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Modification of road networks to reduce the energy use of the transport sector

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A contribution of optimal control theory to the optimization of transport networks

Approved by

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Contents

Abstract

An optimal road capacity exists at which the energy use of the road infrastructure and its traffic is minimal. It implies that 15% of the time, the traffic is jammed. The socio-economic optimal road capacity implies jammed traffic for no more than 5% of the time. The claim that widening roads reduces the environmental damage of transport, because it prevents the formation of traffic jams, is generally speaking incorrect. However, modification of road transport infrastructure with the aim of reducing CO_2 emissions of transport is not efficient, since other economic sector can reduce CO_2 emission at far less costs.

Should one wish to optimize a network on a criterion that is different from the criterion that governs the flows over the network, complex systems behavior looms. This report advocates the use of back casting as most likely approach to optimize complex networks on an infinite horizon basis. In a network that should be build up, the procedure that leads to a (near) optimal construction strategy is demonstrated.

First, one assesses the optimal end state of the system. Second, one determines the time that is required for the construction works to transform the network from the original configuration to the optimal end configuration. Consequently, one back casts the system configuration under the condition that the shadow price of a network link modification is equal for all network links. As long as the attainability domain is respected, this procedure is shown to provide near optimal results. Whether the near optimal results are also unique, optimal results is not mathematically proven.

Acknowledgments

This work follows previous work at IIASA on the application of optimal control theory to minimize energy use due to road infrastructure expansion. The latter work was elaborated upon in a University of Groningen thesis on the energy use of transport infrastructure. A post-doc grant from NWO enabled me to pursue some of the subsequent questions that emerged from the thesis. This report builds on the latter thesis, and includes many of its findings. From a personal point of view, I have found the answers that I was looking for.

Therefore, I am thankful to NWO for financing this research, and to the Center for Energy and Environmental studies (IVEM) of the University of Groningen for the development of the transport related research aspects on which this report stands. Most of all, though, I am indebted to Sergey Aseev and Arkady Kryazhimskiy for guiding me in the application of the optimal control theory.

About the Author

Sander Lensink was a YSSP-student in 2002. He returned to the IIASA Dynamic Systems group in 2005 for half a year as a post-doc. His expertise lies in the field of energy and environmentalism, notably on the energy use in the transport systems in which he gained his PhD at the University in Groningen in 2005. Currently, he is working at the Energy research Centre of the Netherlands (ECN) on renewable energy and biofuels.

Application of optimal control to guide the development of transport networks

Sander M. Lensink (lensink@ecn.nl)

1. Introduction

1.1. About the optimization of energy use in the transportation sector

Transport and traffic are integral parts of human society. Individualized travel has helped shape modern societies by facilitating emancipation. Notwithstanding the importance of transport, transport has detrimental effects on the environment. Partly due to the diversity of the transport system, it is difficult to formulate generalized policy to combat the adverse environmental impact of transport. Nowadays, policy attention seems to be focused on the reduction of emissions of small particles (PM_{10})[1]. Sustainable transport, as essential element of sustainable development, goes beyond the reduction of specific emissions. Sustainable transport is often envisioned as transport that restrictively uses electrical or hydrogen vehicles. However, even with the related 'clean' energy carriers, a more efficient use of energy resources is expected in future transport.

The transport system is, as mentioned, diverse as it consists of both passenger traffic and goods transport using many different, possible interacting, transport modes like road, rail, water and air transport. Generally, transport uses vehicles and infrastructure, whereas most infrastructures can accommodate a range of different vehicles. The efforts to reduce the environmental impacts of transport have been targeted at reduction of the transport volume, change of modal split, the development of more efficient engines and cleaner fuels. The latter two options have proven to be successful, while the primer two have shown themselves to be difficult to realize.

Assuming an increasing scarcity of available energy resources, one can wonder to which extent the scarcity of resources might shape future transport systems. Reversely, one can aim to shape future transport systems in order to minimize the primary energy use of transport. An analysis of the future development of transport infrastructures can limit the variety in development paths of the transport system. By looking at the impact that available transport infrastructures have on the energy use, one

[1] EU-Council Directive 1999/30/EC of 22 April 1999 had let to, for example, speed restrictions in Austria (50 km/h in Vienna), in the Netherlands (80 km/h on urban highways), traffic access measures in Germany and the tendency to impose filters on diesel exhausts.

can determine the effects of long term changes in the transport system under the conditions of increasing energy scarcity.

Control theory, more specifically optimal control theory using the Pontryagin maximum principle [3], is applied to determine the energy-optimal transport infrastructure and explore the options transport policy has or takes to approach this energy optimum. For this, infrastructure construction is taken as governmental control to change the characteristics of the transport system. Section 2 starts with an overview of the application of optimal control theory on a single-road segment. Section 3 outlines the methodology for a system of two competing, parallel road segments. Section 4 looks to formulate the optimal development paths. It shows a generalist approach to finding the solution, and demonstrates that the generalist approach is, at least, nearly optimal.

This paper shows twofold conclusions in Section 6. Based on the conclusions, it supports (or not) the extended use of optimal control theory in the transport and energy related sciences. Section 6 indicates as well whether future efforts to achieve a sustainable transport system should mainly be targeted at engine and fuel efficiencies, or the latter efforts should (also) be directed towards changing the structure of the transport system by modifying infrastructure capacities.

1.2. Introduction to optimal control

Optimal control theory has proven itself as useful tool in economics [4]. With the Pontryagin maximum principle, one can determine the optimal investment strategy that maximizes future profits. Similarly, infrastructure construction can be seen as investments in the transport system, which should increase the future rate of return. As such, the optimal control theory does not help optimizing a profit that is as high as possible, but an energy use that is as low as possible.

Optimal control problems are conventionally presented as the following problem (P), see e.g. [1,3].

Problem (P):

$$\dot{x}(t) = f\big(x(t), u(t)\big); \tag{1.1}$$

$$u(t) \in U;$$

$$x(0) = x_0; \tag{1.2}$$

$$\text{maximize } J(x,u) = \int_0^\infty e^{-\rho t} g\big(x(t), u(t)\big) dt. \tag{1.3}$$

The time in these equations runs in $t \in [0,\infty)$. Initially, the state of the system *x(0)* is known, see (1.2). The development of the state of the system in time *x(t)* is controlled by some control parameter *u(t)*, which can take any value in the set *U*. The precise manner in which *x(t)* changes is determined in (1.1) that is referred to as the state equation. One should find the control *u(t)* that satisfies the objective defined in (1.3).

A pair (u, x) in which x is the trajectory of system (1.1) corresponding to the control u is called admissible. The admissible pair that also meets the objective (1.3) is the optimal pair (u_*, x_*), consisting of the optimal control u_* and optimal trajectory x_*.

1.3. Optimal control in the field of energy and transportation

The information that is required or requested decisions on transport policy is often fairly specific and detailed, like in Environmental Impact Statements (EIS) for transport projects, or quite generic for transport programs[2]. The optimal control methodology conforms to the generic nature of the information needed for the latter. For the methodology applied to be most explanatory in describing the dynamics between infrastructure construction and the energy use of the transport system, it is considered best to use as few parameters as possible in a so-called minimalist model.

By using a minimalist model, the paper chooses implicitly that many transport related phenomena remain unchanged. Principally, only three aspects are taken as dynamic characteristics of the transport system: the capacity of the network infrastructure, the traffic flows over the transport infrastructure and the energy use of the transport system.

Energy research, for as far as Energy Analysis (EA) is concerned, does not commonly include time discounting [5]. EA is closely related to Life Cycle Assessment (LCA), which is formalized to great extent [6]. Historically, LCA and EA are product-based assessments, for which temporal aspects of energy use are hardly relevant. However, the data presented in EISs on energy use of (alternatives in) construction projects are in essence determined in an EA-like assessment. Furthermore, the latter data is sometimes presented in the form of a cost-benefit analysis. Indeed, if one considers energy quantities as monetary values, energy research can benefit greatly from methodologies applied in economics. The rationales for money discounting might also be valid for energy discounting.

1.4. Introduction to the problem

The optimization of transport infrastructure as such is not a clearly defined process. Besides the optimization criteria - this report demonstrates both economic and energy-related criteria - multiple actors are present in infrastructure use and planning. The latter actors may have different objectives and time frames. This report assumes the existence of the following two actors: a road planning authority that optimizes the energy use of the entire transport system, and the travelers, the infrastructure users, that minimize their travel time. The time frame of the road planning authority ranges from months to decennia, while the time frame of travelers ranges from minutes to days.

The study should increase understanding of the dynamics that occur when a transport network is improved. Using the insight, the report attempts to use the optimal control theory to optimize the transport system with an infinite time horizon, while taking the short time travel behavior into account. As dilemmas in infrastructure

[2] See EU-directive (85/337/EEG) and its change in (97/11/EG) of March, 3, 1997.

optimization frequently appear in road infrastructure planning, the outcome of the study should comment on general policy choices.

The research aims are threefold:

1) to characterize the optimal road, given a certain criterion;

2) to identify the optimal network configuration, given certain criteria;

3) to determine the optimal construction strategy, given the latter criteria.

2. System description

2.1. Introduction in the energy use of road transport

This study takes the energy use as main criterion for determining optimal road and network configuration and optimal network development. The energy use includes both the energy that is required for the propulsion of the vehicles and the energy required for constructing, maintaining and discarding the road infrastructure. Of the total energy use of road transport, 85% is roughly attributable to the fuel consumption of the vehicles, while the remaining 15% are infrastructure related, including the construction of major artworks like tunnels and bridges [16].

Diversity in the per vehicle energy use is large. Whereas a passenger car typically uses between the 2 and 4 MJ/vkm, a freight truck has an average energy use of about 10 MJ/vkm, largely dependent on the (vehicle and cargo) mass to be transported.[3]

One kilometer of Western European highway requires circa $90 \cdot 10^6$ MJ/km and can accommodate circa 2000 vehicles per hour per lane [16]. When the traffic intensity approaches the road capacity, a traffic jam may occur. In the Netherlands, traffic jams are for 12% directly caused by traffic intensity/road capacity ratio, and for more than 80% indirectly caused by i/c ratio (e.g. bottlenecks) [10].

Discussions, both societal and scientific, about road construction policies are inclined to include the effect of generated traffic, i.e. traffic that results for travel that would not have been undertaken had a certain road not been improved [11]. Notwithstanding the importance of generated traffic for the effects of road constructing measures for the total energy use of the transport system, the effect of generated traffic is not included in this report. In finding the optimal construction policy, it is methodological of minor importance. Furthermore, the effect of generated traffic is hard to predict and would cloud the results with large uncertainties.

The report assumes that induced traffic, i.e. traffic that uses a different time or route for performing a journey, has no effect on the energy use of the transport system. In other words, it is assumed that deflected traffic can be accommodated elsewhere on the network with same energy intensity, measured in MJ/trip;

Finally, it is assumed that road capacity is a continuous variable, thus road improvements might be infinitesimal small. As a consequence, the results may not be applied straightforwardly to a specific road construction case.

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[3] vkm stands for vehicle times kilometer.

2.2. Single road system description

2.2.1 Capacity as state variable

Infrastructure construction requires the application of energy resources. One could consider the latter application as investment of energy resources in the transport system. A reduction of energy use, or increased efficiency of transport, provides the profit of the investment. For each project considered, one can use an EA to assess the cost-benefit ratio of the energy resource investment. The latter practice is, encouraged through EU-legislation[2], common for large construction projects. The EA is also a suitable tool for assessment of the impacts of largely discrete choices.

Due to the large scale of transport infrastructure, the transport system changes gradually in time, despite the sometimes even enormous discrete changes from the traveler's perspective. The dynamic behavior of the entire transport is suitably illustrated with the use of a continuous state variable. Using a continuous state variable, one should keep in mind the limited meaning the state variable will have for single, specific cases.

The state of the system is described by the capacity of the road segment x, in vkm/h.[4] It is a measure of the transport performance that can be accommodated on a road segment.

The road capacity has an autonomous decay rate δ. It is a crude approximation. For one, because the decay rate partly depends on the cumulative traffic load that occurred since the last maintenance activities, and partly as the decay rate increases as the maintenance-free period becomes longer, see [7,8]. Without construction and maintenance activities, the capacity of the road, or the state of the system is described by (2.1):

$$\dot{x}(t) = -\delta \cdot x(t) + \dots; \qquad (2.1a)$$

Typical values for δ are estimated at 0.15 yr^{-1} [9].

2.3.2 Construction activities as control variable

The capacity can be increased by the maintaining or constructing capacity at a rate of *u(t)*, measured in vkm/h^2. Following (1.1), with $f(x,u) = -\delta \cdot x(t) + u(t)$, the state equation becomes:

$$\dot{x}(t) = -\delta \cdot x(t) + u(t); \qquad (2.1)$$

The energy resources to be committed for construction activities are given by $\alpha \cdot u(t)$, in which α is the energy intensity of construction activities or capacity increase. The energy intensity depends on the type of civil engineering construction required for e.g. road widening. Typical values are $10 \cdot 10^3$ MJ/(vkm·h^{-1}) for ground level highways, and $100 \cdot 10^3$ MJ/(vkm·h^{-1}) for tunnel highways [9].

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[4] vkm stands for vehicle · km.

2.3.3 Life cycle energy use as utility function

The road transport system uses a great variety of road types and road vehicles. For now, it is assumed that any benefit has to be achieved through reduced energy use of the vehicles, thus reduced fuel consumption[5]. Vehicular fuel consumption per kilometer traveled is, not exclusively, influenced by the average vehicular velocity, changes in velocity and altitude differences. Apart from the speed limits set, inner city velocities depend on many road characteristics, like number of crossings, priority regulations, traffic lights and speed bumps. The velocities on rural roads, however, are more often determined by the capacity of the roads in relation to the occurring traffic intensity. In fact, the majority of the traffic jams in the Netherlands is caused by insufficient capacity or by bottlenecks [10]. A bottleneck is formed by two or more converging traffic flows. To some extent, it is also the limited capacity of the road junction or road intersection that causes traffic to flow slower. Therefore, the road capacity is seen as the explaining factor for the occurrence of vehicle velocities well below the speed limit on rural highways. The road capacity might furthermore serve sufficiently well, although being a continuous variable, provided that a sufficiently long road segment or extensive road network is considered.

The variety in road vehicles has presumably three important aspects for the determination of the general vehicular fuel consumption, notably the dependence of energy use on velocity[6], the official speed limits[7], if any, and the length of the vehicle (or the amount of capacity that should be allocated to the vehicle). Part of the variety can be cancelled out by averaging the curves of energy use vs. velocity. Furthermore, calculations can be carried out using the passenger car equivalent (pce), in which often one freight vehicle stands for 2 pce.

Two variables are considered in the determination of the fuel consumption of the vehicles: the traffic intensity and the road capacity. The capacity of the road, or state variable x, is expressed in the dimension of vkm/h.

The traffic is represented by the transport performance (vkm/h), which is the product of the traffic flux φ in veh/h and the length ℓ of the road segment (km). Traffic flux φ is the actual number of vehicles which goes through one km of the road in one hour. The traffic flux φ is implicitly limited to a certain value φ_{max} by the capacity x, but otherwise assumed to be independent of the capacity x. Although hard to quantify, empirical evidence is sufficiently strong to state that expansions of infrastructure have led to an increase in traffic [11], or reversely stated: limited capacity in the transport network causes some traffic not to occur. This phenomenon consists of two effects. One is called induced traffic and implies that people traveling differently after road construction than before; the other is referred to as generated traffic and indicates that people are traveling that would not have been traveling without the construction of the road.

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[5] Where ever this paper uses the term 'fuel consumption', it refers not only to e.g. diesel or gasoline consumption, but also to electricity use by electrical vehicles.

[6] Generally speaking, heavy vehicles have higher fuel consumption at any velocity than light vehicles and vehicles with a high power-to-mass ratio have their optimal velocity at a higher velocity than vehicles with a low power-to-mass ratio.

[7] Freight vehicles often have speed limits in the range of 80-100 km/h, while the passenger car speed limits often are 100-130 km/h.

For sure, there exists a theoretical hard upper bound on personal transport, corresponding to the case where one is continuously traveling at the highest possible velocity. A softer upper bound can theoretically be deduced from the amount of time one is willing to spend traveling and the transport modes to ones disposal, see also [12]. Suppose the practical upper bound for the transport performance is presented by $\varphi_{max}\cdot\ell$, the actual transport performance is thus $\varphi\cdot\ell \leq \varphi_{max}\cdot\ell$ or $\varphi \leq \varphi_{max}$. The difference between the upper bound and the actual traffic $\varphi_{latent} = \varphi_{max} - \varphi$ is called the latent transport. For the current single-road description, it is assumed that $\varphi_{latent} = 0$.

The ratio of intensity over capacity, the so-called i/c-ratio, determines to large extent the velocity v of the vehicles. An often-used approximating function is called the BPR-function [13], from which the velocity (km/h) is easily deduced leading to[8]:

$$v(\varphi, x, \ell) = v_{\max} \Big/ 1 + \tfrac{1}{2}\left(\frac{\varphi\ell}{x}\right)^2 . \tag{2.2}$$

Herein v_{max} represents the highest occurring velocity in km/h.

The energy use $e(v)$ of the vehicles (in MJ/vkm) is on average to be computed out of the velocity. For each type of vehicle and type of fuel used, the energy use function $e(v)$ is different. Hickman [14] gives vehicular emissions that have been empirically determined under controlled conditions. From the CO_2 emissions, the primary energy can be deduced. Figure 1 shows the graph for 1990 cars using EURO-I gasoline, and the graph that shows the average energy use per freight vehicle, with the expected freight vehicle distribution in the Netherlands for 2010. Roughly, the energy use per vehicle kilometer of one freight vehicle (excluding vans) approximates the energy use of five passenger cars.

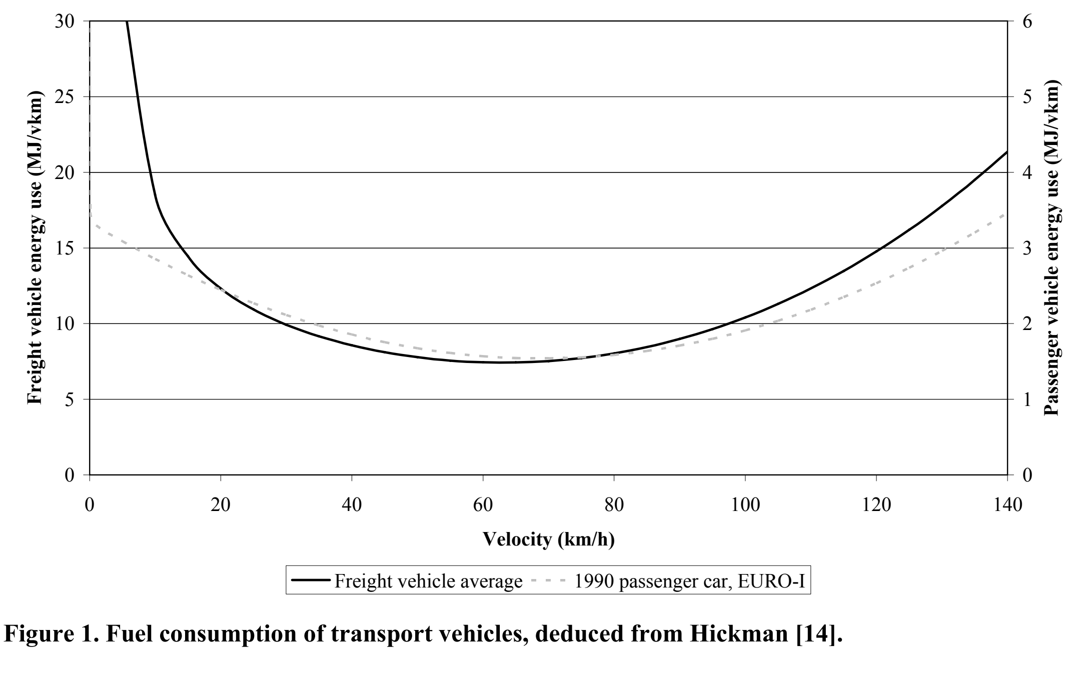


Figure 1. Fuel consumption of transport vehicles, deduced from Hickman [14].

[8] The factor ½ and exponent 2 are basically empirical determent parameters. In literature, the exponent takes values up to 6. As approximating formula, there is no explicit constraint on the i/c ratio that $\varphi\ell/x \leq 1$.

The graph of the fuel consumption of passenger cars represent that of a 1990 built passenger car using EURO-I gasoline. It conforms to the formula:

$$e(v) = 3.338 - 0.0523 \cdot v(x) + 0.00038 \cdot v(x)^2 + 36.50 / v(x); \tag{2.3}$$

The fuel consumption E of the vehicles is thus:

$$E(x) = \varphi \cdot \ell \cdot e(v(x)); \tag{2.4}$$

In the form of (1.3), using (2.4) we get $g(x,u) = -\alpha \cdot u(t) - \varphi \cdot \ell \cdot e(v(x))$ to give

$$\text{minimize } J = \int_0^\infty e^{-\rho t} [\alpha \cdot u(t) + E(x(t))] dt \,. \tag{2.5}$$

The value of the discount rate ρ is based on the expected average functional lifetime of the infrastructure connection at ρ=80 yr^{-1} [9].

2.4. Optimal control solution

The optimal control problem for a system that represents a single road segment, based on (1.2), (2.1) and (2.5), is problem (P^{single}):

Problem (P^{single}):

$\dot{x}(t) = -\delta \cdot x(t) + u(t);$

$0 \le u(t) \le u_{\max}$ for all $t \in [0, \infty);$

$x(0) = x_0;$

$$\text{minimize } J = \int_0^\infty e^{-\rho t} [\alpha \cdot u(t) + E(x(t))] dt.$$

Figure 2 shows that the function *E(x)* is convex for the lower ranges of *x*. [8] and [9] show that these are the relevant ranges for *x* as well in finding the optimal capacity.

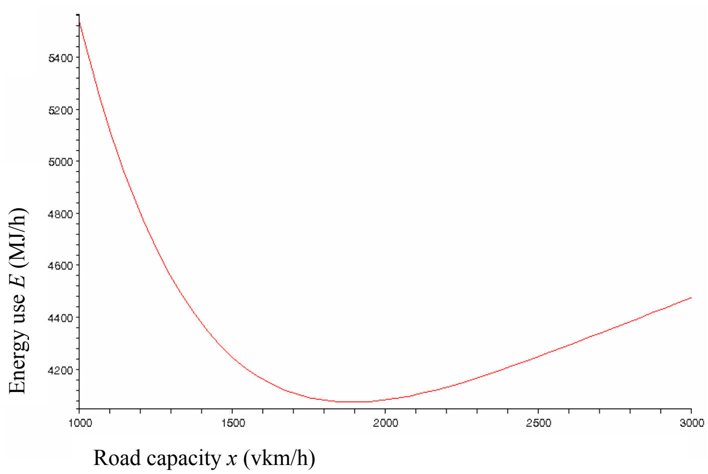


Figure 2. Fuel consumption of all vehicles in a one kilometer road per hour, *E(x)*, in MJ/h as function of the road capacity *x* (vkm/h).

Balder proves that an optimal control $u_*(t)$ exists in a problem like problem (P^{single}), see theorem 3.6 in [2]. Furthermore, if $(u_*(t), x_*(t))$ is an optimal pair, it satisfies to the Pontryagin maximum principle [1,15].

The main tool in the study of problem (P^{single}) involves looking for the necessary optimality conditions in the form of the Pontryagin maximum principle [1]. The theory involves two closely related functions, the Hamilton-Pontryagin function (2.6)

$$\mathcal{H}(t,x,u,\psi) = f(x,u)\psi + e^{-\rho t} g(x,u) = (u - \delta x)\psi - e^{-\rho t}\left(\alpha u + E(x)\right) \tag{2.6}$$

and the Hamiltonian (2.7) which is the maximum of (2.6) under condition of admissible controls

$$H(t,x,\Psi) = \max_{0 \le u \le u_{\max}} \mathcal{H}(t,x,u,\Psi)\,. \tag{2.7}$$

In (2.6) and (2.7) the adjoint variable is ψ and its behavior is defined by the adjoint equation (2.8)

$$\dot{\psi}(t) = -\left(\frac{\partial f(x_*(t),u_*(t))}{\partial x}\right)\psi(t) - e^{-\rho t}\,\frac{\partial g(x_*(t),u_*(t))}{\partial x}\,. \tag{2.8}$$

In the current example, (2.8) is written as

$$\dot{\psi}(t) = \delta\psi + e^{-\rho t}\,\frac{\partial E(x)}{\partial x}\,. \tag{2.9}$$

In short, if the adjoint variable $\psi(t)$ behaves 'properly', the set of equations (2.6) to (2.8) provides the basis for determining the optimal control $u_*(t)$. Section 5 in [8] shows the solution to problem (P^{single}), including proof of sufficient and necessary conditions for optimality.

By substituting $\tilde{\psi} = \psi \cdot e^{\rho t}$, (2.9) can be reformulated into a time independent equation. [8] shows the latter reformulation and establishes the existence of two rest points: one rest point identifies the optimal capacity x_{opt} as the maximum construction effort a divided by the road wear rate δ, it points to the maximal maintainable capacity given limited resources; the other rest point is located at the capacity where the marginal time-discounted energy benefits $\gamma(x)$ of capacity increase equal the marginal costs α of capacity increase. The marginal costs are independent of x, whereas the marginal energy benefits is given by the curve defined by (2.10), see also Figure 3:

$$\gamma(x) = -\frac{1}{\rho+\delta}\frac{\partial E(x)}{\partial x}\,. \tag{2.10}$$

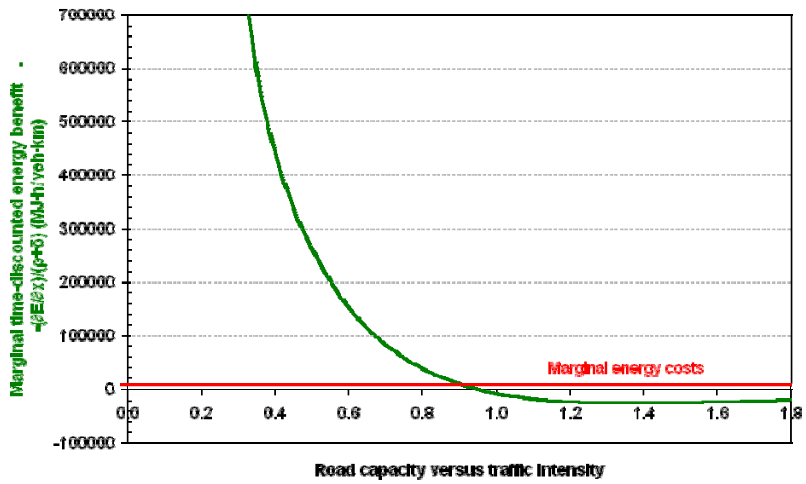


Figure 3. Marginal time-discounted energy benefits and energy costs of capacity increase. The optimal capacity is at a capacity of 0.9 times the traffic intensity.

2.5. Conclusion for a single road

The optimal construction policy for an isolated road is twofold:

If the current capacity is smaller than the optimal capacity, one builds as fast as possible and one builds until the optimal capacity is reached or as far as resources allow; If the current capacity is larger than the optimal capacity, one lets the road wear down until the optimal capacity is reached or down to the highest capacity that can be maintained with the available resources.

2.6. Discussion: energy versus economy

The solution summarized in section 2.4 has been presented as energy optimum in Figure 3. However, the theoretical solution is equally valid for use in an economic analysis. For an economic analysis, different parameter values apply. With a cost of CO_2 emissions of 7.7 €/ton CO_2, and an CO_2 emission factor of 69.3 g CO_2/MJ, the indirect costs due to energy use is $0.53 \cdot 10^{-3}$ €/MJ. The direct costs of construction are $\alpha_{econ}=1.25 \cdot 10^3$ €·h/vkm. The variable costs of travel are $e_{econ}(v)=e(v) \cdot 0.53 \cdot 10^{-3}+0.013+(\ell/v) \cdot 8.17$. The time discount rate is set to 1/15 yr^{-1}. These parameter values have been established in [9] and lead to Figure 4 as the socio-economic analogy of Figure 3.

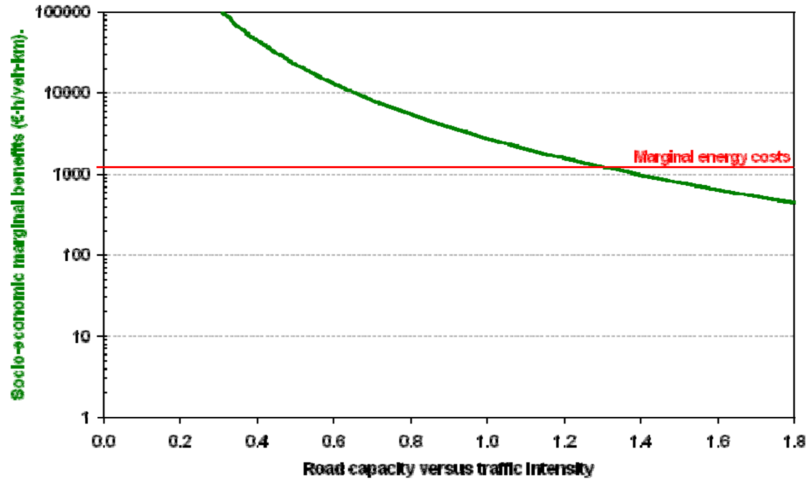


Figure 4. Marginal socio-economic benefits and costs of capacity increase.

Comparison of Figure 3 and Figure 4 shows that - for a single road - the capacity at which the energy use is lowest is significantly[9] less than the capacity at which the road has the largest socio-economic surplus. Figure 5 relates the marginal net energy benefit to the marginal net socio-economic benefit. After conversion of energy figure into CO_2 emission figures, one can use the results to give a macro-level indication of the costs of CO_2 reduction through the reduction of the capacity of the road system.

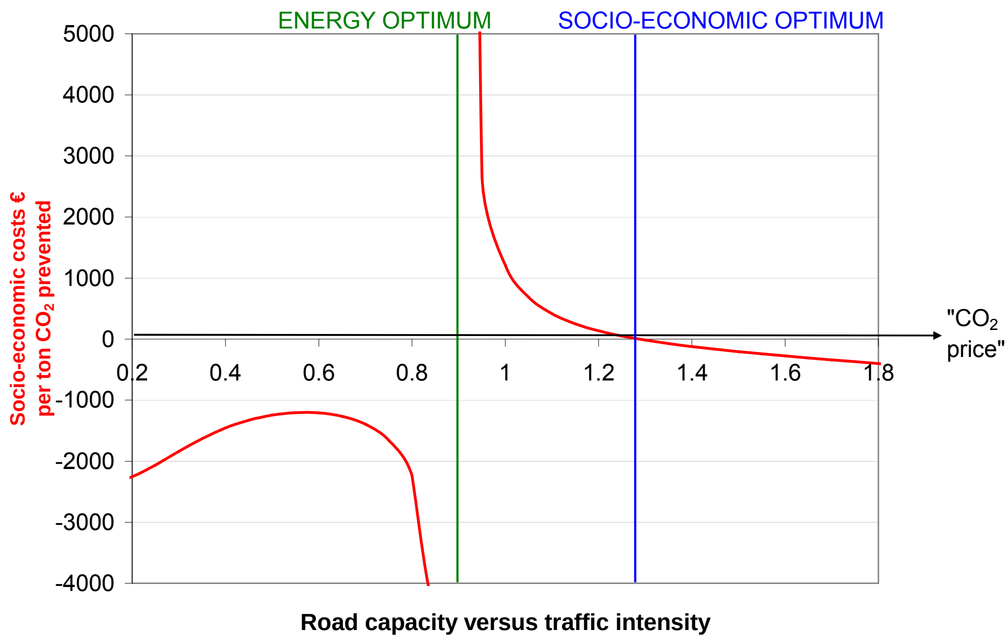


Figure 5. Between the energy and socio-economic optimum, capacity increase leads to addition CO_2 emissions but socio-economic benefits, whereas reduction decreases CO_2 emissions at the expense of high socio-economic costs. Note that expected CO_2 carbon credit prices vary between 10 and 70 €/ton CO_2 in the next decades.

[9] See [9] for an uncertainty analysis.

2.7. Discussion: single road versus network

Using the description of an arbitrary network system, several symbols, variables and definitions are defined in this paragraph. The arbitrary system, taken from [9], in this paragraph serves furthermore as illustration to the description of the further study.

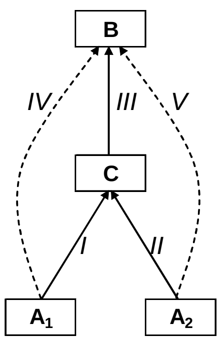
In this paper, the following symbols are used:

$$\ell_1 > 0; \ell_2 > 0; x_1 \geq 0; x_2 \geq 0; \varphi > 0; \varphi_1 \geq 0; \varphi_2 \geq 0; v_{\max} > 0 .$$

Furthermore:

$$x_1 + x_2 > 0; \varphi_1 + \varphi_2 = \varphi .$$

The transport system is represented by a network that consists of fixed nodes (A_1, A_2, B and C) and arcs or links *I, II, III, IV* and *V*. The latter network has to accommodate a fixed transport demand *Y*. The arcs *i* are the road connections that have a certain mono-directional capacity x_i, the nodes are the road intersections.



The network is assumed to be symmetrical in the sense that $x_{A_2C} = x_{CA_2} = x_{II}$.

Travelers want to travel from one node to another node. Their transport demand is shown by a node-to-node matrix. The transport demand, presented by a constant origin-destination matrix, is symmetrical as well:

$$Y = \begin{bmatrix} - & 0 & Y_{BA_1} & 0 \\ 0 & - & Y_{BA_2} & 0 \\ Y_{A_1B} & Y_{A_2B} & - & 0 \\ 0 & 0 & 0 & - \end{bmatrix}.$$

The travellers *j* will minimize their personal cost function, C_j, by choosing the best travel route. The personal cost function might for example represent travel time.

Purpose is to change the road capacities x_i to ensure to lowest possible energy use *J* of the system. The energy use *J* of the system is the sum of the energy required for the production and maintenance of the road infrastructure and the energy use for vehicular movements (thus related to the fuel consumption by the vehicles). The most important notations are:

Length of road	*(constant)*	ℓ_i	km
(traffic) Flux		φ_i	veh/h
(traffic) Intensity		$y_i=\varphi_i \cdot \ell_i$	veh·km/h
(road) Capacity	*(state variable)*	x_i	veh·km/h
Transport demand	*(constant)*	Y	veh/h
Energy use of the transport system		J	MJ
Rate of energy use for infrastructure		G_i	MJ/h
Rate of fuel consumption		E_i	MJ/h

The energy use of the entire system is given by:

$$J = \int_0^\infty \sum_i G_i(x_i(t),t) + E_i(x_i(t), y_i(t))\, dt \,. \qquad (2.11)$$

The energy required for the infrastructure writes as:

$$G_i = \alpha_i \cdot \left(p_i(t) + m_i(t) \cdot x_i(t)\right) = \alpha_i \cdot u_i(t)\,, \qquad (2.12)$$

wherein:

α_i the energy intensity of road production and maintenance (MJ·h/vkm);

p_i the production rate of new capacity (vkm/h^2);

m_i the maintenance rate of existing capacity (h^{-1});

u_i total construction efforts (vkm/h^2).

The energy required for the propulsion of the road vehicles is:

$$E_i\left(x_i(t), y_i(t)\right) = y_i \cdot e\left(v(x_i, y_i)\right), \qquad (2.13)$$

Herein is $v(x_i(t),y_i(t))$ the average velocity of the vehicles;

The energy use per kilometer per vehicle is of form

$$e(v)=\gamma_1+\gamma_2 v+\gamma_3 v^2+\gamma_4/v; \qquad (2.14)$$

The average velocity of the vehicles is:

$$v_i(x_i(t), y_i(t)) = \frac{v_{i,\max}}{1 + \beta_1\left(y_i(t)/x_i(t)\right)^{\beta_2}}\,. \qquad (2.15)$$

v_{max} is the highest allowed velocity – typically 120 km/h; Assumingly, β_1=½ and β_2=2 (actually, in literature β_2 can take values of 6 or larger). It should be noted that $y_i(t)$ can be greater than $x_i(t)$ in this formula[10]. The variable x_i is defined as the highest amount of vehicles that can use a road segment without any significant delay (it corresponds thus to an unperturbed traffic flow). The improvement of capacity is seen as a continuous entity that can take all values. In reality, this is not the case. Construction activities create a jump in available capacity after the works are finished.

If the optimal construction policy for an isolated road is applied to a road network, ignoring the dynamic travel patterns, the optimization method of the previous section fails. Considering the possibility that construction start at the road within the network that has the highest marginal net energy benefit, ignoring the effects of this construction on future travel patterns, one can witness transition inefficiencies, bifurcations and lock-in effects.

[10] If one assumes that traffic, which cannot be accommodated on the road considered, will use other roads with similar energy efficiency (MJ/vkm) and distance (ℓ), one can loosen the constraint that $y_i \leq x_i$.

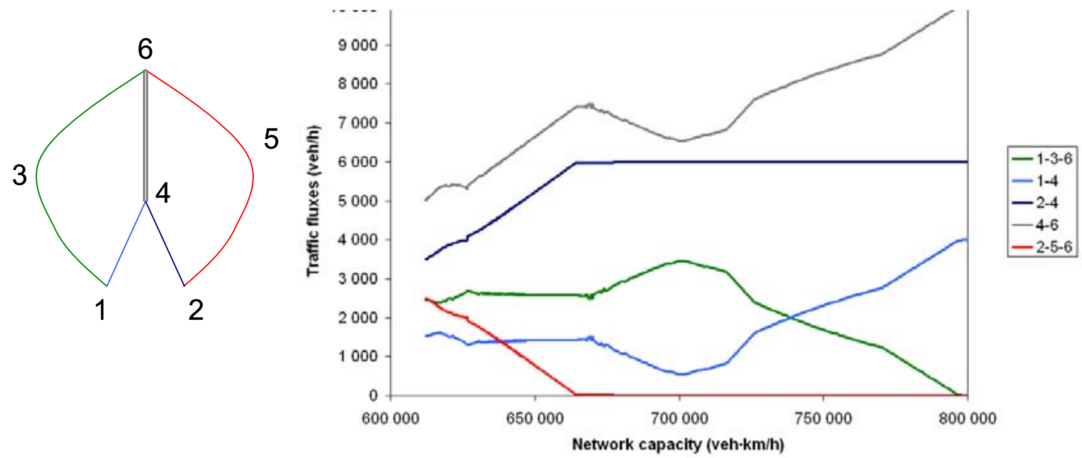


Figure 6. Inefficient network improvement policy: the network on the left-hand side is improved. One can see that some connection are temporarily improved, while at the optimal configuration they are no longer used (e.g. road the connects the nodes 1-3-6).

3. Methodology

3.1. Objective as formalized goal of the study

The objective is to minimize the energy use of the system J by changing the capacities $x_i(t)$ in time. This is to be done by optimally controlling to system through the construction efforts $u_i(t)$. While applying certain construction efforts to the system, the behavior of travelers will autonomously change. In that sense, a *short-time optimization* occurs where the travelers will minimize their individual travel costs. The *long-term optimization* takes place through the application of the control on the road capacities.

3.2. Short term optimization: determining travel routes.

Consider the following road network, see Figure 7. People want to travel from origin ***A*** to destination ***B***: φ_{AB} (veh/h).

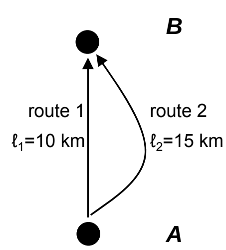


Figure 7. Two-road network.

These travelers choose between route 1, with length ℓ_1 (km) and capacity x_1 (vkm/h) and route 2, with length ℓ_2 (km) and capacity x_2 (vkm/h), resulting in traffic fluxes φ_i over routes i, measured in veh/h. The travelers have full knowledge and make their choice based on the travel times T_i (h) for route i. The traffic fluxes are therefore determined by the road capacities. Consequently, the velocity v_i on road i is determined by the flux φ_i and capacity x_i directly, and ultimately by just the capacities x_1 and x_2.

Either only one route is used, or the travel times over both routes are in equilibrium:

$$T_1 = T_2 \Rightarrow \frac{\ell_1}{v_1(x_1,x_2)} = \frac{\ell_2}{v_2(x_1,x_2)} \tag{3.1}$$

in which $v_i(x_1, x_2)$ is the velocity of the vehicles on route i, and with $\varphi_{AB}=\varphi_1+\varphi_2$, (3.1) becomes:

$$\frac{\ell_1}{v_{\max}} \cdot \left(1+\tfrac{1}{2}\left(\frac{\ell_1\varphi_1}{x_1}\right)^2\right) = \frac{\ell_2}{v_{\max}} \cdot \left(1+\tfrac{1}{2}\left(\frac{\ell_2(\varphi_{AB}-\varphi_1)}{x_2}\right)^2\right)$$

$$\ell_1 + \tfrac{1}{2}\frac{\ell_1^{\ 3}}{x_1^{\ 2}}\varphi_1^{\ 2} = \ell_2 + \tfrac{1}{2}\frac{\ell_2^{\ 3}}{x_2^{\ 2}}\left(\varphi_{AB}^{\ 2} - 2\varphi_1\varphi_{AB} + \varphi_1^{\ 2}\right) \tag{3.2}$$

$$\left(\tfrac{1}{2}\frac{\ell_1^{\ 3}}{x_1^{\ 2}} - \tfrac{1}{2}\frac{\ell_2^{\ 3}}{x_2^{\ 2}}\right)\cdot\varphi_1^{\ 2} + \frac{\ell_2^{\ 3}}{x_2^{\ 2}}\varphi_{AB}\cdot\varphi_1 + \ell_1 - \ell_2 - \tfrac{1}{2}\frac{\ell_2^{\ 3}}{x_2^{\ 2}}\varphi_{AB}^{\ 2} = 0$$

Consider the introduction of auxiliary variables, in short notation:

$$c_1 = \varphi\cdot\frac{\ell_2^{\ 3}}{x_2^{\ 2}} \quad ; \quad c_2 = \tfrac{1}{2}\frac{\ell_1^{\ 3}}{x_1^{\ 2}} - \tfrac{1}{2}\frac{\ell_2^{\ 3}}{x_2^{\ 2}} \quad ; \quad c_3 = \ell_1 - \ell_2 - \tfrac{1}{2}\varphi^2\frac{\ell_2^{\ 3}}{x_2^{\ 2}}; \quad \varphi = \varphi_{AB}\,.$$

If $c_2=0$, the equality in (3.2) has possible solutions:

$$\begin{cases} \varphi_1 = 0, \text{if } -c_3/c_1 \le 0. \\ \varphi_1 = \varphi, \text{if } -c_3/c_1 > \varphi. \\ \varphi_1 = \dfrac{-c_3}{c_1} = \dfrac{\varphi^2\ell_2^{\ 3} + 2(\ell_2-\ell_1)x_2^{\ 2}}{2\varphi\ell_2^{\ 3}} = \tfrac{1}{2}\varphi + \dfrac{(\ell_2-\ell_1)x_2^{\ 2}}{\varphi\ell_2^{\ 3}} \text{ for all other.} \end{cases} \tag{3.3}$$

If $c_2\neq0$, the equality in (3.2) has possible solutions:

$$\varphi_1 = \frac{-c_1 \pm \sqrt{c_1^{\ 2} - 4c_2c_3}}{2c_2}\,. \tag{3.4a}$$

Since $0 \le \varphi_1 \le \varphi$ and from $c_2 > 0 \Rightarrow -c_1/2c_2 < 0$ and from $c_2 < 0 \Rightarrow -c_1/2c_2 > \varphi$, it can further be refined to:

$$\varphi_1 = \frac{-c_1 + \sqrt{c_1^{\ 2} - 4c_2c_3}}{2c_2}\,. \tag{3.4}$$

Taking care of the condition $0 \le \varphi_1 \le \varphi$, we can identify the special situations:

$$\text{If } c_2 > 0 \text{ and} \begin{cases} c_3 \geq 0 \Rightarrow \varphi_1 = 0; \varphi_2 = \varphi; \\ \dfrac{-c_1 + \sqrt{c_1^{\,2} - 4c_2c_3}}{2c_2} > \varphi \Rightarrow \varphi_1 = \varphi; \varphi_2 = 0; \end{cases}$$

$$\text{If } c_2 < 0 \text{ and} \begin{cases} c_3 \geq 0 \Rightarrow \varphi_1 = 0; \varphi_2 = \varphi; \\ 4c_2c_3 > c_1^{\,2} \Rightarrow \varphi_1 = \varphi; \varphi_2 = 0; \quad \textit{no solution, but due to continuity} \\ \dfrac{-c_1 + \sqrt{c_1^{\,2} - 4c_2c_3}}{2c_2} > \varphi \Rightarrow \varphi_1 = \varphi; \varphi_2 = 0. \end{cases} \tag{3.5}$$

For all other cases, the formula becomes: $\varphi = \dfrac{-c_1 + \sqrt{c_1^{\,2} - 4c_2c_3}}{2c_2}$. (3.6)

The lower bound of φ_1 is reached if and only if $c_3 \geq 0$ or $\ell_1 - \ell_2 - \frac{1}{2}\varphi^2 \ell_2^{\,3} / x_2^{\,2} \geq 0 \Rightarrow \varphi_1 = 0.$

Due to the symmetry between road 1 and road 2, the upper bound is reached if and only if $\ell_2 - \ell_1 - \frac{1}{2}\varphi^2 \ell_1^{\,3} / x_1^{\,2} \geq 0 \Rightarrow \varphi_1 = \varphi.$

Now that the traffic fluxes are known, it is possible to give an expression for the (energy use due to) fuel consumption $E(x_1, x_2, \varphi_1(x_1,x_2))$, expressed in MJ/h.

First, the fuel consumption E depends on the traffic flow and the fuel consumption per vehicle $e(v)$, expressed in MJ/vkm.

$$E(x_1,x_2,\varphi_1(x_1,x_2)) = \ell_1\varphi_1(x_1,x_2)\cdot e(v(\varphi_1(x_1,x_2),x_1) + \ell_2(\varphi_{AB} - \varphi_1(x_1,x_2))\cdot e(v(\varphi_{AB} - \varphi_1(x_1,x_2),x_2)$$

in which

$e(v) = \gamma_1 + \gamma_2 v + \gamma_3 v^2 + \gamma_4 / v$ or approximated by $e(v) = 3.338 - 0.0523v + 0.00038v^2 + 36.50/v$

The latter equations are (3.7) respectively (3.8).

The final equations for the fuel consumption $E(\mathbf{x})$ are:

If $\ell_1 - \ell_2 - \frac{1}{2}\varphi^2 \ell_2^{\,3} / x_2^{\,2} \geq 0$:

$$E(\mathbf{x}) = \ell_2\varphi\left(\gamma_1 + \gamma_2\left(\frac{v_{\max}}{1 + \frac{1}{2}(\varphi\ell_2/x_2)}\right) + \gamma_3\left(\frac{v_{\max}}{1 + \frac{1}{2}(\varphi\ell_2/x_2)}\right)^2 + \gamma_4\left(\frac{1 + \frac{1}{2}(\varphi\ell_2/x_2)}{v_{\max}}\right)\right); \tag{3.9a}$$

If $\ell_2 - \ell_1 - \frac{1}{2}\varphi^2 \ell_1^{\,3} / x_1^{\,2} \geq 0$:

$$E(\mathbf{x}) = \ell_1\varphi\left(\gamma_1 + \gamma_2\left(\frac{v_{\max}}{1 + \frac{1}{2}(\varphi\ell_1/x_1)}\right) + \gamma_3\left(\frac{v_{\max}}{1 + \frac{1}{2}(\varphi\ell_1/x_1)}\right)^2 + \gamma_4\left(\frac{1 + \frac{1}{2}(\varphi\ell_1/x_1)}{v_{\max}}\right)\right); \tag{3.9b}$$

For other cases, the even more elaborate formula for *E(**x**)* can be deduced by combining formulas (3.7) and (3.8) with (3.4).

3.3. Methodology

Consider the problem ($\overline{\mathrm{P}^{\text{network}}}$):

$$\begin{cases} \dot{x}_1 = u_1 - \delta_1 x_1; \\ \quad \dots \\ \dot{x}_n = u_n - \delta_n x_n; \end{cases}$$
with n the number of roads;
$$0 \le u_1(t) + \dots + u_n(t) \le u_{\max} \text{ for all } t \in [0,\infty);$$
$$x_i(0) = x_{i,0} \text{ for all } 1 \le i \le n;$$
$$\text{minimize } \overline{J} = \int_0^\infty e^{-\rho t} \left[E(x_1,\dots,x_n) + \sum_{i=1}^n \alpha_i u_i(t) \right] dt.$$

In vector notation, ($\overline{\mathrm{P}^{\text{network}}}$) becomes

$$\dot{\mathbf{x}}(t) = \mathbf{u}(t) - \delta \mathbf{x}(t);$$
$$0 \le \|\mathbf{u}(t)\| \le u_{\max} \text{ for all } t \in [0,\infty);$$
$$\mathbf{x}(0) = \mathbf{x}_0;$$
$$\text{minimize } \overline{J} = \int_0^\infty e^{-\rho t} [E(\mathbf{x}) + \boldsymbol{\alpha} \cdot \mathbf{u}(t)] dt.$$

Remembering the state equation $u_i(t) = \dot{x}_i(t) + \delta_i x_i(t)$, part of the functional can be rewritten as:

$$\int_0^\infty e^{-\rho t} u_i(t) dt = \int_0^\infty e^{-\rho t} \dot{x}_i(t) dt + \int_0^\infty e^{-\rho t} \delta_i x_i(t) dt$$
$$= \int_0^\infty \rho e^{-\rho t} x_i(t) dt + e^{-\rho t} x_i(t) \Big|_0^\infty + \int_0^\infty e^{-\rho t} \delta_i x_i(t) dt$$
$$= \int_0^\infty e^{-\rho t} (\rho + \delta_i) x_i(t) dt - x_i(0).$$

Hence, the functional $\overline{J}$ is equivalent to:

$$\text{minimize } J = \int_0^\infty e^{-\rho t} \left[E(\mathbf{x}(t)) + \sum_{i=1}^n \alpha_i (\rho + \delta_i) x_i(t) \right] dt. \tag{3.12}$$

The problem ($\overline{\mathrm{P}^{\text{network}}}$) is thus equivalent to problem ($\mathrm{P}^{\text{network}}$):

$$\frac{d}{dt}x_i(t) = u_i(t) - \delta_i x_i(t), \quad 0 \le \sum_{i=1}^{n} u_i(t) \le u_{\max};$$

$$x_i(0) = x_{i,0};$$

$$J(x,u) = \int_0^{\infty} e^{-\rho t}\left[\sum_{i=1}^{n}\alpha_i(\rho+\delta_i)x_i(t) + E(x(t))\right]dt \to \min.$$

Here δ_i>0, α_i>0, ρ>0, u_{max}>0 are parameters of the model; $x_{i,0} \ge 0$, $i = 1,2,\ldots,n$ are initial conditions; *E(x)* is continuously differentiable (smooth) positive function.

Due to the existence results proved by Balder [2], an optimal admissible pare $\left(x^*(t), u^*(t)\right)$ exists.

Obviously, all admissible trajectories of the control system are uniformly bounded: $\exists \kappa_0 > 0$ such that for an arbitrary admissible trajectory *x(t)*, we have

$\|x(t)\| \le \kappa_0$ for all $t \ge 0$.

This implies that the gradient $\left\|\frac{\partial}{\partial x}E(x(t))\right\|$ is uniformly bounded on all admissible trajectories: $\exists \kappa_1 > 0$ such that for an arbitrary admissible trajectory *x(t)*, we have $\left\|\frac{\partial}{\partial x}E(x(t))\right\| \le \kappa_1$ for all $t \ge 0$.

Further, the associated system of differential equations in variations is the following:

$$\frac{d}{dt}y_i(t) = -\delta_i y_i(t), \quad y_i(0) = 1.$$

Hence, $y_i(t) = e^{-\delta_i}t$ are uniformly bounded too.

All these mean that we can take λ=0 and as far as ρ>0, the assumption $\rho > (r+1)\lambda$ from Theorem 4 in [1] is satisfied. So, Theorem 4 is applicable to problem ($\mathrm{P}^{\text{network}}$). It implies to following result.

Theorem (Maximum Principle) *Let* $\left(x^*(t), u^*(t)\right)$ *be an optimal admissible pair in (*$P^{network}$*). Then pair* $\left(x^*(t), u^*(t)\right)$ *satisfies the conditions of the Pontryagin maximum principle together with the following current valued adjoint variable*

$$\psi_i(t) = -e^{(\rho+\delta_i)t}\int_t^{\infty} e^{-(\rho+\delta_i)s}\left[\alpha_i(\rho+\delta_i) + \frac{\partial}{\partial x_i}E(x^*(t))\right]dt, \tag{3.13}$$

i.e. the following conditions are true for this function $\psi(t)$*:*

a) $$\frac{d}{dt}\psi_i(t) = (\rho+\delta_i)\psi(t) + \alpha_i(\rho+\delta_i) + \frac{\partial}{\partial x_i}E(x^*(t)); \tag{3.14}$$

b) $$\sum_{i=1}^{n} u_i^*(t)\psi_i(t) = u_{\max}\cdot \max_{i=1,\ldots,n}\{0, \psi_i(t)\}. \tag{3.15}$$

Here we took into account that $z_i(t) = e^{\delta_i t}$, i=1,2,…,n in this case (see (5.4) and (5.5) in [1]).

As an easy corollary of (3.13), we have that for any i=1,…,n the following estimate takes place:

$$\begin{aligned}|\psi_i(t)| &\le \frac{\alpha_i(\rho+\delta_i)+\kappa_1}{\rho+\delta_i} \\ &\le \alpha_i + \frac{\kappa_1}{\rho+\delta_i} \text{ for all } t \ge 0.\end{aligned} \tag{3.16}$$

For a two road system, the control u is distributed according to $u_1\psi_1+u_2\psi_2\rightarrow$max. Chapter 4 operationalizes the methodology in a simple two-road network. As the methodology described does not offer a specific, unique distribution of construction efforts $u_1,\dots,u_n$, the next chapter suggests an alternative approach to determine the distribution.

4. Optimal development path

4.1. Single-road trial run

The optimal construction strategy is estimated by varying the starting values ψ_i^0 in order to reach the lowest value of *J*. In the initial condition, the capacity of the road is lower than optimal. Therefore, initial construction lowers the value of *J*. However, construction should cease as soon as the optimal capacity is reached. If construction continues afterwards, the value of *J* rises again. If no construction takes place at all, the function of *J* is steady at a high level.

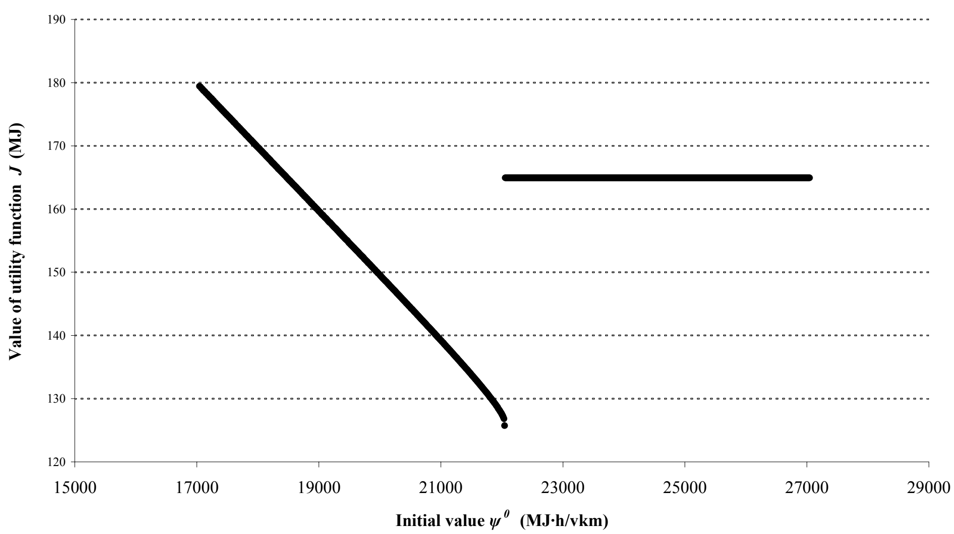


Figure 8. On the left hand side (ψ<approx. 22000), some construction takes place. On the right hand side, no construction takes place. The minimum value of J is located at the discontinuity.

Figure 8 shows the surface of the utility function as function of $\psi(0)$, one-road system (n=1) with ψ the so-called shadow price (in energy terms) of capacity increase. In the numerical example dt=1; u_{max}=1; δ=0.15/8766; ρ=(1/80)·8766; the end time T=10000, α=10000; x_0=2000; φ=10000; ℓ=1.

4.2. Dual road-system run

The same procedure as above is repeated for a two-road case. The system consists of two non-identical independent roads, both with initial capacity lower than optimum and constant traffic flows. Basically, it is the combination of two one-road system, with one exception: the control u has to be distributed over the roads in u_1 and u_2. See Figure 9.

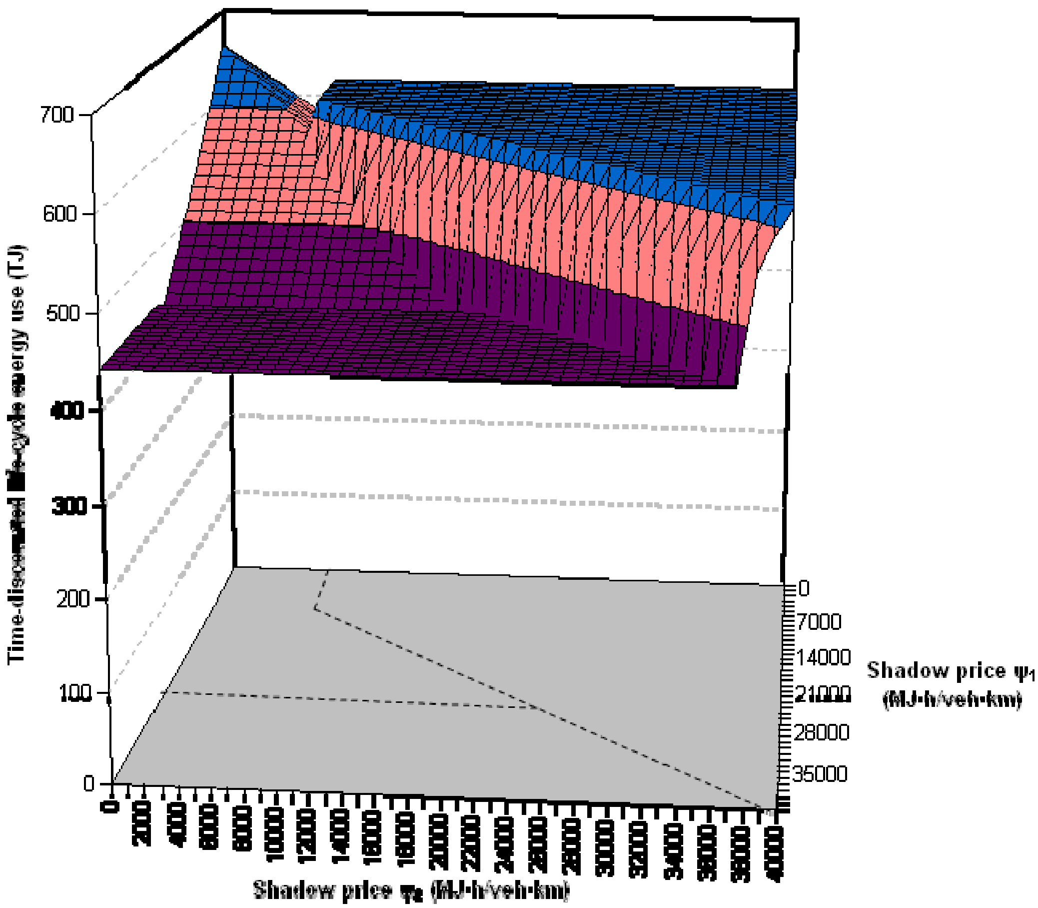


Figure 9. Several 'regimes' or 'construction strategies' are identifiable. On the left-hand side backside (low ψ_1 and lower ψ_2), there is continuous development of x_1. The flat surface on the right-hand side of the foreground (high ψ_2 and lower ψ_1), reflects the case of continuous development of x_2. For very low ψ_1 (foreground), the value of x_2 is balanced, without caring for road 1. The steepest line in the 'valley' around ψ_2=20000 reflects an optimal construction strategy for road 2, while optimality lacks for road 1.

Using the grid of Figure 9, it has shown to be difficult if not practically impossible to find the optimal construction strategy for the entire system. Problem is the discontinuity that is also present in Figure 8.

Figure 10 shows an initial run of a two-road system in which the traffic can change routes. The structure of the results seems to be comparable to that of system of two independent roads. For simplicity, the analysis is subjected to an independent two-road system.

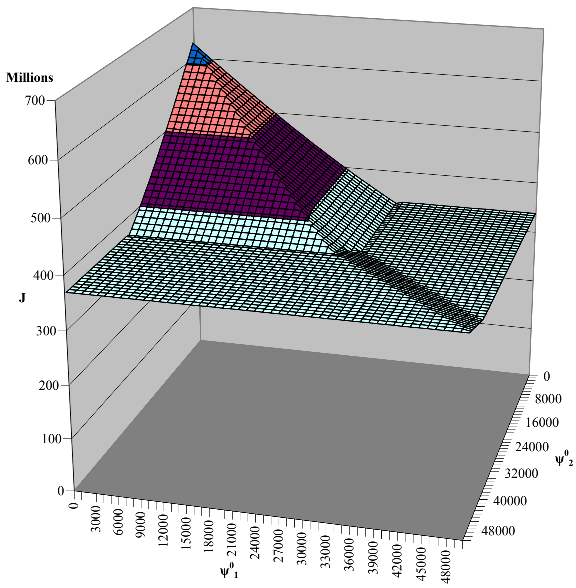


Figure 10. Surface of *g(x)* of a interdependent two-road system.

The 'grid'-procedure has also been repeated in the smoothened form:

$$\mathcal{H}(t,\mathbf{x},\mathbf{u},\boldsymbol{\psi}) = \sum_{i=1}^{n}(u_i - \delta_i x_i)\psi_i - e^{-\rho t}\left[\sum_{i=1}^{n}\alpha_i(\rho+\delta_i)x_i + E(\mathbf{x})\right] - A(u_1{}^2 + u_2{}^2)$$

Thus: $u_1\psi_1+u_2\psi_2+A(u_1^2+u_2^2)\rightarrow$max.

$$\text{minimize } J = \int_0^\infty e^{-\rho t}\left[E(\mathbf{x}(t)) + \sum_{i=1}^{n}\alpha_i(\rho+\delta_i)x_i(t)\right] + A(u_1{}^2 + u_2{}^2)dt\,.$$

It did not help in finding the optimal point within reasonable computing time, with A=100 and a gridsize of Δx=200. Therefore, an alternative smoothing, or regularization procedure, is sought.

4.3. Regularization

The controls $u_1(t),\ldots,u_n(t)$ have thus far been determined as:

$$u_1(t)\psi_1(t) + u_2(t)\psi_2(t) \to \max$$
$$0 \le u_1(t) + u_2(t) \le u_{\max}$$
$$u_1(t) = u_{\max};\, u_2(t) = 0 \text{ if } \psi_1(t) > \psi_2(t);$$
$$u_2(t) = u_{\max};\, u_1(t) = 0 \text{ if } \psi_2(t) > \psi_1(t);$$
$$u_i(t) \text{ is undetermined if } \psi_1(t) = \psi_2(t).$$

The regularization proposal is the replace these control rules with the individual controls $u_1(t)$ and $u_2(t)$ as following:

$$u_1(t) = \frac{\overline{u}}{2}(1 + \frac{\psi_1(t) - \psi_2(t) + \varepsilon^2(\psi_1(t) + \psi_2(t))}{\sqrt{(\psi_1(t) - \psi_2(t))^2 + \varepsilon^2(\psi_1(t) + \psi_2(t))^2}}; \qquad (4.1a)$$

$$u_2(t) = \frac{\overline{u}}{2}(1 + \frac{\psi_2(t) - \psi_1(t) + \varepsilon^2(\psi_1(t) + \psi_2(t))}{\sqrt{(\psi_1(t) - \psi_2(t))^2 + \varepsilon^2(\psi_1(t) + \psi_2(t))^2}}. \qquad (4.1b)$$

Remember the previous results of the problem under consideration: Figure 11 shows the value of the utility function J as function of starting values for ψ_1 and ψ_2. The lowest value in Figure 11 is 421 TJ, whereas the lower, flat plain on the bottom right of the graph is situated at a level of 441 TJ.

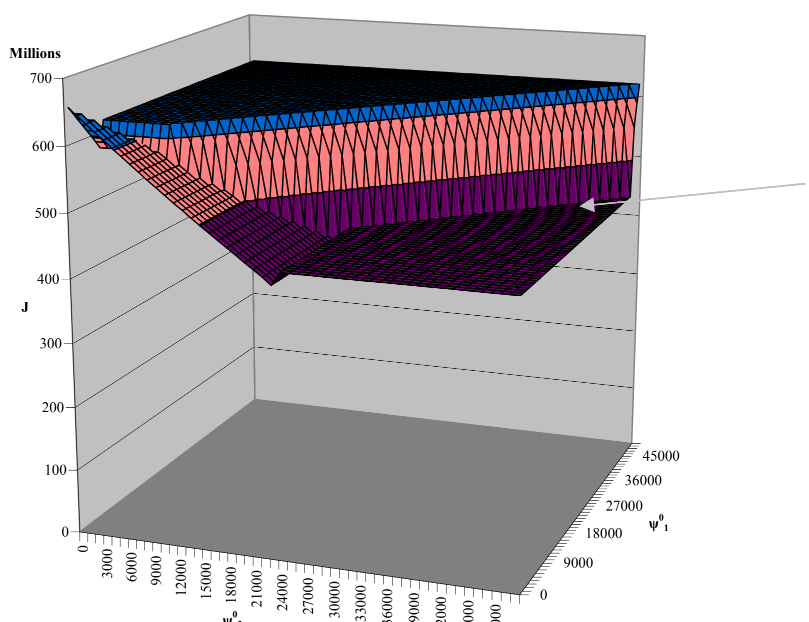


Figure 11. Utility surface without regularization.

Closer observation of the data represented in Figure 11 revealed that the minimum point is located somewhere along a line of $\psi_1^0 \approx c_1 \cdot \psi_2^0 + c_2$, with c_1 and c_2 some constants. This line is pointed to by the arrow in Figure 11. Now, using the regularization algorithm referred to above, we zoom in to that line in Figure 12.

The results using the smoothing algorithm show indeed the lowest values of the utility function along a straight line. Using a grid of $\Delta\psi$=20, the lowest value of the utility function approaches 370 TJ. After the behavior of the shadow prices has been known, it was possible to design an optimizing routine to find the best starting shadow price ψ_0 and (thus) shadow price trajectories. The results (with ε=0.01) are shown in Figure 13.

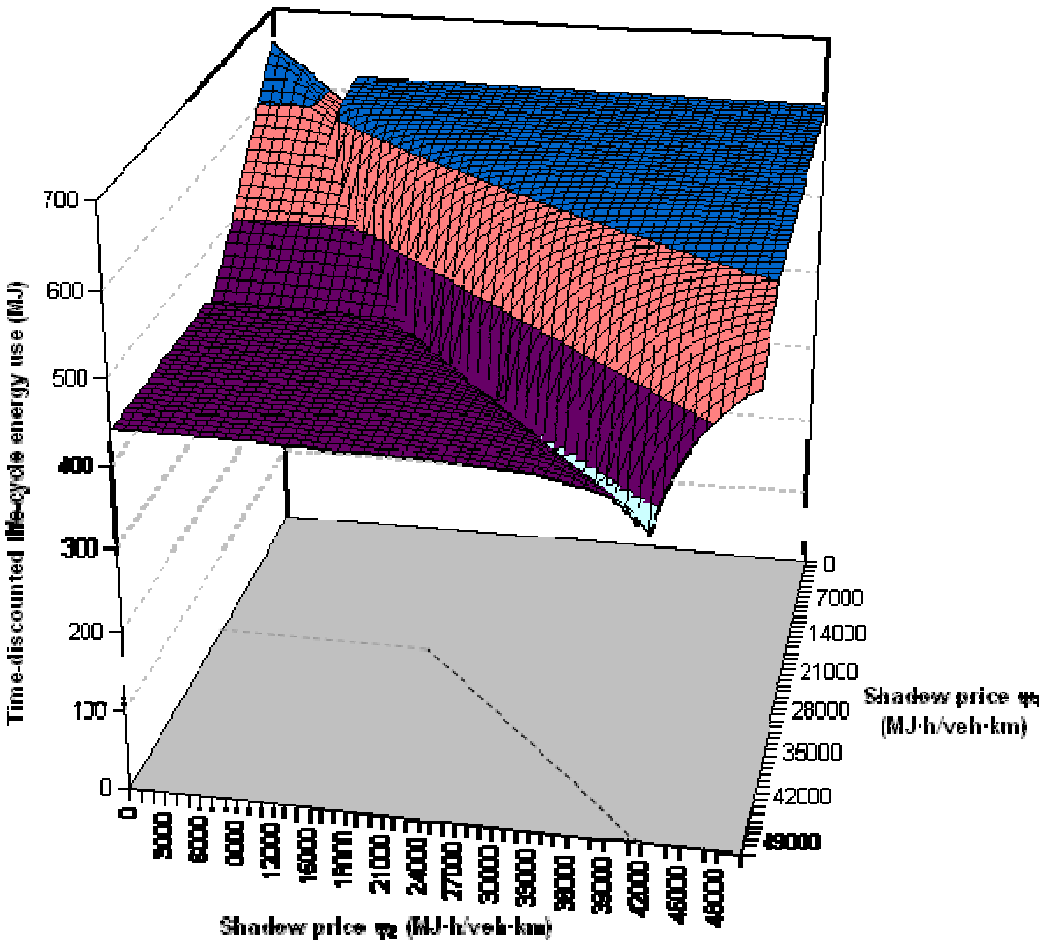


Figure 12. Results of the utility function with the smoothing algorithm.

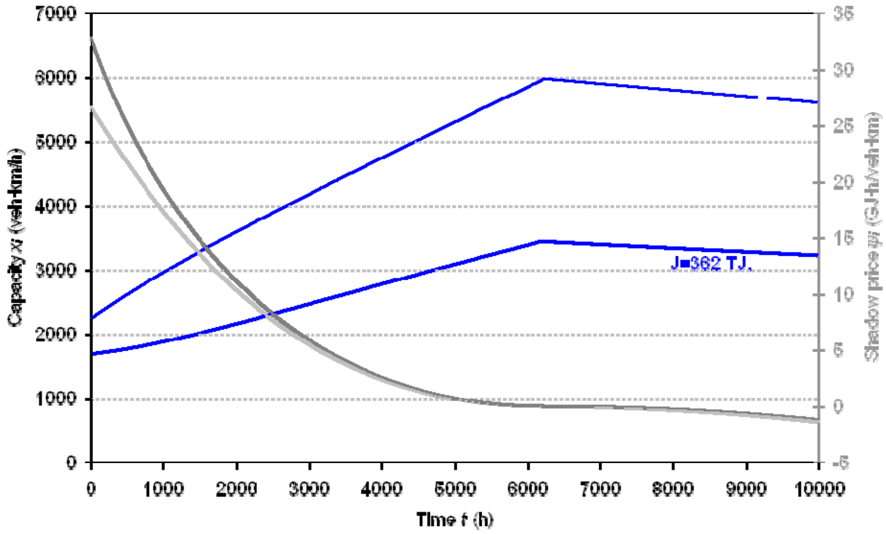


Figure 13. Best trajectories that have been found with the ε=0.01 regularization method.

4.4. Back casting

In order to determine the optimal control, and thus to determine the optimal capacities of the network in time, one may reconstruct the shadow prices function $\psi_i(t)$, back casting it from infinity to the initial configuration $\psi_{i,0}$.

4.4.1 Possible optimal end configurations

If we consider the functional in (3.12), the surface of $g(x_1,x_2)$ in (1.3) indicates the possible end states of the system. First, let us consider the basic functions for velocity *v* and per vehicle fuel consumption *e*. Figure 14 and Figure 15 lie at the roots of the construction of the surface of $g(x_1,x_2)$ and as such suggest the existence of a limited, if not one, local minima.

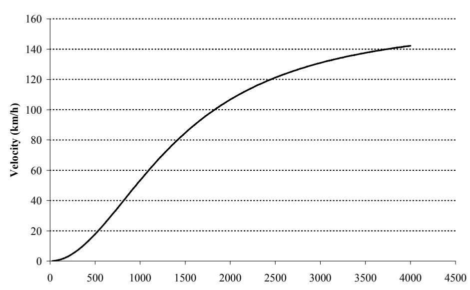


Figure 14. Velocity as function of the capacity.

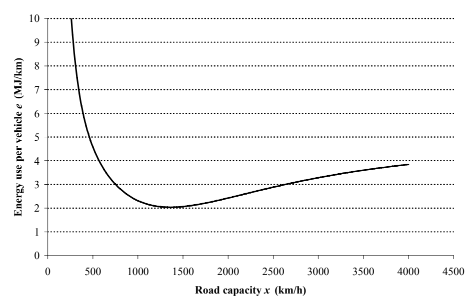


Figure 15. Per vehicle fuel consumption as function of the capacity.

The surface of *g* is given by (4.2):

$$g(x_1(t),x_2(t)) = E(x_1(t),x_2(t)) + \alpha_1(\rho+\delta_1)x_1(t) + \alpha_2(\rho+\delta_2)x_2(t) \tag{4.2}$$

Figure 16 shows the surface for $\alpha_1 = \alpha_2 = 10 \cdot 10^3$ MJ·h/vkm.

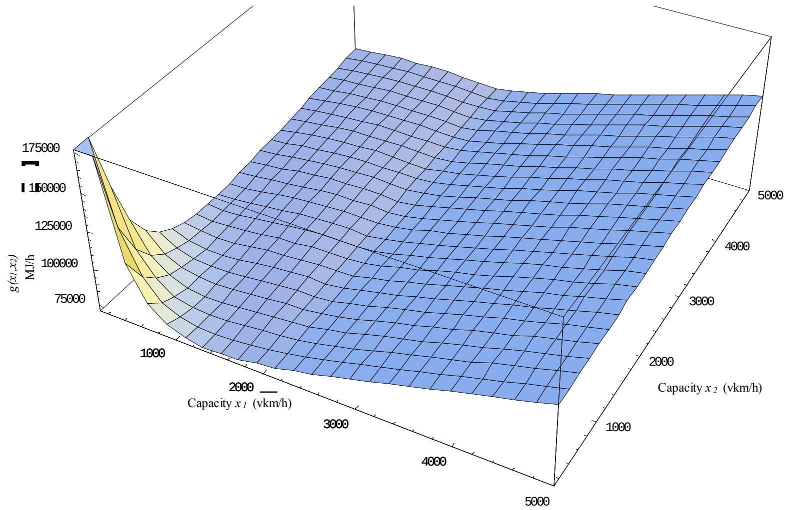


Figure 16. Current-time energy use of the transport system of figure 3, as function of road capacities x_1 and x_2, with φ_{AB}=2000 veh/h, α_1=80 GJ·h/vkm and α_2=10 GJ·h/vkm.

Figure 10 indicates that, depending on the values of α_i and δ_i, either road 1 or road 2 is used in the end state, and not both roads. Varying the parameters α_i shows that the minimum of current-time energy use lies either on the line x_1=0 or on the line x_2=0. If the surface is smooth, the location of the minimum of $g(x_1,x_2)$ coincides with the end states of the system at infinity in terms of x_i. Consider rewriting the utility function as following:

$$J = \int_0^\infty e^{-\rho t}\left(\alpha_1 x_1(t)(\rho+\delta_1) + \alpha_2 x_2(t)(\rho+\delta_2) + E(\mathbf{x}(t))\right)dt = \int_0^\infty e^{-\rho t} \cdot g(\mathbf{x}(t))dt. \tag{4.3}$$

The surface of g, in Figure 17, is a tilted version of the surface of E and Figure 16.

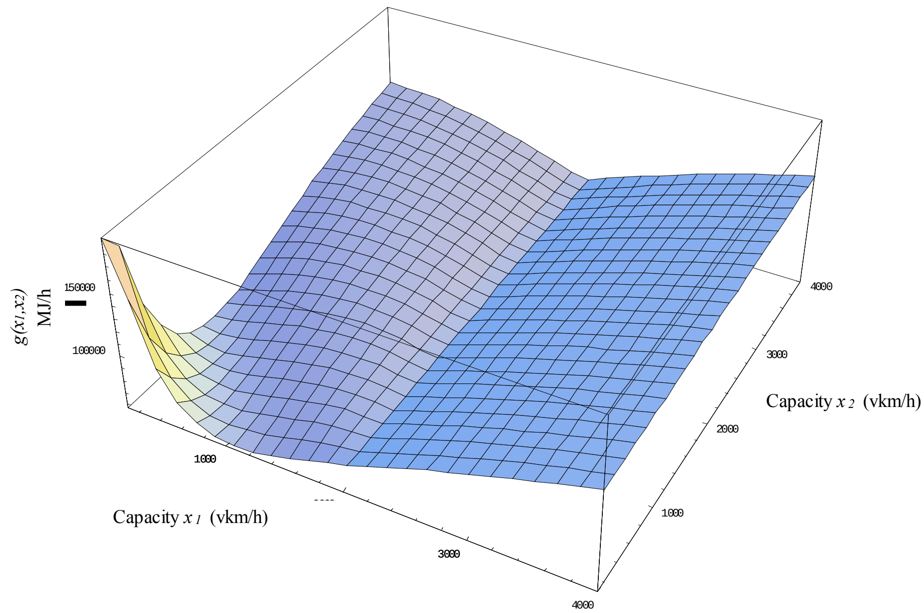


Figure 17. Current-time energy use g as function of the point capacities of road 1 and road 2, for α_1=80000 MJ·h/vkm, α_2=10000 MJ·h/vkm, ρ=80 yr^{-1} and δ_1= δ_2=1/0.15 yr^{-1}.

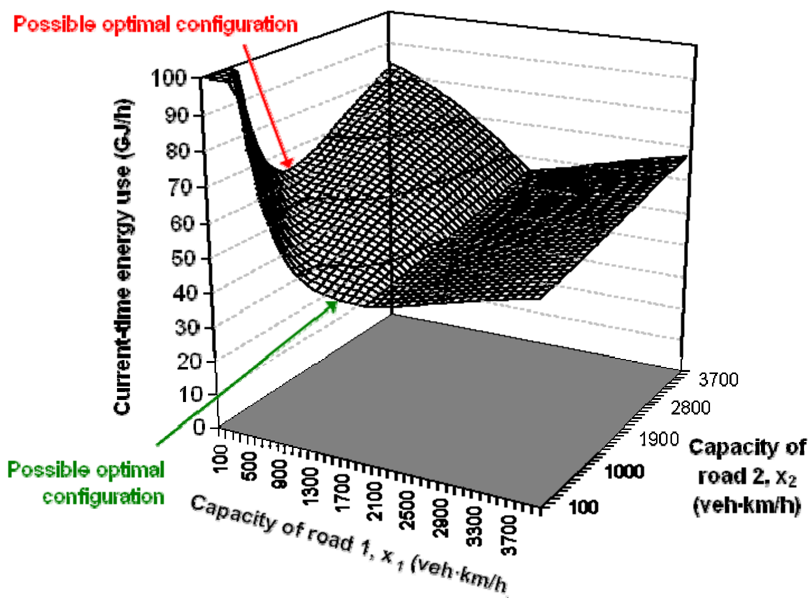


Figure 18. Optimal configuration lies at one of the axes.

The suggestion made on the basis of Figure 16 to Figure 18 implies that in the optimal configuration - if attainable - two concurrent routes do not coexist. The implies e.g. for the more elaborate network of Figure 6 that the possible end configurations can be narrowed down to the ones displayed in Figure 19.

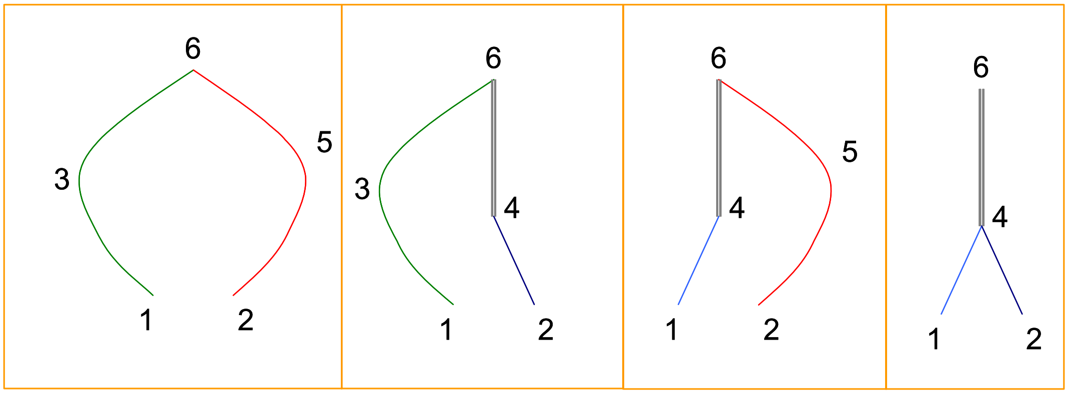


Figure 19. Possible end states of the network portrayed in Figure 6.

4.4.2 Indiscriminate behavior

For $n=1$, the gradient of the surface g can be expanded into series form:

$$-\frac{\partial g(x,u)}{\partial x}=\alpha(\rho+\delta)-\frac{c_4}{v_{\max}}\left(\tfrac{y}{x}\right)^3+\sum_{i=0}^{\infty}\left(\tfrac{y}{x}\right)^{2i+3}\left\{\left(\prod_{j=0}^{i}b_1(j)\right)c_2v_{\max}+b_2(j)c_3v_{\max}^2\right\}$$

$$\text{with } b_1(j)=\begin{cases}1 & \text{if } j=0\\ -\frac{2i-1}{i} & \text{if } j=1,3,5,...\\ -\frac{i-1}{2i} & \text{if } j=2,4,6,...\end{cases}$$

and with $b_2(j)$*=2,-3,3,-2 1/2, 1 7/8, -1 5/16, 7/8, -9/16, ... for j=0,....*

$$\text{or: } \frac{\partial g(x,u)}{\partial x}=-\alpha(\rho+\delta)-\sum_{i=0}^{\infty}B_i\left(\frac{\varphi\ell}{x}\right)^{2i+3}. \tag{4.4}$$

The formula for the shadow prices (3.13) can thus be expressed in terms of the i/c-ratio y/x. In the end state, wherein the traffic fluxes are fixed, the i/c-ratio's are such that the shadow prices are equal. If the physical characteristics of the roads are equal in the sense that the road wear δ_i and the energy intensity of construction works α_i are identical for all roads, the i/c-ratios are equal as well.

In the optimal path, the condition that $\mathbf{u}^*(t)\cdot\boldsymbol{\psi}^*(t)$ is maximal holds for all t. The partial derivative becomes constant: $\frac{\partial g(\mathbf{x}(t))}{\partial x_i(t)}\to 0$ for $t\to\infty$.

From (3.14) and $\frac{\partial g(\mathbf{x}(t))}{\partial x_i(t)}\le 0$ for all $t\in[0,\infty)$, it is clear that $\psi(t)$ is a declining function. It implies the optimal control during the transition is distributed in such a way that all $\psi_i(t)$ are equal in the end state (or $\psi_i(t)$=0). In other words: a road is either not used, or used with certain i/c-ratio that is equal for all remaining roads in use.

Consider now that a fixed i/c-ratio means that the ratio $\varphi(t)\ell/x(t)$ is fixed. If this ratio is to remain equal for all roads during the transition process, and considering the state equation, it follows that the i/c-ratio holds the key for distributing the control efforts. The maximum control is to be applied to the road with the highest shadow price. If more roads have similar high shadow prices, the control should be distributed among them on the basis of length to ensure the equality of the i/c-ratio. Given the initial system $\mathbf{x_0}$, roads also have a minimum capacity (see state equation) of $x_i(t) \geq x_{i,0}(t) \cdot e^{-\delta t}$. In back casting, this minimum should not be exceeded.

4.4.3 Finding the initial position for backcasting

Consider the function ψ_i:

$$\dot{\psi}_i(t) = (\rho + \delta_i)\psi_i(t) + \alpha_i(\rho + \delta_i) + \partial E(\mathbf{x})/\partial x_i \ .$$

It implies an end state of $\psi_i = 0; \quad \partial E(\mathbf{x})/\partial x_i = -\alpha_i(\rho + \delta_i)$. For a one-dimensional system, the function $\partial E(x)/\partial x$ is convex, see Figure 20 to Figure 22 below.

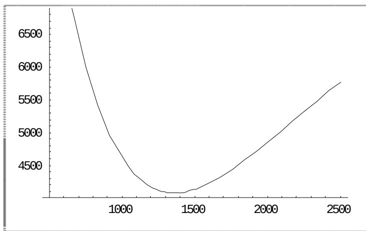


Figure 20. General form of $E(x)$

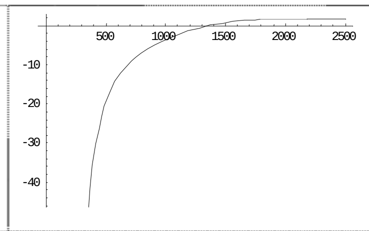


Figure 21. General form of $\partial E(x)/\partial x$

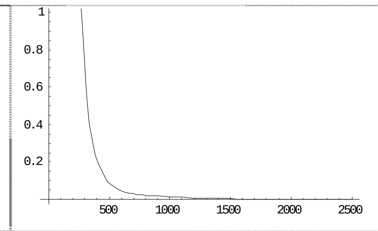


Figure 22. General form of $\dfrac{\partial^2 E(x)}{\partial x^2}$

If $\alpha_i(\rho + \delta_i) + \partial E(\mathbf{x})/\partial x_i$ is convex and negative for $x(t)<x(T)$, and if $\psi_i(T) = 0$, then $\psi(t)$ is declining.

The construction activities are distributed using the rule: $\sum_i u_i \psi_i \to \max$;

Suppose now two roads exist, *i* and *j*, with equal $\psi_i(t)=\psi_j(t)$. Assuming the convexity of $\partial E/\partial x$, the construction efforts $u_i+u_j=u_{max}$ should be distributed over both roads, such that:

$$\alpha_i(\rho + \delta_i) + \partial E(x_i + u_i, x_j + u_j)/\partial x_i = \alpha_j(\rho + \delta_j) + \partial E(x_i + u_i, x_j + u_j)/\partial x_j \ .$$

If construction does not influence traffic flows φ_i, φ_j, thus the roads are independent, it is:

$$\alpha_i(\rho + \delta_i) + \partial E(x_i + u_i, x_j)/\partial x_i = \alpha_j(\rho + \delta_j) + \partial E(x_i, x_j + u_j)/\partial x_j \ .$$

Suppose ψ is concave, any other allocation of construction efforts would lead to shadow prices that are higher than those of the optimal path:

$$\psi_1(t + dt) + \psi_2(t + dt) > \psi_1^*(t + dt) + \psi_2^*(t + dt) \ .$$

At the end time *T*, all roads that are still in use (φ_i>0) and that have been expanded ($x_i(T)>x^0_i \cdot e^{-\delta T}$), must have experienced recent expansion: $u_i(T\text{-}dt)>0$. This again is a result of the declining nature of ψ: The construction efforts u_i leading to only a marginal decrease in ψ: $-\dot{\psi}_i(t) \leq \varepsilon$, with ε some small positive number, are only 'lucrative' at the very last stages of the construction period.

Therefore, if $\dot{\psi}_i(t) < 0$ *for all* $t \in [0,T)$ *and* $\ddot{\psi}_i(T-dt) > 0$ *for all i* then it is possible to back cast, using initial direction for ψ as described above, and the general description for ψ as in (3.14).

4.4.4 System behavior and end state determination

Consider a system of *n* roads. Each road has, at time *t*, a capacity x_i(t), a construction effort u_i(t) and the marginal net energy benefit of capacity increase ψ_i(t). The total construction effort should be allocated over the *n* roads and in time $t \in [0,\infty)$ such that:

$$J = \int_0^\infty e^{-\rho t}\left[E(\mathbf{x}(t)) + \sum_{i=1}^n \alpha_i(\rho+\delta_i)x_i(t)\right]dt \rightarrow \min. \qquad (4.5)$$

The following rules apply:

$$\begin{aligned} &\dot{x}_i(t) = u_i(t) - \delta_i x_i(t); \\ &\sum_{i=1}^n u_i^*(t)\psi_i(t) = u_{\max} \cdot \max_{1\leq i\leq n}\{0, \psi_i(t)\}. \end{aligned} \qquad (4.6)$$

If the *n*-dimensional surface of the part of (4.5) between brackets - or the function $g(\mathbf{x}(t))$ - is smooth, with a single, thus global minimum, the latter minimum defines the optimal end state of the road network. As $x_i(t)\rightarrow 0$ with $x_{i,0}>0$ is only possible if $t\rightarrow\infty$, the optimal end state might not be reached in finite time. The marginal net energy benefit is given by:

$$\begin{aligned} &\psi_i(t) = -e^{(\rho+\delta_i)t}\int_t^\infty e^{-(\rho+\delta_i)s}\left[\alpha_i(\rho+\delta_i) + \frac{\partial E(\mathbf{x}^*(s))}{\partial x_i}\right]ds; \\ &\tfrac{d}{dt}\psi_i(t) = (\rho+\delta_i)\psi(t) + \alpha_i(\rho+\delta_i) + \frac{\partial E(\mathbf{x}^*(t))}{\partial x_i}. \end{aligned} \qquad (4.7)$$

Assumingly, the function $\psi_i(t)$ is convex in the relevant domain.

The end state characteristics of road *i* can be categorized into:

$$x_i(T) > 0 \text{ and } \varphi_i(T) > 0 \Rightarrow \begin{cases} \dfrac{\partial E(\mathbf{x}^*(T))}{\partial x_i} = -\alpha_i(\rho+\delta_i); \\ \psi_i(T) = 0; \end{cases}$$

$$x_i(T) \rightarrow 0 \text{ and } \varphi_i(T) = 0 \Rightarrow \begin{cases} \dfrac{\partial E(\mathbf{x}^*(T))}{\partial x_i} = 0; \\ \psi_i(T) = -\alpha_i; \text{(cannot be reached)} \end{cases}$$

$$x_i(T) \rightarrow 0 \text{ and } \varphi_i(T) \rightarrow 0 \Rightarrow \left\{ \dfrac{\partial E(\mathbf{x}^*(T))}{\partial x_i} < 0 \text{ (but bounded).} \right.$$

$$\left(\lim_{x_1 \to 0} E(\mathbf{x}(t)) = \varphi \ell_2 (a_1 + \frac{a_4}{v_{\max}} + \frac{\varphi^2 a_4 \ell_2^{\;2}}{2 v_{\max} x_2^{\;2}} + \frac{a_3 v_{\max}^{\;\;2}}{\left(1 + \frac{\varphi^2 \ell_2^{\;2}}{2 x_2^{\;2}}\right)^2} + \frac{a_2 v_{\max}}{1 + \frac{\varphi^2 \ell_2^{\;2}}{2 x_2^{\;2}}}) \right)$$ for a two road network.

Furthermore, it is true that the i/c-ratios are constant in the end time T:

$\frac{\varphi_i(T)\ell_i}{x_i(T)} = \frac{\varphi_j(T)\ell_j}{x_j(T)}$ for all $1 \le i \le n; 1 \le j \le n;$ if $x_i(T) > 0; x_j(T) > 0; \alpha_i = \alpha_j; \delta_i = \delta_j$.

4.4.5 Demarcation of attainability domain

To determine the time T at which the end state is reached, one first assesses the time that is required to complete the construction works. For a network, the amount of initial capacity x^0 that is worn down is: ${}^{\delta}x(T) = \sum_{i=1}^{n} x_i^0 (1 - e^{-\delta T})$;

The amount of worn down newly constructed capacity, supposing continuous and maximal construction efforts u_{max} is: ${}^{\delta}u(T) = u_{\max} \int_0^T 1 - e^{-\delta(T-t)} dt = u_{\max} \left(T + \frac{-1 + e^{-\delta T}}{\delta} \right)$.

The total time can therefore be computed with the equality ($T<\infty$):

$$T = \frac{\sum_{i=1}^{n} (x_i(T) - x_i^0) + {}^{\delta}x(T) + {}^{\delta}u(T)}{u_{\max}} \Leftrightarrow T = \frac{\sum_{i=1}^{n} (x_i(T) - x_i^0(T) e^{-\delta T})}{u_{\max}} + T + \frac{-1 + e^{-\delta T}}{\delta}.$$

The time needed for the construction works to be completed, taking the instantaneous road wear into account, is:

$$T_{\text{infrastructure}} = \frac{1}{\delta} Ln \left(\frac{u_{\max} - \delta \sum_{i=1}^{n} x_i^0}{u_{\max} - \delta \sum_{i}^{n} x_i(T)} \right). \qquad (4.8)$$

The behavior of ψ proposed ensure that for all roads *i,* the inequality $x_i(T) \ge x^0{}_i \cdot e^{-\delta T}$ holds for all $T\ge 0$. Equation (4.8) is necessary to guarantee that $x_i(T)_{T=0} = x^0{}_i \cdot e^{-\delta T} \Leftrightarrow x_i(0) = x^0{}_i$ for all i. And thus, the back casting procedure ensures that the original configuration is reached at T=0.

Furthermore, at time $T_{traffic}$, the roads to be abandoned should no longer be in use: $\varphi_i(T_{traffic})=0$ if $x_i(\text{T}) \to 0$. The actual end time T is now determined by the highest of the latter Ts: $T=\text{Max}(T_{infrastructure}, T_{traffic})$. If $T_{traffic}>T_{infrastructure}$, it implies that the residual capacities of the roads to be abandoned are still influencing the traffic flows over the network when the construction works are completed as fast as possible. It means that the optimal end state of the network at $t \rightarrow \infty$ is outside the attainability domain at $t=T$. Figure 23 illustrates the latter reasoning.

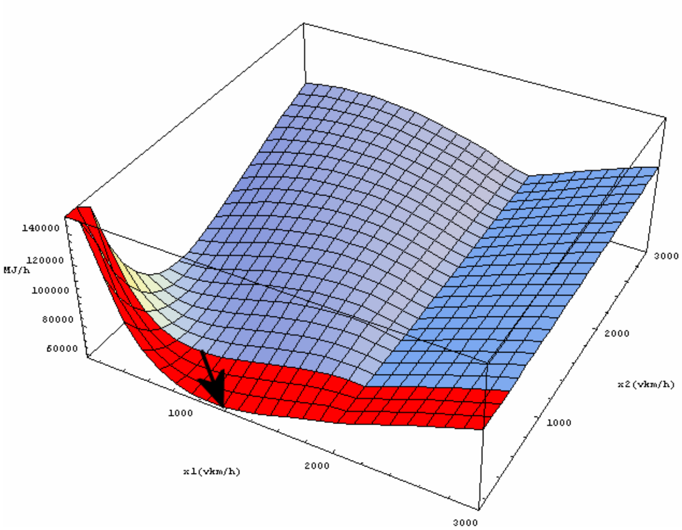


Figure 23. The shadow (red) surface of point capacity x_2/ℓ_2<600 vkm/h shows the domain that is unreachable at a certain moment τ. The arrow indicates the behavior of the system from τ until the final time T. The capacity of road1 x_1 should be increased slowly, to match the reduction of road capacity on road2 x_2 due to road wear. Therefore, between τ and T the construction activities are larger than zero, but might be less than the maximum construction effort: $0 < u(t) \leq u_{max}$ if $\tau \leq t \leq T$.

4.4.6 Distributing the construction works over the roads

The system has a certain inflexibility towards deliberate reduction of capacity. Consider now the previously described 2-road-network. Deducible from Figure 23, Figure 24 plots the location of the minimum of the total energy use, or $\frac{\partial g(x_1, x_2)}{\partial x_1} = 0$ as function of the capacity of road2 x_2.

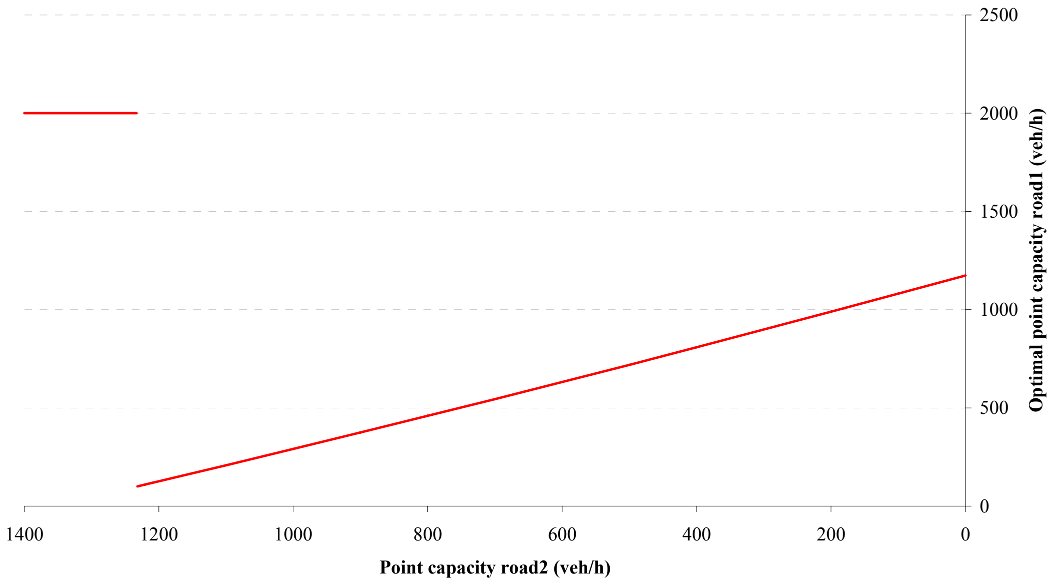


Figure 24. Optimal capacity of road1 as function of a given capacity of road2. See also Figure 23.

For a large initial capacity x_2 (x_2/ℓ_2>1232 veh/h), the lowest energy use in the attainable domain is reached at x_1/ℓ_1=2000 veh/h. As road wear on road 2 takes its toll, the point of lowest energy use jumps suddenly to x_1/ℓ_1=100 veh/h. The system cannot follow this sudden jump. Therefore, we should only consider networks in which the roads to be used should be upgraded. In the following reasoning, roads that need downgrading are removed from the control system:

$$x_i(T) < x_i^0 \Rightarrow u_i(t) = 0 \text{ for all } t \in [0,T].$$

In the period from t=$T_{traffic}$ down to t=$T_{infrastructure}$, the construction activities are allocated such that the system remains in the configuration of the lowest total energy use within the attainable domain at time t. From t=$T_{infrastructure}$ down to t=0, the maximum construction effort u_{max} is applied at all times t.

Consider now the behavior of the marginal energy benefit curves:

$$\dot{\psi}_i(t) = (\rho+\delta_i)\psi_i(t) + \alpha_i(\rho+\delta_i) + \frac{\partial E(\mathbf{x}(t))}{\partial x_i} \text{ or}$$

$$\dot{\psi}_i(t) = (\rho+\delta_i)\psi_i(t) - g(x(t)).$$

The function $\psi_i(t)$ is convex within certain limits of x(t), especially when $x(T)>x^0$. In a numerical approximation, the back casting of this function to t=T-Δt might occur through[11]:

$$\frac{\psi_i(T)-\psi_i(T-\Delta t)}{\Delta t} \approx (\rho+\delta_i)\cdot\psi_i(T-\Delta t) + f(x_i(T-\Delta t))$$

$$\psi_i(T-\Delta t) \approx -\frac{1}{\Delta t(\rho+\delta_i)}\left(\psi_i(T) - \Delta t\cdot\left\{\alpha_i(\rho+\delta_i) + \frac{\partial}{\partial x_i}E(x_i(T-\Delta t))\right\}\right).$$

---

[11] The examples shown in this report use a Runga-Kutta approximation.

The construction works $u_i(t)$ now take place at the road with the lowest marginal energy benefit, due to the back casting. These construction works imply that the capacity at $t=T$ is larger than at $t=T-\Delta t$, thus that the i/c-ratio is larger. A largest i/c-ratio generally implies larger net energy benefits. It also leads, through possible redistribution of traffic flows, to larger traffic flows on other roads, therefore also larger i/c-ratio's and larger net energy benefits on the other roads. Finally, the all time largest net energy benefit will occur on the road that is upgraded at time t=0, according with the theory.

Due to the convexity of ψ, the construction effort be distributed over roads with equal ψ such, that $\alpha_i(\rho+\delta_i)+\frac{\partial}{\partial x_i}E(x_i(T)-u_i(T-\Delta t))$ is equal for all roads considered. This uniform behavior continues until a road reaches the limit of its attainable domain, or: $x_i(t)\le x_i^0\cdot e^{-\delta_i t}$. From this point backwards, no construction activities are allocated to that road.

4.5 Results

The backward-procedure has also been implemented for the two road system. In that case, it is assumed that ψ^T=0 for both roads, and that the values of ψ_i are kept identical (i.e. $\psi_1(t)=\psi_2(t)$) as long as possible, thus as long as $x_i(t)>x^0_i(t)\cdot e^{-\delta t}$. The result is that initial values of ψ^0_1=28907.21 and ψ^0_2=31338.33 give a final value of the functional of J=355·10^6 MJ.

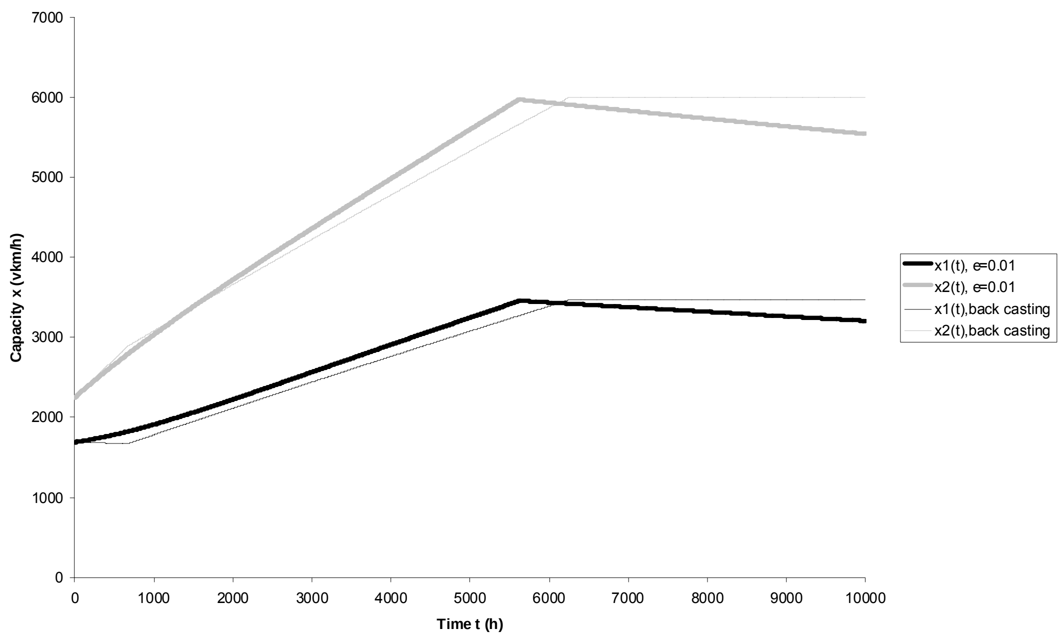


Figure 25. Comparison of the best 'smoothing' result with the time-backward method.

Finally, Figure 25 shows the comparison. The best 'smoothing' result has a value for the utility function of 351 TJ, whereas the value of the time backward method was 355 TJ. However, Figure 25 seems to indicate that this difference is a result of a slightly higher control effort. The modified time-forwards algorithm seems to allow for a slightly higher total construction efforts than is allowed (u_1+u_2>u_{max}). The net result is that the optimal network configuration is sooner reached. A better comparison would therefore be with the best result of the grid approach of section 4.2 and Figure 11. For this comparison, see Figure 26.

In conclusion, figure 4 does not seem to contradict with a time-backward algorithm. The modified time-forward algorithm seems suitable in further study.

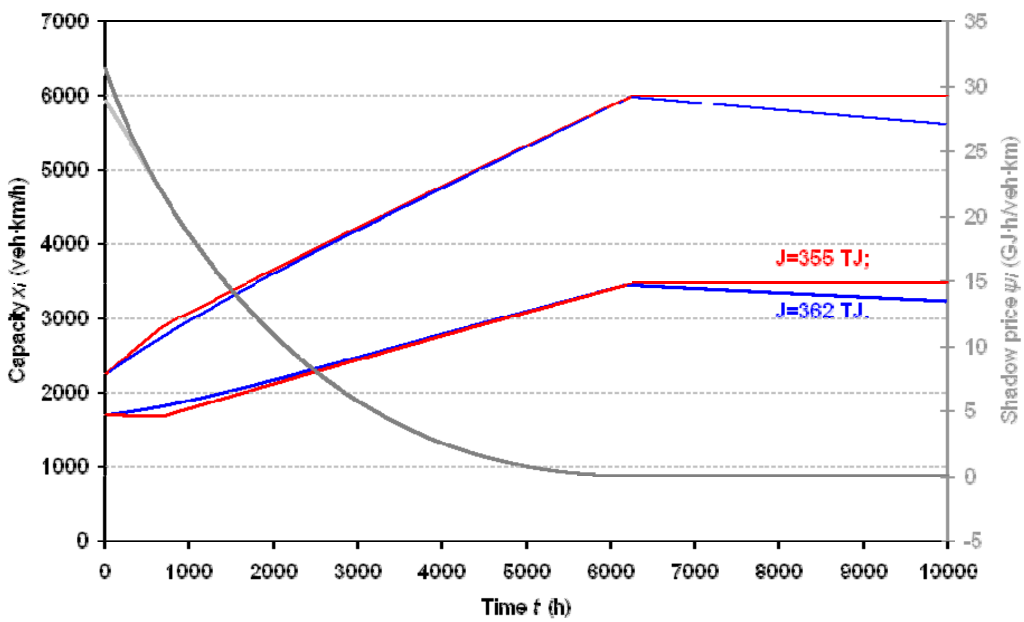


Figure 26. Comparison of back casting and forecasting methods.

5. Conclusion and discussion

5.1. Methodological conclusion

Grand road construction schemes can be devised using optimal control methodology. The model fabricated to demonstrate an application of the latter cannot be used for specific road projects. The optimal control trajectory can be tracked down by the following procedure. First, one should identify the possible and reachable end states. If the lowest value of the integrand of the utility function is within the attainability domain, the latter value coincides with the optimal end state. Second, one should determine the time that is needed to transform the original network to the desired network. Third, one should while backcasting, keep the marginal net energy benefit of all links in the network equal as long as possible; finally, one should do so until the boundary of the attainability domain is reached. From that point backward, the system will move along the edges of the attainability domain.

The report shows that the procedure described provides a trajectory with a very low value for the utility function. However, the report does not prove that this good trajectory is also the optimal trajectory. Furthermore, several assumptions have been made, most importantly that the optimal end state can be reached in finite time and that the road network should be build up and should not be worn down.

The real life equivalents of the described mathematical procedure are that policy makers should be proactive in designing transport networks, should one want to use the infrastructure to minimize the energy use of transport (or the maximize the socio-economic benefits of transport).

5.2. Transport infrastructure change and the slowing of global warming

Generally, construction and maintenance of road infrastructure play a limited role in an attempt to minimize the energy use of road transport. However, if one wants to enhance the capacity of a road connection to eliminate traffic jams that occur only during a limited period of time, the relevance of the infrastructure related energy use increases.

Given common traffic intensity distributions, traffic jams for 15% of the time have been shown to be energetically optimal. Traffic jams for 5% of the time are economically optimal, which adequately reflects current Dutch policy practice.

Policy measures that aim the reduce CO_2 emissions by modifying transport infrastructure are not economically sound, as they are relatively expensive.

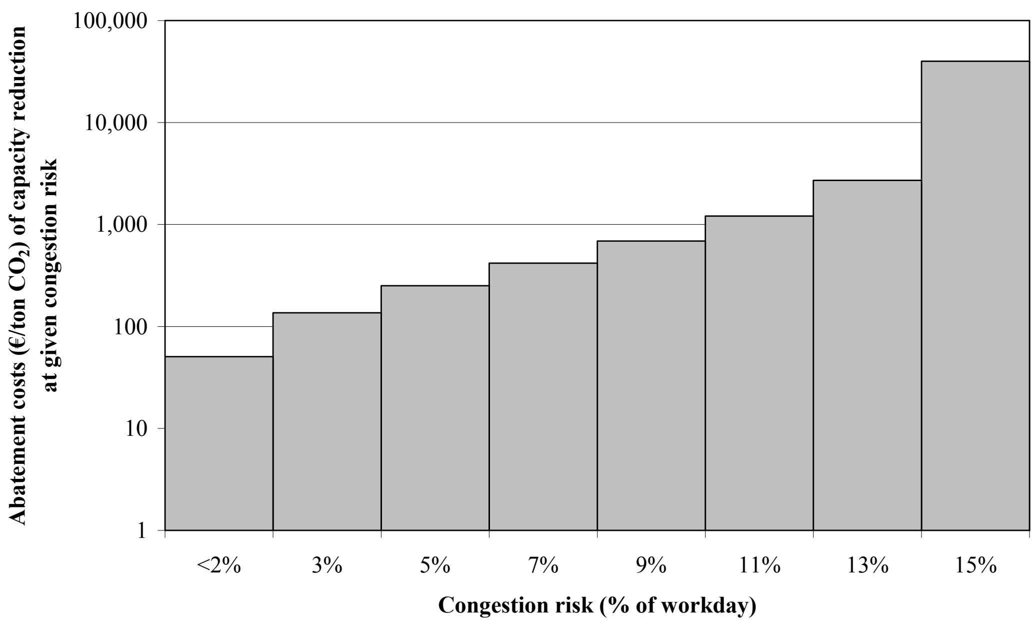


Figure 27. CO_2 abatement costs as function of congestion risk. The figure implies that letting traffic jam occurences increase exist is not an economically-efficient approach to reduce CO_2 emissions.

One should keep the economic importance of infrastructure in mind in interpreting Figure 27. It is generally assumed that other economic sector can reduce CO_2 emissions are costs below 100 €/ton CO_2. From an environmental and economic perspective, one can expand the road capacity to (near) economic optimal values, *provided that* the increased CO_2 emissions are compensated in other economic sectors.

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