sim}{K}(x + 5)$, given particular numerical values for $\underset{\sim}{S}(x)$ and $_0\underset{\sim}{K}(x)$. Now, however, we consider the application of (14) to derive $\underset{\sim}{S}(x)$ given numerical values for $_0\underset{\sim}{K}(x)$ and $_0\underset{\sim}{K}(x + 5)$. Clearly,

$$\hat{\underset{\sim}{S}}(x) = \,_0\underset{\sim}{K}(x + 5)\,_0\underset{\sim}{K}(x)^{-1}$$

and for a 10-year age and time interval

$$_{10}\hat{\underset{\sim}{S}}(x) = \,_0\underset{\sim}{K}^{(t+1)}(x + 10)\,_0\underset{\sim}{K}^{(t)}(x)^{-1} \quad . \qquad (15)$$

Note that (15) is the matrix expression of (13):

Having found crude initial estimates of the various regional survivorship and outmigration proportions by means of the PRPB method, we may construct the associated life table to obtain the regional expectations of life at birth that are implied by these proportions (Rogers, 1975, pp. 85-88.) Then, as in the single-region case, we may "adjust" our initial estimates of probabilities of outmigration and death by interpolating in an appropriate set of model multiregional life tables (Rogers, 1975, pp. 185-189).

4. MODEL MIGRATION SCHEDULES: THE FERTILITY APPROACH

Fertility schedules have long been recognized as exhibiting a fundamental pattern that persists over a wide range of human populations. This recognition has fostered two related research efforts: 1) one concerned with the analytic graduation of fertility curves (Keyfitz, 1968, Chap. 6; Hoem and Berge, 1975) and 2) the other focused on the construction of model fertility schedules (Coale and Demeny, 1966, p. 30).

In a recent paper, Coale and Trussell (1974) combine these two lines of research to provide an analytic graduation of a standard fertility schedule from which a wide variety of fertility patterns can be derived by a simple transformation involving four parameters. Observed fertility patterns are defined with respect to the natural fertility of married women. Fertility is held to be a function of nuptiality patterns (characterized by two parameters), contraception (characterized by one parameter), and a fertility level (characterized by one parameter). Such a model seems to provide a good fit, and readily leads to methods for obtaining an appropriate fertility schedule on the basis of inadequate information regarding the fertility regime of an observed population.

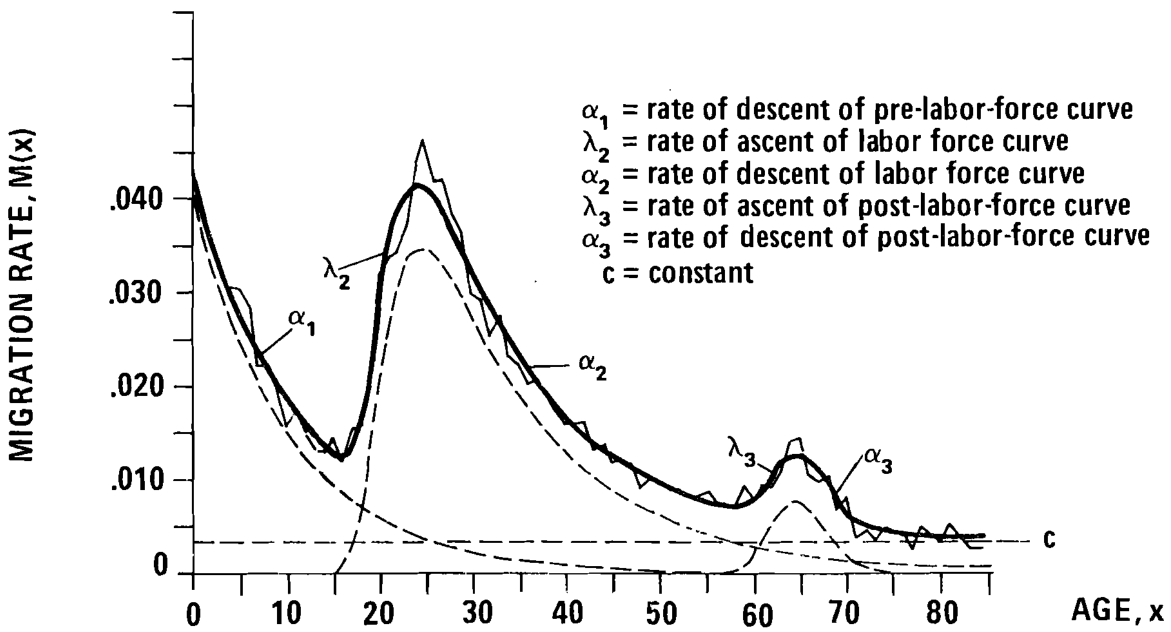
In this section of our paper, we explore the potential utility of the Coale-Trussell fertility approach for constructing model migration schedules.

4.1 The Fundamental Components of Migration Schedules

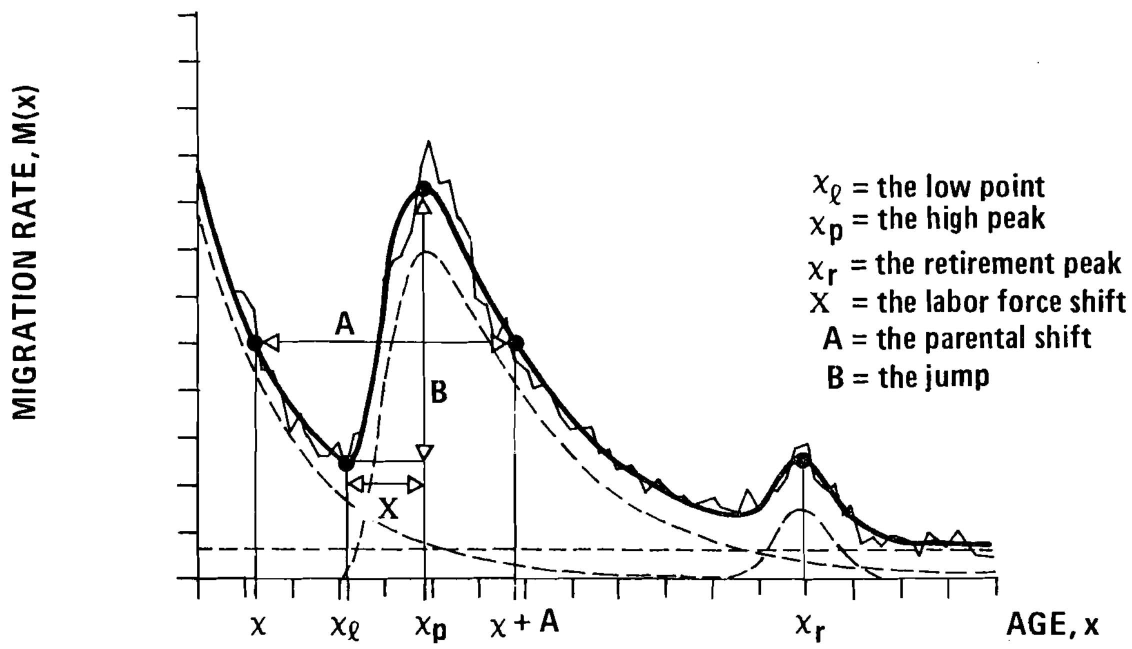
Regularities in observed age-specific schedules of migration may be examined in a number of interesting ways. A particularly useful approach is to decompose the migration schedule into three parts, separating the migration rates of persons in the labor force age groups from those of individuals in the pre- and post-labor force ages, respectively. Such a decomposition gives rise to the three fundamental curves illustrated in Figure 13A:

1) the single negative exponential curve of the pre-labor force ages with its rate of descent, α_1 ;

FIGURE 13. DECOMPOSITION OF THE MIGRATION SCHEDULE



13.A THE MODEL MIGRATION SCHEDULE: ITS FUNDAMENTAL COMPONENTS AND THEIR PARAMETERS



13.B THE SHIFTS AND THE JUMP

2) the left-skewed unimodal curve of the labor force ages with its rates of ascent and descent, λ_2 and α_2 , respectively; and

3) the almost bell-shaped curve of the post-labor force ages with its rates of ascent and descent, λ_3 and α_3 , respectively. (When no retirement peak is exhibited by the data, this last curve is suppressed.)

For future reference, Figure 13A also includes the constant curve c, to which we shall refer later in the paper. Its inclusion improves the quality of fit provided by the mathemetical model schedule.

Figure 13B illustrates several important points along the age profile of a migration schedule: its *low point*, x_l, its *high peak*, x_p, and its *retirement peak*, x_r. Associated with the first two points is the labor force shift, X, which is defined as the difference in years between the ages of the low point and the high peak, i.e. $X = x_p - x_l$. Associated with this shift is *the jump*, the increase in the migration rate of individuals aged x_p over those aged x_l.

Another important shift in observed migration schedules arises out of the close correspondence between the migration rates of children and those of their parents. If, for each point x on the pre-labor force part of the migration curve, we obtain by interpolation the point, $x + A_x$, say, on the labor force curve that yields the identical rate of migration, then the average of the values of A_x, calculated for the first 14 years of age,

$$A = \frac{1}{14} \sum_{x=0}^{13} A_x \quad , \tag{16}$$

will be defined to be the observed *parental shift*.

Table 5 presents numerical approximations of the observed parental shift for eight Swedish regions (viksområden), with single year age-specific migration and population data for 1974

Table 5. Observed values of the parental shift: Swedish regions, 1974

The Parental Shift	Regions of Sweden[a]							
	1. Stockholm	2. Upper East	3. Lower East	4. South	5. West	6. Lower North	7. Middle North	8. Upper North
Observed,[b] males	27.61 (3.19)	29.77 (0.86)	29.58 (1.75)	28.98 (0.58)	29.42 (0.23)	28.97 (0.27)	30.07 (0.60)	30.33 (0.83)
Observed,[b] females	25.16 (0.50)	26.50 (0.31)	27.48 (0.96)	27.79 (0.60)	27.45 (0.23)	26.59 (0.97)	27.21 (0.82)	29.52 (2.01)
Mean age of child-bearing, females	27.59	26.95	27.16	27.23	27.34	26.83	27.17	27.38

[a] The annual migration data by single-years of age and the regional delineations were kindly provided by Arne Arvidsson of the Swedish Central Bureau of Statistics.

[b] Values in parentheses are variances. The method for calculating the observed parental shift is described in the text.

(Stochastic variations in the rates were first smoothed out by Michael Stoto using a method described in Tukey (1977) called "non-linear smoothing"). The results indicate that the observed parental shift was roughly 26 to 28 years for females with about an additional two years for males. The last row in the table suggests that the parental shift may be closely approximated by the mean age of childbearing, (Stoto, 1977).

4.2 Summarizing the Regularities: Curve-Fitting

Our examination of the regularities in observed migration schedules suggested a decomposition into three curves: a single negative exponential and two skewed unimodal bell-shaped functions. The obvious mathematical expression for the first is $ae^{-\alpha x}$; to represent the other two, we have adopted the "double exponential" developed by Coale and McNeil (1972):

$$ae^{-\alpha(x-\mu)-e^{-\lambda(x-\mu)}} .$$

And, because observed migration rates do not drop to zero within the range of post-labor force ages normally recorded, an additional constant term (the c in Figure 13A) needs to be included. We have then a model migration schedule that is the simple sum of four curves, namely:

$$\begin{aligned} \hat{M}(x) &= a_1 e^{-\alpha_1 x} \\ &+ a_2 e^{-\alpha_2(x-\mu_2)-e^{-\lambda_2(x-\mu_2)}} \\ &+ a_3 e^{-\alpha_3(x-\mu_3)-e^{-\lambda_3(x-\mu_3)}} \\ &+ c \qquad\qquad , x = 0,1,2,\ldots \end{aligned} \tag{17}$$

The "full" model schedule in (17) has 11 parameters: a_1, α_1, a_2, μ_2, α_2, λ_2, a_3, μ_3, α_3, λ_3, and c. Migration schedules without a retirement peak may be represented by a "reduced" model with 7 parameters, because in such instances the third component of (17) is omitted. Illustrative values for the model schedule's parameters are set out in Table 6.

Table 6. Fundamental parameters of the model migration schedule[a]

Parameters and Statistics	UNITED STATES, 1966-1971				POLAND, 1974		SWEDEN, 1968-1973	
	Males	GMR=1.00 $\bar{m}$ 29.63	Females	GMR=1.00 $\bar{m}$ 28.94	Males GMR=1.00	Females GMR=1.00	Males GMR=1.00	Females GMR=1.00
	l.s.e.	min.chi.	l.s.e.	min.chi.	$\bar{m}$ = 31.97	$\bar{m}$ = 33.66	$\bar{m}$ = 26.63	$\bar{m}$ = 25.40
a_1	0.0192	0.0191	0.0200	0.0196	0.0272	0.0266	0.0300	0.0290
α_1	0.1186	0.1141	0.1242	0.1177	0.1972	0.1956	0.1187	0.1261
a_2	0.0437	0.0427	0.0481	0.0462	0.0771	0.0786	0.0639	0.0730
μ_2	19.5378	19.4613	18.4032	18.2979	22.8367	20.3397	21.4133	19.2624
α_2	0.1037	0.0999	0.1275	0.1216	0.1611	0.1707	0.1059	0.1252
λ_2	0.5354	0.5335	0.5600	0.5868	0.4460	0.4496	0.3846	0.5060
a_3	-	-	-	-	0.0001	0.0067	-	-
μ_3	-	-	-	-	112.5498	116.2804	-	-
α_3	-	-	-	-	0.2995	0.0945	-	-
λ_3	-	-	-	-	0.0546	0.0325	-	-
c	0.0060	0.0059	0.0066	0.0065	0.0051	0.0042	0.0027	0.0032
$\delta_2 = \alpha_2/\lambda_2$	0.1937	0.1872	0.2277	0.2072	0.3612	0.3797	0.2754	0.2474
$\delta_3 = \alpha_3/\lambda_3$	-	-	-	-	5.4866	2.9085	-	-
χ^2	0.0021	0.0020	0.0027	0.0026	0.0080	0.0017	0.0036	0.0028
MAE/M	3.33	3.41	3.89	3.96	4.34	2.91	3.79	2.73
GMR(est.)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\bar{m}$(est.)	29.71	29.68	29.04	29.00	32.09	33.65	26.71	25.48

[a]Least-squares parameter estimates except in the case of the U.S. schedule, for which minimum chi-square estimates are also presented. Data sources are: for the U.S., Long (1973) and Long (1976, personal communication); Polish Central Bureau of Statistics (1974); and Swedish Central Bureau of Statistics (1974). All migration schedules were first scaled to a gross migraproduction rate (GMR) of unity.

Having chosen the particular function in (17) to represent age-specific migration schedules, one then is faced with the problem of selecting a method for fitting the function to observed migration data. Previous research in the analytic graduation of fertility schedules has shown that moment type estimators may be inconsistent and do not compare favorably with functional minimization methods such as minimum chi-square or least squares estimation procedures (Hoem, 1972,and Hoem and Berge, 1975).

Least squares parameter estimates are presented in Table 6. Minimum chi-square estimates are also included for the U.S. data, for purposes of comparison. The differences between the two sets of parametric estimates tend to be small, and because the latter give more weight to age groups with smaller rates of migration, we use minimum chi-square estimators in the remainder of the paper.

To assess the quality of fit that the model schedule provides when it is applied to observed data, two indices of goodness-of-fit have been included in Table 6: the chi-square statistic, χ^2, and the"mean absolute error as a percentage of the observed mean",

$$MAE/M = \frac{\frac{1}{n} \sum_x \left| \hat{M}(x) - M(x) \right|}{\frac{1}{n} \sum_x M(x)} \cdot 100 \quad .$$

Both measures indicate that the fit of the model to the data is remarkably good.

The numerical values in Table 6 suggest possible simplifications of the model:

1) To the extent that the migration rates of children mirror those of their parents, the parameter α_1 should be approximately equal to α_2. Table 6 indicates that this is indeed the case for the migration schedules of the U.S., Poland and Sweden. Thus a reasonable

simplification of the model is to assume that $\alpha_1 = \alpha_2$.

2) Experiments with a wide range of empirical migration schedules suggest that the ratio of the rate of descent, α, to that of the rate of ascent, λ, does not vary greatly, particularly for the retirement peak. We assume, therefore, that

$$\delta_3 = \frac{\alpha_3}{\lambda_3} = \text{a constant} = 5\text{, say.}$$

The above two simplifications reduce the number of parameters in the full model to 9 and in the reduced model to 6. Table 7 compares the fits of the original model with those of the simplified model for data on the U.S., Sweden, and eight Swedish regions defined by Arne Arvidsson of the Swedish Central Bureau of Statistics (who also kindly provided that data). Figures 14 and 15 illustrate graphically the closeness of the fit of the simplified model migration schedule to the Swedish regional data. It appears that little information is lost by simplifying the model, and we therefore adopt the simplified full and reduced models for all analyses described in the rest of this paper.

4.3 Families of Model Migration Schedules

Model migration schedules of the original form specified in (17), or of the simplified form described above, may be classified into *families* according to the values taken on by their principal parameters. For example, we may distinguish those schedules with a retirement peak from those without; or we may refer to schedules with relatively low or high values for the rate of ascent α_2. In many applications, it is also meaningful and convenient to characterize the model schedules in terms of several of the fundamental measures illustrated in Figure 13, such as the low point x_l, the high peak x_p, the labor force shift X, the parental shift A, and the jump B.

The simplified model migration schedule has a built-in parental shift which can be defined analytically. Shortly after

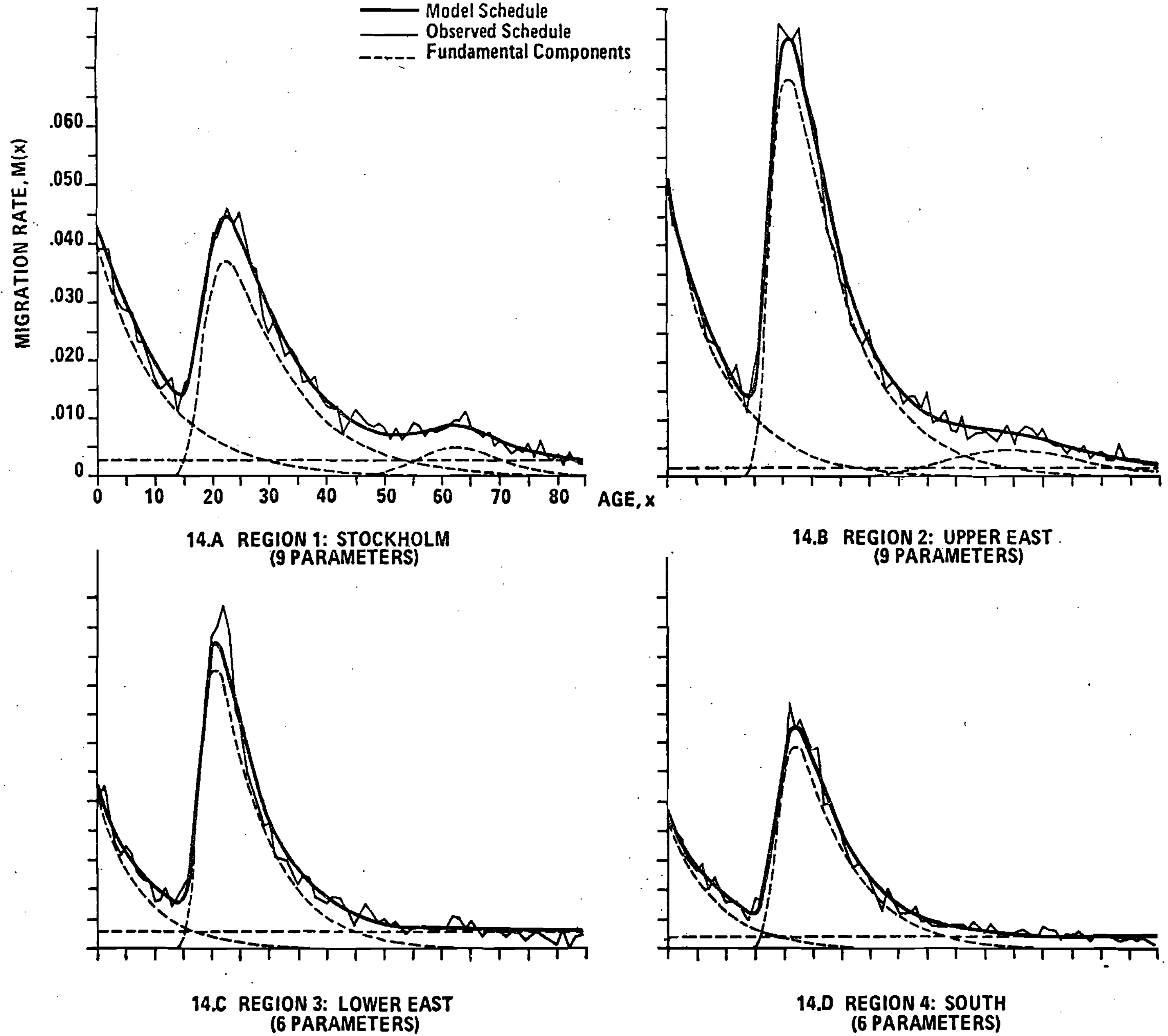


FIGURE 14. OBSERVED AND SIMPLIFIED MODEL MIGRATION SCHEDULES: FEMALES, SWEDISH REGIONS, 1974.

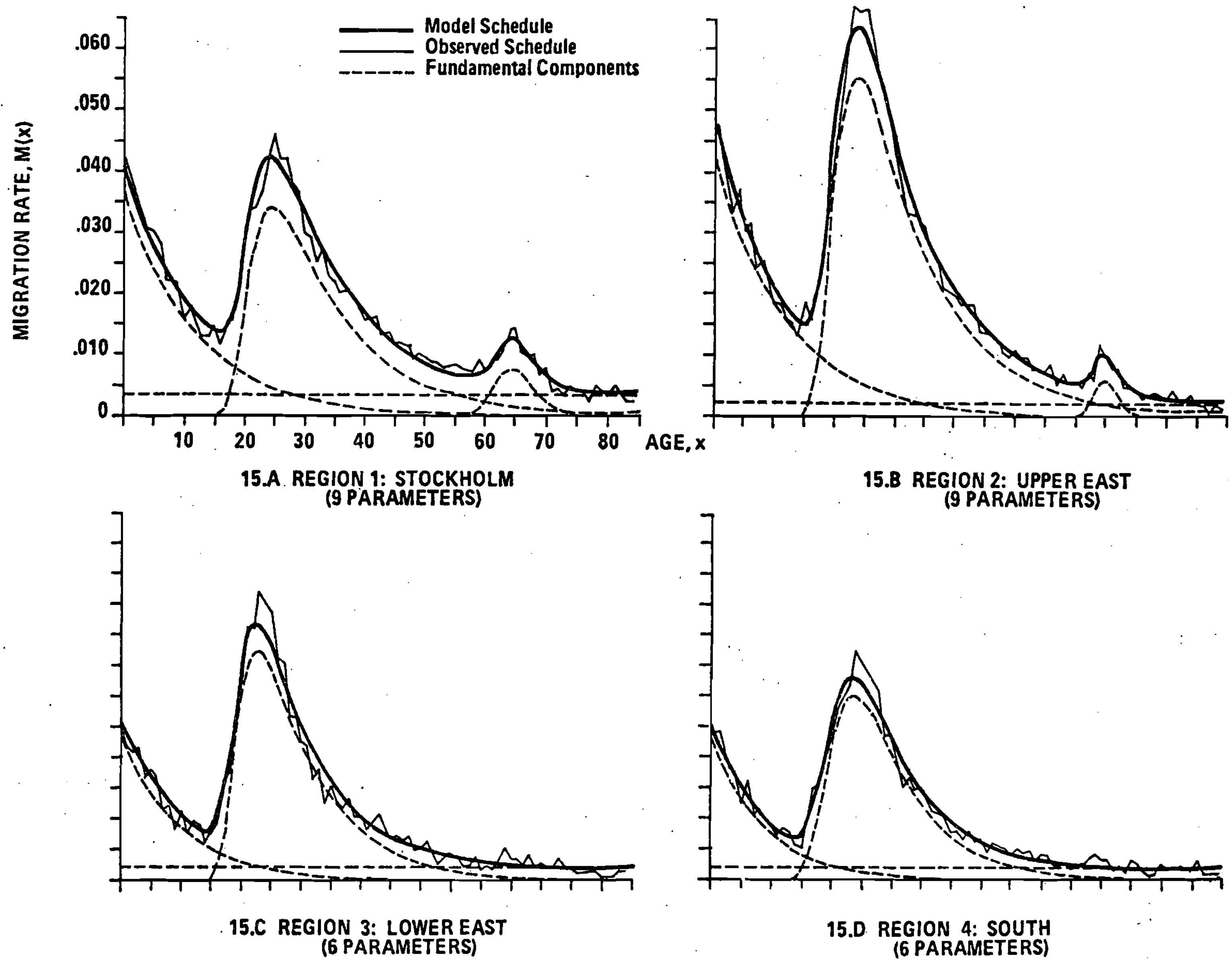


FIGURE 15. OBSERVED AND SIMPLIFIED MODEL MIGRATION SCHEDULES: MALES, SWEDISH REGIONS, 1974

Table 7. The original and the simplified model migration schedules[a]: goodness-of-fits and rates of descent

Region	Original Model[b]			Simplified model[c]	
	MAE/M	α_1	α_2	MAE/M	$\alpha_1 = \alpha_2$
United States, 1966-71					
Males	3.41	0.1141	0.0999	3.60	0.1038
Females	3.96	0.1177	0.1216	3.95	0.1203
Sweden, 1968-73					
Males	3.83	0.1169	0.1019	4.39	0.1076
Females	3.08	0.1251	0.1170	3.13	0.1195
Swedish Regions, 1974					
1. Stockholm					
Males	6.93	0.0971	0.0776	7.48	0.0856
Females	7.29	0.0905	0.0919	7.32	0.0903
2. Upper East					
Males	6.42	0.0811	0.0858	6.46	0.0846
Females	7.37	0.1000	0.1030	7.23	0.1042
3. Lower East					
Males	12.23	0.0984	0.1046	12.45	0.1033
Females	10.82	0.1086	0.1284	11.26	0.1243
4. South					
Males	11.13	0.1170	0.1143	11.07	0.1153
Females	8.77	0.1043	0.1290	9.34	0.1216
5. West					
Males	9.40	0.0895	0.0914	9.39	0.0909
Females	9.31	0.1056	0.1140	9.28	0.1110
6. Lower North					
Males	10.84	0.1037	0.1032	10.83	0.1033
Females	11.66	0.0995	0.1364	12.14	0.1289
7. Middle North					
Males	11.78	0.1228	0.1178	11.72	0.1189
Females	11.40	0.1185	0.1480	11.94	0.1424
8. Upper North					
Males	14.88	0.1356	0.1140	14.85	0.1177
Females	13.28	0.1261	0.1417	13.13	0.1398

a Minimum chi-square estimates

b 11 parameter full model for Stockholm and Upper East only; in all other cases the 7 parameter reduced model was used.

c 9 parameter ($\delta_3 = 5$) simplified full model for Stockholm and Upper East only; in all other cases the 6 parameter simplified model was used.

the high peak, the labor force curve can be closely approximated by the function $a_2 e^{-\alpha_2 (x_2 - \mu_2)}$. Recalling that the pre-labor force curve is given by $a_1 e^{-\alpha_2 x_1}$ when $\alpha_1 = \alpha_2$, we may equate the two functions and solve for the difference in ages, $x_2 - x_1$, to find

$$A = x_2 - x_1 = \mu_2 + \frac{1}{\alpha_2} \ln \frac{a_2}{a_1} \quad , \tag{18}$$

our new analytical definition of the parental shift.

Table 8 compares the values of this analytically defined parental shift with the corresponding observed parental shifts set out earlier in Table 5. The two definitions appear to produce similar numerical values, but the analytical definition has the advantage of being simpler to compute, and it is a more rigorous definition.

In addition to the parental shift, three other measures are sufficient to characterize the profile of a simplified model migration schedule without a retirement peak. They are: the low point, the high peak, and the rate of descent.[2] Taken together the four measures vary in a regular manner, and by using an appropriate chain of inferences, it is possible to identify the particular age profile that they specify.

Figure 16 shows that for a given value of the parental shift, the labor force shift, X, varies as a function only of the rate of descent α and the rate of ascent λ. For a given set of values for x_l, x_p, α and A, it is therefore possible to infer the values of λ and μ. Entering Figure 16 with $x_p - x_l$, α, and A, we obtain λ. With values for λ, α, and A it is possible to enter Figure 17 to

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[2]No direct analytical expression seems to exist for computing the low point and the high peak. However, their values may be calculated by means of an iterative numerical procedure that seeks the age at which the sum of two derivatives along the migration age profile is zero.

Table 8: Observed and model values of the parental shift: Swedish Regions, 1974

The Parental Shift	Regions of Sweden							
	1. Stockholm	2. Upper East	3. Lower East	4. South	5. West	6. Lower North	7. Middle North	8. Upper North
Observed,[a] males	27.61	29.77	29.58	28.98	29.42	28.97	30.07	30.33
Model, males	26.67	28.97	29.63	29.74	28.84	29.43	29.74	30.59
Observed,[a] females	25.16	26.50	27.48	27.79	27.45	26.59	27.21	29.52
Model, females	24.49	26.33	27.51	28.21	27.19	27.69	27.53	28.59

[a] Source: Table 5.

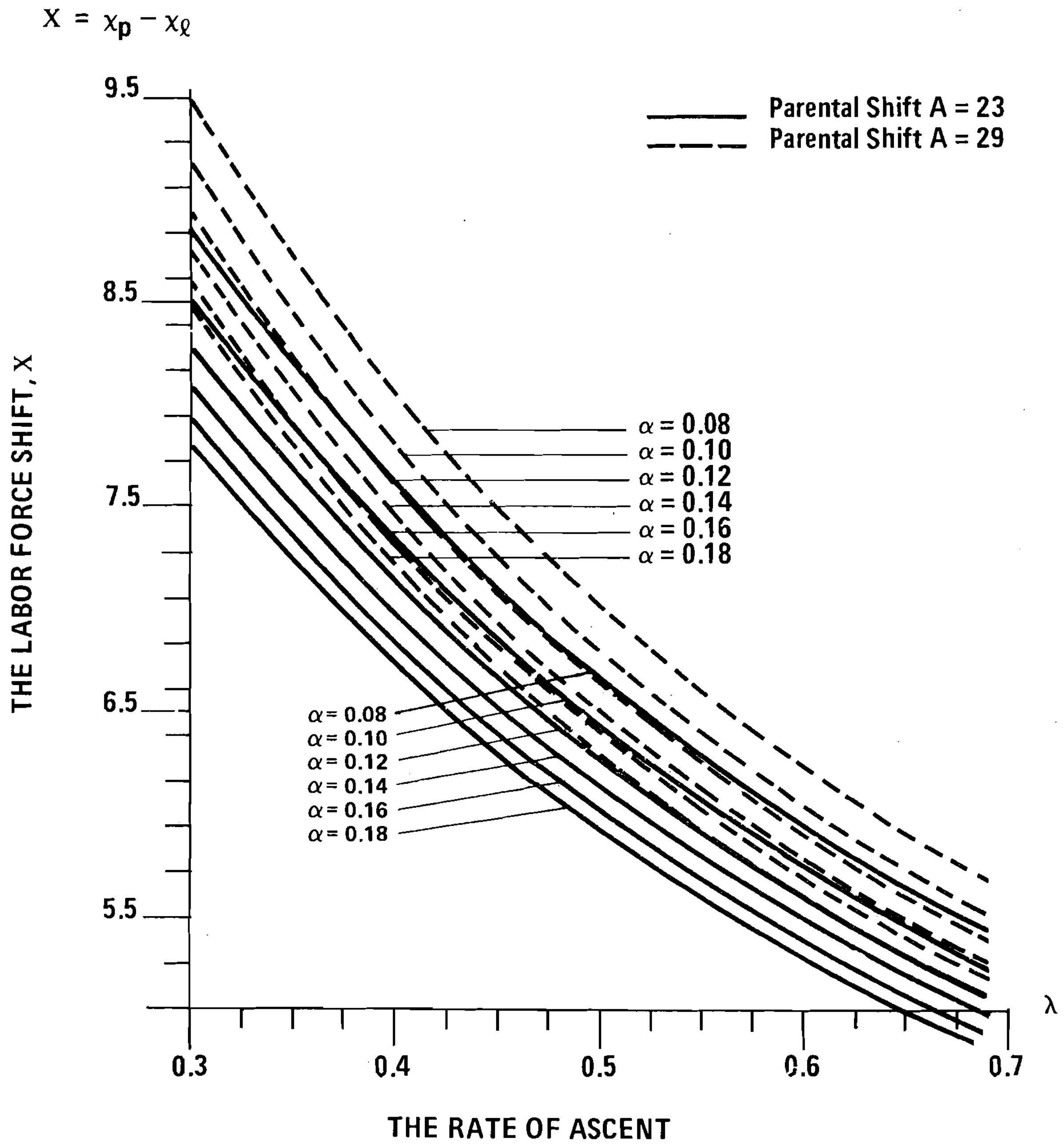

FIGURE 16. GRAPH OF THE LABOR FORCE SHIFT AGAINST THE RATE OF ASCENT FOR TWO VALUES OF THE PARENTAL SHIFT

obtain the values of $x_p - \mu$, and therefore of μ. With values for α, λ, μ we have defined the profile (but not the level) of a model migration schedule. To obtain the level we also need values for a_1, a_2, and c.

Preliminary empirical explorations indicate that *profile* indices such as the low point, the high peak, and the two shifts are somewhat more "robust" descriptive measures of regularities in empirical migration schedules than are *level-related* indices, such as the fundamental model parameters α and λ. Perhaps, this is because the former are "purer" indicators of profile: they do not confound measures reflecting levels (such as, for example, the GMR and the jump) with measures indicating locations along the age axis. This attribute of such profile indices is suggested in Table 9, in which both sets of indicators are presented for inter-regional migration alone and for inter- plus intra-regional (intercommunal) migration taken together. The results are by no means clear-cut, but they do suggest a possibly fruitful direction for further study.

4.4 Application: Graduation and Interpolation

Among the various analytical and practical applications of the model migration schedule concept described above, the most immediately obvious one is the estimation of single-year migration rates from data reported only by five-year age intervals. As a by-product of this operation one also obtains the various fundamental parameters and profile indices described earlier.

Migration rates for five-year age intervals are weighted linear combinations of the corresponding single-year rates, where each particular weight is the proportion of the population in the five-year age interval that falls within a particular single year of age inside of the interval. For purposes of interpolation, these weights may be derived from an observed age composition, or their values may be assumed to follow those of some "standard" population composition, e.g., a stable population.

Given a range of population weights by single years of age and a set of observed migration rates by five-year age intervals, we may search for the model migration schedule that reproduces

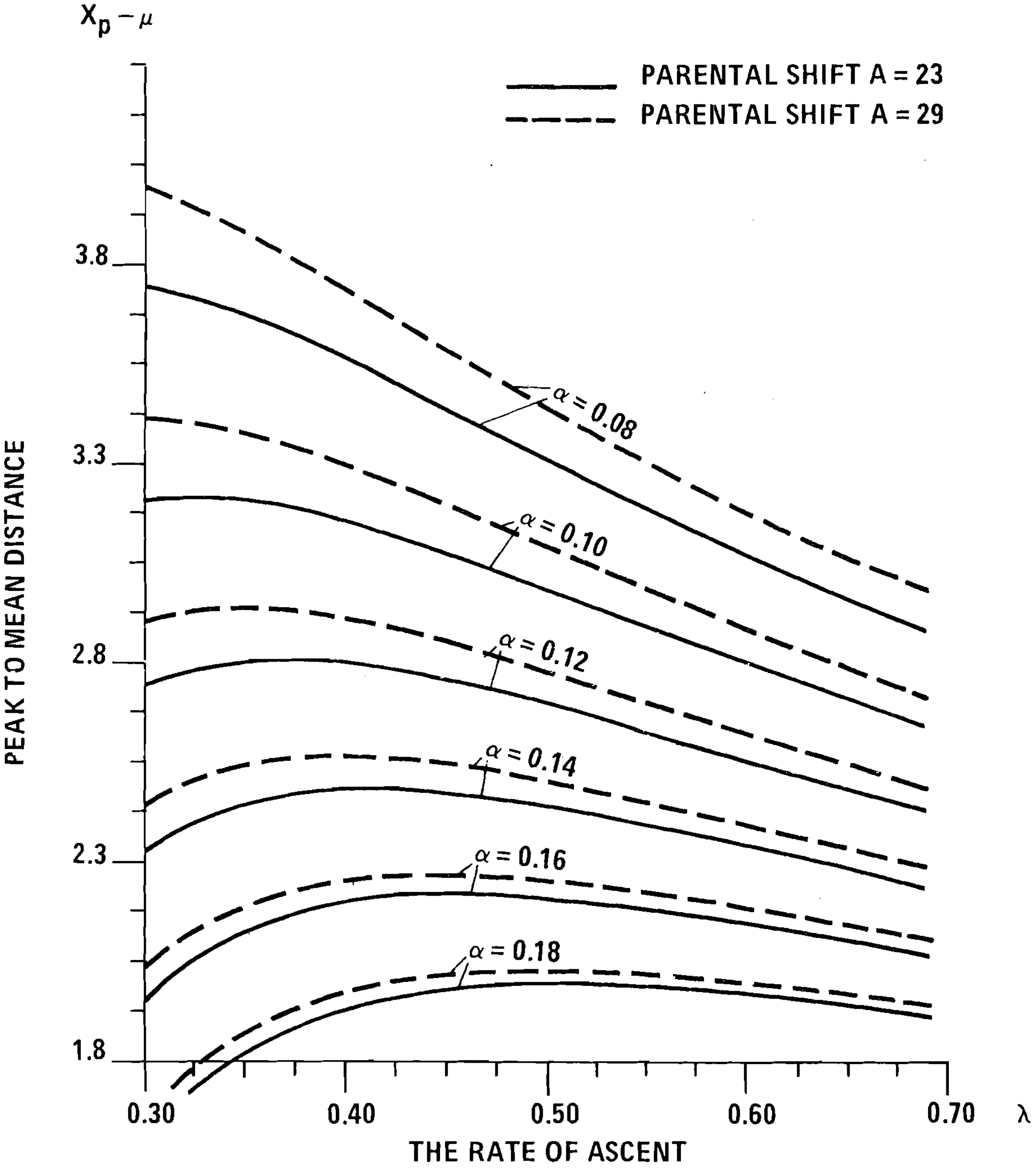

FIGURE 17. GRAPH OF THE PEAK TO MEAN DISTANCE AGAINST THE RATE OF ASCENT FOR TWO VALUES OF THE PARENTAL SHIFT

Table 9. Parameter variation under geographical aggregation

PARAMETERS	Region of Sweden											
	2. Upper East				3. Lower East				4. South			
	Males		Females		Males		Females		Males		Females	
	Inter	All	Inter	All	Inter	All	Inter	All	Inter	All	Inter	All
GMR	1.76	2.32	1.81	2.40	1.01	1.24	1.07	1.31	0.88	1.47	0.85	1.47
$\alpha_1 = \alpha_2$	0.0846	0.0900	0.1042	0.1073	0.1033	0.0958	0.1243	0.1183	0.1153	0.1086	0.1216	0.1195
λ_2	0.4050	0.4164	0.4643	0.4945	0.4061	0.4485	0.5613	0.5880	0.2687	0.3113	0.4482	0.4414
x_p, the high peak	23.74	23.66	21.47	21.40	23.03	22.90	20.97	20.91	24.16	23.70	22.48	21.65
x_1, the low point	15.82	15.94	14.57	14.77	15.30	15.58	14.94	15.03	14.53	14.77	15.46	14.60
X, the labor force shift	7.92	7.72	6.90	6.63	7.72	7.32	6.03	5.88	9.63	8.93	7.02	7.05
A, the parental shift	28.97	28.97	26.33	26.62	29.63	29.36	27.51	27.30	29.74	29.31	28.21	27.38
B, the jump	0.0494	0.0689	0.0621	0.0836	0.0343	0.0404	0.0456	0.0525	0.0264	0.0426	0.0327	0.0525

best (in the least squares or minimum chi-square sense) the set of observed migration rates. Formally, the estimation algorithm is precisely the same as before; only the criterion function to be minimized is slightly altered.

Table 10 presents the results of four such graduation-interpolation experiments using the regional data for Sweden. In it are contrasted the goodness-of-fit statistics, parameter estimates, and level and profile indices produced in the course of fitting the model schedule to migration data by single-year and five-year intervals of age. The results show that to a remarkable extent, five-year age interval data may be used in place of the generally scarcer migration data by single years of age. Information contained in 85 one-year age group rates may be inferred quite accurately from 17 five-year age group rates.

Figure 18 illustrates graphically the quality of the fit provided by the graduation-interpolation procedure to data on the migration of males out of Stockholm and includes, for purposes of contrast, the fit provided to the same input data of a cubic spline (McNeil, Trussell, and Turner, 1977).[3] Note that the spline interpolation is less accurate in identifying the retirement peak, and it introduces a break in the curve (an inflection point) at age 35.

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[3]As an additional indicator of the quality of the fit, we include a plot of the residuals at the bottom of the graph. These may be reduced to zero by further interpolation, but the model schedule then loses its smooth regularities.

Table 10. Graduation-interpolation with the simplified model migration schedule: goodness-of-fit and parameter estimates, Swedish males, 1974

Parameters and Statistics	Region of Sweden							
	1. Stockholm		2. Upper East		3. Lower East		4. South	
	MS[a]	GI[b]	MS[a]	GI[b]	MS[a]	GI[b]	MS[a]	GI[b]
MAE/M	7.48	7.66	6.46	6.82	12.45	12.92	11.07	11.12
GMR	1.46	1.45	1.76	1.75	1.01	1.00	0.88	0.87
$\alpha_1 = \alpha_2$	0.0856	0.0824	0.0846	0.0835	0.1033	0.0963	0.1153	0.1100
λ_2	0.3529	0.3747	0.4050	0.4263	0.4061	0.5974	0.2687	0.2870
x_p, the high peak	24.93	24.92	23.74	23.50	23.03	22.26	24.16	24.04
x_1, the low point	16.53	16.78	15.82	15.81	15.30	16.18	14.53	14.69
X, the labor force shift	8.40	8.15	7.92	7.69	7.72	6.08	9.63	9.35
A, the parental shift	26.67	26.49	28.97	29.01	29.63	29.37	29.74	29.59
B, the jump	0.0290	0.0295	0.0494	0.0505	0.0343	0.0372	0.0264	0.0264

[a] Model Schedule fitted to data by single-years of age (Regions 1 and 2 fitted with 9 parameter model; regions 3 and 4 with 6 parameter model).

[b] Graduation-Interpolation to single-years of age using data by five-year age groups and the simplified model migration schedule.

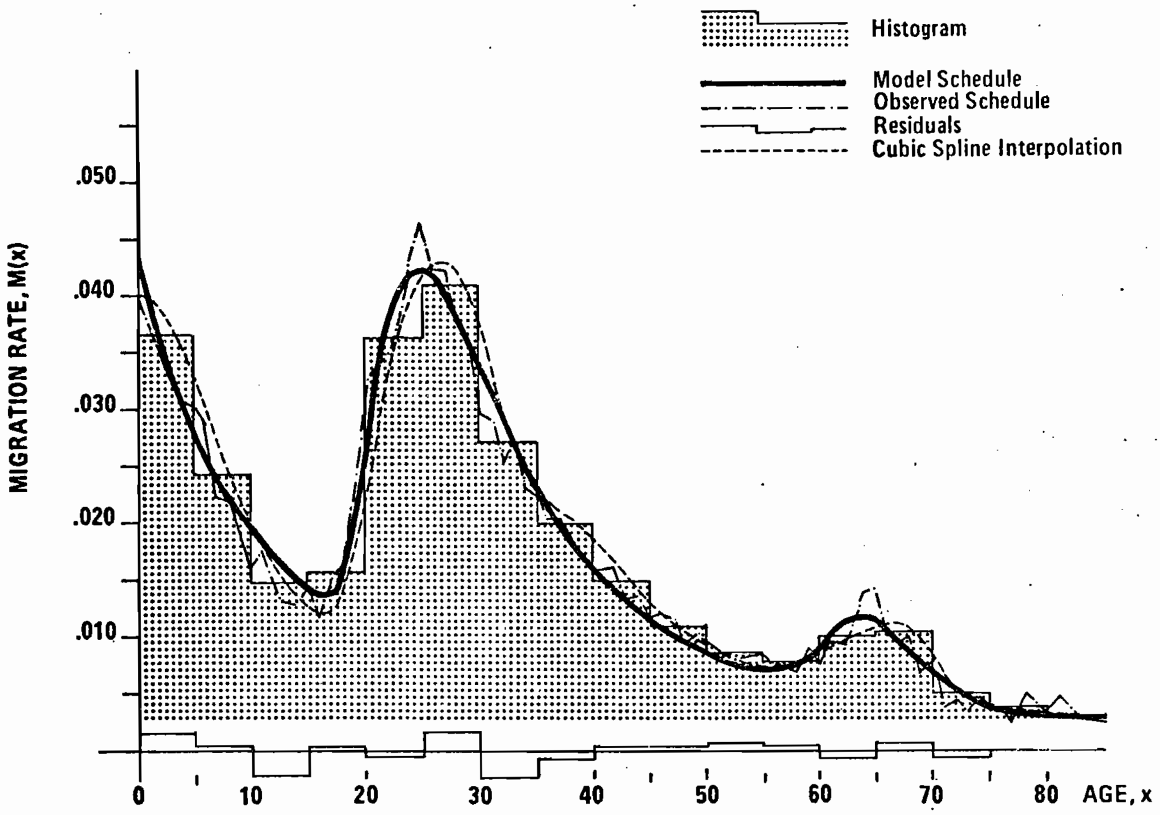

FIGURE 18. TWO ALTERNATIVE GRADUATION – INTERPOLATIONS: THE CUBIC SPLINE AND THE MODEL MIGRATION SCHEDULE, MALES, STOCKHOLM, 1974

5. CONCLUSION

This paper has examined two alternative approaches for summarizing and exploiting the regularities exhibited by empirical migration schedules: the mortality approach and the fertility approach. In developing both we have elected to generalize and extend the standard paradigms put forward by Ansley Coale and his associates, specifically his early work on model mortality schedules (Coale and Demeny, 1966) and his subsequent work on model nuptiality and fertility schedules (Coale, 1971, Coale and McNeil, 1972, and Coale and Trussell, 1974). Our initial exploratory efforts are not yet complete, but they do suggest several observations that will guide our future efforts.

Both approaches have their strengths and weaknesses. The strength of the mortality approach lies in its ability to infer migration flows from two consecutive censuses that contain population data disaggregated by age, region of residence, and region of birth. Its weakness is that it leads to a classification of families of schedules that may have little analytical validity.

The strength of the fertility approach lies in its ability to represent the migration schedule in terms of dimensions that are intuitively appealing and analytically robust. The approach may be used to study the fundamental properties of migration schedules, to identify and smooth-out errors in observed data, and to interpolate within observed migration rates. Its weakness is that it does not suggest a ready method for inferring migration measures on the basis of distributional data alone. The method requires at least some crude estimates of age-specific interregional flows, and these are hard to come by in most developing and several developed countries.

It may well be that a combination of the two approaches will produce the most useful perspective. Such a perspective will demand the integration of concepts associated with model multiregional life tables with those defining model migration schedules. Fractions of a lifetime lived in a particular region will need to be expressed in terms of shifts, jumps, and migra-production rates, and vice versa.

Finally, arguing by analogy, it seems probable that Coale's model of nuptiality and fertility can be transformed into one of labor force participation and migration by reinterpreting

1) entry into the marriage market as entry into the job market,

2) marital search as job search,

3) first marriage frequency as first job frequency, and

4) proportion ever married as proportion ever active.

The menu for future research is a rich one.

References

Arriaga, E. (1968), *New Life Tables for Latin American Populations in the Nineteenth and Twentieth Centuries* (Berkeley: Institute of International Studies, University of California).

Coale, A.J. (1971), "Age Patterns of Marriage", *Population Studies*, 25, 193-214.

Coale, A.J. (1972), *The Growth and Structure of Human Populations* (Princeton, J.J.: Princeton University Press).

Coale, A.J. and P. Demeny (1966), *Regional Model Life Tables and Stable Populations* (Princeton, N.J.: Princeton University Press).

Coale, A.J. and D.R. McNeil (1972), "The Distribution by Age of the Frequency of First Marriage in a Female Cohort," *Journal of the American Statistical Association*, 67, 743-749.

Coale A.J. and T.J. Trussell (1974), "Model Fertility Schedules: Variations in the Age Structure of Childbearing in Human Populations," *Population Index*, 40, 2, 185-258.

Hoem, J.M. (1972), "On the Statistical Theory of Analytic Graduation," *Proceedings of the Sixth Berkeley Symposium on Mathematics, Statistics, and Probability, Vol.1.* (Berkeley, Cal.: University of California Press), 569-600.

Hoem, J.M. and E. Berge (1975), "Some Problems in Hadwiger Fertility Graduation," *Scandinavian Actuarial Journal*, 129-144.

Keyfitz, N. (1968), *Introduction to the Mathematics of Population* (Reading, Mass.: Addison-Wesley).

Long, L.H. (1973), "New Estimates of Migration Expectancy in the United States," *Journal of the American Statistical Association*, 68, 37-43.

Lowry, I.S. (1966), *Migration and Metropolitan Growth: Two Analytical Models* (San Francisco: Chandler).

McNeil, D.R., T.J. Trussell, and J.C. Turner (1977), "Spline Interpolation of Demographic Data," *Demography*, 14, 245-252.

Polish Central Bureau of Statistics (1974), *Rocznik Demograficzny, 1974*, No.33, Warsaw.

Rele, J.R. (1967), *Fertility Analysis Through Extension of Stable Population Concepts* (Berkeley: Institute of International Studies, University of California).

Rogers, A. (1973), "The Multiregional Life Table," *Journal of Mathematical Sociology*, 3, 1, 127-137.

Rogers, A. (1975), *Introduction to Multiregional Mathematical Demography* (New York: John Wiley).

Rogers, A. and L.J. Castro (1976), *Model Multiregional Life Tables and Stable Populations, RR-76-09*, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rogers, A. and J. Ledent (1976), "Increment-Decrement Life Tables: A Comment," *Demography*, 13, 287-290.

Stoto, M.A. (1977), *On the Relationship of Childhood to Labor Force Migration Rates, RM-77-00*, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Swedish Central Bureau of Statistics, (1974), *Internal Migration in Sweden 1968-1973*, No.9, Stockholm.

Tukey, J.W. (1977), *Exploratory Data Analysis*, (Reading, Mass.: Addison-Wesley).

United Nations (1955), *Age and Sex Patterns of Mortality: Model Life Tables for Underdeveloped Countries* (New York: United Nations).

United Nations (1967), *Methods of Estimating Basic Demographic Measures from Incomplete Data*, (New York: United Nations).

Wilber, G.W. (1963), "Migration Expectancy in the United States," *Journal of the American Statistical Association*, 58, 444-453.

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