THE USE OF ALTERNATIVE PREDICTIONS IN LONG-TERM INFERENCE INTO THE FUTURE (WITH SPECIAL REFERENCE TO WATER DEMAND)

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SUMMARY

Let Y be a variable for which prediction pertaining to a future time period T is sought while using a model $Y = f(X_1, X_2, \ldots, X_k, \xi)$, where $X_1, X_2, \ldots, X_k$ are explanatory variables and ξ represents a random component. If period T lies far in the future then usually the values of explanatory variables of the model are also not known for time T. The author outlines some methods of avoiding this difficulty. After presenting the main points of several approaches already known in the literature he concentrates on two procedures. First, how the so-called optimistic and pessimistic predictions of Y_T could be used, the optimistic prediction being defined as the one that has been computed under the assumption of very favorable values of X_{iT}, while the pessimistic one assumes the existence of very unfavorable conditions. Then, he outlines the concept of alternative predictions. If the predicted phenomenon Z can be realized in time T by realization of one of different variants $Z_i (i = 1,2, \ldots, r)$ then an alternative prediction of Z is defined as the prediction that in time T will occur as one of several possible subsets of the set $(Z_1, Z_2, \ldots, Z_r)$. Sections 4, 5 and 6 of the paper are devoted to the presentation of how the concept of alternative predictions can be applied to the prediction of explanatory variables of the model that, finally, is to give a prediction of variable Y in time T.

PREFACE

Interest in water resources systems has been a critical part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social, and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

In the years of 1976 and 1977, IIASA initiated a concentrated research effort on the modeling and forecasting of water demands. Our interest in water demands derived itself from the generally accepted realization that these fundamental aspects of water resources management have not been given due consideration in the past.

This paper, the eighth in the IIASA water demand series, reports on the use of alternative predictions in long-term inference into the future, with special reference made to forecasting water demands. Following an outline of several standard approaches for determining the values of explanatory variables of an econometric model, a new method of building "optimistic" and "pessimistic" predictions is presented. The interval defined by these two predictions provides information on what can be expected when extreme cases are excluded from consideration.

Based on this material, the concept of alternative predictions is introduced and illustrated by several examples that refer explicitly to water demand forecasting. This approach can be used for assessing the future values of explanatory variables of the econometric model as well as for final prediction of the future values of the endogenous variable. The paper ends by presenting a method of building alternative predictions that minimizes the sum of expected losses due to incorrect prediction and of costs due to the initiation of some actions on the assumption that an alternative prediction will prove correct. Practical applicability of the proposed methods is demonstrated and recommendations are made as to how they could be extended further.

The Use of Alternative Predictions in
Long-Term Inference into the Future
(With Special Reference to Water Demand)

1. INTRODUCTORY REMARKS

We shall assume that a prediction is sought for a variable Y in the future time T when the model used for prediction is of the form

$$Y = f(X_1, X_2, \ldots, X_k, \xi) \quad , \tag{1}$$

where X_i's are different explanatory variables and ξ is a random component with expected value equal to zero. Once some assumptions about the values of the explanatory variables at time T are made it is easy to find prediction y_{Tp}. When the prediction lead (i.e., the distance between the predicted time period T and the present one t_o) is large, problems usually arise as to what the values of X_i's will really be. In the present paper we shall try to show how the concept of the so-called alternative predictions[1*] can be used to assess values of explanatory variables at time T. Before going into details of possible uses of alternative predictions, it seems worthwhile to make a short review of other approaches for determining the values of X_i's.

2. AN OUTLINE OF SEVERAL STANDARD APPROACHES

There is a great variety of ways of determining the values of explanatory variables of an econometric model to be used for inference about the future. Among others, reference should be made to the following:

(a) Extrapolation of Trends

Economic and technological explanatory variables usually exhibit some trends. If deviations from such trends in the past were not large and if one can reasonably assume that no major structural changes (technological, institutional or social) will have occurred before time T, then one can extrapolate these trends of

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*The superscript numbers refer to notes on pp. 26-30.

X_i's for time T, then take their resulting values, X_{iT}'s say, and then insert them in model (1) to get the prediction for Y_T. The rationale of this approach is obviously conditioned by the degree of fit of X_{iT}'s to their trends. Moreover, it should be observed that this approach fails to be admissible when among the explanatory variables are such whose values are fixed by administrative, institutional or political acts, which cannot be expected to follow a "smooth" pattern.

(b) Using Information Pertaining to Decision Making

This approach is often used in countries with planned economies. If an economic plan assumes some specified levels for the X_i's in time T, then an obvious course of action is to use these plan data by substituting them for the explanatory variables of the model[2]. Sometimes this approach is combined with the previous one, trend extrapolation being used for variables not controlled by the economic plan.

(c) Maximum Probability Approach

Let us assume that some of the explanatory variables are random variables whose probability distribution functions are at last approximately known[3]. With such an assumption is it advantageous to use for prediction of Y_T the most probable values of the X_i's. Two further remarks must be made here. First, it should be noted that when a continuous distribution is symmetric and unimodal, then the most probable value coincides with the expected value of the variable. Second, note that, ideally, the vector of most probable values of the X_i's should be obtained from the k-dimensional joint distribution function of explanatory variables. Since construction of such a joint distribution function would usually be very difficult, except in the cases of normality, one is obliged to seek the most probable values from one-dimensional distribution functions instead, which will involve some error[4]. Let us also note that this approach is especially recommended when prediction is to be done but once, and when for this particular unique act of inference about the future one wishes to determine the "external conditions" for the formation of Y_T as accurately as possible[5].

(d) Using Expected Values of Explanatory Variables

This approach consists of substituting for the X_i's in (1) their expected values or the estimates thereof. This procedure is often used in practice. Two examples can be given. First, the use of trend values described above under (a) can be shown to be a particular case of the approach discussed now. The second example refers to the chain prediction method while using dynamic econometric models with lagged endogenous variables[6]. The procedure consisting in the use of expected values of explanatory variables can be shown to minimize the variance of prediction when the explanatory variables are random and the model is linear. For this reason, it is recommended in situations when inference about future values of Y_T is a process which recurs with a given frequency over time[7]. Of course, a precondition to the possible use of approach (d), is that there is adequate information about the distribution of the variables involved so that an estimate of the expected value of the explanatory variable X_i can be provided.

(e) Tabulation of Predictions Under Different Sets of Values of X_i

Another approach, advocated by some people in cases where there is very little information available about the values of explanatory variables in period T, consists in singling out a number of possible vectors of likely values of the X_i's and in computing predictions corresponding to the particular sets of values of explanatory variables. Thus, instead of a single prediction, a table of prediction values is obtained.

This approach is not to be recommended because it does not answer the question: "What will happen to the variable Y in time T", but evades the answer by substituting for it information about the possible behavior of the predicted variable according to different possible formations of explanatory variables.

(f) Construction of Optimistic and Pessimistic Predictions of the Predicted Variable Y_T

Since this procedure will be explained in more detail in the next section of this paper, reference to it is only made here for the sake of completeness of exposition.

To conclude these preliminary remarks, one should note that the approaches presented above are essentially relevant--while either point or interval prediction of Y_T is to be made--in cases where the prediction maker is seeking information about future values of explanatory variables before building the prediction he is basically interested in, i.e. before prediction of variable Y_T. The various procedures outlined above refer both to the case when X_i's are continuous or discrete variables, the only reservation in the latter case being that any resulting value of an explanatory variable is rounded to its nearest really possible value[8]. As will be seen later in Section 4, such a point approach to values of explanatory variables is not necessary and, in cases of far-reaching predictive inference[9], it may even be very embarassing.

3. OPTIMISTIC AND PESSIMISTIC PREDICTIONS OF Y_T

For the sake of simplicity of exposition we shall assume the model (1) to be a linear one, i.e. to have the form

$$Y = \sum_{i=1}^{k} \beta_i X_i + \xi \quad . \qquad (2)$$

Let us assume that the values of explanatory variables of the model in time T cannot be known in advance, so that X_i's must be treated as random variables. The problem is how to predict Y_T.

If there is some knowledge available as to the probability distribution[10] of X_i's in time T or if at least one can make reasonable guesses as to the range of variation of explanatory variables, one of the possible ways to solve the problem is to build the so-called optimistic and pessimistic predictions of Y_T.

Let us assume that the explanatory variables of the model (2) can be classified into two sets A and B. Variable X_i will belong to the set A if and only if its coefficient β_i is positive.

Variable X_i will belong to the set B if and only if its coefficient is negative[11]. Let us assume further that the utility connected with achieving a level y_T of the variable Y_T is an increasing function of y_T.

As is easily seen, under these assumptions, A contains explanatory variables whose high values will result in (relatively) high values of Y_T. Set B, on the other hand, contains such variables whose low values will be advantageous while high values will tend to decrease the level of Y_T and, hence, the level of utility achieved. Therefore, it will be reasonable to denote by the optimistic prediction one which was computed under the assumption of favorable conditions, i.e. of high values of explanatory variables belonging to A and of low values of explanatory variables belonging to B. By similar argument, a pessimistic prediction is one which resulted from assuming low values of variables belonging to A and high values of explanatory variables belonging to B[12]. Since the expressions "high values", and "low values" are too vague, we shall rather make use of the concept of "ε-probability high value" and of "ε-probability low value" of an explanatory variable. Formally, x_{ih} will be called ε-probability high value of X_i, if at time T the following relation holds true:

$$P\{X_{iT} \geqslant x_{ih}\} = \varepsilon \quad , \tag{3}$$

where ε is some preassigned small positive number. Similarly, $x_{i\ell}$ will be called ε-probability low value of X_i if at time T there is

$$P\{X_{iT} < x_{i\ell}\} = \varepsilon \quad . \tag{4}$$

To determine the numbers x_{ih} and $x_{i\ell}$ one obviously must make use of the (assumed) knowledge of marginal probability distributions of the explanatory variables. If no such knowledge is available and the only information refers to finite ranges of variation of X_i's, it becomes necessary to make use of the assumption that within these ranges every value is equally likely. This means that if X_i is a discrete variable all its possible values are assumed to have equal probabilities of occurrence. If, on the other hand, X_i

is a continuous variable then it is assumed to have a rectangular distribution, i.e. a distribution with constant density function over the interval of possible variation of this variable.

Let us adopt the principle unbiased prediction which consists in putting prediction equal to the expected value of the predicted variable Y_T. Under this assumption, and since $E(\xi_T) = 0$, an optimistic prediction can be formally written as

$$y^o_{Tp} = \sum_{i\varepsilon J_A} \beta_i x_{ih} + \sum_{i\varepsilon J_B} \beta_i x_{i\ell} \quad , \tag{5}$$

where J_A and J_B denote, respectively, the sets of indexes of explanatory variables which belong to the sets A or B. By the same argument, a pessimistic unbiased prediction is obtained by using the formula

$$y^p_{Tp} = \sum_{i\varepsilon J_A} \beta_i x_{i\ell} + \sum_{i\varepsilon J_B} \beta_i x_{ih} \quad . \tag{6}$$

It follows from the way they have been defined that $y^o_{Tp} > y^p_{Tp}$, the size of this difference depending on ε. Since the user of the prediction cannot reasonably count on the occurrence of the most favorable conditions nor would he be justified in expecting the worst, he should rather expect the true value of Y_T to fall somewhere between y^p_{Tp} and y^o_{Tp}, the emphasis being on the word "somewhere". For this reason the interval $[y^p_{Tp}, y^o_{Tp}]$ will be referred to as the interval of indeterminacy of prediction. The length of this interval will be denoted by $L(\varepsilon)$, the symbol ε being included as a reminder that L is a decreasing function of the probability level.

So far ε has been treated as a preassigned number without any reference as to how its value should be determined. Although the choice of ε depends on the decision of the prediction user, one must observe that both very low and very high values of ε are not very useful. Values of ε which are close to zero reflect well indeed the concept of favorable and unfavorable conditions. Furthermore, with ε close to zero there is virtually no risk of encountering in practice still more favorable or still worse conditions for the formation of the predicted variable Y_T. This guarantees that the pessimistic and the optimistic predictions truly represent the two extremes. On the other hand, however, it should be noticed that with $\varepsilon \rightarrow 0$ the difference between the two types of predictions increases indefinitely giving a decision maker a high level of uncertainty as to what really may happen. Hence, this advocates keeping off very small values of ε.

A reasonable solution to the dilemma of how to set ε is provided by the rule that [13]

$$L^{-1}(d) \leq \varepsilon \leq \frac{1}{2k} \quad , \tag{7}$$

where k is the number of all explanatory variables of the model and $L^{-1}(d)$ is the value of ε for which the corresponding length of undeterminacy of prediction is equal to a predetermined positive number d. Since decision makers can usually determine the maximum level of uncertainty they can cope with, the value of d may be assumed to be easily found out, and thus also $L^{-1}(d)$.[14]

To conclude our remarks on the theory of optimistic and pessimistic predictions let us add that the assumption of linearity of the model was not necessary, although in practical applications it does simplify the procedure. If the model is nonlinear but the dependence of Y on its explanatory variables is monotone with respect to each variable, the X_i's are still classified into two sets A and B, defined as in the linear case. If the relations are not monotone, however, A and B have to be redefined. The set A becomes then the Cartesian product of k sets of such values of each of the X_i's which are considered favorable with respect to Y_T, while the set B is a Cartesian product of the remaining possible values of the explanatory variables. An optimistic prediction is then obtained by assuming X_i's values from set A while the pessimistic prediction is computed by making use of appropriate values belonging to set B[15].

To make the exposition complete we shall add just a few remarks about the way the optimistic and the pessimistic predictions are built when utility is a decreasing function of y_T. Keeping the same definitions of sets A and B as in the linear case we shall assume for an optimistic prediction low values of X_i's belonging to A and high values of explanatory variables belonging to set B. On the other hand, while making a pessimistic prediction, we shall use high values of variables classified into set A and low values of variables from set B.

As is easily verified, in the case of linear models and symmetric probability distributions, the optimistic prediction computed under the assumption of utility being an increasing function of y_T coincides with the pessimistic prediction obtained under the assumption of utility being a decreasing function of y_T. Similarly, a pessimistic prediction built under the assumption of increasing utility function is equal to the optimistic prediction obtained for the case when utility is a decreasing function of y_T.

To visualize better the concept of optimistic and pessimistic predictions let us consider a simple example. Let

$$Y = 0.5X_1 + 1.2X_2 - 0.3X_3 - 40.0 + u \qquad (8)$$

be a model of water demand estimated from appropriate statistical data pertaining to middle size cities and let all the variables be measured as indexes with 1960 as the base year. The variables in the model have the following meaning: Y = total water demand, X_i = population number, X_2 = industrial output volume, X_3 = price of water per unit of volume, and u = the random term[16].

Let us suppose that for a future time period T, for which prediction of water demand is sought, the true values of the explanatory variables are not known but that experts' opinions have provided information about the possible variation of X_1, X_2 and X_3. For the sake of simplicity all three variables will be treated as continuous. Let us suppose for instance, that it is accepted that in time T there will be

$$105 < X_1 < 110 \quad ,$$

$$120 < X_3 < 130 \quad ,$$

while the variable X_2 is thought to have normal distribution N(130,2). Using these data we can build the optimistic and the pessimistic prediction of water demand. Just for the sake of determining the problem we shall assume the utility to be a decreasing function of water demand. From this it follows that the optimistic prediction must be lower than the pessimistic one[17].

We shall set ε equal to 0.1 which obeys the right-hand side of the double inequality (7)[18]. Since it follows from (8) that Y is positively correlated with X_1 and X_2, and is negatively correlated with X_3, the first two explanatory variables will belong to the set A while the third will make up the set B. Using (3) and (4) we shall find ε-probability high and low values of the explanatory variables. Since no detailed information about the possible variation of X_1 and X_3 is available, we shall assume them to have rectangular distributions over their possible ranges of variation. Variable X_2 has been assumed to have normal distribution with specified parameters. For $\varepsilon = 0.1$ we have

$$P\{X_{1T} \geq 109.5\} = 0.1 \quad ,$$

$$P\{X_{2T} \geq 133.3\} = 0.1 \quad ,$$

$$P\{X_{3T} \leq 121.0\} = 0.1 \quad .$$

Hence, the pessimistic prediction is

$$y^{p}_{Tp} = 0.5 \times 109.5 + 1.2 \times 133.3 - 0.3 \times 121.0 - 40.0 = 138.41 \quad . \tag{9}$$

Similarly, for building an optimistic prediction, we find

$$P\{X_{1T} \geq 105.5\} = 0.1 \quad ,$$

$$P\{X_{2T} \geq 126.7\} = 0.1 \quad ,$$

$$P\{X_{3T} \leq 129.0\} = 0.1 \quad ,$$

and the optimistic prediction is

$$y^{o}_{Tp} = 0.5 \times 105.5 + 1.2 \times 126.7 - 0.3 \times 129.0 - 40.0 = 126.09 \quad . \tag{10}$$

The interval of uncertainty of prediction is thus [126.09, 138.41]. Although this was but an example, it can still easily be seen that it is very important to assess with accuracy the ranges of possible variation of the explanatory variables. The smaller their possible variation, the shorter the length of the interval induced by the two predictions.

4. THE CONCEPT OF ALTERNATIVE PREDICTIONS

Still another approach to the problem of determining the formation of explanatory variables in time T may consist in using the concept of alternative predictions[19]. Before we get into details of this approach it is necessary to give some preliminary information about alternative predictions in general.

Let Z be a phenomenon whose future formation (in time T) is as yet unknown and one wishes to make a prediction about it. It is assumed that in time T one and only one of the g different variants of Z may happen, the probability of occurrence of the variant Z_i being known and equal $P(Z_i)$. Symbol C will denote the set of all possible variants of the phenomenon Z, i.e.

$$C = \{Z_1, Z_2, \ldots, Z_g\} \quad . \tag{11}$$

The variants Z_i may have at least three different interpretations: a) they may represent subintervals of variation of a continuous random variable, b) they may represent different possible values (or sets of values) of a discrete random variable[20], and c) they may represent different variants of a qualitative phenomenon.

If the maximum value of the probabilities $P(Z_i)$ is low (which will usually be the case when g is large) then it would be unsafe to single out only one element of C as the prediction of the behavior of phenomenon Z in time T. A more realistic attitude, stemming from the obvious requirement that in the long run the frequency of right predictions be high, suggests using alternative rather than single predictions.

Definition 1. An alternative prediction is a prediction stating that one of the specified alternatives of possible variants of the phenomenon Z will occur in time T.

<u>Definition 2.</u> An alternative prediction is said to be based on set A if the set A contains all the elements Z_i specified by the alternative prediction and only such elements.

<u>Definition 3.</u> An alternative prediction is called proper when the number of variants belonging to set A is larger than one and is less than g.

<u>Definition 4.</u> An alternative prediction based on set A = C is called a trivial one, while a prediction based on set A containing only one variant is called a single prediction[21].

An alternative prediction will be denoted by Z_{Ap}. If Z_{pi} stands for the (single) prediction that the variant Z_i will occur in time T, then

$$Z_{Ap} = \bigcup_{i \varepsilon J_A} Z_{pi} \quad , \tag{12}$$

where J_A denotes the set of indexes of variants belonging to set A.

The probability of a prediction coming true will be referred to as the likelihood of that prediction[22]. The main advantage of using alternative predictions consists in the possibility of increasing the likelihood as compared with single predictions. An alternative prediction will be called admissible if its likelihood is equal to or greater than a predetermined number γ, where $0 < \gamma < 1$.

It is possible to prove a number of theorems concerning the properties of alternative predictions[23].

<u>Theorem I</u> Proper alternative predictions exist if and only if the set C contains at least three elements.

<u>Theorem II</u> The number of possible different proper alternative predictions is equal to $2^g - g - 2$, where g denotes the number of elements in set C, i.e. the number of different possible variants of Z.

<u>Theorem III</u> A necessary and sufficient condition for the existence of at least one proper alternative prediction, admissible at γ probability level, is that

$$\min_{Z_i \varepsilon C} P(Z_i) \leq 1 - \gamma \quad . \tag{13}$$

<u>Theorem IV</u> The number of proper admissible alternative predictions which can be built from a given set C is a non-increasing function of probability level γ.

<u>Theorem V</u> Let Z_{A_1P} be an alternative prediction based on set A_1 and admissible at γ_1 level and and let Z_{A_2P} be another alternative prediction based on set A_2 and admissible at γ_2 probability level. Then the alternative prediction Z_{Ap} based on set $A = A_1 \cup A_2$ is admissible at level γ, such that

$$\gamma = \gamma_1 + \gamma_2 - P(Z \varepsilon A_1 \cdot A_2) \quad . \tag{14}$$

Since one can usually build several different alternative predictions with the same set C--as follows from the theorems stated above--there arises the problem of some reasonable principles which could lead to the choice of A. In connection with the chief problem of this paper , i.e. of assessing values for explanatory variables in time T, we shall present two possible solutions[24].

The first possible approach consists in predetermining a required level γ of likelihood of alternative prediction and then in constructing the set A in such a way that its diameter[25] d be minimum among all sets fulfilling the condition imposed on the likelihood. The second approach consists of imposing the requirement that the diameter be not greater than a given number d_o and finding then such a set A for which the likelihood of alternative prediction be as high as possible. As is easily seen the two approaches put emphasis on the possibility of alternative predictions coming true and on compactness of such predictions; i.e. they tend to avoid including into the same alternative prediction variants which are too much unlike i.e. are too distant from each other.

Different taxonomies can be used for measuring distances between variants. For the purpose of this paper we shall assume that:

a) If Z_i's represent disjoint intervals of variation of a continuous random variable then the distance of two variants is equal to the absolute difference of expected values of that variable, the expectations being computed from distributions truncated to the respective intervals[26].

b) If Z_i's represent variants of a discrete variable then the distance between two variants is equal to the absolute value of the difference of respective values of the variable[27].

c) If Z_i's represent different qualitative variants of a phenomenon the distance of two variants Z_i and Z_j, say, is provided by an appropriate taxonomic measure obeying all the conditions imposed normally on a measure of distance[28].

Two examples will be given. The first one is rather simple and refers to the case when variable X must be used in a model describing the behaviour of the endogenous variable Y, and there is the problem of assessing the value of this explanatory variable in time T. Let us suppose that using experts' judgements it was possible to attach the following probabilities to different intervals of variation of X:

Table 1.

Interval of variation of X	0-3	3-6	6-9	9-12	12-20	20-30
Probability	0.10	0.30	0.30	0.20	0.07	0.03

For the sake of simplicity we shall assume the conditional mathematical expectations of variation within given intervals to coincide with interval mid-ranges. Let us assume also that the required likelihood level is $\lambda = 0.7$. Different alternative predictions obeying this constraint are listed in Table 2 below[29]:

Table 2.

Variants forming alternative predictions with likelihood equal to at least 0.7	Diameter of A	Likelihood
1,2,3	6.0	0.70 (optimum)
1,2,3,4	9.0	0.90
1,2,3,5	14.5	0.77
1,2,3,4,5	14.5	0.97
1,2,3,6	23.5	0.73
1,2,3,4,6	23.5	0.93
1,3,4,5,6	23.5	0.70
2,3,4	6.0	0.80
2,3,4,5	11.5	0.87
2,3,4,5,6	20.5	0.90
2,3,5,6	20.5	0.70
1,2,3,4,5,6	23.5	1.00 (trivial)

As can be seen, there are two different alternative predictions obeying the conditions imposed on their likelihood and having the least diameter of the set A. These are the predictions using variants Z_1, Z_2 and Z_3 or using Z_2, Z_3, Z_4. Since the diameters of the two respective sets A are equal it is reasonable to adopt finally the alternative prediction $Z_A p$: $Z \varepsilon (Z_2 \cup Z_3 \cup Z_4) = Z \varepsilon (3,12)$, since the likelihood of the latter prediction is higher. Having made a prediction about the behavior of the explanatory variable X in time T one can then proceed to predict Y_T, using for this purpose one of the methods outlined in Section 2 or the concept of optimistic and pessimistic predictions[30]. The choice of one of those methods will depend on the character of variable X and on the additional available information about the behavior of X in time.

Let us note also that one might start with fixing the diameter of set A. If, for instance, ones wishes to consider sets A with diameters not exceeding 10 then one must choose among the different alternative predictions listed in Table 3. As is easily seen, the best alternative prediction is now composed of variants Z_1, Z_2, Z_3, Z_4.

Table 3.

Variants forming alternative predictions with $d \leq 10$	Likelihood of prediction
1,2	0.40
1,2,3	0.70
1,2,4	0.60
1,2,3,4	0.90
1,3,4	0.60
1,3	0.40
1,4	0.30
2,3	0.60
2,3,4	0.80
2,4	0.50
3,4	0.50
3,4,5	0.57
3,5	0.37
4,5	0.27
5,6	0.10

It should be noted, however, that in some cases, when the required size of the diameter of set A is small, it may be that only a single prediction will provide the solution. For instance, setting the diameter to be not greater than 2.5 one easily finds that there are no variants with so little distance between each other. This shows that constraints should be formulated with due regard to real conditions of the problem[31].

In the second example, we shall assume that the aim of prediction is to find the level of water demand in an urban area and that for making a prediction one must take into account the factors influencing this demand. The exact levels of these factors in future time T are not known but experts have singled out some alternatives as to their joint behavior. These alternatives differ among each other with respect to assumptions concerning population growth (X_1), industrial output (X_2) and price of water (X_3), these being the three principal factors accounted for[32]. For the sake of example let us assume that four typical situations have been distinguished. These have been schematically presented in Table 4 where arrows pointing upwards or downwards represent assumed directions of change. Double arrows correspond to very significant changes while a horizontal arrow denotes an expected stationary state of variable. Quite obviously, because of the simultaneous approach to the three factors, the different alternatives can be viewed as variants of a qualitative type.

Table 4.

Situations (variants)	Behavior of variable x_1	x_2	x_3	Experts' opinion of probability
z_1	→	↗	→	0.20
z_2	↗	↗	→	0.50
z_3	↗	⇗	↗	0.25
z_4	→	↘	**→**	0.05

Let us assume also that by applying an appropriate taxonomy to quantitative characteristics of these varients (i.e. to envisaged rates of change) the matrix of normalized distances[33] of the variants was computed. Let it be the following one:

$$D = \begin{bmatrix} 0 & 0.3 & 0.5 & 0.6 \\ 0.3 & 0 & 0.2 & 0.8 \\ 0.5 & 0.2 & 0 & 1.0 \\ 0.6 & 0.8 & 1.0 & 0 \end{bmatrix} \qquad (15)$$

Let us assume also that the required level of likelihood of alternative prediction is $\lambda = 0.7$. Without counting the trivial alternative prediction $z \in z_1 \cup z_2 \cup z_3 \cup z_4$ there are five other proper alternative predictions which fulfill the condition imposed on their likelihood. These predictions, their respective likelihood levels and diameters of A are given in Table 5.

Table 5.

Admissible alternative predictions	Likelihood	Distances	Diameter of A
Z_1, Z_2	0.70	0.3	0.3
Z_1, Z_2, Z_3	0.95	0.3, 0.5, 0.2	0.5
Z_1, Z_2, Z_4	0.75	0.3, 0.6, 0.8	0.8
Z_2, Z_3	0.75	0.2	0.2
Z_2, Z_3, Z_4	0.80	0.2, 0.8, 1.0	1.0

The best alternative prediction is then that composed of variants Z_2 and Z_3. Hence, when inferring about the level of water demand in time T one should assume that there will be either some[34] growth of population and of industrial activity coupled with constant prices, or that with some growth of population and of price level will be coupled a very rapid growth of industrial output. The result of alternative prediction of variants Z_i provides us with information about what to expect at time T as to the behavior of explanatory variables of a water demand model. Having reached this result one may turn to one of the methods outlined in Sections 2 and 3 for making the final prediction, i.e. for finding the level of water demand in time T.

5. ALTERNATIVE PREDICTION OF BOTH THE EXPLANATORY VARIABLES AND OF THE ENDOGENOUS VARIABLE

In this section a method of prediction will be shown which will use the concept of alternative predictions both for assessing the levels of explanatory variables in time T and for making a final prediction of the predicted endogenous variable Y_T. For this purpose it will be assumed that there exists an alternative prediction Z_{Ap} of explanatory variables in time T, this alternative being composed of v elements[35]. For every element of set A an alternative prediction of Y_T variable is then done, according, for instance, to one of the principles presented in Section 4. Without any loss of generality we can assume that the range of possible variation of Y_T has been divided into r intervals, the corresponding variants of Y_T being hence denoted by $Y_1, Y_2, \ldots, Y_r$ and r being at least equal to 3.

Let us now denote $A_y(i)$ the set of variants of the variable Y_T which enter into the alternative prediction of Y_T when account is taken of variant Z_i belonging to A; also let W_i be the number of elements of $A_y(i)$. The final alternative prediction[36] of Y_T is then defined as the sum of all sets $A_y(i)$. Denoting this final prediction by Y_{Ayp}, we have

$$Y_{Ayp} = \bigcup_{i \varepsilon J_A} \bigcup_{j \varepsilon J_{Ay(i)}} Y_{pj} \quad , \tag{16}$$

where J_A denotes the set of indexes of variants of explanatory variables belonging to A, $J_{Ay(i)}$ denotes the set of indexes of variants of variable Y_T belonging to $A_y(i)$ and Y_{pj} denotes the single prediction that in time T the variable Y_T will assume a value corresponding to interval Y_j.

The important point is to find the likelihood of prediction (16). As is easily seen, this is equal to the sum of probabilities of all variants Y_j belonging to any of the sets $A_y(i)$. Hence

$$P\{Y_T \ \varepsilon \ Y_{Ayp}\} = \sum_j P(Y_j) \quad , \tag{17}$$

where the summation extends to all the variants of Y_T which enter into the final alternative prediction defined by (16).

It may be observed that some of the variants Y_j appearing in the final prediction will usually belong to several sets $A_y(i)$. If this is so, one can infer that the final prediction is not very sensitive to the way the variants Z_i have been defined. Although this normally inspires much confidence in the user of prediction, it may also be viewed as a warning signal that perhaps not all factors of genuine influence on Y_T have been accounted for when choosing the explanatory variables entering the variants Z_i's. A convenient measure of sensitivity of final prediction with respect to the adopted system of variants of explanatory variables is provided by the following ratio:

$$\delta_A = \frac{W_o}{\sum_{i \varepsilon J_A} W_i} \quad . \tag{18}$$

In this formula W_o stands for the number of elements belonging to A_y, i.e. for the number of variants Y_j which enter into the final prediction. It can be shown that if only proper alternative predictions are considered then there is always $\frac{1}{v} \leq \delta_A \leq 1$. The case $\delta_A = \frac{1}{v}$ occurs only when for every $Z_i \varepsilon A$, the corresponding alternative prediction of Y_T leads to choosing the same two variants of the endogenous variable. Hence, this case can be referred to as one of complete insensitivity of Y_T to different variants of explanatory variables. The other extreme case, namely, when $\delta_A = 1$, takes place when for every $Z_i \varepsilon A$ such and only such variants of Y_T are chosen which do not enter into other sets $A_y(i)$. This is the case of perfect sensitivity of Y_T with respect to its explanatory variables[37].

To conclude our theoretical remarks let us note that--in addition to information whose availability has thus far been assumed--it is now necessary to know also the conditional probabilities of occurrence of different Y_j's for given Z_i's.

We shall present next a short example of the procedure outlined above. Let us assume that four variants of formation of explanatory variables have been singled out, namely Z_1, Z_2, Z_3, Z_4, with probabilities 0.22, 0.32, 0.41, 0.05 respectively, and that for making a prediction of formation of explanatory variables the likelihood of at least 0.6 is required, while keeping the diameter of set A as small as possible. The matrix of distances Z_i's is assumed to be as follows

$$D = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.6 \\ 0.4 & 0 & 0.1 & 0.5 \\ 0.3 & 0.1 & 0 & 1.0 \\ 0.6 & 0.5 & 1.0 & 0 \end{bmatrix} .$$

In addition to the trivial one, there are five alternative predictions fulfilling the condition imposed on the likelihood of prediction, i.e. (Z_1, Z_3) (Z_1, Z_2, Z_3), (Z_1, Z_3, Z_4), (Z_2, Z_3), (Z_2, Z_3, Z_4). Using matrix D it can be found that the best alternative prediction for the explanatory variable is that for which the set A is composed of variants Z_2 and Z_3. For this set the likelihood of a prediction being right is 0.73 and the diameter of the set (Z_2, Z_3) is equal to 0.1.

Let us now assume that the conditional probabilities of occurrence of Y_j for various Z_i's--as given by experts--are as quoted in Table 6 and that again one requires the likelihood of prediction of Y_T to be at least 0.6.

Table 6.

Variants of Y_T	Variants of explanatory variables				Experts' Marginal Probabilities of Y_j
	Z_1	Z_2	Z_3	Z_4	
	Experts' probabilities				
Y_1	0.10	0.05	0.12	0.25	0.09
Y_2	0.15	0.08	0.26	0.40	0.19
Y_3	0.35	0.40	0.30	0.15	0.34
Y_4	0.30	0.30	0.20	0.10	0.25
Y_5	0.08	0.10	0.09	0.05	0.09
Y_6	0.02	0.07	0.03	0.05	0.04

Suppose[38] that for Z_2 the set $A_y(2)$ is composed of elements Y_3 and Y_4, while for Z_3 the set $A_y(3)$ contains three elements, i.e. Y_2, Y_3 and Y_4. Then, the final alternative prediction of the variable Y_T, corresponding to formula (16), is of the form

$$Y_{Ayp} = Y_2 \cup Y_3 \cup Y_4 \quad .$$

Using (17) we now find the likelihood of this prediction to be

$$P\{Y_T \varepsilon Y_{Ayp}\} = \sum_{j=2}^{4} P(Y_j) = 0.19 + 0.34 + 0.25 = 0.78 \quad .$$

6. AN ECONOMIC APPROACH TO BUILDING ALTERNATIVE PREDICTIONS

Finally, we shall consider yet another approach to building alternative predictions, this one seeming to be especially suitable for inference about the future behavior of factors affecting the endogenous variable, i.e. the behavior of explanatory variables. In the case of the present approach the leading underlying assumption is that the results of prediction will be used to serve some specific practical purposes, that is, that every prediction will be followed by an action which, in turn, will induce some costs. The failure of being prepared for occurrence of a specific variant induces some loss, the amount of which can be estimated beforehand. The problem reduces then to building such predictions which will minimize the sum of expected losses due to not being prepared for the occurrence of various variants and of costs connected with actions undertaken on the assumption that one of an alternative of variants will happen[39].

Let $S_k(A)$ denote the loss which will result when the alternative prediction is based on set A while in time T variant Z_k occurs which does not belong to A. Further, let $S_{ij}(A)$ be the loss when the alternative prediction was based on set A with practical actions being particularly concentrated on the possibility of occurrence of Z_j, whereas variant Z_i actually occurred, with both variants belonging to A. Finally, let K_A denote the total costs of actions undertaken to meet the situation characterized by prediction $Z \varepsilon A$.

Under these assumptions the best alternative prediction, i.e. the best set A, is obtained by minimizing, with respect to A, the following expression

$$E[S(A)] = \sum_{Z_k \varepsilon \bar{A}} S_k(A) \cdot P(Z_k) + \sum_{Z_i \varepsilon A} \sum_{Z_j \varepsilon A} S_{ij}(A) \cdot P(Z_i) + K_A \quad (19)$$

with $\bar{A}$ denoting the set of all variants of Z which do not belong to A and $i \neq j$. Since it can be assumed that the losses $S_{ij}(A)$ will usually be close to zero[40], one can use--instead of (19)--a simpler formula

$$E(S) = \sum_{Z_k \varepsilon \bar{A}} S_k(A) \cdot P(Z_k) + K_A \quad . \tag{20}$$

When the number of possible variants is small, the choice of the best set A can be done in a straightforward way by going over all possible subsets which can be had from a given set C. When the number of elements of C is large such a procedure is no longer feasible[41]. It is, however, possible to determine the best set A by using mathematical binary programming.

It should be noted that the best solution may result either in finding a proper alternative prediction or sometimes the best A set may prove to coincide with C or even to be an empty set. If the solution is that $A = C$, this means that one has to prepare oneself for all possible variants. If, on the other hand, one finds A to be an empty set this must be interpreted as a sign that from the point of view of costs and losses no action should be undertaken. From (20) it is obvious that the solution $A = C$ is likely to hold when the cost K_A is low as compared with losses $S_k(A)$. On the other hand, when losses are negligible and costs of actions are high then the optimum solution will tend toward the case of A being an empty set[42].

To conclude, we shall give a simple example. Let us suppose that four different variants of behavior of explanatory variables have been singled out[43] and that for different possible A sets the corresponding costs and losses are as they have been presented in Table 7 below.

For the sake of example we shall calculate E(S) for two cases, namely when $A = \{Z_1\}$ and when A is an empty set. From Table 4 it is known that the probabilities of different variants are equal to 0.20, 0.50, 0.25 and 0.05 respectively. In the first case we have

Table 7.

Different Possible Sets A	Costs of Action K_A	Losses			
		$S_1(A)$	$S_2(A)$	$S_3(A)$	$S_4(A)$
Z_1	5	0	5	10	12
Z_2	10	4	0	8	15
Z_3	4	7	4	0	10
Z_4	20	8	6	10	0
Z_1, Z_2	12	0	0	10	14
Z_1, Z_3	9	0	8	0	16
Z_1, Z_4	22	0	7	10	0
Z_2, Z_3	13	5	0	0	17
Z_2, Z_4	28	4	0	8	0
Z_3, Z_4	24	10	5	0	0
Z_1, Z_2, Z_3	18	0	0	0	12
Z_1, Z_2, Z_4	29	0	0	12	0
Z_1, Z_3, Z_4	35	0	6	0	0
Z_2, Z_3, Z_4	30	8	0	0	0
Z_1, Z_2, Z_3, Z_4	38	0	0	0	0
A = empty set	0	9	11	8	13

$$E(S) = 5 \cdot \frac{5}{10} + 10 \cdot \frac{25}{100} + 12 \cdot \frac{5}{100} + 5 = 10.6 \quad .$$

On the other hand, in the case when no action is undertaken (and therefore A is an empty set[44]), we have

$$E(S) = 9 \cdot \frac{2}{10} + 11 \cdot \frac{5}{10} + 8 \cdot \frac{25}{100} + 13 \cdot \frac{5}{100} = 9.95 \quad .$$

Exploring all the possible sets A and computing the corresponding values of E(S) one will eventually find the best prediction, i.e. the best set A , which is $A = \{Z_3\}$.

7. CONCLUDING REMARKS

Although the title of this paper refers explicitly to water demand prediction it seems nevertheless that the range of possible uses of methods which have been presented here is much wider. Whenever the inference into the future refers to far-distant periods of time (i.e. when prediction lead is large) there arise problems of how to define the levels of factors determining the behavior of the predicted variable. For the far-distant future not only the predicted variable Y_T but also its "environment conditions" remain unknown.

One of the possible ways to by-pass this difficulty is to use simple forecasting, instead of prediction, i.e. to base the inference on trend or autoregressive models. In most practical situations, however, this approach would not be adequate since the aim of inference into the future is to ascertain the impact of different variants of actions (different policies) which can be pursued. This obviously calls for using causal models with a number of explanatory variables.

Several approaches to prediction have been presented in this paper when there is doubt about the levels of explanatory variables in time T. First (section 2) a critical outline of a number of known methods has been given. Next (section 3) we have presented a new method, consisting of building two predictions, one based on the assumption that "favorable" conditions will occur in time T and one computed when it is expected that the explanatory variables will behave in an "unfavorable" way. The interval defined by these two predictions gives the user of the prediction information about what to expect when the extreme cases are dropped from consideration. In section 4 the concept of alternative predictions was introduced and it was shown how this type of prediction can be used for predicting the levels of explanatory variables in time T. It seems worth adding that the concept of alternative predictions is of special interest in situations when different possible ways of formation of explanatory variables are singled out as a number of qualitative variants (see Table 4). The concept of alternative predictions can be used also for predicting not only the explanatory variables of the model but also the predicted variable Y_T. This is shown in section 5 of the paper while section 6 deals with a special way of building alternative predictions of Y_T. This special feature of prediction consists of building it in such a way that it be the best from the point of view of economic efficiency, minimizing thus the sum of expected losses due to wrong predictions and of costs connected with actions started on the assumption that alternative prediction will prove correct.

NOTES

1. The concept of alternative predictions was first presented in my paper, "Przyczynek do teorii prognoz alternatywnych" (A Contribution to the Theory of Alternative Predictions), published in the journal, Przeglad Statystyczny (Statistical Review), 2, 1975, in Polish.

2. This method can usually be refined by correcting the data assumed by the relevant economic plan by a coefficient expressing the average level of fulfillment of targets set by the plan.

3. An obvious question arises: where is the information about the probability distribution obtained from? Three cases must be analyzed separately. First, a variable X_i has a distribution stationary in time. An appropriate analysis of its past values provides then an estimate of the probability distribution function. Second, variable X_i is nonstationary, its parameters varying in time while the functional form of the distribution function remains constant. Here again, a sample of past data can be used to extrapolate the parameters of the distribution function in time T. Third, variable X_i does not behave in a "smooth" way, or there are no past data on its previous behavior. In this case, a pool of experts' opinions can be expected to shed some light on the distribution of X_i in time T.

4. In the general case, the mode of a k-dimensional distribution does not have to coincide with mode values corresponding to one-dimensional (marginal) distributions of the X_i variables.

5. Let us note here that the same principle applies to choosing the best prediction of Y_T. Under the assumption of a unique act of inference about the future the best prediction of the predicted variable is then provided by the mode of its distribution in time T (see Z. Pawlowski, "Prognozy Ekonometryczne" [Econometric Predictions], PWN, Warsaw (1973)).

6. As is well known, in that case one substitutes for lagged endogenous variables, for all intermediate time periods following t_o and prior to T, the estimates of their expected values, these estimates being equal to theoretical values of the respective variables ,i.e. to values computed from the model.

7. It is to be noted, however, that using expected values of explanatory variables instead of their (unknown as yet), true values in time T results in increasing the variance of prediction by an amount equal to a quadratic form of the

true coefficients of linear form of (1) and of variances and covariances of X_i's (see reference quoted in note 5).

8. When extrapolating trends, it may happen that the value of the trend for $t = T$ is equal to a noninteger number, while from its very essence the explanatory variable can assume only integer values. Then rounding of the computed trend value is evidently necessary. As another example, one should note that when using procedure (d), the mathematical expectation of X_i in time T may be equal to a number whose probability of occurrence is zero. In this case again rounding of the result is necessary.

9. That is, in cases when prediction lead is large.

10. With reference to the problem of estimation of the probability distribution function we refer the reader again to note 3.

11. Let us note that in some cases one of the sets A and B may prove to be empty.

12. The procedure when utility is a decreasing function of Y_T will be explained later in this same section. As will be seen it is strictly analogous to the approach discussed now, i.e. the higher the Y_T the better.

13. The right-hand part of the inequality (7) stems from the requirement that the frequency of situations when an explanatory variable exceeds its high value (as defined by (3)), or falls short of its low value (as defined by (4)), be kept close to zero. For the sake of brevity, we omit here the mathematical justification.

14. On the other hand, a long interval of indeterminacy provides a high probability that in fact the value of Y_T will fall within that interval. It may be interesting to find the probability p of $Y_T \in [y_{Tp}^{(p)}, y_{Tp}^{(0)}]$. This probability can easily be determined if the distribution function $G_T(y)$ is known. Then $p = G_T(y_{Tp}^{(0)}) - G_T(y_{Tp}^{(p)})$.

15. These values are--as in the linear case--chosen in such a way that the probability of getting still better (or still worse) values of explanatory variables is equal to a predetermined number ε.

16. Since this is merely an artificial example designed for the illustration of the concept of optimistic and pessimistic predictions, not much attention was paid to the question of whether parameter estimates are realistic, although they perhaps do not deviate very much from results which one would obtain using real data.

17. If the reverse were true, i.e. high demand levels were desirable, the only difference would consist in reversing the values of the two predictions, the interval of the uncertainty of prediction remaining the same. It should be noted that decision whether an optimistic prediction should be a high one and the pessimistic a low one or vice versa depends on the nature of the problem and on the point of view of the user of the prediction. Let us add also that in order to eliminate a somewhat ambiguous sense of the words pessimistic and optimistic one could refer to minimalistic and maximalistic predictions instead.

18. It is almost impossible to get in practice, when the assumed distributions differ from the rectangular one, an explicit formula for $L = L(\varepsilon)$. Therefore, one is obliged to choose ε obeying the constraint $\varepsilon \leq \frac{1}{2K}$, and after computing the predictions to check if the length of the interval is small enough for practical purposes.

19. Let us note also that alternative predictions may be used not only for predicting levels of explanatory variables but may also be directly applied for predicting the endogenous variable Y_T.

20. Treating a set of possible values of a discrete random variable as a single variant provides a way of dealing with such variables which--in theory at least--can assume an infinite (but countable) number of values, as is the case of a Poisson distribution, for instance.

21. Hence, all classical point predictions can be viewed as single predictions.

22. If the alternative prediction is Z_{AP}, then its likelihood is $P\{Z \varepsilon A\}$. If the variants Z_i are disjoint then of course $P\{Z \varepsilon A\} = \sum_{i \varepsilon J_A} P(Z_i)$.

23. For proofs, see the author's paper quoted in note 1.

24. A third possible solution will be presented in Section 5 when we shall be concerned with direct application of the concept of alternative predictions for predicting not only the explanatory variables but also the predicted variable Y_T.

25. The diameter of a set is usually defined as the maximum distance between two elements belonging to that set.

26. Hence if the mathematical expectation of a variable truncated to an open (first or last) interval happens to be nonexistent, we shall say that the appropriate variant is located infinitely far from the others.

27. If some variants represent a set of at least two different possible values of the discrete variable one must use the procedure under heading (a) i.e. to compute the mathematical expectation of that variable under the condition that it is truncated to the set of values contained in the variant considered.

28. That is, nonnegativity, symmetry, and triangularity.

29. For brevity, in Table 2 (and Table 3) variants are denoted simply by their numbers, i.e. 1 stands for Z_1, 2 represents variant Z_2, etc.

30. In this case, the interval (3,12) represents the range of expected variation of X and favorable or unfavorable values of this variable are selected according to the chosen value of ε and to the character of X (i.e. whether it is positively or negatively correlated with Y).

31. Let us observe that any single prediction would be a rather poor one since the maximum probability of a single variant is but equal to 0.3.

32. Let us note that these are the explanatory variables already introduced in our argument in the example given in Section 3.

33. That is, of distances defined in such a way that the maximum observed distance is equal to 1.

34. It has been implicitly assumed that experts who have singled out the alternatives presented in Table 4 know in quantitative (interval) terms what they mean by stationary state, moderate or fast growth of different factors taken into account.

35. Which means that the set A contains exactly v elements.

36. The word "final" when referring to prediction of Y_T is adopted because in this paragraph we are dealing with a two-stage procedure. First, the procedure of alternative prediction is applied to explanatory variables and then (finally) to the endogenous variable of the model.

37. It should be emphasized that the procedure outlined in this section is expecially suitable when assumptions about the formation of explanatory variables lead to distinguishing a number of variants corresponding to joint formation of all explanatory variables, the Z_i's being of complex, "qualitative" character.

38. For the sake of brevity we omit quoting here information pertaining to the distances, which are, however, essential when deciding which alternative of variants Y_j should finally be adopted as the prediction.

39. One can also consider a more general framework in which it is possible not only to incur losses but also to achieve gains by actions started on information stemming from predictions which later proved to be right. Since this generalization is very straightforward we shall not be concerned with it.

40. An interesting, but still not completely solved problem is how to distribute a given finite amount of outlays on actions aimed at meeting the occurrence of different variants included in A - so as to achieve the highest efficiency of actions based on alternative prediction.

41. The total number of combinations which can be made out of a set containing g different elements equals--as is well-known--2^g.

42. A good example of a predictive situation leading to inclusion of all possible variants in A is that of a man going to a country with a tropical climate where he can contact one out of g lethal diseases $Z_1, Z_2, \ldots, Z_g$. The cost of action connected with meeting such variants is small (vaccination) while the loss if very substantial (death of the person contracting the disease).

43. These are identical with variants presented in Table 4 of Section 4 of this paper.

44. An empty set A is not to be interpreted that nothing will happen in time T but that it is economically profitable to take no action whatever to meet the future.

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