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WAITING FOR THE BREEDER

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Abstract

This paper is addressed to the selection of an optimal mix of electricity generating plants. The focus is on the problem of uncertainty with respect to the date of availability of breeder nuclear reactors. Sequential probabilistic linear programming is employed. This makes it possible to optimize the mix of fossil, nuclear, and peaking plants to be installed during the 1980's--assuming that breeder technology will become available at some randomly determined later date. The model allows for the effects of exhausting our reserves of uranium ore. The exhaustion of these resources does not lead to disaster in the 21st century for an economy or a world with a backstop technology such as coal-fired electricity plants.

There seems to be a low value of information on the breeder availability date, for the initial policy is rather insensitive to this date. This conclusion holds not only when future demands are taken as fixed parameters, but also when they are dependent upon the price of electricity.

On environmental grounds (climate changes, radioactivity hazards, air and water pollution),

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there may be good reasons to slow the rate of growth of electricity demand. These are quite different issues than exhausting the resources of low-cost uranium ore. If our numerical assumptions are correct, it is not optimal to slow down the electricity growth rate up to 1990 just because of possible delays in the arrival of the breeder and hence a rapid rise in the price of uranium. For the year 2000, the decision on demands can be deferred until the time arrives to make capital investment decisions for the decade following 1990. By that point, some of the breeder's uncertainties will have been resolved.

I. Introduction

Within the Doomsday community, one of the more popular scenarios is the exhaustion of natural resources for energy production. We are told, for example, that at current rates of consumption the world has only 30 years of remaining reserves of oil and gas, plus perhaps 30 more in the form of uranium. Ergo, by the year 2050--plus or minus one or two decades--the human race is headed toward overshoot and collapse. Today's Cassandras remain unimpressed with the observation that technological progress has continually come to the rescue during the century and a half since Malthus. They ask to be shown specifically what resource-saving developments are likely during the next century.

Solar, fusion, and breeder fission are the most plausible contenders for large-scale future supplies of energy, and each would be virtually independent of the earth's finite stock of fossil and nuclear fuels.[1] To the Doomsday prophets, neither solar, fusion, nor fission are convincing possibilities. Solar electricity generation is feasible today, but is exceedingly high-cost (see Hottel and Howard [14]). Fusion has not yet passed the threshold of scientific feasibility, and fission leads to the inevitable hazard of radioactive releases (see e.g. Gofman and Tamplin [9]).

Throughout the balance of this paper, we shall concentrate upon only one of many aspects of the energy problem--the race between the development of breeder fission and the exhaustion of low-cost natural uranium resources. Sequential probabilistic linear programming is employed to calculate optimal electricity plant-mix decisions during the decade of the 1980's--given the uncertainty on the date of availability of the breeder. The model allows for the possibility that future uranium resource scarcities will lead to an increase in electricity prices and hence a reduction in the projected demands.

Somewhat surprisingly, the near future decisions are insensitive to the distant future uncertainties.[2] There seems to be a low value of information on the date of availability of the resource-saving breeder technology. This insensitivity to future events may be connected with the use of a 10% discount rate. Were we to have used a much lower rate, e.g. 3%, the advantages of nuclear over fossil fuel would have been even greater than those estimated here. Critics of nuclear power would then have raised a series of objections.

First, there are those who would have pointed out that a low discount rate would lead to an inefficient allocation of U.S. federal funds--too much investment in nuclear power and too little in other governmental programs. They would have cited the former director of the Office of Management and Budget, George Schultz. In a letter to the heads of executive departments and establishments, he directed all agencies of the Executive Branch of the federal government, except the U.S. Postal Service, to use a 10% discount rate for program analyses submitted to OMB in support of legislative and budget programs, except where some other rate is prescribed by or pursuant to law, Executive Order, or other relevant circulars.[3]

Second, there is a more fundamental objection. Suppose that the economy as a whole were to employ a 3% discount rate, and that it was prepared to undertake the corresponding reduction in present consumption levels so as to release resources for savings and investment. Then there would be major changes in the relative prices of fuels and generating equipment. The analysis could no longer be a partial equilibrium model of the electricity sector--with input prices given at today's conventionally measured levels. Instead, the sectoral analysis would have to be embedded within an economy-wide general equilibrium model. For sectoral planning, therefore, a 10% discount rate has been employed--even though it is known that this tends to speed up the exhaustion of some energy resources that would otherwise be available to our yet unborn descendents.

II. Structure of Decision Tree

For major electricity generating units, an investment decision must be taken some 5-10 years in advance of the date when a new plant's capacity first becomes available for use. It is for this reason that the first decision period (t = 1) refers to plant capacities that are to become available during the middle 1980's. This is the earliest point to be significantly affected by decisions taken during the middle 1970's. The 1980 capacity mix is already (as of the middle 1970's) virtually determined.

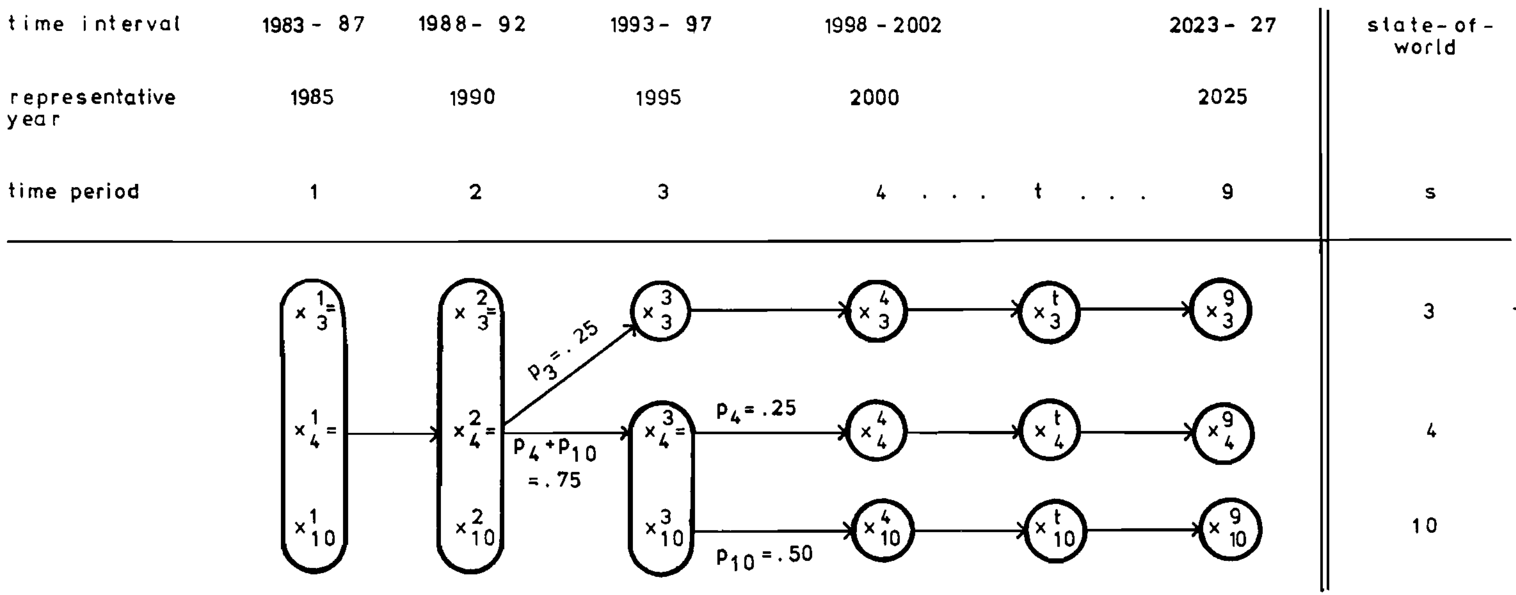
Over the future planning horizon, it is convenient to employ five-year time intervals, t = 1,2,.... More exactly then, the initial decisions are those that affect capacities to be brought onstream during the period of 1983 through 1987. For short, we shall generally refer to this five-year time period (t = 1) in terms of the representative midpoint year 1985. Similarly, the second investment decisions (t = 2) are those that must be taken during the late 1970's. These affect capacities during 1990, the representative year for the five-year interval 1988-92.

In line with the Doomsday viewpoint, we have taken a pessimistic view as to the date when a safe breeder will first become competitive with conventional LWR (light water reactor) nuclear units.[4] Until this point is reached, breeder capacity will not be installed in significant quantities. For our cost target definition of "commercially competitive," see Table 4 below.

The planning horizon extends through the year 2025 (period 9). It will be shown that the U.S. has sufficient supplies of coal so that electricity can be generated from this exhaustible fossil fuel until well after the horizon date. Coal is viewed as a "backstop technology" in the event of failure to develop a safe and competitive breeder. Moreover, by the horizon date, there is a good chance that either fusion or solar energy will have been developed on a large scale--whether or not breeder fission turns out to be successful. According to our calculations, natural resource scarcities will not be a bottleneck to the electricity sector and are not "essential" limits to growth.

In this sequential decision model, the breeder availability date is viewed as a random variable, s. Let p_s denote the probability that the breeder will first become available during period s. For illustrative calculations, it has been supposed that this event will occur either in period 3 or 4, or else not at all within the planning horizon. As a pessimistic technology forecast, the following numerical values have been adopted for the subjective probabilities: $p_3 = .25$; $p_4 = .25$; $p_{10} = .50$ (see Figure 1).

Figure 1. Structure of decision tree
(pessimistic breeder technology forecast)



Note: The unknown vector x_s^t denotes the strategy to be adopted during period t, contingent upon knowledge of the state-of-world s.

On the decision tree diagram, the vector x_s^t denotes the decisions to be adopted for period t, given that the state-of-the-world is s. Because it is supposed that the breeder will not be available during the two initial time periods, the identical values must be assigned to x_s^1 for all values of s, and similarly for x_s^2. That is, the initial decisions must be taken under complete uncertainty with respect to s. Subsequently, with respective probabilities p_3, p_4, and p_{10}, the stochastic process will lead to the top, middle, or bottom branch of the decision tree.

If the breeder becomes competitive during period 3--that is, if s = 3--the uncertainties will be resolved directly at that date. Suppose, however, that s ≠ 3. Then during period 3 it is not known whether the breeder will first become commercially available during period 4 or not at all until after period 9. Therefore x_4^3 must be identical with x_{10}^3 (see Figure 1). According to this tree, the uncertainty on the breeder's date has been resolved by the time that decisions must be taken for period 4. Hence, for $t \geq 4$, there is no sequential uncertainty restriction that $x_4^t = x_{10}^t$.

To connect this notation with the linear programming variables identified in the next section, note that those variables will be written in a slightly different form: CP(i,t,s), UT(i,k,t,s), WT(ℓ,t,s), and RX(m,t,s). For compatibility with computer format requirements, the time period index t and the state-of-world index s will be written on the same line as the rest of the identification for the individual unknowns. Nonetheless, these variables have the same logical structure as the decision vectors x_s^t. Each denotes a strategy to be adopted during period t, contingent upon knowledge of the state-of-the-world s. The strategy is calculated so as to minimize the expected discounted cost of meeting electricity demands, including shortage penalty costs for unsatisfied demands.

III. Activities and Constraints for Capacity Utilization

Given the sequential uncertainty restriction defined by Figure 1, we are ready to define the activities and the constraints for each time period t and state-of-world s. See Table 1 for a summary of the indices employed. The index i helps to distinguish between six alternative processes for generating electricity: LWR's (light water reactors), breeders, peak storage, and three types of fossil units. Each of these plant types has a somewhat different comparative advantage in producing the joint outputs of the electricity industry: energy[5] available at different points of time in the annual cycle of operations (see Massé and Gibrat [15]).

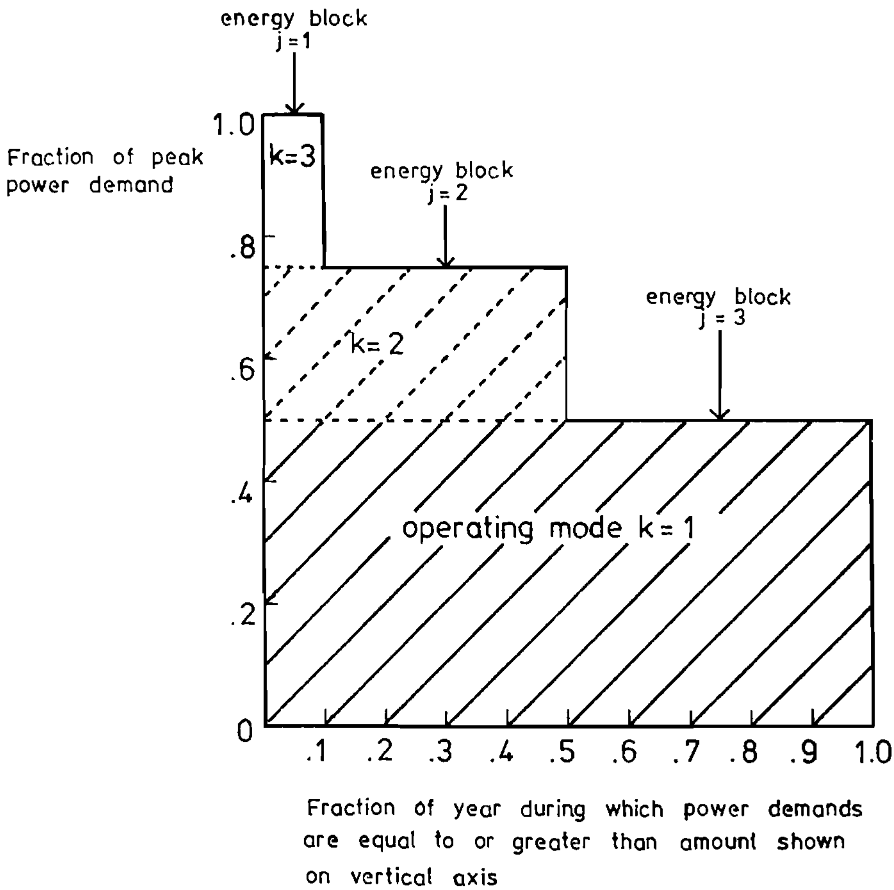
The annual cycle is summarized by the load-duration curve shown in Figure 2. This cumulative distribution is approximated by a three-step function.[6] It is supposed

Table 1. Definition of indices

index	representative year	time interval
t = 1	1985	1983 - 87
2	1990	1988 - 92
3	1995	1993 - 97
4	2000	1998 - 2002
5	2005	2003 - 07
6	2010	2008 - 12
7	2015	2013 - 17
8	2020	2018 - 22
9	2025	2023 - 27
s = 3	1995	state-of-world: alternative dates for a safe breeder to become less expensive than the LWR; p_s is the subjective probability that this cost target is <u>first</u> reached in period s; random variable s is unknown in periods 1 and 2, but is known with certainty in period 4 and thereafter.
4	2000	
10	after 2025	
i = 1 . . 6		plant types
j = 1 . . 3		energy blocks: 10, 40, 50% intervals along load-duration curve
k = 1 . . 5		mode of operation of generating plants
ℓ = 1 . . 5		level of demand (for piecewise linear approximation to demand shortage cost function)
m = 1 . . 5		uranium ore cost category

Figure 2. Load-duration curve for power demands and operating modes

key	operating mode k		energy demands under operating mode k
(blank)	3	peak-load	(.1)(.25) = .025
(dotted hatch)	2	intermediate	(.5)(.25) = .125
(solid hatch)	1	base-load	(1.0)(.50) = .500
		mean energy demand, ratio to maximum	.650



that the peak demands occur at a constant rate during energy block j = 1--that is, during 10% of the 8760 hours in a year. Similarly, the intermediate demands are at 75% of the maximum, and they occur at a constant rate during 40% of the year. The base-load demands (block j = 3) are at 50% of the maximum, and occur during 50% of the annual cycle. By measuring the vertical areas beneath each step of the load curve, it can be seen that 10/65 of the annual energy requirements are consumed in the peak period (j = 1), 30/65 in the intermediate period (j = 2), and 25/65 in the off-peak (j = 3). It will seen below (in Table 7) that there may be a ratio of 5:1 or more between the economic value of a kilowatt-hour in the peak and the off-peak blocks of time.

Table 2 summarizes the submatrix for a typical period t and state-of-world s. Each of these submatrices is identical except that the breeder construction activities CP(5,t,s) are omitted for t < s. Altogether, including the sequential uncertainty constraints, the model contains over 300 rows, 1100 variables, and 5000 non-zero elements.

The capacity utilization rows UT(i,t,s) are written as weak inequalities to allow for the possibility that it may be optimal to leave some of the older equipment idle.[7] These rows represent a link between successive points of time. That is, the amount of capacity available at period t is the sum of the quantities installed during each of the

Table 2. Programming submatrix for period t and state-of-world s.

row identification \ column identification	capacity increments, prior periods τ = 1..t-1: CP(i,τ,s) (i=3,4,5)	CP(6,τ,s)	← activities for period t → capacity increments: CP(i,t,s) (i=3,4,5)	CP(6,t,s)	peak capacity utilized: UT (i,k,t,s) (i=1..5), k= 1 2 3	UT(6,k,t,s), k= 4 5	demand interpolation weights WT(ℓ,t,s) (ℓ=1..5)	current annual requirements for uranium ore RX(m,t,s) (m=1..5)		right-hand side constants
UT (i,t,s) capacity utilization (10^9KW) (i = 1..5)	-1		-1		B_i B_i B_i				≤	initial
(i = 6)		-1		-1		B_6=1.05			≤	capacities
DM (j,t,s), demand requirements, energy block j (10^{12}KWH) j = 1					8.76 (.1 .1 .1	.1 .1)	$-(10/65)q_{\ell,t}$		≥	fixed hydro-electric energy supplies
j = 2					.4 .4	-.15	$-(30/65)q_{\ell,t}$		≥	
j = 3					.5	-.15	$-(25/65)q_{\ell,t}$		≥	
WT (t,s), demand interpolation weights							1		=	1
RX (t,s), current year's requirements for uranium ore (10^6 metric tons)					.216 .108 .0216 (applies only to LWR, plant type i=4)	0 0		-1	≤	0
CX (m,s) cumulative total requirements for uranium ore, cost level m = 1..5 (10^6 metric tons)								5	≤	uranium ore resources available, cost level m
PV, minimand, present value of costs ($10^9$$),			$p_s\left[C_i\right]\left[\frac{\beta^{t-.5}-\beta^{9.5}}{(1-\beta^6)}\right]$		$p_s\left[OM_i + H_k \cdot F_{i,t}\right] 5\beta^t$, where H_k = 8.760 4.380 .876	0 0	$p_s\left[u(q_{\ell,t})\right] 5\beta^t$	$p_s\left[r_m\right] 5\beta^t$		

prior periods τ. The available capacity must be sufficient to handle not only the peak requirements but also to provide a reserve for scheduled and unscheduled shutdown. The reserve capacity factors B_i are listed in Table 4 below. For example, plant type 6 requires 5% reserve capacity, and so $B_6 = 1.05$.

There is some flexibility in the number of hours per year that each of the plant types may be operated. To distinguish alternative modes of operations, we employ the index k. Three modes are defined for steam plants (i = 1..5), and these correspond to the three horizontal strips along the load curve of Figure 2.[8] Two other modes are defined for the peak storage units (i = 6).[9] Each of the activities UT(i,k,t,s) utilize capacity, and they help to satisfy demands in one or another of the three energy blocks j = 1,2,3.

IV. Demand Projections

The programming matrix is written so that future demands may be specified in either of two ways: as independent of or as dependent upon future prices. Let the parameter $q_{1,t}$ denote the aggregate energy demands for period t--assuming that demands are independent of future supply costs. These projections are adapted from Federal Power Commission [8, pp. I-18-23-29]. Hereafter, this document will be abbreviated as 1970 NPS (The 1970 National

Power Survey). This source is out-of-date, but has the advantage of providing a U.S. total demand forecast for 1980 and 1990, along with a consistent set of estimates of installed capacities for those years.[10] We have estimated the quantities $q_{1,t}$ by assuming that the growth of electricity demand will slow down after 1990, and that the demand increments will grow at the rate of 3% per year thereafter. This implies that the total annual growth rate will slow down from 6.6% during the 1980's to 5.3% during the 1990's, and that it will gradually approach 3%--approximately the same long-term growth rate as national income.

For price-dependent future demands, we shall consider only the case in which prices rise so that demands are reduced below the reference levels $q_{1,t}$. The costs of these shortages are then inferred through a series of assumptions related to the demand curve for electricity as a function of the price in period t. It is supposed that future electricity prices are high enough to limit demands to the supplies available, that prices are equal to the level of marginal supply costs, and that income distribution consequences may be neglected.[11]

For short, we may omit the time subscript and refer to the independently projected demand level as q_1. It is supposed that q_1 corresponds to a future reference price $p_1 = \$10/10^3$ KWH, and that the price elasticity of demand for electricity generation is $\eta = -.5$.[12] For simplicity, no distinction is drawn between the short- and the long-run elasticity. The market demand curve for year t is extrapolated from the reference values p_1, q_1 and the elasticity η. This is casual econometrics!

Let q denote the future quantity of electricity demanded--assuming that prices are raised to cover the future marginal supply costs. Let the money value of benefits be an isoelastic function of q. That is,

$$u(q) = aq^b + c \quad ,$$

where a, b, and c are constants to be estimated from the market demand curve. Setting the incremental benefits equal to the price p, the demand curve is related to the benefit function as follows:

$$p = abq^{b-1}$$

$$\therefore \quad \eta = \frac{dq}{dp} \cdot \frac{p}{q} = \frac{1}{b-1}$$

$$\therefore \quad b = 1 + \frac{1}{\eta} = -1 \quad .$$

It can also be seen that

$$a = (p_1/b)\, q_1^{1-b} \quad .$$

Finally, the arbitrary constant c is chosen so that zero benefits are associated with the reference demand level q_1

$$u(q_1) = aq_1^b + c = 0 \quad .$$

With this normalization, the expected discounted costs are comparable--both for the independent demand projections and for those that are price-dependent. That is, if the industry's output $q < q_1$, the level of benefits $u(q) < 0$. These negative "benefits" are subtracted from the other components of the minimand, and are in effect treated as shortage penalty costs.

Once the parameters a, b, and c have been estimated for each time period t, this problem may be formulated as a nonlinear mathematical programming model in which we solve for optimal levels of the unknown demand levels q_t. The constraints would be linear, but the objective function would include the term $u_t(q_t)$. To handle this nonlinearity and yet retain the advantages of linear programming computations, our unknowns are not the levels q_t, but rather the interpolation weight variables $WT(\ell,t,s)$. These unknowns are nonnegative, and they are constrained to add to unity (see row WT(t,s) in Table 2). The grid points $\ell = 1..5$ are chosen so as to allow for reductions in demand below the reference level $q_{1,t}$, but so as to ensure that there will be no decrease below $q_{1,0}$ (the projection for 1980). The five alternative levels of demand are defined

$$q_{\ell,t} = q_{1,t}\left[1 - .1(\ell - 1)\right] + q_{1,0}\left[.1(\ell - 1)\right]$$

$$(\ell = 1..5) \ .$$

Thus, for the price-dependent[13] cases, we focus upon the range between 60 and 100% of the independently projected increment in demand between period 0 and t. For example, it might be optimal to set WT(2,t,s) = .6 and WT(3,t,s) = .4. This solution would be interpreted as a reduction in demand to 86% of the projected increment between years 0 and t.

For each demand level ℓ in year t, there is a shortage cost coefficient: $-u(q_{\ell,t})$. These are readily calculated by inserting the quantity $q_{\ell,t}$ into the utility function $u(q_t)$.

In order to see how the interpolation weight variables affect the demands for energy in each block, we recall the size of the <u>vertical</u> areas under the load-duration curve. Figure 2 stipulated that the aggregate quantity $q_{\ell,t}$ is to be distributed among demand blocks in the proportions 10/65, 30/65, and 25/65 respectively (see the entries in rows DM(j,t,s)). This completes the derivation of the coefficients for the demand interpolation weight activities WT(ℓ,t,s).

V. Uranium Ore Supplies and Demands

The only plants that use significant quantities[14] of natural uranium are the LWR's, plant type 4. The uranium inputs into these reactors are the positive coefficients shown in the programming matrix rows RX(t,s). The reader can verify that these numbers are calculated as follows:

$$\left(\begin{array}{c}\text{180 metric tons}\\ \text{of uranium ore/}\\ \text{year for } 10^6 \text{ KW}\\ \text{of capacity}^{15}\end{array}\right)\left(\begin{array}{c}\text{LWR capacity}\\ \text{to utilization}\\ \text{ratio, } B_4 = 1.20\end{array}\right)\left(\begin{array}{c}\text{fraction of}\\ \text{year operated,}\\ \text{mode k}\end{array}\right)$$

To allow for increases in uranium extraction costs as a function of cumulative production, there are five ore extraction cost categories m = 1..5. The annual amounts to be supplied by category m are denoted by RX(m,t,s) (again, see Table 2). These unknowns are multiplied by 5 in rows CX(m,s) in order to convert the annual amounts into the cumulative total resource requirements for each 5-year period. The optimization procedure automatically ensures that each of the lower-cost resources will be exhausted before the next higher cost ore is utilized.

Table 3 contains the right-hand side constants for rows CX(m,s), the uranium resources available at cost level m. These are taken from WASH-1243, Atomic Energy Commission. In that document, Robert Nininger cautions the reader that there are wide margins of uncertainty in

Table 3. U.S. resources of uranium ore [a]

Extraction cost category m	Cutoff supply cost $/pound	10^3 \$/metric ton, r_m	Reserves plus potential within category m (10^6 metric tons)
1	15	33	1.4
2	30	66	.6
3	50	110	5.0
4	100	220	8.0
5	250	550	∞

[a] Source: WASH-1243, Atomic Energy Commission [4, Figures 4 and 10].

these availability estimates. He points out that "at present we have a limited knowledge of the real potential resources of the country" [4, p. 15].

Our model includes one high-cost source that does not appear in WASH-1243: uranium available in unlimited quantities from sea water at $250/pound. At this cost, it turns out that uranium-fueled LWR's are not competitive with coal-fueled fossil plants. It therefore makes no difference whether we assume that there is a finite or an infinite amount of uranium in the oceans.

VI. Structure of Cost Coefficients

In this constrained optimization model, the minimand PV denotes expected discounted costs[16] (see the lowest row of Table 2). Note that this is the only point in the entire tableau where the probabilities p_s enter explicitly. The probabilities are the left-most term in each cost coefficient. They serve to transform costs that are conditional upon state s into their expected value.

Each term in the minimand contains a present-value factor, β^t. The coefficient β denotes a 5-year discount factor for a uniform annual rate of 10%. That is, $\beta = \left(\frac{1}{1.10}\right)^5 \approx .62$.

The PV row of Table 2 provides additional details on the calculation of the cost coefficients. First, consider the variables CP(i,t,s), the five-year capacity increments of plant type i in period t under state-of-world s. There are initial capital outlays of C(i) per unit of capacity. The expected capital costs are discounted by the factor $\beta^{t-.5}$, that is, as of the date that lies 2.5 years before the midpoint of period t.[17] As an approximation to reduce horizon effects, there is a terminal value credit of $\beta^{9.5}$ for each of these capacity cost variables. In addition, to allow for an infinite chain of replacement investments at intervals of 6 periods (30 years), each capital cost coefficient is divided by the term $(1-\beta^6) \approx 1/1.06$. In other words, with a 10% discount rate, there is only a 6% capital cost difference between a 30-year and an infinite service life.

The capacity utilization variables UT(i,k,t,s) lead to annually recurring costs. For these, the midpoint is taken to be representative of the entire five-year interval. Hence the annual costs are multiplied by the factor of 5, and are discounted by the factor β^t. The term OM_i denotes the operating and maintenance costs. These are incurred whenever the unit is operated at any point during the year. The annual fuel costs are proportional to H(k), the number of hours operated for mode k.[18]

Annually recurring costs are associated with the demand interpolation weight variables WT(ℓ,t,s) and with the requirements for uranium ore RX(m,t,s). Their cost coefficients are therefore also multiplied by the factor $5\beta^t$. For the derivation of the demand shortage costs $u(q_{\ell,t})$ and the uranium ore costs r_m, see respectively section 4 and Table 3 above.

VII. Numerical Values of Cost and Performance Factors

There is no objective way to estimate the cost and performance factors of new technologies. Inherently there is a subjective element in the parameters given in Table 4.[19] For example, a critic of the breeder development program will point out that LWR's are a more conventional technology than breeders. He will then say that LWR's have proved quite unreliable, and that the reserve capacity factor B(4) should be much higher than 1.20. A fortiori, it is hopelessly optimistic to set the breeder reserve factor B(5) = 1.25. Similar questions can and should be raised about each of these numbers in Table 4. They must be viewed as illustrative, and they do not represent an industry-wide consensus.

Given the cost and performance factors of the LWR in Table 4, it is fairly straightforward to see what targets must be achieved for the breeder to satisfy high safety standards, and yet reach a commercially competitive position. This technology will not be competitive if its capital costs are much more than \$50/KW above those of the LWR, or if its operating and maintenance costs are higher than \$8/KW-year, or if its fuel costs are higher than $\$1.00/10^3$ KWH. This is the reasoning that underlies the breeder performance factors of Table 4. Clearly these costs will not be achieved by the initial demonstration plants during the early 1980's. A lengthy period will be required for learning-by-doing before it becomes economical to install large amounts of breeder capacity.

Table 4. Cost and performance factors (in 1974 prices)

Plant type i	1 fossil, before 1970	2 fossil, 1970-80	3 fossil, after 1980	4 LWR (light water reactor)	5 breeder	6 peak storage
BTU(i), heat rate, fossil plants (10^3 BTU/KWH)	10.4	9.5	8.5	—	—	—
F(i,1), fuel cost ($\$/10^3$ KWH)	10.4 a/	9.5 a/	8.5 a/	$1.5+.8\left(\frac{\text{ore price}}{\$15/\text{pound}}\right)$ b/	1.0	0 c/
OM(i), operating and maintenance ($/KW-year)	6.0	5.0	4.0	6.0	8.0	3.0
C(i), initial investment ($/KW)	d/	d/	300	500	550	300
B(i), reserve capacity factor; reciprocal of maximum plant factor	1.15	1.15	1.15	1.20	1.25	1.05
Average costs for base-load operations, mode 1; ($\$/10^3$ KWH) e/	—	—	13.1	10.2*	10.2*	—
Average costs for intermediate operations, mode 2; ($\$/10^3$ KWH) f/	—	—	17.6*	18.2	19.5	—
Average costs for peaking operations, modes 3,4 or 5; ($\$/10^3$ KWH) g/	—	—	54.8	81.8	93.3	45.0*

Table 4 (continued)

Notes:

a) For fossil plants, the fuel costs in period t are obtained by multiplying the heat rate BTU(i) by the fuel price in year t. In this table, the fuel costs F(i,1) are based upon a 1985 fossil fuel price of $\$1.00/10^6$BTU. In the dynamic optimization model, it is supposed that fossil fuel prices will increase at the annual rate of $\$.01/10^6$BTU after 1985.

b) The cost of uranium ore will vary with the cumulative production. The ore cost component of F(4,1) is calculated as follows:

$$\left(.216\ \frac{10^6 \text{ metric tons}}{8.76\ 10^{12} \text{ KWH}}\right)\left(\frac{2.2\ 10^3 \text{pounds}}{\text{metric ton}}\right)\left(\frac{\$15}{\text{pound}}\right) \approx \frac{\$.8}{10^3 \text{KWH}}$$

On this basis, LWR fuel costs would be $\$1.9/10^3$KWH with U_3O_8 at $8.00/pound, but they would rise to $2.3 or more with the gradual exhaustion of low-cost uranium reserves. See National Petroleum Council (1972, p.176). For calculating average energy costs in the three bottom rows of this table, the ore price is taken to be $15/pound.

c) For peak storage, the direct fuel cost is zero, but there is an indirect cost: an input of 1.5 KWH during offpeak hours per KWH generated during the peak hours.

d) These two fossil plant types are not candidates for new investment after 1980.

e) Average costs include capital charges at 10%/year for a 30 year service life, hence a capital recovery factor of 10.6%/year. Average costs for base-load operations are calculated as follows from the operating, maintenance, and fuel costs, together with the B(i) factor for reserve capacity:

$$\frac{\left[(10.6\%/\text{year})C(i)\right]\ B(i) + OM(i)}{8.76\ 10^3 \text{ hours/year}} + F(i,1) \ .$$

Asterisk denotes process with lowest average cost.

f) Same as e, except that fixed annual charges are divided by $(.5)(8.76)\ 10^3$ hours/year.

g) Same as e, except that fixed annual charges are divided by $(.1)(8.76)\ 10^3$ hours/year. Peak storage fuel costs are taken as 1.5 times the LWR fuel cost.

Before running these data through a dynamic decision model, it is helpful to make quick calculations such as those in the three lowest lines of Table 4. For this average cost comparison, it is supposed that each post-1980 type of plant (i = 3,4,5,6) will be operated in <u>one fixed mode</u> throughout a 30-year service life. The asterisked entries denote the process with the lowest average costs.[20] According to Table 4, LWR's and breeders will be the least expensive units for base-load duty (mode 1), fossil plants for intermediate duty (mode 2), and peak storage for modes 3,4, or 5. This is a static comparison. By contrast, the dynamic optimization model allows for endogenously determined shifting between base-load, intermediate, and peaking service.

The optimal choice of base-load equipment is quite sensitive to the assumption with respect to fossil fuel prices. If these costs lie below $\$.50/10^6$BTU--and all other factors remain the same--fossil plants will be less expensive than LWR's or breeders. No elaborate optimization model is then needed to evaluate these competing technologies.

Despite the large quantities of coal available in the U.S., low future fossil fuel costs seem unlikely. This is partly because of tightened air pollution standards and partly because of the gradual exhaustion of eastern reserves. For these reasons, we have supposed that the 1985 average fossil fuel cost will be $\$1.00/10^6$BTU, and that it will rise thereafter at the annual rate of $\$.01/10^6$BTU. This is a rough estimate of the price that will be needed to cover the cost of low-sulfur western coal that will be strip-mined in accordance with increasingly stringent land use regulations and then shipped for distances of 1500 or more miles. Alternatively, this price would be needed to cover the costs of coal gasification or of solvent refining,[21] or to cover the cost of the cleaner fossil fuels--natural gas and low-sulfur residual fuel oil.

VIII. Numerical Results--Sensitivity Analysis and the Value of Information

With these cost parameters, an optimal strategy may be calculated through the sequential probabilistic programming model. For each time period t and state-of-world s, there is an optimal value for each of the unknowns: CP(i,t,s), UT(i,k,t,s), WT(ℓ,t,s) and RX(m,t,s). Rather than list the numerical solution for each of these 1100 variables, it seems preferable to describe only the aggregate results.

Let $\overline{PV}$ denote the numerical value of the minimand, expected discounted costs (see the lowest line of Table 2). Similarly, let PV_s denote the minimum discounted costs if it is known that the state-of-world is s in advance of choosing any of the unknowns. On the basis of this advance information, a deterministic solution may be calculated separately for each possible state-of-world. These deterministic solutions provide a sensitivity analysis for the breeder availability date.

Table 5 indicates the numerical values for $\overline{PV}$ and PV_s--both when the future demands are independent and also when

Table 5. Discounted costs (unit: 10^9\$; discounted to 1980 at 10% per year; 1974 general price level).

	Basic case, probabilistic (expected costs)	Advanced estimate of breeder availability date (deterministic sensitivity analysis)			Expected value of perfect information
		1995	2000	after 2025	
Discounted costs	$\overline{PV}$	PV_3	PV_4	PV_{10}	$\overline{PV} - \sum_s p_s PV_s$
Independent demands	628.72	598.83	610.18	652.46	.24
Price-dependent demands	609.98	587.96	594.59	628.25	.22
Subjective probabilities p_s (pessimistic breeder technology forecast)		.25	.25	.50	

these demands are dependent upon the costs of electricity supply. In the latter case, suppose it were certain that the breeder will become available in 1995. The discounted costs would then be $587.96 billions. It can also be seen that $PV_4 - PV_3 = 594.59 - 587.96 \approx 7$ billions. If all other data were reliable, this would mean that the U.S. could afford to spend up to $7 billions on research and development in order to have the breeder available in 1995 (period 3) rather than in 2000 (period 4).

For the case of independent demands, $PV_4 - PV_3 \approx \$11$ billions. Since no demand shortages are permitted, there are higher demands for electricity, higher derived demands for uranium ore, and hence higher benefits from the breeder's savings on ore resources.

For some models, it is sufficient to perform a series of sensitivity analyses, and then to demonstrate that the optimal <u>initial</u> decisions are unaffected by the uncertainties that will be resolved at a later date. In this case the one set of initial decisions is said to be "dominant." There is a zero value of information on the state-of-world s, and it is unnecessary to calculate an optimal sequential decision strategy.

In our numerical model, the optimal initial decisions are not invariant with the state-of-world, but nonetheless the value of information is low. The maximum that could be afforded for a perfect forecast is the difference between $\overline{PV}$ (the expected costs of following an optimal strategy without advance information) and $\sum_s p_s PV_s$ (the expected costs with this information). According to the rightmost column of Table 5, electric utility decision makers could afford to pay no more than \$.24 billions for perfect information on the breeder date.[22] (Note added in proof: Much the same results have been obtained from the same type of model, but with an entirely different set of data from Electricité de France.)

To understand why the value of information is low but positive, it is necessary to examine some of the detailed numerical results--here reported only for the case of price-dependent demands.[23] According to the sensitivity analysis, the breeder date seems to affect the time-phasing but not the broad aggregates for the decade of the 1980's. During period 1 (the five years centered about 1985), it would be optimal to install either 307 or 288 or 220 GW of LWR capacity--depending upon whether one had advance information that the breeder would become available in 1995 or 2000 or after 2025 respectively. For periods 1 and 2 together, the total LWR capacity is virtually unaffected by the breeder date (see the three middle columns of Table 6). In no case does the model suggest that it is optimal to install net new fossil capacity during the 1980's. There are sufficient numbers of old units to provide intermediate and peaking capacity during that decade.

IX. Numerical Results--Natural Resource Demands and Prices

On environmental grounds (climate changes, radioactivity hazards, air and water pollution), there may be good reasons to slow the rate of growth of electricity demand. These are

Table 6 . Capacity available (GW)

plant type i \ year t	1970 National Power Survey [a)]			Optimal solutions - for 1990 and 2000 - price-dependent demand projections						
				advance estimate of breeder availability date (sensitivity analysis)			basic case (probabilistic)			
								state-of-world s =		
	actual	estimated		1995	2000	after 2025		1995	2000	after 2025
	1970	1980	1990	1990	1990	1990	1990	2000	2000	2000
1.fossil, before '70	259	259	259	259	259	259	259	259	259	259
2.fossil, '70-'80	0	131	131	131	131	131	131	131	131	131
3.fossil, after '80	0	0	168	0	0	0	0	0	0	152
4.LWR	6	140	475 (4 and 5)	546	555	555	555	555	720	883
5.breeder	0	0		0	0	0	0	764	640	0
6.peak storage	4	27	70	27	27	27	27	32	117	119
7.conventional hydro [b)]	52	68	82	82	82	82	82	82	82	82
total capacity [c)]	321	625	1185	1045	1054	1054	1054	1823	1949	1626
total capacity, annual growth rate over preceding decade, %		6.9	6.6	5.3	5.4	5.4	5.4	5.6	6.3	4.4

Notes:

a) See p. I-18-29

b) It is supposed that negligible amounts of conventional hydro will be added after 1990.

c) Excludes gas turbines and internal combustion generators. In 1970, their capacity was 6% of the total, but their energy generation was only 1.4%. This total also excludes geothermal plants.

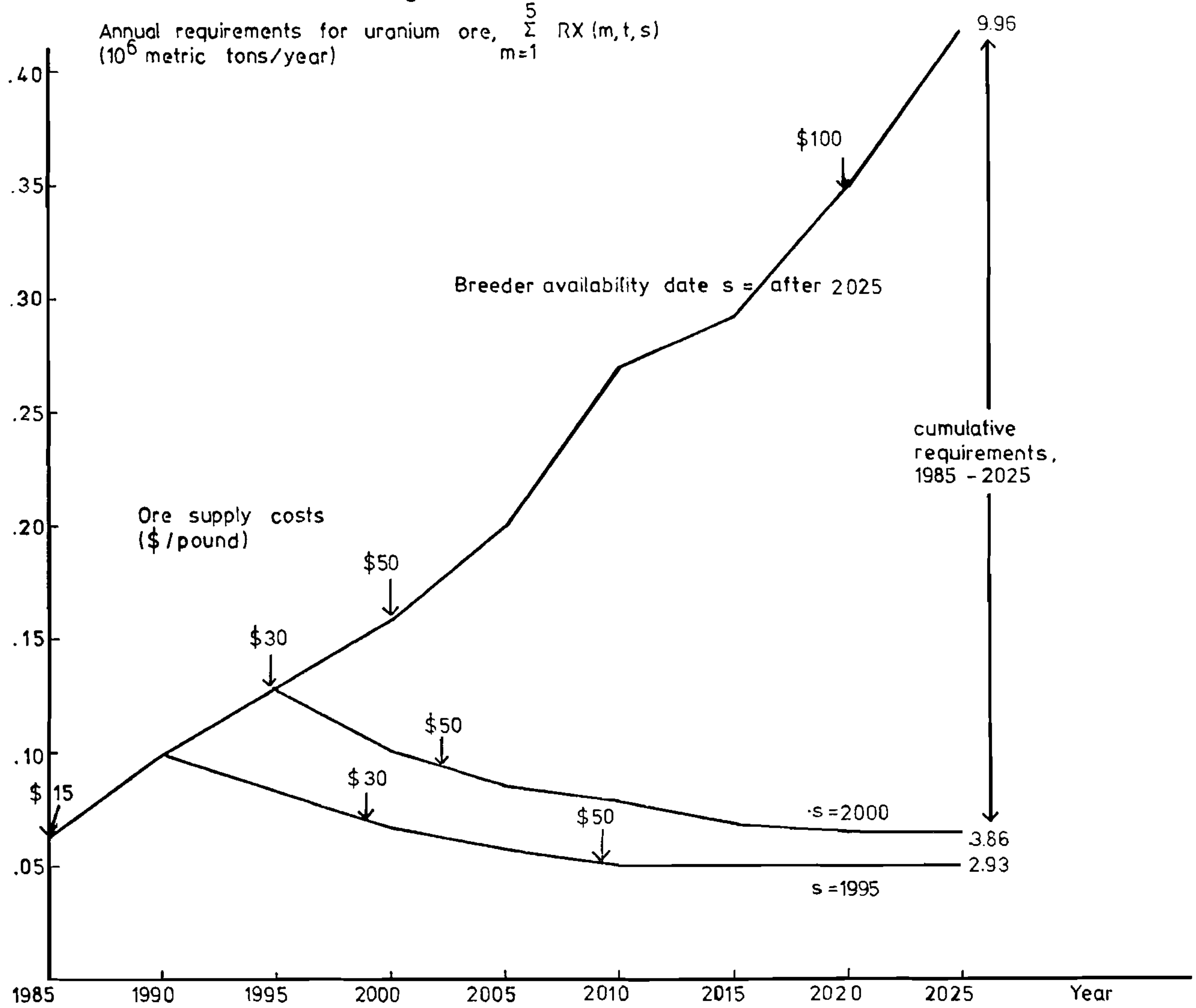
quite different issues than exhausting the resources of low-cost uranium ore. If our numerical assumptions are correct, it is not optimal to slow down the electricity growth rate up to 1990 just because of possible delays in the arrival of the breeder and hence a rapid rise in the price of uranium. For the year 2000, the decision on demands can be deferred until the time arrives to make capital investment decisions for the decade following 1990. By that point, some of the breeder's uncertainties will have been resolved. If the breeder is available early (say in 1995), it would be optimal to plan for an annual demand growth of 5.6% during the 1990's. If the breeder is greatly delayed (e.g. by a major accident at one of the demonstration plants), the demand growth could be slowed to 4.4% (see the three rightmost columns of Table 6).

Because of the uncertainty on the date of arrival of the breeder, there are three different tracks shown on Figure 3 for the annual requirements of uranium ore. The lowest amount of ore is needed if the breeder becomes available early, and the greatest amount if it is late (after 2025). Note that in no case is it optimal to exhaust the reserves of $100 per pound ore, nor to use any of the $250 ore from sea water.

Along Figure 3, there are arrows pointing to the <u>first</u> date at which it becomes necessary to extract ore with the indicated supply cost. Beginning in 1985, the ore extraction costs are $15 (cost category m = 1). Suppose that the breeder does not become available until after 2025. Then in the year 2000, Figure 3 indicates that it will become necessary to

Figure 3.

Annual requirements for uranium ore, $\sum_{m=1}^{5} RX(m,t,s)$
(10^6 metric tons/year)



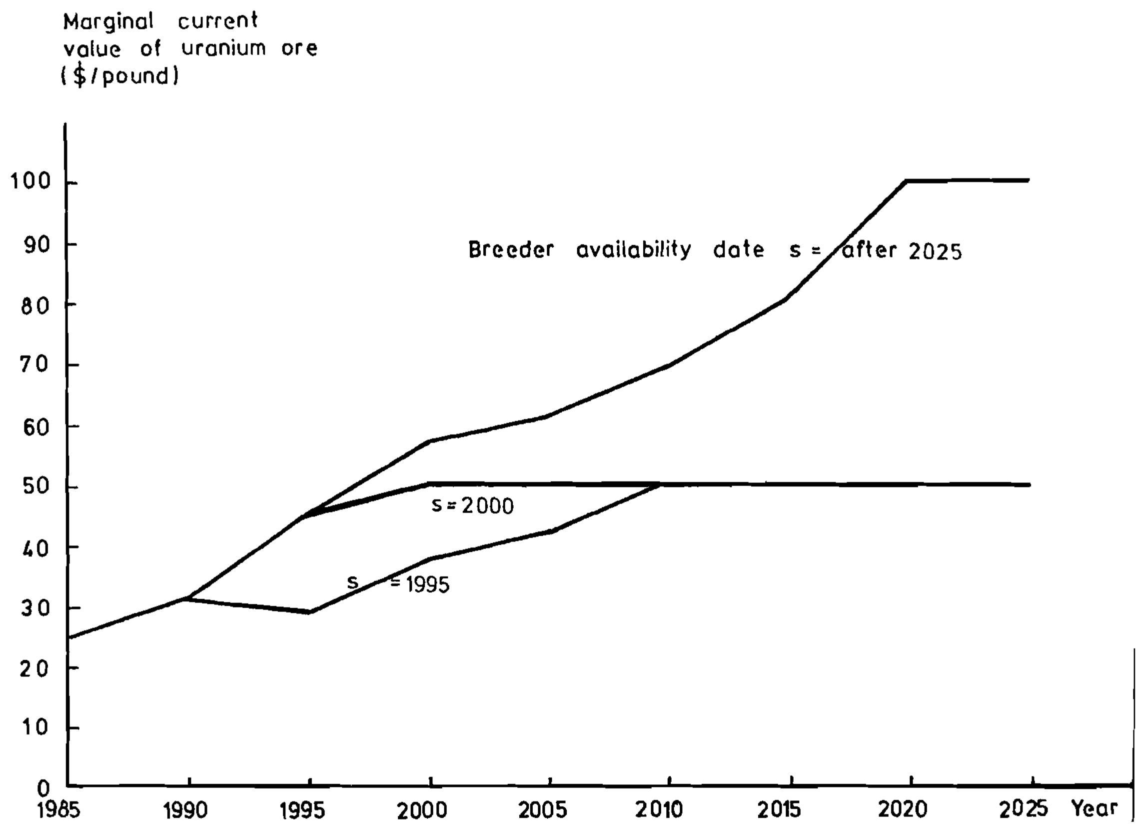
mine ore with a \$50 extraction cost (category m = 3). This corresponds to an annual rate of increase of 8% between 1985 and 2000.

The shadow prices of ore (marginal current values) are shown in Figure 4. In all cases, these are equal to or greater than the ore supply costs of Figure 3. The difference may be interpreted as the scarcity rent on ore deposits (see Nordhaus [17]). These rents depend upon anticipations of future prices. With our pessimistic assessment of breeder availability, the 1985 price is \$25--well above the extraction cost of \$15. In the year 2000, the contingency price is \$57 if the breeder does not become available until after 2025. In this case, the price of uranium is initially much higher than the extraction costs, but it rises at a slower annual rate, 6% per year between 1985 and 2025.

Despite these increases in uranium prices, there are only second-order effects upon the marginal cost of electrical energy--and hence upon the demands for electricity. This conclusion does depend upon the existence of a backup technology--coal-fired fossil plants. Eventually there could be a resource exhaustion problem with coal as well as with uranium. Note, however, that even in the most pessimistic case shown on Figure 5, the cumulative coal consumption would be 655 10^{15} BTU between 1985 and 2025. This is less than 5% of the minable U.S. coal reserves (see Table 8).

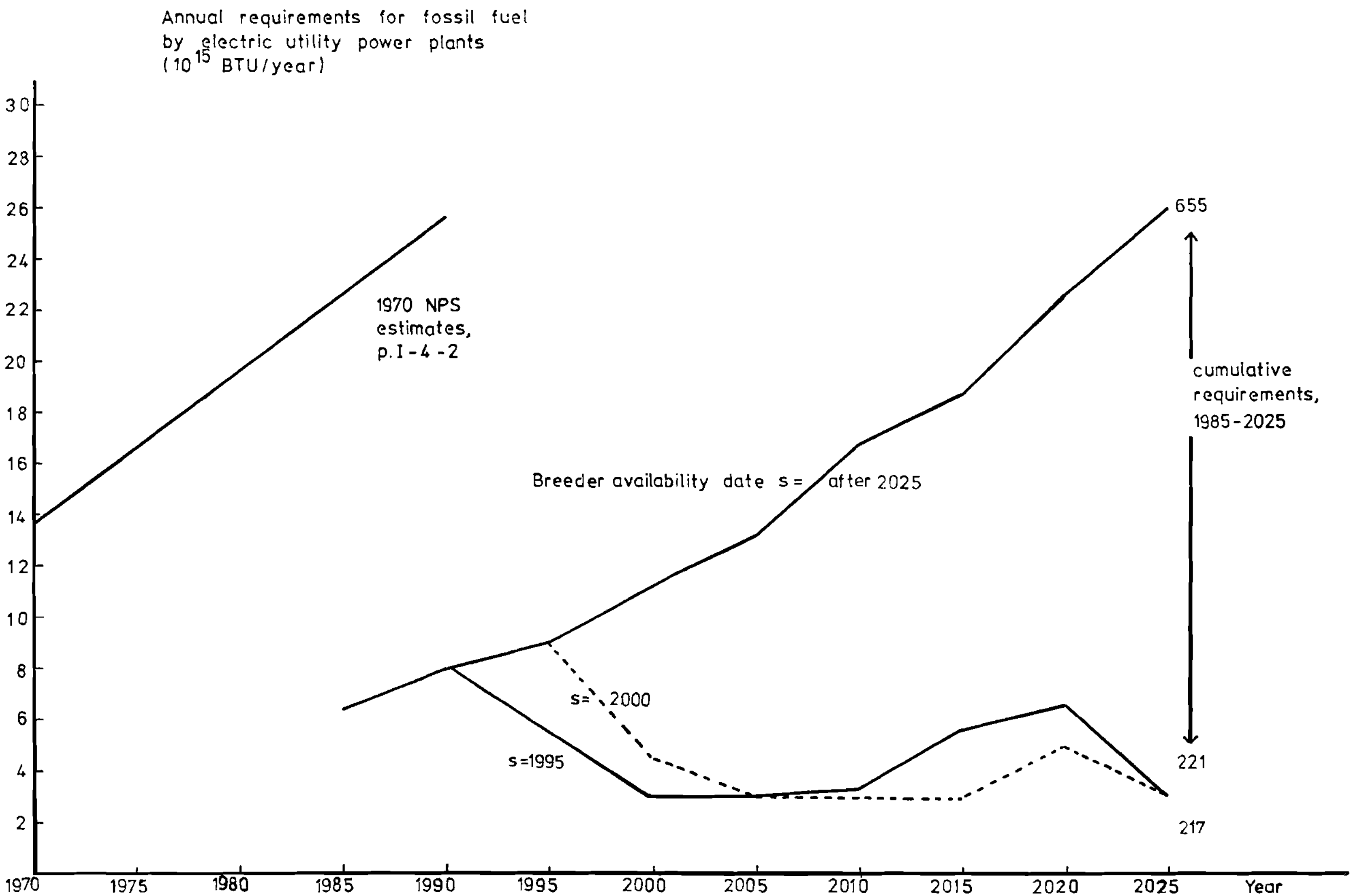
One caution in interpreting the rightmost column of Table 8, the number of "years" of reserves. Production is likely to grow

Figure 4.



Notes: This figure presents the <u>conditional</u> value of the dual variable for row RX (t, s) - given the occurrence of state-of-world s. Present values at time 0 are converted into current costs in year t.

Figure 5.


Annual requirements for fossil fuel
by electric utility power plants
(10^{15} BTU/year)
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
1970 NPS
estimates,
p.I-4-2
Breeder availability date s = after 2025
s= 2000
s=1995
655
cumulative
requirements,
1985-2025
221
217
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
2025
Year

above the 1970 level. In the U.S., for example, suppose that only 50% of the coal is minable, and that production grows at the annual rate of 3% (more or less equal to the long-term growth rate of national income). Then our reserves of coal would last for another 115 years, rather than the 1000 shown in the rightmost column. This still provides ample time to learn how to build either fusion or solar power plants or a safe breeder.

Table 8 indicates that the world's resources are being exploited at very different rates between countries. Over 90% of the coal reserves lie in the USSR, the U.S.A., and China. In an autarchic world, coal will not be a universal backstop to the breeder.

Table 7. Marginal value of electrical energy[a] ($/$10^3$KWH)

year / period t / energy block j	1990 (t=2)	2000 (t=4)	2010 (t=6)	2020 (t=8)	state-of-world s
1.peak[b]	32.7	45.4	42.8	41.6	3
		34.4	46.4	41.9	4
		48.9	49.7	53.2	10
2.intermediate	10.9	9.0	11.9	13.1	3
		10.9	7.9	12.8	4
		11.2	12.0	12.2	10
3.base	7.3	3.5	1.8	1.0	3
		4.2	4.2	1.2	4
		5.9	6.3	8.8	10
average of marginal values[c]	12.9	12.5	12.8	12.8	3
		11.9	12.4	12.8	4
		15.0	15.6	17.2	10

Table 7 (continued)

Notes:

a) This table presents the conditional value of the dual variable for row DM(j,t,s) - given the occurrence of state-of-world s. Present values at time 0 are converted into current costs in year t.

b) Pumped storage provides the possibility of converting 1.5 KWH off-peak into 1 KWH of peak energy. Nonetheless, because of capital and operating costs, it is optimal for the ratio of their marginal values to be much higher than 1.5.

c) This is calculated as a weighted average of the marginal values for peak, intermediate and base-load energy. Following the load-duration curve of Figure 2, these weights are, respectively: 10/65, 30/65 and 25/65.

From the value of the average KWH for year t and state-of-world s, it is possible to infer the reduction of demand below the reference level q_1. Recall that the reference price p_1=\$10/KWH, and that the assumed elasticity of demand is $\eta = -.5$. For example, in the year 2000 the average value is 15.0 $\$/10^3$KWH in state-of-world 10. Since this is 50% above the reference price p_1, the optimal level of demands is therefore 82% of the reference quantity q_1.

Table 8. International comparison of coal reserves and production a)

Country	Total reserves (billion metric tons)	Production, 1970 (million metric tons/year)	Minable b) reserves ÷ 1970 production (years)
USSR	4,122	433	4,800
USA	1,100	550	1,000
China	1,011	360	1,400
India	106	74	700
South Africa	72	55	700
Germany, Federal Republic	70	111	300
Canada	61	12	2,500
Poland	46	140	160
Japan	19.2	39.7	240
Australia	16.0	44.3	180
United Kingdom	15.5	144.6	50
Colombia	12.5	3.3	1,900
Czechoslovakia	11.6	28.1	210
Brazil	10.7	2.4	2,200
Total, countries with more than 10 billion tons of reserves	6,673.5	1,997.4	1,700
World total	6,641 c)	2,131	1,600

Notes:

a) Source: United Nations [20] (pp 177-178). Includes both bituminous and anthracite, but excludes lignite. Refers to the total of measured plus inferred reserves.
Approximate conversion factor: 25 million BTU/metric ton of coal.

b) Assuming that 50% of total reserves are minable.

c) Source document does not indicate why the world's total reserves are less than those of the 14 countries listed here. This appears to be the result of an ambiguity in the U.N. tabulation for the Federal Republic of Germany.

Footnotes

[1]For a review of the pros and cons of these three alternative technologies, see Häfele [11] and Häfele and Starr [12]. Other possibilities each have their small but dedicated groups of enthusiasts--energy from wind, from garbage, and from geothermal sources, including the heat content of the earth's crust plus the oceans.

[2]This insensitivity refers to decisions on electricity demands and supplies--not on the level of funding for breeder research and development. Clearly, the R. & D. decisions depend upon the date of breeder availability. So also are decisions related to the supply of nuclear fuels--uranium enrichment and plutonium recycling. Through side calculations, we have found that these fuel supply options do not imply major changes in the nuclear cost parameters shown below in Table 4.

[3]Circular No. A-94, revised, March 27th, 1972, to the heads of executive departments and establishments from George P. Schultz, Director, Office of Management and Budget. Cited by Cochran [5].

[4]For conflicting views on whether the breeder will eventually become a safe and competitive technology, see Atomic Energy Commission [1], [2], [3], and also Cochran [5].

[5]Power refers to the output of an electricity plant per unit of time. Energy refers to the integral of power output over time.

Example: One kilowatt (KW) of power capacity operated for, say, 10% of the 8760 hours in a year will produce 876 kilowatt-hours (KWH) of electrical energy. The following unit of measurement will also be employed: 10^6KW = 1 GW = 1 gigawatt.

[6] Our 3-step approximation to the load curve is similar to that of Hoffman [13, especially pp. 30]. This particular approximation leads to a bias against one type of generating equipment: gas turbines. Because of their low capital and high fuel costs, it is advantageous to employ gas turbines for reserve peaking capacity, but to plan to operate them for less than 10% of a year--e.g., in the event of an unscheduled shutdown of a thermal plant during the peak-load period. To evaluate gas turbines properly, it would require a fourth energy block, e.g. a 1-3% interval along the load-duration curve. In the interests of simplicity, this fourth block has been omitted, and therefore gas turbines have also been omitted from the model.

[7] Typically, it is optimal to employ a thermal plant as a base-load unit during its early years of operation (mode k=1) and then gradually retire it toward peak-load service (mode k=3) in its old age. This pattern can be observed indirectly through the time series of fuel requirements for fossil plants and for LWR's in Figures 3 and 5 below.

[8] For example, if a steam plant is operated in the intermediate mode 2, this means that it is to be operated at its capacity during block 1 (10% of the year) and also at capacity during block 2 (40% of the year), but that it is to produce no energy during the base-load period (block 3). Since linear programming models allow for convex combinations of activities, other modes of operation are incorporated implicitly. E.g., an optimal solution might specify that 70 GW of capacity are to be operated in mode 2 and 30 GW in mode 3. This would be interpreted as follows: all 100 GW are to be fully utilized during energy block 1, but are only 70% utilized during block 2.

[9] Perhaps the most conventional form of such storage

equipment is hydroelectric pumped storage. Alternatively, high-energy batteries might be used. Still another possibility would consist of electrolytic cells for the production of hydrogen, together with facilities for the storage of hydrogen and for its conversion into peak electrical energy through fuel cells or turbines. Whatever the particular route, these devices are likely to lead to similar overall results--an input of 1.5 KWH of energy off-peak per KWH produced during the peak period. This structure of inputs and outputs is reflected in the two columns labeled UT(6,k,t,s). Note the negative entry of -.15 in the off-peak rows DM(2,t,s) or DM(3,t,s together with the corresponding positive entry of .1 in row DM(1,t,s).

[10]The 1970 NPS is our source for two sets of right-hand side constants: the "initial capacities" in rows UT(i,t,s) and the "fixed hydroelectric energy supplies" in rows DM(j,t,s) of Table 2. It is also our source for the ratio of the mean to the maximum energy demand, .650. (See Figure 2.)

[11]This formulation was suggested by Samuelson [18], who pointed out that a competitive equilibrium could be computed through an optimizing model in which the sum of consumers' and producers' surpluses is to be maximized. For subsequent empirical applications of this idea, see Takayama and Judge 19 and also Duloy and Norton [7, pp. 311 ff.].

[12]Doctor et al. [6, pp. 38] estimated a price elasticity of -.85 for residential consumption of electricity, but their calculations refer to a delivered price, including transmission and distribution. Clearly there will be a lower absolute value of the elasticity of the derived demand for generation.

The reference price p_1 = \$10/10^3KWH is of the same order of magnitude as the generating cost forecast in the 1970 NPS, pp. I-19-2.

[13]To calculate the price-independent cases, the interpolation weight variables WT(ℓ,t,s) are set equal to zero for all demand levels other than ℓ = 1.

[14]Breeders also use uranium, but the quantities are negligible compared with the demands of the LWR's.

[15]Source: Golan and Salmon [10], Table 1. This fuel coefficient is based on enrichment with a tails assay of .25%.

Our calculations omit the natural uranium required for the initial loading of each LWR. According to Golan and Salmon, the initial loading requires 500 tons per GW. This is only 2.8 years' worth of the annual fuel requirements for base-load operations, and will constitute a much smaller part of the ore demands after 1985 than in earlier years.

[16]Income and property taxes are omitted. These are private but not social costs.

[17]In a more refined calculation, one would allow for the fact that not all of the capacity needed in period t must be installed at the beginning of that period. One would also allow for an initial period of start-up difficulties for each new plant. These factors are largely offsetting, and so we have omitted them here.

[18]For peak storage, the direct fuel cost is zero, but there is an indirect cost: the input of 1.5 KWH during off-peak hours per KWH generated during the peak hours.

[19]All cost comparisons are made in terms of the 1974 U.S. price level, and are not adjusted for general inflation thereafter. No allowances are made for the gradual improvements that will be made in each of these processes over time.

[20]These average costs all lie somewhat above the reference price assumed for the interdependent demand calculations.

From section 4, recall that $p_1 = \$10/10^3$KWH.

[21]For a review of the costs of alternative coal conversion processes, see Hottel and Howard [14, ch. 3]

[22]Within the nuclear fuels industry, it is likely that there would be a higher value for such information. E.g., the profitability of uranium mining and enrichment will depend upon the number of years that LWR's remain as base-load units before being displaced by breeders. (See Figure 3 below.)

[23]In the case of independent demands, the optimal 1990 capacity mix is similar to that shown in Table 6. The total, however, is higher: 1215 GW. Except for minor variations introduced by the reserve capacity factors B(i), this total is constrained to be identical with the 1185 GW estimated by the 1970 NPS.

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