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FORECASTING INDUSTRIAL WATER USE

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Preface

Interest in water resources systems has been a critical part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design and operational decisions.

In the years of 1976 and 1977 IIASA initiated a concentrated research effort focusing on the modeling and forecasting of water demands. Our interest in water demands developed from the generally accepted realization that these fundamental aspects of water resources management have not been given due consideration in the past.

This paper, the fifth in the IIASA water demand series, reports on various methods of forecasting industrial water use. Essentially, two basic approaches are distinguished. The first approach is to directly project water use by trend extrapolation, by application of fixed water use coefficients, or by application of multiple regression analysis. The second is to develop an explanatory model of industrial water use (statistical or engineering-economic models), and then project changes in the variables of the model in order to forecast future water use. The paper also discusses how to estimate and use demand functions for forecasting water use.

Janusz Kindler
Task Leader

Abstract

This paper provides an introduction to the theory and analytical methods of forecasting industrial water use. Two basic approaches to the problem are presented--forecasting with and without the use of demand functions for water. Section 2 discusses the widely used water use coefficient method and regression techniques for forecasting water use. Section 3 reviews the statistical and economic-engineering models for deriving industrial water demand functions. Section 4 illustrates how water demand functions can be utilized in forecasting exercises and discusses the difficulties of forecasting industrial water use.

Acknowledgements

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1 Introduction

In the last twenty years much professional effort has been invested in both theoretical and applied water resources planning. This interest is easy to understand; in both the developed and developing countries water resources projects constitute a significant portion of public capital investments. This paper surveys the methodologies and state of the art of one of the most neglected and yet important aspects of many water resources projects-- modeling and forecasting industrial water use.

The term "water demand" as commonly used in the water resources literature is ambiguous and often misleading. When referring to "industrial water demand" it is necessary to make a distinction between the "demand function" and "supply and demand equilibrium." The amount of water an industry will use in the future depends on numerous factors, one of which is the price of water itself, (or more generally the marginal cost of water use). If we assume that all other factors remain constant and we raise the price of water, two things could happen [1]. Since the price of water is higher, the industrial plant may use less water. This is represented by a standard downward-sloping demand curve. Alternatively, when the price of water rises, the plant's use of water may remain unchanged. The quantity of water demanded remains the same whatever the price charged.

Which of these two situations is most accurate in any given case over the relevant range of price changes is an empirical question. For example, if we define the commodity "water" broadly to include all H_2O in food and drink, the human body has a very inelastic demand for water. There are no substitutes. On the other hand, the evidence for most industrial plants indicates that industrial water use declines when prices rise. In both cases, however, there is a relationship between the quantity of water demanded and the price of water. Economic theory also includes the prices of other factors of production and the level of output as determinants of the quantity of a "factor of production" demanded. Thus, the standard industrial demand function for water states that the quantity of water demanded is a function of the price of water, the prices of other factors of production, and the level of output.

The quantity of water actually used and the price of water are, however, simultaneously "determined" by both this demand function and the supply function. Over time both the supply and demand functions will change due to changes in population, tastes, technology, income, and numerous other factors. As illustrated in Figure 1, the demand function is commonly assumed to shift upward and to the right due to increases in income or output. The supply curve is commonly assumed to shift downward and to the right due to improvements in technology. Over time the shifting supply and demand functions trace out a series of equilibria.

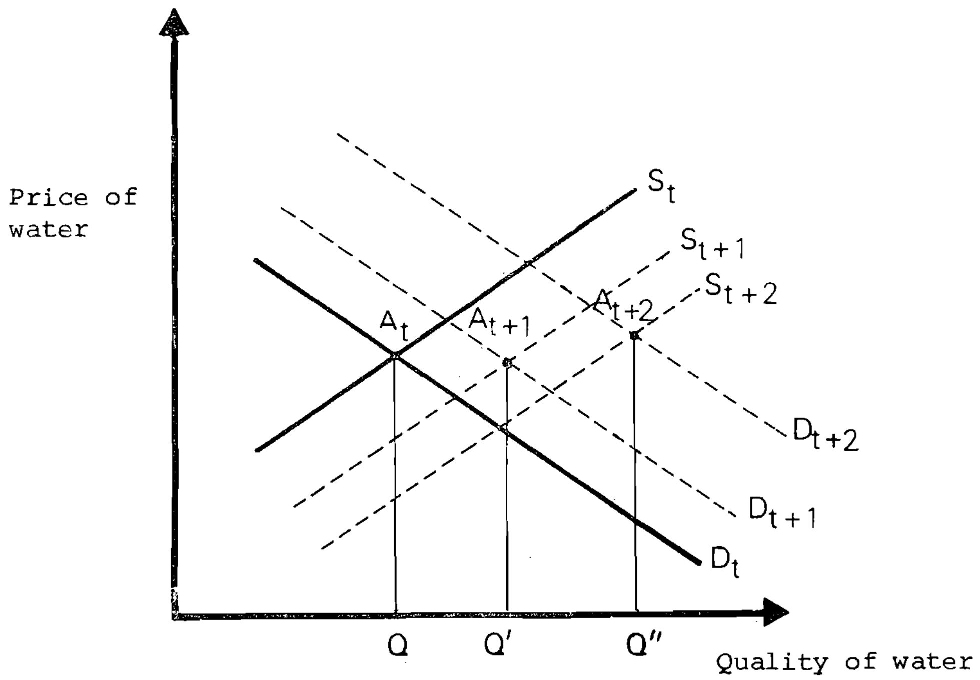


FIGURE 1

Thus, the industrial facility uses quantity of water Q at time t, Q' at t+1, and Q" at t+2. The upward shift of the demand curve and the downward shift of the supply curve are merely commonly observed trends for many commodities. Demand could, of course, shift leftward due to a fall in income or output, or technological change. Both the direction and the magnitude of the shifts are questions for empirical investigation.

There are essentially two approaches to forecasting industrial water use. The first is to attempt to project directly the equilibrium points A_{t+1}, A_{t+2}, etc. The second is to estimate both the supply and demand functions, project their movement over time, and then solve for the supply and demand equilibrium in each period. Either approach can be used for short run or long run forecasting.

This second approach is essentially a two step process. The analyst first develops an explanatory model of industrial water use and then projects changes in the variables in the model in order to forecast future water use. If the analyst has estimated the demand and supply functions, short run forecasting may be no more than an exercise in comparative statics. For example, if the only change expected is the increased cost of supplying a given quantity of water, one can estimate the resulting decrease in the quantity of water used (Figure 2a). Alternatively, if the demand function for water is expected to shift due to economic growth and the supply curve remains unchanged, one can estimate the increase in the quantity of water used and the increased price necessary to choke off excess demand (Figure 2b).

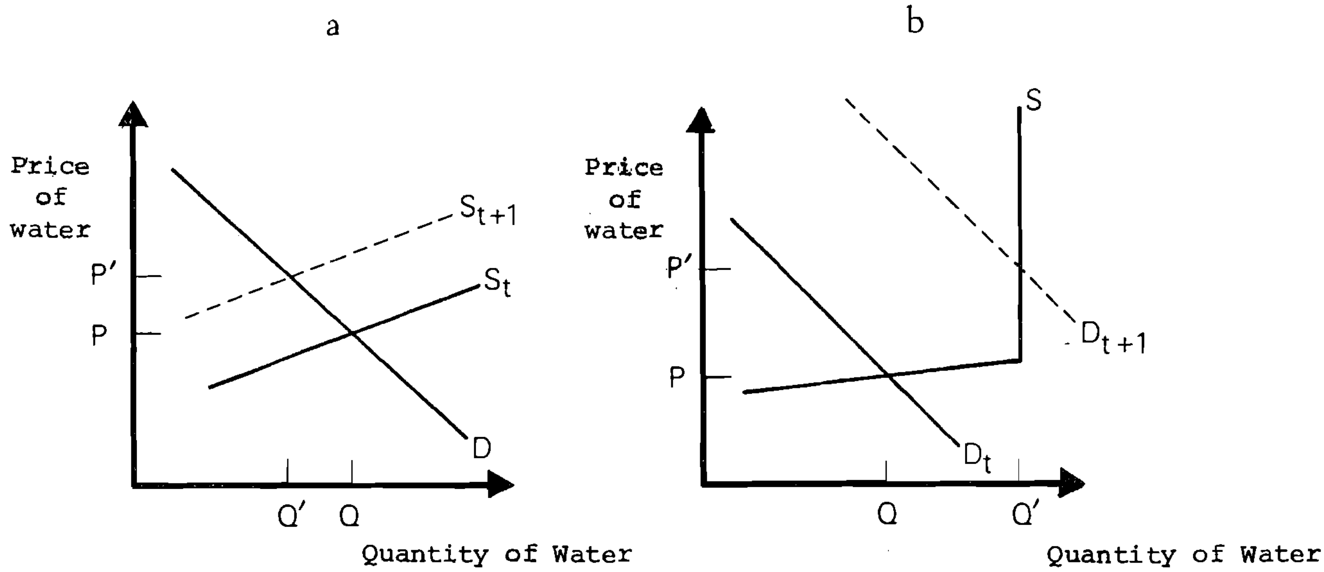


FIGURE 2a & 2b

This paper describes these two forecasting methodologies and the analytical techniques commonly used to implement them. The primary focus is on the estimation of industrial water demand functions because we feel that this is an area of analysis which must be further developed to ensure more effective planning of water resources development. The supply side of the problem has received extensive effort, but water resources planners have typically handled the demand side in a very ad hoc manner. The quantity of water used has been assumed to be insensitive to the price of water (or, more generally, the marginal cost of water use) and thus simply a "requirement". This has recently proved inadequate in many countries [2].

This situation in water resources planning is in many ways analogous to the difficulties faced by utilities in forecasting electricity demand. Historically electricity use grew at a steady, constant rate. Relative price changes were small, and, in fact, real prices often declined [3]. Forecasts based on an extrapolation of this historical growth trend proved very accurate. Recent increases in relative prices, however, have resulted in reductions in actual use. Similarly, water use forecasts which assume constant relative prices are becoming very unreliable. The high costs of bringing additional water supplies from areas farther and farther away from the area of use, and the imposition of water pollution control requirements have greatly increased the costs of water use in many countries and seem to ensure even higher costs in the future.

Water demand functions are not simply an added analytical sophistication to be tacked on to sound water resources engineering. They provide necessary information for the determination of more socially optimal decisions regarding water resources development. Until the relevant demand functions are known, the benefits of a policy or project cannot be estimated. This point is often overlooked when one thinks in terms of water "requirements" because the water requirements approach assumes that the benefits from water use are infinite, and the requirements must be met whatever the cost.

Much of the confusion in the water resources literature concerning the economic aspects of water use and the simultaneous determination of quantity and price arises from the different meanings and interpretations which can be attached to the terms "water supply function" and "price of water." In the classical competitive equilibrium, the supply curve is characterized as upward sloping--the higher the price, the greater the quantity producers will be willing to supply. Competitive supply and demand conditions, assuming no externalities, ensure that the price is equal to the marginal cost of supplying the commodity. Thus, the supply curve is simply the upward sloping portion of the marginal cost curve where profits are nonnegative.

This simple competitive model is not an accurate description of the water supply industry in either market or centrally planned economies. Historically, water supplies have either been available to industry free of charge or regulated by governmental authorities. Governments face no market pressures to charge the marginal cost for water supplies; few have ever considered doing so. Nevertheless, the simple supply and demand model is important in water resources planning because it illustrates the determination of the socially optimal level of water use. If the supply curve reflects the marginal social costs of supplying water and the demand curve represents the marginal social benefits of water use, then at the supply and demand equilibrium water supplies are developed to the point where the marginal costs equal the marginal benefits. If any more water is provided, the social costs to society of supplying the water exceed the social benefits of using it. Thus, this conceptual framework defines the optimal quantity of water use and is common to both market and centrally planned economies. The price mechanism is only a means of achieving this optimal quantity of water. If the economic planning unit knew the optimal quantities of water for each user and could enforce its directives, it could simply order that the optimal quantity be used.

The problem for the water resources planner is then to ensure that the industry uses the optimal quantity of water or, alternatively, that the industry faces the appropriate incentives to use the socially optimal quantity of water. In this context the term "price of water" generally means the marginal social cost of the industry using a gallon of water, including charges for withdrawal (assuming they reflect the marginal cost of supply), intake treatment, pumping, and ultimately disposal of the water and the residuals it may be carrying. One can, however, conceptualize water pollution control requirements as a tax which impacts either the supply or demand side of this "market for water." If the impact of water pollution abatement requirements is included on the demand side, then the marginal benefits to the industry of using water are reduced due to this "pollution tax." The demand curve shifts downward to the left, as shown in Figure 3.

This case corresponds to the policy of most market economies of "polluter pays," i.e., the industry is responsible for paying the pollution "tax". Alternatively, if we view the impact of water pollution control requirements on the supply side, then the marginal social

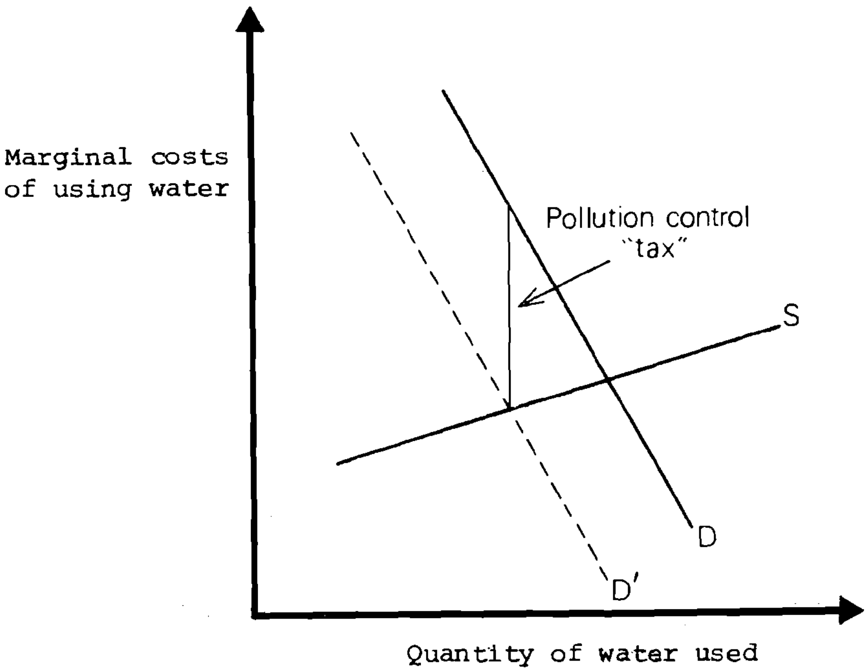


FIGURE 3

costs of using water increase, and the supply curve shifts upward to the left. This particular example simply illustrates the economic principle that the effect of a tax upon quantities and relative prices is identical whether the tax is levied on the suppliers or the demanders.

This paper does not investigate different means of separately forecasting such interrelated concepts as water intake or withdrawal, gross water use (the amount of intake water which would be used if there was no recirculation), water consumption, and wastewater discharges (the difference between water intake and water consumption), unless the methodology requires this level of sophistication. The analyst must, of course, utilize a model relating these concepts if he is to forecast more than one of these variables. Conceptually we can distinguish between three factor inputs to the production process: 1) water intake, 2) gross water applied, and 3) waste disposal services of the environment. However, unless the model handles these inputs within an optimization framework, we simply refer to 'water use' in the general sense in which water is an input to production and the 'price of water' is simply all the costs of using a unit of water.

One final introductory comment should be made about forecasting industrial water use. The water resources analyst should clearly distinguish between forecasting water use given the current institutional structure for pricing and regulating water supplies and wastewater discharges, and forecasting the socially optimal level of water use. The distinction is thus between forecasting what will happen and what should happen. Presumably the analyst is forecasting water use because his organization has some influence on the determination of water policy. The analyst is thus not simply interested in forecasting what will happen, but is rather concerned with the choice between alternative policies given the constraints within which he and his institution operate.

For example, suppose the fundamental decision confronting the water resources authority is whether to develop additional water supplies or to restrict water use through higher prices or rationing. Suppose the analyst projects future equilibria of the demand curves of the water users and the supply curves reflecting current institutional arrangements. Assume users are paying less than marginal social costs. He finds a gap between currently available supplies and his forecast level of water use. If the analyst can change the existing institutional arrangements so that users pay the marginal social costs of water use, he should not then argue for increased reservoir capacity or inter-regional transfers because the forecast level of water use is higher than is socially optimal. However, suppose the analyst feels that it is politically infeasible to increase the cost of water and that no matter what his organization does the users will continue to pay less than the marginal social costs of using water. He also feels that when this forecast shortfall in water capacity occurs, the available water will be utilized by low priority users and high priority users will bear the effects of nonprice rationing. In this case the water resources analyst may be justified in developing the additional water supplies. Even though this is a suboptimal solution, it may be the best the analyst can do given the institutional and political constraints he faces.

2 Methods of Forecasting Water Use which do not Utilize Demand Functions for Water.

2.1 Water Use Coefficients

The simplest approach to forecasting water use is to find one or more variables which have been highly correlated with water use in the past and to assume they will continue to be correlated with water use in the future. The most obvious such variable is time, and the simplest forecasts of water use assume that the water use will continue to grow at the same rate as it has in the past. As illustrated in Figure 4, this involves an extrapolation of a trend fitted to the data for quantity of water used at a specific time. The trend can be linear or nonlinear.

This projection method attempts to forecast supply and demand equilibria. No demand function is required, and there is no assurance that the forecast points will, in fact, represent stable equilibria. Other variables closely correlated with time, such as economic or population growth, would be preferable if they provided a better fit to the data. There is, however, no behavioral theory of water use underlying this projection method. Although economic and population growth appear more intuitively reasonable, the increase in crime or number of airports might better explain the historical water quantity data and could be a better predictor of water use.

A slightly more sophisticated version of this basic method is to assume that water use is a fixed relationship with some other variable such as product output and then to independently project this other variable. For example, suppose that an industrial facility withdraws 1,000 gallons of water per ton of product, and we have the following projection of product output as shown in Figure 5.

Thus, in 1977, the facility is withdrawing 150,000 gallons per day,

(150 tons/day)(1000 gallons/ton) = 150,000 gallons/day

and we project that it will be withdrawing 250,000 gallons per day in 1980.

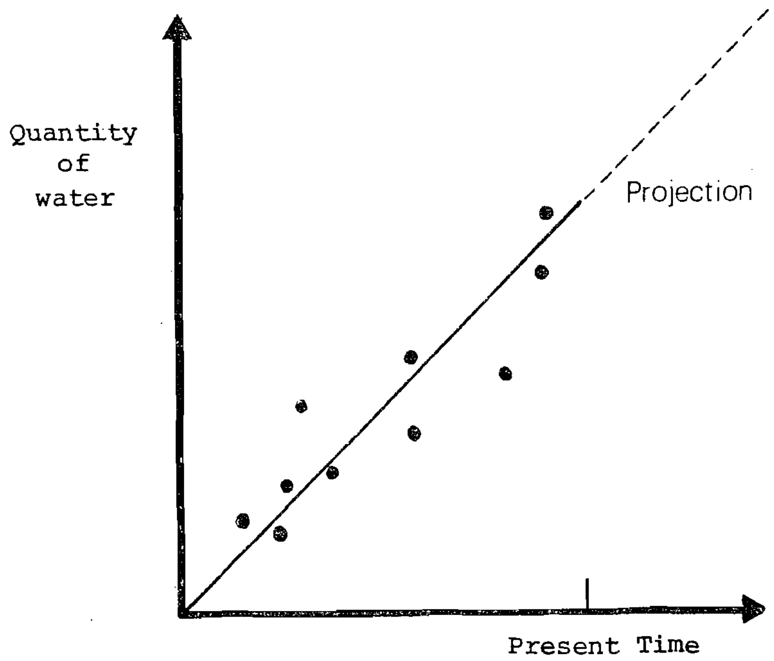


FIGURE 4

This method assumes that water use is fixed in terms of the variable selected. Numerous variables could be used, such as number of employees, value added per ton of product, or tons or volume of input or output (e.g., gallons of water per barrel of crude oil input to a petroleum refinery). For a particular industry some variables would seem preferable on theoretical grounds, but it is an empirical question which variable explains the past variations in the data most accurately. The coefficient approach can be elaborated in several ways; we review several.

The coefficients of water use can be related to more detailed classifications of products and production technology, depending on the data available. For example, suppose we know that gallons of water used per ton of product A is 125 gallons per ton and of product B is 175 gallons per ton. In 1977 the industrial facility is producing 75 tons of A and 75 tons of B. We may not only project the total output of (A + B) to be 250 tons per day in 1980, but also that the product mix will change so that

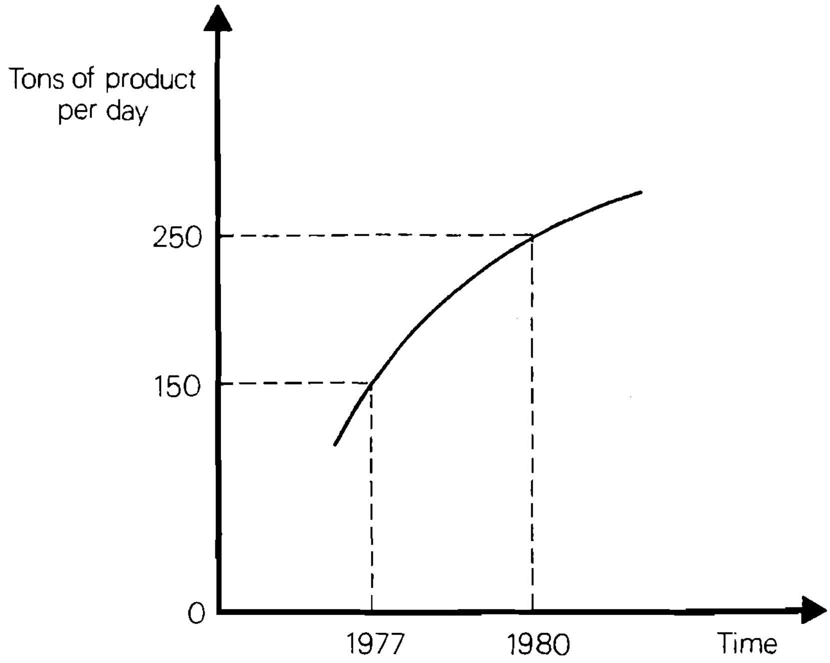


FIGURE 5

by 1980 product A accounts for 75% of the output and product B for 25%. In this case, total water use per day in 1980 will be:

water use in the production of Product A per day in 1980

(.75)(250 tons/day)(125 gallons/ton) = 23,437.5

plus the water use in the production of Product B per day in 198

(.25)(250 tons/day)(175 gallons/ton) = 10,937.5

Therefore, total water use per day in 1980 = 34,375 gallons per day. We could also, of course, predict A and B independently and add the projections to obtain the total.

In addition to projecting product mix, water use coefficients can be associated with different production technologies. Suppose we want to project water use in the iron and steel industry, and we know that water use per ton of steel is different for different steel-making technologies, e.g., open hearth, basic oxygen, and electric furnace. Thus, in addition to the projection of total steel output, we need a forecast of the percentage of the output produced by each technology. Figure 6 summarizes a hypothetical technological forecast.

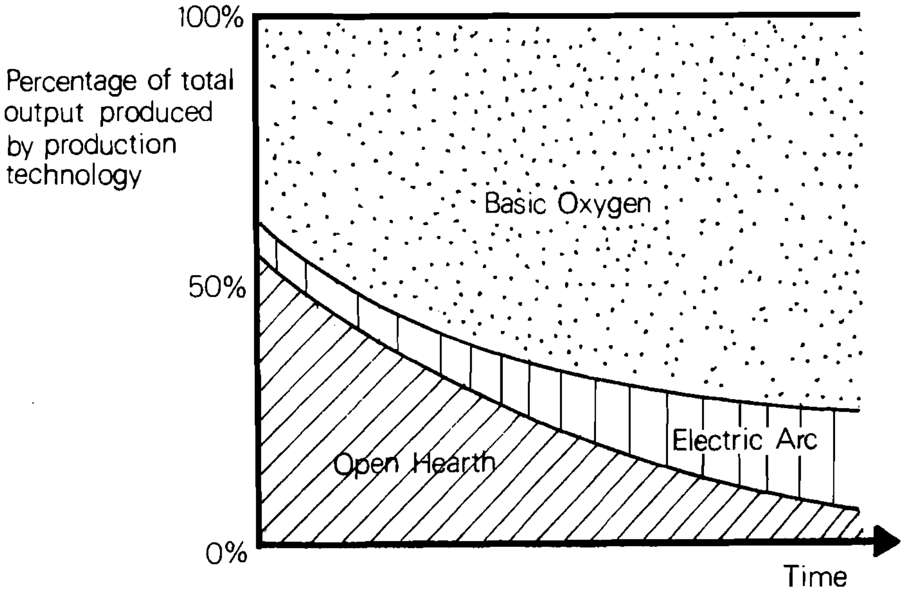


FIGURE 6

Another extension of the coefficient approach is to include time trends in the coefficients themselves. Suppose we observe that water use per ton of product has been decreasing at a constant rate of 1% per year over the last 10 years, and we wish to forecast water use five years into the future. We obviously do not have to assume that the current number of gallons per ton will be used five years from now. We could use a different coefficient; one way of obtaining it would be to assume that the 1% per year decline in water use per ton will continue over the next five years. This is obviously a judgment decision, but so also is the assumption that the current coefficient will remain constant. All of these modifications can, of course, be incorporated in the same forecast of water use.

When we assume a linear relationship between water use and outputs, the logical culmination of this coefficient approach is simply an input-output analysis in which one of the industries is water supply. Input-output tables describe the interrelationships in an economy between the flows of inputs and outputs and can be constructed for national economies, regional economies, or river basins. Each transaction between two sectors involves a sale of output from one sector and a purchase of input by the other. For example, the provision would be a sale of output for the water supply industry and a purchase of an input by the industrial water user.

The basic assumption of input-output analysis is that there is a fixed linear relationship between the inputs to an economic activity and its output. In input-output analysis this assumption of fixed input coefficients is not only made for water, but also for all other factors of production. Input-output analysis can, however, include some procedure for changing the water use coefficients over time. Including the water supply industry as one of the sectors enables the analyst to make several interesting types of calculations. In the simple water use coefficient method future water use is associated with projections of final demand for one sector. Through the use of input-output analysis the analyst can calculate both the direct and indirect effects on water use of forecast changes in the final demand in one or several sectors. For example, if pulp and paper production is forecasted to increase, this will not only increase water use in the pulp and paper industry, but also in the industries which must increase their production to supply the pulp and paper industry with the increased inputs necessary to produce the forecast pulp and paper production. Input-output analysis provides a comprehensive, general equilibrium framework for assessing the impacts on regional water use of forecast increases in various economic activities in a region or river basin. Water use by one industry is not viewed in isolation from economic interrelationships in the region.

Some form of this coefficients method is widely used in many countries. The data for a simple model are usually available and projections of water use can be made relatively inexpensively. The National Institute for Water Supply (The Netherlands) is utilizing the coefficients approach in their study of industrial water consumption [4]. Coefficients of water use per unit of production are derived from statistical analysis and consultation with industrial firms. The authors note that these coefficients have recently declined due to the higher costs of industrial water management, and they plan to revise their long-term forecasts every three years to incorporate changes in the water use coefficients.

Some work within Canadian government in forecasting industrial water use has focused on refinements of the coefficient approach. In their paper, "Industrial Water Demand Forecasting", D.M. Tate and R. Robichaud [5] develop a simulation model for forecasting industrial water use which explicitly incorporates the impact of technological change and effluent standards on future water use. Their model provides a framework for including projections of numerous variables which will change the coefficients in the future.

The Water Resources Bureau of the National Agency in Japan developed a simulation model which incorporates water use per unit of production coefficients for their forecasts of industrial water use [6]. Economic activity for each industrial sector is forecast as a function of the following exogenous variables: economic growth, birth and mortality rate, the pattern of consumption activities, and the structure of international trade. The forecasts of economic activity are more detailed for such major water using industries as pulp and paper, steel, and chemicals. Each industrial sector is subdivided into new and existing production, and different water use coefficients are often used for each. The water use coefficients themselves are forecast to change according to the rate of change of the percentage of factories recycling water and the changes in the amount of water recycled by a factory.

The United States Water Resources Council uses a similar method for the projection of United States industrial water use [7]. Although the industrial water use data base is more extensive and the calculations more detailed than those in many other countries, the Council's projections are based upon coefficients estimated by industrial water use specialists. The Council's specific forecasts depend upon a number of assumptions intended to improve the simple coefficient approach. By 2000 all cooling water used by major industrial water users is assumed to be recirculated through the use of cooling lagoons or cooling towers. Gross water use, however, is projected to grow in direct proportion to the gross product. This assumes the experience of the past twenty years will continue for the foreseeable future.

Other major forecasts of industrial water use in the United States have also used the coefficient approach, with varying degrees of sophistication. The Report of the North

Atlantic Regional Water Resources Study Coordinating Committee modified water use coefficients to hopefully reflect technological changes, increased costs of using water, and water pollution control regulations [8]. In a series of reports to the National Commission on Water Quality entitled Water Use in Manufacturing, the Conference Board analysed the economic impact of U.S. water pollution control requirements. One of the steps in their analysis was the projection of industrial water use and wastewater discharges for which they utilized a coefficients approach incorporating trends in recirculation, product mix, and production technology. Perhaps the most well known U.S. study of future water use is The Outlook for Water by Nathaniel Wollman and Gilbert W. Bonem, which again used a simple water use coefficient method and assumed that water intake coefficients would decline by either 5% or 10% per decade depending on the industrial sector.

2.2 Regression Techniques

The simple coefficient method assumes that the variation in water use is explained by one variable, usually production. Other variables such as product mix or recycling technology only explain variations in water use through their impact on this water use coefficient. Regression analysis provides a more technically sophisticated technique for forecasting water use [9]. Multiple regression techniques enable the analyst to correlate past variations in water use with numerous variables. Variations in historical water use data on a specific plant or an industry may be explained by variations in such variables as production level, age of plant, mean summer temperature, cooling water as a percent of total water use, or region of a country. The list of potential variables is long, and many may explain much of the same variation in the data [10].

Assume we want to forecast water use for a specific industrial facility. The first step is to estimate the reduced form coefficients [11] of the principal variables which explain the past variations in water use. We assume that future variations in water use will follow the same pattern as those in the past, i.e., that the reduced form coefficients remain constant. The second step is to independently project the explanatory variables; we assume they are determined independently of water use. We can thus derive an estimate of future water use from the regression equation and the projection of the exogenous explanatory variables.

Suppose, for example, that 95% of the variation in water use data is explained by three variables A, B, and C and there is a residual variation U. A reduced form linear relationship can be represented as

$$Qw = \beta + \alpha_1 A + \alpha_2 B + \alpha_3 C + U \qquad (1)$$

where Qw equals the mean volume of water use per day in gallons by a specific industrial plant. The estimated coefficients β, α_1, α_2, and α_3 will by definition be in the units necessary to convert the right hand side of the equation into gallons per day. Assume the estimated equation is

$$Qw = 75 + 1.2\ A + 280\ B - 93\ C + U \qquad (2)$$

We assume this estimated relationship holds both for current values of the variables, say at time t, and for future values at time t + 10. Thus,

$$Qw_t = 75 + 1.2\ A_t + 280\ B_t - 93\ C_t \qquad (3)$$

and

$$Qw_{t+10} = 75 + 1.2A_{t+10} + 280\ B_{t+10} - 93C_{t+10} \qquad (4)$$

Subtracting (3) from (4) we have

$$Qw_{t+10} - Qw_t = 1.2\ (A_{t+10} - A_t) + 280\ (B_{t+10} - B_t)$$

$$+ (-93)(C_{t+10} - C_t) \qquad (5)$$

$$\Delta\ Qw = 1.2\ \Delta A + 280\ \Delta B + 93\ (-\ \Delta C) \qquad (6)$$

where

$$\Delta C = (C_{t+10} - C_t) \quad .$$

Let A be the dollar value of mean daily production, B the mean daily temperature, and C some standardized measure of the quality of the principal raw material input to the production process. Assume at time t that A = \$50,000/day, B = 20 degrees centigrade and C = 100. Thus, at time t the quantity of water used per day is

$$Qw_t = 75 + (1.2 \text{ gallons/\$})(50{,}000 \text{ \$/day})$$

$$+ (280 \text{ gallons/day C°})(20°\text{C}) - (93 \text{ gallons/day})(100)$$

$$= 56{,}375 \text{ gallons/day} \quad . \qquad (7)$$

We independently project the following values for A, B, and C at time t + 10:

$$A = 90{,}000 \text{ \$/day}; \quad B = 21°\text{C}; \quad C = 110$$

Therefore at time t + 10, we forecast Qw to be

$$Qw_{t+10} = 75 + (1.2)(90{,}000) + (280)(21) - (93)(110)$$

$$= 103{,}725 \text{ gallons/day} \quad . \qquad (8)$$

It is important to be clear about the precise specifications of the dependent water use variable and what exactly is being modeled. There are several possibilities, e.g., total annual water use, mean water use per day, water use per ton, or mean water use per ton of input or output per day. Since some measure of total production is usually a principal determinant of water use, the most convenient variable is often some mean unit water use per unit of input or output per day (e.g., per ton of raw tomatoes processed per day or per ton of finished steel per day).

In her book Industrial Demand for Water: A Study of South East England [12], Judith Rees reports the results of regressions for numerous industries. Rees regressed the "quantity of water purchased", "quantity of water used but not purchased from a water supply authority", and "the total quantity of water withdrawn" on several explanatory variables. Table 1 presents the equations for each industry which best explained the variation in the data. Table 2 presents more detailed results for the chemical industry. Rees concluded that tonnage of raw materials and number of persons employed were highly correlated with water use in virtually all the industry groups studied, although which of the two provided the highest level of explanation varied from industry to industry. Although in some cases price is found to be a statistically significant explanatory variable, tons of input or number of employees is much more important.

There are several problems with this method of modeling and then forecasting water use. First, we have assumed that the reduced form coefficients remain constant. Our forecast is thus based on historical relationships among the variables which may well change in the future. Second, if the price of water is omitted from the explanatory variables, on theoretical grounds we would expect to have specification problems. In this case, our coefficient estimates are biased and inconsistent. If there has been no variation in the relative prices, this may be of minor significance.

Third, as discussed in more detail in Sections 3.2.1 and 3.2.2, this reduced form forecasting technique combines the influences of supply and demand relationships [13]. Under some circumstances this may be adequate for forecasting water use. However, as noted in the introduction, from a policy point of view, the water resources problem is often whether to decrease demand through price increase or rationing, or whether to increase supply. The reduced form equation is of no use in this task of determing the optimal level of water supply because the analyst cannot use it to estimate the benefits of water use.

TABLE 1

Comparison of the Best-Fit Explanatory Equations in the Industry Groups

	purchased water		privately abstracted water		total water taken	
industry group	best-fit explanatory equation	level of explanation (R^2)	best-fit explanatory equation	level of explanation (R^2)	best-fit explanatory equation	level of explanation (R^2)
1	2	3	4	5	6	7
		%				
A chemicals	$Q^2 = a + bT$	97.2	$QA = a + bT$*	100	$Qt = a + bT$*	100
B food	$Q^2 = a + bT$	B6.7	$QA^2 = a + bT$	90.7	$Qt^2 = a + bT$	99.4
C drink	$Q = a + b \log P_m$	36.3	$\log QA = a + bT$	77.4	NS	–
D plastics and rubber	$Q^2 = a + bE - cT$	41.5	NS	–	NS	–
E paper and products	$\log Q = a + b \log P + c \log QA$	55.3	$QA = a + bT$	60.31	$Qt = a + bT$	91.8
F non-metallic minerals	$Q = a + bE^2$	67.2	$\log QA = a + bT$	27.5	NS	–
G metals and products	$Q = a + bE^2$	96.1	$QA = a + bE$	81.7	$Qt = a + bE$	B2.8
H engineering (precision plus mechanical)	$\log Q = a + b \log P + c \log QA$	44.1	$QA^3 = a + bE$	25.1	$Qt^3 = a + bE$	25.1
I other (leather and fur, clothing and textiles, timber and furniture, printing)	$Q^2 = a + bE$	87	NS	–	NS	–
total	$\log Q = a + b \log E + c \log T + d \log P$	34.9	$\log QA = a + b \log P + c \log E + d \log T + c \log C$	16.6	$\log Qt = a + b \log E + c \log T + d \log P$	47.3

*The equations $QA^2 (Qt)^2 = a + bT$ also produced an R^2 of 100% in this group.
NS: No equation achieved significance at the 0.95 or 0.90 levels of probability.

Source: Rees, Judith, *Industrial Demand for Water: A Study of South East England*, p. 100.

Methodologically, this regression forecasting approach is identical to the previously explained water use coefficient approach except that there may be more explanatory variables. If we regress water use on production or number of employees assuming a linear relationship, we simply have another means of calculating the water use per ton of product or water use per employee coefficient.

The principal difficulty with both the simple coefficient and multiple regression approaches is that neither presents a theory explaining water use. No behavioral model is tested. We simply note that in the past variations in water use have been correlated with variations in production or other variables. We have no sound theory on which to argue that this correlation will exist in the future.

TABLE 2

Determinants of the Quality of Water taken by Chemical Firms

dependent variable	significant independent variables	best-fit equation form	regression coefficient (b)	standard error	'f' test value (significance level)	degrees of freedom	level of explanation (R^2)
1	2	3	4	5	6	7	8
A purchased water	raw materials: tonnage	$Q^2 = a + bT$	15121172.5	510685.7	876.73	48	97.3%
	employment	$Q^2 = a + b E^2$	0.03895	0.00528	54.44	48	53.1
	price of metered water	$\log Q = a + b P_m$	–0.0302	0.0145	4.312	35	10.9
	multiple regression: price of bought water	$\log Q = a + b P + \log QA$	–2.05798	0.43887	21.99	47	37.0
	+ quantity of water abstracted		0.10626	0.06987	2.31		
B abstracted water	raw materials: ton-input	$QA = a + b T$	999.69	1.9119	273418.8	48	100 *
	employment	$QA = a + b E$	2115.56	318.86	44.03	48	47.8
C total water taken	raw materials: ton-input	$Qt = a + b T$	994.216	4.232	55184.6	48	100 *
	employment	$Qt = a + b E$	2122.52	317.196	44.78	48	48.3
	price of bought water	$\log Qt = a + b P$	–2.164	0.4389	46.22	48	34.1
D purchased water	abstracted water	$Q^2 = a + b QA$	15501.85	371.53	1740.92	48	97.4

*The equation forms QA^2 (or Qt) = a + b T also gave a 100% level of explanation, but for interpretive problems see text.
No multiple regression model using tonnage and employment was attempted due to multicollinearity.

Source: Rees, Judith, *Industrial Demand for Water: A Study of South East England*, p. 69.

3 Estimation of Demand Functions for Water

Economic theory offers a simple behavioral model to explain industrial water use in competitive free market economies. For a given technology the quantity of water demanded by an industry is assumed to be a function of the price of water, the price of other inputs, and the level of industrial output. The supply of water is assumed to be a function of the price of water and various institutional factors. In equilibrium we assume the supply of water equals the demand for water. In this simple model the quantity of water used is determined just as any other factor input.

If we want to test the usefulness of this model for forecasting industrial water use, the first step is to estimate the demand function for water. In this section we discuss the derived demand for a factor of production and three methods of estimating the water demand function for an industrial facility: 1) statistical; 2) process engineering; and 3) mathematical programming.

3.1 Derived Demand for a Factor of Production

The purpose of this section is to describe qualitatively the determinants of an industrial facility's demand function for a factor input such as water. The mathematical framework of the problem is presented in the Appendix.

What determines how an industry's use of water will change if the costs of using water change? To get a feel for the issues involved, we consider the case where there are only two factor inputs to the production process. Assume that the industrial facility produces a product X with only two inputs: water W and capital K. If the cost of using water increases either or both of two things can happen. First, the increased cost of water will increase the average cost of producing X. If the firm is to remain in business, in the long run it must cover its costs. Therefore, the price of X will rise and the demand for X will fall. This fall in the production of the product will reduce the quantity of water used by the firm. Thus, the more the demand for the firm's product falls as the price of the product rises, the more the firm's demand for water falls as the costs of using water rise [15].

Second, the industry will try to minimize its costs of production for any level of output. If the costs of using water increase, the firm will attempt to use less water and use more capital instead. The actual degree of substitution which occurs between water and the capital depends upon two factors. The first is the technical feasibility of substituting one factor for another at different factor ratios. There are two extreme possibilities: (1) there is no substitution possible between the two factors; and (2) the two factors are perfect substitutes for each other. The degree of substitution is commonly assumed to be somewhere between these two extremes; the less of one factor we have, the harder it is to substitute the other factor.

The extent to which substitution between water and other factor inputs is technically feasible will depend in large part upon the specific use of water in the industrial facility. Industrial plants use water for any of the following reasons [16]:

1. cleaning of inputs, intermediate products, and outputs;
2. cooling;
3. disposal of residuals from the production process;
4. transport of materials through a sequence of production stages;

5. component of the final product;
6. output of the production process, i.e., the process is a net producer of water;
7. sanitary facilities.

There are few generally applicable guidelines as to which uses of water are more readily amendable to substitution by other factor inputs. Plants which withdraw large amounts of water for once-through cooling can usually greatly reduce their water withdrawal by investing capital in a closed-cycle cooling system. Thus, there is some price of intake water at which the facility is indifferent between paying the charge for intake water and making the capital investment necessary for recirculation. If the facility is actually a net producer of water, reducing wastewater discharges below some level may be very expensive without changing the production process. In general, however, the degree of substitution which can be achieved can only be determined by a technical evaluation of the actual situation and the options open to the specific plant.

The second factor which affects the actual degree of substitution which occurs between water and capital is the elasticity of supply of capital. Consider the example of air versus water cooling. It may be technically feasible to switch from water to air cooling. If the price of water rises, at the initial price of air cooling systems it may be economically justified to switch. But as more firms switch, the demand for air cooling systems increases and the price of air cooling systems may rise, thus making the switch less and less attractive economically. Or firms might like to switch to air cooling, but the systems may simply not be available in the short run. Either the production capacity for air cooling equipment is fixed in the short run or the skilled labor and contractors might not be available to install the air cooling systems. Such short term capacity bottlenecks can easily occur if many industrial facilities attempt to substitute the same factor for water at the same time, due perhaps to a uniform pollution abatement deadline.

To summarize, if the price of water rises, we expect that the quantity of water used will fall because both the output effect (we produce less of the product and thus use less water) and the substitution effect (for a given production level we use less water and more of other factor inputs) are negative. The quantity of capital may either rise or fall depending on the relative magnitudes of the positive substitution effect (less water is used and thus more capital) and the negative output effect (less of the product is produced and thus less capital is required). When there are three or more factors of production, an increase in the price of one factor may increase or decrease the quantity of other inputs, even holding output constant. In other words, with a given level of production, inputs may be either substitutes or complements.

3.2 Statistical Estimation of Industrial Water Demand Functions

The estimation of industrial water demand functions with ordinary least squares regression techniques is a special case of the estimation of regression equation in which the choice of the explanatory variables is based upon microeconomic theory. For a given technology, the explanatory variables are hypothesized to be the price of water (or costs of using water), the prices of other factor inputs, and the level of output. Since the product mix and product specifications vary between industrial facilities and at the same facility over time, this must also be taken into consideration. The expectations are that the coefficient for the price of water will be negative (the higher the costs of water use, the less water used) and for output will be positive (the more output, the more water used). The signs of the coefficients on the prices of other factor inputs can be either negative or positive depending upon whether they are complements or substitutes respectively.

There is no a priori correct functional form for a demand function. In statistical work, however, the demand function is commonly assumed to be either additive, multiplicative, or a combination of the two, yielding the following linear, logarithmic, or semi logarithmic forms [18]:

$$Q_w = a_1 + a_2 Z_1 + \ldots a_{n+1} Z_n + u \qquad (9)$$

$$\ln Q_w = b_1 + b_2 \ln Z_1 + \ldots b_{n+1} \ln Z_n + u \qquad (10)$$

$$Q_w = c_1 + c_2 \ln Z_2 + \ldots c_{n+1} \ln Z_n + u \qquad (11)$$

where Q_w is the quantity of water used, $Z_1 \ldots Z_n$ are the explanatory variables. The log-linear form (10) is often preferred on theoretical grounds because it implies a Cobb-

Douglas production function. These forms are convenient because they allow the use of ordinary least squares regression techniques if it can be assumed that the explanatory variables are determined independently of the quantity of water used. Although more complicated functional forms could be used, neither the theory nor the available data justify greater sophistication [19].

In most industries Q_w has been primarily determined by the level of production activity, indicated by such variables as tons of output, volume of output, or number of employees. The water resources analyst is often particularly interested, however, in what variables other than industrial output can be changed to alter water use. The standard procedure is thus to remove the influence of output level on total water use and to estimate the variables explaining water use per ton of output or per ton input...

$$Q_w / \text{ unit of output} = a_1 + a_2Z_1 + a_3Z_2 + a_{n+1}Z_n + u \qquad (12)$$

In this case, if one of the explanatory variables is output, its estimated coefficient will not measure the effect of the level of economic activity, but the effect of another variable such as economies of scale in production or operating efficiency of the plant on water use per unit of output.

3.2.1 Some Practical Considerations and Difficulties with the Application of Regression Techniques to Modeling Industrial Water Demand Relationships

There are a few examples in the literature of the application of regression techniques to the modeling of industrial water use which include the "price of water" as one of the explanatory variables, but the approach has not been widely used [20]. Although the use of regression techniques to estimate industrial water demand appears straightforward, there are numerous difficulties involved.

The first is the classic problem in the statistical estimation of demand functions: the identification of the demand function itself. In this case, the identification problem is concerned with whether it is possible to distinguish the following demand relationship:

$$Q_d = \alpha_1 + \alpha_2 P_w + \alpha_3 Z_1 + \ldots + \alpha_{n+2} Z_n$$

where P_w = the price of water; Q_d = quantity of water demanded and $Z_1 \ldots Z_n$ = additional explanatory variables

from a supply relationship of the form

$$Q_s = \beta_1 + \beta_2 P_w + \beta_3 Z'_1 + \ldots + \beta_{m+2} Z'_m$$

where Q_s = quantity of water supplied

and $Z'_1 \ldots Z'_m$ = additional explanatory variables, some of which may or may not equal $Z_1 \ldots Z_n$.

The data are in the form of price-quantity observations. The "price of water" may be determined simultaneously by both supply-side and demand-side forces. If the "price of water" changes and both the supply and demand relationships change, it may be difficult to disentangle the shifts in one from the shifts in the other. More formally, a necessary condition for a structural equation to be identified is that the number of exogenous or predetermined variables in the complete model which are excluded from the equation must be greater than or equal to the number of endogenous variables included in the equation minus one [21]. This is a difficult condition to meet in the estimation of industrial water demand functions. The next section on the statistical modeling of water use in the Dutch paper industry discusses the identification problem in more detail.

A <u>second</u> difficulty concerns the direct estimation of the water demand relationship when some of the right-hand side explanatory variables are not predetermined. Two variables which are endogenous in the more complete model are the quantity of production and the "price of water". Even if the demand function is identified, if the "price of water" or level of production is not predetermined, ordinary least squares estimation will yield biased and inconsistent estimates because the "price of water" or level of production variables will be correlated with the error term.

Standard methods for estimating a structural equation in which one or more of the explanatory variables are endogenous in the model include instrumental variables or two stage least squares [21]. A good instrument in this case will be highly correlated with the price of water variable and independent of the error term, i.e., the instrument is predetermined with respect to the model. Unfortunately, in this problem of estimating demand functions for industrial water use, good instruments are very hard to come by. Similarly, two stage least squares appear inappropriate because, as Ginn et al note, there are too few predetermined variables in the model of industrial water use.

The <u>third</u> difficulty concerns the data generally available for the estimation. There may be little experience in the historical data for the kinds of changes which may be likely to take place in the future. Industry has traditionally treated water as a free good. Many of the impacts of environmental regulations have yet to be fully reflected in the historical data of many countries. Recent increases in the costs of using water or absolute quantity restrictions may cause technological changes which will shift the demand curve for water. The estimated coefficients will thus not remain constant over the projection period. In short, the past may be a poor guide to the future.

Moreover, there are virtually no publicly available time series or cross-section data for individual industrial plants which include the following variables: quantity of water withdrawn; quantity of water discharged; gross water use; marginal costs of water use including withdrawal and disposal charges; prices of other factor inputs; and level of production. Cross-section data are more often available than time series, but they are frequently of questionable reliability because they are based on simple surveys or questionnaires. In addition the sample sizes are invariably small because such data collection is tedious and difficult [22].

The "price of water" variable, in particular, presents data problems. The information which should be included in the water demand function is the marginal benefit of water to the plant at the specified quantity of water. A plant is assumed to use water until the marginal benefits of water equal the marginal costs of using water. Since the data reflect equilibrium values, the marginal costs of water use can be used for the "price of water" variable in the demand function.

Since many large water-using industries obtain their own supplies directly from surface or groundwater, the marginal costs of using water must usually be calculated by the analyst. The usual procedure is to attempt to derive the "costs of using water" from information on water intake charges, pumping costs, pretreatment costs, wastewater treatment costs, and effluent charges. Industries may not only be reluctant to release such information, but also they may not have accurate estimates of such water-related costs themselves. Some of the components of these costs of using water are calculated by amortizing capital expenditures for such items as pumps, piping, and waste treatment plants. The costs of using one unit of water are usually derived by dividing the total annual costs by the total quantity of water used during the year. The procedure yields average costs per unit of water rather than marginal costs of using water, which is the appropriate variable to use in the estimation of the demand function [23].

Data on the prices of other factor inputs are also difficult to obtain. There again appear to be few ways around this problem except by a thorough data collection effort. Appropriate proxy variables are difficult to find. De Rooy has suggested the use of the price of recycled water as a substitute for the prices of other factor inputs, arguing that recycled water is the closest substitute for water intake. This would appear to be a reasonable variable to include when attempting to explain water intake (it does not solve the problem of lack of data on other factor input prices when attempting to explain gross water use). There are likely, however, to be problems with the data for this variable. Unless the cost estimates are based upon detailed engineering design studies for individual industrial facilities, the estimated costs of recirculation are unlikely to show significant variation over time or from plant to plant. The failure to include the prices of other factor inputs as variables in the estimation will lead to biased and inconsistent estimates [24]. For cross-section analysis for plants in a small region this lack of factor input prices may not be a serious problem because prices for such inputs as energy and labor are likely to be relatively constant across the sample. Moreover, some of the influence of variations in factor input prices may be accounted for by a variable for type of technology.

The reliance on cross-section data highlights a fourth problem generally associated with regression models of industrial water use: aggregation. The coefficients of regression equations estimated with cross-section data for different plants may be useful as average or representative values for the industry or for the particular subset of the industry from which the sample is drawn. They may be of little value, however, for the estimation of water use by a particular facility. The estimated equation is more likely to be useful for a specific plant if such variables as age of plant, type of process technology, geographic region, and product mix are either included in the regression or are constant across the sample. However, variations in water use and wastewater discharges are often large even if for plants with the same process technology and product mix, and the analyst should keep clearly in mind the level of aggregation of industrial water use which is actually being modeled. Regression estimates with cross-section data do not yield much information about water use patterns at the activity level of the specific industrial plant.

A fifth point to consider in the use of cross-section or time series data to estimate models of industrial water use relates to the time period being modeled. If the quantity of water used is regressed on various explanatory variables such as the price of water, level of output, etc., the time period must be explicit in the equation--i.e., cubic meters of water used per month or per year, tons of output per month, etc. Data available for such a regression would rarely be precise daily values, and if they are reported as daily, such as cubic meters of water per day, they should usually be interpreted as mean values for a longer time period. Even if the data are precise daily values, the analyst should not assume that such simple functional relationships and small sets of explanatory variables as indicated here are capable of explaining daily variations in water use by an industrial facility. Thus, an equation estimated with "daily" data should not be misconstrued as actually explaining water use at a particular plant on a given day; this would require a much more detailed understanding of water use patterns at the particular plant or a much more extensive set or precise daily data.

When cross-section or time series data are used to regress the quantity of water used per ton on various explanatory variables, the time period is not always made explicit. It is still, of course, implicit in the values of the data for the explanatory variables. To estimate water use by a given plant, the analyst can substitute the values of the explanatory variables for that particular plant into the estimated water demand relationship. This will yield water use per unit of output for that plant as a mean or representative value over some time period determined by the data for the explanatory variables. The total water used by the plant over some time period is then the water use per unit of output times the output of the plant.

3.2.2 Statistical Estimations of Industrial Water Demand Relationships: An Analysis of the Dutch Paper Industry

Data were collected on water use in the Dutch paper industry (see Table 3) in order to illustrate some of the concepts and problems involved in the statistical modeling of industrial water demand relationships. Few solutions are offered to the problems raised in the previous section; the purpose of this example is rather to show the existing practice in the field and its limitations.

As a basis for the analysis, the following model of water intake per unit of production was used:

$$(IW/_Q) = f\ (Q,\ P_w,\ T,\ Ty,\ Pr)$$

where

IW = water intake or water withdrawals;

Q = physical output of the final product;

Pw = "price of water intake";

T = technology, classified as old, average, or advanced;

Ty = type of production process, classified as simple, average, or complicated; and

Pr = product type.

Ordinary least squares regression techniques were used to estimate linear, semi-logarithmic, and logarithmic forms of the equation explaining water withdrawals per unit of production in the sample of 22 plants.

The question of the identification of this equation concerns the type of relationship actually obtained from this regression model. The analyst may only be interested in the estimated parameters in the individual facility's demand function for water. In order to identify the demand function, however, care must be taken with the assumptions concerning the specifications of a more complete model which details how each of the variables in the model is

TABLE 3

Plant No.	Water Intake IW (m^3)	Gross Water Use GW (m^3)	Final Product Output Q (1000 kg)	Price of Water Intake Pw (cents/m^3)	Production Technology[a] T	Type of Production Process[b] Ty	Type of Product[c] Pr
1	2	3	4	5	6	7	8
1	2.847.500	5.677.000	43.450	15,6	A	A	3
2	46.935.250	46.935.250	133.400	7,2	A	C	1,3,5
3	38.910.900	38.910.900	91.000	4,3	AD	S	1
4	4.090.000	4.090.000	123.000	42,8	AD	A	5
5	5.663.000	5.663.000	78.800	29,1	A	A	4,5
6	11.037.000	11.037.000	86.750	30,5	AD	A	4,5
7	22.278.000	29.278.000	175.558	10,0	AD	C	3,5
8	2.450.000	4.900.000	28.200	26,7	A	A	4,5
9	1,750.000	1,750.000	43.000	25,7	AD	S	5
10	2.940.000	2.940.000	55.647	17,9	A	A	4
11	4.918.000	4.918.000	62.000	20,3	A	A	3,5
12	663.000	663.000	17.200	39,9	A	S	5
13	1.010.000	1.010.000	33.500	86,7	O	A	4,5
14	1.662.350	1.662.350	57.000	12,0	O	S	2,4
15	1.053.000	1.053.000	50.000	12,1	O	S	2,4
16	5.670.000	5.670.000	66.000	48,1	O	C	2,4
17	3.548.000	6.405.680	131.000	95,5	A	C	4,5
18	500.000	500.000	17.800	108,8	A	S	3,4,5
19	500.000	500.000	12.000	69,4	A	A	4,5
20	840.000	840.000	12.200	52,4	A	C	3
21	2.212.000	2.212.000	49.000	23,3	O	A	2,4
22	1.241.000	1.241.000	11.000	42.1	A	S	5

Note: (a) O = old
A = average
AD = advanced

(b) S = simple
A = average
C = complicated

(c) 1 = newsprint
2 = straw-board
3 = printing and writing paper and board
4 = packing paper and board
5 = special paper and board
(the most important product is underlined)

Source: Rijksinstituut voor Drinkwatervoorziening, the Netherlands.

determined--whether it is exogenous or endogenous to the model. For example, if the "price of water" variable is a function of the quantity of water used and the production technology, it is not predetermined in this model. If the "price of water" variable is endogenous in the model, the analyst must distinguish between a supply relationship and a demand relationship.

In general, an equation is identified by excluding predetermined variables from the equation which are included in the complete model. In this example of the Dutch paper industry, the exclusion of predetermined variables from the demand function is difficult because there are no data on predetermined variables which belong to the supply relationship which are not already included in the demand relationship. Ginn et al identify the industrial water demand function in their model by assuming that the price variable is exogenous to the model. Their system of equations becomes recursive and thus identified. The identification of the industrial water demand function rests upon this assumption.

Economists frequently assume that the demand function has been estimated when the coefficient on the price variable is negative. This example of the Dutch paper industry illustrates the difficulty with this presumption in the estimation of industrial water demand functions. These plants purchase water from local water authorities, but the marginal costs of using water are partly determined within the plant. Water is in some respects a self-supplied factor input. As previously discussed, the marginal costs to society of supplying water to users is currently increasing in many countries. This is rarely, however, reflected in the marginal costs which an industrial plant faces in its use of water. The paper plants in this sample, for instance, are most probably operating on the downward sloping portion of their marginal cost curve for using water. This simply means that there are economies of scale in the use of water; the more water a facility uses, the lower the marginal cost per unit.

The difficulty this creates for identifying the demand function for water for paper plants in our sample is illustrated in Figure 7 and Figure 8. Figure 8 illustrates the case in which the regression model estimates the supply relationship--the more water a facility uses, the lower the marginal cost of water. The shifting demand functions for the different plants trace out the downward sloping marginal cost curve. In Figure 7 the regression model estimates the desired demand relationship. The shifting supply functions for the different plants trace out the downward sloping demand curve--the higher the marginal costs of water use,

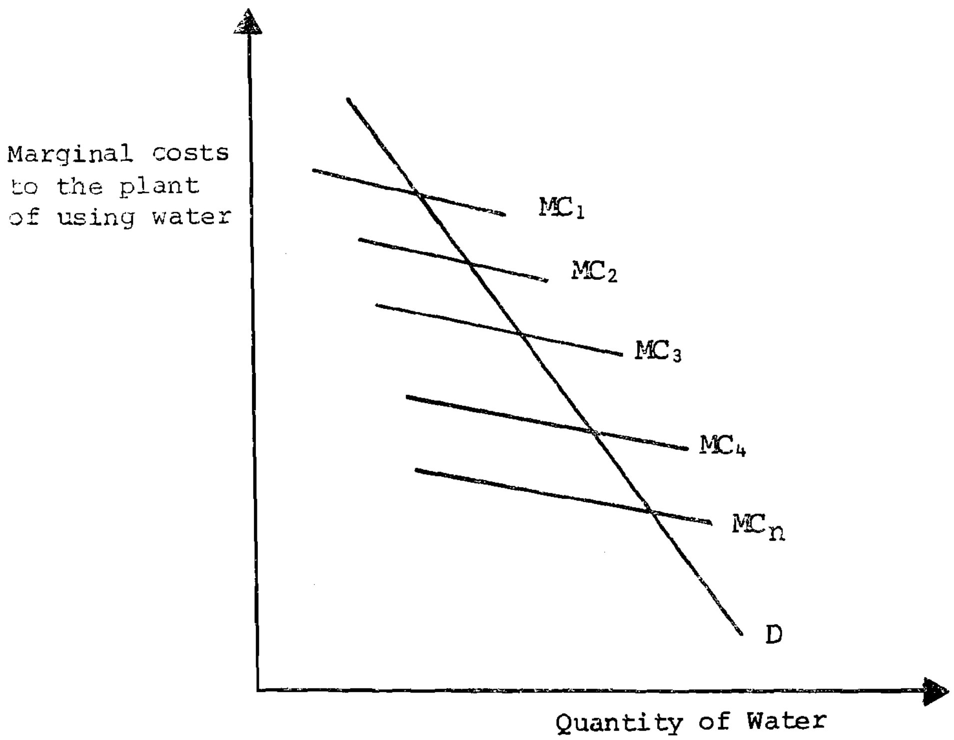


FIGURE 7

the less water demanded by the plant. Unless the demand function can be properly identified, the analyst cannot distinguish between these two cases.

Several additional aspects of this model should be noted. <u>First</u>, there is no term for the prices of other input factors because these data were not available. This specification error will lead to biased estimates of the coefficients if factor input prices vary across the sample and if they influence water intake. This is not anticipated to be a major problem, however, because the data are from a relatively small, homogeneous region and the prices of such inputs as energy, labor, and capital would not be expected to vary significantly between plants at a given time.

<u>Second</u>, only water intake per unit of output is being modeled; nothing is said about the relationship between intake, water consumption, discharge, and gross water use. In this sample, gross water use equals water intake in 18 of the 22 plants; it is thus not possible to estimate a relationship between the two with this data set.

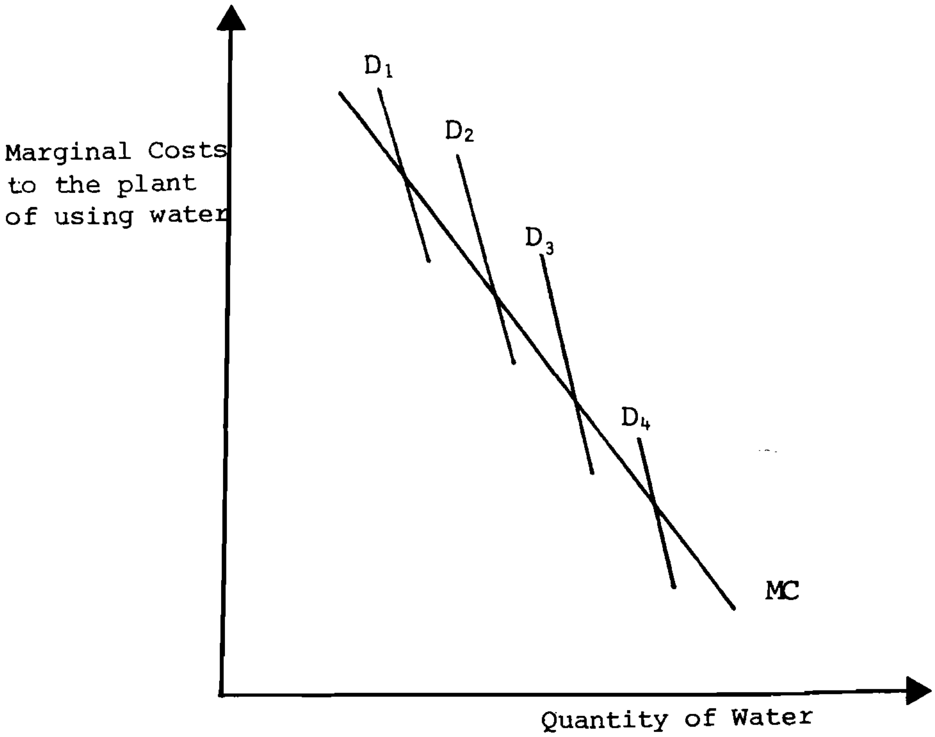


FIGURE 8

Third, data for the "price of water" variable are actually estimated as the average cost to the plant of withdrawing one cubic meter of water--including pumping costs, intake charges, water intake and wastewater treatment costs, and effluent charges--rather than as marginal costs. To the extent that there are significant economies of scale in the use of water, this problem will result in the estimated price elasticities of demand for water being biased upward. Moreover, if the average costs of using water are specified as an endogenous variable in the model, then a simultaneity problem may arise, and ordinary least squares techniques will yield biased and inconsistent estimates. This problem is not addressed in the estimation.

Fourth, the variables T, Ty, and Pr are all dummy variables. Technology T and type of production process Ty can each take three values, and thus each requires the estimation of two regression coefficients. The sample of plants is also categorized by five principal product types. Ten of the 22 plants are, however, product type "special paper and board", so two categories for the estimation were created: (1) special paper and board, and (2) not special paper and board. Thus, only one coefficient needed to be estimated. The expected signs of these dummy variables are not clear a priori. The use of so many dummy variables is obviously a costly practice in terms of the limited degrees of freedom available with such a small sample, and during the analysis of the data ways were explored to reduce the number of coefficients to be estimated.

Table 4, Table 5, and Table 6 present the results for the three functional forms for the case in which all the explanatory variables are included. Considering first the F-value for the linear and semi-logarithmic functional forms, it is not possible to reject the hypothesis at the 95% confidence level that all the estimated coefficients = 0. The equation as a whole is significant for the logarithmic form at the 95% confidence level but not at the 99% level. An examination of the t-values for the individual coefficients reveals that only the "price of water" variable in the log-linear form is statistically different from zero at the 95% confidence level. The sign of the coefficient of the price of water has the expected negative sign in all three functional forms. The percentage of the variation in the data explained by the three functional forms improves moving from linear to semi-logarithmic to logarithmic (R^2 = .47; .56; .62).

The estimated coefficient for output is insignificant and close to zero in all functional forms. Similarly, the t-values for the dummy variables for type of production process and product type are rarely above 1 and often close to zero. These variables are thus dropped from the estimation. In addition, the linear forms of the equation perform consistantly worse than either the semi-logarithmic or logarithmic in terms of R^2, F-statistics, and the t-values for the individual coefficients.

The results for the following equation, presented in Table 7, are considered the best of the analysis in terms of the test statistics:

$$\ln(IW/Q) = a_1 + a_2 \ln(Pw) + a_3 (T_{average}) + a_4 (T_{advanced}) + u \quad .$$

The equation as a whole is significant at the 99% level. Each of the coefficients is significant at the 90% level; the coefficient on the "price of water" variable is significantly different from zero at the 99% level.

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG. COEF.	COMPUTED T VALUE
2	37.29091	29.34446	-0.43465	-0.67990	0.86784	-0.78344
3	62613.86328	45144.61719	0.39023	0.00058	0.00072	0.80599
5	0.54545	0.50965	-0.05594	118.03237	63.03536	1.87248
6	0.22727	0.42893	0.32762	105.42657	71.40348	1.47649
7	0.45455	0.50965	-0.24744	-62.62472	49.35072	-1.26897
8	0.22727	0.42893	0.22475	-43.88855	82.80082	-0.53005
9	0.45455	0.50965	-0.31450	-91.41328	58.97450	-1.55005
DEPENDENT						
1	90.56500	102.62262				

INTERCEPT	71.23164
MULTIPLE CORRELATION	0.68862
STD. ERROR OF ESTIMATE	91.13763

Variable No.	Symbol	Variable No.	Symbol
1	IW/Q	6	Tad
2	PW	7	Tyav
3	Q	8	Tycm
5	Tav	9	Pr

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	7	104874.49219	14982.07031	1.80375
DEVIATION FROM REGRESSION	14	116284.96094	8306.06836	
TOTAL	21	221159.45312		

TABLE 4

Results for Linear Form

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COEF.	COMPUTED T VALUE
2	3.30818	0.84954	-0.64926	-67.57259	35.93573	-1.88037
3	10.76091	0.82420	0.34487	-4.15136	37.33162	-0.11120
5	0.54545	0.50965	-0.05594	78.19137	65.60384	1.19187
6	0.22727	0.42893	0.32762	93.64211	61.84970	1.51403
7	0.45455	0.50965	-0.24744	-20.84352	52.54983	-0.39664
8	0.22727	0.42893	0.22475	33.45975	76.22895	0.43894
9	0.45455	0.50965	-0.31450	-31.75743	62.29122	-0.50982
DEPENDENT						
1	90.56500	102.62262				

INTERCEPT 311.15277

MULTIPLE CORRELATION 0.75156

STD. ERROR OF ESTIMATE 82.91119

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	7	124919.75000	17845.67773	2.59601
DEVIATION FROM REGRESSION	14	96239.70312	6874.26465	
TOTAL	21	221159.45312		

TABLE 5

Results for Semi-Logarithmic Form

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COEF.	COMPUTED T VALUE
2	3.30818	0.84954	-0.58681	-0.63812	0.25969	-2.45727
3	10.76091	0.82420	0.28552	-0.28079	0.26977	-1.04085
5	0.54545	0.50965	0.04469	0.57829	0.47408	1.21981
6	0.22727	0.42893	0.31618	0.90143	0.44695	2.01685
7	0.45455	0.50965	-0.10392	0.35397	0.37975	0.93211
8	0.22727	0.42893	0.27677	0.86857	0.55086	1.57675
9	0.45455	0.50965	-0.28043	-0.09459	0.45014	-0.21014
DEPENDENT						
1	4.14273	0.78876				

INTERCEPT 8.43972

MULTIPLE CORRELATION 0.78443

STD. ERROR OF ESTIMATE 0.59915

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	7	8.03912	1.14845	3.19919
DEVIATION FROM REGRESSION	14	5.02572	0.35898	
TOTAL	21	13.06484		

TABLE 6

Results for Logarithmic Form

VARIABLE NO.	MEAN	STANDARD DEVIATION	CORRELATION X VS Y	REGRESSION COEFFICIENT	STD. ERROR OF REG.COEF.	COMPUTED T VALUE
2	3.30818	0.84954	-0.58681	-0.54056	0.16582	-3.25997
5	0.54545	0.50965	0.04469	0.67599	0.32700	2.06728
6	0.22727	0.42893	0.31618	0.72967	0.39327	1.85537
DEPENDENT						
1	4.14273	0.78876				

INTERCEPT	5.39643
MULTIPLE CORRELATION	0.69759
STD. ERROR OF ESTIMATE	0.61042

ANALYSIS OF VARIANCE FOR THE REGRESSION

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	MEAN SQUARES	F VALUE
ATTRIBUTABLE TO REGRESSION	3	6.35773	2.11924	5.68745
DEVIATION FROM REGRESSION	18	6.70711	0.37262	
TOTAL	21	13.06484		

TABLE 7

Results for Logarithmic Form

If the estimated relationship is, in fact, the desired demand function for water intake, the estimated elasticity of intake water per unit of output with respect to the "price of water" (average costs of using water withdrawals) is -.54. A priori this would be considered a very reasonable result. It seems unlikely that economies of scale in the costs of using water could be large enough to account for an inverse relationship between costs of using water and the quantity of water which is this significant. Nevertheless, the demand curve is not identified, and this type of result should be used cautiously. Additional data are required in order to understand the plants' costs of water use and to interpret the price-quantity shifts in the data. Even if the estimated relationship is the desired demand function, however, the results indicate that there are substantial variations in IW/Q between plants which are not explained by this simple regression model.

3.3 Engineering-Economic Models of Industrial Water Use

Statistical models of industrial water use estimate the parameters associated with variables which are related by relatively simple functional forms. The statistical models do not explain how or why the water is used in the industrial plant. The technological aspects of water use within the plant are in a sense a black box abstractly represented by the functional form relating the variables and the estimated coefficients.

A second approach to modeling water use is to actually analyze the engineering details of using water--what purposes it serves, where it goes, how can it be changed technically, and how much such changes cost. We refer to this second approach to modeling water use as engineering-economic. We discuss two methods of using such an engineering-economic approach: (1) process engineering and (2) linear programming.

3.3.1 Process Engineering

The process engineering approach to modeling industrial water use is to first identify the technological options open for changing existing water use patterns in the plant and then to estimate the engineering costs of making such changes. The next step is to determine whether the savings in water use and wastewater discharge costs justify the increased costs associated with making the change. If the price of water withdrawals increases, the question is whether it is cheaper to continue using the same amount of water and pay the increased price or to spend the funds to reduce water usage. Alternatively, if pollution control requirements are imposed, the problem is whether to pay an effluent tax, or to treat the current flows to meet the standards, or to reduce the flows and wasteload and treat the remainder. The engineer must determine how flows and wasteloads can be reduced, what different levels of reduction of each cost, and how these changes impact other aspects of the production process. The process engineering method is thus essentially a search procedure over a production surface. The decision rule is to change water use patterns whenever it lowers the cost of producing the output. Points on the demand function for water withdrawals can be estimated if the engineer attempts to find the quantity of water associated with the least cost means of production for several different prices of water withdrawals.

The next stage of the process engineering analysis is to broaden the scope to include not only the technical options for changing existing water use within the plant, but also the impact on the least cost solution of changes in the prices and quality of factor inputs, process technology, and product specification. The number of possible solutions quickly becomes immense as the number of factors considered increases and, if done correctly, the process engineering analysis of water use and wastewater discharges requires the estimation of cost relationships which are rarely developed in the engineering literature.

The process engineering approach to the study of industrial residual management problems is commonly utilized. Most studies do not, however, consider a wide range of possible alternatives. The Resources for the Future industry program conducted three studies which illustrate this method:

(1) Water Demand for Steam Electric Generation--An Economic Projection Model, by Paul Cootner and George O. Löf;

(2) The Economics of Water Utilization in the Beet Sugar Industry, by George O. Löf and A.V. Kneese;

(3) Residuals Generation in the Pulp and Paper Industry, by Blair T. Bower, George O. Löf, and W.M. Hearon.

3.3.2 Linear Programming

Linear programming techniques offer a systematic way of organizing and analyzing the engineering data used in the process engineering approach in order to estimate industrial demand functions for water. It is outside the scope of this paper to present a detailed review of the application of linear programming methods to the theory of the firm [25]. Rather we briefly review how a demand function for a factor input can be derived by parametric programming from the dual of the firm's profit maximization problem. We then discuss the application of linear programming to the study of water use by an industrial facility and note some of the existing work in this field.

Many textbooks [33] illustrate how a demand function for a factor input can be derived from the profit maximization conditions for the firm. If the production process technologies can be adequately characterized as constant returns to scale over the relevant range of production, linear programming techniques provide a way to proceed from this abstract theory of the firm to the empirical estimation of demand functions for factor inputs such as water. The generalized form of a linear programming problem is to maximize a linear function subjected to a set of linear inequalities and nonnegativity requirements. Linear programming has been applied to a wide variety of industrial problems [26]. In the context of the theory of the firm, the objective function may be to maximize profits subject to the linear production activities and input availability constraints. The following simple example is developed in numerous microeconomics and linear programming texts [27].

Consider an industrial plant using two "activities" or processes and three inputs--capital K, labor L, and water w, to produce two outputs x_1, and x_2. Assume for simplicity that one activity produces only one of the two outputs. Let a_{ij} equal the units of input i required to produce one unit of output j, where in this case i = 1,2,3 and j = 1,2. The expression for the primal problem is....

$$\text{Maximize Profits} = \pi_1 x_1 + \pi_2 x_2$$

$$\text{Subject to} \quad a_{11}x_1 + a_{12}x_2 \leq Z_1^\circ$$

$$a_{21}x_1 + a_{22}x_2 \leq Z_2^\circ$$

$$a_{31}x_1 + a_{32}x_2 \leq Z_3^\circ$$

$$x_1, x_2 \geq 0$$

where π_1 and π_2 are profit per unit of output x_1 and x_2, respectively, and Z°_1, Z°_2, and Z°_3... are given levels of the resources water, capital, and labor. The solution is in terms of the optimal levels of output of the two products x_1 and x_2; the nonnegativity restrictions ensure that those output levels are not less than zero. In matrix notation each coefficient column in the set of constraints represents an "activity" and each row represents the requirements of all activities for one resource, such as water.

For the given resource availability constraint, the value to the firm of each factor input in terms of the objective function is implicit in this statement of the profit maximization problem. The dual of the primal problem, however, allows the analyst to solve directly for those values of the factor inputs, or shadow prices. The dual problem is thus;

$$\text{Minimize Cost} = Z^{\circ}_1 p_L + Z^{\circ}_2\, p_K + Z^{\circ}_3\, p_W$$

$$\text{Subject to} \quad a_{11}p_L + a_{21}p_K + a_{31}p_W \geq \pi_1$$

$$a_{12}p_L + a_{22}p_K + a_{32}p_W \geq \pi_1$$

$$p_L,\ p_K,\ p_W,\ \geq 0$$

where p_L, p_K and p_W are the shadow prices of labor, capital and water, respectively. The economic interpretation of the objective function of the dual problem is to minimize the opportunity cost of the use of resources. The constraints require that the value of resources used in the production of a unit of output be greater than or equal to the net profit per unit of output. Otherwise, profits can be increased by increasing output.

These shadow prices of the factor inputs indicate the real value to the industrial operation if one more unit of an input is available, or alternatively the opportunity cost to the industry if one less unit of the input is available. If increasing the quantity of a resource does not effect the objective function, the resource constraint is not binding, and its shadow price is zero.

For each amount of water, there is a corresponding solution of the dual problem for the shadow price of water. By systematically, or "parameterically" varying the amount of water available and then solving the dual problem for the associated shadow price of water, we can trace out a demand curve for water, i.e., a relationship between the quantity of water and its marginal benefit to the industrial facility.

The linear programming framework can be elaborated to encompass the aspects of industrial activities which are of particular interest in the analysis of water use and environmental problems. The basic idea is to account for all the inputs and outputs of each column, or activity, in the set of constraints, explicitly including all residuals of environmental interest, and also to include in this objective function social costs associated with the removal of resources from the environment and ultimate disposal into the environment of all residuals of the production process. As generally formulated, the objective function of such a linear programming model of an industrial facility is to minimize the costs of producing a given level of output; constraints require that for each resource or input used by the industrial plant that the total input to all "activities" equal the amount purchased plus the amount produced within the plant plus the amount released to the environment. Thus, rows of the constraint matrix represent materials balance equations for inputs and outputs. Each column describes the technical relationship between the inputs and outputs of one activity. The model selects the combination of activities which minimizes the total costs for producing the given output level.

The basic concepts and structure of this model, as well as the first application, were presented by Clifford Russell in Residuals Management in Industry--A Case Study of Petroleum Refining. Six types of activities are included in the model: (1) production alternatives; (2) by-product production; (3) materials recovery; (4) treatment and transport of residuals; (5) discharge of residuals; and (6) sale of products. The constraints require that inputs be available, all products produced are sold, output meets specified quality standards, and that all residuals be accounted for by material recovery, by-product production, treatment, or discharge. Table 8 presents Russell's formulation in more detail. The model structure can, of course, be altered in a variety of ways to meet the specific objectives of different analysts.

The primary task in model development is the construction of the activity vectors which requires

TABLE 8

Model of Industrial Residuals Management

Rows \ Columns	Production alternatives $X_1 \dots X_H$	By-product production $B_1 \dots B_J$	Raw material recovery $W_1 \dots W_K$	Treatment and transport of residuals $T_1 \dots T_L,\ V_1 \dots V_M$	Sale of products $Y_1 \dots Y_N$	Discharges of residuals $D_1 \dots D_g \dots D_G$		Right-hand side
Production and sale	$-\bar{e}_{X_1} \dots +\bar{e}_{X_H}$	$+\bar{b}_1 \dots +\bar{b}_J$			$-\bar{e}_{Y_1} \dots -\bar{e}_{Y_N}$		$\geqslant$	0
Input availability	$-\bar{p}_{X_1} \dots -\bar{p}_{X_H}$		$+\bar{w}_1 \dots +\bar{w}_K$				$\geqslant$	$-\bar{P}$
Output quality	$+\bar{q}_{X_1} \dots +\bar{q}_{X_H}$						$\geqslant$ $<$	$\bar{Q}$
Primary residuals	$-\bar{r}_{X_1} \dots -\bar{r}_{X_H}$	$+\bar{e}_{B_1} \dots +\bar{e}_{B_J}$	$+\bar{e}_{W_1} \dots +\bar{e}_{W_K}$	$+\bar{e}_{T_1}$ $+\bar{e}_{V_1} \dots$		$\dots +\bar{e}_{D_1}$	=	0
Secondary residuals		$-\bar{r}_{B_1} \dots -\bar{r}_{B_J}$	$-\bar{r}_{W_1} \dots -\bar{r}_{W_K}$	$-\bar{r}_{T_1} \dots +\bar{e}_{T_L} \dots +\bar{e}_{V_M}$		$+\bar{e}_{D_g}$	=	0
				$-\bar{r}_{V_1}$ $-\bar{r}_{V_M}$		.		.
				$-\bar{r}_{T_L}$		.		.
				Etc.		$+\bar{e}_{D_g}$	=	0
Possible discharge constraints						+1 +1 +1	$\leqslant$	$\bar{F}$
Objective function	Costs of production	Costs of production	Costs of recovery	Costs of treatment and transport	Prices of output	Possible effluent charges		

Schematic of models of industrial residuals management. The $\bar{e}$ are column vectors of zeroes and ones. A particular vector $\bar{e}$ has the number of row elements corresponding to the constraint set in which it appears. The occurrence of ones is determined by the function of the column in which the vector appears. Thus in $\bar{e}_{X_1}$, a one appears in the row corresponding to the output of process X_1.

Source: *Residuals Management in Industry: A Case Study of Petroleum Refining*, Clifford S. Russell, The Johns Hopkins University Press, Baltimore, USA, 1973. p. 20–21.

knowledge of the material flows throughout the production process and thus engineering data on inputs and outputs for specific process and residuals treatment options. Calloway [28] provides a useful simple illustration of the construction of such activity vectors for ammonia production. Russell presents a detailed account for petroleum refining in Residuals Management in Industry [29].

Industrial process modeling is a powerful tool for estimating demand curves for factor inputs and for analyzing the generation and disposal of residuals. It provides a means to estimate separate demand functions for water withdrawals, water consumption, water discharge, and waste disposal rights. Nevertheless, in addition to the usual limitations of linear programming such as linear approximations of nonlinear relationships, a few words of caution are necessary.

The first concerns the treatment of capital. One of the principal resource inputs to production processes is obviously the capital necessary to purchase the equipment. This capital equipment must be paid for and depreciated. The usual practice for handling capital costs in the model is to annualize the capital costs and calculate a capital cost per unit of output based upon the annual output of the plant. The annualization factor, however, tends to be simply a rule of thumb. The real cost of capital to a firm or plant and the appropriate depreciation rate for the capital are often very difficult to determine. Both can vary significantly between plants and between countries due to such factors as imperfect capital markets, different expectations about inflation, and different tax arrangements. Unfortunately, the results of such models are often sensitive to variations in capital costs well within the margin of error of the annualization factor.

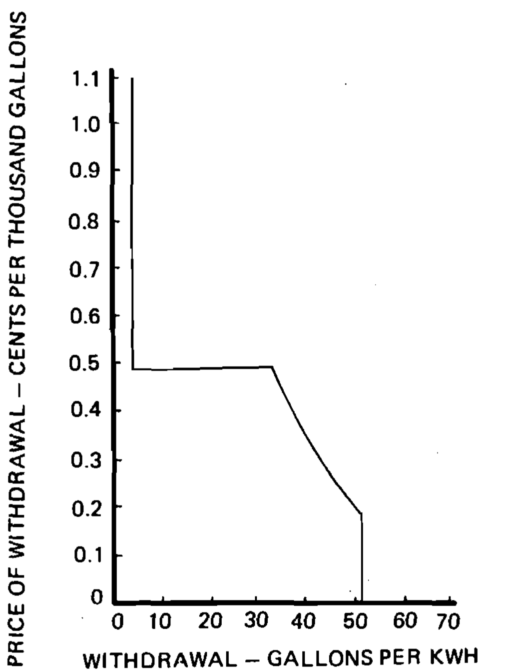
The treatment of capital in the model is further complicated in market economies because even if the cost of capital to the firm can be approximated by the real market rate of interest for different classes of credit risk, the market rate of interest cannot serve as an approximation of the social opportunity cost of capital or the social rate of discount. If the purpose of developing such models is to explain the behavior of industrial plants, then an examination of capital markets and tax policies at least leads the analyst toward the appropriate annualization factor. If the purpose of the analysis, however, is to provide a basis for making recommendations concerning water resources or environmental management policy, the welfare significance of the model results is unclear unless capital is valued at its social opportunity cost. Of course, any other factor inputs purchased in markets should also be assessed at their social value and not their market prices.

The second point concerns the difficulty of applying the model to existing facilities. The majority of model applications to date are for new "grassroots" facilities. In this case the model selects the optimal levels of the activities assuming a new plant is to be designed and all possibilities are open. If the modeling approach is adapted to an existing facility, the options are more limited. The costs of continuing to operate existing equipment may be very different than for new facilities. The model can reflect such considerations, but data on the economic value of existing capital can be very difficult to obtain and are subject to serious measurement problems.

The third point is that the minimization of total costs for producing a given output determines only one point on the firm's total cost function. This does not, of course, yield the optimal solution for the firm unless the minimum cost solution for the given output also maximizes profit to the firm. The problem of the firm is to determine the minimum cost of producing every level of output and then, given this total cost function, determine the optimal level of output. Thus, if this cost minimization objective function is used, the analyst must either explore the profitability of alternative levels of output or very carefully select the initial level of output. The assumption generally made is that the size of the most recently built plants in the industry is optimal. This is reasonable, but the optimal size of plants in an industry does change over time, between locations, and from one production process to another. When the model coefficients are unitized to reflect the inputs required to produce one unit of final output, the analyst must be very careful that the aggregation of the results to the level of the plant does not violate the engineering assumptions implicit in the original determination of the coefficients.

Finally, the model optimization attempts to explain what should happen given the assumption and data limitations if the plant minimizes cost or maximizes profits. The model cannot explain what will happen if the plant owners or managers cannot competently achieve their objective or if they pursue other objectives. Thus, model verification can be very difficult.

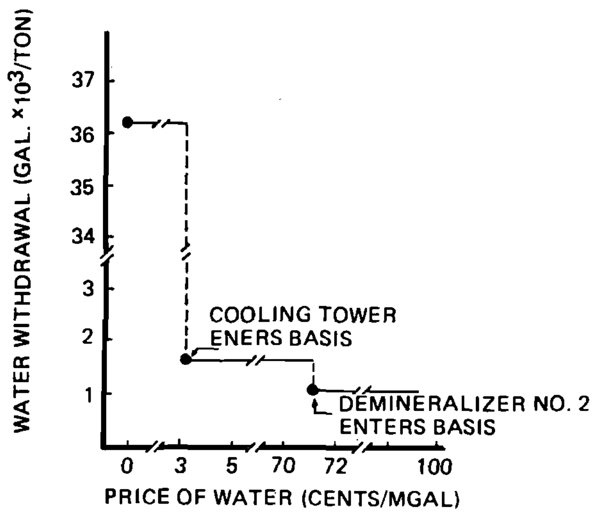
It is felt, however, that linear programming methods of estimating industrial water demand functions have significant potential for use in water resources planning. Indeed, a limited number of industrial applications already exist. In the United States, work in this area of industrial process modeling for the analysis of environmental and energy issues has been carried out by two groups: Resources for the Future (C. Russell and J. Vaughn) and the University of Houston (Russell G. Thompson et al.). Industrial plants in the following industries have been modeled: petroleum refining; iron and steel; electric power generation; ammonia; chlor-alkali; alkali; and ethylene. Figure 9, Figure 10, Figure 11, and Figure 12 present some samples of water demand curves developed by the University of Houston project.



Imputed demand curve with cooling tower option ($p_1 = 30¢/10^6$ Btu, $p_s = 0$).

Source: Thompson & Young "Forecasting Water Use For Policy Making: A Review" Water Resources Research August, 1973 p 796

FIGURE 9 Estimate of a Water Demand Function in Steam Power Generation

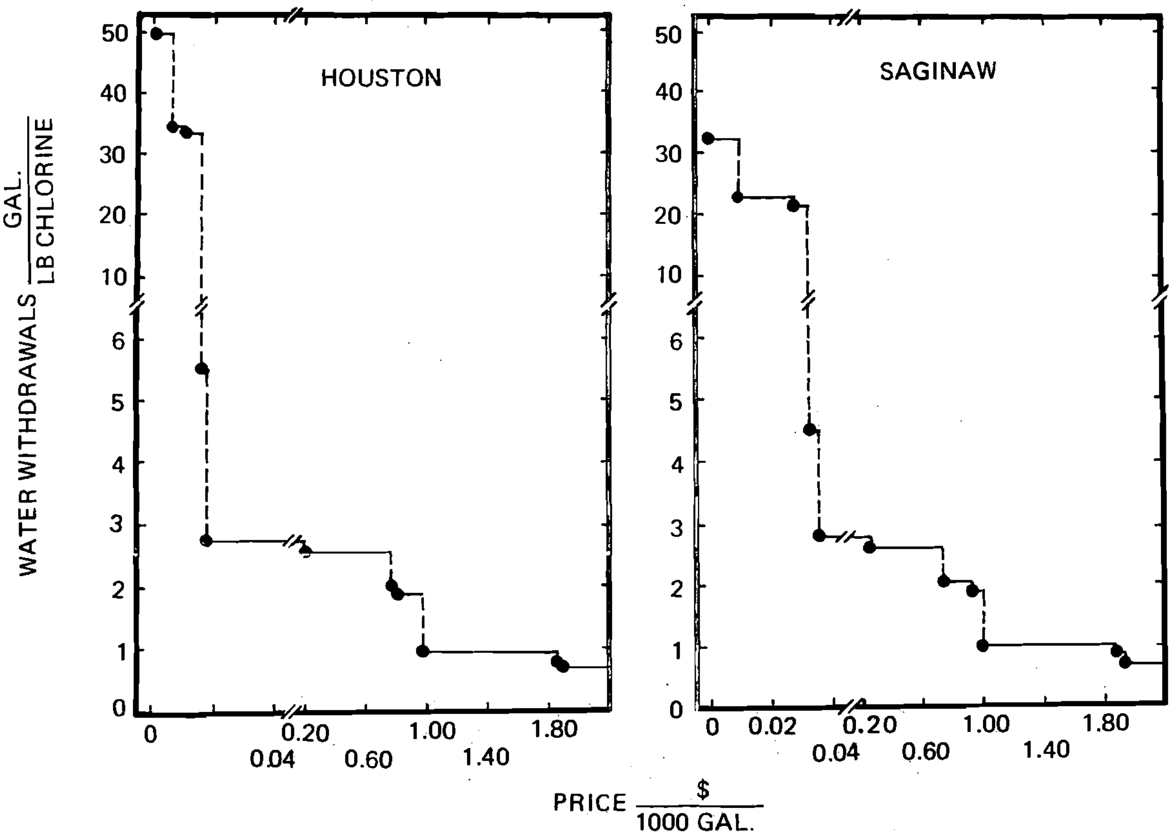


Water withdrawal versus water price.

Source: Calloway et al. "Industrial Economic Model of Water Use and Waste Treatment for Ammonia" Water Resources Research August, 1974 p 657

FIGURE 10 Estimate Water Demand Function for Amonia Production

FIGURES 11 & 12 Estimates of Water Demand Functions in Chlor-Alkali Production



Comparison of water withdrawals versus water price for large diaphragm cell plants in Houston, Texas, and Saginaw, Michigan.

Source: Singleton et al. "Integrated Power Process Model of Water Use and Waste Water Treatment in Chlor-Alkali Production"; Water Resources Research; August, 1975, p 521

4 Use of Water Demand Functions for Forecasting Water Use

Linear programming techniques can thus be utilized to estimate demand curves for water for industrial facilities or representative plants. The use of these plant level demand curves for forecasting water use is a straight forward application of microeconomic theory, assuming firms are price takers. As noted in the introduction, we want to project the shifts over time in the supply and demand functions, and then solve for the supply and demand equilibrium in each period. The analyst will hopefully have information on projected changes in several types of variables on which to base his forecast of water use. These include projections of economic growth, changes in government policy such as the imposition of increasingly stringent water pollution abatement requirements, estimates of the future costs of providing additional water supplies, changes in factor input availability and prices, anticipated changes in product specification and product mix, and changes in process and waste treatment technologies [30].

Assume the analyst knows the costs of providing additional water, and thus knows the supply curve for the region. Assume for simplicity that the cost of providing water is the same for all users in the region. This is relatively realistic if we are considering adding additional reservoir capacity upstream from a number of users concentrated in one area and the delivery costs of the water supplies are the same for all users. We also assume that the real costs of supplying water to the intake of the users remain constant over the period of projection, i.e., that the supply curve does not shift downward to the right due to improvements in the technology of supplying water. We thus have a supply curve for a river basin or region which will likely increase sharply after easily available resources are exhausted and additional supplies become increasingly costly to obtain (see Figure 13).

The analyst's task is to project the individual demand curves (D_1, D_2,...D_N in the Figure 13, ordered from smallest to largest), to add together the demands of all the users, and then to compare the total regional demands with the regional supplies [27]. The demand curves of different industrial facilities are added horizontally to obtain the total industrial demand for the region. For example, consider the situation in which at price p_1, industrial plant A demands Q_1; plant B demands Q_2 and plant C demands Q_3 (see Figure 14). If these were the only three users, the regional industrial demand at price p_1 would be $Q_1 + Q_2 + Q_3$. By varying the price, we can trace out the regional industrial demand curve.

Suppose the analyst wants to project water use 10 years ahead. He can easily incorporate his projections of water

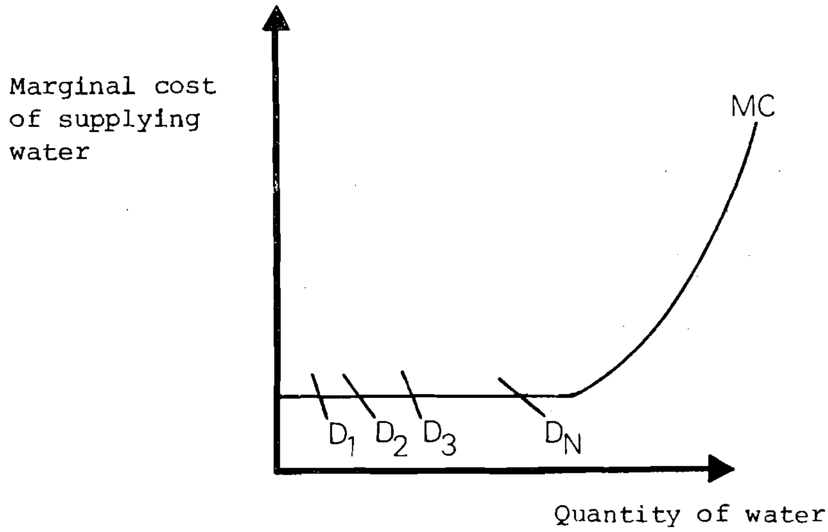


FIGURE 13

pollution abatement requirements by solving the linear programming model for each representative plant subject to the constraint that wastewater discharges meet certain standards, and thus trace out a new demand curve for water. This is the demand curve for water which would exist given current technology if the projected pollution abatement requirements were now enforced.

The next step in the analysis is to incorporate the projection of economic growth. The projection of economic growth must be disaggregated to the level of the model or representative plants for each industry. For example, suppose steel production is forecast to double in 10 years. If the analyst only has a LP model of one type of iron and steel facility, he will probably be forced to estimate the demand curve for the steel industry in the region at t + 10 by simply scaling up the water demand curve of the model plant. In other words, the analyst calculates a figure for water use at the model plant for each price of water. Since he knows the output of the model plant, he can calculate the water use per ton of steel production for each price of water. To estimate the demand curve for the total industrial production in the region at t + 10, the analyst multiplies the water use per ton of production coefficient by the projected output for each price of water.

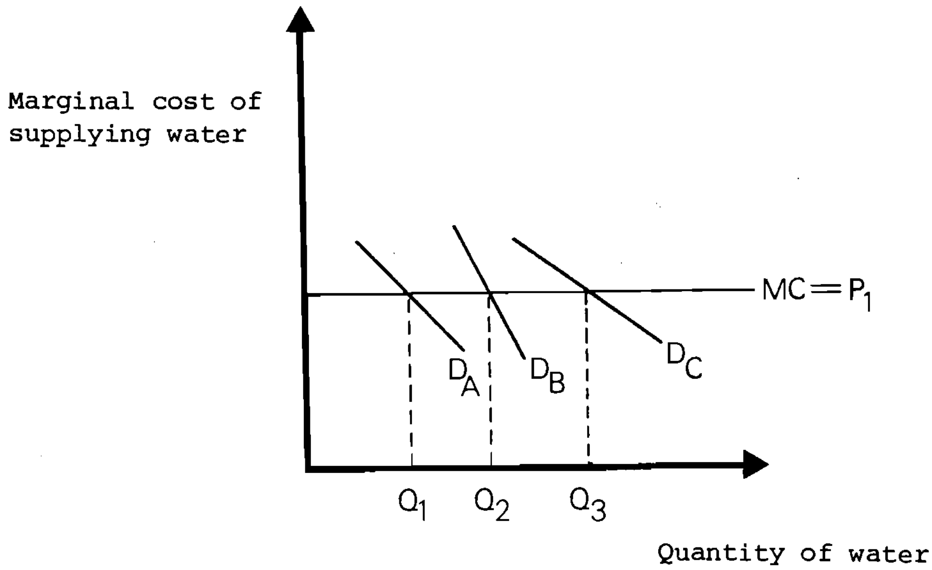


FIGURE 14

If more than one representative plant model exists and projection of output at t + 10 can be divided between these two types of facilities, then the regional demand curve for each type can be estimated for t + 10. The more model plants available and the more accurately the representative plants characterize the other plants in the industry in the region, the better will be the projection of the demand curve for water for the entire industry at t + 10. This aggregation of plant-level water demand curves to an industry water demand curve can introduce large errors into the analysis. The analyst should carefully examine the representativeness of the model plants and the likely characteristics of future industrial capacity to assess the likely bias introduced as a result of this extrapolation [31].

Forecasting industrial water use also requires demand functions for both agricultural and municipal users because the marginal cost of water supply is dependent upon the total water use in the region. In order to forecast industrial water use independently of other water users, we must choose the "price of water" prevailing at time t + 10, or assume that the supply curve at t + 10 is perfectly elastic at the given "price of water". For example, we could assume that the current relative price of water will continue until period t + 10. In this case the forecast of industrial water use (assuming anticipated water pollution control requirements are not incorporated) will yield very similar results to a forecast based upon a regression of quantity of water used on some economic variable because there will often be little variation in the relative price of water in the historical data.

The question of how best to incorporate technological change into water use forecasting is a difficult one. There are actually two aspects to consider. First, changes in the relative prices of factors of production will affect the choice of techniques, i.e., which technology is selected as the least cost means to produce a given commodity. To the extent that some future trends in relative prices are discernable, the analyst may predict which of the existing techniques will be used in the future and thus the ensuing pattern of water use. Second, new techniques may be developed. Forecasting such technological breakthroughs is much more difficult to incorporate into a model of water use. If the analyst knows the current research and development expenditures, he may be able to determine the likely water use characteristics of new techniques and thus introduce this knowledge into the forecast.

Changes in technology can affect water demand functions in numerous ways. We can distinguish between changes in the technology of using and treating water itself and changes in production technologies which also impact the ways water is used. Advances in water use and wastewater treatment technologies change the cost of using water and are thus captured in the price of water variable. Changes in production technologies, however, shift the demand function for factor inputs such as water. Different industrial processes producing the same or similar products use water in different ways and thus have different demand functions for water. The literature is full of examples. Wastewaters from Kraft pulp mills have different characteristics than sulfite or groundwood mills. Basic oxygen furnaces generate different residuals and have different cooling requirements than electric arc or open hearth furnaces. Modern petroleum refineries have different water use patterns than older, simpler refineries. The point is simply that water demand functions for an industry cannot be assumed to remain fixed as production technologies change. Water demand functions for a single industrial facility cannot be assumed to remain fixed as individual process units are replaced with more modern equipment. Technological change can either increase or decrease the industry's demand for water withdrawals; the matter is again one for investigation.

In addition to technological changes, projections of future water use should include an examination of changes in the quality of factor inputs, product specifications, and product mix of an industry or industrial facility due either to changes in the relative prices of products or changes in tastes. Changes in relative prices not only influence the choice of techniques, but also the vector of final demands. If the price of plastics falls relative to paper, the switch to plastic packaging will have implications for industrial water use and wastewater discharges. At the plant level the imposition of pollution controls will increase the relative differences in costs of producing bleached paper vs. unbleached paper. Increased demands for specialty, high-quality steel will alter the amount of gross water applied per ton and the character of the wastewater stream. Changes in tastes may increase the demand for throwaway bottles and cans. Sources of crude oil may contain more sulphur and thus entail higher pollution control costs. The characteristics of agricultural products received by food processing plants may change due to the introduction of automated harvesting techniques or new varieties of crops. The ores received at benefication facilities may become progressively poorer as the richer sources are exhausted and require additional processing and water use. Such trends and changes in product mix, input factor quality and product specifications are often difficult to quantify and incorporate in projections of water uses, but at a minimum the analyst should be aware of the assumptions being made.

5 Additional Remarks and Conclusions

We have several additional observations on the forecasting of industrial water use. The first is perhaps an obvious methodological point. Any forecast is subject to error from unforeseen events and random shocks to the system. A model of industrial water use attempts to explain the future pattern of water use in terms of projected changes in a limited number of variables. The model is always incomplete. We should expect surprises. Forecasting models of industrial water use are only a useful planning tool if the analyst understands their limitations and is prepared to adjust to unanticipated circumstances.

Second, knowledge of forecasting techniques is no substitute for an understanding of the actual situations in industries and water supply systems. The deficiencies in the data in this field are often so severe that only an analyst with such an understanding can judge how the data can be used. For example, in his paper "Modelling of Water Demands and wastewater Discharges in England and Wales", Robert J. Smith notes that the trend in per capita consumption of unmetered water since 1961 is significantly influenced by very high data for 1963 and 1964. These data are attributable to severe winter weather in early 1963, which caused a large number of pipes to burst and consequently a high level of leakage. Only someone familiar with such data would know how to interpret them.

The third concerns the forecasting of peak loads. Throughout this paper we have implicitly talked of industrial water use in terms of average daily or monthly flows. The problem of peak load demands is generally a more critical issue for municipalities than for industries. Many large water-using industrial facilities operate 24 hours a day, 7 days a week and are not subject to the same type of daily cycles of water use as municipalities. Moreover, some industrial facilities with peak load demands store enough water themselves to average out some stochastic variations in water use. However, Hanke and Bower argue that there is very substantial stochastic variation in the demand for intake water and wastewater discharges.

> "The variability occuring in 'normal' operations has been underestimated and overlooked. These variations reflect changing qualities of raw material - not only from day to day but within the day -, weekly and seasonal changes in product mix, ambient temperatures and varying conditions of operating equipment. In addition to these variations under normal operating conditions, there are substantial variations associated with 'start-up' and 'shut-down' periods and accidential spills and breakdowns [32]."

The reasons for the peak to average ratio are likely to vary widely between plants in one industry and between different industries. The significance of such peak load variations in industrial water use depends, of course, upon the particular policy issue the water resources analyst wishes to address. The standard engineering procedure for estimating municipal peak load demands is to simply multiply average flows for some time period by some factor which depends upon the size of population and possibly a few other variables. Use of such a rule of thumb may well prove very inaccurate for forecasting industrial peak load demand. We do not know of any model which explains peak load industrial water use except as a simple multiple of average flows. The study of the stochastic variations in industrial water withdrawals and wastewater discharges is certainly an area which deserves further research. Detailed forecasting of such variations seems impracticable to incorporate in the planning process in the near future.

To summarize, industrial water use forecasts are prepared throughout the world with simple statistical methods. More sophisticated forecasting techniques utilizing regression and input-output analysis are becoming more widely used, but they do not offer explanatory models of industrial water use. Water demand functions derived from statistical models of industrial water use would theoretically be useful in forecasting industrial water use, but the limitations of the available data render them of limited utility. Although subject to numerous limitations, linear programming models of industrial plants offer the only systematic means of analyzing both engineering and economic data to derive industrial water demand functions for forecasting purposes. The development of industrial process models is constrained by both limited manpower and financial resources. The benefits of such models are not, however, limited to water resources planning, and as they become more widely used in economic planning and policy analysis, industrial process models promise to be a useful forecasting tool in the water resources field.

FOOTNOTES

[1] We assume the perfectly elastic demand curve to be a totally unrealistic case.

[2] See Hanke and Bower "Economic Aspects of Attaining Efficiency in the Use and Reuse of Water".

[3] Relative prices reflect real exchange ratios between different goods and services and are thus not dependent on the nominal price level.

[4] National Institute for Water Supply--The Netherlands, Quarterly Report No. 8, January 1977.

[5] Social Science Series No. 10, Inland Waters Directorate, Water Planning and Management Branch, Ottawa, Canada, 1973. See also "Water Use and Demand Forecasting in Canada: A Review" by Donald M. Tate, September 1977, Inland Waters Directorate, Water Planning and Management Branch, Economic Analysis Section, Ottawa, Ontario.

[6] "Creating A Water Resources Model", March 1977, Water Resources Bureau, National Land Agency, Japan.

[7] "Methodologies and Assumptions for Water Use and Supply Data" and "Modified Central Case--Water Requirements; Water Supplies and Comparison Thereof", preliminary documents, U.S. Water Resources Council, Suite 800, 2120 L Street NW, Washington, D.C. 20037.

[8] "Water Resource Models: A Historical Summary" by Nathaniel Wollman in Economic Modeling for Water Policy Evaluation.

[9] For an excellent discussion of the methodological and practical problems and advantages of using multiple regression analysis for forecasting, see Forecasting Recreation in the United States by Charles Cichetti, Lexington Books, 1973.

[10] The estimation problems often encountered in regression analysis, such as multicolinearity, are detailed in many statistics and econometrics texts. See, for example Econometric Methods by J. Johnston, McGraw-Hill, 1972; or Applied Econometrics by Potlori Rao and Roger LeRoy Miller, Wadsworth Publishing Company, 1971.

[11] The reduced form model expresses each dependent (endogenous) variable as a function of independent (exogenous) variables and an error term. The endogenous variables are determined within the model; the exogenous variables are determined independently outside the functional relationships described by the model.

[12] See Industrial Demand for Water: A Study of Southeast England by Judith Rees, Weidenfeld and Nicolson, 1969; and Water Use in Manufacturing, The Conference Board, 1975, for examples of this type of calculation.

[13] The implications of including the 'price' of water in the regression will be discussed in more detail in the next section on "Statistical Estimation of Water Demand Functions".

[14] See Industrial Demand for Water: A Study of Southeast England by Judith Rees, Weidenfeld and Nicolson, 1969, and Water Use in Manufacturing, The Conference Board, 1975, for examples of this type of calculation.

[15] This is the one of four rules presented by A. Marshall to explain the elasticity of demand for a factor of production in the two factor case. See Marshall's Principles.

[16] See Blair T. Bower, "The Economics of Industrial Water Utilization", in Water Research, edited by A.V. Kneese and S.C. Smith, The John Hopkins University Press, Baltimore, 1966.

[17] See Appendix - *Mathematical Derivation of Water Demand Functions*.

[18] In these equations (9)-(11), the exogenous variables Z_1, Z_2, and Z_3 correspond to the exogenous variables A, B, and C in equations (1)-(6).

[19] See *Consumer Demand in the United States; Analysis and Projections with Application to Other Countries*. Second Edition, Harvard University Press, Cambridge, Mass., 1976.

[20] See De Rooy, 1969; Ginn et al, 1975; Rees, 1969.

[21] See *The Identification Problem in Econometrics*, F.M. Fisher; *Econometric Methods*, Johnston; or any standard econometrics text.

[22] For an interesting effort at dealing with such data, see "The Price Elasticity of Demand for Water" by D.M. Wood and R.M. Bell, February 1974.

[23] Gibbs, Kenneth, "Price Variable in Residential Water Demand Models (Paper 7W0761), *Water Resources Research*, Vol.14, No.1, February 1978.

[24] Ginn et al, 1975.

[25] See Dorfman, R., P.A. Samuelson, and R. Solow, *Linear Programming and Economic Analysis*, New York, McGraw-Hill, 1958.

[26] Much of the pioneering work in this field is by A. Charnes and W.W. Cooper. See, for example, *Management Models and Industrial Applications of Linear Programming*, John Wiley & Sons, Inc., New York, 1961.

[27] Here we generally follow the notation presented in *Microeconomic Theory* by R. Layard and A.A. Walters, McGraw-Hill, 1978.

[28] "Process Modeling Using Linear Programming", James A. Calloway, 1977, Proceedings of the First Workshop on Modelling of Water Demands, January 17-21, 1977, J. Kindler (Editor), International Institute for Applied Systems Analysis, CP-78-06, Laxenburg, Austria.

[29] See also *Steel Production, Processes, Products and Residuals*, by C.S. Russell and J. Vaughn.

[30] For examples of regional water supply and demand analysis, *Water Supplies and Economic Growth in an Arid Environment ; An Arizona Case Study* by Maurice M. Kelso, William E. Martin, and Lawrence E. Mack and *Interbasin Water Transfer--A Case Study in Mexico* by Ronald C. Cummings.

[31] See "Sufficient Conditions for Exact Aggregation in Linear Models", by T. Miller *Agricultural Economics Research*, 18, 1966, p.52-57.

[32] Noted in "Economic Aspects of Attaining Efficiency in the Use and Reuse of Water" by Steve Hanke and Blair T. Bower; a paper presented to the UN Panel of Experts on Wastewater Reuse, Tel Aviv, Israel, 1974.

[33] The derivation of demand functions for factor inputs is often presented in microeconomic texts. See for example, *Microeconomic Theory: A Mathematical Approach* by J. Henderson and Richard Quandt, McGraw-Hill, 1971. For a discussion of the water demand function in particular, see "Forecasting Water Use for Policy Making: A review" by R.G. Thompson and H.P. Young *Water Resources Research*, August 1973 and *The Industrial Demand for Water Resources--An Econometric Analysis* by Jacob DeRooy.

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APPENDIX

MATHEMATICAL DERIVATION OF WATER DEMAND FUNCTIONS [33]

The objective of the firm is commonly characterized as profit maximization subject to the constraints that profits be nonnegative and that output be feasible in terms of the production function. Consider a firm with three inputs--capital (K), labor (L), and water (W)--producing a product X with a production function X = f (K,L,W). The problem of the firm is thus

Maximize Profits = Total Revenue - Total Costs

$$\text{MAX } \pi = P_x X - [P_k \cdot K + P_l \cdot L + P_w \cdot W]$$

Subject to $X = f(K,L,W)$ and $\pi \geq 0$ where P_x, P_k, P_l and P_w

are the prices of the product and the inputs capital, labor, and water, respectively.

We can substitute the production function for X:

$$\text{MAX } \pi = P_x\, [f(K,L,W)] - [P_k \cdot K + P_l \cdot L + P_w \cdot W]$$

The first order optimality conditions are:

$$\frac{\partial \pi}{\partial K} = P_x \cdot \partial f(K,L,W)/\partial K - P_k = 0$$

$$\frac{\partial \pi}{\partial L} = P_x \cdot \partial f(K,L,W)/\partial L - P_l = 0$$

$$\frac{\partial \pi}{\partial X} = P_x \cdot \partial f(K,L,W)/\partial W - P_w = 0$$

Since P_x is some function g (P_k, P_l, P_w), the first order conditions are the implicit demand functions for the factor inputs. If we knew the production function, we could straightaway solve these equations for the explicit demand functions for the inputs:

$$K = h^1 \ (P_k, P_l, P_w, X)$$

$$L = h^2 \ (P_k, P_l, P_w, X)$$

$$W = h^3 \ (P_k, P_l, P_w, X)$$

The quantity of water demanded is thus some function of the price of water, the prices of the other input factors, and the level of output.

The demand functions for the input factors can be similarly derived from the first order conditions of the dual problem:

$$\text{Minimize Costs} = P_k \cdot K + P_l \cdot L + P_w \cdot W$$

subject to the production function, holding output constant. In this case we only observe the substitution effect of a change in the price of water, and thus cannot develop the complete measure of the own price elasticity of demand for water.

To derive the own and cross price elasticities of the demand function for water $(\frac{\partial W}{\partial P_i}$, Pi where i = w,l,k) .

We first take the total derivatives to obtain:

$$P_x \ [f_{KK}dK + f_{KL}dL + f_{KW}dW] + f_K dP_x = dP_k$$

$$P_x \ [f_{LK}dK + f_{LL}dL + f_{LW}dW] + f_L dP_x + dP_1$$

$$P_x \ [f_{WK}dK + f_{WL}dL + f_{WW}dW] + f_W dP_x = dP_w$$

We then solve for dW by Cramer's rule:

$$\begin{vmatrix} P_x f_{KK} & P_x f_{KL} & (-f_K dP_x + dP_x) \\ P_x f_{LK} & P_x f_{LL} & (-f_L dP_x + dP_l) \\ P_x f_{WK} & P_x f_{WL} & (-f_W dP_x + dP_w) \end{vmatrix}$$

$$dW = P_x \begin{vmatrix} f_{KK} & f_{KL} & f_{KW} \\ f_{LK} & f_{LL} & f_{LW} \\ f_{WK} & f_{WL} & f_{WW} \end{vmatrix}$$

To obtain the own and cross price elasticities of demand for water we simply divide the value for dW by dP_w, dP_k, or dP_l and then multiply by P_w, P_k, or P_l, respectively. The point of this exercise is to illustrate that the computation of the own and cross price elasticities of the demand function for water involves a detailed understanding of the firm's production function and the market for its product. The signs of the cross-price elasticities are ambiguous; only the sign of the own price elasticity is known (i.e., negative). These results generalize to the n-factor case. It is difficult to distinguish between output and substitution effects from a change in the price of water without assuming a particular form of production function such as constant returns to scale. Although the examination of the two-factor case does give some feel for the problems involved in determining the elasticity of demand for a factor input, the results for the two-factor case do not all generalize to the n-factor case because inputs may be either substitutes or complements.

Blair Bower has suggested the following conceptual relationship of the quantity of water withdrawn, consumed, and discharged to factors of production function and the economic environment of firm [35]:

$$Q_{It}, Q_{Dt}, C_t, Q_{Et}, W_{Dt}, W_{Et} = f[Q_t, q_t, T, PP, L, OR, poqr, R, S$$

$$= E_c, A_c, Q_{dt}, q_{dt}, D, C_w/C_t] \quad ,$$

where

QIT = the quantity and time pattern of water intake;

Ct = the quantity and time pattern of consumptive use;

QDt and WDt = the quantity and time pattern of wastewater discharge and residuals in the wastewater, respectively;

QEt and WEt = the quantity and time pattern of final effluent water, and residuals in the effluent water, respectively;

Q_t and q_t = the quantity and quality and their corresponding time patterns of water available at the intake;

T = the water and waste treatment processes within the production unit;

PP = the technology of the production process;

L = the physical layout of the plant;

OR = the operating rate;

porg = the product output quality requirements;

R = the degree of recirculation;

S = the solid wastes from the production process;

E_c = the limitations on the final liquid effluent;

A_c = the limitations of the final gaseous effluent;

Q_{dt} and q_{dt} = the quantity and quality and their corresponding time patterns of water available for dilution at the effluent point;

D = availability places for final disposal of wastes; and

C_w / C_t = the ratio of total water utilization costs to total production costs.

Although obviously not an explicit functional relationship, the formulation does illustrate the kinds of factors which would have to be included in our generalized production function x = f (K,L,W) in the previous example.