

# Optimal Forest Management with Stochastic Prices & Endogenous Fire Risk

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**Abstract** – Earth observations are one way to reduce the risk to standing forests from damages caused by wild fires, since they enable early warning systems, preventive actions and faster extinguishing of fires, before they spread out. Another channel through which fire hazard can be reduced is the thinning of the forest, so the risk of a fire occurring becomes partially endogenous. In order to shed more light on optimal forest management under such endogenous fire risk, we develop a real options model, where the price of biomass is stochastic and the harvesting decision needs to be timed optimally in the face of these uncertainties. We find that there is a positive value of information. In other words, there is a positive willingness to pay for Earth observations by forest managers.

**Keywords:** value of information; Earth observation; endogenous risk; real options

## 1. INTRODUCTION

The recent upsurge in large-scale wild fires as for example in Austria has raised the public's awareness of the need to establish mechanisms to accelerate fire extinguishing, evacuation and – if possible – the prevention of the fire in the first place. However, such mechanisms are useless, if there is not sufficient information about the incidence and location of the fires. Khabarov, Moltchanova and Obersteiner (2008) conduct simulation studies to estimate the benefits of a finer grid of weather stations and more frequent patrols in forest areas, so that wild fires can be detected earlier and – if not prevented – at least limited or extinguished before they can spread to a larger area and thus cause economic damage and endanger the life of humans and animals. They find that the addition of more weather stations indeed reduces the fraction of the area burnt by wild fires. The value of Earth observations is thus substantial for the information of wild fire containment activities and will have decisive influence on forest management issues such as optimal rotations.

### 1.1 Motivation

The forest sector is characterized by irreversible decision-making (the cutting of trees) under uncertainty (the risk of fire and the price of biomass). The problem setup therefore qualifies for the application of real options theory (Dixit & Pindyck, 1994). The main idea behind real options is that sufficiently large uncertainty makes it worthwhile to postpone irreversible decisions. In the case of forestry management, the decision to cut trees will be postponed until more information about the price that can be earned by selling the wood becomes available. In other words, the option to cut has a “waiting” value, which initially exceeds the immediate profits from cutting and selling trees. The real options approach then enables the determination of the optimal timing of this decision.

The other source of uncertainty we want to consider and actually focus upon in this study is fire hazard and the potential impact of Earth observations in this context. In fact, if the incidence of fire can be reduced due to quick extinguishing or even prevention actions, then this will have substantial impacts on profits and – as will be shown in the analysis – behavioral patterns will change as well.

In this paper we model fire risk to be increasing in stand age, which is of course dependent on the rotation decisions. The fire risk is therefore partially endogenous to the optimization of the forest manager's behavior, as will be seen in the results. The analysis is thus not only useful to evaluate the benefits from Earth observation, but it is also interesting from the theoretical point-of-view: González Olabarria (2006), for example, finds in his doctoral thesis that an increase in fire risk leads to shorter rotations if fire risk is purely exogenous. If the risk is considered to be endogenous to stand management, a clear effect of risk on the thinning regime was observed, with earlier and more intensive thinning as fire risk rises. This shows that the endogeneity of fire risk is a decisive factor in forest management and should therefore not be neglected in optimal rotation analyses.

This study builds on earlier work by Huang and Jana Szolgayova (2007), conducted at IIASA. The framework is extended to include stochastic biomass prices and models fire risk differently. This makes the model suitable to investigate the benefits of Earth observation in a novel way, while at the same time making a contribution to the existing literature on real options modeling of forest management.

### 1.2 References to Related Work

Optimal forest management modeling has a long history in forestry economics, going back as far as the mid-19<sup>th</sup> century, where Faustman's (1854) seminal work set off an ongoing debate about the right approach for determining optimal rotations in forests. An excellent overview of the related literature can be found in Chladna (2007). In this section we rather want to focus on the application of real options to forestry management and on the literature concerned with fire risk in particular.

Morck et al. (1989) was probably the first paper introducing real options to forestry economics. While Morck et al. (1989) used the contingent claim approach to value forestry lease under uncertainty about timber prices and timber inventories, later work by Thomson (1992) modeled these decisions in a discrete framework and used the lattice method. Most of the more recent real options literature in forestry management decision-making focuses on stochastic biomass prices and forest growth or combinations of the same (e.g. Insley, 2002; or Saphores, 2003). Alvarez and Koskela (2004) have further investigated the impact of stochastic interest rates. A more elaborate review of these studies can be found in Chladna (2007). Her work differs from the other studies insofar as she considers the optimal rotation period

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under the assumption of stochastic wood and stochastic carbon prices, regarding the forest as a potential carbon sink giving it some value beyond the supply of biomass.

With respect to fire risk and its impact on forestry management, there are several studies computing the rotation age maximizing the profits from selling timber. Seminal work includes Martell (1980), Routledge (1980) and Reed (1984). While Martell (1980) and Routledge (1980) use discrete time and fire probabilities, which are dependent on stand age, Reed (1984) considers optimal rotation in a continuous time framework, where fire risk is independent of stand age. Later work also included other aspects to the analysis. Caulfield (1988), for example, incorporated risk aversion.

As regards the methodologies employed in these frameworks, a diversity of approaches can be observed: Cohan et al. (1986) use decision trees to analyze different sources of uncertainty, including fire, to find optimal decisions concerning fuel and timber management. Reed and Errico (1986), make use of Monte Carlo simulations to determine the optimal harvesting schedule. Gassmann (1989) optimizes the expected harvested timber volume of timber over a finite planning horizon, where random parts of the forest can be lost due to fires. Boychuk and Martell (1996) employ multistage stochastic programming methods to optimize forest management given that timber supply should be maintained in the long run.<sup>1</sup> A more comprehensive overview can be found in González Olabarria (2006).

The work presented in this paper will differ from the reviewed literature in various points: it will include both a stochastic timber price and stochastic endogeneous incidence of fire dependent on stand age and density; at the same time maintaining a wide range of decisions available to the forest owner. It will also explicitly employ a real options approach and the associated valuations and interpretations thereof; and the model will be formulated so as to enable an analysis of decreased fire hazard due to Earth observation and the impact of this on income and forestry management behavior.

## 2. MODELING FRAMEWORK

### 2.1 Forest management data

Similar to Huang (2007), we use the Forest Inventory and Analysis (FIA) database, documentation on which can be found at ([http://www.ncrs2.fs.fed.us/4801/FIADB/fiadb\\_documentation/S\\_NAPSHOT\\_DB\\_V2pt1\\_JULY\\_2006.pdf](http://www.ncrs2.fs.fed.us/4801/FIADB/fiadb_documentation/S_NAPSHOT_DB_V2pt1_JULY_2006.pdf)). FIA data are collected on a periodic basis. The database has a uniform data structure for forestry inventories. It contains extensive data on stand age, stand size, diameter, stocking status, height, species and other attributes. Table A shows the data extracted from the FIA database for one the 12 southern states of US. Using a statistical software package, plot level data were used to generate descriptive statistics for loblolly pine.

Table A. Descriptive statistics for loblolly pine

Variables	unit	mean	std. dev.
Growing stock volume	cubic feet/acre	1333.5	1110.89
Stand age	Years	18	7.771
Stand density	100 trees/ acre	3.92	3.396
Site productivity class	-	3.8	0.992

For the estimation of the forest's growth, we first use the basic S-

shaped Richard's function:  $GSV_i = a \cdot e^{-b \cdot \ln^2 \frac{X_i}{c}}$  to estimate the average growing stock volume (GSV) per tree on plot  $i$  depends on the stand age on plot  $i$ , where  $a$  is the maximum value of GSV per tree, which is about 143 cubic feet in our case. Parameter  $b$  refers to the shape and  $c$  is the maximum age.

Since a single tree on a forest stand will obviously grow faster (because of the ample supply of water, light and nutrients) than a tree in a full-stocked stand, we extend the single tree model to a model of a stand by employing the self-thinning line used in Huang (2007). Finally, since two thinnings of prescribed intensity during one rotation period are available as a managerial decision in the real options model, we extend the volume function to describe the volume for each stand age and each thinning decision possible.

We also use the same method as Huang (2007) to estimate the diameter as a function of GSV per tree, which is an increasing relationship (but at a diminishing rate) given the data.

### 2.1 Real Options Model

Given the growth model referred to in the last section, we can now turn to the real options model to determine the optimal forest management schedules in the face of fire risk.  $X$  defines the current status of the forest stand. It is thus a vector including stand age and thinning status, where the latter is described by the year/years of thinning. Knowing  $X_t$  therefore implies knowing the site-specific stand GSV and the average diameter at time  $t$ .

The wood price is modeled as a stochastic, mean-reverting process, where the stumpage price data are assembled from Timber Mart-South (TM-S). The data is for three product classes, which are specified as (1) pulpwood (PW) at a d.b.h. of 4 to 9 inches, (2), chip-n-saw (CNS) at a d.b.h of 9 through 11 inches, and (3) saw timber (ST) with a d.b.h greater than 11 inches. For products that are smaller than pulpwood the biomass value is considered. The price per tin in US\$ is 6.42 for PW, 25.8 for CNS, 40.97 for ST and 1 for the biomass value. Using these data, the product price is modeled as a function of the diameter, where we will use both a step-function and a continuous function and compare the results.

We consider both the state  $X$  and price  $P$  to be Markov processes, which means that the information for determining the probability distribution of future values of  $X$  is summarized in the current state  $X_t$  and is independent of past states.

Following Huang (2007), planting costs are linear in planting density. Per-acre costs of growing trees are based on current loblolly pine plantation practice. The cost on burned land is lower than that on unburned land because less soil preparation is required.

Forest fires occur according to a Poisson process with arrival rate  $\lambda$ . The impact of the fire is the destruction of the total stock volume. The probability that a fire occurs is  $(1 - e^{-\lambda})$ .  $\lambda$  is modeled as a function of stand age and stand density.  $\lambda$  is assumed to be decreasing with stand age, since older trees will be more resistant

<sup>1</sup> Multi-stage optimisation problems can be formulated in such a way as to answer the same questions as real options models. However, they have the tendency to become computationally intensive when there are many periods and scenarios, since it requires decision-making at each stage depending on the prior history of states. Cheng et al (2004) compare the two approaches and their advantages and disadvantages in more detail.

to fire. As fires are more probable on a denser forest stand,  $\lambda$  is increasing with stand density. Thinning thus helps to reduce fire risk in addition to fostering forest growth.

We assume the decisions can be made on a yearly basis. There are three managerial actions the forest manager can perform in each time period: (1) thin, (2) harvest and (3) do nothing. The action performed and the resulting state of the forest stand will then determine the immediate profits, i.e. the difference between the income from selling the harvested wood and the costs of doing so. The model determines the decisions (in the optimal control sense) that maximize the sum of expected discounted profits.

The associated optimal control problem can be solved recursively by fixing the terminal value to zero and choosing the optimal action,  $a_t^2$ , to maximize the following value function for all possible future states (depending also on fire occurrence) and wood price instances:

$$V(X_t, P_t) = \max_{a \in A(X_t)} \{ \pi(X_t, a(X_t, P_t), P_t) + e^{-r} E[V(X_{t+1}, P_{t+1}) | X_t, P_t] \}$$

This so-called Bellman function is composed of the immediate profits denoted by  $\pi(\bullet)$  that are received upon harvesting and the continuation value (second part of the sum), which is the expected value of the value function in all possible future states and for all possible future prices given today's prices and state. The second part is thus the value from waiting and we compute it by using Monte Carlo simulation. This method was chosen, since it remains computationally efficient for a high degree of complexity and is rather precise when the discretization of the price is sufficiently fine. The output of the process is a table with the optimal action for each time period  $t$ , for each possible state  $X$  and for each possible wood price at that time,  $P_t$ . To obtain the final outcome, we conduct 10,000 simulations and extract the corresponding decisions from the output table.

### 3. RESULTS: OPTIMAL ROTATIONS & LOSSES DUE TO LACK OF EARTH OBSERVATIONS

#### 3.1 Optimal rotations & the impact of different product price functions

The optimal rotations have been computed with the above model for both a step-wise price function and a continuous price function for a range of different fire arrival rates. Figures 1 and 2 show the median of optimal forest management decisions for these continuous and step-wise price functions respectively.

What can clearly be seen from both Figures is that higher fire risk (i.e. a larger probability that a fire will destroy the stand) leads to earlier harvesting, i.e. shorter rotations, and also to earlier thinning. In the continuous case, these relationships are smoother than in the step-wise case, since the harvesting time drops more drastically, once a threshold is surpassed, and stays constant until the next "step" of the price is reached. Note also that the continuous case starts out with an optimal harvesting time of 33, which falls to 29, as the fire risk increases, while the step-wise case starts out at 40, dropping to 26 with higher risk. The magnitude of the impact is thus also larger when stepwise prices are used.

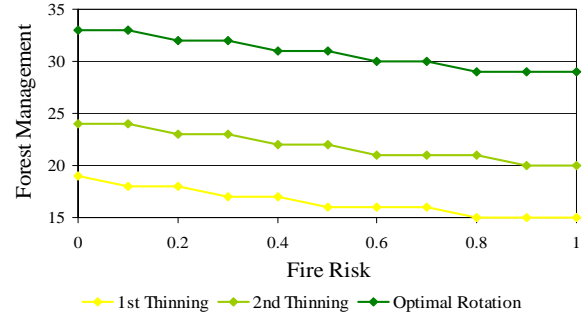


Figure 1. Fire risk impact on decisions with continuous prices.

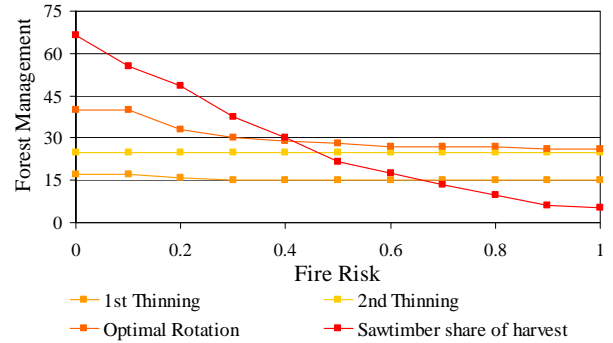


Figure 2. Fire risk impact on decisions with step-wise prices.

Reducing fire risk by obtaining better information through EO will therefore lead to longer rotations and thus also higher-quality wood output: in Figure 2, the share of saw timber *rises* with *falling* fire risk. Since saw timber commands a higher price than chip-n-saw wood, this could lead to higher expected profits. The following section will therefore be devoted to analyzing the impact of fire risk on expected profits and the associated distributions.

#### 3.2 Value of information & analysis of distributions

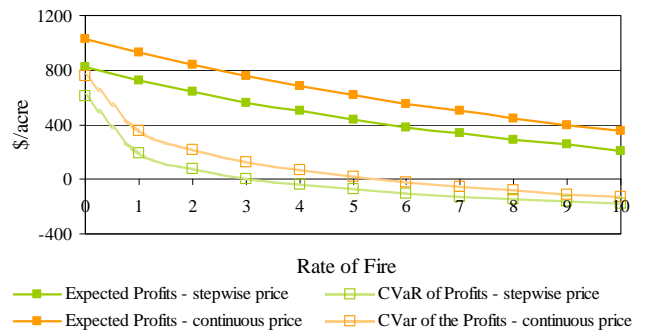


Figure 3. Impact of fire rate on profits in \$/acre.

Earth observation can be represented in the model by its impact on fire risk. As already mentioned, earlier studies (Khabarov et al., 2008) show that Earth observation shortens response times, i.e. fires can be extinguished before major damage has occurred. Since fire risk in our model implies the loss of the whole stand, Earth observation results in lower fire risk. Therefore, we can determine the value of information from Earth observation by comparing profits for different fire risk rates.

<sup>2</sup> The actions are element of the set of feasible actions,  $A_t$ , where e.g. thinning is not a feasible action in the year following a harvest, i.e. it is not element of  $A_t$  in that time period. The action determined is a function of state and price in that year.

Figure 3 shows that expected profits fall, as the rate of fire increases, which is in line with our considerations from the previous section. In addition, we can analyze the the impact of Earth observation (through a decrease in the rate of fire) on risk in terms of expected profits, measured by the Conditional Value-at-Risk (CVaR). The CVaR of profits is a risk indicator calculated as the average of the lowest 5% of profits and Figure 3 shows that CVaR-risk is rising with increasing fire risk. For high rates of fire risk, we see that the CVaR of the profits might even be negative, since for simulations with more frequent forest fires, you would harvest so early that the costs exceed the income from selling chip-n-saw wood.

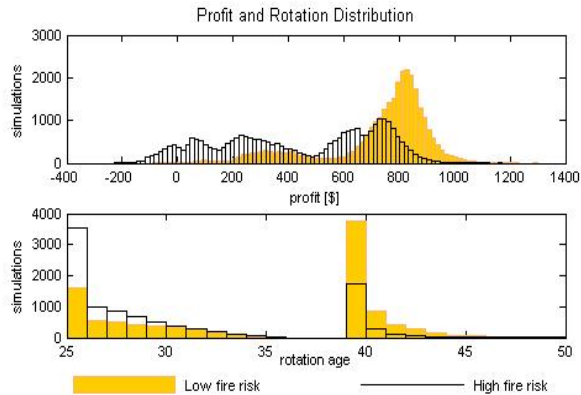


Figure 4. Profit distributions with different degrees of fire risk.

These insights are confirmed in Figure 4, where the distribution of expected profits for low fire risk (yellow) is much narrower than the one for larger fire risk (upper panel). Furthermore, the lower panel indicates that the average harvesting time increases substantially, as fire risk decreases. Together with the fact observed in the previous section – that the share of saw timber increases with falling risk – this observation explains the gains in expected profits that can be seen in the upper panel of the same panel and in Figure 3.

#### 4. CONCLUSION

This paper has presented the results of a study on optimal rotations in a real options framework, where the major source of uncertainty is fire risk. The fire risk being defined as loss of a forest stand in case of fire, the results have shown that Earth observation can lead to considerable gains in terms of expected profits and risk by reducing the fire risk. Rotations will be longer as a result of more security and the share of saw timber can be increased substantially.

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