ESTIMATION AND INTERPRETATION OF A NONLINEAR MIGRATION MODEL

Kao-Lee Liaw

Cornelis P.A. Bartels

July 1981
WP-81-88

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

FOREWORD

Sharply reduced rates of population and industrial growth have been projected for many of the developed nations in the 1980s. In economies that rely primarily on market mechanisms to redirect capital and labor from surplus to deficit areas, the problems of adjustment may be slow and socially costly. In the more centralized economies, increasing difficulties in determining investment allocations and inducing sectoral redistributions of a nearly constant or diminishing labor force may arise. The socioeconomic problems that flow from such changes in labor demands and supplies form the contextual background of the Manpower Analysis Task, which is striving to develop methods for analyzing and projecting the impacts of international, national, and regional population dynamics on labor supply, demand, and productivity in the more-developed nations.

The subtask that focuses on regional and urban labor markets includes investigations of spatial labor mobility over time. This study proposes a two-level migration model that is considered attractive for the analysis of spatial and temporal characteristics of aggregate migration data. The authors focus on the description of the estimation procedure for their nonlinear model. The model has been applied to Dutch data on internal labor migration; this application is described more extensively in a companion paper (Bartels and Liaw 1981).

Publications in the Manpower Anlaysis Task series are listed at the end of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ACKNOWLEDGMENTS

The authors are grateful to Dimiter Philipov for making available useful reference material. We are also indebted to Gerard Evers for his assistance in data collection.

ABSTRACT

This paper provides a practical guide to using a two-level logistic model to analyze macro migration data. It explains the estimation method, provides subroutines for carrying out the estimation through a program in the BMDP package, and uses an empirical example to show how the parameters are to be estimated and interpreted.

CONTENTS

ESTIMATION AND INTERPRETATION OF A NONLINEAR MIGRATION MODEL

1. INTRODUCTION

From various perspectives, social scientists in recent decades have developed migration models that are more than mere analogies of models in the physical sciences. Using information theory, Wilson (1971) derived the constrained entropy models of spatial interactions, which subsume migration as a special case. Based on the interdependent notions of opportunity and competition, Alonso (1976) completed the conceptual refinement of his general migration model. Along the line of utility maximization theory, Moss (1979) translated a version of McFadden's logistic model for travel choice (McFadden 1974) into another general model of migration. In a less rigorous fashion, Grant and Vanderkamp (1976) also developed a logistic model of migration from the theory of human capital investment. All these models have one feature in common - they are all *nonlinear*.

Unfortunately, empirical applications of these nonlinear models to the explanation of migration in terms of socioeconomic variables have been hindered by the nonexistence or complexity of a consistent nonlinear statistical theory. In many cases, *ad hoc* procedures are used to linearize the model (usually

through the log-transformations), and then one of the widely available computer programs for linear least-squares regression analysis is used for estimation and statistical inference. Beside the doubt that the linearized model can satisfy the restrictive assumptions of the standard linear model, these procedures sometimes lead to the nonsensical result of negative outmigration rates. Sometimes the nonnegativity property is preserved by using the log of the odds of migration as the dependent variable which in turn breaks down when some observed migration frequencies are zero (Grant and Vanderkamp 1976). With respect to a model of destination choice, all linear estimation procedures fail to guarantee that the sum of estimated choice probabilities across all destinations be equal to one, unless for every origin, one of the destinations is arbitrarily suppressed from the data set and is allowed to absorb all estimation errors.

In this paper, we will focus on the use of a *two-level logistic model of migration*, which has an appropriate maximum likelihood estimation method and a relatively well-developed, albeit asymptotic, statistical theory. The model is a specific form of the "production constrained" model of Wilson and Alonso.* It is also a special case of Moss's migration model with the assumption that the decision to move preceeds the decision to choose a destination. In fact we believe that the logistic model is a simple and practical nonlinear model of migration that will remain popular, at least until the statistical problems of the more complicated migration models are solved.

In using a quantitative model of migration, it is important to find the best estimates of the unknown parameters. But these estimates would not be very useful, if they could not be used to evaluate the relative importance of the explanatory variables. Is it more likely that migration would respond to wage differentials than to unemployment differentials? Would a unit increase

*Ledent (1980) has shown that Wilson's models are actually equivalent to Alonso's general migration model with various "inputs".

in housing opportunity affect migration more than a unit increase in job opportunity does? These are the type of questions that must be dealt with by an empirically useful statistical methodology.

Without a readily accessible computational algorithm, an elegant statistical methodology is not worth much to a migration researcher who has no time to write his own computational program. Those who have *micro* migration data (i.e., data with individual persons or households as the observation units) and want to use logistic models are relatively fortunate, because there are computer programs for travel choice problems such as those described in McFadden (1976) which can be easily adopted. However, many migration researchers (e.g., Grant and Vanderkamp 1976; Schultz 1977; and Rempel 1980) who recently used logistic models for *macro* data (i.e., those with geographical areas as the units of observation) were unable to use the appropriate maximum likelihood estimation method, presumably because of the lack of a suitable computer program. Even Da Vanzo who has used the maximum likelihood method for her micro migration data (Da Vanzo 1976) was not helpful in saying that "with aggregate data, the politomous logit model can be estimated by OLS (ordinary least-squares) once the data are appropriately transformed" (Da Vanzo 1980:16).

This paper is written mainly for migration researchers who have a set of macro origin-destination migration data to explain. We will first describe and justify the two-level logistic migration model in Section 2. We then provide a digest of the maximum likelihood method of estimation and the relevant statistical theory in Section 3. The evaluation of the relative importance of explanatory variables is discussed in Section 4. We then explain in Section 5 the use of a versatile program in the widely available BMDP package (Dixon and Brown 1977) for carrying out the estimation procedure. More importantly, in Section 6, an empirical example is used to show the actual implementation of the model. A short conclusion in Section 7 completes the paper.

2. THE TWO-LEVEL MODEL OF MIGRATION

Let the probability that a person in region i will migrate to region j in period t be M_{tij}. Assuming that migration is the result of two successive decisions--first the decision to move out of the current residence and then the decision to choose a destination--we write

$$M_{tij} = p_{ti} p_{tij} \tag{1}$$

where p_{ti} is the person's probability of migrating out of region i in period t; and p_{tij} is the person's conditional probability of choosing region j as his destination, given that he has decided to move.* It is assumed that within each region; the propensity of every person to migrate to any other region is governed by equation (1).

The decomposition of M_{tij} into the product of p_{ti} and p_{tij} has been advocated by many migration researchers, e.g., Morrison (1973), Cordey-Hayes and Gleave (1973), and Moss (1979). Furthermore, our data on the annual interprovincial migration of Dutch labor force between 1971 and 1978 suggest that p_{ti} and p_{tij} have different temporal patterns: the former has fluctuated markedly, whereas the latter has remained quite stable. This suggests that the two aspects of migration may be related to different sets of determinants and hence can be analyzed by a two-level model.

By definition, p_{ti} and p_{tij} must satisfy the constraints

$$0 \leq p_{ti} \leq 1 \tag{2}$$

$$0 \leq p_{tij} \leq 1 \tag{3}$$

---

*If the user's data is for only one period, then the subscript t can be dropped. However, in order to have enough degrees of freedom for the statistical inference about the determinants of the departure probabilities, the number of origins will then have to be large. If the data are stratified in terms of relevant attributes such as age and labor force status, then equation (1) can be applied to each relatively homogeneous subpopulation.

and

$$\sum_{j=1}^{G} p_{tij} = 1 \tag{4}$$

where G is the number of origins.

To satisfy these constraints, we adopt the satistically convenient logistic formulations:

$$p_{ti} = \frac{e^{\alpha_0 + \alpha_1 x_{ti1} + \ldots + \alpha_K x_{tiK}}}{1 + e^{\alpha_0 + \alpha_1 x_{ti1} + \ldots + \alpha_K x_{tiK}}} \tag{5}$$

and

$$p_{tij} = \frac{e^{\beta_1 x_{tij1} + \beta_2 x_{tij2} + \ldots + \beta_K x_{tijK}}}{\sum_{\ell=1}^{D} e^{\beta_1 x_{ti\ell 1} + \beta_2 x_{ti\ell 2} + \ldots + \beta_K x_{ti\ell K}}} \tag{6}$$

where $x_{ti1}, \ldots, x_{tiK}$ are observable factors controlling the departure probabilities p_{ti}; $x_{tij1}, \ldots, x_{tijK}$ are observable determinants of the destination choice probabilities; and D is the number of all alternative destinations. The fact that the exponential functions in equations (5) and (6) are linear in the unknown vectors of parameters $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ makes the tasks of estimation and inference relatively simple. However, these logistic models are quite flexible in that the explanatory variables x_{tik} and x_{tijk} may be monotonic or non-monotonic transformations of such variables as housing and job opportunties or dummy variables representing specific cultural ties or barriers between regions. We will call equation (5) the *departure model* and equation (6) the *destination choice model*. Since the parameters and explanatory variables are assumed to be finite, both p_{ti} and p_{tij} are not equal to zero or one. But this does not imply that the observed relative frequencies cannot assume these extreme values.

3. THE ESTIMATION METHOD AND RELEVANT STATISTICAL THEORY

The *maximum likelihood method* is appropriate for the estimation of unknown parameters of the two-level logistic model for several reasons. First, it guarantees that the estimated values of p_{ti} and p_{tij} satisfy the constraints (2), (3), and (4). Second, under relatively mild conditions, the maximum likelihood estimators are consistent and asymptotically efficient (McFadden 1974). Third, the maximum likelihood method leads to a computational algorithm that can handle efficiently a relatively large data set (e.g., we found that it takes a computer less than three minutes to apply the estimation method to a data set with 880 cases and 10 explanatory variables).

To make the statistical problem simple, we will consider the nature of the statistics of the destination choice model to be conditional to the departure model. In other words, the randomness of one process is not entered into the investigation of the other. Since our description of the estimation method is intended to be brief, the reader is referred to Ginsburg (1972), McFadden (1974), and Jennrich and Moore (1975) for more detailed information.

Let N_{ti} be the population size in region i at the beginning of period t; and let Y_{ti} be the number of migrants moving out of region i during period t, among whom Y_{tij} migrants choose region j as the destination. Assuming that the migrants are random samples from the population, the likelihood functions of models (5) and (6) are, respectively,

$$L_1 = \prod_{t=1}^{T} \prod_{i=1}^{G} \left[\frac{p_{ti}}{1 - p_{ti}}\right]^{Y_{ti}} \left[1 - p_{ti}\right]^{N_{ti}} \frac{N_{ti}}{Y_{ti}!\,(N_{ti} - Y_{ti})!} \qquad (7)$$

$$L_2 = \prod_{t=1}^{T} \prod_{i=1}^{G} \left[\prod_{j=1}^{D} p_{tij}^{Y_{tij}} / Y_{tij}!\right] Y_{ti}! \qquad (8)$$

where T is the number of periods, G is the number of origin regions, and D is the number of all destination regions. Note that it is not necessary that G and D be equal. Both of these likelihood functions belong to the *regular exponential* family. That is, they can be rewritten in the form:

$$L = e^{\underset{\sim}{\gamma}(\underset{\sim}{\theta})' \underset{\sim}{y} + \delta(\underset{\sim}{\theta}) + h(\underset{\sim}{y})} \tag{9}$$

where $\underset{\sim}{y}$ is a column vector of random variables; $\underset{\sim}{\theta}$ is the parameter vector (representing $\underset{\sim}{\alpha}$ for the departure model and $\underset{\sim}{\beta}$ for the destination choice model); $\underset{\sim}{\gamma}(\underset{\sim}{\theta})'$ is a row vector that depends on $\underset{\sim}{\theta}$ but is independent of $\underset{\sim}{y}$; $\delta(\underset{\sim}{\theta})$ is a scalar function of $\underset{\sim}{\theta}$ and is independent of $\underset{\sim}{y}$; and $h(\underset{\sim}{y})$ is a scalar function of $\underset{\sim}{y}$ and is independent of $\underset{\sim}{\theta}$. Note that the order of $\underset{\sim}{y}$ is TG x 1 for the departure model and TGD x 1 for the destination choice model.

Let the expectation of $\underset{\sim}{y}$ be $\underset{\sim}{\mu}$ and the convariance matrix of $\underset{\sim}{y}$ be $\underset{\sim}{\Lambda}$. Two remarkable properties of the regular exponential likelihood function have been derived by Jennrich and Moore (1975). First, the vector of first-order partial derivatives are related to $\underset{\sim}{y}$, $\underset{\sim}{\mu}$, and $\underset{\sim}{\Lambda}$ according to

$$\frac{\partial \ln L}{\partial \underset{\sim}{\theta}} = \frac{\partial \underset{\sim}{\mu}'}{\partial \underset{\sim}{\theta}} \underset{\sim}{W}(\underset{\sim}{y} - \underset{\sim}{\mu}) \tag{10}$$

where $\underset{\sim}{W}$ is a generalized inverse of $\underset{\sim}{\Lambda}$ such that

$$\underset{\sim}{\Lambda}\,\underset{\sim}{W}\,\underset{\sim}{\Lambda} = \underset{\sim}{\Lambda} \tag{11}$$

Second, the information matrix $\underset{\sim}{I}(\underset{\sim}{\theta})$ is related to $\underset{\sim}{\mu}$ and $\underset{\sim}{\Lambda}$ according to

$$\underset{\sim}{I}(\underset{\sim}{\theta}) = \frac{\partial \underset{\sim}{\mu}'}{\partial \underset{\sim}{\theta}} \underset{\sim}{W} \frac{\partial \underset{\sim}{\mu}}{\partial \underset{\sim}{\theta}} \tag{12}$$

The significance of the information matrix is that its inverse is the asymptotic covariance matrix of the maximum likelihood estimator $\hat{\underset{\sim}{\theta}}$ of the unknown parameter vector $\underset{\sim}{\theta}$. Note that both equations (10) and (12) are derived without using any approximation. The true first-order condition for maximization is therefore

$$\frac{\partial \mu'}{\partial \underset{\sim}{\theta}} \underset{\sim}{W}(\underset{\sim}{y} - \underset{\sim}{\mu}) = \underset{\sim}{0} \tag{13}$$

which, being nonlinear, does not provide an explicit solution of $\hat{\underset{\sim}{\theta}}$.

The solution may be obtained iteratively by the Newton-Raphson algorithm in the following manner. It is assumed that the log of the likelihood function can be approximated around some guessed solution $\underset{\sim}{\theta}_o$ by the second-order Taylor series:

$$\ln L(\underset{\sim}{\theta}) = \ln L(\underset{\sim}{\theta}_o) + \Delta\underset{\sim}{\theta} \frac{\partial \ln L}{\partial \underset{\sim}{\theta}} + \frac{1}{2} \Delta\underset{\sim}{\theta}' \frac{\partial^2 \ln L}{\partial \underset{\sim o}{\theta} \partial \underset{\sim o}{\theta}'} \Delta\underset{\sim}{\theta} \tag{14}$$

where $\Delta\underset{\sim}{\theta} = \underset{\sim}{\theta} - \underset{\sim o}{\theta}$. To move from one guessed solution to another, the increment $\Delta\underset{\sim}{\theta}$ is chosen such that $\ln L(\underset{\sim}{\theta})$ is maximized. That is,

$$\frac{\partial \ln L(\underset{\sim}{\theta})}{\partial \Delta\underset{\sim}{\theta}} = \frac{\partial \ln L}{\partial \underset{\sim o}{\theta}} + \frac{\partial^2 \ln L}{\partial \underset{\sim o}{\theta} \partial \underset{\sim o}{\theta}'} \Delta\underset{\sim}{\theta} = \underset{\sim}{0} \tag{15}$$

and hence

$$\Delta\underset{\sim}{\theta} = -\left[\frac{\partial^2 \ln L}{\partial \underset{\sim o}{\theta} \partial \underset{\sim o}{\theta}'}\right]^{-1} \frac{\partial \ln L}{\partial \underset{\sim o}{\theta}} \tag{16}$$

Since it turns out that for our logistic models, the matrix of second-order partial derivatives in equation (16) does not depend

on the random vector $\underset{\sim}{y}$ and hence is equal to its expectation $\underset{\sim}{I}(\underset{\sim}{\theta})$, we can substitute equations (10) and (12) into equation (16) to obtain

$$\Delta\underset{\sim}{\theta} = \left[\frac{\partial\underset{\sim}{\mu}'}{\partial\underset{\sim}{\theta}}\underset{\sim o}{W}\frac{\partial\underset{\sim}{\mu}}{\partial\underset{\sim}{\theta}}\right]^{-1}\frac{\partial\underset{\sim}{\mu}'}{\partial\underset{\sim}{\theta}}\underset{\sim o}{W}(\underset{\sim}{y} - \underset{\sim o}{\mu}) \tag{17}$$

where the right-hand-side quantities are evaluated at the most recent guessed value of $\hat{\underset{\sim}{\theta}}$. The iterative procedure is terminated when $\Delta\underset{\sim}{\theta}$ is sufficiently close to zero. Since setting equation (17) to zero implies equation (15), we see that when $\Delta\underset{\sim}{\theta} = \underset{\sim}{0}$, the true first-order condition is indeed satisfied. For the logistic models, McFadden (1974) has proved that the nonsingularity of the information matrix guarantees the *uniqueness* of the maximum likelihood solution $\hat{\underset{\sim}{\theta}}$; but the *existence* of the solution is relatively difficult to ascertain from inspecting the data. Usually, small sample size and multicollinear explanatory variables are the main reasons for failing to find the correct solution.

It is also true for the logistic models that under relatively mild conditions, the maximum likelihood estimator $\hat{\underset{\sim}{\theta}}$ is asymptotically normally distributed, with mean $\underset{\sim}{\theta}$ and covariance matrix $\underset{\sim}{I}(\underset{\sim}{\theta})^{-1}$ (McFadden 1974). Thus, when the sample size (i.e., TG for the departure model and TGD for the destination choice model) is very large, significance tests about individual parameters can be carried out by considering the "t-ratio" (i.e., the estimator of a parameter divided by the corresponding standard error) as the standard normal variate. Just like other nonlinear statistical models, the logistic models do not have a tractable sampling theory for a finite sample size. When sample size is small, Monte Carlo simulations of some hypothetical migration processes are necessary before much confidence can be put in any inferential procedure of testing hypothesis about the unknown parameters. Simulation results of a couple of very simple logisitc models are shown in McFadden (1974), indicating that when the sample size is 200, the biases in the expectation of $\hat{\underset{\sim}{\theta}}$ and the corresponding

variances are less than 5%. However, it may be dangerous to generalize from such simple examples.

According to the multidimensional generalizations of the well-known Cramer-Rao inequality, the inverse of the information matrix is a *lower* bound of the covariance matrix of a regular unbiased maximum likelihood estimator of the unknown parameter vector (Theil 1971:389). This suggests that the estimated asymptotic standard errors obtained from $\underset{\sim}{I}(\hat{\underset{\sim}{\theta}})^{-1}$ would tend to understate the values of the actual standard errors of the estimators of the unknown parameters. We consider it advisable to correct this tendency by multiplying the asymptotic standard errors by the square root of the *weighted residual mean square* $\hat{S}^2$ before the t-ratios are computed. Note that

$$\hat{S}^2 = (\underset{\sim}{y} - \hat{\underset{\sim}{\mu}})' \underset{\sim}{W}(\underset{\sim}{y} - \hat{\underset{\sim}{\mu}})/V \tag{18}$$

where the number of degrees of freedom V equals the number of elements in $\underset{\sim}{y}$ minus the number of elements in $\hat{\underset{\sim}{\theta}}$. The motivations for this correction are that the results are analogous to the standard errors in nonlinear least squares problems, and that it does not affect the nice asymptotic properties, because $\hat{S}^2$ approaches one as the sample size approaches infinity (Jennrich and Moore 1975). Note that without this correction, a variable that contributes practically nothing to the reduction in $\hat{S}^2$ is sometimes found to have a t-ratio of large magnitude, say, about 4 or 5. However, we cannot deny the possibility that the correction may occasionally be too much.

To test the model's overall goodness-of-fit, we observe that for a large sample, the quadratic form $\hat{\underset{\sim}{\theta}}' \underset{\sim}{I}(\underset{\sim}{\theta}) \hat{\underset{\sim}{\theta}}$ tends to be chi-square distributed with the degrees of freedom being the number of parameters, if the null hypothesis that $\underset{\sim}{\theta} = \underset{\sim}{0}$ is true (McFadden 1974).* Since $\underset{\sim}{I}(\underset{\sim}{\theta})$ depends on the unknown vectar $\underset{\sim}{\theta}$,

*Note that for the departure model, the appropriate null hypothesis is $\alpha_1 = \alpha_2 = \ldots = \alpha_K = 0$. In other words, α_o should not be included in the hypothesis. Thus, the first element of $\hat{\theta}$ and the first row and column of $\underset{\sim}{I}(\theta)$ are to be deleted in specifying the quadratic form. Of course, the number of degrees of freedom must be adjusted correspondingly.

the quadratic form is first approximated by $\hat{\underset{\sim}{\theta}}\, \underset{\sim}{I}\, (\hat{\underset{\sim}{\theta}})\hat{\underset{\sim}{\theta}}$ and then compared with a critical chi-square value at, say, $\alpha = 0.05$. If the value of the quadratic form is larger than the critical value, then the null hypothesis is rejected. However, if the sample size is large, the null hypothese ($\underset{\sim}{\theta} = \underset{\sim}{0}$) can also be rejected when one of the elements in $\hat{\underset{\sim}{\theta}}$ has a t-ratio that is greater in magnitude than the critical value of the standard normal variate. Since the program we recommend does not print out the value of the quadratic form, we will rely only on the t-ratios for statistical inference.

To convey the goodness-of-fit at the *intuitive* level, we may use the coefficient of determination R^2, where R is the simple correlation coefficient between $\underset{\sim}{y}$ and $\hat{\underset{\sim}{\mu}}$. There are two other indices discussed in McFadden (1974). One index is

$$\rho_1^2 = 1 - \hat{S}^2/S_h^2 \tag{19}$$

where $\hat{S}^2$ is the weighted residual mean square defined in equation (18), and S_h^2 is the weighted residual mean square computed under the null hypothesis that all parameters are zero. For the destination choice model, the value of ρ_1^2 is similar to that of R^2. For the departure model, ρ_1^2 can, however, assume a misleadingly large value even when the model fits very poorly. This is because the expected departure probability under the null hypothesis is 0.5, which is usually much larger than the observed departure rates. This drastic contrast results in a very large S_h^2, which in turn causes ρ_1^2 to be large. Therefore, for the departure model we will not use ρ_1^2 as a simple index of the goodness-of-fit. The other index is

$$\rho_2^2 = 1 - \frac{\ln L\,(\hat{\underset{\sim}{\theta}})}{\ln L\ \underset{\sim}{\theta}} \tag{20}$$

where $L(\hat{\underset{\sim}{\theta}})$ is the value of the likelihood function evaluated at $\hat{\underset{\sim}{\theta}}$, and $L(\underset{\sim}{\theta}_h)$ is the value of L evaluated under the above-mentioned null hypothesis. We will not use ρ_2^2 in our empirical example,

because it tends to understate substantially the goodness-of-fit. For example, it is reported in McFadden (1979) that values of 0.2 to 0.4 for ρ_2^2 represent an excellent fit.

4. RELATIVE IMPORTANCE OF EXPLANATORY VARIABLES

There are two distinct types of criteria to evaluate the relative importance of explanatory variables. The first (*intensity*) criterion is the relative *average* amounts of change in the dependent variable due to a unit change in different explantory variables. When the explanatory variables are measured in comparable units, the relative importance is simply reflected by the relative magnitude of the partial derivatives of the dependent variable with respect to the explanatory variables. For the departure model, we have

$$\frac{\partial p_{ti}}{\partial x_{tik}} = \alpha_k \, p_{ti}(1 - p_{ti}) \tag{21}$$

and

$$\frac{\partial p_{ti}}{\partial x_{tik}} \Big/ \frac{\partial p_{ti}}{\partial x_{ti\ell}} = \frac{\alpha_k}{\alpha_\ell} \tag{22}$$

Thus, the relative importance of the k^{th} variable over the ℓ^{th} explanatory variable is indicated by the relative magnitudes of the coefficients α_k and α_ℓ. For the destination choice model, we have

$$\frac{\partial p_{tij}}{\partial x_{tijk}} = \beta_k \, p_{tij}(1 - p_{tij}) \tag{23}$$

and

$$\frac{\partial p_{tij}}{\partial x_{tijk}} \Big/ \frac{\partial p_{tij}}{\partial x_{tij\ell}} = \frac{\beta_k}{\beta_\ell} \tag{24}$$

which are similar in form to equations (21) and (22). When the explanatory variables are not measured in comparable units, it is common practice to substitute the partial derivatives by elasticities or "beta weights" (i.e., the estimated values of the parameters obtained by standardizing all explanatory variables). The use of beta weights is based on the assumption that one standard deviation in one variable is comparable to one standard deviation in another variable; while the use of elasticities is based on the assumption that a 1% increase in one variable is comparable to a 1% increase in another variable. Note that for all logistic models, the elasticities are not constant across the observations and are usually evaluated only at some representative points like the mean.

The second (*likelihood*) criterion is the relative likelihood of *some* change in the dependent variable caused by changes in different explanatory variables. For the logistic models, the probability that the dependent variable will respond to a change in an explanatory variable is assumed to be positively related to the magnitude of the t-ratio of the coefficient associated with the explanatory variable. This assumption is based on the t-ratio (1) being indeed a t-statistic in the standard linear model and (2) having a standard normal distribution in the logistic model. It is worth noting that in the standard linear model, the magnitude of the t-statistic is monotonically related to (and hence equivalent to) the partial F-statistic, the magnitude of partial correlation coefficient and the incremental contribution in R^2. However, it is important to remember that a large t-ratio *need not* indicate that a unit change in the corresponding explanatory variable will cause a *large* change in the dependent value.

It is now clear that the importance of an explanatory variable must be judged by both intensity and likelihood criteria. Conceptually, the likelihood criterion is relatively straightforward, because probabilities (i.e., levels of significance according to the t-statistics or partial F-statistics) are not influenced by the different choices of the physical units for the explanatory

variables. The intensity criterion is more troublesome; an explanatory variable with a relatively large elasticity may or may not have a relatively large beta weight. When the absolute truth is beyond reach, conventions are the second best. Most sociologists rely on beta weights, whereas most economists favor elasticitics. In geography, beta-weights are in relatively frequent use.

Finally, in evaluating the relative importance of explanatory variables, we should keep a *complementarity* as well as a *competition* perspective. The inclusion of an additional explanatory variable into the migration model may increase rather than decrease the importance of an existing explanatory variable. By adding economic variables into his gravity model of intermetropolitan migration, Lowry (1966:14-17) managed to increase substantially the importance of the distance variable in terms of elasticity as well as partial correlation. To infer if two explanatory variables are mutually complementary or competitive, one should choose a computer program that allows easy selections of arbitrary subsets of input variables to be included in the model.

5. ESTIMATION OF THE UNKNOWN PARAMETERS BY BMDP3R

The iterative algorithm described by equation (17) can be implemented without undue difficulties by the P3R program in a recent version of the BMDP package (Dixon and Brown 1977). The program was originally designed to solve nonlinear weighted least-squares problems, using the Gauss-Newton algorithm (Jennrich and Ralston 1979). However, it is fortunate that in our departure and destination choice models, the matrix $\underset{\sim}{W}$ is diagonal so that the Newton-Raphson algorithm for the maximum likelihood method becomes identical to the Gauss-Newton algorithm for the nonlinear weighted least-squares problems, *except* that the former requires the matrix of weights $\underset{\sim}{W}$ to depend on the unknown parameters, whereas the latter does not. The modification to accommodate this subtle difference is accomplished by a subroutine that allows the user to specify the computational formulas for $\underset{\sim}{\mu}$, $\frac{\partial \underset{\sim}{\mu}}{\partial \underset{\sim}{\theta}}$, and $\underset{\sim}{W}$. These computational

formulas are shown in Table 1. However, for diagnostic and interpretational convenience, it is better to measure migration in proportions rather than in volumes. Therefore, we will measure the dependent variables in proportions and use the computational formulas in Table 2. Note that the estimated parameters, the t-ratios, and the weighted residual mean square are not affected by the different ways of measuring migration.

The subroutine to implement the departure model is shown in Figure 1. It assumes that the first three columns of the input data contain respectively the observed departure rates, the arbitrary initial values of the weights, and the at-risk population sizes. All the explanatory variables to be included in the model then occupy consecutive columns starting from the fourth one. If the input data were not arranged in this way, we could use transformation instructions in the file of control statements to rearrange the variables in the data set. In each iteration, the subroutine is called to evaluate p_{ti}, $N_{ti}/[p_{ti}(1 - p_{ti})]$, and $p_{ti}(1 - p_{ti})x_{tik}$ in terms of the most recent estimate of $\hat{\underset{\sim}{\alpha}}$. Without any modification, the subroutine can accommodate a data set of any size, provided there is enough space in the computer.

The subroutine to implement the destination choice model is shown in Figure 2. The arrangement of variables in the input data is assumed to be similar to that of the departure model (i.e., the observed choice proportions followed by the arbitrary initial weights, etc.). Futhermore, the observations (cases) corresponding to all the destinations for each origin and period must be in neighboring rows. In other words, the rows of the input data matrix must be nested in the order of time-origin-destination or origin-time-destination. In each iteration, the subroutine will be passed twice: the first pass is for computing the partial sums in equation (6) and Table 2 across all destinations for each t, i, and k; and the second pass is for computing the estimates of the expected values, weights, and partial derivatives. If the number of parameters is no more than 10, and if the product of the number of periods and the number of origins does not exceed 88, then the user only has to make sure that the

Table 1. Computational formulas for the Newton-Raphson algorithm, using number of migrants as the dependent variable.

	Departure Model	Destination Choice Model
Random Variable	Y_{ti}	Y_{tij}
Expected Value	$N_{ti}p_{ti}$	$Y_{ti}p_{tij}$
Weight	$[N_{ti}p_{ti}(1-p_{ti})]^{-1}$	$[Y_{ti}p_{tij}]^{-1}$
Partial Derivative	$N_{ti}p_{ti}(1-p_{ti})x_{tik}$	$Y_{ti}p_{tij}(x_{tijk} - \sum_{l=1}^{D} p_{til}\ x_{tilk})$

Table 2. Computational formulas for the Newton-Raphson algorithm, using proportion of migrants as the dependent variable.

	Departure Model	Destination Choice Model
Random Variable	Y_{ti}/N_{ti}	Y_{tij}/Y_{ti}
Expected Value	p_{ti}	p_{tij}
Weight	$N_{ti}/[p_{ti}(1-p_{ti})]$	Y_{tij}/p_{tij}
Partial Derivative	$p_{ti}(1-p_{ti})x_{tik}$	$p_{tij}(x_{tij} - \sum_{l=1}^{D} p_{til}x_{tilk})$

```
      subroutine funp3r(f,df,p,x,n,kase,nvar,npar,ipass,
     *xloss,idep)
c  assumed data setup--
c  x(1)=observed departure rates.
c  x(2)=weights.
c  x(3)=at-risk populations.
c  x(4) and beyond=all explanatory variables followed by
c                  any potentially useful indices.
      dimension df(npar),p(npar),x(nvar)
      implicit real*8 (a-h,o-z)
      sum=p(1)
      do 20 j=2,npar
      j1=j+2
   20 sum=sum+p(j)*x(j1)
      xnum=dexp(sum)
      denom=1.0+xnum
      f=xnum/denom
      df(1)=f/denom
      do 40 j=2,npar
      j1=j+2
   40 df(j)=f*x(j1)/denom
      x(2)=x(3)*denom/f
      return
      end
```

Figure 1. The subroutine for BMDP3R to implement the departure model.

right-hand-side of the statement nr=10 is made to equal the actual number of destinations. For a larger model, the only necessary additional change is to replace the subscripts in the second dimension statement according to the comments in the subroutine.

One particularly attractive feature of BMDP3R is its ability to plot the observed and predicted values of the dependent variable against any variable that may or may not be an explanatory variable of the model. By plotting these values against such variables as time and an index of origin or destination, it is easy to see the temporal and spatial patterns of the migration process. Furthermore, the plots can be used to identify outliers quickly and to improve the structure of the model. Another useful feature of P3R is that various types of transformations are available. Through these transformations, the user can change

```
      subroutine funp3r(f,df,p,x,n,kase,nvar,npar,ipass,
     *xloss,idep)
      dimension df(npar),p(npar),x(nvar)
      dimension part(10,33),sum(33)
c  the subscript of "sum" and the second subscript of
c  "part" must be no less than (no. of cases/no. of
c  destinations).
c  the first subscript of "part" must be no less than
c  the number of parameters.
      implicit real*8 (a-h,o-z)
      nr=10
c  "nr" must equal the actual number of destinations.
      igrp=1+(kase-1)/nr
      ikase=kase-(igrp-1)*nr
      if(ipass.eq.2) go to 100
      if(ikase.gt.1) go to 20
      do 10 j=1,npar
   10 part(j,igrp)=0.0
      sum(igrp)=0.0
   20 temp=0.0
      do 30 j=1,npar
      j1=j+3
   30 temp=temp+x(j1)*p(j)
      temp=dexp(temp)
      sum(igrp)=sum(igrp)+temp
      do 40 j=1,npar
      j1=j+3
   40 part(j,igrp)=part(j,igrp)+x(j1)*temp
      return
  100 temp=0.0
      do 50 j=1,npar
      j1=j+3
   50 temp=temp+x(j1)*p(j)
      f=  dexp(temp)/sum(igrp)
      x(2)=x(3)/f
      do 120 j=1,npar
      j1=j+3
  120 df(j)=f*(x(j1)-part(j,igrp)/sum(igrp))
      return
      end
```

Figure 2. The subroutine for BMDP3R to implement the destination choice model.

volumes into proportions and vice versa, combine old variables to form new ones, and rearrange the order of the input variables for alternative specifications of the model.

6. AN EMPIRICAL EXAMPLE

We have used the two-level migration model to study the 1971-1978 data on annual labor force migration among the eleven provinces of the Netherlands (Figure 3). Here we present one of the several specifications that we tried in an attempt to develop a parsimonious explanatory model (for more details, see Bartels and Liaw 1981). Briefly a migrant is defined as a member of the Dutch labor force who had a known occupation and was observed to have changed the province of residence during a year.

6.1 The Departure Model

Our data and model permit us to investigate simultaneously the temporal and spatial aspects of the departure probabilities. We first intend to explain the temporal pattern by changes in the *national* housing and job opportunities, because we suspect that when these opportunities are generally poor, the incentive to move will be weak. We then assume that the interregional contrast in departure propensity may depend on *regional* housing and job conditions. Perhaps a province with relatively good housing and job conditions would have a relatively low departure rate; but we recall that Lowry (1966) has provided a vivid counter example in the contrast between San Jose, California and Albany, New York.

The change in national housing opportunity is represented by the national annual percentage rate of increase in housing stock. The proxy for the change in national job opportunity is the inverse of national annual unemployment rate. Regional housing opportunity is defined as the ratio of regional housing increase to national housing increase. Similarly, regional job opportunity is the inverse of the ratio of regional unemployment rate to national unemployment rate. All these explanatory variables are

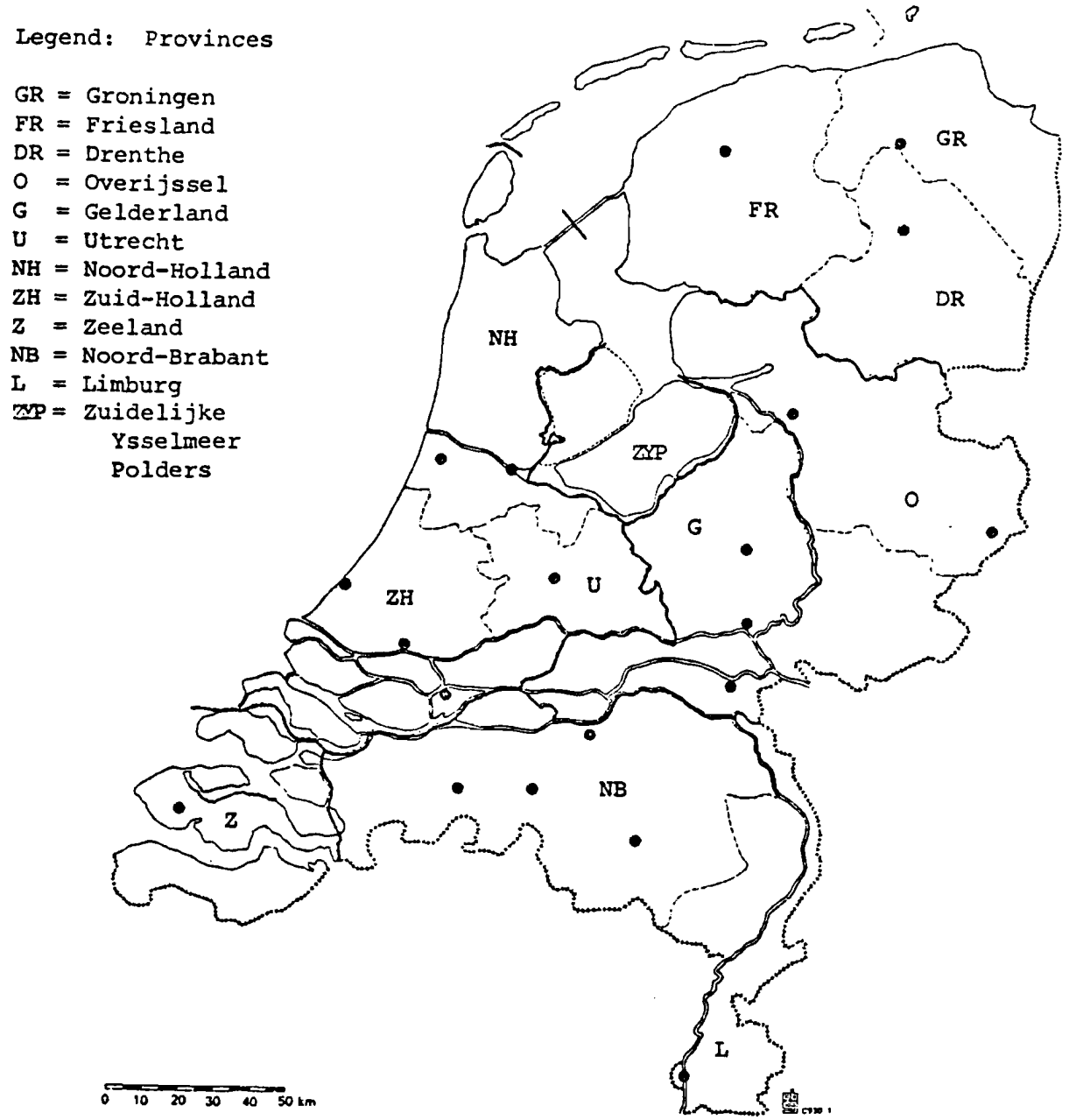

Figure 3. Regional demarcation of the Netherlands according to provinces. (The dots represent the locations of major cities.)

evaluated on a yearly basis. To eliminate persistent regional biases in the estimated departure probabilities, three regional dummy variables are used: the first to reflect the fact that the province of Groningen has a relatively high departure rate due to the high concentration of its population near the southern border; the second to reflect the high departure rate of Utrecht probably due to its small area and its location near the gravity center of the national population; and the third to reflect the low departure rate of Overijssel perhaps due to its high concentration of blue collar workers whose mobility is typically low. The dependent variable (the observed regional departure rate) is the annual number of regional migrants divided by the size of regional labor force in the relevant year.

The input data matrix has 88 cases (8 periods times 11 provinces) and 10 variables (departure rate, weight, size of labor force, and seven explanatory variables). To show temporal and spatial patterns graphically, we augmented the input matrix by two more variables: one is the year, the other is the province index. The matrix is arranged such that the cases are rows, and the variables are columns. The control statements to analyze this data matrix are shown in Figure 4. The number of iterations is set at 10, but usually it takes only five or six iterations to converge to the optimum solution. For precise meanings of the control statements, the reader is referred to the BMDP Manual (Dixon and Brown 1977).

The fit of the model is quite good ($R^2 = 0.79$). The t-ratios in Table 3 indicate (1) that the temporal fluctuations in the departure rates are more likely to be caused by changes in national housing conditions than by changes in national job opportunities, (2) that the interprovincial contrasts in housing and job opportunities do not have a clear relationship with the interregional contrast in departure propensity, and (3) that there is little doubt that the spatial contrast in mobility level is related to the underlying factors represented by the three dummy variables. Ignoring the two most uncertain variables (i.e., those with the smallest t-ratios), we see that all the explanatory variables have coefficients of the "right" signs.

```
/problem   title is 'departure model:holland,1971-78'.
/input     variables are 12.
           format is '(12f10.0)'.
           cases are 83.
/variable  names are dptrt,wt,prisk,hincn,jobn,hincp,jobp,
                     dumgr,dumov,dumut,year,origin.
/regress   dependent is dptrt.
           parameters are 8.   number is 20.
           iterations are 10.  convergence is -1.0.
           weight is wt.       halving is 0.
           meansquare is 1.0.
/parameters initial are 8*0.0.
/plot      residual.
           variable=dptrt,hincn,jobn,hincp,jobp,dumgr,
                    dumov,dumut,year,origin.
           size=45,40.
/end
```

Figure 4. The control statements to request BMDP3R to carry out the maximum likelihood estimation of the departure model of the Dutch labor force.

Since the explanatory variables are not all measured on comparable units, the relative intensity of the influence of these variables on the departure propensity will be judged in terms of elasticity and beta weight (Table 4). The most influential explanatory variable is unequivocally the national housing increase. National job opportunity may or may not be more important than the three dummy variables, depending on whether elasticity or beta weight is used as the criterion. It is best to ignore the elasticities and beta weights of the provincial housing and job opportunity variables, because the influences of these two variables have been shown by the t-ratios to be most uncertain.

6.2 The Destination Choice Model

As we have indicated earlier, the spatial pattern of the destination choice probabilities in the Netherlands appeared to remain quite stable through the 1970s. This observation suggests that the important explanatory variables should also be stable in nature. Two variables with such stability are distance and the spatial pattern of employment size. Thus, the distance between

Table 3. The estimated values of the parameters and their reliability measures: departure model of the 1971-78 Dutch labor force.

Explanatory Variable	Estimated Parameter	Asymptotic Std. Error	t-ratio*
National Housing Increase	0.143	0.0017	8.42
National Job Opportunity	0.112	0.0069	1.60
Provincial Housing Increase	-0.019	0.0039	-0.48
Provincial Job Opportunity	0.055	0.0038	1.44
Groningen Dummy	0.145	0.0039	3.71
Utrecht Dummy	0.212	0.0038	5.57
Overijssel Dummy	-0.131	0.0031	-4.28
Constant Term	-3.332	0.0083	-40.07

*The asymptotic standard errors are multiplied by 8.305 (the square root of the weighted residual mean square) before they are used to compute the t-ratios. For test of significance, these ratios may be compared with $z = \pm 1.65$ which are the critical values of the standard normal variate at the significance level of $\alpha = 0.10$.

Table 4. The indices for evaluating the relative importance of the explanatory variables in terms of the intensity criterion: departure model of the 1971-78 Dutch labor force.

Explanatory Variable	Partial Derivative*	Elasticity*	Beta-weight
National Housing Increase	0.0074	0.371	0.083
National Job Opportunity	0.0058	0.033	0.016
Provincial Housing Increase	-0.0010	-0.018	-0.004
Provincial Job Opportunity	0.0028	0.051	0.018
Groningen Dummy	0.0075	0.012	0.042
Utrecht Dummy	0.0110	0.018	0.061
Overijssel Dummy	-0.0068	-0.011	-0.038

*The partial derivatives and elasticities are evaluated at the mean.

origin and destination and the size of employment at the destination are natural choices as explanatory variables. The former is represented by the physical distance between the gravity centers of two provinces divided by the average distance of all pairs of provinces; whereas the latter is represented by the ratio of the destination employment size to the origin employment size.

To check if the destination choice probabilities are influenced systematically by changes in the conditions of housing and job markets, we use two additional explanatory variables: "destination housing increase" expressed as the ratio of housing increase at the destination to housing increase at the origin, and "destination job opportunity" expressed as the ratio of origin unemployment rate to destination unemployment rate.

Three dummy variables are also used to account for persistent biases in the estimated destination choice probabilities. Dum1 is used to accommodate the strong preference of the outmigrants from Drenthe to Groningen presumably due to a heavy share of return migrants. Dum2 is used to account for the relatively strong preference for Gelderland among the outmigrants from the neighboring Overijssel and Utrect perhaps due to the availability of the newly created land which is included as part of Gelderland in our data base. Dum3 is used to account for the lower-than-expected preference for Zuid Holland among the outmigrants from the neighboring Zeeland due to the fact that the distance variable fails to reflect the additional transportation distance between the two provinces because of the intervening waters. The dependent variable (the observed destination choice proportions) is the annual number of migrants who moved from province i to province j divided by the annual number of total outmigrants from province i.

The input data matrix has 880 cases (8 periods times 11 origins times 10 destinations) and 13 variables (dependent variable, arbitrary weight, volume of migrants at origin, seven explanatory variables, year, origin index, and destination index). The last three variables are for showing temporal and spatial patterns in the plots. The file of control statements for

analyzing the data by BMDP3R is shown in Figure 5. It is essential to set the value of "pass" at 2. For detailed explanations, the reader is again referred to the BMDP manual.

The fit of the model is very good ($R^2 = 0.89$ and $\rho_1^2 = 0.90$). From the t-ratios in Table 5, we are quite certain that the migrants prefer nearby places with large employment. There is practically no evidence that destination choice probabilities are related to interprovincial difference in the housing increase. The t-ratio of -2.22 associated with the destination job opportunity suggests that some migrants prefer provinces with relatively poor job opportunity; for this result we do not have a good explanation, except that the relationship may be spurious because the provinces with relatively high unemployment tend to be those with more relatively attractive types of housing (e.g., single family dwelling units) and with better natural environments. Finally, we are reasonably sure that the destination choice probabilities are influenced by the underlying factors represented by the three dummy variables, because the corresponding t-ratios are quite large in magitude.

```
/problem   title is 'destination choice model:holland,1971-78'.
/input     variables are 13.
           format is '(13f10.0)'.
           cases are 380.
/variable  names are rfreq,wt,totmig,dist,employ,hincp,jobp,
                     dum31,dum4565,dum98,year,origin,dest.
/regress   dependent is rfreq.
           parameters are 7.  number is 20.
           weight is wt.      iterations are 10.
           halving is 0.      convergence is -1.
           meansquare is 1.   pass is 2.
/parameters initial are 7*0.0.
/plot      residual.
           variable=rfreq,dist,employ,hincp,jobp,dum31,
                    dum4565,dum98,year,origin,dest.
           size=45,40.
/end
```

Figure 5. The control statements to request BMDP3R to carry out the maximum likelihood estimation of the destination choice model of the Dutch labor force.

Table 5. The estimated values of the parameters and their reliability measures: destination choice model of the 1971-78 Dutch labor force

Explanatory Variable*	Estimated Parameter	Asymptotic Std. Error	t-ratio
Distance	-2.235	0.0043	-62.54
Destination Employment Size	0.344	0.0013	32.39
Destination Housing Increase	-0.001	0.0050	- 0.02
Destination Job Opportunity	-0.067	0.0036	- 2.22
Dum 1	1.122	0.0127	10.60
Dum 2	0.589	0.0058	12.13
Dum 3	-1.571	0.0162	-11.65

*Dum 1 represents the flow from Drenthe to Groningen.
Dum 2 represents the flow from Overijssel to Gelderland and from Utrecht to Gelderland.
Dum 3 represents the flow from Zeeland to Zuid-Holland.

The relative intensity of the response of the destination choice probability to the explanatory variables is shown in Table 6. Again, since the variables are not measured in comparable units, their relative importance will be judged in terms of elasticity and beta weight. Clearly, distance is by far the most important variable. The second important variable is the destination employment size. In terms of elasticity, the three dummy variables are less important than destination job opportunity; in terms of beta-weight, however, the opposite is true. The elasticity and beta weight of destination housing increase are practically zero.

From the methodological point of view, the most significant finding of our empirical example is that the departure probabilities are most strongly influenced by an unstable national variable (housing increase), whereas the destination choice probabilities are determined mainly by very stable regional variables (distance and destination employment size). It is through the use of the two-level logistic model that this kind of interesting contrast is revealed.

7. CONCLUSION

We have argued that the two-level logistic model is a useful and practical migration model that can be used to analyze macro as well as micro migration data. Since the applications of logistic models to macro data are often found to be rather unsatisfactory from the statistical point of view, we have made the model immediately useable for macro data. To increase the probability of other migration researchers using the same kind of model, we have (1) explained an appropriate estimation method that can be implemented by a program in the BMDP package, and (2) provided an empirical example to show the implementation of the estimation method and the interpretation of the statistical output.

We realize that in some situations the logistic model may be too simplistic or restrictive. However, it seems rather senseless

Table 6. The indices for evaluating the relative importance of the explanatory variables, according to the intensity criterion: destination choice model of the 1971-78 Dutch labor force.

Explanatory Variable	Partial Derivative*	Elasticity*	Beta-weight
Distance	-0.2011	-2.012	-1.027
Destination Employment Size	0.0310	0.555	0.702
Destination Housing Increase	-0.0001	-0.001	-0.000
Destination Job Opportunity	-0.0060	-0.067	-0.035
Dum 1	0.1010	0.009	0.107
Dum 2	0.0530	0.010	0.079
Dum 3	-0.1414	-0.013	-0.149

*The partial derivatives and elasticities are evaluated at the mean.

to combine a complex model with a primitive estimation model. Without an adequate statistical theory, a complex model of migration may produce results that are easily misinterpreted.

REFERENCES

Alonso, W. (1976) A Theory of Movements, in *International Perspective on Structure, Change, and Public Policy* edited by N. Hansen. Cambridge, Mass.: Ballinger.

Bartels, C.P.A., and K-L. Liaw (1981) *The Dynamics of Spatial Labor Mobility in the Netherlands*. Forthcoming Working Paper. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Cordey-Hayes, M., and D. Gleave (1973) Migration Movements and the Differential Growth of City Regions in England and Wales. *Papers of the Regional Science Association* 33:99-123.

DaVanzo, J. (1976) *Why Families Move: A Model of the Geographical Mobility of Married Couples*. R-1972-DOL. Santa Monica, California: Rand Corporation.

Da Vanzo, J. (1980) *Microeconomic Approaches to Studying Migration Decisions*. N-1201-NICHD. Santa Monica, California: Rand Corporation.

Dixon, W.J., and M.B. Brown, eds. (1977) *BMDP Biomedical Computer Programs*. Berkeley, California: University of California Press.

Ginsberg, R.B. (1972) Incorporating Causal Structure and Exogenous Information with Probabilistic Models: With Special Reference to Choice, Gravity, Migration, and Markov Chains. *Journal of Mathematical Sociology* 2:83-103.

Grant, E.K., and J. Vanderkamp (1976) *The Economic Causes and Effects of Migration: Canada, 1965-71*. Ottawa: Economic Council of Canada.

Jennrich, R.I., and H. Moore (1975) Maximum Likelihood Estimation by Means of Nonlinear Least Squares. Pages 57-65 in *Proceedings of Statistical Computing Section.* Washington, D.C.: American Statistical Association.

Jennrich, R.I., and M.L. Ralston (1979) Fitting Nonlinear Models to Data. *Annual Reviews of Biophysics and Bioengineering.* 8:195-239.

Ledent, J. (1980) On the Relationship Between Alonso's Theory of Movement and Wilson's Family of Spatial-Interaction Models. *Environment and Planning A*, 13:217-224.

Lowry, I.S. (1966) *Migration and Metropolitan Growth: Two Analytic Models.* San Francisco: Chandler Publishing Co.

McFadden, D. (1974) Conditional Logit Analysis of Qualitative Choice Behavior, in *Frontiers in Econometrics* edited by P. Zarembka. New York: Academic Press.

McFadden, D. (1976) Quantal Choice Analysis: A Survey. *Annals of Economic and Social Measurement* 5(4):363-390.

McFadden, D. (1979) Quantitative Methods for Analyzing Travel Behavior of Individuals: Some Recent Developments, in *Behavioral Travel Modeling* edited by D. Hensher and P. Stopher. London: Groom Helm.

Morrison, P.A. (1973) Theoretical Issues in the Design of Population Mobility Models. *Environment and Planning A*, 5:125-134.

Moss, W.G. (1979) A Note on Individual Choice Models of Migration. *Regional Science and Urban Economics* 9:1-11.

Rempel, H. (1980) Determinants of Rural-to-Urban Migration in Kenya. Pages 281-307 in *IIASA Reports* 2(2). Laxenburg, Austria: International Institute for Applied Systems Analysis.

Schultz, T.P. (1977) *A Conditional Logit Model of Internal Migration: Venezuelan Lifetime Migration with Educational Strata.* Discussion Paper 266. New Haven, Conn.: Yale University.

Theil, H. (1971) *Principles of Econometrics.* New York: John Wiley and Sons.

Wilson, A.G. (1971) A Family for Spatial Interaction Models and Associated Development. *Environment and Planning A*, 3:1-32.

PUBLICATIONS IN THE MANPOWER ANALYSIS SERIES

1. Cornelis P.A. Bartels, *Diminishing Qualitative Discrepancies in Regional Labor Markets: A Discussion of Some Policy Options.* WP-81-27.

2. Klaus Neusser, *Fertility and Female Labor-Force Participation: Estimates and Projections for Austrian Women Aged 20-30.*

3. Jesse Abraham, *Earnings Growth without Investment.* WP-81-56.

4. Cornelis P.A. Bartels, William R. Nicol, and Jaap J. van Duijn, *Estimating Impacts of Regional Policies: A Review of Applied Research Methods.* WP-81-59.

5. Cornelis P.A. Bartels and Jaap J. van Duijn, *Implementing Regional Economic Policy: An Analysis of Economic and Political Influences in the Netherlands.* WP-81-61.

6. Cornelis P.A. Bartels and Jaap J. van Duijn, *Regional Economic Policy in a Changed Labor Market.* WP-81-64.

7. Ilkka Leveelahti, *A Time Series Analysis of Regional Migration in Finland.* Forthcoming Working Paper.