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DYNAMIC STANDARD SETTING FOR CARBON DIOXIDE

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Preface

Standard setting is one of the most commonly used regulatory tools to limit detrimental effects of technologies on human health, safety, and psychological well being. Standards also work as major constraints on technological development, particularly in the energy field. The trade-offs to be made between economic, engineering, environmental, and political objectives, the high uncertainty about environmental effects, and the conflicting interests of groups involved in standard setting make the regulatory task exceedingly difficult.

Realizing this difficulty, the Volkswagenwerk Foundation sponsored a research subtask in IIASA's Energy Systems Program entitled *"Procedures for the Establishment of Standards"*. The objectives of this research are to analyze existing procedures for standard setting and to develop new techniques to improve the regulatory decision making process. The research performed under this project include:

i) policy analyses of the institutional aspects of standard setting and comparisons with other regulatory tools;

ii) case studies of ongoing or past standard setting processes (e.g. oil discharge standards or noise standards);

iii) development of formal methods for standard setting based on decision and game theory;

iv) applications of these methods to real world standard setting problems.

The present research memorandum is one in a series of papers dealing with the application of game-theoretic methods to standard setting. It presents a formal model for the conflict situation arising from carbon dioxide pollution.

Wolf Häfele

Abstract

Under the assumption that a continuous increase in atmospheric carbon dioxide beyond a critical value, caused by the combustion of fossil fuel, will lead to irreversible and large changes of the climate of the earth, the problem of limiting CO_2 emission becomes an urgent concern. The subject of how to determine and adapt an emission standard for carbon dioxide is treated as a three-person infinite stage game, the players of which are the decision units of regulators, producers, and population. After the description of the model solutions are derived for several solution concepts and discussed. In special cases the solutions differ substantially from each other.

Dynamic Standard Setting for Carbon Dioxide

INTRODUCTION

The emission of carbon dioxide into the atmosphere resulting from fossil fuel use has been increasing at an exponential rate for more than one century. If this expansion continues, the concentration of carbon dioxide in the atmosphere may be doubled in about the next 60 years according to R.M. Rotty, 1977. The effects on the global climate may well appear suddenly and could get out of control before remedial actions become effective.

Since easily accessible fossil fuels contain such big amounts of carbon there is a strong tendency to use them as a source of energy that could last for nearly two more centuries. This is much more so since the competing nuclear energy meets increasing resistance by citizen groups. But it is the vastness of this carbon reserve that causes deep concern within the climatological community. The amount of carbon in recoverable fossil reserves is ten times the amount now contained as carbon dioxide in the entire global atmosphere.

As these reserves are being used, the concentration of carbon dioxide in the atmosphere will surely increase; and because carbon dioxide absorbs a portion of the infrared radiation emitted by the earth, it is generally believed that a higher atmospheric temperature will result ("greenhouse effect"). Although it is uncertain how much warming is produced by a given increase, the increased atmospheric carbon dioxide could have a considerable impact on man's environment.

Significant physical effects that may be expected with high fossil use are the melting of polar sea ice and/or decreasing precipitation in mid-latitude regions. Major socio-political impacts could plausibly attend a substantial increase of carbon dioxide, for example:

- large and persistent fluctuations in global food supply, due to repeated crop failures in various regions of the world which are caused by chronic and severe weather variability;

- increasingly regulated demographic migration between regions and across national borders, due to a climate-related collapse of selected webs in regional economies; shifts in the power balance among nations due to physical effects stimulating the economic and cultural decline in some regions and stimulating increased growth and prosperity elsewhere.

At the present time the physical processes causing variations of temperature are poorly understood (see J. Williams, 1978 and T. Augustsson et al., 1977), and changes due to atmospheric carbon dioxide increases are impossible to detect since there is no accurate knowledge of the natural variability of the global average temperature. As outlined by O.W. Markley et al., 1977, and R.M. Rotty, 1977, the other physical and sociopolitical effects are also highly uncertain.

Although a large part of the climatological community shares the opinion that mankind needs and can afford a time window between five and ten years for vigorous research and planning in order to narrow the uncertainties sufficiently so as to justify a major change in energy policies, the model analyzed in this paper excludes an increase of relevant knowledge about the physical effects. Thus the model deals with the pessimistic view of the climatic aspects of carbon dioxide. It is global in character because the global effects seem to dominate the local or regional ones.

Given these substantial uncertainties about the development of climate, the problem of what energy policies governments should choose, becomes important. This problem is approached as a conflict situation among the groups of governments, producers emitting carbon dioxide, and population. In order to work out the global aspects this conflict situation has been formalized as a multistage three person game, the players of which are called regulator, producer, and impactee. Thus we neglect conflicting interests among governments, producers, and different groups of populations, such as of developed and developing countries. The regulator stands for an international agency, the producer for an organization of all producers, and the impactee for the community of people possibly affected by the carbon dioxide problem.

The paper is based on the assumption that a continuous increase of atmospheric carbon dioxide beyond a critical value will lead to irreversible and large changes of the climate which are regarded as a catastrophe. All three players have their subjective probability of the level of the critical value. Since, by assumption, there is no increase of knowledge about the climatological process, the regulator can only be concerned about the reactions of the producer and especially of the impactee.

After the specification of the model the results for several solution concepts are derived. These are quite different in general but can all be interpreted in terms of fair play or power. Given that the model allows prescriptive answers although it is primarily descriptive.

Since data are often unknown or scarcely available or arbitrary--as in the case of the regulator where the utility function may be conceived of as reflecting a trade-off between the interests of producer and impactee--solutions are derived as functions of the parameters. Hence parameter analysis can reveal the crucial parameters. For the purpose of illustration a small numerical example is added.

THE MODEL

The conflict situation is described by a three-person dynamic or multistage game in extensive form (see G. Owen, 1968, or J.C.C. McKinsey, 1952) which resembles stochastic games. At each stage a component game of perfect information is played which is completely specified by a state. The players' choices control not only the payoffs but also the transition probabilities governing the game to be played at the next stage. Each player has his own subjective estimate of the transition probability due to his subjective probability of the "true critical value".

The set of states of the game is

$$S = \{(C,L) \mid C_p \geqslant C \geqslant 0, L \geqslant 0\} \cup \{k \geqslant 0\}$$

C being the amount of carbon dioxide in the atmosphere;
C_p the maximal amount of carbon dioxide if all fossil fuel is burnt;
L the upper bound of carbon dioxide emission during a period;
k the critical value for a catastrophe.

Let (C^1,L^1) denote the first state. Then C^1 can be assigned the present amount of atmospheric carbon dioxide, and L^1 the present maximal emission of CO_2 or some multiple of it.

The perfect information of the component games is specified as follows:

For state (C,L) the regulator's set of choices is

$$M_R(C,L) = \{\ell \mid 0 \leqslant \ell \leqslant L \leqslant L\} \quad ,$$

where ℓ denotes the upper bound of the emission of carbon dioxide by the producer.

Then the producer chooses the amount of carbon dioxide to be emitted. His set of choices or measures equals

$$M_p(C,L,\ell) = \{a \mid 0 \leqslant a \leqslant \ell, \ a \leqslant \frac{C_p - C}{\beta}\} \quad ;$$

$0 < \beta < 1$ is defined below. The impactee's set of measures equals

$$M_I(C,L,\ell,a) = \{p \mid 0 \leqslant p \leqslant 1\} \quad .$$

Knowing the choices ℓ and a he chooses the degree p of the pressure he wants to exert on the regulator. p can denote the probability of a vote to suspend the government or of an aggression against institutions.

The sets of measures in the case of k, i.e. a catastrophe has occureed at amount k of carbon dioxide in the atmosphere, equal

$$M_R(k) = \{0\} \quad ;$$

$$M_P(k,0) = \{0\} \quad ;$$

$$M_I(k,0,0) = \{0\} \quad ;$$

which means that there is no pressure.

Given state (C,L) and the choices (ℓ,a,p) the following states are possible at the next stage:

$$(C + \beta a,\ L), \qquad (C + \beta a,\ \frac{L}{2}), \qquad \{k \geqslant C\} \quad .$$

The first component of the first and second states indicates that the constant share βa of emitted carbon dioxide is added to the amount of carbon dioxide in the atmosphere. This is consistent with results of box models for the CO_2 cycle of the earth (see R. Avenhaus, et al., 1978) if a is emitted at a constant rate during the time period. The estimates for β range between 0.01 and 0.5. Amount $(1 - \beta)a$ is assumed to disappear into the biosphere, the upper mixed layer of the sea, and the deep sea. The second components express that the old upper bound either remains or is reduced by half. It is assumed that there is a probability pv that L is replaced by $\frac{L}{2}$, where $0 < v < 1$ is a parameter provided that the catastrophe will not occur. $k \geqslant C$ denotes the amoung of carbon dioxide in the atmosphere at which the catastrophe occurs.

All three players are assumed to have subjective probabilities relating to the critical amount k of carbon dioxide. They characterize the transition probabilities. For simplification of the model we assume that the subjective probabilities concentrate on points denoted by C_R, C_P, and C_I for regulator, producer, and impactee. We assume $C_R < C_P$, $C_I < C_P$ thus allowing the producer to neglect a possible catastrophe.

The subjective probabilities P_R, P_P, P_I for the transition from (C,L) to the possible new states are

New state t	$P_R(t\|C,L,\ell,a,p)$	$P_P(t\|C,L,\ell,a,p)$	$P_I(t\|C,L,\ell,a,p)$
$(C+\beta a,L)$	0 if $C\leqslant C_R<C+\beta a$ or $C_R<C<C+\beta a$	$1-pv$	0 if $C\leqslant C_I<C+\beta a$ or $C_I<C<C+\beta a$
	$1-pv$ if $C+\beta a\leqslant C_R$ or $C_R<C=C+\beta a$		$1-pv$ if $C+\beta a\leqslant C_I$ or $C_I<C+\beta a$
$(C+\beta a,\frac{L}{2})$	0 if $C\leqslant C_R<C+\beta a$ or $C_R<C<C+\beta a$	pv	0 if $C\leqslant C_I<C+\beta a$ or $C_I<C<C+\beta a$
	pv if $C+\beta a\leqslant C_R$ or $C_R<C=C+\beta a$		pv if $C+\beta a\leqslant C_I$ or $C_I<C=C+\beta a$
C_R	1 if $C\leqslant C_R<C+\beta a$ 0 else	0	1 if $C\leqslant C_R=C_I<C+\beta a$ 0 else
C_I	1 if $C\leqslant C_I=C_R<C+\beta a$ 0 else	0	1 if $C\leqslant C_I<C+\beta a$ 0 else

If the inequality $C\leqslant C_j<C+\beta a$ holds, player j thinks that with probability 1 catastrophe C_j will occur since with the scheduled emission a the critical threshold is passed. The probability for $C_j<C<C+\beta a$ is only defined so that the scope of the definition covers all possible states and choices. Nevertheless, the probability is defined such as to express the idea of player j that although C_j has turned out as a view too pessimistic, $C_j<C$ and any further increase $C<C+\beta a$ will result in a catastrophe. From the results below it is obvious that the specific definition of $C_R<C$ has no consequence.

State k cannot be changed: $P_j(k|k,o,o,o) = 1$ $(j=R,P,I)$.

Since no utility functions are known for the three players, we start with linear ones which are simplest to assess. Let the transition from state s and measures (l,a,p) to state t have the utility $U_j(s;\ l,a,p,t)$ for players j=R,P,I.

$$U_R(C,L,l,a,p;\ C+\beta a,M) = c_1 l+c_2 a+c_3 p\ , \quad (M=L,\tfrac{L}{2})\ ;$$

$$U_R(C,L,l,a,p;\ k) = c_1 l+c_2\frac{k-C}{\beta} + c_3 p+c_R\ ;$$

$$U_R(k_4 0,0,0;\ k) = 0;$$

$$U_P(C,L;\ 1,a,p;\ C+\beta a,M) = c_4 a \ , \qquad (M=L,\tfrac{L}{2}) \ ;$$
$$U_P(C,L,;\ 1,a,p;\ k) = c_4 \frac{k-C}{\beta} + c_P \ ;$$
$$U_P(k;\ o,o,o;\ k) = 0 \ ;$$
$$U_I(C,L;\ 1,a,p;\ C+\beta a,M) = c_5 a + c_6 p \ , \qquad (M=L,\tfrac{L}{2}) \ ;$$
$$U_I(C,L;\ 1,a,p;\ k) = c_5 \frac{k-C}{\beta} + c_6 p + c_I \ ;$$
$$U_I(k;\ o,o,o;\ k) = 0 \ .$$

The parameters are assumed to have the signs $c_1 \geqq 0$, $c_2 > 0$, $c_3 < 0$, $c_4 > 0$, $c_5 > 0$, $c_6 < 0$. $c_j (j=R,P,I)$ is the additional payoff to player j due to catastrophe and therefore regarded as largely negative. $c_1 \geqslant 0$ reflects the regulator's internal difficulties in setting small standards, $c_2 > 0$, $c_4 > 0$, $c_5 > 0$ the benefits of energy production; $c_3 < 0$ the damage to the regulator due to pressure exerted on him; and $c_6 < 0$ the burden of organization. The term $\frac{k-C}{\beta}$ expresses that energy production is only valuable up to the critical amount. Thus the idea is excluded that in the case of a slowly developing catastrophe energy production by combustion of fossil fuel may give additional benefits during the initial stages of the catastrophe.

A play π of the game is given by an infinite sequence

$$\pi = (s^1, l^1, a^1, p^1;\ s^2, l^2, a^2, p^2; \ldots)$$

of states, measures of the regulator, producer, and impactee, respectively. According to the list of transition probabilities, there are only sequences where

$$C^1 \leqslant C^i \leqslant C_P \text{ and } L^i \varepsilon \{L^1, \frac{L^1}{2}, \frac{L^1}{4}, \ldots\} \ ,$$

and $a^i = \frac{C^{i+1} - C^i}{\beta}$ if $s^{i+1} = (C^{i+1}, L^{i+1})$.

Furthermore if $s^i = k$ then $s^m = k$ for $m > i$. As a first approach we define the utility of a play as the undiscounted infinite sum of the transition utilities:

$$\underline{U}_j(\pi) = \sum_{i=1}^{\infty} U_j(s^i, l^i, a^i, s^{i+1}) \quad .$$

Since the summed-up internal utilities $\Sigma c_1 l^i$ can become infinite we omit them by specifying $c_1 = 0$. Let $(s^1, l^1, a^1, p^1, \ldots)$ denote a play where $s^i = (C^i, L^i)$ and $s^{i+1} = k$.

Then

$$\underline{U}_R(s^1\ldots) = \sum_{j=1}^{i}(c_2a^j+c_3p^j) + c_2\frac{k-C^i}{\beta} + c_3p^{i+1} + c_R$$

$$= c_3\sum_{j=1}^{i+1} p^j + c_2\frac{k-C^1}{\beta} + c_R \quad .$$

In the case of $s^j = (C^j,L^j)$ for $j=1,2,\ldots$

$$\underline{U}_R(s^1,\ldots) = c_3\sum_{j=1}^{\infty} p^j + \lim_{j\to\infty}\frac{C^j-C^1}{\beta} \quad .$$

Admitting $-\infty$ as a payoff then $\underline{U}_R(s^1,\ldots)$ is well defined because of $C^i \leqslant C_P$.

The same argument gives

$$\underline{U}_P(\ ) = c_4\frac{k-C^1}{\beta} + C_P \quad ,$$

$$U_P(\ ) = c_4 \lim \frac{C^j-C^1}{\beta} \quad ;$$

and

$$\underline{U}_I(\ ) = c_5\frac{k-C^1}{\beta} + c_6\sum_{j=1}^{i+1} pj + c_I \quad ,$$

$$\underline{U}_I(\ ) = c_5 \lim \frac{C^i-C^1}{\beta} + c_6\sum_{j=1}^{\infty} p^i \quad ;$$

respectively.

The game is now completely described except for the definition of strategies. For simplification we admit only stationary strategies where the choices depend only on the last state and last measures of the other players.

Definiton: A strategy σ_R of the regulator is a map:

$$\sigma_R : S \to \mathbb{R}$$

such that

$$\sigma_R(C,L) \in M_R(C,L) = \{1 | 0 \leqslant 1 \leqslant L\} \quad ,$$

$$\sigma_R(k) = 0 \quad .$$

A strategy σ_P of the producer is a map

$$\sigma_P:\{(s,l)\,|\,s\varepsilon S, l\varepsilon M_R(s)\} \to \mathbb{R} \quad ,$$

such that

$$\sigma_P(C,L,l) \;\varepsilon\; M_P(C,L,l) = \{a\,|\,0\leqslant a\leqslant l, \frac{C_P - C}{\beta}\} \quad ,$$

$$\sigma_P(k,0) = 0 \quad .$$

A strategy σ_I of the impactee is a map

$$\sigma_I:\{(s,l,a)\,|\,s\varepsilon S, l\varepsilon M_P(s), a\varepsilon M_P(s,l)\} \to [0,1] \quad ,$$

such that

$$\sigma_I(C,L,l,a) \;\varepsilon\; [0,1] \quad ,$$

$$\sigma_I(k,0,0) = 0 \quad .$$

The sets of strategies are denoted by Σ_j $(j = R,P,I)$.

Due to the list of transition probabilities defined above infinitely many plays can occur. The appropriate σ-algebra over the set Π of all possible plays is defined as the minimal σ-algebra containing all cylinders with finite bases (see M. Loève, 1955, 8.3). Due to the theorem of Tulcea there exist probability measures $P_j(\cdot|\sigma_R,\sigma_P,\sigma_I)$ on this σ-algebra where $P_j(\cdot|\sigma_R,\sigma_P,\sigma_I)$ stems from the iteration of given subjective probabilities.

The payoff function to player j is defined as his high subjective expected utility

$$V_j(\sigma_R,\sigma_P,\sigma_I) = \int \underline{U}_j(\pi)\,dP_j(\pi|\sigma_R,\sigma_P,\sigma_I) \quad (j=R,P,I) \quad .$$

The formalism allows to derive a sharp upper bound for $V_j(\sigma_R,\sigma_P,\sigma_I)$. Due to the definition of the transition probability P_R the set of plays with a component state $s^m=(C^m,L^m)$, such that $C^m>C_R$ has probability $P_R(\cdot|\sigma_R,\sigma_P,\sigma_1) = 0$.

Hence only plays $\pi = (s^1,l^1,a^1,p^1;\ldots)$ have to be considered where a component state s^m either equals (C^m,L^m) such that $C^m<C_R$ or C_R. Hence

$$\underline{U}_R(\pi) = c_3 \sum_{j=1}^{i+1} p^m + c_2 \frac{C_R - C^1}{\beta} + c_R \qquad \text{if } C^i \leqslant C_R < C^i + \beta\, a^i \quad ,$$

or

$$\underline{U}_R(\pi) = c_3 \sum_{j=1}^{i+1} p^j + \lim c_2 \frac{C^j - C^1}{\beta} \quad \text{if } C^j \leqslant C_R \ (j=1,\ldots) \quad .$$

In both cases $\underline{U}_R(\pi) \leqslant c_2 \frac{C_R - C^1}{\beta}$ is obvious. Hence

$$V_R(\sigma_R, \sigma_P, \sigma_I) \leqslant c_2 \frac{C_R - C^1}{\beta} \quad .$$

The analogous argument yields $V_I(\sigma_R, \sigma_P, \sigma_I) \leqslant c_5 \frac{C_I - C^1}{\beta}$ whereas $\underline{U}_P(\pi) \leqslant c_4 \frac{C_P - C^1}{\beta}$ immediately implicates

$$V_P(\sigma_R, \sigma_P, \sigma_I) \leqslant c_4 \frac{C_P - C^1}{\beta} \quad .$$

The bounds are sharp in the sense that strategy triples exist yielding the bounds as payoffs.

Let $\sigma_R(C,L) = L$, $\sigma_P(C,L,l) = \min(l, \frac{C_P - C}{\beta})$, $\sigma_I(C,L,l,a) = 0$.

Then $V_R(\sigma_R, \sigma_P, \sigma_I) = c_4 \frac{C_P - C^1}{\beta}$.

We give examples for V_R and V_P below. If the establishment of the payoffs as expected payoffs over Π were more elaborated (see e.g. J. Kindler, 1971) it would be obvious that we arrive at the same payoffs $V_j : \Sigma_R \times \Sigma_P \times \Sigma_I \to \mathbb{R}$ if we replace the component utility U_I by $U_{I,r}$:

$$U_{I,r}(C,L;\ l,a,p;\ C+\beta a,M) = \begin{cases} c_5 a & \text{if } M = L \ ; \\ c_5 a + \frac{c_6}{v} & \text{if } M = \frac{L}{2} \ ; \end{cases}$$

$$U_{I,r}(C,L;\ 1,a,p;\ k) = U_I(C,L;\ l,a,p;\ k) \ ;$$

$$U_{I,r}(k;\ 0,0,0;\ k) = U_I(k;\ 0,0,0,k) \ .$$

This remark permits to shorten proofs in the next section.

THE GAME-THEORETIC SOLUTION

Except for two-person zero-sum games or equivalent games, there is no unanimous solution concept. Instead there are a variety. Therefore we shall first give brief definitions of the solution concepts (for a broader discussion see R. Avenhaus and E. Höpfinger, 1978), and later on describe strategy three-tuples satisfying them.

Definition: A three-tuple $(\sigma_R^+, \sigma_P^+, \sigma_I^+) \varepsilon \Sigma_R x \Sigma_P x \Sigma_I$ of strategies is called a (weak) *equilibrium point* if

$$V_R(\sigma_R^+, \sigma_P^+, \sigma_I^+) \geqslant V_R(\sigma_R, \sigma_P^+, \sigma_I^+) \quad (\sigma_R \varepsilon \Sigma_R) \quad ;$$
$$V_P(\sigma_R^+, \sigma_P^+, \sigma_I^+) \geqslant V_P(\sigma_R^+, \sigma_P, \sigma_I^+) \quad (\sigma_P \varepsilon \Sigma_P) \quad ;$$
$$V_I(\sigma_R^+, \sigma_P^+, \sigma_I^+) \geqslant V_I(\sigma_R^+, \sigma_P^+, \sigma_I) \quad (\sigma_I \varepsilon \Sigma_I) \quad .$$

Definition: The payoff vector $(V_j(\sigma_R, \sigma_P, \sigma_I))j = R,P,I$ is called *Pareto-optimal* if there is no other payoff vector $(V_j(\tau_R, \tau_P, \tau_I))$ where $\tau_j \varepsilon \Sigma_j (j = R,P,I)$, such that

$$V_j(\sigma_R, \sigma_P, \sigma_I) \geq V_j(\tau_R, \tau_P, \tau_I) \quad (j=R,P,I) \quad ,$$

and at least one inequality strictly holding.

Definition: Let $(W_R, W_P, W_I) \varepsilon R^3$ denote the point of maximal possible payoffs which is called *bliss point*, i.e. $W_j = \max(V_j(\sigma_R, \sigma_P, \sigma_I) | \sigma_i \varepsilon \Sigma_i (i = R,P,I))$. The payoff vector (v_R, v_P, v_I) is called bliss-optimal if

$$\sum_{j=R,P,I} (v_j - W_j)^2 = \min\left(\sum_j (V_j(\sigma_R, \sigma_P, \sigma_I) - W_j)^2 \,|\, (\sigma_R, \sigma_P, \sigma_I) \varepsilon \Sigma_R x \Sigma_P x \Sigma_I\right)$$

Definition: Let (d_R, d_P, d_I) be a triple of payoffs the players obtain in case they cannot reach an unanimous agreement on the choice of a payoff vector. Then the Nash *solution* is the point (W_R, W_P, W_I) which maximizes the term $(u_R - d_R)(u_P - d_P)(u_I - d_I)$ subject to the requirements $u_j = V_j(\sigma_R, \sigma_P, \sigma_I)(j = R,P,I)$ for some strategy three-tuple and $u_j \geq d_j$ $(j = R,P,I)$.

<u>Definition</u>: A *hierarchic solution* is a triple (τ_R, τ_P, τ_I) consistent of a strategy $\tau_R \varepsilon \Sigma_R$, and two maps

$$\tau_P : \Sigma_R \to \Sigma_P \quad ,$$

$$\tau_I : \Sigma_R x \Sigma_P \to \Sigma_I \quad ,$$

such that $V_I\left(\sigma_R, \sigma_P, \tau_I(\sigma_R, \sigma_P)\right) = \max\limits_{\sigma_I \varepsilon \Sigma_I} V_I(\sigma_R, \sigma_P, \sigma_I) \quad ;$

$$V_P\left(\sigma_R, \tau_P(\sigma_R), \tau_I\left(\sigma_R, \tau_P(\sigma_R)\right)\right) = \max_{\sigma_P \varepsilon \Sigma_P} V_P\left(\sigma_R, \sigma_P, \tau_I(\sigma_R, \sigma_P)\right) \quad ;$$

$$V_R\left(\tau_R, \tau_P(\tau_R), \tau_I\left(\tau_R, \tau_P(\tau_R)\right)\right) = \max_{\sigma_R \varepsilon \Sigma_R} V_R\left(\sigma_R, \tau_P(\sigma_R), \tau_I\left(\sigma_R, \tau_P(\sigma_R)\right)\right)$$

The game has a huge variety of equilibrium points. In the following we give three equilibrium points, the first two of which have Pareto-optimal payoffs, whereas the third is only given as an indicator of the variety of equilibrium points.

<u>Theorem</u>: The tuples of strategies given below are equilibrium points:

1) $\sigma_R^1\ (C,L) : = \min\left(L, \max(0, \frac{C_R - C}{\beta})\right) \quad ;$

$\sigma_P^1\ (C,L,l) : = 1 \quad ;$

$\sigma_I^1\ (C,L,l,a) : = 0 \quad .$

The inherent utilities are

$$V_R(\sigma_R^1, \sigma_P^1, \sigma_I^1) = c_2 \frac{C_R - C^1}{\beta} \quad ;$$

$$V_P(\sigma_R^1, \sigma_P^1, \sigma_I^1) = c_4 \frac{C_R - C^1}{\beta} \quad ;$$

$$V_I(\sigma_R^1, \sigma_P^1, \sigma_I^1) = \begin{cases} c_5 \frac{C_R - C^1}{\beta} & \text{if } C_R \leqslant C_I \quad , \\ c_5 \frac{C_I - C^1}{\beta} + c_I & \text{if } C_R > C_I \quad . \end{cases}$$

2) $\sigma_R^2(C,L) : = \min\left(L, \max(0, \frac{C_I - C}{\beta})\right) \quad ;$

$\sigma_P^2(C,L,l) : = 1 \quad ;$

$$\sigma_I^2(C,L,l,a) := \begin{cases} 0 & \text{if } l = \min(L, \frac{C_I - C}{\beta}) \text{ and } C \leqslant C_I \quad , \\ 1 & \text{if } l \neq \min(L, \frac{C_I - C}{\beta}) \text{ or } C > C_I \quad . \end{cases}$$

The inherent utilities are

$$V_R(\sigma_R^2, \sigma_P^2, \sigma_I^2) = \begin{cases} c_2 \frac{C_I - C^1}{\beta} & \text{if } C_I \leqslant C_R \quad , \\ c_2 \frac{C_I - C^1}{\beta} + c_R & \text{if } C_I > C_R \quad . \end{cases}$$

$$V_P(\sigma_R^2, \sigma_P^2, \sigma_I^2) = c_4 \frac{C_I - C^1}{\beta} \quad ;$$

$$V_I(\sigma_R^2, \sigma_P^2, \sigma_I^2) = c_5 \frac{C_I - C^1}{\beta} \quad .$$

3) *Keep quiet point*

$$\sigma_R^3(C,L) = 0 \quad ;$$

$$\sigma_P^3(C,L,l) = 0 \quad ;$$

$$\sigma_I^3(C,L,l,a) = \begin{cases} 0 & \text{if } l=0 \text{ and } C=C^1 \quad , \\ 1 & \text{if } l>0 \text{ or } C>C^1 \quad ; \end{cases}$$

with utilities $V_j(\sigma_R^3, \sigma_P^3, \sigma_I^3) = 0 \quad (j=R,P,I)$.

Proof: In order to avoid descriptions that are cumbersome but not illustrative we give sketches only.

1) Let $i_R \in \{1,2,...\}$ be defined by $C^1 + \beta(i_R - 1)L^1 \leqslant C_R < C^1 + \beta i_R L^1$.
One can show by iteration on i that

$$C^{i+1} = C^1 + \beta i L^1 \; (i=0,1,\dots,i_R-1), \quad a^i = L^1 \; (i=1,\dots,i_R-1) \quad ,$$
$$a^{i_R} = \frac{C_R - C^{i_R}}{\beta} \quad ,$$
$$C^{i+1} = C_R \; (i=i_R, i_R+1, \dots), \quad a^i = 0 \; (i=i_R+1, i_R+2, \dots) \quad ,$$

due to the regulator's strategy. Hence

$$V_R(\sigma_R^1, \sigma_R^2, \sigma_R^3) = c_2 \sum_{i=1}^{i_R - 1} L^1 + c_2 \frac{C_R - C^{i_R}}{\beta} = c_2 \frac{C_R - C^1}{\beta} \quad ;$$

analogously

$$V_P(\sigma_R,\sigma_P,\sigma_I)=c_4\frac{C_R-C^1}{\beta} \quad .$$

In the case of $C_R \leqslant C_I$, (C_R,L^1) will be the state of the play for $i = i_R + 1, i_R + 2,\ldots$ also due to the subjective probability of the impactee. However, if $C_R > C_I$, catastrophe C_I will be the final state resulting in a payoff $c_5\frac{C_I-C^1}{\beta} + c_I$.

The regulator's condition for an equilibrium is obviously satisfied since the strategy triple gives him the maximal possible utility. Just as is obvious, there is no better payoff for the producer with another strategy, and this is also true for the impactee in the case of $C_R \leqq C_I$.

Only $C_R > C_I$ requires more sophistication. Let σ_P' denote a different strategy of the impactee. Then a play π with $\lim C^i \leqq \leqq C_I$ is only possible if the reduction of L^i to its half takes place an infinite number of times. But then $\underline{U}_{I,r}(\pi) = \infty < c_5\frac{C_I-C^1}{\beta} + c_I$. If the reduction of L^1 takes place only a finite number of times then $\underline{U}_{I,r}(\pi) \leqq c_5\frac{C_I-C^1}{\beta} + c_I$. Hence any other strategy cannot yield a better payoff.

2) In the case of $C_I < C_R$ the regulator can only get a better payoff if plays π with states (C^i,L^i) where $C^i > C_I$ occur with a subjective probability greater than zero. But then $\sigma_I^2(C^i,L^i,1,a) = 1$ infinitely often yielding the payoff $-\infty$ to the regulator. Thus he cannot get a better payoff with a different strategy. Obviously the producer cannot get a better payoff, whereas the impactee gets his maximal payoff.

In the case of $C_I = C_R$ regulator and impactee receive their maximal payoffs, whereas the producer has no better response. In the case of $C_I > C_R$ the regulator may want to escape catastrophe by applying a strategy like the one of the first equilibrium point. But then he is punished an infinite number of times by pressure from the impactee and gets a smaller payoff. Again it is obvious that producer and impactee cannot do better.

3) The impactee's ability to exert pressure infinitely often again makes the strategy triple $(\sigma_R^3,\sigma_P^3,\sigma_I^3)$ an equilibrium point.

The question arises: Which of these equilibrium points yield Pareto-optimal payoffs? The answer can immediately be deduced from the following:

<u>Theorem:</u> The set of payoffs

$$\left\{\left(V_R(\sigma_R,\sigma_P,\sigma_I),V_P(\sigma_R,\sigma_P,\sigma_I),V_I(\sigma_R,\sigma_P,\sigma_I)\right) \mid \sigma_j \varepsilon \Sigma_j\ (j=R,P,I)\right\}$$

is a subset of the following domain $D \supseteq \mathbb{R}^3$.

1) Let $C_R < C_I < C_P$. Then D consists of all $(x,y,z) \varepsilon \mathbb{R}^3$ such that a pair (p_R,p_I) of real numbers exists such that $o \leqq p_R$, $o \leqq p_I$, $0 \leq p_R + p_I$ and the following inequalities hold:

$$x \leq c_2 \frac{C_R - C^1}{\beta} + (1-p_R)c_R \quad ;$$

$$y \leq c_4 \left\{ p_R \frac{C_R - C^1}{\beta} + p_I \frac{C_I - C^1}{\beta} + (1-p_R-p_I)\frac{C_P - C^1}{\beta} \right\} \quad ;$$

$$z \leq c_5 p_R \frac{C_R - C^1}{\beta} + c_5 p_I \frac{C_I - C^1}{\beta} + (1-p_R-p_I)\left(c_5 \frac{C_I - C^1}{\beta} + c_I\right) \quad .$$

2) Let $C_R = C_I < C_P$. Then D consists of all $(x,y,z) \varepsilon \mathbb{R}^3$ which are part of a solution $(x,y,z,p) \varepsilon \mathbb{R}^4$ of the following system of inequalities:

$$0 \leq p \leq 1 \quad ;$$

$$x \leq c_2 \frac{C_R - C^1}{\beta} + (1-p)c_R \quad ;$$

$$y \leq c_4 \left\{ p \frac{C_R - C^1}{\beta} + (1-p)\frac{C_P - C^1}{\beta} \right\} \quad ;$$

$$z \leq c_5 p \frac{C_R - C^1}{\beta} + (1-p)\left(c_5 \frac{C_R - C^1}{\beta} + c_I\right) \quad .$$

3) Let $C_I < C_R < C_P$. Then D consists of all $(x,y,z) \varepsilon \mathbb{R}^3$ which are part of a solution $(x,y,z,p_I p_R) \varepsilon \mathbb{R}^5$ of the following system of inequalities:

$$0\leq p_I,\ 0\leq p_R,\ 0\leq 1-p_I-p_R \quad ;$$

$$x\leq c_2 p_I \frac{C_I-C^1}{\beta} + c_2 p_R \frac{C_R-C^1}{\beta} + (1-p_I-p_R)(c_2 \frac{C_R-C^1}{\beta} + c_R) \quad ;$$

$$y\leq c_4 \left\{ p_I \frac{C_I-C^1}{\beta} + p_R \frac{C_R-C^1}{\beta} + (1-p_I-p_R) \frac{C_P-C^1}{\beta} \right\} \quad ;$$

$$z\leq c_5 \frac{C_I-C^1}{\beta} + (1-p_I)c_I \quad .$$

<u>Sketched proof</u>: Let $(\sigma_R,\sigma_P,\sigma_I)$ denote a strategy triple. In the case of $C_R < C_I \leq C_P$ let p_R denote the probability $(P_P(T_R|\sigma_R,\sigma_P,\sigma_I)$ that a play with states (C^i,L^i), $C^i \leq C_R$ will be realized, i.e. T_R is the set of all plays $(s^1,l^1,a^1,p^1,\ldots)$ such that $C^i \leqq C_R$ for all component states $(C^i,L^i)(i = 1,2,\ldots)$. Let $p_I = P_P(T_I|\sigma_R,\sigma_P,\sigma_I)$ denote the probability for the set of plays $(s^i,l^i,a^i,p^i)(i = 1,2,\ldots)$ such that $C^i \leq C_I$ for all i, where $s^i = (C^i,L^i)(i = 1,2,\ldots)$ but $C^j > C_R$ for at least one j. Obviously

$$V_P(\sigma_R,\sigma_P,\sigma_I) \leq c_4 \left\{ p_R \frac{C_R-C^1}{\beta} + p_I \frac{C_I-C^1}{\beta} + (1-p_R-p_I)\frac{C_P-C^1}{\beta} \right\} \quad .$$

By definition of the regulator's transition probability, $P_R(T_R|\sigma_R,\sigma_P,\sigma_I) = p_R$, but with probability $1 - p_R$ the catastrophe will occur. Hence

$$V_R(\sigma_R,\sigma_P,\sigma_I) \leq c_2 \frac{C_R-C^1}{\beta} + (1-p_R)c_R \ .$$

The impactee's probabilities for plays with only state components below C_R, and between C_R and C_I are p_R and p_I respectively. Therefore

$$V_I(\sigma_R,\sigma_P,\sigma_I) \leq c_5 p_R \frac{C_R-C^1}{\beta} + c_5\, {}_I\frac{C_I-C^1}{\beta} + (1-p_R-p_I)(c_5 \frac{C_I-C^1}{\beta} + c_I) \ .$$

The proofs for the two remaining cases follow the same line of argumentation. One has only to consider that p is the producer's subjective probability that a play will occur where $C^i \leq C_R = C_I$ for all component states C^i. In the last case p_I denotes the producer's probability for a play with component states not greater that C_I, p_R, the probability for a play with a component state greater than C_I, and all component states not greater than C_R.

Corollary: The first and the second equilibrium point of the last but one theorem have Pareto-optimal payoff vectors. In the case of $C_R > C^1$ and $C_I \geq C^1$ the keep-quiet point has no Pareto-optimal payoff vector.

Proof: Having chosen either $p_I = 1$ or $p_R = 1$ and $p = 1$, immediately verifies that the payoff vectors of the first and second equilibrium points belong to the boundary plane given on the right-hand side of the inequalities of the last but one theorem. Hence the payoff vectors are Pareto-optimal.

Under the given conditions the keep-quiet point is dominated by the first or the second equilibrium point. The results are illustrated by Figures 1 and 2 showing the projection of subset D of the last theorem.

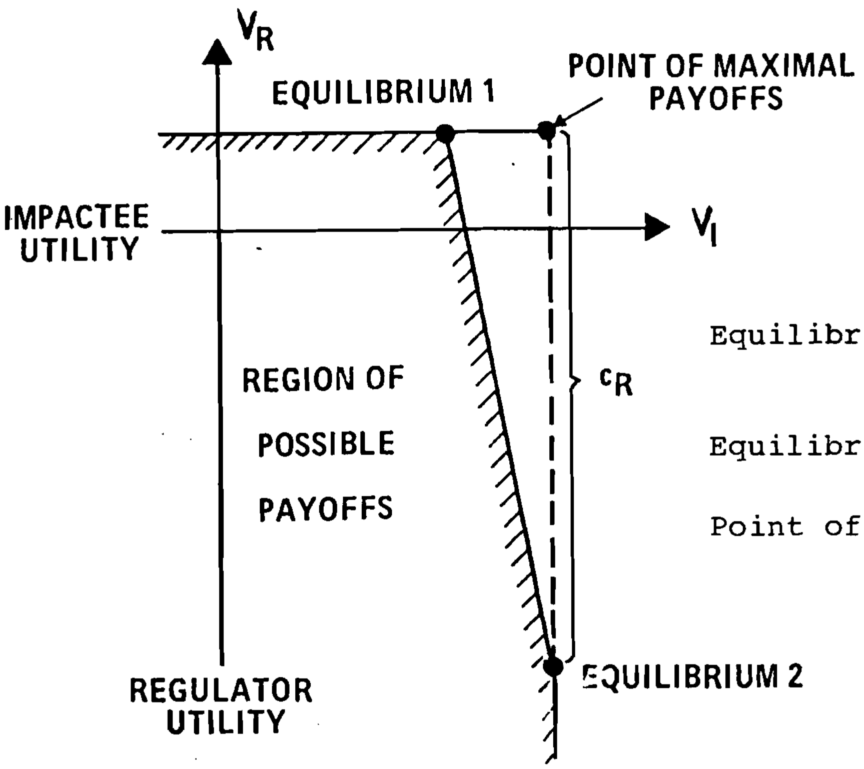
As can be seen from the figures even the combined solution concepts of equilibrium point and Pareto-optimality do not yield an unanimous solution. But what about the remaining solution concepts? In order to discuss them we give the boundary plane of the last theorem after elimination of the parameters for the case of $C_R < C_I < C_P$ by the following equation:

$$\frac{y}{c_4} + \frac{z}{c_I}\;\frac{C_P - C_I}{\beta} + \frac{x}{c_R}\left\{\frac{C_R - C_I}{\beta} + \frac{c_5}{c_I}\;\frac{C_R - C_I}{\beta}\;\frac{C_P - C_I}{\beta}\right\} = \text{constant} \quad .$$

Since by assumption c_I and c_R are huge negative numbers the equation is dominated by the first term $\frac{y}{c_4}$. Hence the payoff vector $(c_2\frac{C_R - C^1}{\beta},\, c_4\frac{C_R - C^1}{\beta},\, c_5\frac{C_R - C^1}{\beta})$ is either bliss-optimal or very close to the bliss-optimal payoff vector. Hence we can regard it as approximately bliss-optimal.

The same holds for $C_R = C_I < C_P$, and in the case of $C_I < C_R < C_P$ for $(c_2\frac{C_I - C^1}{\beta},\, c_4\frac{C_I - C^1}{\beta},\, c_5\frac{C_I - C^1}{\beta})$.

Without proof we state that the two approximate bliss-optimal points are Nash solutions for $d_j = o$ $(j = R,P,I)$ as soon as the absolute values of c_I and c_R are large enough. This means that the bliss-point concept as well as the Nash solution favor a behavior based on the most pessimistic estimate min (C_R, C_I) of the critical value.



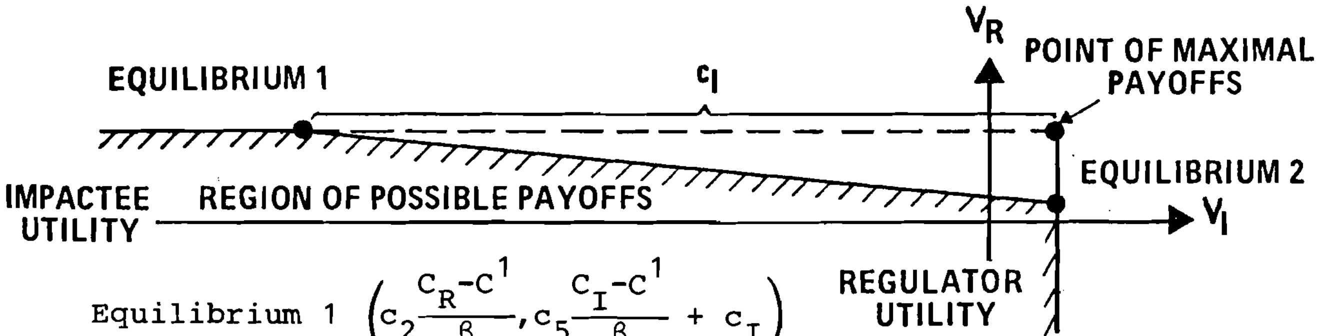
Equilibrium 1 $\left(c_2\frac{C_R-c^1}{\beta}, c_5\frac{C_R-c^1}{\beta}\right)$

Equilibrium 2 $\left(c_2\frac{C_R-c^1}{\beta} + c_R, c_5\frac{C_I-c^1}{\beta}\right)$

Point of Maximal Payoffs

$$\left(c_2\frac{C_R-c^1}{\beta}, c_5\frac{C_I-c^1}{\beta}\right)$$

Figure 1. Payoff diagram for regulator and impactee ($C_R < C_I$).



Equilibrium 1 $\left(c_2\frac{C_R-c^1}{\beta}, c_5\frac{C_I-c^1}{\beta} + c_I\right)$

Equilibrium 2 $\left(c_2\frac{C_I-c^1}{\beta}, c_5\frac{C_I-c^1}{\beta}\right)$

Point of Maximal Payoffs

$$\left(c_2\frac{C_R-c^1}{\beta}, c_5\frac{C_I-c^1}{\beta}\right)$$

Figure 2. Payoff diagram for regulator and impactee ($C_R > C_I$).

The hierarchic solution concept is much more complicated than the preceding ones since it involves maps from strategy spaces into strategy spaces. We circumvent the mathematical optimization problem specifying only the resulting strategies.

Theorem: Let $(\sigma_R^1, \sigma_P^1, \sigma_I^1)$ be the first equilibrium point of the last but one theorem, i.e.,

$$\sigma_R^1(C,L) = \min\left(L, \max(0, \frac{C_R - C}{\beta})\right) \quad ;$$

$$\sigma_P^1(C,L,l) = 1 \quad ;$$

$$\sigma_I^1(C,L,l,a) = 0 \quad .$$

Let $(\tau_R^1, \tau_P^1, \tau_I^1)$ denote a hierarchic solution. Then (τ_R, τ_A, τ_I) defined by

$$\tau_R = \sigma_R^1 \quad ;$$

$$\tau_P(\sigma_R) = \tau_P^1(\sigma_R)(\sigma_R \varepsilon \Sigma_R - \{\sigma_R^1\}), \tau_P(\sigma_R^1) = \sigma_P^1 \quad ;$$

$$\tau_I(\sigma_R, \sigma_P) = \tau^1(\sigma_R, \sigma_P)(\sigma_j \varepsilon \Sigma_j - \{\sigma_j^1\}(j=R,P) \quad ;$$

$$\tau_I(\sigma_R^1, \sigma_P^1) = \sigma_I^1 \quad ;$$

is also a hierarchic solution.

Proof: $V_I(\sigma_R^1, \sigma_P^1, \sigma_I^1) = \max_{\sigma_I} V_I(\sigma_R^1, \sigma_P^1, \sigma_I^1)$ since $(\sigma_R^1, \sigma_P^1, \sigma_I^1)$ is an equilibrium point. The next step is the verification of $V_P(\sigma_R^1, \sigma_P^1, \sigma_I^1) = \max_{\sigma_P} V_P\left(\sigma_R^1, \sigma_P, \tau_I(\sigma_R^1, \sigma_P)\right)$. The regulator's strategy σ_R^1 prevents a larger amount than C_R of carbon dioxide in the atmosphere, whereas the producer's utility is the larger the more dioxide is in the atmosphere. Therefore $V_P(\sigma_R^1, \sigma_P^1, \sigma_I^1) = c_4 \frac{C_R - C^1}{\beta} = \max_{\sigma_P, \sigma_I} V_P(\sigma_R^1, \sigma_P, \sigma_I)$, which is even stronger. The last condition is trivially satisfied since $V_R(\sigma_R^1, \sigma_P^1, \sigma_I^1)$ gives the maximal possible utility $c_2 \frac{C_R - C^1}{\beta}$ to the regulator.

It should be remarked that the theorem is independent of whether $C_R < C_I$ or not. It simply states that the regulator is strong enough to push through his standpoint.

The following example serves to illustrate the order of magnitude. Let $C^1 = 6 \cdot 10^{16}g$, $C_I = 18 \cdot 10^{16}g$, $L^1 = 0.2 \cdot 10^{16}g$, $\beta = 0.3$, $c_2 = 0.002\$/g$, $c_4 = 10^{-4}c_2$, $c_5 = 0.7\ c_2$. C^1 is in the order of magnitude of the present amount of carbon dioxide in the atmosphere, and L^1 in the order of magnitude of the present release of carbon dioxide. $\$3.6 \cdot 10^{12}$ is an estimate of the gross world product of 1970. Then production is possible for 200 years and the payoff vector equals ($\$8 \cdot 10^{14}$, $\$8 \cdot 10^{10}$, $\$5.6 \cdot 10^{14}$).

CONCLUSION

The game has been analyzed for different solution concepts. It turns out that the Nash solution and the bliss-optimal concept yield solutions that are basically different from the hierarchic solution. In the case of $C_I < C_R$ where the impactee's view is more pessimistic than that of the regulator, the Nash solution and the bliss-optimum concept, by their tendency to fair bargains, favor the second equilibrium point based on the estimate C_I. Contrary to this the hierarchic solution yields the first equilibrium point which is based on the estimate C_R as critical value.

The results heavily depend on the fact that the summed up component payoffs are not discounted. Thus the impactee can principally push the regulator's payoff down to minus infinity. Actually he cannot exert pressure infinitely often since then he would also receive the payoff minus infinity. Hence this capability to punish or to exert pressure only yields a vastness of equilibrium points. It seems that the results may change substantially if discounting is included. Then the regulator may be able to resist pressure, and on the other side the impactee may be able to afford pressure. Another way would be to assume the game to be stopped as soon as the upper bound L is below a given limit, e.g., if L is less than ten percent of the carbon dioxide produced by the biosphere during one year. Again the question arises whether the impactee can enforce a total release that is less than $\frac{C_I - C^1}{\beta}$.

So far the impactee has been represented as a rational player with a utility function. Another possibility would be to represent him by a response function based on his perception of the regulator's and the producer's decisions, i.e., to prescribe one strategy of the impactee. Then we would actually have a regulator-producer game, and as soltuion concept we might take the hierarchic solution. But which response should we use? Our analysis of the three-person game offers us two responses:

$$\sigma_I^1(C,L,l,a) = 0 \quad ;$$

$$\sigma_I^2(C,L,l,a) = \begin{cases} 0 & \text{if } l = \min(L,\frac{C_I - C}{\beta}) \text{ and } C \leq C_I \quad , \\ 1 & \text{if } l \neq \min(L,\frac{C_I - C}{\beta}) \text{ or } C > C_I \quad . \end{cases}$$

If we assume the first, then the impactee is actually a dummy player. Then equilibrium point one is part of the hierarchic solution. In the case of σ_I^2 however, the hierarchic solution yields the second equilibrium point as can be verified very easily. Thus, the three-person game can provide for ideas how to formalize a response function.

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