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Endogenous Risks and Learning in Climate Change Decision Analysis

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Abstract

We analyze the effects of risks and learning on climate change decisions. A two-stage, dynamic, climate change stabilization problem is formulated. The explicit incorporation of ex-post learning induces risk aversion among ex-ante decisions, which is characterized in linear models by VaR- and CVaR-type risk measures. Combined with explicit introduction of "safety" constraints, it creates a "hit-or-miss" type decision-making situation and shows that, even in linear models, learning may lead to either less- or more restrictive ex-ante emission reductions. We analyze stylized elements of the model in order to identify the key factors driving outcomes, in particular, the critical role of quantiles of probability distributions characterizing key uncertainties.

Acknowledgments

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1. Introduction

Discussions of climate change policies are often framed as a choice between acting now or waiting until we know more about the problem (Manne and Richels, 1992; Webster 2002; Wright and Erickson, 2003). The climate change policy dilemma poses a "hit-or-miss" type of decision making situation. Emissions of greenhouse gases (GHG) are associated with the production and consumption of goods and services, and the atmospheric concentration (stock) of these long-lived pollutants may cause irreversible damages. On one hand, postponing the reduction of GHG emissions may lead to potentially irreversible climate-related impacts, for example, reorganizations of large-scale ocean circulation patterns or increased frequency of extreme weather-related events. On the other hand, if the problem turns out to be less severe than expected, the delay will avoid irreversible investments in capital for emissions abatement.

These discussions are often supported by a traditional expected utility maximization model, suggesting either risk-averse *ex ante* (anticipative) decisions or risk-prone *ex post* (adaptive) decisions that are made only after receiving full information. If uncertainty about climate change is resolved over time, the robust strategy would definitely be to make only partial *ex ante* commitments and to keep options open until new information is revealed. This approach requires so-called two-stage recourse stochastic optimization (STO) models (Dantzig and Madansky, 1961; Ermoliev and Wets, 1988; Kall and Wallace, 1994; Yastremskij 1983) incorporating both *ex ante* and *ex post* decisions within a single model. Accordingly, the climate change stabilization problem (Section 2) can be formulated as the choice of an *ex ante* risk reduction (stage 1) strategy over a random time horizon taking into account that it may need to be adjusted after new information is revealed (stage 2). This decision-making framework implies, in particular, that the capacity for adaptive decision making in stage 2 has to be created *ex ante*.

In economics literature, the importance of learning was first discussed in connection with irreversible investments in 1974 in Arrow and Fisher (1974) and Henry (1974) without an overall two-stage model being formulated. Arrow and Fisher (1974), Henry (1974), and Chichilnisky and Heal (1993) have concluded that when future damages are uncertain and irreversible, the ability to learn should lead to more active *ex ante* emission reductions. On the other hand, irreversibility of capital may lock an

economy into a wasteful use of resources. Viscusi and Zeckhauser (1976), Dixit and Pindyck (1994), Ulph and Ulph (1997), and Pindyck (1999) showed that the ability to learn in this case should lead to less active *ex ante* emission reduction. These competing effects imply that the net effect of learning on *ex ante* decisions is an empirical question. Nordhaus (1994) and Kolstad (1996) examined the effects of learning by using empirically-calibrated integrated assessment models. They concluded that, in fact, learning has insignificant effects on *ex ante* abatement policies because the damage losses are not severe enough. A reason for this is that in most integrated climate and economics models, climate changes are considered as if they occur continuously and as if they can eventually be reversed through *ex post* adjustments (Wright and Erickson, 2003). These models also use average damages (i.e., they cannot properly capture the effects of abrupt climate change and catastrophic risks (Ermoliev et al, 2000; O'Neill and Oppenheimer, 2002; Wright and Erickson, 2003)). A paper by Fisher and Narain (2003) analyzes a two-period model with risk characterized by a parameter introducing high or low climate change damages. Because overall impacts are evaluated by using expected values, the effects of capital irreversibility dominate catastrophic damages in a similar way to other models. Epstein (1980) demonstrated that the effects of learning on *ex ante* decisions depend in general on the convexity or concavity of marginal costs, which are very restrictive for climate change policy analysis (Ulph and Ulph, 1997).

In this paper we take a different approach. Instead of using expected damages we explicitly introduce safety constraints by formulating the climate change problem within the framework of stabilization. We develop a two-stage dynamic STO model and deliberately analyze only stylized linear versions of this model. We show that the combination of safety constraints and perspectives of learning induces potentially strong risk aversion among *ex ante* decisions that is characterized by quantile-based VaR and CVaR risk measures. As a result we show that, even with a linear net cost function, learning may lead to either less or more restrictive emission reductions, depending on mitigation costs and probability distributions describing key uncertainties.

Section 2 characterizes climate change risk by the probability of total atmospheric CO_2 concentrations exceeding a vital random threshold associated with potential ranges of global temperature. It outlines a two-stage dynamic climate change stabilization STO model with random durations of stages. In general, this model can be solved only numerically and therefore the key factors driving results are difficult to identify. For these reasons, we analyze only stylized aspects of the model; these provide a clearer picture of the various driving forces and show why the ability to learn in the future can lead to either less-restrictive or more-restrictive *ex ante* abatement policies today.

Section 3 uses a very simple linear two-stage STO model to illustrate that the results from empirical models can be rather contradictory, because optimal solutions depend on complex nonsmooth interactions among *ex ante* and *ex post* decisions, costs, and probability distributions. In particular, they induce potentially strong risk aversion characterized by risk measures that are used for regulating the safety of nuclear plants and insolvency of insurance companies, but also in financial applications, extremal value theory (Embrechts et al, 2000), and catastrophic risk management (Ermoliev et al, 2000).

Section 4 analyzes the effects of uncertain costs and random duration of stages. It emphasizes the importance of quantiles of probability distributions characterizing key uncertainties and shows that without aggressive *ex ante* commitments, a misperception of limited adaptive second-stage capacity may provoke a catastrophe (i.e., exceedance of the safety constraint).

Sections 4 and 5 discuss the controversial effects of incomplete learning and nonlinear costs, which more strongly and even unconditionally require coexistence of *ex ante* anticipative and *ex post* adaptive risk-management decisions.

A more realistic but still linear dynamic two-stage climate change stabilization STO model will be analyzed in Section 6. Similar to Section 3, the explicit incorporation of *ex ante* and *ex post* decisions induces risk aversions characterized by a dynamic version of a CVaR type risk measure. This may create a wrong impression of truly risk-based policy analysis and, without explicit introduction of adaptive capacity and additional safety constraints, may provoke a catastrophe. In conclusion, Section 7 emphasizes the importance of the proper treatment of uncertainty, risks, and robust decisions, as well as the development of adequate computational approaches.

2. Endogenous Climate Change Risk

Climate change integrated assessment models (see, e.g., (Nordhaus, 1994)) incorporate economic and geophysical processes that link economic growth models with the accumulation of GHG emissions in the atmosphere. The accumulation of CO_2 emissions is the main driving force behind global climate change. The process involves complex interactions between the atmosphere, the upper oceans and the biosphere, and the deep oceans. Current integrated assessment models use different carbon cycle parameterizations for computing changes in atmospheric concentrations M(t) resulting from CO_2 emissions $e(t)$ (Schultz and Kasting, 1997). In general, these models are of the form

$$M(t+1) = f[M(t), e(t), \beta], \ t = 0, 1, 2, \ldots \tag{1}$$

where β is a vector of model parameters. Values $M(t)$ are used in integrated assessment models to compute the increase in the global average temperature as a smooth function of $M(t)$, and damages are typically computed in the form of smooth functions of this temperature increase. The resulting smooth dependence of damage functions on global average temperature does not permit the proper modeling of abrupt climate changes. A serious underestimation of climate change impacts may also result from the use of average global temperature. It is possible that changes in average global temperature would be associated with an increase in the frequency of extreme weather-related events or could trigger singular events with widespread consequences (e.g., the disintegration of the West Antarctic Ice Sheet (O'Neill and Oppenheimer, 2002)). Therefore, proper modeling of risks is crucial for evaluating climate-change policies.

Emissions $e(t)$ in (1) depend on a vast variety of uncertainties, denoted by ω, and policy variables, denoted by x. In this paper, we assume that emissions $e(t, \omega)$, $t = 0, 1, \ldots$, are characterized as random variables defined in a probability space Ω of scenarios ω, $\omega \in \Omega$, with a probability measure $P(d\omega)$. Thus, for $\Omega = \{1, \ldots, N\}$,

$P(d\omega) = p(s) := P[\omega = s]$, $\sum_{s=1}^{N} p(s) = 1$. Frequently we do not indicate the dependence of random variables on ω if this is clear from the context.

We introduce risk by imposing a safety constraint in the form of an atmospheric stabilization target, beyond which it is assumed that the risks of high-impact outcomes substantially increase. Let us denote by $L(\omega)$ the uncertain target level of CO_2 concentration in the atmosphere. The main problem can be formulated as the choice of a cost-efficient emission-reduction path that satisfies probabilistic safety constraints on vital but uncertain levels of concentrations

$$P\left[M(t,x,\omega) \leq L(\omega), t = \overline{1,T}\right] \geq 1 - \gamma, \tag{2}$$

where γ is a risk factor, $0 \leq \gamma < 1$, T is a time horizon that may also be uncertain. In the insurance industry, constraints of type (2) regulate risk reserves to prevent insolvency. The typical approach to choosing γ in this industry is not based primarily on evaluating potential damages, but rather on limiting the chance that the insolvency may occur to, say, once in 800 years, $\gamma = 1/800$. Similarly, the major failure of a nuclear plant is allowed once in 10^7 years, $\gamma = 10^{-7}$. Note that these are expected time horizons, and therefore there is the possibility that events may occur at any time.

The concentration-stabilization problem can be formulated in a similar way to the catastrophic risk management problem discussed in (Ermoliev et al, 2000). At stage 1, the emission-reduction path is defined by *ex ante* decisions $x(t)$, $t = 1,2,...$ until a random time moment τ when new information about uncertain variables is revealed. The new information may also include a new critical time horizon $T(\omega)$ for stage 2 *ex post* emission reductions $x(t,\omega)$, $t = \tau+1,...,T(\omega)$. The problem is to minimize total emission reduction cost.

The resulting model can only be solved numerically. Here, instead of using numerical simulations, we take a different approach. In the following sections we formulate various stylized elements of the model and evaluate them analytically. This allows us to keep the discussion on a simple level, which provides a clear picture of the potential results.

Remark 1: The abrupt climate change in (2) is modeled by random $L(\omega)$, which is revealed as a "shock" at random moment $\tau(\omega)$. Despite a smooth and even linear dependence of function $M(t,x,\omega)$ on x, the left-hand side of (2) is, in general, a nonsmooth and often even a discontinuous risk function (Ermoliev and Norkin, 1997; Ermoliev and Wets, 1998; Marti, 2005). Endogenous catastrophic collapse is modeled as a violation of constraint (2). In general, the learning may not reveal full information but perhaps only shift ranges of probability distributions. The learning may also not occur at $\tau(\omega) \leq T$, or it may occur very close to $T(\omega)$. Since the inertia of the system may not allow constraints (2) to be fulfilled quickly, the probability of a catastrophe conditional on revealed information may drop rapidly below the vital level γ (i.e., constraint (2) emphasizes the importance of proper *ex ante* actions).

3. Linear Cost Functions

The following two-stage model is relatively easy to analyze. It provides suggestive results and serves as a building block of more general two-stage dynamic models (Section 7). Even in its simplest form, it already shows that results from purely empirical models can be contradictory regarding the effects of learning on *ex ante* decisions, since optimal solutions depend on complex nonlinear interactions among decisions, costs, and probability distributions that characterize uncertainties.

3.1. Two-stage model

A stylized climate change stabilization problem can be formulated as follows: assume that there are only two time intervals or periods $t = 1,2$. Define by x_t, $x_t \geq 0$, $t = 1,2$, a feasible level of emission reduction that can be chosen in period t; C_t, $C_t > 0$ is the known expected abatement cost per unit of emission reduction in period t; $\theta(\omega)$ is the uncertain target value of cumulative emission reductions for two periods. In this problem formulation, $\theta(\omega)$ serves as the safety constraint, $x_1 + x_2 \geq \theta(\omega)$. The constraint on minimum emissions reduction can be thought of as a concise way to represent several factors (and their uncertainties) that come into play in meeting a target based on environmental outcomes such as atmospheric concentrations, global average temperature levels, or particular impacts. For example, emissions reductions required to meet a target will depend on the target itself (i.e., whether a concentration or temperature target is high or low), on reference emissions e(t) (because the absolute size of required emissions reductions will depend on the magnitude of uncontrolled emissions in the reference case), and on the system mapping emissions to environmental outcomes (e.g., parameters of the carbon cycle or climate system). Uncertainty in $\theta(\omega)$ can be thought of as reflecting uncertainty in one or more of these different factors.

Assume uncertainty in $\theta(\omega)$ is resolved between periods 1 and 2. The *ex ante* decision x_1 is made before the uncertainty in θ is resolved, whereas the *ex post* decision x_2 is based on known θ (i.e., x_2 is a function of θ, $x_2(\theta)$). Assume that the ability to fulfill risk constraint $x_1 + x_2 \geq \theta(\omega)$ in period 2 is unbounded. The impacts of this rather unrealistic assumption (which is often a standard assumption of existing integrated assessment model) are analyzed in Section 4. The problem is formulated as the minimization of total linear costs

$$C_1 x_1 + C_2 E x_2(\theta) \tag{3}$$

subject to safety constraints

$$x_1 + x_2(\theta) \geq \theta, \text{ for all } \theta. \tag{4}$$

Clearly, if the *ex ante* decision x_1 is irreversible, then the optimal period 2 decision is $x_2^*(\theta) = \max\{0, \theta - x_1\}$, that is, it nonsmoothly depends on period 1 decision x_1 (path dependence) and θ, providing potentially strong cross-period random

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interactions among decisions. Optimal period-1 decision x_1^* solves the stochastic minimax problem: minimize

$$F(x) = C_1 x + C_2 E \max \{0, \theta - x\}, \ x \geq 0. \tag{5}$$

Remark 2: Although the initial model (3)-(4) is linear in (x_1, x_2), the introduction of *ex post* decision x_2 induces risk aversion among *ex ante* decisions that is defined by implicit nonsmooth (in general) function (5). The following *Proposition* summarizes some important facts about stochastic minimax problem (5). It shows that the induced risk attitudes are characterized by VaR (critical quantile) and CVaR risk measures (Rockafellar and Uryasev, 2000). In extremal value theory (Embrechts et al, 2000), CVaR is also known as Mean Shortfall and Mean Excess Loss.

Proposition:

(i) $F(x)$ is a convex function. If $H(z) = P[\theta \leq z]$ is a continuously differentiable function, then $F(x)$ is a strictly convex continuously differentiable function.

(ii) If $C_1 > C_2$, then $x_1^* = 0$ and $x_2^*(\theta) = \theta$. If $C_1 < C_2$, then the necessary and sufficient condition for optimal x^* reads: x^* is the quantile satisfying equation

$$P[\theta \geq x] = C_1 / C_2. \tag{6}$$

(iii) The optimal value $F(x^*)$ has two important representations:

$$F(x^*) = C_1 \bar{\theta} + C_1 E[x^* - \theta \mid \theta \leq x^*] = C_2 E \theta I(\theta \geq x^*), \tag{7}$$

where $E[\cdot \mid \cdot]$ denotes the conditional expectation, the indicator function $I(\theta > x) = 1$ if $\theta \geq x$ and $I(\theta \geq x) = 0$ otherwise.

Let us outline the proof.

(i) The convexity of $F(x)$ follows from the convexity of function $\max \{0, \theta - x\}$ which is preserved under expectation operation. The strict convexity of $F(x)$ follows from the continuous differentiability of $F(x)$.

(ii) The minimization of $F(x)$ is a specific case of so-called stochastic minimax problems [10]. From the general results it follows that $F'(x) = C_1 - C_2 P[\theta \geq x]$. From $C_1 < C_2$, it follows that $F'(0) < 0$, i.e., $x^* > 0$ (assuming $x^* = 0$ we can derive a contradiction with assumption $C_1 < C_2$ for small x). As $F(x)$ is a strictly convex function, it follows that (6) is indeed a necessary and sufficient optimality condition.

(iii) The first representation in (7) follows from (6) and the following rearrangements:

$$F(x^*) = C_1 x^* + C_2 E \max \{0, \theta - x^*\} = C_1 x^* + C_2 E[\theta - x^* \mid \theta > x^*] P[\theta > x^*] =$$

$$= C_1 x^* + C_1 \left(E[\theta - x^*] - E[\theta - x^* \mid \theta \leq x^*] \right) = C_1 \bar{\theta} + C_1 \left(E[x^* - \theta \mid \theta \leq x^*] \right).$$

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The second representation in (7) follows from (6) and

$$E \max\{0, \theta - x^*\} = E\theta I(\theta \geq x^*) - x^* P(\theta \geq x^*).$$

Remark 3: The critical quantile in (6) defines the VaR risk measure, i.e., it indicates the magnitude of emission reduction in stage 1 that, with probability 1- C_1/C_2, will be sufficient to meet the safety constraint with no additional reduction required in stage 2. The second equation in (7) defines the CVaR risk measure; i.e., the expected value of abatement costs that will be necessary in stage 2 if emissions reductions in stage 1 are not sufficient to meet the safety constraint. For some distributions it is possible to derive x^* from (6) explicitly. If θ is uniformly distributed on $[a,b]$, then it is easy to see that $x^* = \dfrac{C_1}{C_2} a + \left(1 - \dfrac{C_1}{C_2}\right) b$ (i.e., x^* is between optimistic and pessimistic scenarios of emissions with weights defined by ratio of costs C_1 and C_2).

3.2. Comparative analysis

The *Proposition* of Section 3.1 allows the comparison of cases of perfect information, full uncertainty, and uncertainty with learning. Equation (6) shows the critical dependence of period 1 optimal decision on the probability distribution H. Assume that $C_1 < C_2$. In the case of perfect information, i.e., when θ is known at the beginning of the first period, both x_1 and x_2 can be chosen as a function of observable θ. Clearly, the optimal solution is $x_1^* = \theta$, $x_2^* = 0$. Thus, the first term $C_1 \bar{\theta}$ of the first equation in (7) represents the expected cost under perfect information. The second term represents the expected value of perfect information because this cost would be eliminated if θ were known before the first-period emission-reduction decision had to be made (rather than afterward as in the learning case). In the case of full uncertainty ("without learning"), the optimal decision $x_1 = \bar{\theta}$, $x_2 = 0$ is also known as the certainty equivalent. The possibility of learning combined with explicit introduction of *ex post* decisions specifies optimal period 1 abatements by the quantile satisfying (6). It may exceed the certainty equivalent $\bar{\theta}$ or it may be below this level. As equation (6) shows, this depends on the relative values of costs C_1, C_2, and the probability distribution H. For example, if $C_1 / C_2 = 1/2$ and θ has a normal distribution, then optimal *ex ante* abatement coincides with the certainty equivalent $x_1^* = \bar{\theta}$. For non-normal probability distributions, the optimal abatements can be below or above $\bar{\theta}$.[1]

Remark 4: The certainty equivalent solution in the case of full uncertainty (no learning) does not satisfy (4) for all θ, which may lead to a catastrophic collapse of high probability. The only way to fulfill the safety constraint (4) is to choose x_1 from the

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[1] An asymmetric probability distribution can be caused, for example, by the interaction of a symmetric probability distribution with an environmental constraint. For example, if the probability density function for *e(ω)* is normal, the distribution for *θ(ω)* can still be asymmetric if there is an atmospheric concentration constraint that does not require emissions reductions for all *ω*.

worst-case scenario as $\max_{\omega} \theta(\omega)$, $\omega \in \Omega$. Clearly, this is an unrealistic and extremely costly solution. This calls for the explicit introduction of safety constraint (2) to provide a trade-off between cost effectiveness and risk. The optimal solution under full uncertainty is now defined as minimal x_1, $x_1 = x_\gamma$, satisfying equation $P[x_1 \geq \theta] = 1 - \gamma$ (since $C_1 < C_2$, $x_2 = 0$). Clearly, the risk-based solution under full uncertainty $x_1 = x_\gamma$ may be greater or less than $\bar{\theta}$, depending on γ, C_1 / C_2, and probability distribution H. Imperfect learning (Section 4.2) may shift the ranges (the support) of the probability distribution requiring *ex ante* abatement that is more or less-aggressive than x_γ.

4. Uncertain Costs and Outcomes

4.1. Uncertain cost

This section illustrates that the use of expected costs (a common assumption of standard integrated assessment models (Wright and Erickson, 2003)) can be misleading, which calls for the use of quantiles rather than expectations. Assume that stage 2 cost C_2 in (3) is observable (ex-post) random variable, $C_2(\omega)$, $C_2 = EC_2(\omega)$, $C_1 > 0$, $C_2 > 0$. Function (5) is written as $F(x) = C_1 x + EC_2(\omega) \max\{0, \theta(\omega) - x\}$. The optimal *ex ante* abatements satisfy equation $F'(x) = C_1 - EC_2(\omega)I(\theta \geq x) = 0$, i.e., risk attitudes induced among *ex ante* decisions are now characterized by more complicated risk measures than the CVaR risk measure of Section 3.1. Let us simplify the analysis by assuming as in Arrow and Fisher, (1974) that $\theta = d$, where d is a known (deterministic) positive number. Then, without learning of cost $C_2(\omega)$, the problem is to minimize the expected cost $C_1 x_1 + C_2 x_2$, where $x_1 + x_2 = d$, $x_1 \geq 0$, $x_2 \geq 0$. If $C_1 < C_2$, then the solution $x_1^* = d$, $x_2^* = 0$ is optimal. The ability to learn before the period 2 decision is made implies that a decision maker may reject emissions reduction in period 1, with a likelihood given by the probability that $C_2(\omega) < C_1$. This poses the important question about the applicability of expected costs, which is also the key issue for evaluating the potential catastrophic impacts characterized by skewed and multimodal distributions (see, e.g., Figures 5, 7 in Ermoliev et al, 2000). This requires the use of quantiles, say, the median rather then expected values.

Consider random two-period costs $C_1 x_1 + C_2(\omega)(d - x_1)$. The median of $C_1 x_1 + C_2(\omega)(d - x_1)$ is $C_1 x_1 + med C_2(\omega)(d - x_1)$. Assume that $C_1 = 3.5$, $C_2(\omega) = -3$ with probability 1/3, $C_2(\omega) = 15$ with probability 1/3, and with probability 1/3 it is uniformly distributed in the interval [2,4]. Thus, $EC_2(\omega) = 5$, $med C_2(\omega) = 3$. Therefore, the optimal solution using expectations without learning is $x_1^* = d$, $x_2^* = 0$. Let us notice that the expected value $EC_2(\omega) = 5$ does not occur in reality, i.e., unlike the median it does not belong to the support of the distribution of $C_2(\omega)$. The use of the median leads to the optimal solution $x_1^* = 0$, $x_2^* = d$, i.e., it reverses the previous

conclusion regarding the optimal solution. Assume now that $C_1 = 11$, $C_2(\omega) = -5$ with probability 1/5, $C_2(\omega) = 5$ with probability 1/5 and with probability 3/5 it is uniformly distributed in interval [14,16]. In this case, $EC_2(\omega) = 9$, i.e., the optimal solution without learning is $x_1^* = 0$, $x_2^* = d$. Since $medC_2(\omega) > 14$, then the use of the median leads to the opposite optimal solution $x_1^* = d$, $x_2^* = 0$.

Remark 5: An approach for minimizing quantiles follows from Section 3.1. It shows that the minimization of a quantile of $C_1 x_1 + C_2(\omega) \max\{0, \beta - x\}$ can be reduced to minimization of a convex function $y + \mu E \max\{0, C_1 x + C_2(\omega) \max\{0, \beta - x\} - y\}$ with respect to (x, y). The solution of this STO problem (x^*, y^*) satisfies constraints

$$P[C_1 x + C_2(\omega) \max\{0, \beta - x\} \geq y] = \frac{1}{\mu}.$$

4.2. Uncertain durations of stages: Limited adaptive capacity

The general two-stage problem (Section 2) with constraint (2) has a random duration of stages. There are at least two reasons this duration may be uncertain. First, in cases with learning, the timing of new information may be uncertain. For example, if learning is slow, the second period may occur late, while if learning is fast the second period will begin early. In addition, inertia in socio-economic systems may affect the duration of stages. For example, a given emissions reduction in period 1 may take a long time in systems with substantial inertia, but less time in more flexible systems (Ha-Duong et al, 1997). The path-dependencies (inertia) of the socio-economic systems producing greenhouse gasses are critical for dealing with abrupt changes. Without inertia, the switching from one emission path to another would be instantaneous. In reality, energy production systems cannot be changed overnight. As a result, preparedness programs may be only partially implemented because of the potential lack of time in both stages. The proper treatment of these effects requires the explicit introduction of random durations of stages. The following show that a delay with proper *ex ante* abatements may result in a violation of constraint (2).

In its simplest form, the uncertain duration of stages can be modeled by constraints $x_2 \leq \beta$ with positive random β which becomes known from learning at stage 2. In other words, it is assumed that the uncertain duration of stage one (with fixed two period time horizon T) affects the capacity for reductions in period 2 (for example, a longer period 1 reduces the scope for reductions in period 2 given its shorter duration).

Without the safety constraint of type (2), the optimal stage 2 decision $x_2 = \min\left[\beta, \max\{0, \theta - x_1\}\right]$ cannot in general satisfy safety constraints (4) for all θ. As a consequence, the probability of a catastrophe can be rather high, calling for explicit introduction of type (2) safety constraint $P[x_1 + x_2 \geq \theta] = 1 - \gamma$. Since $x_2 \leq \beta$, this requires *ex-ante* emission reduction commitments $x_1 \geq x_\gamma$, where x_γ is minimal non-negative x satisfying equation $P[x \geq \theta - \beta] = 1 - \gamma$. Therefore, in order to prevent a

catastrophic collapse, there must exist regulations on minimal *ex-ante* emission reductions sufficient to keep open the possibility of satisfying the safety constraint in stage 2, which can be evaluated by analyzing STO problems with safety constraints (2).

4.3. Incomplete learning and safety constraints

Consider a very realistic case where the learning affects only the prior distribution $H(z)$, in other words, it shifts the range of uncertainty. As the optimal period 1 decision of Section 3.1 is a quantile of $H(z)$, learning may dramatically affect this decision in different directions. Let us assume that $H(z) = P[\theta \leq z]$ is a mixture $H(z) = E_\xi H(\xi, z) = \int H(y, z) dG(y)$ of distribution $H(\xi, z)$ with unknown ξ characterized by a probability distribution $G(y) = P[\xi \leq y]$, which may reflect polarized views on scenarios of climate changes. The learning reveals only ξ at the beginning of period 2. For example, $H(z)$ can be a mixture of distributions $H(\xi, z)$ with probability mass concentrated in different subregions from the support of $H(z)$. If the support of $H(\xi, z)$ is a singleton, then the learning of ξ reveals the true value of θ. For the sake of illustration, let $H(z)$ be a mixture of two distributions $H_0(z)$ and $H_1(z)$, that is, $\xi H_0(z) + (1 - \xi) H_1(z)$, where $\xi = 0$ with probability p and $\xi = 1$ with probability $1 - p$, that is, $H(z) = p H_0(z) + (1 - p) H_1(z)$. Since only ξ is observed, the stage 2 decision $x_2(\xi)$ can not fulfill constraints (4), and the safety constraint has to be written as in (2):

$$P\left[x_1 + x_2(\xi) \geq \theta(\xi)\right] \geq 1 - \gamma, \tag{8}$$

where $\theta(\xi)$ has the probability distribution $H_\xi(z)$. For a given ξ and γ, let us define $z_\gamma(\xi)$ as minimal z, satisfying equation $P\left[z \geq \theta(\xi)\right] = 1 - \gamma$. Then equation (8) is equivalent to the equation $x_1 + x_2(\xi) \geq z_\gamma(\xi)$, which is similar to (4). The optimal period 2 decision $x_2 = \max\left\{0, z_\gamma(\xi) - x_1\right\}$, and optimal x_1 has to minimize

$$F(x) = C_1 x + C_2 \left[p \max\left\{0, z_\gamma(0) - x\right\} + (1 - p) \max\left\{0, z_\gamma(1) - x\right\}\right].$$

As ξ has a discrete probability distribution, function $F(x)$ does not have continuous derivatives. Therefore, the optimality condition cannot be derived from the *Proposition* of Section 3.1. Assume that $z_\gamma(0) < z_\gamma(1)$. $F(x)$ is a piece-wise continuous linear function. Namely, for $0 \leq x < z_\gamma(0)$,

$$F(x) = C_1 x + C_2 \left[p\left(z_\gamma(0) - x\right) + (1 - p)\left(z_\gamma(1) - x\right)\right] = (C_1 - C_2) x + C_2 \overline{z}_\gamma(\xi).$$

For $z_\gamma(0) \leq x < z_\gamma(1)$,

$$F(x) = C_1 x + C_2(1-p)\big(z_\gamma(1) - x\big) = \big(C_1 - C_2(1-p)\big)x + C_2(1-p)z_\gamma(1), \quad \text{and}$$

for

$$x \geq z_\gamma(1), \; F(x) = C_1 x.$$

The optimal ex-ante solution hedges against different contingencies. It is characterized as follows: $x = 0$, if $C_1 > C_2$. Otherwise, $x = z_\gamma(0)$, if $C_1 - C_2(1-p) > 0$, and $x = z_\gamma(1)$, if $C_1 - C_2(1-p) < 0$.

Since $C_1 < C_2$, in general it pays to make reductions in period 1 that are as large as possible. After learning takes place at the end of period 1, the optimal solution is to make reductions such that the total reduction is either $z(0)$ or $z(1)$. Thus, the minimal first period reduction is $z(0)$. If first period costs are very low, or the chance that $\xi = 1$ is very high, then it is optimal to make larger first period reduction $z(1)$, accepting the chance that $\xi = 0$ and that reductions $z(1) - z(0)$ will have been unnecessary.

Let us compare this *ex ante* period 1 optimal "with-learning" solution with the optimal "without-learning" solution $x_1^* = z_\gamma$, $x_2^* = 0$ derived from minimization of (4) under safety constraint $P[x_1 + x_2 \geq \theta] \geq 1 - \gamma$, i.e., $x_1 + x_2 \geq z_\gamma$, where z_γ is the minimal z satisfying constraint $P[z \geq \theta] \geq 1 - \gamma$. Assume that $H_0(z)$, $H_1(z)$ have continuous derivatives, the support of distribution $H_0(z)$ is interval $[a_0, b_0]$, and the support of $H_1(z)$ is interval $[a_1, b_1]$, where $a_1 > b_0$. If $C_1 - C_2(1-p) < 0$, then the optimal "with-learning" period 1 solution $x = z_\gamma(1)$. If $x \in [a_1, b_1]$, then $P[x \geq \theta] = p + (1-p)H_1(x)$. Since $H_1(z_\gamma(1)) = 1 - \gamma$, then $P[x \geq \theta] = p + (1-p)(1-\gamma) = 1 - \gamma + \gamma p$ for $x = z_\gamma(1)$. As $\gamma p > 0$, then the optimal "without-learning" decision $x = \theta_\gamma$ satisfying $P[x \geq \theta] = 1 - \gamma$ is less demanding (smaller) than $x = z_\gamma(1)$, i.e., learning increases the optimal *ex ante* emission reductions. This conclusion is reversed in the case $C_1 - C_2(1-p) > 0$. Indeed, let $x \in [a_0, b_0]$. Then, $P[x \geq \theta] = pH_0(x)$, $H_0(z_\gamma(0)) = 1 - \gamma$ and for $x = z_\gamma(0)$, $P[x \geq \theta] = p(1-\gamma)$ (i.e., the optimal "without-learning" decision $x = \theta_\gamma$ is greater than the optimal "with-learning" decision $x = z_\gamma(0)$).

5. Nonlinear Abatement Cost

It is well known that abatement-cost functions are nonlinear (Ha-Duong et al, 1997). This section illustrates that, in contrast to the linear case, nonlinear cost functions call more strongly for the coexistence of *ex ante* and *ex post* decisions. It is interesting to compare the case of linear functions with the quadratic cost functions used in some integrated assessment models. Assume that the cost functions of both periods $C_i(x) = C_i x^2$ with positive C_1, C_2. Cost function (4) takes on the form

$$F(x) = C_1 x^2 + C_2 E\left(\max\{0, \theta - x\}\right)^2 \quad \text{and} \quad F'(x) = 2C_1 x - 2C_2 E(\theta - x) I[\theta \geq x].$$

Therefore, $F'(0) = -2C_2 E\theta < 0$, assuming that $E\theta > 0$ with a positive probability. Thus, this case calls for coexistence of period 1 and period 2 decisions independently of C_1, C_2. Compare this to the case of linear costs in Section 3.1, where $F'(0) = C_1 - C_2$ if $C_1 < C_2$; i.e., non-zero first period reductions are called for only if costs are less in period 1. With quadratic costs, period 1 reductions are optimal even if $C_1 > C_2$ because marginal costs can still be lower in period 1 if larger period 2 reductions will be required.

Optimal *ex ante* abatements are characterized by more complicated than (6) equation $C_1 x = C_2 E(\theta - x) I(\theta \geq x)$. In the case of normally distributed θ, $F(x) = C_1 x^2 + \frac{1}{2} C_2 E(\theta - x)^2$. From the optimality $2C_1 x + C_2 (x - E\theta) = 0$ it follows that optimal $x = C_2 \bar{\theta}/(2C_1 + C_2)$, i.e., it is defined only by mean value $\bar{\theta}$. This conclusion is false for non-normal distributions, although standard integrated assessment models often use only average values $\bar{\theta}$.

It is important to illustrate some typical situations that may occur in the case of non-smooth, piece-wise linear functions commonly used in emission-control problems where technology switches may call for more intensive emission reduction. These functions implicitly impose upper or lower bounds on *ex ante* emission reductions. Assume that $C_2(x) = C_2 x$ and $C_1(x)$ is a piece-wise linear function $C_1(x) = C_1^1 x$ for $0 \leq x \leq a$ and $C_1(x) = C_1^2(x - a) + C_1^1 a$ for $x \geq a$, where $C_1^2 > C_2$ and $C_1^1 < C_2$. It is easy to see that the optimal *ex ante* solution has to satisfy the additional requirement $x_1 \leq a$. As $C_1^1 < C_2$ and $C_1^2 > C_2$, the optimal *ex ante* decision is defined as follows: let $\bar{x}$ be the solution of equation $P[\theta > x] = C_1^1 / C_2$. The optimal period 1 decision $x_1^* = a$ if $\bar{x} > a$, and $x_1^* = \bar{x}$ for $\bar{x} \leq a$. Assume that $C_1(x) = C_1 x$, and $C_2(x) = C_2^1 x$ for $0 \leq x \leq a$; $C_2(x) = C_2^2(x - a) + C_2^1 a$ for $x \geq a$, where $C_1 > C_2^1$, $C_1 < C_2^2$. Consider solution $\underline{x}$ of the equation $P[\theta > x] = C_1^1 / C_2$. It is easy to see that the optimal period-1 decision $x_1^* = \underline{x}$ for $\underline{x} \geq a$ and $x_1^* = 0$ for $\underline{x} < a$.

6. A Dynamic Stabilization Problem

The general stabilization problem is outlined in Section 2. Its proper formulation requires a catastrophe-generating submodel (i.e., a submodel of CO_2-generating activities). Therefore, in its rather general form, the problem becomes similar to catastrophic-risk-management problems discussed in [9]. In its simplest form, the dynamic two-stage model has strong connections with dynamic versions of CVaR risk measures.

Assume that CO_2 emission paths are characterized by exogenous scenarios as in Section 3. Let us consider $R_t = \sum_{k=0}^{t} x_k$, where decision variables $x_k \geq 0$, $k = 0,1,...,t$, $t \leq T$. We can think of x_k as a feasible level of CO_2 emission reduction at the beginning of period k. At time $t = 0$ the target value on total emission reduction R_t in period t is given as a random variable ρ_t. It is assumed that the exact value of ρ_t is revealed at a random time $t = \tau$. Since τ is uncertain, the decision path $x = (x_0, x_1,..., x_T)$ for the whole time horizon has to be chosen *ex ante* in period $t = 0$ to "hit" the target ρ_t, $R_\tau \geq \rho_\tau$, at $t = \tau$ in a sense specified further by (10). At random $t = \tau$, the decision path can be revised for the remaining available time. Similar to the model of Section 3.1, consider a stream of linear random costs $v(x) = \sum_{t=0}^{T} [c_t x_t + d_t \max\{0, \rho_t - R_t\}] I_{\tau=t}$, where $c_t > 0$, $d_t > 0$, $t = 0,1,...,T$ are known ex-ante and ex-post abatement costs. The expected value of $v(x)$ can be written as

$$V(x) = \sum_{t=0}^{T} c_t x_t + Ed_\tau \max\left\{0, \rho_\tau - \sum_{t=0}^{\tau} x_t\right\}. \tag{9}$$

Let us consider a path x^* minimizing $V(x)$ subject to $x_t \geq 0$, $t = 0,1,...,T$. Assume that $V(x)$ is a continuously differentiable function (e.g., a component of random vector $\rho = (\rho_0, \rho_1,..., \rho_T)$ has a continuous density function). Assume also that, so far, there exists a positive optimal solution $x^* = (x_0^*, x_1^*,..., x_T^*)$, $x_t^* > 0$, $t = 0,1,...,T$. Then, from the optimality condition for stochastic minimax problems (see, e.g., discussion in [4], [10]), it follows that for $x = x^*$, $V_{x_t} = c_t - \sum_{k=0}^{t} p_k d_k EI\left(\sum_{k=0}^{t} x_k \leq \rho_k\right)$ or

$V_{x_t} = c_t - \sum_{k=0}^{t} p_k d_k P[\sum_{k=0}^{t} x_k \leq \rho_k] = 0$, $t = 0,1,...,T$, where p_t is the probability that τ occurs first time at t. From this sequentially for $t = 0,1,...,T$, it follows that

$$P[x_0 < \rho_0] = c_0 / p_0 d_0, \quad P[\sum_{k=0}^{t} x_k < \rho_k] = (c_t - c_{t-1}) / p_t d_t, \quad t = 0,1,...,T. \tag{10}$$

It is easy to see that from (9) it follows that

$$V(x^*) = c_0 E\left[\rho_0 I(\rho_0 > R_0^*)\right] + (c_1 - c_0) E\left[\rho_i I(\rho_1 > R_1^*)\right] + ... + (c_T - c_{T-1}) E\left[\rho_T I(\rho_T > R_T^*)\right]$$
which can be viewed as a dynamic CVaR risk measure.

Remark 6: Equations (9) are derived from the existence of the positive optimal solution x^*. It is easy to see that this solution follows from $c_0 / p_0 d_0 < 1$, $(c_t - c_{t-1}) / p_t d_t < 1$ and the monotonicity of quantiles β_t, $\beta_0 < \beta_1 < ... < \beta_T$ defined by equations

$$P[\beta_0 < \rho_0] = c_0 / p_0 d_0, \quad P[\beta_t < \rho_t] = (c_t - c_{t-1}) / p_t d_t, \quad t = 0,1,...,T.$$

If probability p_t rapidly decreases to 0, then from (9) it follows that *ex-ante* abatements are positive for a relatively short initial interval defined by inequality $(c_t - c_{t-1})/p_t d_t < 1$. This misleading conclusion is due to a strong assumption of unlimited capacity for emissions reductions, which is a standard assumption of climatic-economic integrated assessment models (see discussion in Ha-Duong, 1997 and Wright and Erickson, 2003). Similar to conclusions of Section 4.2, this requires an adequate treatment of safety constraints (2) to prevent catastrophes.

7. Concluding Remarks

This paper analyzes the effects of risks and learning on climate change decisions using a two-stage, dynamic model that assumes a stabilization constraint. It shows that learning can lead either to larger or smaller first period emissions reductions, compared to the optimal reduction under uncertainty without learning, and that this effect can either be large or small. The direction and magnitude of the learning effect is determined by a number of interacting factors. For example, in a simple linear model with deterministic mitigation costs but uncertainty in total required emissions reductions, the learning effect depends on how mitigation costs evolve over time, the shape of the uncertainty distribution in required emissions reductions, the confidence with which the safety constraint (i.e., stabilization level) is desired to be met, and, in the case of incomplete learning, the probability distribution describing the anticipated learning possibilities. If costs are uncertain, but the emissions reduction target is known, the problem emphasizes the use of risk adjusted costs (e.g., quantiles) rather than expected values. Introducing a more realistic nonlinear cost function with increasing marginal costs induces a higher level of first period emissions reductions compared to the linear case. We also analyze the case of random duration of stages as a proxy either for uncertain timing of learning or uncertain inertia in socio-economic systems, showing how this consideration can induce a minimum level of first period reductions. Finally, framing the problem in dynamic terms as a multi-period problem with an uncertain time path of required cumulative emissions reductions shows that the problem has strong connections with dynamic versions of VaR risk measures. Given the multiple influences on the learning effect, we conclude that drawing practical conclusions on the likely effect of learning on climate change decisions is an empirical question requiring analysis with models capable of adequately representing endogenous risks, abrupt changes, in particular, abrupt learning, inertia, and path dependent costs.

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