

Analysis of stability to cheaters in models of antibiotic degrading microbial communities

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Abstract

Antibiotic resistance carried out by antibiotic degradation has been suggested recently as a new mechanism to maintain coexistence of microbial species competing on a single limiting resource, even in well-mixed homogeneous environments. Species diversity and community stability, however, critically depend on resistance against social cheaters, mutants that do not invest in production, but still enjoy the benefits provided by others. Here we investigate how different mutant cheaters affect the stability of antibiotic producing and degrading microbial communities. We consider two cheater types, production and degradation cheaters. We generalize the mixed inhibition-zone and chemostat models introduced previously (Kelsic et al., 2015) to study the population dynamics of microbial communities in well-mixed environment, and analyze the invasion of different cheaters in these models. We show that production cheaters, mutants that cease producing antibiotics, always destroy coexistence whenever there is a cost of producing these antibiotics. Degradation cheaters, mutants that lose their function of producing extracellular antibiotic degrading molecules, induce community collapse only if the cost of producing the degradation factors is above

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a critical level. Our analytical studies, supported by numerical simulations, highlight the sensitivity of antibiotic producing and degrading communities to loss-of-function mutants.

Keywords: rock-paper-scissors, social parasite, evolutionary instability, antibiotic-mediated microbiome, degradation resistance

1 **1. Introduction**

2 Unraveling mechanisms that maintain high genetic and functional diversity
3 of microbial communities has become one of the most challenging problems in
4 theoretical and evolutionary ecology (Costello et al., 2012; Morris et al., 2012;
5 Cordero and Polz, 2014). A great variety of bacteria form stable communi-
6 ties in relatively homogeneous environments, competing for only a few limiting
7 resources (Hibbing et al., 2010), seemingly contradicting with the competitive
8 exclusion principle, which states that the number of species cannot be higher
9 than the number of limiting resources (Gause, 1934).

10 In bacteria, the most common forms of interactions are carried out by
11 molecules secreted into the extracellular environment, such as exoenzymes to
12 digest nutrients (Arnosti, 2011), iron scavenging siderophores (Ross-Gillespie
13 et al., 2009), signaling molecules (Miller and Bassler, 2001), virulence factors
14 (Hacker and Carniel, 2001), antibiotics (Bernier and Surette, 2013), or antibiotic
15 degrading molecules (Wright, 2005). Via these molecules, microorganisms can
16 be in competitive, antagonistic, or cooperative relationships (West et al., 2001;
17 Coyte et al., 2015). Interestingly, these molecules are public goods, meaning
18 that not only the producers, but all nearby individuals can enjoy the benefits
19 delivered by them (West et al., 2001). Cheaters, individuals that do not pro-
20 duce such molecules and hence pay no cost of production, can also enjoy these
21 benefits. Thus cheaters have higher fitness and can outcompete producers, lead-
22 ing to the **loss of the diversity** by ceasing the production of the public good
23 (West et al., 2001). These antagonistic interactions carried out by the extra-
24 cellular antibiotics make cyclic competition dominance possible, for example,

25 among antibiotic sensitive, producer, and resistant types. Since producing of an
 26 antibiotic and being resistant to it are both costly, the resistant strain wins over
 27 the producer, similarly the sensitive wins over the resistant, and the producer
 28 can take over the sensitive population. This 'rock-paper-scissors' interaction
 29 cycle is the simplest example of cyclical competition dominance network, where
 30 each species is superior to one, but inferior to another (Fig. 1.a). Coexis-
 31 tence of species in such cyclical interaction networks is documented in spatially
 32 structured environments, in which interaction and dispersion are limited to the
 33 immediate neighbors of the focal individual (Kerr et al., 2002; Czárán et al.,
 34 2002; Károlyi et al., 2005; Müller and Gallas, 2010), but coexistence is much
 35 less prevalent in unstructured environments where individuals mix intensively
 (Kerr et al., 2002; Károlyi et al., 2005).

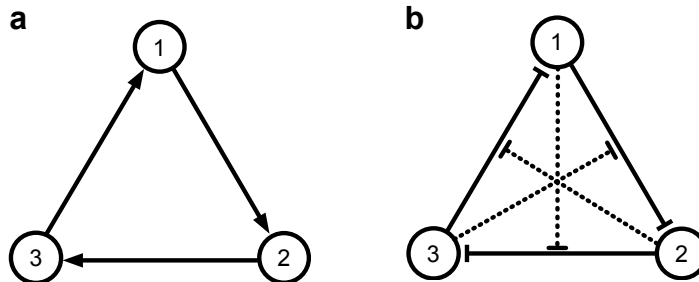


Figure 1: Cyclical competition dominance of three species. (a) Topology of a general 'rock-paper-scissors' type interaction. Here species 1 wins over species 2, species 2 wins over species 3, and species 3 wins over species 1, as indicated by the arrows. (b) The interaction topology where each species inhibits another by producing antibiotic (solid lines) and decomposes antibiotic produced by that species (dotted lines) according to a cyclical interaction topology.

36

37 Recently, Kelsic et al. (2015) (KEA) employed theoretical models to demon-
 38 strate that bacterial species with different antibiotic production, intrinsic re-
 39 sistance, and extracellular degradation factors can coexist even in well-mixed
 40 microbial communities competing for one common limiting factor. Including
 41 degradation resistance has a key role in their model, since excreting antibiotic
 42 degrading molecules can weaken the inhibitory interaction between other species
 43 thus balance the fitnesses through the community. Their study focuses mainly

44 on three species systems, in which species produce one type of antibiotics and
45 reduce the effect of another type via degrading molecules (Fig. 1.b). The au-
46 thors showed that coexistence of species in this system is robust to variation
47 of model parameters even in well-mixed environment. They further demon-
48 strated that analogous systems with four or five species producing 4-6 different
49 antibiotics and degradation factors can have coexistence, although robustness
50 is significantly less prevalent in these richer communities (Kelsic et al., 2015).
51 However, the explanatory power and significance of degradation resistance in
52 explaining microbial diversity largely depends on whether these communities
53 prove to be resistant to the invasion of mutants, mainly against the invasion of
54 social cheaters. A community is defined to be resistant or robust to the invasion
55 of a mutant if its species composition does not change significantly after the
56 invasion. That is, the mutant will be present in the community only transiently,
57 and after its disappearance, the community returns to its pre-invasion state.

58 In the following, we study the generalized versions of KEA’s so-called mixed
59 inhibition-zone and chemostat models (Kelsic et al., 2015), and show analytically
60 that bacterial communities, independently of the interaction topology, are not
61 robust against the invasion of social cheaters. More precisely, we show that
62 mutant cheaters, loosing the costly function of antibiotic production, destroy any
63 diverse community either in one step, or following a cascade of invasion steps.
64 The other type of social cheaters considered in the model, the mutants loosing
65 their functions of producing extracellular antibiotic degrading molecules have
66 less dramatic effect on community stability, but species diversity still declines
67 after the invasion of such mutants.

68 **2. Model description**

69 We assume that there are n_s phenotypically different species and n_a different
70 antibiotics that can be produced by these species. A phenotype (or species) is
71 defined by its relation to an antibiotic: it can produce, can be resistant to, or can
72 be sensitive to the given antibiotic. Naturally, a species producing an antibiotic

73 is also resistant to it, where the resistance is carried out either by removing
 74 antibiotic molecules from the cell via efflux mechanisms, or by neutralizing these
 75 molecules within the cell (Kumar and Schweizer, 2005). Accordingly, a cell
 76 producing an antibiotic l (P_l) is also intrinsically resistant (R_l) to this antibiotic.
 77 Non-producing species can have two types of resistance: intrinsic resistance (R_l)
 78 and degradation resistance (D_l). Bacteria with degradation resistance produce
 79 molecules and secrete to the extracellular matrix which diffuse and degrade the
 80 target antibiotic molecules in a given neighborhood of the cell (Wright, 2005;
 81 Bastos et al., 2015). Phenotypes which are not resistant to antibiotics l carried
 82 out either by intrinsic or by degradation resistance, are considered sensitive
 83 (S_l) and the presence of this antibiotic in the locality reduces their fitnesses.
 84 Thus, every species $i = 1, 2, ..n_s$ is characterized by any of the four phenotypes
 85 P_l, R_l, D_l, S_l for each antibiotic $l = 1, 2, ..n_a$.

86 Let x_i be the abundance of species i per unit area, and assume that cells are
 87 dispersed randomly on a two-dimensional surface. The fitness w_i of species i is
 88 determined by its intrinsic replication rate g_i and the fraction of area $1 - A_i^{(kill)}$
 89 in which individuals of species i are not killed by antibiotics, that is

$$w_i = g_i(1 - A_i^{(kill)}). \quad (1)$$

90 Antibiotic l is effective within area $K_l^{(P)}$ around the cell producing it and, sim-
 91 ilarly, degrading molecules protect every sensitive cell within area $K_l^{(D)}$ around
 92 a cell producing this degrading molecule. A sensitive cell is killed if there is
 93 at least one cell producing antibiotic l within its $K_l^{(P)}$ neighborhood and there
 94 is no bacterium producing degrading molecules for antibiotic l within its $K_l^{(D)}$
 95 neighborhood. Since the aim of this model is to show that coexistence is possi-
 96 ble in unstructured environment, it is assumed that bacteria are dispersed
 97 randomly, so the number of cells follows Poisson distribution within the defined
 98 areas. Thus, the probability that at least one antibiotic producer cell is in the
 99 $K_l^{(P)}$ neighborhood of a cell is $1 - e^{-K_l^{(P)}x_p}$, where x_p is the abundance of species
 100 producing antibiotic l . This value gives the fraction of area in which sensitive
 101 cells are killed except if they are protected by individuals producing degrading

102 molecules within area $K_l^{(D)}$. If the abundance of species producing degrading
 103 molecules is x_d , then the probability of having no cells in this area is $e^{-K_l^{(D)}x_d}$.
 104 So, species i is killed by antibiotic l in the fraction of area is as follows

$$A_{i,l}(x_d, x_p) = e^{-K_l^{(D)}x_d} \left(1 - e^{-K_l^{(P)}x_p}\right). \quad (2)$$

105 Since not only one species can produce antibiotics l or molecules degrading it,
 106 the total area where at least one molecule of antibiotic l kills the sensitive species
 107 i is written as a product of the probabilities of all possible occurrences

$$A_{i,l}(x_1, x_2 \dots x_{i-1}, x_{i+1} \dots x_{n_s}) = A_{i,l}(\mathbf{x} \setminus x_i) = \prod_{j=1}^{n_s} e^{-\delta_{jl}K_l^{(D)}x_j} \left(1 - \prod_{j=1}^{n_s} e^{-\epsilon_{ijl}K_l^{(P)}x_j}\right), \quad (3)$$

108 where $\delta_{jl} = 1$ if the j -th species degrades antibiotic l , otherwise $\delta_{jl} = 0$. Simi-
 109 larly, $\epsilon_{ijl} = 1$ if species i is sensitive to antibiotic l which is produced by species
 110 j , otherwise $\epsilon_{ijl} = 0$ (for P and D type cells). Consequently, the fraction of
 111 area where individuals of species i are not killed by any antibiotics of any other
 112 species is

$$1 - A_i^{(kill)}(\mathbf{x} \setminus x_i) = \prod_{l=1}^{n_a} (1 - A_{i,l}(\mathbf{x} \setminus x_i)). \quad (4)$$

113 Thus, the fitness of species i will be

$$w_i = g_i \left(1 - A_i^{(kill)}(\mathbf{x} \setminus x_i)\right), \quad (5)$$

114 and the average fitness is

$$\bar{w} = \sum_{i=1}^{n_s} w_i x_i. \quad (6)$$

115 By knowing fitness functions for every species, the population dynamics of
 116 the system can be described by the following discrete-time replication dynamics:

$$x_i(t+1) = \frac{c + w_i(t)}{c + \bar{w}(t)} x_i(t), \quad (7)$$

117 where the $c > 0$ constant depends on the time unit (Weibull, 1997). For the
 118 continuous time counterpart of the dynamics, see Appendix A.

119 We note here that KEA have pointed out previously, that the three-species
 120 coexistence (see Fig 1.b) is robust if the areas of chemical activities ($K_l^{(P)}$ and

121 $K_i^{(D)}$) and replication rates (g_i) of all the three species are relatively similar.
122 KEA have also shown that the same dynamics can be observed in the agent-
123 based and the chemostat versions of the mixed inhibition-zone model (Kelsic
124 et al., 2015). The detailed analyses of the generalized chemostat model can be
125 found in Appendix C. **They studied a system where $K_i^{(P)} = K^{(P)}$ and**
126 **$K_i^{(D)} = K^{(D)}$ are constants for every antibiotic which assumption does**
127 **not have to hold in our generalized model.**

128 Besides the ecological stability of three species models, KEA investigated
129 the invasion of "production cheaters", that is, the mutants which do not pro-
130 duce antibiotics and "degradation cheaters" which do not produce degrading
131 molecules. Losing these functions results in fitness increase for mutants, which
132 is then translated into higher replication rates. Based on numerical simulations
133 including cheaters in the community, they concluded that "These interactions
134 enable coexistence that is robust to substantial differences in inherent growth
135 rates and to invasion by 'cheating' species that cease to produce or degrade
136 antibiotics." **Our** discussions with the authors clarified that they studied the
137 evolutionary stability of this system in the spatially extended agent-based ver-
138 sion of the mixed inhibition zone model, and analyzed **it** numerically for 3- and
139 4-species networks (Kelsic et al., 2015, 2016). They found that networks are
140 resistant to both degradation and production parasites in these systems if the
141 colonization radius is small enough. In the following sections, we show that
142 cheater mutants crash such communities not only in the three-species 'rock-
143 paper-scissors' interaction topology in the mixed inhibition model, but in the
144 generalized mixed inhibition model, and similarly in the chemostat model with
145 any interaction topology. In the discussion we explain briefly why the agent-
146 based model with short range colonization behaves differently from the analyt-
147 ical model studied here.

148 **3. Results**

149 *3.1. Evolutionary instability in the mixed inhibition-zone model: introducing*
150 *social cheaters*

151 Species having resistance D_l protect not only themselves but any other
152 strains S_l in the neighborhood from the antibiotics, and similarly a strain P_l
153 producing antibiotic l generates empty space by killing sensitive individuals not
154 only for itself but for non-producing strains R_l as well. Therefore these de-
155 grading molecules and antibiotics are *public goods*, so strains not producing the
156 costly degradation or antibiotic molecules have advantage over producers; thus
157 these are *social cheaters* (Hardin, 1968; Cordero et al., 2012b). We consider two
158 types of mutants, "production cheaters" that fail to produce antibiotics but re-
159 tain intrinsic resistance to this antibiotic ($P_l \rightarrow R_l$), and "degradation cheaters"
160 that lose their resistance through antibiotic degradation and become suscepti-
161 ble to the antibiotics ($D_l \rightarrow S_l$). The benefit of non-producing extracellular
162 materials results in higher replication rates for cheaters, that is the growth rate
163 of mutant increases with $(1 + \alpha)$, where α is an arbitrary, but generally small,
164 positive number.

165 *3.1.1. Invasion of antibiotic production cheaters*

166 Assume that an antibiotic production cheater evolves in a community in
167 which n_s species are in a stable coexistence. (According to KEA, any type
168 of species coexistence is possible from stable fixed points through limit cycles
169 to chaotic behaviors. Our analysis remains valid for every type of dynamical
170 coexistence.) Let us denote the mother species by m , and assume this species
171 produces antibiotic l . The mutant m' of the mother loses the costly production
172 of antibiotic l and consequently its replication rate increases as $g_{m'} = g_m(1 + \alpha)$.
173 It follows from the definition of the model that the fitness function of species m
174 depends only on the abundances of the two types of species affecting survival:
175 the species producing antibiotics for which the focal species is sensitive, and
176 the species producing the molecules degrading this particular antibiotic (see

177 Eq. 3). Since m' remains sensitive to the same antibiotic as m , its replication
 178 rate increases, but its fitness function does not change. Thus, the dynamics of
 179 mother and mutant species are

$$x_m(t+1) = \frac{c + w_m(t)}{c + \bar{w}'(t)} x_m(t) \quad (8)$$

$$x_{m'}(t+1) = \frac{c + w_{m'}(t)}{c + \bar{w}'(t)} x_{m'}(t), \quad (9)$$

180 where $\bar{w}'(t)$ is the average fitness in the population including the mutant. Di-
 181 viding Eq. (8) by Eq. (9)

$$\frac{x_m(t+1)}{x_{m'}(t+1)} = \frac{c + w_m}{c + (1 + \alpha)w_m} \frac{x_m(t)}{x_{m'}(t)} \quad (10)$$

182 that is

$$\frac{x_m(t+1)}{x_{m'}(t+1)} = \left[\frac{c + w_m(t)}{c + (1 + \alpha)w_m(t)} \right]^t \frac{x_m(0)}{x_{m'}(0)}. \quad (11)$$

183 Since $0 < [c + w_m(t)]/[c + (1 + \alpha)w_m(t)] < 1$ for any $c \geq 0$ then

184 $\lim_{t \rightarrow \infty} ([c + w_m(t)]/[c + (1 + \alpha)w_m(t)])^t = 0$ and consequently

$$\lim_{t \rightarrow \infty} x_m(t)/x_{m'}(t) = 0. \quad (12)$$

185 According to (12) three scenarios are possible: (i) both m and m' are selected
 186 against in the community, but species m goes extinct faster than species m' ; (ii)
 187 species m is selected against, and the invading mutant m' is getting fixed in the
 188 community, but **mutant m' triggers the loss of another species besides**
 189 **the mother strain;** (iii) species m is selected against, and species m' replaces
 190 it in the community, so the number of coexisting species remains unchanged.
 191 In case of scenarios (i) and (ii), the number of coexisting species decreases after
 192 the invasion of the mutant. In scenario (iii) a non-producing cheater merely
 193 replaces a producer.

194 Let us assume a sequence of production cheaters invading according to (iii).
 195 **The number of coexisting species doesn't change in this scenario,**
 196 **however if there were l number of different antibiotics in the commu-**
 197 **nity then the number of antibiotics decreases to zero after the l number of**
 198 **such a species replacements.** As a result, neither of the coexisting species

199 produces antibiotics any more **in this new community**. However, survival of
 200 more than one species becomes impossible in this situation, since the replication
 201 rate will become $w_i = g_i$ for every i as there are no more interactions between
 202 the species, and thus only the species with the highest g_i will survive (survival
 203 of the fittest). Consequently, in any of the above mentioned possible scenarios,
 204 species m (and consequently the community) is *not resistant* against the inva-
 205 sion of mutant m' that has any replication benefit ($\alpha > 0$) due to its loss of
 206 antibiotic producing function. We show that continuous time replicator dynam-
 207 ics and the chemostat model lead to completely similar results (see Appendix
 208 A and C for details).

209 3.1.2. Invasion of degradation cheaters

210 The other type of social cheater is the degradation cheater m' , which ceases
 211 the production of degradation molecule synthesized by the mother species m
 212 against antibiotic l . By loosing this function, m' becomes sensitive to antibiotic
 213 l if it is present in the environment but its replication rate increases as $g_m(1 + \alpha)$
 214 at the same time. Thus, the equations of the mother and the mutant species
 215 dynamics are

$$x_m(t+1) = \frac{c + w_m(t)}{c + \bar{w}'(t)} x_m(t) \quad (13)$$

$$x_{m'}(t+1) = \frac{c + (1 + \alpha)(1 - A_{m',l}(\mathbf{x} \setminus x_{m'}))w_m(t)}{c + \bar{w}'(t)} x_{m'}(t). \quad (14)$$

216 Dividing Eq. (13) by Eq. (14) we get

$$\frac{x_m(t+1)}{x_{m'}(t+1)} = \left[\frac{c + w_m(t)}{c + (1 + \alpha)(1 - A_{m',l}(\mathbf{x} \setminus x_{m'}))w_m(t)} \right]^t \frac{x_m(0)}{x_{m'}(0)} \quad (15)$$

217 The fate of a mutant depends on the values of both α and $A_{m',l}(\mathbf{x} \setminus x_{m'})$,
 218 thus the advantage of the invading mutant m' is insufficient yet. By defining
 219 $A_{m',l}^{(max)} = \text{Max}\{A_{m',l}(\mathbf{x} \setminus x_{m'}) \mid x_i \in [0, 1], \sum_i x_i = 1\}$ a sufficient condition for
 220 the invasion of mutant m' can be set. For $\lim_{t \rightarrow \infty} x_m(t)/x_{m'}(t) = 0$ to be valid,
 221 the expression in the square bracket on the right hand side of (15) must be in

222 the $(0, 1)$ interval which leads to the following sufficient condition:

$$\alpha > \frac{A_{m',l}^{(max)}}{1 - A_{m',l}^{(max)}}. \quad (16)$$

223 Consequently, one of the above mentioned three possible scenarios describes
 224 the fate of mutant m' in this case as well. However, besides the loss of species
 225 diversity, according to the above described three invasion scenarios, it is possible
 226 that the degradation-molecule producer and the sensitive mutant strains coexist.
 227 To prove this we show that it is possible that m' invades the community where
 228 type m is resident, but m invades the community where m' is resident. Let us
 229 assume first that m is resident in a stably coexisting community. For the sake of
 230 simplicity, we assume that coexistence is characterized by a stable fixed point,
 231 denoted by $\hat{\mathbf{x}}^{(1)}$. The mutant m' emerges in small abundance, that is $x'_m \ll \hat{x}_i^{(1)}$
 232 for every $i \neq m'$, $\hat{x}_i^{(1)} > 0$. Since $x_i(t+1) = x_i(t)$ for every i , $\hat{x}_i^{(1)} > 0$ at the
 233 equilibrium the abundance of the rare mutant m' increases in the community if
 234 (cf. Eq. (14))

$$\frac{c + (1 + \alpha)(1 - A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'}))w_m(t)}{c + \bar{w}'(t)} > 1, \quad (17)$$

235 which leads to the condition

$$\alpha > \frac{A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})}{1 - A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})}. \quad (18)$$

236 Let us consider now m' as the resident species of **the same** community **but**
 237 **m is replaced by m' and thus m is the rare mutant.** Let $\hat{\mathbf{x}}^{(2)}$ denote the
 238 equilibrium abundances before invasion, so the rare mutant m spreads if

$$\frac{c + \frac{w_{m'}(t)}{(1+\alpha)(1-A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'}))}}{c + \bar{w}'(t)} > 1, \quad (19)$$

239 (cf. Eq. (14) that is if

$$\alpha < \frac{A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'})}{1 - A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_m)}. \quad (20)$$

240 Consequently, if $A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'}) < A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})$ then both (18) and (20)
 241 can be satisfied simultaneously, thus the rare m and m' mutants mutually invade

242 the communities in which the other is resident, which guarantees the coexistence
 243 of these species. Naturally, this analysis assumes that beside species m and
 244 m' there is at least one another species that produces an antibiotic lethal for
 245 species m' . **Furthermore, it is assumed that residents m and m' are in**
 246 **coexistence with the same species, but their densities can be different.**
 247 Identical conditions determine the invasion of mutants in a model based on
 248 continuous replicator dynamics (see Appendix B for details). Thus, according to
 249 our analytical investigation, degradation cheaters can coexist within the resident
 250 community, and can degrade resident community only if their replication rate
 251 is above a critical level. This level can be arbitrarily low or high depending on
 252 the parameters. In the next section, we will test the generality of our results
 253 using numerical investigations.

254 3.2. Numerical studies

255 Next, we run numerical investigations to test the effect of social cheaters, and
 256 for comparison we followed the methodology and parameters used by KEA in
 257 their simulations. In the first series of experiments we generated a statistically
 258 representative sample of ecologically stable communities of 3-5 coexisting species
 259 producing 2-5 different antibiotics, where the initially selected five species can
 260 be any of the four phenotypes (S_l, D_l, R_l, P_l) for each antibiotic $l = 1, 2, \dots, 5$
 261 and the intrinsic replication rate for species i is: $g_i = 1 + (i - 1) \cdot 0.005$. The area
 262 of chemical activities were either $K_l^{(P)} = K^{(P)} = 10$ and $K_l^{(D)} = K^{(D)} = 3$ or
 263 $K_l^{(P)} = K^{(P)} = 30$ and $K_l^{(D)} = K^{(D)} = 10$. We randomly assembled communi-
 264 ties with five interacting species by assigning randomly selected phenotypes for
 265 each antibiotic l to each of the species. The initial abundances were $1/n_s$ for
 266 each species. We repeated $T = 10.000$ update steps according to Eq. (7) with
 267 $c = 0$ and determined the number of coexisting species and the type of equilib-
 268 rium at the end (fixed point, limit cycle or chaotic behavior). (We note that
 269 $c = 0$ is the standard parameter choice used by KEA as well, although $c > 0$
 270 fits the mathematical deduction of the dynamics (Weibull, 1997). However, this
 271 modification does not alter the qualitative behavior of the model.) A species

272 was considered to be extinct if its frequency went below $0.01/n_s$ (Kelsic et al.,
273 2015).

274 In agreement with Kelsic et al. (2015, Extended data Figure 8), we experi-
275 enced that only an extremely small fraction of possible interaction topologies
276 were suitable to maintain complex communities. While three species remain
277 in coexistence from the the initial five species networks in 1 out of $10^2 - 10^3$
278 randomly selected networks, five species could coexist only in 1 out of $10^4 - 10^6$
279 randomly selected networks on average (depending on the $K^{(P)}$ and $K^{(D)}$ pa-
280 rameters). That is, in line with the Extended Data Figure 8 of Kelsic et al.
281 (2015), we found that the fraction of stable communities decreases dramatically
282 as the number of coexisting species increases.

283 After generating the sample of ecologically stable 3-5 species communities
284 we tested the resistance of these communities against the production and degra-
285 dation cheaters but only one function and only in one species could be lost at
286 a time, thus either $P \rightarrow R$ or $D \rightarrow S$ mutants could emerge in the community
287 for each possible case. The mutants with fitness of $(1 + \alpha)g_i$ were introduced
288 at the 10.000th time step with density of 10^{-3} , and the density of the corre-
289 sponding mother species was decreased by the same amount. After subsequent
290 10.000 update steps the coexistence was monitored again, and we recorded the
291 communities that could not resist invasion and hence diversity declined. We
292 declared communities not being resistant to the invasion of mutants if at least
293 one mutant type caused the number of coexisting species (with frequency higher
294 than 0.01) to be smaller after T time steps compared to the number of species
295 before the invasion. That is, we consider only the cases when the invasion of
296 mutants decreases the number of coexisting species within one step (scenarios
297 (i) and (ii)).

298 We tested the resistance of three, four, and five-species communities against
299 the cheater mutants as the function of the α growth-rate advantage of the mu-
300 tants. There is a critical α above which the fraction of unstable communities
301 increases abruptly in a sigmoid manner (Fig. 2a). Species diversity declines
302 dramatically in the majority of these communities even at as little as 0.1% rela-

303 tive growth-rate advantage of mutants $\alpha^* = \alpha/\bar{g}_i$ where \bar{g}_i is the average growth
 304 rate in the community. The rapid decline of diversity results in the exclusion
 305 of all but one species in most of the cases (around 70% of the outcomes in the
 306 case of five species communities in Fig 1a). Production cheaters are responsible
 for the decline of diversity in more than 99% of the cases.

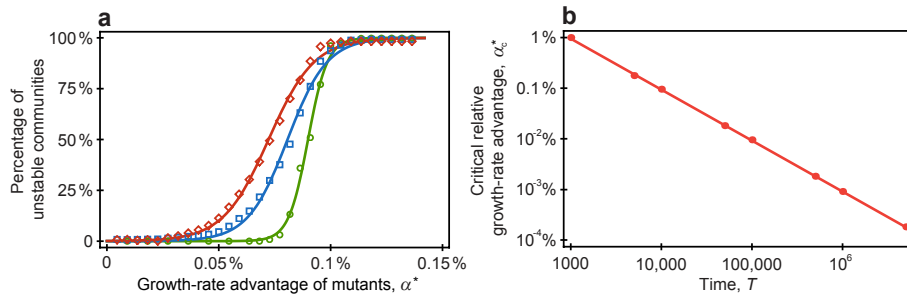


Figure 2: Measures of community instability fostered by cheater mutants. **(a)** The fraction of unstable communities increases in a sigmoid manner (depicted by colored lines) as the relative growth-rate advantage of cheater mutants increases. At 0.1% growth-rate advantage, the majority of the modeled communities become unstable. Statistics are based on 10^3 randomly selected communities composed of three (green circles), four (blue rectangles), and five (red diamonds) species. **(b)** The critical level of relative growth-rate advantage of mutants (where at least 99% of communities are not resistant to the invasion of at least one mutant type) decreases as the duration of simulations (T) increases for 10^3 randomly selected interaction network topologies composed of 5 species. Parameters are: $g_i = 1 + (i - 1) \cdot 0.05$, $K_j^{(P)} = K^{(P)} = 30$, $K_j^{(D)} = K^{(D)} = 10$.

307

308 In our second analysis, we studied the dependence of community resistance
 309 on simulation time. According to Eq. (11), it is straightforward to assume
 310 that it takes more time to observe competitive exclusion if fitness differences
 311 are smaller. To test this hypothesis, we repeated the numerical experiments
 312 in five-species communities with parameters used in Figure 2a but for differ-
 313 ent simulation times (T), and measured the critical α_c^* , that is the α^* value
 314 for which at least 99% of the communities proved to be unstable. As Figure
 315 2b demonstrates, α_c^* decreases continuously as the duration of the simulations
 316 increases according to $\alpha_c^* \propto T^{-1.05 \pm 0.01}$. This relation is in concordance with

317 our analytical results, since the necessary condition to detect collapse of com-
 318 munity is that $x_m(t)/x_{m'}(t) \leq x_c$ where x_c is a critical frequency below which
 319 the species is selected out by definition. It follows from Eq. (11) that

$$\ln(x_c) = T \ln \left(\frac{1}{1 + \alpha} \right). \quad (21)$$

320 For $\alpha \ll 1$ $\ln[1/(1+\alpha)] \approx -\alpha$, consequently $\alpha \propto 1/T$ determines the relationship
 321 between these two variables in the extinction dynamics.

322 To investigate the different invasion scenarios discussed previously, we nu-
 323 merically analyzed the invasion dynamics of different production and degrada-
 324 tion cheaters in a community with the topology shown in Figure 3a. Note that
 325 in this case antibiotic production—sensitivity combinations are not cyclic as in
 326 Figure 1, but still each antibiotic is degraded by one of the species. This topol-
 327 ogy enables us to demonstrate all possible invasion events starting from the same
 328 community. We iterated the dynamics for 1000 time steps and then introduced
 329 mutants into the system. The number of coexisting species was monitored until
 330 $t = 2000$ (except in Fig. 4d in which case due to slow invasion dynamics the
 331 mutant was added at $t = 2000$ and the simulation was terminated at $t = 4000$).

332 Investigating the three invasion scenarios in the numerical model discussed
 333 previously (see Eq. (12) and afterwards) confirms that the invasion of mutants
 334 can (i) result in the extinction of both the mutant and the mother species (Fig.
 335 3b); (ii) result in the exclusion of mother species leading to a decrease in species
 336 diversity (Fig. 3c); and (iii) exclude the mother species but the mutant remains
 337 in coexistence with the other species (Fig. 3d).

338 Figure 3b shows the effect of the invasion of production cheater mutant
 339 for species 2 (mutant ceases producing the antibiotic that inhibits species 5).
 340 Although the invasion of this mutant is unsuccessful it triggers a community
 341 collapse and only one resident species (species 5 in this case) remains in the end.
 342 In Figure 3c the other possible production cheater mutant of species 2 (mutant
 343 ceases producing the antibiotic that inhibits species 4) invades the system and
 344 reduces the number of coexisting species (to an odd number smaller than the
 345 original number of species; in our case to one). Finally, in Figure 3d the same

346 type of mutant with lower fitness advantage invades the community and replaces
 347 the mother species preserving the number of coexisting species but reducing the
 348 number of interactions by one. In accordance with Eq. (12) and discussions
 349 afterwards, these results suggest that the invasion of cheater mutants can result
 350 in the loss of species diversity, antibiotic diversity, or both.

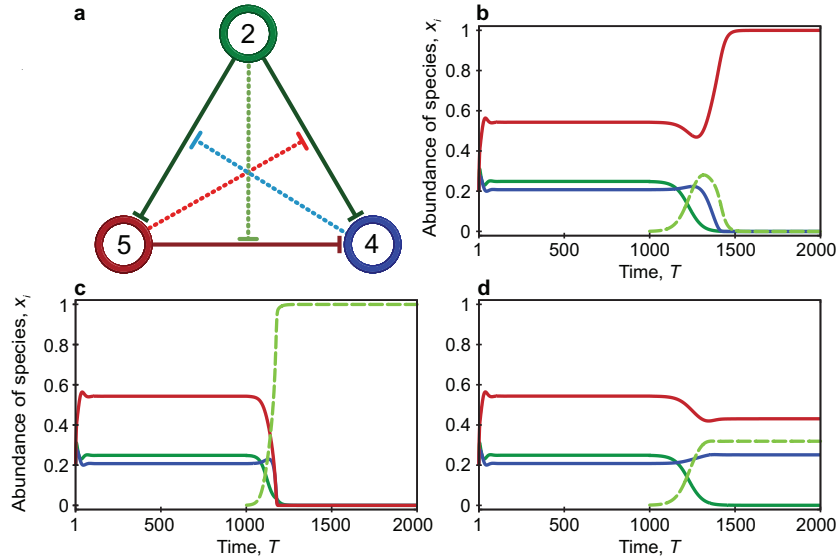


Figure 3: Invasion dynamics of different production cheaters in a model community. (a) The interaction topology of the model community. Each species produces different antibiotics, and species numbering represents the increments in reproduction rates as described in Methods. Species 2 is not affected by any antibiotic, species 5 is inhibited by antibiotic produced by species 5, and species 4 is inhibited by two different antibiotics produced by species 2 and 5. Three different scenarios of production cheater mutant invasions: (b) both the introduced mutant and the corresponding mother species go extinct after the invasion of production cheater mutant for species 2, (c) the invasion of production cheater mutant of species 2 that ceases producing the antibiotic that inhibits species 4 results in the exclusion of the mother type and triggers further species loss, and finally (d) the production cheater mutant of species 2 that ceases producing the antibiotic that inhibits species 4, similar as in the previous numerical experiment, but with lower fitness advantage, replaces the mother lineage. Parameters are the same as in Fig. 2, $\alpha = 0.05$ for (b,d), $\alpha = 0.1$ for (c). Red, green, blue solid lines correspond to species 5, 2, 4, respectively. Dashed line denotes the actual mutant.

351 In case of degradation cheater invasion experiments (in model community

352 with the same topology as in Fig. 3a) we found the four different outcomes in
 353 line with expectations from Eq. (16) and the discussion afterwards. In contrast
 354 to production cheater mutants, degradation cheaters cannot always invade the
 355 system, thus the community structure can remain intact, or the mutants can
 356 coexist with the original coalition (Fig. 4). In line with the first scenario of the
 357 production mutants, the degradation cheater (mutant of species 5) can destroy
 358 the coexistence and one of the original species survives (Fig. 4c), or the cheater
 359 (mutant of species 2) survives only after the community collapses (Fig. 4d).

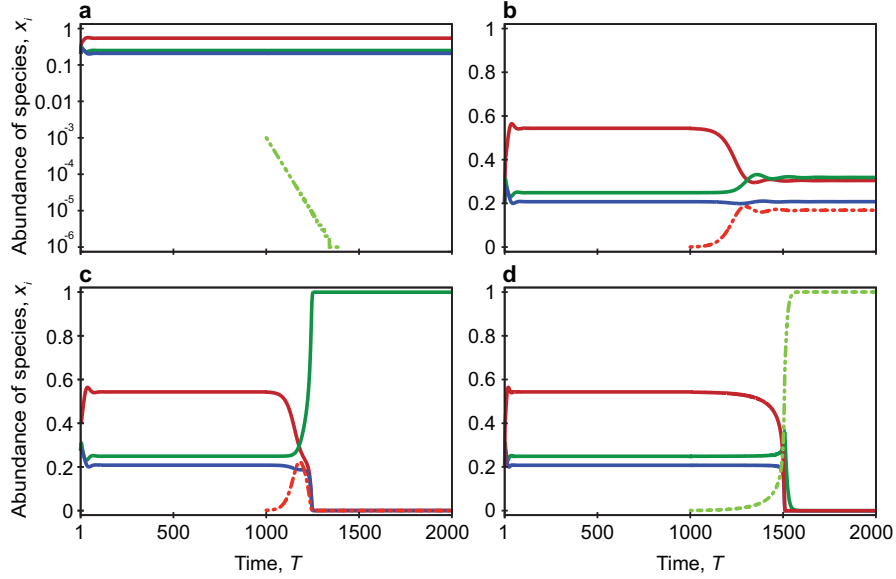


Figure 4: Four different scenarios for the invasion of degradation cheater mutants in model communities depicted by Figure 3a. (a) Unsuccessful invasion of the degradation mutant of species 2, where the resident community remains unchanged after the invasion attempt. (b) Successful invasion of degradation mutant of species 5 leading to the coexistence of all species, the residents and the mutant. (c) The invasion of degradation mutant of species 5 fails, but triggers species extinctions in the community, and one resident species survives in the end. (d) The mutant of species 2 successfully invades a stable community and excludes all other species. Parameters and color coding are the same as in Figure 3, $\alpha = 0.05$ for **a** and **b**, $\alpha = 0.08$ for **c**, and $\alpha = 0.1$ for **d**.

360 **4. Discussion**

361 Our results imply that the counteraction of antibiotic production by ex-
362 tracellular antibiotic degradation does not in itself guarantee high diversity in
363 antibiotic producing microbial communities. In particular, we pointed out that
364 production cheaters with increased reproduction rate demolish the coexistence
365 of interacting species in well-mixed models. According to our studies, three
366 scenarios are possible: in two cases (scenarios (i) and (ii)) the invasion of pro-
367 duction cheaters causes immediate decrease of the number of coexisting species.
368 In scenario (iii) it takes more than one invasion events to decrease the number
369 of coexisting species, but eventually a sequence of invasion events also leads to
370 the decrease of species diversity. **The intuitive explanation is that when**
371 **non-producing mutants invade no cell produces any antibiotics in the**
372 **end, and their competitive interactions are now driven only by their**
373 **reproduction rates. Unless these reproduction rates are identical,**
374 **eventually only one will survive (survival of the fittest).** These results
375 are valid for the mixed inhibition-zone model and the chemostat model with
376 any interaction topology and even if the different antibiotics and degradation
377 molecules have different diffusion abilities (different $K_i^{(D)}$ and $K_i^{(P)}$ parame-
378 ters). It follows that the invasion success of production cheaters is independent
379 of the model details. Our conclusions remain valid for any other systems where
380 the fitness of phenotype i is described by $g_i f_i(x_1(t), x_2(t), x_{i-1}(t), x_{i+1}(t), \dots)$,
381 where $f_i(\mathbf{x} \setminus x_i)$ is an arbitrary continuous function and the replicator dynamic
382 describes the selection among the different phenotypes (see Eqs. (9-12)). We
383 found that the emergence of degradation cheaters causes less dramatic changes
384 in the community; they are able to invade a stable community only if their fit-
385 ness benefit is above a critical level, and in some cases the coexistence of mutant
386 and resident types is possible after invasion.

387 Our numerical simulations show (in line with Kelsic et al. (2015) Extended
388 Data Figure 8.) that the proportion of ecologically stable communities among
389 randomly selected interaction topologies becomes negligibly low as the number

390 of coexisting species increases to five or more. As in the current study the
391 focus was on the evolutionary stability of microbial communities against invasion
392 by cheaters, this aspect of ecological stability received less attention in our
393 analyses. Similarly, in the study of KEA this behavior of the system did not
394 receive sufficient attention. However, we would like to emphasize that it becomes
395 increasingly unlikely that stable communities can emerge when the number of
396 species increases. That is, besides the evolutionary instability, the robustness
397 of ecological stability of these communities is also problematic in well-mixed
398 models without additional mechanisms promoting diversity.

399 A more recent investigation by (Kelsic et al., 2016) pointed out that the
400 spatially extended agent-based version of the mixed inhibition model exhibits
401 resistance to invasion of cheaters. The crucial difference is that in this spatial
402 extended model empty sites are colonized from a finite distance. A producer
403 cell creates empty sites by killing sensitive cells in its neighborhood. Such cells
404 have a greater chance for colonizing these empty sites than the non-producing
405 cheaters being in the vicinity of the empty site. Thus producer cells have higher
406 replication success than non-producers which can balance the higher per-capita
407 replication rate of non-producer ones. The smaller the colonization distance
408 the higher the benefit of producers compared to non-producers, and since the
409 colonization distance tends to be infinite in the well-mixed models studied here
410 this effect disappears.

411 We assumed in the analysis that the production of antibiotics and molecules
412 degrading antibiotics is costly for the cells. In line with this assumption, there
413 are numerous experiments demonstrating that the inactivation or loss of such
414 genes have a significant positive effect on the fitness of such mutant types in a
415 given environment (Lee and Marx, 2012; Koskiniemi et al., 2012; D'Souza et al.,
416 2014). Moreover, other investigations reveal that such antibiotic resistance fac-
417 tors can be the by-products of the general metabolism and thus the production
418 costs are practically negligible (Melnik et al., 2014). In some cases, switching
419 off such gene can even be beneficial for the cell due to pleiotropic effects of the
420 regulating genes (Dandekar et al., 2012; Mitri and Foster, 2016). However, the

421 high population size which is typical in bacterial communities enhances selection
422 and thus it can dominate over genetic drift even for small fitness differences.

423 The mixed inhibition-zone and chemostat models consider the dynamics of
424 well-mixed individuals producing diffusive antibiotics and degrading molecules.
425 The assumptions behind these models enable us to handle the problem analyt-
426 ically, however, these assumptions oversimplify some aspects of the dynamics.
427 First and foremost a more realistic diffusion dynamics and chemical interactions
428 among the dispersed molecules and cells are not taken into account. It is known
429 from other studies that even minor modifications in the dynamics describing
430 diffusion of public goods molecules, interaction of these molecules with cells,
431 the non-linear relation between the molecule concentration and the fitness, and
432 even timing of death and birth events in population dynamics can have signifi-
433 cant effect on selection between producers and non-producers (Borenstein et al.,
434 2013; Scheuring, 2014; Archetti, 2014).

435 Recent studies pointed out that the secreted extracellular molecules are not
436 completely mixing public goods, because due to the restricted motion of cells and
437 of molecules in real bacterial communities, only the immediate neighborhood of
438 the producer is able to enjoy the benefits (Morris, 2015). As the close neighbors
439 of the producer are most probably the clones of the producer, non-producers
440 further away from the source can benefit much less. According to the exper-
441 iments, these definite spatial effects establish density-dependent and negative
442 frequency-dependent selection which stabilizes the coexistence of the producers
443 and social cheaters (Kerr et al., 2002; Cordero et al., 2012a; Drescher et al., 2014;
444 Kümmerli et al., 2014; Morris, 2015). In addition, our results highlight that in-
445 teractions of antibiotic production and attenuation are insufficient in effectively
446 stabilizing bacterial communities in well-mixed environments. Presumably mi-
447 croscale spacial structure of the habitat, negative frequency-dependent selection,
448 pleiotropy, auxotrophy, and top down control by phages play more significant
449 role in maintaining microbiome diversity (Cordero and Polz, 2014; Morris et al.,
450 2012, 2014; Morris, 2015; Koskiniemi et al., 2012; D'Souza et al., 2014; Velend,
451 2010; Ross-Gillespie et al., 2007, 2009; Dandekar et al., 2012; Mitri and Foster,

452 2016; Kelsic et al., 2016).

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458 Appendix A. Continuous replicator dynamics: invasion of produc- 459 tion cheaters

460 The continuous replication dynamics of bacterial strains is generally written
461 as

$$\dot{x}_i(t) = (w_i(t) - \bar{w}(t))x_i(t), \quad (\text{A.1})$$

462 where $w_i(t)$ and $\bar{w}(t)$ are the fitness values of individuals and the population
463 average as defined in the main text. Let us denote the mother and production
464 cheater mutant with m and m' , respectively. Thus, the dynamics of these two
465 types are

$$\dot{x}_m(t) = (w_m(t) - \bar{w}'(t))x_m(t) \quad (\text{A.2})$$

$$\dot{x}_{m'}(t) = ((1 + \alpha)w_m(t) - \bar{w}'(t))x_{m'}(t). \quad (\text{A.3})$$

466 Dividing the two equations by $x_m(t)$ and $x_{m'}(t)$, respectively, and subtracting
467 Eq. (A.3) from Eq. (A.2), after some rearrangement we get

$$\frac{\dot{x}_m(t)}{x_m(t)} - \frac{\dot{x}_{m'}(t)}{x_{m'}(t)} = -\alpha w_m(t), \quad (\text{A.4})$$

468 which leads to

$$\frac{x_m(t)}{x_{m'}(t)} = e^{-\alpha \int_0^t w_m(\tau) d\tau}. \quad (\text{A.5})$$

469 Since $w_m(t) > w_{min} > 0$, where w_{min} is a constant, we have $\lim_{t \rightarrow \infty} \int_0^t w_m(\tau) d\tau =$
470 ∞ . Therefore, equation (12), and consequently the three scenarios described in
471 the main text remain valid in continuous time dynamical systems as well.

472 **Appendix B. Continuous replicator dynamics: invasion of degrada-**
 473 **tion cheaters**

474 In case of continuous replicator dynamics, the time evolution of m and m'
 475 species is

$$\dot{x}_m = (w_m(t) - \bar{w}(t)) x_m \quad (\text{B.1})$$

$$\dot{x}_{m'} = ((1 + \alpha)w_m(t)(1 - A_{m',l}(\mathbf{x} \setminus x_{m'})) - \bar{w}'(t)) x_{m'}, \quad (\text{B.2})$$

476 where m' denotes the degradation cheater. Following the algebraic steps de-
 477 scribed in the previous subsection, we get

$$\frac{\dot{x}_m(t)}{x_m(t)} - \frac{\dot{x}_{m'}(t)}{x_{m'}(t)} = [1 - (1 + \alpha)(1 - A_{m',l}(\mathbf{x} \setminus x_{m'}))] w_m(t). \quad (\text{B.3})$$

478 The sign of the right hand side of (B.3) depends on α and $A_{m',l}(\mathbf{x} \setminus x_{m'})$. As be-
 479 fore, a sufficient condition for the invasion of mutant m' can be determined with
 480 the help of the maximum value of $A_{m',l}(\mathbf{x} \setminus x_{m'})$: if $[1 - (1 + \alpha)(1 - A_{m',l}^{(max)})] <$
 481 0 , that is if

$$\alpha > \frac{A_{m',l}^{(max)}}{1 - A_{m',l}^{(max)}}. \quad (\text{B.4})$$

482 To determine the criterion of mutual invasibility, let us assume first that
 483 type m is the resident species and type m' invades the community. For sake
 484 of simplicity (as in the discrete model presented in the main text), we assume
 485 that the dynamics of the resident population is in fixed point, the abundances
 486 before invasion are denoted by $\mathbf{x}^{(1)}$. Mutant m' spreads if

$$\dot{x}_{m'}(t) = \left((1 + \alpha)(1 - A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'}))w_m(t) - \bar{w}(t) \right) x_{m'}(t) > 0 \quad (\text{B.5})$$

487 which leads to

$$\alpha > \frac{A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})}{1 - A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})}. \quad (\text{B.6})$$

488 Let us consider now m' as the resident species in a community and m as the
 489 rare mutant. Let $\hat{\mathbf{x}}^{(2)}$ denote the equilibrium abundances before invasion, so
 490 the rare mutant m spreads if

$$\dot{x}_m(t) = \left(\frac{w_{m'}(t)}{(1 + \alpha)(1 - A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'}))} - \bar{w}'(t) \right) x_m(t) > 0, \quad (\text{B.7})$$

491 which leads to the condition

$$\alpha < \frac{A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'})}{1 - A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'})}. \quad (\text{B.8})$$

492 Again, as in the discrete time dynamics, if $A_{m',l}(\hat{\mathbf{x}}^{(2)} \setminus x_{m'}) < A_{m',l}(\hat{\mathbf{x}}^{(1)} \setminus x_{m'})$
 493 then both (B.6) and (B.8) can be satisfied simultaneously, thus the rare m
 494 and m' mutants mutually invade each other which guarantees the coexistence
 495 of these species. (Naturally, this analysis assumes that beside species m and
 496 m' at least one similar a species is present in the community which produces
 497 antibiotic affecting species m' .)

498 **Appendix C. Invasion of production cheaters in the chemostat model**

499 Here we review the chemostat model version of microbial community with
 500 interference competition. Following Kelsic et al. (2015), it is assumed that
 501 bacteria compete for a common limiting resource z and there is a constant
 502 dilution d from the chemostat. The dynamics of the resource is

$$\dot{z}(t) = (z_0 - z(t))d - \frac{\sum_{i=1}^{n_s} w_i(t)x_i(t)}{\mu}, \quad (\text{C.1})$$

503 where z_0d is the constant inflow into the chemostat, $w_i(t)$ is the actual growth
 504 rate of species i with concentration x_i and μ is a conversion factor between
 505 resource and species concentration. The species concentrations change according
 506 to

$$\dot{x}_i(t) = (w_i(t) - d)x_i(t), \quad (\text{C.2})$$

507 with

$$w_i(t) = g_i \frac{z(t)}{k_z + z(t)} \prod_{j=1}^{n_a} e^{-\sigma_{i,j} K_j^{(P)} c_j(t)}, \quad (\text{C.3})$$

508 that is the growth rate $w_i(t)$ is determined by the intrinsic growth rate g_i , the
 509 concentrations of the resource and the antibiotics $z(t)$ and $c_j(t)$, respectively.
 510 The effect of z is saturated in line with the standard Michaelis-Menten kinetics
 511 with half saturation constant k_z and the antibiotics cause exponential decay on
 512 total growth rate, $\sigma_{i,j} = 1$ if species i is sensitive to antibiotic j otherwise $\sigma_{i,j} =$

513 0. The concentration of the antibiotics changes because of the production, the
 514 degradation, and the dilution of antibiotics, thus the dynamics can be written
 515 as

$$\dot{c}_j(t) = \rho \sum_{i=1}^{n_s} \eta_{i,j} w_i(t) x_i(t) - K_j^{(D)} c_j(t) \sum_{i=1}^{n_s} \delta_{i,j} x_i(t) - d c_j(t), \quad (\text{C.4})$$

516 where ρ is the amount of antibiotics produced by unit concentration of cells,
 517 $\eta_{i,j} = 1$ if antibiotic j produced by species i , otherwise $\eta_{i,j} = 0$. Similarly
 518 $\delta_{i,j} = 1$ if species i produces degradation molecules for antibiotic j , otherwise
 519 $\delta_{i,j} = 0$. It follows from (C.1) and (C.2) that

$$\frac{d}{dt} \left(\sum_{i=1}^{n_s} \frac{x_i(t)}{\mu} + z(t) - z_0 \right) = -d \left(\sum_{i=1}^{n_s} \frac{x_i(t)}{\mu} + z(t) - z_0 \right), \quad (\text{C.5})$$

520 thus after a transient time

$$z(t) = z_0 - \sum_i \frac{x_i(t)}{\mu}. \quad (\text{C.6})$$

521 Therefore (C.1) can be eliminated when we study the stationary solutions of
 522 the system by substituting (C.6) into (C.3) (Kelsic et al., 2015).

523 Let us assume that dynamics of a bacterial community is described by (C.1-
 524 C.4), and a species m is a member of a community ($\bar{x}_m > 0$ in the stationary
 525 state), and produces at least one type of antibiotic. The mutant m' species
 526 loses the production of this antibiotic, thus it has an increased growth rate
 527 ($g_{m'} = (1 + \alpha)g_m, \alpha > 1$) as above. Thus, the difference of relative growth rates
 528 of m and m' species is

$$\frac{\dot{x}_m(t)}{x_m(t)} - \frac{\dot{x}_{m'}(t)}{x_{m'}(t)} = w_m(t) - w_{m'}(t) = -\alpha \frac{z(t)}{k_z + z(t)} \prod_{j=1}^{n_a} e^{-\sigma_{m,j} K_j^{(P)} c_j(t)}. \quad (\text{C.7})$$

529 Our aim here is to show that $z(t)/(k_z + z(t)) \prod_j e^{-\sigma_{m,j} K_j^{(P)} c_j(t)} > W_0 > 0$ if
 530 $t > t_c$ which guarantees that $\lim_{t \rightarrow \infty} x_m(t)/x_{m'}(t) = 0$. It follows from (C.2)
 531 that $x_i(t) \geq 0$ if $x_i(0) > 0$ and thus because of (C.6) $z(t) \leq z_0$ and $x_i < \mu z_0$ for
 532 every i . Therefore, $w_i(t) < g_i z_0 / (k_z + z_0)$ and the right hand side of (C.4) can
 533 be estimated above with

$$\dot{c}_j(t) < \rho \mu \frac{z_0^2}{k_z + z_0} n_s g_{\max} - \left(K^{(D)} \mu z_0 n_s + d \right) c_j(t) = \alpha_1 - \alpha_2 c_j(t) \quad (\text{C.8})$$

534 where $g_{\max} = \max\{g_i, i = 1, \dots, n_s\}$, $\sum_{i=1}^{n_s} \eta_{i,j}$ and $\sum_{i=1}^{n_s} \eta_{i,j}$ can be estimated
 535 above by n_s . Here α_1, α_2 are positive constants. By introducing function $C(t)$
 536 in such a way that its derivative estimates over $\dot{c}(t)$, we get

$$\dot{c}_j(t) < \dot{C}_j(t) = \alpha_1 - \alpha_2 C(t) \quad (\text{C.9})$$

537 This estimation is valid as the ordering between derivatives guarantees $C(t) >$
 538 $c(t)$ if $t > t^*$. It is easy to show that $\lim_{t \rightarrow \infty} C_i(t) = C^*$ where C is a finite
 539 positive constant, thus $\lim_{t \rightarrow \infty} c_i(t) \leq C^*$ for every i . Similarly, knowing that
 540 $\sum_{i=1}^{n_s} x_i/\mu \leq z_0$ and using the estimation introduced above Eq. (C.1) can be
 541 estimated below with

$$\dot{z}(t) \geq \dot{Z}(t) = (z_0 - Z(t))d - g_{\max} \frac{z_0}{\mu(k_z + z_0)} Z(t), \quad (\text{C.10})$$

542 Since $\lim_{t \rightarrow \infty} Z(t) = Z^* > 0$, thus $\lim_{t \rightarrow \infty} z(t) \geq Z^*$. That is, $z/(k_z +$
 543 $z)\Pi_j e^{-\sigma_{i,j} K_i^{(P)} c_j(t)} > Z^*/(k_z + Z^*)\Pi_j e^{-\sigma_{i,j} K_i^{(P)} C^*} = W_0 > 0$ for every t greater
 544 than a critical time t_c . Thus

$$\lim_{t \rightarrow \infty} x_m(t)/x_{m'}(t) = 0 \quad (\text{C.11})$$

545 as in the mixed inhibition model. We note here that the calculation remains
 546 valid if we use any monotonously decreasing function to model the effect of the
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