OPTIMIZATION MODELS IN HEALTH CARE SYSTEM PLANNING

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Preface

The aim of the IIASA Modeling Health Care System Task is to build a National Health Care System model and apply it in collaboration with national research centers as an aid to health service planners. The modeling work is proceeding along the lines proposed in earlier papers. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource supply, and resource allocation.

This paper is a joint product of the Human Settlements and Services and the System and Decision Sciences Areas of IIASA. Its purpose is to discuss the advantages and limitations of dynamic linear programming as applied to health care system planning. The approach is illustrated by formulating a model for Health Care Resource planning, in particular, manpower planning. Such a model could serve as a resource supply submodel in the linked system of submodels mentioned above.

Recent related publications of the IIASA Modeling Health Care Systems Task are listed on the back pages of this paper. Other IIASA publications on Dynamic Linear Programming are also listed in the same place.

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Abstract

This paper discusses the possibilities of applying the dynamic linear programming (DLP) approach to different optimization problems of health care modeling. A basic manpower supply model is considered, as well as its modifications, including a three-level manpower planning model, a model of educational system development, and a model for improving professional skills. The other models described in this paper are connected with the planning of different disease control and prevention programs or with national health care system planning. The variety of problems that can be written in the DLP format, shows that this approach might be an efficient tool for the elaboration and implementation of optimal policies in health care systems.

Contents

Optimization Models in Health Care System Planning

1. INTRODUCTION

The health care system is a large and complex dynamic system consisting of a set of interrelated subsystems that are closely related to external systems (population, socio-economic systems etc.) [1,2].

In investigating such a complex system different techniques can be used [1-6]. They can be classified into three types: macro-econometric, simulation and optimization [3].

Optimization methods, particularly linear programming (LP), have proved to be valuable tools in decision making, especially when a very large number of variables and constraints must be accounted for in an optimal way [7,8].

But now it is becoming difficult to make a decision without taking into account the consequences of this decision for some specific time period. For example, a decision to increase the enrollment of students in a particular speciality will cause an increase in the number of such specialists only after some definite period of time; the building of a large hospital in a region can be implemented only after one period of time and the improvement in quality of health care in this region can be noticeable only after another period of time. Thus the majority of optimization problems nowadays become multistage, dynamic ones.

Dynamic linear programming (DLP) may be considered as a new stage of LP development and is aimed at the elaboration of methods for optimal planning in different large-scale systems [8].

It should be noted that in some cases the direct application of deterministic methods for single health care activities may seem to be embarrassing because of the stochastic nature of some of these activities associated with the illness or recovery processes. However, consideration of health care systems on a large-scale basis allows us to operate with mean values and thus bypass these difficulties.

The purpose of this paper is to discuss the possibilities of the DLP approach to planning problems in different health care systems. Typical problems here are connected with manpower planning, planning of programs for prevention and control of diseases, or the long-range allocation of resources for total care.

In the first section some manpower supply models are considered. These models can be used for planning enrollment to different types of educational systems in order to yield the required labor force stock for a given planning horizon. The other group of models is connected with optimal planning of different disease control and prevention programs at national or regional levels.

2. HEALTH MANPOWER PLANNING MODELS

Manpower planning is one of the most important problems in health care systems [9-12]. Absolute or relative increases in the quantity of health manpower will not determine the quality of the health care system, and therefore sophisticated planning of different categories of health care specialists is needed.

There is a large variety of manpower planning problems on different levels. Here we shall consider several types of manpower supply models at the national (or regional) level. That is, a plan for socio-economic development of a country (or region) implies a certain manpower demand structure, which in turn can be translated into an optimal structure of outputs from the educational system.

As a rule the training of health care personnel requires many years, and in addition it is necessary for most medical workers to specialize. Thus the manpower planning problems of health personnel are both large-scale and long-term in nature.

The most effective technique now available for dealing with large-scale dynamic problems is linear programming, in particular dynamic linear programming [7,8]. Hence manpower models will be formulated here in the framework of dynamic linear programming (DLP).

We shall first consider the simple aggregate model.

2.1 The Simple Model

In formulating DLP models it is useful to single out and consider separately [8] :

1. *State equations* of the system with the distinct separation of *state* and *control* variables;
2. *Constraints* imposed on these variables;
3. *Planning period*, T - the number of steps during which the system is considered and the *length* of each step t; and
4. *Performance index* (objective function) which quantifies the quality of a plan (control).

State Equations The general scheme of the simple model considered is presented in Figure 1.

Let

$x_i(t)$ be the number of health care specialists of type i $(i = 1,\ldots,n)$ (speciality, level, etc.) at step t (e.g. one year, three years, six years, etc.) $(t = 0,1,\ldots,T)$, and

$u_k(t)$ be the number of entrants to the medical educational system of type k $(k = 1,\ldots,r)$ (medical schools, institutes, faculties, etc.).

It is assumed that τ_k steps are needed for graduating from the educational system of type k (k = 1,...,r).

Thus vector $x(t) = \{x_i(t)\}$ (i = 1,...,n) represents the distribution of health care specialists at step t, and vector $u(t) = \{u_k(t)\}$ (k = 1,...,r) represents the distribution of new enrollments at step t over different types of educational systems. Vector x(t) is the state of the system and vector u(t) is the control variable.

The state equations describing the development of the manpower system can be written as follows:

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{r} b_{ik}(t-\tau_k)u_k(t-\tau_k) \tag{1}$$

$$(i = 1,\ldots,n;\ t = 0,1,\ldots,T-1)$$

In the state equations (1):

$a_{ij}(t)$ is the coefficient which shows how many specialists of type j progress to group i between steps t and t+1 (i,j = 1,...,n; t = 0,1,...,T-1). Usually $1 - a_{ij}(t)$ is called the manpower stock attrition rate. In many cases

$$a_{ij}(t) = \begin{cases} 1 - \tilde{a}_{ii}(t), & i = j \\ 0, & i \neq j; \end{cases}$$

$b_{ik}(t-\tau_k)$ is the coefficient which shows how many enrollments of type k at year $t-\tau_k$ will obtain the speciality i at year t (k = 1,...,r; i = 1,...,n; t = 0,1,...,T-1, $\tau_k < T$). These coefficients denote the ratio of graduates of type i to the total number of students enrolled in the type k educational system.

It is assumed that for the state equations (1) the initial conditions are:

$$x_i(0) = x_i^o$$

$$u_k(t - \tau_k) = u_k^o(t - \tau_k) \tag{2}$$

where x_i^o $(i = 1,\ldots,n)$, $u_k^o(t - \tau_k)$ $(k = 1,\ldots,r;\ 0 \leqslant t \leqslant \tau_k - 1)$ are given numbers.

If the length of each step t is equal to the maximum duration of training in the educational system, then the state equations (1) with time delay τ_k will be transformed into the ordinary form of state equations without time delays.

Definitions. The sequence of variables $u = \{u_k(0),\ldots, u_k(T-1-\tau_k)\}$ denotes *control* of the system (or the *enrollment plan* for a given planning horizon T). The sequence of variables $x = \{x_i(0),\ldots,x_i(T)\}$ is the *trajectory* of the system (or the *manpower plan*).

Choosing different controls (enrollment plans) u, we can define with the state equations (1) the corresponding trajectory x (the manpower plan). The problem is to find such an enrollment plan as will satisfy all the constraints of the system and be optimal in some sense (for example, the corresponding manpower plan at each step t will be as close as possible to the demand requirements on different types of specialists).

Thus the second stage of DLP model building is to delineate the constraints on the variables.

Constraints Basically constraints on the variables may be broken down into three types: physical, resource and goal.

a. Physical Constraints: It is evident that the number of people cannot be negative:

$$x_i(t) \geqslant 0 \qquad (i = 1,\ldots,n) \tag{3}$$

$$u_k(t) \geqslant 0 \qquad (k = 1,\ldots,r) \tag{4}$$

b. Resource Constraints: In rather general form these can be written as:

$$\sum_{k=1}^{r} d_{sk}(t)u_k(t) \leqslant f_s(t) \qquad (s = 1,\ldots,m) \tag{5}$$

where $f(t) = \{f_1(t),\ldots,f_s(t),\ldots,f_m(t)\}$ is the vector of given resources (capacities) for training (teachers, buildings, equipment, etc); the coefficients $d_{sk}(t)$ $(s = 1,\ldots,m;\ k = 1,\ldots,r)$ show the amount of resources of type s needed per unit for education of type k at year t.

In many cases it is necessary to single out the constraints on the availability of teachers or instructors of different types.

Let the $y_j(t)$ $(j = 1,\ldots,J)$ be the number of available teachers of type j at step t and $\gamma_{jk}(t)$ be the ratio of required teachers of type j to students enrolled in the educational system of type k. Then the constraints on the teachers' availability can be written as

$$\sum_{k=1}^{r} \gamma_{jk}(t)u_k(t) \leqslant y_j(t) \qquad (j = 1,\ldots,J) \tag{6}$$

These constraints should be added to the constraints (5).

In this simple model the numbers $f_s(t)$ $(s = 1,\ldots,m)$ and $y_j(t)$ $(j = 1,\ldots,J)$ are the given exogenous variables. In more detailed models these variables are considered as state variables which are governed by some controllable activities in training teachers and building other educational facilities.

c. Goal Constraints: Usually goals for control of a system are associated with the value of a performance index (objective function). However, sometimes only some of them are introduced into the objective function. The others are considered as into the objective function. The others are considered as additional constraints on the system. For example, it is required to keep the numbers of specialists of some types $i \in I_1 \subset I = \{1,\dots,n\}$ at the given levels:

$$x_i(t) = \bar{x}_i(t) \qquad i \in I_1 \subset I \quad ,$$

or

$$x_i(t) \geq \bar{x}_i(t) \qquad i \in I_2 \subset I \quad .$$

In some cases one of the goals may be to provide the system with the desired distribution of specialists at the end of planning period

$$x(T) = x_T \quad , \tag{7}$$

where x_T is a given vector (terminal conditions).

The general form of constraints on the system's variables can be given in the form

$$\begin{aligned} G(t)x(t) + D(t)u(t) &\leq f(t) \\ x(t) \geq 0 \quad , \qquad u(t) &\geq 0 \end{aligned} \tag{8}$$

where $f(t)$ is the given m-vector, and $G(t)$ and $D(t)$ are the given matrices with dimension $(m \times n)$ and $(m \times r)$ respectively.

Performance Index The ultimate goal of a manpower supply model is to meet the projected demand requirements in manpower, thus increasing the quality of the health care system. In the models considered the projected figures of required specialists of all types i are supposed to be known for each step t of the planning period T, that is, the numbers $\bar{x}_i(t)$ are given for each $i = 1,\dots,n$ and $t = 1,\dots,T$. The goal of control of the system is to bring the manpower stock plan $\{x_i(t)\}$ as close as possible, under given dynamic (1),(2) and static (3)-(8) constraints, to the desired distribution of specialists $\{\bar{x}_i(t)\}$.

In the framework of linear objective functions this closeness can be evaluated by

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t) |x_i(t) - \bar{x}_i(t)| \quad . \tag{9}$$

The $\alpha_i(t)$ are the given weight coefficients, $|x|$ is the absolute value of x, or, in more general form,

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \phi_i^t\left(x_i(t) - \bar{x}_i(t)\right) \quad , \tag{10}$$

where

$$\phi_i^t(x) = \begin{cases} \alpha_i^1(t)x \;, & \text{if} \quad x \geq 0 \\ \alpha_i^2(t)x \;, & \text{if} \quad x \leq 0 \end{cases}$$

If $\alpha_i^1(t) = -\alpha_i^2(t)$ for all i and t, then the performance index (10) reduces to (9).

Evidently, the objective functions (9) and (10) can easily be reduced to some linear case. If the shortage of specialists is excluded,

$$x_i(t) \geq \bar{x}_i(t) \quad (i = 1,\ldots,n;\; t = 0,1,\ldots,T) \quad , \tag{11}$$

then the objective function can be directly written in linear form:

$$J(u) = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t)[x_i(t) - \bar{x}_i(t)] \quad , \tag{12}$$

with additional constraints (11).

The other group of objective functions is associated with the minimization of expenditure for education. If $\beta_k(t)$ is the cost of training per student of speciality k at year t, then the total expenditure for education will be the following:

$$J(u) = \sum_{t=0}^{T-1} \sum_{k=1}^{r} \beta_k(t)u_k(t) \quad . \tag{13}$$

Finally, if it is necessary to develop a special program for training the greatest feasible number of given specialists $i \in I_1 \subset I$ by the end of this program, then the problem can be reduced to maximization of the objective function

$$J(u) = \sum_{i \in I_1} \alpha_i(T)x_i(T) \quad , \tag{14}$$

where $\alpha_i(T)$, $i \in I_1$ are weight coefficients for the eligible specialities.

Summarizing, we can state the typical problem for the considered model as follows.

Problem 1 Given the initial conditions

$$\begin{aligned} x_i(0) &= x_i^0 && (i = 1,\dots,n) \\ u_k(t-\tau_k) &= \bar{u}_k(t-\tau_k) && (k = 1,\dots,r;\ 0 \leqslant t \leqslant \tau_k - 1) \end{aligned} \tag{15}$$

and the state equations

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{r} b_{ik}(t-\tau_k)u_k(t-\tau_k) \tag{16}$$

$$(i = 1,\dots,n;\ t = 0,1,\dots,T-1)$$

with constraints

$$\sum_{k=1}^{r} d_{sk}(t)u_k(t) \leqslant f_s(t) \tag{17}$$

$$(s = 1,\dots,m;\ t = 0,1,\dots,T-1-\tau_k;\ k = 1,\dots,r)$$

$$x_i(t) \geqslant 0 \qquad (i = 1,\dots,n;\ t = 1,\dots,T) \tag{18}$$

$$u_k(t) \geqslant 0 \ , \qquad (k = 1,\dots,r;\ t = 0,1,\dots,T-\tau_k-1) \tag{19}$$

find a control

$$u = \{u_k(0), u_k(1), \ldots, u_k(T-1-\tau_k) \quad (k = 1, \ldots, r)\}$$

and a corresponding trajectory

$$x = \{x_i(0), x_i(1), \ldots, x_i(T) \quad (i = 1, \ldots, n)\}$$

which minimize the performance index

$$J(u) = \sum_{t=1}^{T} \alpha_i(t) \left| x_i(t) - \bar{x}_i(t) \right| \quad . \tag{20}$$

Even in this simple form the model may be useful in practice, as it takes into account in some optimal way the main features of manpower planning models; the dynamics of the process of training specialists and the limits of available resources.

In this model only a single level education is considered, and the investments to the system are supposed to be fixed. Thus a more detailed model is needed.

2.2 The Three-Level Manpower Planning Model

In this section we consider the three-level manpower planning model, which incorporates three subsystems of specialist training: nurses who graduate from medical schools, practical physicians who graduate from medical institutes, and medical specialists of high level who are trained in special professional courses (for example, postgraduate) (Figure 2).

Some of the second-level specialists can be teachers for the first-level educational subsystem, and all the third-level specialists are supposed to be instructors either for the second level or for the third level educational subsystem.

We now consider the subsystems separately.

First Level

Let

$x_i^1(t)$ be the number of nurses of type i $(i = 1,\dots,I^1)$ at step t (the health care specialists of the first level),

$u_k^1(t)$ be the number of entrants to the nurses' schools of type k $(k = 1,\dots,K^1)$ at step t,

$a_{ii}^1(t)$ be the proportion of nurses of type i $(i = 1,\dots,n)$ who leave the stock of the first level labour force during step t because of death or retirement (the first level manpower stock attrition rate),

$b_{ik}^1(t)$ be the ratio of graduated nurses of type i to the total number of students enrolled in the nurses' schools of type k at step t (the graduating ratio of nurses' schools),

$u_k^2(t)$ be the number of entrants to the medical institutes (the second educational level) of type k $(k = 1,\dots,K^2)$ at step t,

$c_{ik}^{1,2}(t)$ be the rate of entrance to the second educational level of type k $(k = 1,\dots,K^2)$ for those who graduated from the first educational level with speciality of type i $(i = 1,\dots,I^1)$ at step t,

τ_k^1 be the training period for the first educational level of type k $(k = 1,\dots,K^1)$.

Then the state equations for the first educational level will be as follows:

$$x_i^1(t+1) = \{1 - a_{ii}^1(t)\}x_i^1(t) + \sum_{k=1}^{K^1} b_{ik}^1(t-\tau_k^1)u_k^1(t-\tau_k^1) - \sum_{k=1}^{K^2} c_{ik}^{1,2}(t)u_k^2(t) \qquad (i = 1,\dots,I^1) \quad .. \quad (21)$$

We have the usual conditions and constraints on variables $x^1(t)$, $u^1(t)$ (see section 1.1):

$$x_i^1(0) = \bar{x}_i^1 \qquad (i = 1,\dots,I^1) \tag{22}$$

$$u_k^1(t - \tau_k^1) = \bar{u}_k^1(t - \tau_k^1) \qquad (k = 1,\dots,K^1;\ 0 \leqslant t \leqslant \tau_k^1 - 1)$$

and

$$x_i^1(t) \geqslant 0 \qquad (i = 1,\dots,I^1;\ t = 1,\dots,T) \tag{23}$$

$$u_k^1(t) \geqslant 0 \qquad (k = 1,\dots,K^2;\ t = 0,\dots,T - \tau_k^1 - 1) \tag{24}$$

$$\sum_{k=1}^{K^2} d_{sk}^1(t) u_k^1(t) \leqslant f_s^1(t) \tag{25}$$

The given resource vector $f^1(t)$ does not now include the number of teachers available in medical specialities. In the three-level model under consideration these constraints can be treated separately.

Let

$$x_{i_1}^2(t),\ i_1 \in I_1^2 \subset I^2 \ ; \qquad I = \{1,\dots,I^2\}$$

be the number of teachers available in the medical speciality $i_1 \in I_1^2 \subset I^2$ at step t.

It is supposed that some of the second level specialists are teachers for the first level educational system. The set of all such specialities is denoted by I_1^2. The set I^2 of all second level specialities contains the teacher set I_1^2: $I^2 \supset I_1^2$.

Generally speaking, a teacher spares only part of his working time for teaching; the other part is left for practising.

Let

$\delta^2_{i_1}(t)$ be the ratio of teaching time to total working time for the second level specialists of type $i_1 \in I^2_1$ $(0 \leq \delta^2_{i_1}(t) \leq 1)$ and

$g^{2,1}_{i_1 k}(t)$ be the ratio of required type i_1 teachers to students enrolled in the first educational level of type k $(i_1 \in I^2_1;\ k = 1,\ldots,K^1)$.

Then the requirements for teachers of medical specialities $i_1 \in I^2_1$ necessary for training the first level students can be written in the form

$$\sum_{k=1}^{K^1} g^{2,1}_{i_1 k}(t) u^1_k(t) \leq \delta^2_{i_1}(t) x^2_{i_1}(t) \qquad (26)$$

$$(i_1 \in I^2_1 \subset I^2;\ t = 0,1,\ldots,T - \tau^1_k - 1)$$

Second Level

Let

$x^2_i(t)$ be the number of second level specialists (e.g. physicians) of type i $(i = 1,\ldots,I^2)$ at step t,

$u^2_k(t)$ be the number of entrants to medical institutes of type k $(k = 1,\ldots,K^2)$,

$a^2_{ij}(t)$ be the proportion of second level specialists of type j who pass to the group of second level specialists of type j during a step t $(i,j = 1,\ldots,I^2)$,

$b^2_{ik}(t)$ be the ratio of second level graduates of type i $(i = 1,\ldots,I^2)$ to the total number of students enrolled in medical institutes of type k $(k = 1,\ldots,K^2)$ at step t,

$x^{3,2}_i(t)$ be the number of second level specialists of type i who enter the third level educational subsystem at step t $(i = 1,\ldots,I^2)$,

τ_k^2 be the training time for the second educational level of type k $(k=1,\dots,K^2)$.

Then the state equations for the second educational level subsystem will be the following:

$$x_i^2(t+1) = \sum_{j=1}^{I^2} a_{ij}^2(t)x_j^2(t) + \sum_{k=1}^{K^2} b_{ik}^2(t-\tau_k^2)u_k(t-\tau_k^2) -$$
$$- x_i^{3,2}(t) + \sum_{j=1}^{I^3} [1-b_{ji}^3(t-\tau_j^3)]x^{3,2}(t-\tau_j^3) \quad (27)$$
$$(i=1,\dots,I^2;\; t=0,1,\dots,T-1)$$

if $i \notin I_1^2$, and

$$x_{i_1}^2(t+1) = \sum_{j=1}^{I^2} a_{i_1 j}^2(t)x_j^2(t) + \sum_{k=1}^{K^2} b_{i_1 k}^2(t-\tau_k^2)u_k^2(t-\tau_k^2)$$
$$- x_{i_1}^{3,2}(t) - \delta_{i_1}^2(t)x_{i_1}^2(t) +$$
$$+ \sum_{j=1}^{I^3} [1-b_{ji_1}^3(t-\tau_j^3)x_{ji_1}^{3,2}(t-\tau_j^3)] \quad (28)$$
$$(i_1 \in I_1^2,\; t=0,1,\dots,T-1)$$

Equations (27) and (28) are valid if the teachers do not practise and thus really leave the second level manpower stock. If the teachers divide their working time between practising and teaching we must introduce the "actual" manpower stock by

$$\tilde{x}_i(t) = \begin{cases} x_i(t) , & \text{if } i \notin I_1^2 \\ [1-\delta_i^2(t)]x_i(t) , & \text{if } i \in I_1^2 \end{cases} , \quad (28a)$$

where $\{x_i(t)\}$ $i \in I^2$ are now determined by state equations (27) in both cases: $i \notin I_1^2$ and $i \in I_1^2$.

The meaning of the last term in equations (27) and (28) will be explained later.

It should be noted that transitions from one group of specialists of type i to another group of type j are admissible during each step t for the second level model under consideration.

If these transitions are not permitted:

$$a_{ij}(t) = \begin{cases} 1 - a_{ii}(t) & i = j \\ 0 & \text{otherwise} \end{cases} ,$$

then we come to the case of equations of type (21).

The initial conditions

$$x_i^2(0) = \bar{x}_i^2 \qquad (i = 1,\dots,I^2) \tag{29}$$

$$u_k^2(t - \tau_k^2) = \bar{u}_k^2(t - \tau_k^2) \tag{30}$$

$$(k = 1,\dots,K^2;\ 0 \leqslant t \leqslant \tau_k^2 - 1)$$

and constraints

$$x_i^2(t) \geqslant 0 \qquad (i = 1,\dots,I^2;\ t = 1,\dots,T) \tag{31}$$

$$u_k^2(t) \geqslant 0 \qquad (k = 1,\dots,K^2;\ t = 0,\dots,T - \tau_k^2 - 1) \tag{32}$$

$$\sum_{k=1}^{K^2} d_{sk}^2(t)\, u_k^2(t) \leqslant f_s^2(t) \tag{33}$$

$$(s = 1,\dots,S^2;\ t = 0,\dots,T - \tau_k^2 - 1)$$

have similar form to those for the first level subsystem.

In (33), $f^2(t) = \{f_s^2(t)\}$ $(s = 1,\dots,S^2)$ is the vector of given resources (capacities) for the second level educational subsystem (excluding teachers), and $d_{sk}^2(t)$ $(s = 1,\dots,S^2,\ k = 1,\dots,K^2)$ is the amount of resource of type s which is needed for the education of each specialist of type k ($k = 1,\dots,K^2$) at step t.

Constraints in teaching capacities of the second level subsystem will be considered later.

Third Level

The third level educational subsystem is the highest level in the considered model. It is supposed that each second level specialist of type i ($i = 1,\dots,I^2$) who enters the third level educational subsystem at step t in speciality j ($j = 1,\dots,I^3$) graduates from it (e.g., obtains a degree of Doctor of Medical Science), with an attrition rate of $1 - b^3_{ji}(t)$ at step $t + \tau^3_j$.

Let

$x^3_j(t)$ be the number of third level specialists of type j ($j = 1,\dots,I^3$) at step t,

$a^3_{jj}(t)$ be the proportion of third level specialists of type j ($j = 1,\dots,I^3$) who leave the stock of the third level labor force during step t because of death or retirement (the third level manpower stock attrition rate),

$b^3_{ji}(t)$ be the ratio of third level graduates of type j ($j = 1,\dots,I^3$) to the total number of second level specialists of type i ($i = 1,\dots,I^2$) enrolled in the third level educational subsystem at step t,

$x^{3,2}_{ji}(t)$ be the distribution of second level entrants over the specialities of the third level educational subsystem at step t,

τ^3_j be the training time for third level specialists of type j ($j = 1,\dots,I^3$).

Then the state equations for the third level educational subsystem will be the following:

$$x_j^3(t+1) = [1 - a_{jj}^3(t)]x_j^3(t) + \sum_{i=1}^{I^2} b_{ji}^3(t-\tau_j^3)\, x_{ji}^{3,2}(t-\tau_j^3) \qquad (34)$$

$$(j = 1,\dots,I^3;\ t = 0,1,\dots,T-\tau_j^3 - 1) \quad ,$$

where

$$\sum_{j=1}^{I^3} x_{ji}^{3,2}(t) = x_i^{3,2}(t) \qquad (34a)$$

and

$$\sum_{j=1}^{I^3} [1 - b_{ji}^3(t-\tau_j^3)]x_{ji}^{3,2}(t-\tau_j^3) = \tilde{x}_i(t)$$

is the number of specialists of type i who do not graduate from the third level educational subsystem and therefore return to the second level manpower stock (see equations (27) and (28)).

For the third level subsystem the initial conditions are:

$$x_j^3(0) = \bar{x}_j^3 \qquad (j = 1,\dots,I^3) \qquad (35)$$

$$x_{ji}^{3,2}(t-\tau_j^3) = \bar{x}_{ji}^{3,2}(t-\tau_j^3) \qquad (35a)$$

$$(i = 1,\dots,I^2;\ j = 1,\dots,I^3;\ 0 \leq t \leq \tau_j^3 - 1) \quad .$$

Each specialist at the third level is supposed to be a teacher either for the second level or for the third level educational subsystem.

Let

$\delta_j^3(t)$ be the ratio of teaching time to total working time for the third level specialists of type j $(j = 1,\dots,I^3)$,

$g_{jk}^{3,2}(t)$ be the ratio of required third level teachers of type j $(j = 1,\dots,I^3)$ to students enrolled in the second level educational subsystem of type k ($k = 1,\ldots,K^2$) at step t,

$g^3_{ji}(t)$ be the ratio of third level teachers of type j ($j = 1,\ldots,I^3$) required for training the second level specialists of type i ($i = 1,\ldots,I^2$) enrolled in the third level educational subsystem at step t.

Then the requirements for teachers for both the second and the third level educational subsystems can be written in the form

$$\sum_{k=1}^{K^2} g^{3,2}_{jk}(t)u^2_k(t) + \sum_{i=1}^{n} g^3_{ji}(t)x^{3,2}_i(t) \leqslant \delta^3_j(t)x^3_j(t) \qquad (36)$$

$$(j = 1,\ldots,I^3,\ t = 0,1,\ldots,T-1)$$

The real manpower stock of third level specialists will be defined by the expression

$$\tilde{x}^3_j(t) = [1 - \delta^3_j(t)]x^3_j(t) \qquad (37)$$

$$(j = 1,\ldots,I^3,\ t = 0,1,\ldots,T-1)$$

Other constraints for the third level subsystem are written in the ordinary form:

$$x^3_j(t) \geqslant 0 \qquad (j = 1,\ldots,I^3,\ t = 1,\ldots,T) \qquad (38)$$

$$x^{3,2}_i(t) \geqslant 0 \qquad (i = 1,\ldots,I^2;\ t = 1,\ldots,T) \qquad (39)$$

$$\sum_{i=1}^{I^2} d^3_{si}(t)x^{3,2}_i(t) \leqslant f^3_s(t) \qquad (40)$$

$$(s = 1,\ldots,S^3;\ t = 0,1,\ldots,T)$$

where $f^3(t) = \{f^3_s(t)\}$ ($s = 1,\ldots,S^3$) is the vector of given resources (capacities) for the third level educational subsystem (excluding teachers) and $d^3_{si}(t)$ is the amount of the resource of type s which is needed for training one specialist of type i ($i = 1,\dots,I^2$) at step t.

Performance Index

Considering the state equations and constraints for these three levels, one can see that if the number of steps T for state variables $\{x^1(t), x^2(t), x^3(t)\}$ is fixed, then the duration of control sequences will be different for controls of each subsystem. There are several ways to treat this situation. For example, one can choose sufficiently large T and consider all the variables only for the period which is equal to min $\{T-1-\tau_k^1, T-1-\tau_k^2, T-1-\tau_k^3\}$, that is, in the case for $t = 0,1,\dots,T-1-\tau_k^2$, $k = 1,\dots,r$.

Furthermore, to simplify notation we shall assume that the number of steps for all state variables is the same and is equal to T.

We shall now formulate the performance index for this model, which quantifies the quality of a chosen plan of enrollments for all three educational subsystem levels.

It is supposed that the projected figures of demand for each level of health care specialists are available for all steps t of planning period T, that is, the numbers

$$X_{i_1}^1(t),\ X_{i_2}^2(t),\ X_{i_3}^3(t) \tag{41}$$

$$(i_1 = 1,\dots,I^1;\ i_2 = 1,\dots,I^2;\ i_3 = 1,\dots,I^3)$$

are given for each $t = 1,\dots,T$.

The goal in planning the three-level model under consideration is to determine a plan for enrollments to all three educational subsystems

$$u_{k_1}^{1}(t),\ u_{k_2}^{2}(t),\ x_i^{3,2}(t)$$

$$(k_1 = 1,\ldots,K^1;\ k_2 = 1,\ldots,K^2;\ i = 1,\ldots,I^2)$$

for each $t = 0,1,\ldots,T^{\nu} - 1$ where T^{ν} is different for each ν-th ($\nu = 1,2,3$) subsystem, which satisfies all the dynamic and static constraints of the system and yields the manpower stock

$$\tilde{x}_{i_1}^{1}(t),\ \tilde{x}_{i_2}^{2}(t),\ \tilde{x}_{i_3}^{3}(t)$$

$$(i_1 = 1,\ldots,I^1;\ i_2 = 1,\ldots,I^2;\ i_3 = 1,\ldots,I^3)$$

which is as close to demand (40) as possible for all steps $t = 1,\ldots,T$.

Thus the performance index can be written as

$$\begin{aligned} J = & \sum_{t=1}^{T^1} \sum_{i=1}^{I^1} \alpha_i^1(t)\,|\tilde{x}_i^1(t) - X_i^1(t)| + \\ & + \sum_{t=1}^{T^2} \sum_{i=1}^{I^2} \alpha_i^2(t)\,|\tilde{x}_i^2(t) - X_i^2(t)| + \\ & + \sum_{t=1}^{T^3} \sum_{i=1}^{I^3} \alpha_i^3(t)\,|\tilde{x}_i^3(t) - X_i^3(t)| \quad , \end{aligned} \tag{42}$$

where $\alpha_i^1(t)$ $(i = 1,\ldots,I^1)$, $\alpha_i^2(t)$ $(i = 1,\ldots,I^2)$, $\alpha_i^3(t)$ $(i = 1,\ldots,I^3)$ are some weighting coefficients, and $\tilde{x}_i^2(t)$ and $\tilde{x}_i^3(t)$ are defined by (28a) and (37).

The three level optimization problem, like Problem 1, allows different modifications and variants. Some of them were discussed when the simple model was considered (section 2.1).

We shall now consider two important extensions of these models. The first is connected with incorporating the investment submodel into the model, the other with consideration of the subsystem for improvement of professional skills.

The Investment Submodel

In both of the models which were considered above, the volumes of resources (buildings, equipments, etc.) were supposed to be given beforehand, that is, the variables $f_s^\nu(t)$ $(s = 1,\dots,S^\nu;$ $\nu = 1,2,3;$ $t = 0,\dots,T-1)$ were considered to be exogenous. In many practical cases it is preferable to incorporate into the manpower model the subsystem for planning the development of training capacities (e.g. construction of buildings, etc.).

Let

$z_n(t)$ be the vector of available training capacities (buildings, equipment, etc.) at step t $(n = 1,\dots,N)$. For example,

$$z(t) = \{z_1(t),\dots,z_N(t)\} = \{f_1^1(t),\dots,f_{S^1}^1(t); f_1^2(t),\dots,f_{S^2}^2(t); f_1^3(t),\dots,f_{S^3}^3(t)\}$$

or vectors $z(t)$ are connected with vectors $\{f^1(t),f^2(t),f^3(t)\}$ by some linear transformation.

There are M options (activities) in increasing training capacities.

Let

$v_m(t)$ $(m = 1,\dots,M)$ be the funds allocated to the m-th activity at step t $(t = 0,1,\dots,T-1)$, and

$q_{nm}(t)$ be the increase of the n-th capacity per unit of m-th activity at step t $(n = 1,\dots,N;$ $m = 1,\dots,M)$.

Then the state equations which describe the development of the available training capacities will be the following:

$$z_n(t+1) = p_n(t)z_n(t) + \sum_{m=1}^{M} q_{nm}(t)v_m(t-\tau_m) \qquad (43)$$

$$(n = 1,\dots,N;\ t = 0,1,\dots,T-1)$$

where $1 - p_n(t)$ is the depreciation rate for the n-th resource, τ_m is the time lag for investments in the m-th activity.

The initial conditions

$$z_n(0) = z_n^o \qquad (n = 1,\dots,N) \tag{44}$$

and

$$v_m(t - \tau_m) = v_m^o(t - \tau_m) \tag{45}$$

$$(m = 1,\dots,M;\ 0 \leqslant t \leqslant \tau_m - 1)$$

are supposed to be known.

Evidently,

$$\begin{aligned} z_n(t) &\geqslant 0 \\ v_m(t) &\geqslant 0 \end{aligned} \tag{46}$$

The budget constraints can be given either for each step t:

$$\sum_{m=1}^{M} v_m(t) \leqslant c_t(t) \tag{47}$$

or for the total planning period:

$$\sum_{m=1}^{M} \sum_{t=0}^{T-\tau_m} v_m(t) \leqslant C \quad , \tag{47a}$$

where C is the given budget for the whole planning period.

In the considered case $v_m(t)$ there are additional control variables, and constraints (25),(33),(40) should be replaced by the constraints

$$\sum_j d_{nj}(t) u_j(t) \leqslant z_n(t) \qquad (n = 1,\dots,N) \tag{48}$$

where vector u(t) is defined by

$$u(t) = \{u_j(t)\} = \{u_k^1(t);\ u_k^2(t);\ x_i^{3,2}(t)\} \quad ,$$

and $z(t) = \{z_n(t)\}$ is the additional state-vector of the system.

2.3 System for Improving Qualifications

Systems for improving professional skills play an important role in health manpower systems, and in planning health care it is necessary to take into account the continuing progress of each health care specialist.

The general scheme of professional skill improvement is presented in Figure 3.

Let all health manpower be broken down into n different groups. The transition of a specialist from one group to another depends on whether this specialist enters courses for improving his qualification or not.

Let

$x_i(t)$ be the total number of health care specialists of group i at the beginning of step (year) t,

$u_i^k(t)$ be the number of health care specialists of group i $(i = 1,\ldots,n)$ who at step t enter courses for improving qualifications of type k $(k = 1,\ldots,r)$,

$A(t) = \{a_{ij}(t)\}$ be the transition matrix for specialists who do not enter any courses for improving qualifications at step t,

$B^k(t) = \{b_{ij}^k(t)\}$ be the transition matrix for specialists who enter courses at step t, τ_k being the training time for courses of type k $(k = 1,\ldots,r)$, and

$v_i(t)$ be the new entrants to the type i manpower stock at step t.

Then the equations which describe the dynamics of the distribution of health care specialists over different groups will be the following:

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)\left[x_j(t) - \sum_{k=1}^{r} u_j^k(t)\right] + \\ + \sum_{j=1}^{n}\sum_{k=1}^{r} b_{ij}^k(t-\tau_k)u_j^k(t-\tau_k) + v_i(t) \qquad (49)$$

$$(i = 1,\ldots,n)$$

Here $x_i(t)$ are the state variables and $u_j^k(t)$ are the control variables.

A simpler version of the model is

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)\left[x_j(t) - u_j(t)\right] + \\ + \sum_{j=1}^{n} b_{ij}(t-\tau_j)u_j(t-\tau_j) \quad . \qquad (50)$$

The initial conditions for (49):

$$x_i(o) = x_i^o \qquad (i = 1,\ldots,n) \qquad (51)$$

$$u_j^k(t-\tau^k) = u_j^{ok}(t-\tau^k) \qquad (52)$$

$$(k = 1,\ldots,r;\ j = 1,\ldots,n;\ 0 \leqslant t \leqslant \tau^k - 1)$$

The constraints can also be written in the usual form:

$$\sum_{k=1}^{r}\sum_{j=1}^{n} d_{sj}^k(t)u_j^k(t) \leqslant f_s(t) \qquad (53)$$

$$u_j^k(t) \geqslant 0 \qquad (54)$$

$$(j = 1,\ldots,n;\ k = 1,\ldots,r)$$

$$x_i(t) \geqslant 0 \qquad (55)$$

$$(i = 1,\ldots,n)$$

This subsystem can be incorporated into the models of sections 2.1 and 2.2 or can be treated separately.

It should also be noted that the model for improvement of professional skill can be effectively used for planning training goals for a given group $I_1 \subset I$ specialists. In this case the performance index should be of the type set out in (14).

Problems of manpower planning are very important, but they cannot be treated separately from the total health care system. We consider below a general model of optimal development of a health care system.

3. OPTIMIZATION MODEL FOR HEALTH CARE SYSTEM PLANNING

The general interrelations between health care, health manpower and economic systems might be roughly sketched as follows. The state of a population's health determines the demand on the manpower stock of health care specialists, while the state of the economy sets the constraints on development of educational subsystems for training health care specialists. In its turn the optimal mix of required health care specialists increases the quality of the health care system, thus increasing the welfare of a country.

The problem of health care system planning may be reduced to finding a plan to allocate funds, manpower and other resources over time among different disease treatment activities in such a way as to yield the optimal result in terms of reduced mortality, morbidity and other losses [1-3,5,6,13-19].

Let us consider, for example, the problem of planning different activities in a treatment program for some disease (e.g. heart diseases, cancer, tuberculosis, etc.). First, we break down the whole population of a country or region into n different categories according to age, sex and degree of disease. This division must be accomplished with respect to the corresponding treatment activities needed by each group.

Thus,

$Z_i(t)$ is the number of people of group i at step (year) t, $i = 1,\ldots,n$.

It is supposed that r ways of treating the disease are at the disposal of the health care system. The number of people in group i $(i = 1,\ldots,n)$ receiving treatment of type k $(k = 1,\ldots,r)$ in year t will be denoted by $u_i^k(t)$. Then the total number of people in group i who receive some treatment in year t will be

$$\sum_{k=1}^{r} u_i^k(t) \qquad (i = 1,\ldots,n) \quad .$$

The transition matrix for those people who did not receive any treatment in year t is denoted by

$$A(t) = \{a_{ij}(t)\} \qquad (i,j = 1,\ldots,n) \quad .$$

Application of type k treatment to different groups of the population changes the transition matrix A(t) on

$$B^k(t) = \{b_{ij}^k(t)\} \qquad (i,j = 1,\ldots,n;\ k = 1,\ldots,r) \quad .$$

Matrix $B^k(t)$ defines the efficacy of type k treatment $(k = 1,\ldots,r)$ with respect to different population groups.

We can now write the equations which define the change of state of the system (that is, the vector $Z(t) = \{Z_i(t)\}$) over time:

$$Z_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)\left[Z_j(t) - \sum_{k=1}^{r} u_j^k(t)\right] + \sum_{k=1}^{r}\ \sum_{j=1}^{n} b_{ij}^k(t) u_j^k(t) \qquad (56)$$

$$(i = 1,\ldots,n;\ t = 0,1,\ldots,T-1)$$

We shall associate vector $u^k(t) = \{u_1^k(t),\ldots,u_n^k(t)\}$ with the intensity of the k-th activity in year t; $u^k(t)$ is the control vector, $Z(t)$ is the state vector. Hence in matrix form the state equations (58) can be rewritten as

$$Z(t+1) = A(t)\left[Z(t) - \sum_{k=1}^{r} u^k(t)\right] + \sum_{k=1}^{r} B^k(t)u^k(t) \qquad (57)$$

It should be noted that the state equations (56), which describe the process of improving the state of the population's health, have just the same form as the state equations (49), which describe the process of improving the professional skill of manpower.

The estimation of the matrices $A(t)$ and $B^k(t)$ may prove difficult in some cases because of the lack of reliable data on the prognosis of many diseases as a function of treatment given [15].

However, for some diseases estimation of similar quantities has been made and used in practice (see, e.g., [17,18]).

Initial conditions are supposed to be known:

$$Z(o) = Z^o \quad . \qquad (58)$$

Each treatment activity $u^k(t)$ requires some amount of different types of resources. Let

$f(t) = \{f_s(t) \ (s=1,\ldots,S)\}$ be the vector of given resources (e.g., number of beds in hospitals, amount of equipment and other facilities), and let

$x(t) = \{x_m(t) \ (m=1,\ldots,M)\}$ be the vector of health care manpower of different categories available at year t.

We denote:

$d_s^k(t) = \{d_{si}^k(t)\}$ as the vector which defines the requirement for resources of type s $(s=1,\ldots,S)$ for treatment activity k $(k=1,\ldots,r)$ at year t, and

$g_m^k(t) = \{g_{mi}^k(t)\}$ as the vector which defines the requirement of manpower resource of group m $(m = 1,\ldots,M)$ at year t $(i = 1,\ldots,n)$.

Then the constraints on the available resources can be written in the form

$$\sum_{k=1}^{r} \sum_{i=1}^{n} d_{si}^k(t) u_i^k(t) \leq f_s(t) \tag{59}$$

$$(s = 1,\ldots,S;\ t = 0,\ldots,T-1)$$

$$\sum_{k=1}^{r} \sum_{i=1}^{n} g_{mi}(t) u_i^k(t) \leq x_m(t) \tag{60}$$

$$(m = 1,\ldots,M;\ t = 0,1,\ldots,T-1)$$

or in the matrix form

$$\sum_{k=1}^{r} D^k(t) u^k(t) \leq f(t) \tag{59a}$$

$$\sum_{k=1}^{r} G^k(t) u^k(t) \leq x(t) \tag{60a}$$

In addition the physical constraints must be introduced:

$$Z_i(t) \geq 0 \qquad (i = 1,\ldots,n) \tag{61}$$

$$u_i^k(t) \geq 0 \qquad (k = 1,\ldots,r;\ i = 1,\ldots,n) \tag{62}$$

$$Z_i(t) - \sum_{k=1}^{r} u_i^k(t) \geq 0 \qquad (i = 1,\ldots,n) \tag{63}$$

Condition (60) defines the link of the considered health-care model with the manpower model. It should be noted that in many cases the standards defining the relations between the numbers of population in different disease/age groups and the required resources are known. Thus we can set

$$x(t) = \Delta^x(t) Z(t) \tag{64}$$

$$f(t) = \Delta^f(t) Z(t) \tag{65}$$

where matrices $\Delta^x(t)$ and $\Delta^f(t)$ define the requirements for health manpower resources $x(t)$ and "commodity" resources $f(t)$ by the population distribution $Z(t)$ over different disease/age groups.

In this case the constraints (59a) and (60a) should be replaced by

$$\Delta^f(t)Z(t) - \sum_{k=1}^{r} D^k(t)u^k(t) \geq 0 \tag{66}$$

$$\Delta^x(t)Z(t) - \sum_{k=1}^{r} G^k(t)u^k(t) \geq 0 \tag{67}$$

with

$$Z(t) - \sum_{k=1}^{r} u^k(t) \geq 0 ;$$

$$Z(t) \geq 0 ; \quad u^k(t) \geq 0$$

The performance index can be expressed as some linear function from the health state of the system. Hence in the simple case we can define

$$J = \sum_{t=1}^{T} \sum_{i=1}^{n} \alpha_i(t)Z_i(t) , \tag{68}$$

where $\alpha_i(t)$ is the weight coefficient associated with the value of the i-th group from the point of view of total estimation of the population's state of health.

In many ways it is more desirable to maximize the relative number of some disease/age groups. Let vector $Z(t)$ have the structure

$$Z(t) = \{Z_{ij}(t) \qquad i = 1,\ldots,n; \; j = 1,\ldots,N)\} ,$$

where i is the age group number and j is the disease group number. Then we can define numbers

$$\sum_{j=1}^{N} Z_{ij}(t) / \bar{Z}_i(t)$$

where $\{\bar{Z}_i(t)\}$ is the projected population distribution over age and time, which is supposed to be known. In this case the performance index can be written as

$$J = \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{j=1}^{N} \frac{\alpha_i(t) Z_{ij}(t)}{\bar{Z}_i(t)} \tag{69}$$

Another type of performance index is the minimization of total expenditure:

$$J = \sum_{t=0}^{T-1} \sum_{k=1}^{r} \sum_{i=1}^{n} \beta_i^k(t) u_i^k(t) \quad .$$

Problem 2 Given the initial state of health of the population (58) and equations (57), which describe the development of the state of health of the population under different treatment programs (activities) $u^k(t)$, find a combination of these programs $\{u^k(t)\}$ which maximizes the total evaluation (69) of the population's health (68) (or (69)) for the planning period T.

The parameters of the model, $d_s^k(t)$ and $g_m^k(t)$ can be regarded as norms for standards of treatment. The model suggested here reveals some of the consequences of implementing such norms in practice. It can therefore be regarded as a development of the IIASA Health Care Resource Requirement Model AMER [16]. It is expected that in practice the model will need to be run several times with different values for the $d_s^k(t)$ and $g_m^k(t)$ parameters so that the decision maker can observe the likely consequences of adopting different norms. It is considered very important that this approach be applied together with the IIASA resource allocation submodel, 'DRAM' [15], which suggests what values these quantities would take in the future if medical practice remains unchanged.

The considered model allows different modifications and variants and may be a basis for developing more sophisticated models for planning resource allocation in a health care system either for treatment of different diseases (control of heart dieases, cancer, tuberculosis, etc. [15 - 17]) or for the health care system as a whole [1,2,13,14].

4. CONCLUSION

In this introductory paper some different DLP models for planning health manpower systems or for planning control of different diseases are considered. More realistic models require cooperative efforts by both health care specialists and systems analysts, and the DLP approach might be an effective tool for elaboration and implementation of optimal policies in health care systems.

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REFERENCES

[1] Venedictov, D.D. (Ed.) *Health Systems Modelling and the Information System for the Coordination of Research in Oncology*, CP-77-4, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[2] Shigan, E.N., and R. Gibbs (Eds.) *Modeling Health Care Systems*, CP-77-8, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[3] Gibbs, R., *Health Care Resource Allocation Models - A Critical Review*, RM-77-53, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[4] Gray, P., and C. Cullinan-James, Applied Optimization - A Survey, *Interfaces*, 6, 3 (1976), 24-41.

[5] Boldy, D., A Review of the Application of Mathematical Programming to Tactical and Strategic Health and Social Services Problems, *Operational Research Quarterly*, 27, 2, ii (1976), 439-448.

[6] McDonald, A.G., G.C. Cuddeford and E.M.L. Beale, Balance of Care: Some Mathematical Models of the National Health Service, *British Medical Bulletin*, 30, 3 (1974), 262-270.

[7] Dantzig, G.B., *Linear Programming and Extensions*, Princeton University Press, Princeton, N.J., 1963.

[8] Propoi, A., *Problems of Dynamic Linear Programming*, RM-76-78, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

[9] U.S. Department of Health, Education and Welfare, *An Analysis of Health Manpower Models*, v.I.II, 1974.

[10] Schneider, D.R., and K.E. Kilpatrick, An Optimum Manpower Utilization Model for Health Maintenance Organization, *Operations Research*, 23, 5 (1975), 869-889.

[11] Shuman, L., J.P. Young and E. Naddov, Manpower Mix for Health Services: A Prescriptive Regional Planning Model, *Health Service Research*, 6 (1971), 103-118.

[12] Wandel, S.E., and J.C. Hershey, Evaluation of Nurse Staffing Policies Using a Manpower Planning and Scheduling Model, in K.B. Haley, ed., *Operational Research '75*, North-Holland, 1976, pp. 561-580.

[13] Zemach, R., A Model of Health Service Utilization and Resource Allocation, *Operations Research*, 18 (1970), 1071-1086.

[14] Belanger, P. et al., On the Modelling of Large-Scale Health Care Systems, *Behavioural Science*, 19 (1974), 407-414.

[15] Gibbs, R., *A Disaggregated Health Care Resource Allocation Model*, RM-78-1, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

[16] Klementiev, A.A., and E.N. Shigan, *Aggregate Model for Estimating HCS Resource Requirements (AMER)*, RM-78-21, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978,

[17] Feldstein, M.S., M.A. Piot and T.K. Sundaresan, *Resource Allocation Model for Public Health Planning*, WHO, Geneva (1973).

[18] Revelle, C., Feldman, F. and W. Lynn, An Optimization Model Tuberculosis Epidemiology, *Management Science*, B.16 (1969), 190-211.

[19] Modi, J.A., Test of Technique for Planning Medical Procedures in Preventive Health Examination Programs, *Economic Planning Science*, 6 (1972), 173-185.

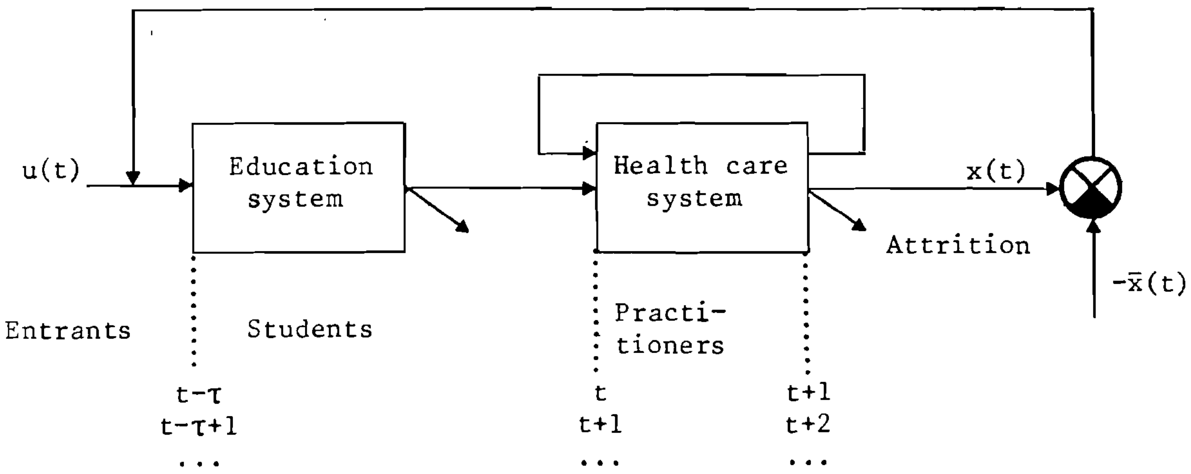

Figure 1. The simple manpower model.

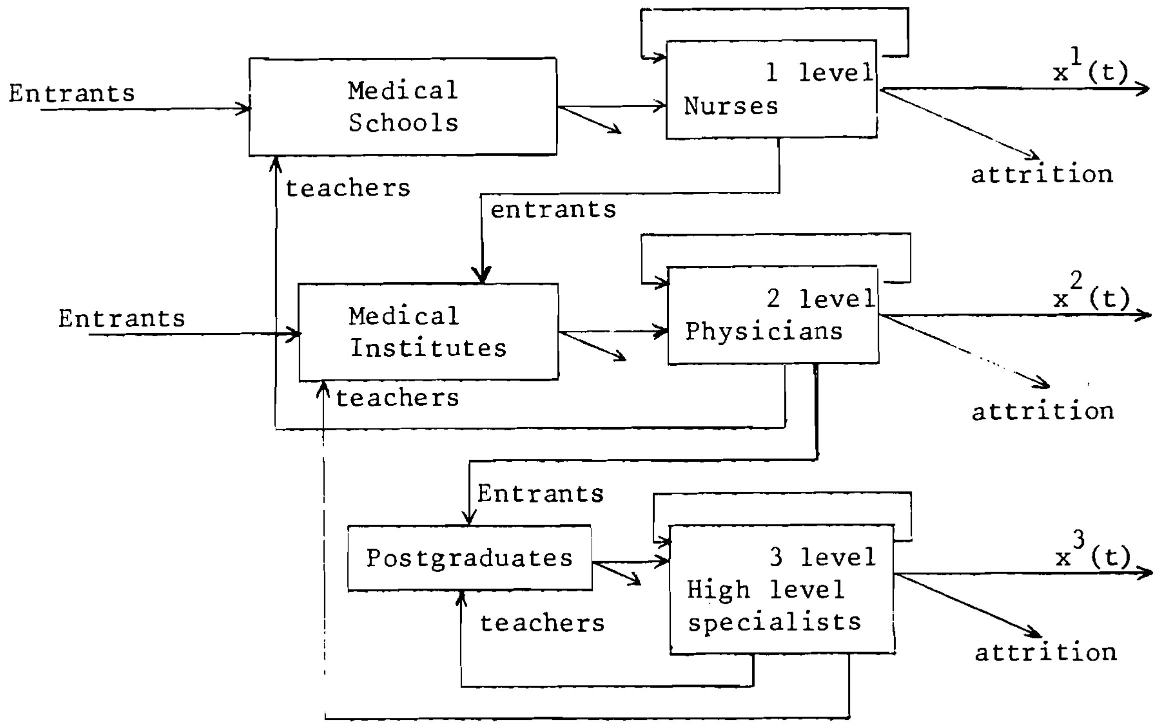

Figure 2. The three-level manpower planning model.

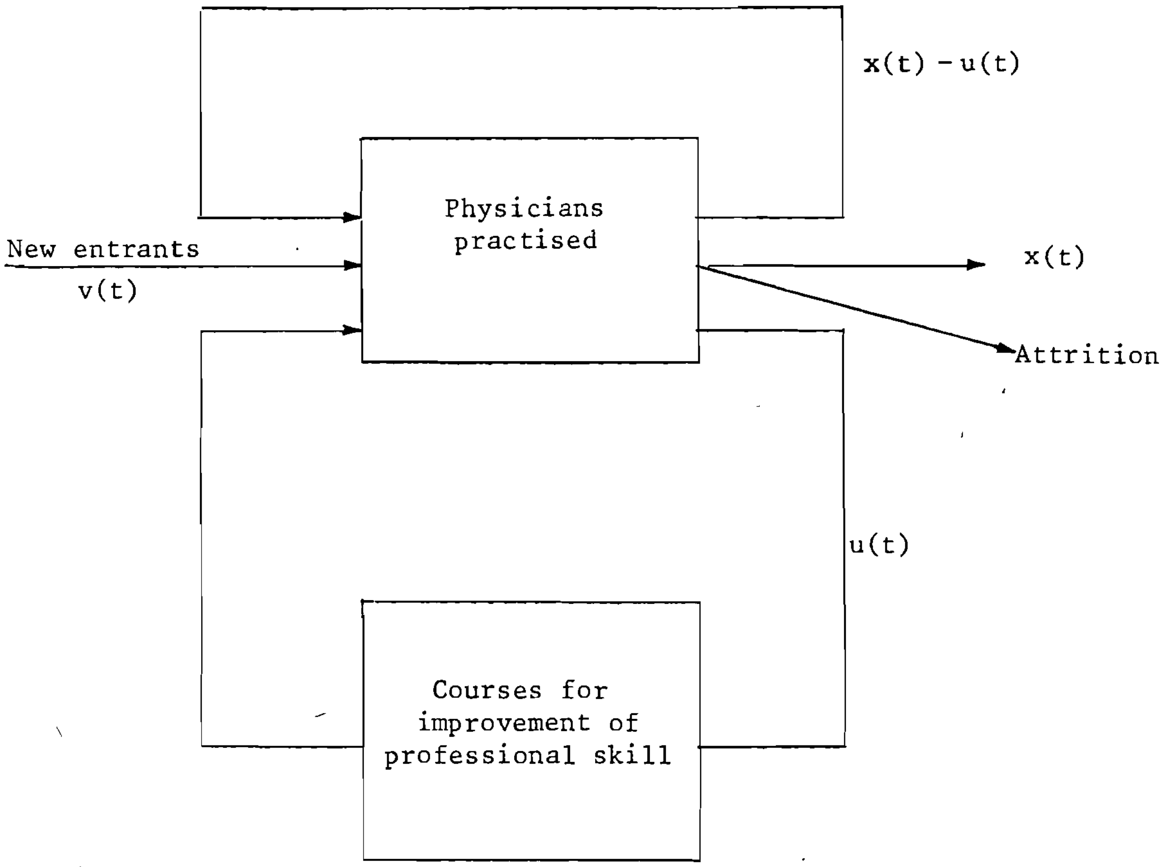

Figure 3. General scheme of the professional skill improvement system.

Papers of the Modeling Health Care Systems Study

March 1978

Venedictov, D.D., Modeling of Health Care Systems, in IIASA Conference '76, Vol.2, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Kiselev, A., A Systems Approach to Health Care, RM-75-31, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

Fleissner, P., Comparing Health Care Systems by Socio-Economic Accounting, RM-76-19, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Klementiev, A.A., A Computer Method for Projecting a Population Sex-Age Structure, RM-76-36, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Klementiev, A.A., Mathematical Approach to Developing a Simulation Model of a Health Care System, RM-76-65, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Kaihara, S., et al., An Approach to Building a Universal Health Care Model: Morbidity Model of Degenerative Diseases, RM-77-06, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

Shigan, E.N., Alternative Analysis of Different Methods for Estimating Prevalence Rate, RM-77-40, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Klementiev, A.A., On the Estimation of Morbidity, RM-77-43, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Fleissner, P., and A. Klementiev, Health Care System Models: A Review, RM-77-49, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Gibbs, R.J., Health Care Resource Allocation Models - A Critical Review, RM-77-53, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

Gibbs, R.J., A Disaggregated Health Care Resource Allocation Model, RM-78-1, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Kaihara, S., et al., Analysis and Future Estimation of Medical Demands Using a Health Care Simulation Model: A Case Study of Japan, RM-78-3, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

Papers in Dynamic Linear Programming and Related Topics

I. MODELS

[1] Haefele, W., and A.S. Manne, *Strategies for a Transition from Fossil to Nuclear Fuels*, RR-74-7, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1974.

[2] Suzuki, A., *An Extension of the Haefele-Manne Model for Assessing Strategies for a Transition from Fossil Fuel to Nuclear and Solar Alternatives*, RR-75-047, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

[3] Dantzig, G.B., A Pilot Linear Programming Model for Assessing Physical Impact on the Economy of a Changing Energy Picture, IIASA Conference '76, May 1976.

[4] Propoi, A.I., *Problems of Dynamic Linear Programming*, RM-76-78, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

[5] Haefele, W., and A. Makarov, Modeling of Medium and Long-Range Energy Strategies, Discussion paper given at the workshop on Energy Strategies, International Institute for Applied Systems Analysis, May 1977.

[6] Agnew, M., Schrattenholzer, L. and A. Voss, MESSAGE: A Brief Description, Discussion paper given at the workshop on Energy Strategies, International Institute for Applied Systems Analysis, May 1977.

[7] Grenon, M., and I. Zimin, Resources Model, Discussion paper given at the workshop on Energy Strategies, International Institute for Applied Systems Analysis, May 1977.

[8] Zimin, I., Aggregated Dynamic Model of a National Economy, Discussion paper given at the workshop on Energy Strategies, International Institute for Applied Systems Analysis, May 1977.

[9] Propoi, A., and F. Willekens, *A Dynamic Linear Programming Approach to National Settlement System Planning*, RM-77-8, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[10] Propoi, A., *Dynamic Linear Programming Models for Live-Stock Farms*, RM-77-29, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[11] Csåki, C., *Dynamic Linear Programming Model for Agricultural Investment and Resources Utilization Policies*, RM-77-36, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[12] Carter, H., Csåki, C. and A. Propoi, *Planning Long Range Agricultural Investment Projects: A Dynamic Programming Approach*, RM-77-38, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[13] Propoi, A., *Models for Educational and Manpower Planning: A Dynamic Linear Programming Approach*, RM-78-20, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978.

II. METHODS

[1] Winkler, C., *Basis Factorization for Block Angular Linear Programs: Unified Theory of Partitioning and Decomposition Using the Simplex Method*, RR-74-22, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1974.

[2] Propoi, A., *Problems of Dynamic Linear Programming*, RM-76-78, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1976.

[3] Propoi, A., *Dual Systems of Dynamic Linear Programs*, RR-77-9, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[4] Propoi, A., and V. Krivonozhko, *The Dynamic Simplex Method*, RM-77-24, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1977.

[5] Propoi, A., and V. Krivonozhko, *Simplex Method for Dynamic Linear Programs*, RR-78- , International Institute for Applied Systems Analysis, Laxenburg, Austria, 1978, (forthcoming).