ABATEMENT OF AIR POLLUTANTS AND COGENERATION: SEARCH FOR AN OPTIMAL SOLUTION

M. Posch
E. Runca

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

The goal of air quality management is to minimize exposure of man and environment to pollutants released in the atmosphere. Although it is generally possible to reduce pollution below a level no longer detrimental, there are economic constraints to be met.

This study discusses in view of achieving a given air quality minimum costs solutions in relation to: a) abatement of air pollutants from a thermal power plant and b) the adoption of a centralized heating system to reduce pollution in a city. It has been done as a part of IIASA research activities on management of environmental resources and quality, with the support of ICSAR funds.

ABSTRACT

In this paper atmospheric diffusion modelling and nonlinear optimization techniques are used for the analysis of minimum cost alternatives of air pollution control strategies. Two cases are considered: a) control of air pollution from a large point source and b) reduction of existing pollution levels in an urban area utilizing the heat cogenerated by a thermal power plant for district heating.

As to a) a program has been built to compute the minimum cost function for the chosen abatement techniques (including stack height) under the constraint of keeping the ground level concentration of N pollutants (gaseous or particulates) at specified values.

Cost functions for stack height and abatement techniques are input to the program. As an example, results are presented for the control of two different pollutants controlled by two abatement techniques plus stack height.

As to b) an interactive program has been developed to identify minimum cost network for heat conveyance necessary to supply a set of residential areas to achieve a given reduction of pollution in the urban area. Results are presented for the city of Vienna.

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INTRODUCTION

Every human activity involves the chemical and physical transformation of materials, thus generating a certain amount of residuals which cannot be economically reused and which must therefore be disposed in the environment. The residuals released into the atmosphere are transported, transformed and accumulated through complex meteorological, physical and chemical processes, which result in temporal and spatial patterns of ambient concentrations. These concentration patterns represent a nuisance, or cause a damage, either to man and his property or to some important ecological subsystem. Formally a damage function relates any level of pollution concentration with the corresponding damage it would produce.

To reduce the damage caused by air pollution a control strategy must be adopted. Let us call $\Gamma_I(C_1,C_2,\ldots,C_N)$ the minimum cost associated with control strategy I to keep the pollution caused by N residuals at their respective concentration levels $C_1,C_2,\ldots,C_N$, and let us assume that we can identify

$$\Gamma(C_1,C_2,\ldots,C_N) = \min_I \Gamma_I(C_1,C_2,\ldots,C_N) \quad ,$$

where I ranges over all the possible control strategies. If we could estimate the cost damage function $\Lambda(C_1, C_2, \ldots, C_N)$ associated with the set $\{C_1, C_2, \ldots, C_N\}$ then the concentrations $\{C_1^{opt}, C_2^{opt}, C_N^{opt}\}$ minimizing the sum of Γ and Λ yields the maximum benefit to man's well-being. This is true under the implicit assumption that Λ and Γ represent respectively the total loss of man's well-being and the minimum pollution control cost (see, e.g., Guldmann and Shefer, 1980).

In practice, pollution control cannot be based on maximum benefit to man's well-being because Γ and Λ are unknown. It is generally based on maximum concentration values which must not be exceeded. These standards are necessarily defined with some degrees of arbitrariness since the complete spectrum of the effects caused by a given pollutant or set of pollutants is generally not known. In order to account for the dependency of effects on the duration of exposure to a given concentration level, standards are given for exposure periods of different length. They are generally defined for short-term (30 minutes - 24 hour) and long-term average concentrations (1 month - 1 year), and, to account for synergistic effects, are mutually constrained for given pollutants (see, e.g., Schedling and Baumann, 1975).

Once the standards are defined, the goal of the related environmental policies becomes the attainment of the chosen standards. This can be achieved by different manners such as regulations, taxes, incentives, etc., (see, e.g., Downing, 1971), all leading to the adoption of control strategies, which, in principle, are required to operate at minimum cost. Recalling the above notation, this can be deduced from the knowledge of:

$$\Gamma_I(C_1, C_2, \ldots, C_N) = \min_{x_1, x_2, \ldots, x_L} \gamma_I(x_1\ x_2, \ldots, x_L) \quad ,$$

where $\{x_1, x_2, \ldots, x_L\}$ is the set of parameters from which the I-th control strategy depends and γ_I is the cost associated with every admissible set $\{x_1, x_2, \ldots, x_L\}$. To illustrate this point let us consider a set of pollutants of only two elements, in addition Γ_I be a monotonic decreasing function (in each argument) for increasing values of its arguments as shown in Figure 1.

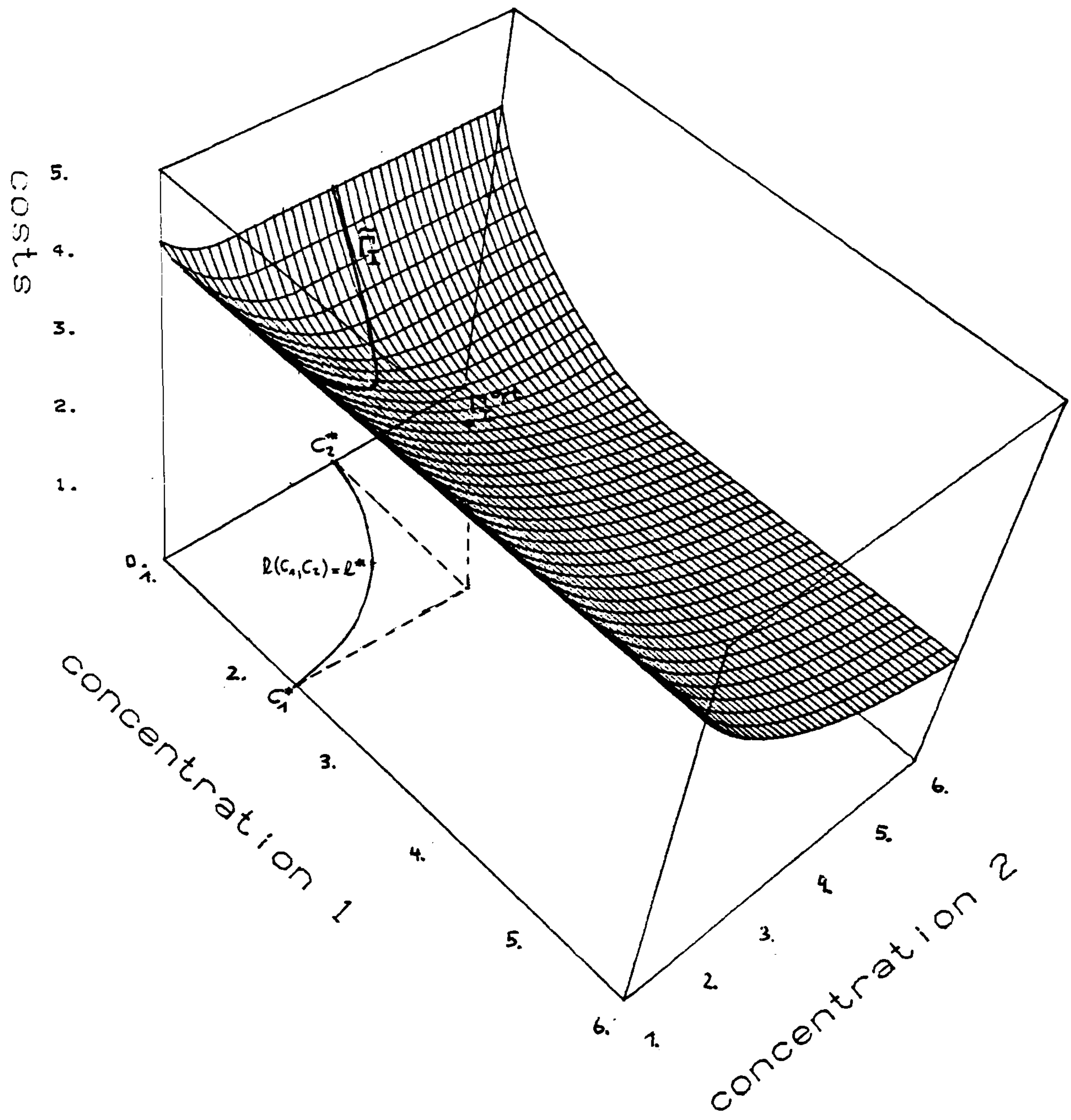

Figure 1. Hypothetical Minimum Cost Function Associated with the I-th Control Strategy for Two Pollutants (units in the figure are arbitrary for both concentration and cost)

For pollutants having independent effects the condition to be verified is:

$$C_1 \leq C_2^*$$
$$C_2 \leq C_2^* \quad ,$$

where C^* indicate a standard value. If, as assumed, Γ_I is monotonically decreasing, the solution of minimum cost is (see Figure 1):

$$\Gamma_I^{opt} = \Gamma_I(C_1^*, C_2^*) \quad .$$

For pollutants having synergistic effects, the above inequalities must be replaced by

$$C_1 \leq C_2^*$$
$$C_2 \leq C_2^*$$
$$\ell(C_1, C_2) \leq \ell^* \quad ,$$

where $\ell(C_1, C_2)$ accounts for the combined action of the two pollutants. The optimal solution now lies on the curve $\tilde{\Gamma}_I$ corresponding to the function $\ell(C_1, C_2)$ as illustrated for a hypothetical case in Fgiure 1.

In this study, first, in relation to the installation of a thermal power plant in a given urban-industrial area, we will analyze the minimum cost solutions of combined abatement techniques and stack height ensuring that pollution caused by the power plant does not exceed a prefixed percent increase of the ambient air pollution already existing in the area. Second, in relation to the adoption of a centralized heating system, built in order to reduce the pollution in the urban area, we will examine the minimum cost networks of heat conveyance necessary to supply a number of city districts chosen in such a way that a preestablished percent reduction of the existing pollution is achieved.

In this study, standards are not defined in an absolute way. They are themselves parameters of the policy, chosen in terms of either increase or reduction of the actual concentration in the controlled area. The present work has been done with reference to the area of Vienna where both installation of a new coal-fired power plant and centralized heating are planned by the local authority. The study has been focused on the effects of these control strategies on air pollutants concentration averaged over the heating period.

In order to identify optimal solutions for the two strategies, costs of abatement, stack and district heating must be specified. The latter, has been assumed proportional to the length and location of the steam pipeline, the formers have been considered to be known functions of the efficiency of the abatement technique (that is, the rates of the output and input flow of pollution) and of stack height, respectively.

Before proceeding to the formulation and application of the mathematical optimization programs associated with the considered control strategies, the pollutants concentration must be related to the emitted quantities through modelling of the atmospheric system. This is done in the next section.

THE ATMOSPHERIC SYSTEM

The rate of diffusion of pollutants depends on the intensity of atmospheric turbulence. This is strongly influenced by the rate of decrease of temperature with height, the so-called "temperature lapse rate". The reference rate is the adiabatic lapse rate which corresponds to a hydrostatically neutral atmosphere (no buoyancy forces act on the air parcels). When the temperature gradient is less than the adiabatic lapse rate, the atmosphere is unstable (buoyancy forces enhances the motion of air parcels). On the contrary when the temperature gradient is greater than the adiabatic lapse rate, the atmosphere is stable (buoyancy forces reduces the motion of air parcels). This situation is referred to as "inversion". It can be either ground based or occur at a given height called "mixing height" because diffusion is suppressed at the inversion height.

Atmospheric turbulence is also strongly influenced by the wind profile. In contrast to the temperature profile, wind profile is always a source of energy for the turbulent motion of air parcels (see, e.g., Dobbins, 1979). Drawing on these concepts, Pasquill and Gifford (see Pasquill, 1974; Gifford, 1961, 1976) proposed to classify atmospheric turbulence into six classes in terms of wind speed, insolation and cloudiness (the latter two parameters being an indirect estimation of the vertical thermal structure); and gave for each class plots of downwind growth of the crosswind and vertical standard deviations of the distribution of matter in a pollutant cloud. This approach provided the basis for the computation of the downwind concentration in many studies; we will also make use of it as specified later.

As the pollutants leave the stack they generally undergo an upward motion, called plume rise, caused by both initial ejection speed and thermal differences between the plume and the ambient atmosphere. The plume rise, bringing the pollutants to the upper layers of the lower atmosphere is effective in reducing local pollution (however, together with high stack contributes to transform pollution from a local to a regional problem) and must therefore be taken into account in the computation of downwind concentrations. Neglecting the turbulence induced by ejection velocity and temperature during the initial stage of the plume, their effects on the downwind concentrations can be accounted for by increasing the geometric height h of the stack of the quantity Δh due to the plume rise. In other words, it is assumed that the computation is done as if the release occurred at $h_e = h + \Delta h$ (h_e: effective stack height) in thermodynamic equilibrium with the ambient atmosphere. Following Briggs (1971, 1975), Δh has been computed in this study by the equations given in Appendix A.

Due to the inherent stochastic nature of turbulence, the concentration is a stochastic quantity of which only statistics can be estimated. Mostly we are interested in the evaluation of its ensemble average, which is an approximation to the time average recorded in monitoring stations (see, Venkatram, 1981).

If $P(\underset{\sim}{x}, t|\underset{\sim}{x}_s,0)$ is the probability that a pollutant parcel be in the volume $\underset{\sim}{x} \pm \Delta\underset{\sim}{x}/2$ after a time t from being released at $\underset{\sim}{x}_s$, the ensemble average concentration is given by:

$$\langle C(\underset{\sim}{x},t)\rangle = Q_I \lim_{\Delta\underset{\sim}{x}\to 0} \frac{P(\underset{\sim}{x},t|\underset{\sim}{x}_s,0)}{\Delta\underset{\sim}{x}} \quad , \qquad (1)$$

where $\langle\cdot\rangle$ denotes ensemble average and Q_I is the pollutant quantity released instantaneously at $\underset{\sim}{x}_s$ at time $t = 0$. In other words, for an instantaneous point source, the ensemble average concentration is obtained by multiplying the quantity released with the probability density

$$G(\underset{\sim}{x},t|\underset{\sim}{x}_s,0) = \lim_{\Delta\underset{\sim}{x}\to 0} \frac{P(\underset{\sim}{x},t|\underset{\sim}{x}_s,0)}{\Delta\underset{\sim}{x}} \quad . \qquad (2)$$

For a continuous point source of rate Q we get by overimposing the effects:

$$\langle C(\underset{\sim}{x},t)\rangle = \int_0^t G(\underset{\sim}{x},t|\underset{\sim}{x}_s,t')Q(t')dt' \quad . \qquad (3)$$

The evaluation of $G(\underset{\sim}{x},t|\underset{\sim}{x}_s,t')$ is the fundamental issue in modelling of air pollutants diffusion (see, e.g., Runca et al., 1981).

For the purpose of this study let us assume that horizontal diffusion is negligible with respect to wind advection. Then by taking the reference frame with the x-axis along the wind direction and making the additional assumption that the wind velocity can be approximated by a uniform value $\bar{u}$ (turbulence homogeneous) we can write the probability density function in the form:

$$G(\underset{\sim}{x},t|\underset{\sim}{x}_s,t') = G_{yz}(y,z,t|y_s,h_e,t')\,\delta(x-x_s-\bar{u}(t-t')) \quad , \qquad (4)$$

where $\delta(\cdot)$ is the Dirac's function and $G_{yz}(y,z,t|y_s,h_e,t')$ is the probability density that a particle released from (y_s,h_e) at t' will be in (y,z) at t, moving in a plane perpendicular to the wind direction.

Using (4) in (3) and replacing the integration variable t' by

$$\xi = \bar{u}(t-t') \quad ,$$

we get

$$\langle C(x,y,z,t)\rangle = \frac{1}{\bar{u}} \int_0^{\bar{u}t} G_{yz}(y,z,t|y_s,h_e,t - \frac{\xi}{\bar{u}})\,\delta(x-x_s-\xi)\,Q(t-\frac{\xi}{\bar{u}})\,d\xi \tag{5}$$

Recalling that

$$\int_a^b f(\eta)\delta(\eta-\eta_o)\,d\eta = \begin{cases} f(\eta_o) & a \leq \eta_o \leq b \\ 0 & \text{else} \end{cases}$$

(5) gives:

$$\langle C(x,y,z,t)\rangle = \frac{Q(t-(x-x_s)/\bar{u})}{\bar{u}} G_{yz}(y,z,t|y_s,h_e,t-\frac{(x-x_s)}{\bar{u}}); \quad \text{for } (x-x_s) \leq \bar{u}t \tag{6a}$$

$$\langle C(x,y,z,t)\rangle = 0 \qquad \text{for } (x-x_s) > \bar{u}t \tag{6b}$$

Assuming that the stochastic process is stationary the probability density $G_{yz}(\cdot)$ depends only on the time lapse $(x-x_s)/\bar{u}$. Furthermore since in our idealization the pollutant particles are rigidly moving in the wind direction, we recognize that $G_{yz}(\cdot)$ depends only on the distance $(x-x_s)$, that is the probability that a particle will be in (y,z) after a time $(x-x_s)/\bar{u}$ is equal to the probability that the particle will be in (y,z) after travelling a distance $(x-x_s)$. Thus, with the additional assumptions that crosswind and vertical diffusion are independent and Q is constant, (6) takes the simplified form:

$$\langle C(x,y,z,t)\rangle = \frac{Q}{\bar{u}} G_y(y,x-x_s|y_s)\,G_z(z,x-x_s|h_e), \quad (x-x_s) \leq \bar{u}t \tag{7a}$$

$$\langle C(x,y,z,t)\rangle = 0 \qquad (x-x_s) \geq \bar{u}t \tag{7b}$$

We now consider the application of (7) to the computation of the average concentration at (x,y,z) over a long period of length T. To proceed in this computation we divide T in intervals of length T_i, such that each interval represents the total duration of the i-th meteorological condition. With this definition we can write:

$$\bar{C}(x,y,z) = \sum_i \frac{\langle C\rangle_i T_i}{T} \quad , \qquad (8)$$

where $\bar{C}(x,y,z)$ is the average concentration over T, $\langle C\rangle_i$ is the ensemble average given by (7) occurring with the i-th meteorological condition and $\frac{T_i}{T}$ is the probability of occurrence of the i-th meteorological condition during $T\,(\sum_i (T_i/T) = 1)$.

The probabilities $\frac{T_i}{T}$ can be easily computed by standard routinely measured meteorological data once a suitable division in classes has been defined for the relevant meteorological parameters. The classification adopted in this study will be presented later. The main difficulties in the application of (8) is the computation of $\langle C\rangle_i$. In principle we need to know $[G_y(\cdot)]_i$ and $[G_z(\cdot)]_i$ for every meteorological condition.

Noting that the wind direction can be taken uniformly distributed in each sector of the wind rose over a sufficiently long period of time, we deduce that $[G_y]_i$ is a uniform distribution independent of the i-th meteorological condition. If N_d is the number of sectors of the wind rose, it follows:

$$G_y(y,x-x_s|y_s) = \frac{N_d}{2\pi(x-x_s)} \quad \text{for} \quad -\frac{\pi}{N_d}(x-x_s) \le y \le \frac{\pi}{N_d}(x-x_s) \qquad (9)$$

otherwise

$$G_y(y,x-x_s|y_s) = 0 \quad .$$

In writing (9) it has been taken that $\tan(\pi/N_d) \sim \pi/N_d$.

The derivation of (7) has been done under the assumptions of turbulence--homogeneous and stationary. Under these assumptions $[G_z(\cdot)]_i$ can be taken as Gaussian (Monin and Yaglom, 1971).

For an unbounded atmosphere it has the form:

$$[G_z(z,x-x_s|h_e)]_i = \frac{1}{\sqrt{2\pi}\ \sigma_{z,i}} \exp\{-\frac{(z-h-\Delta h_i)^2}{2\ \sigma^2_{z,i}}\} \tag{10}$$

In (10), the standard deviation of the distribution $\sigma_{z,i}$ and the plume rise Δh_i, both depend on the intensity of the turbulence associated with the i-th meteorological situation, in addition Δh_i depends also on the ambient air temeperature attributed to the i-th meteorological condition and σ_{zi} depends on the travelled distance $(x-x_s)$.

In reality vertical diffusion is limited below by the ground and in some meteorological conditions above by an elevated inversion. If ground and inversion base act as perfect reflectors of the diffusing matter then $[G_z]_i$ can be easily deduced by adding all the contributions of the infinite number of image sources generated by the two mirrors: ground and inversion base. Then, the form of $[G_z(\cdot)]_i$, calling H the mixing height, becomes:

$$[G_z(z,x-x_s|h_e)]_i = \frac{1}{\sqrt{2\pi}\sigma_{z,i}} \sum_{n=-\infty}^{+\infty} \{\exp[-\frac{(z-h-\Delta h_i+2nH)^2}{2\sigma^2_{z,i}}] + \exp[-\frac{(z+h+\Delta h_i+2nH)^2}{2\sigma^2_{z,i}}]\} \ . \tag{11}$$

Equation (11) has been deduced for a gaseous pollutant. For the simulation of dispersion of particulates or droplets with significant gravitational settling velocities, which will be also considered in this study, (11) must be modified. Following Dumbauld and Bjorklund (1975) gravitational settling is assumed to result in a tilted plume with the plume axis inclined to the horizontal at an angle given by arctan $\frac{V_s}{\bar{u}}$, where V_s is the gravitational settling velocity. With the additional assumption that only a fraction β of the material reaching the ground is reflected from the surface, (11) is transformed to:

$$[P_z(z,x-x_s|h_e)]_i = \frac{1}{\sqrt{2\pi}\sigma_{z,i}} \sum_{n=o}^{\infty} \{\beta^n \exp[-\frac{(z-h-\Delta h_i+\frac{V_s}{\bar{u}_i}(x-x_s)+2nH)^2}{2\sigma_{z,i}^2}$$

$$+ \beta^{n+1} \exp[-\frac{(z+h+\Delta h_i-\frac{V_s}{\bar{u}_i}(x-x_s)+2nH)^2}{2\sigma_{z,i}^2}]\} +$$

$$+ \sum_{n=1}^{\infty} \{\beta^n \exp[-\frac{(z-h-\Delta h_i+\frac{V_s}{\bar{u}_i}(x-x_s)-2nH)^2}{2\sigma_{z,i}^2}]+$$

$$+ \beta^{n-1} \exp[-\frac{(z+h+\Delta h_i-\frac{V_s}{\bar{u}_i}(x-x_s)-2nH)^2}{2\sigma_{z,i}^2}]\},$$

(12)

where β is the reflection coefficient for the particulates (β=0: no reflection; β=1: complete reflection).

If the composition of the particulates emitted by a given source covers a too wide range of settling velocities, the emitted mass can be divided into N fractions ϕ_n ($\sum_{n=1}^{N}\phi_n = 1$) with respective reflection coefficients β_n and settling velocities V_{sn} (n = 1,2,...,N). The vertical probability density function is then the weighted sum of the probability density of each category, that is:

$$[P_z(\cdot)]_i = \sum_{n=1}^{N} \phi_n\{[P_z(\cdot)]_i\}_n \qquad (13)$$

where $\{[P_z(\cdot)]_i\}_n$ is given for the n-th category by (12).

The quantities V_s, β and ϕ in the above equations are not independent variables. The settling velocity can be computed from the particles mass-mean diameter d using Stokes' law:

$$V_s = \frac{g\rho}{18\mu} d^2, \qquad \text{for } d \leq 80\ \mu m$$

where g is the gravity acceleration (980 cm/s^2); μ is the absolute air viscosity (~ $1.83.10^{-4}$ g/cm.s); ρ is the particle density (g/cm^3) and d is the mass-mean diameter given by

$$d = \left(\frac{d_1{}^3 + d_1^2 d_2 + d_1 d_2^2 + d_2^3}{4}\right)^{1/3}$$

where d_1 and d_2 are the lower and upper bounds for the given particle size category.

The mass fraction ϕ depends on the particle diameter. In this study ϕ has been computed on the assumption that size distribution of the emitted particulates is lognormal (see, NATO-CCMS, Vol.). For the relationship between the gravitational settling velocities and the reflection coefficient we adopted the suggestion by Dumbauld et al. (1976).

In order to apply (8) with the specified G_y and G_z or P_z, the probability of occurrence of a given meteorological condition has to be computed. Following Runca et al., (1976) wind direction, wind speed, atmospheric stability and temperature were divided into N_d, N_w, N_s, and N_t classes, respectively. These classes were used to build the frequency matrix of occurrence of a particular set of the chosen parameters, over the considered period. By normalizing over all the observations the frequency matrix is transformed to a joint probability matrix F, for which the sum over all the elements must obviously be equal to unit, that is:

$$\sum_{id=1}^{N_d} \sum_{iw=1}^{N_w} \sum_{is=1}^{N_s} \sum_{it=1}^{N_t} F(id, iw, is, it) = 1 \quad .$$

Making use of this matrix and assuming for simplicity that the source is located at the origin of the reference frame the concentration along the center line of the id-th wind sector has been computed by the following approximation to (8):

$$[\bar{C}(x,o,z)]_{id} = \frac{QN_d}{2\pi x} \sum_{iw=1}^{N_w} \sum_{is=1}^{N_s} \sum_{it=1}^{N_t} \frac{F(id,iw,is,it)}{\bar{u}(id,iw,is,it)} [G_z(z,x|h_e)]_{is,it} \qquad (14)$$

in which $[G_z(\cdot)]_{is,it}$ is replaced by $[P_z(\cdot)]_{is,it}$ for particulates.

For points not falling in the center line, the concentration value has been determined by interpolating along the arc of radius x between the two adjacent sectors.

In the application of equation (14) done in the next section the following classes have been chosen:

wind direction	: $N_d = 8$	
wind speed (m/s)	: $N_w = 6$	$0<\bar{u}_1<1.57$; $1.57<\bar{u}_2<3.14$; $3.14<\bar{u}_3<5.24$; $5.24<\bar{u}_4<8.38$; $8.38<\bar{u}_5<11.0$; $11.0<\bar{u}_6$
atmospheric stability	: $N_s = 6$	1 = very unstable; ...; 6 = very stable
ambient air temperature (°C)	: $N_t = 4$	$T_{a,1}<0$; $0<T_{a,2}<10$; $10<T_{a,3}<20$; $T_{a,4}>20$

MINIMUM COST SOLUTIONS

Abatement and Stack Height

Following the notations given in the introduction let us assume that we have chosen a certain control strategy I; depending on L control parameters $x_i (i = 1,2,...,L)$, with which concentration patterns of N pollutants can be influenced. If γ_I denotes the total cost due to a given set of the L parameters $x_1, x_2, ..., x_L$, the general optimization problem can be stated as follows (for the sake of simplicity we omit from now on the subscript I):

$$\min_{x_1,...,x_L} \gamma(x_1,...,x_L) \quad , \tag{15}$$

subject to

$$\begin{aligned} &C_j(x,y;x_1,...,x_L) \leq C_j^* \qquad \text{for all } (x,y)\varepsilon A \\ &x_i^{min} \leq x_i \leq x_i^{max} \quad , \quad \text{and } j = 1,2,...,N \quad , \end{aligned} \tag{16}$$

where A is the geographical area under consideration and C_j^* is the standard for the j-th pollutant (at ground level).

To become able to tackle this constrained non-linear optimization problem we will make the following simplifications:

(i) The overall cost function is separable, i.e., the total costs are the sum of the costs for each x_i;

$$\gamma(x_1,\ldots,x_L) = \sum_{i=1}^{L} K_i(x_i) \quad , \tag{17}$$

(ii) The set of control parameters $x_1,\ldots,x_L$ can be divided into N+1 groups:

$$\underline{x}^{(1)} = (x_1,\ldots,x_{n_1}{}^{(1)}) \text{ affect only concentration } C_1$$
$$\vdots$$
$$\underline{x}^{(N)} = (x_1^{(N)},\ldots,x_{n_N}^{(N)}) \text{ affect only concentration } C_N \quad ,$$

and finally $\underline{x}^{(0)} = (x_1{}^{(0)}\ldots x_{n_0}^{(0)})$ affect all concentrations like for example the stack height of a plant. To differentiate it from the other groups and due to the reported example we use the following notation $\underline{x}^{(0)} = \underline{h}$. Note that

$$\sum_{k=o}^{N} n_k = L \quad .$$

To proceed further we assume the principle of independency, that means if C_j is the initial concentration and $x_1{}^{(j)},\ldots,x_n{}^{(j)}$ are the (normed to unity) control parameters (efficiencies) of the applied abatement techniques, then the concentration is given by

$$C_j(x,y,\underline{x}^{(j)},\underline{h}) = C_j(x,y,\underline{h}) \prod_{k=1}^{n_j} (1-x_k{}^{(j)}) \tag{18}$$

$$j = 1,\ldots N$$

The optimization problem now reads:

$$\min_{\underline{x}^{(1)},\ldots,\underline{x}^{(N)},\underline{h}} \left\{ \sum_{j=1}^{N} \sum_{k=1}^{n_j} K_{kj}(x_k{}^{(j)}) + \sum_{k=1}^{n_o} K_k(h_k) \right\} \tag{19}$$

subject to

$$C_j(x,y;\underline{h}) \prod_{k=1}^{n_j} (1-x_k^{(j)}) \leq C_j^* \tag{20a}$$

for all $(x,y)\varepsilon A$ and $j=1,\ldots,N$

$$0 \leq \underline{x}^{(j)}_{min} \leq \underline{x}^{(j)} \leq \underline{x}^{(j)}_{max} \leq 1 \tag{20b}$$

$$\underline{h}_{min} \leq \underline{h} \leq \underline{h}_{max} \quad . \tag{20c}$$

The dependency of the concentration-functions on the parameters $h_1,\ldots,h_{n_o}$ might be very complicated and even not differentiable. Therefore, we will proceed in two steps:

(1) We keep the parameter $\underline{h}$ constant; then the optimization problem (19)-(20) splits in N subproblems:

$$\min_{\underline{x}^{(j)}} \sum_{k=1}^{n_j} K_{kj}(x_k^{(j)}) \tag{21}$$

subject to

$$\prod_{k=1}^{n_j} (1-x_k^{(j)}) \leq B_j(\underline{h}) \quad , \tag{22a}$$

$$\underline{x}^{(j)}_{min} \leq \underline{x}^{(j)} \leq \underline{x}^{(j)}_{max} \quad , \tag{22b}$$

where

$$B_j(\underline{h}) = \min_{(x,y)\varepsilon A} \frac{C_j^*}{C_j(x,y;\underline{h})} \quad . \tag{23}$$

(2) with a sequence of N sub-optimal solutions $\underline{x}^{(j)}(\underline{h})$ $(j=1,\ldots,N)$ from step (1) we compute by a suitable search algorithm the minimum of the function:

$$Z(\underline{h}) = \sum_{j=1}^{N} \sum_{k=1}^{n_j} K_{kj}(x_k^{(j)}(\underline{h})) + \sum_{k=1}^{n_o} K_k(h_k) \quad . \tag{24}$$

To be able to perform this in a reasonable amount of computer time, it is necessary to use a fast algorithm for solving the subproblem (21)-(22). We have proceeded as explained in Appendix B.

To illustrate results achievable by solving (20)-(21), an application has been done to the case of two pollutants, one gaseous and the other composed of particulates, released by the same stack. Consistently with the above formulation it was assumed that the emission rate of the two pollutants could be controlled independently by two adequate abatement techniques.

Concentration at the ground for the two pollutants were provided by (14) with the joint probability frequency matrix F(id, iw, is, it) computed by the meteorological data recorded in Vienna for the period October 1977 - April 1978. Information for the computation of the plume rise with the equation of Appendix A were taken from a thermal power plant operating in the area. The particulate emission was assumed to be composed of three fractions in the ratio 1:1.6:0.5 having average diameters (in μm) 3.39, 7.77 and 33.9, respectively. The emission rate was taken for both pollutants equal to (1000 g/s).

The results reported below were achieved by assuming that both costs of stack and abatement techniques were growing with the square of the stack height and abatement techniques efficiencies respectively. The ratio of the cost increase relative to an increase of the stack height of 20 meters and of the efficiency of 0.1 was taken to be 1:1.2:3 respectively for stack, abatement of the gaseous pollutant and abatement of the particulate matter.

The minimum cost Γ_I (see the notation adopted in the introduction) is displayed in Figure 2. Values of the ground concentration (averaged over the period October-March) for both pollutants are given in micrograms/m^3.

The optimal efficiencies and stack height corresponding to the minimum cost function Γ_I displayed in Figure 2 are reported in Figures 3, 4 and 5, respectively.

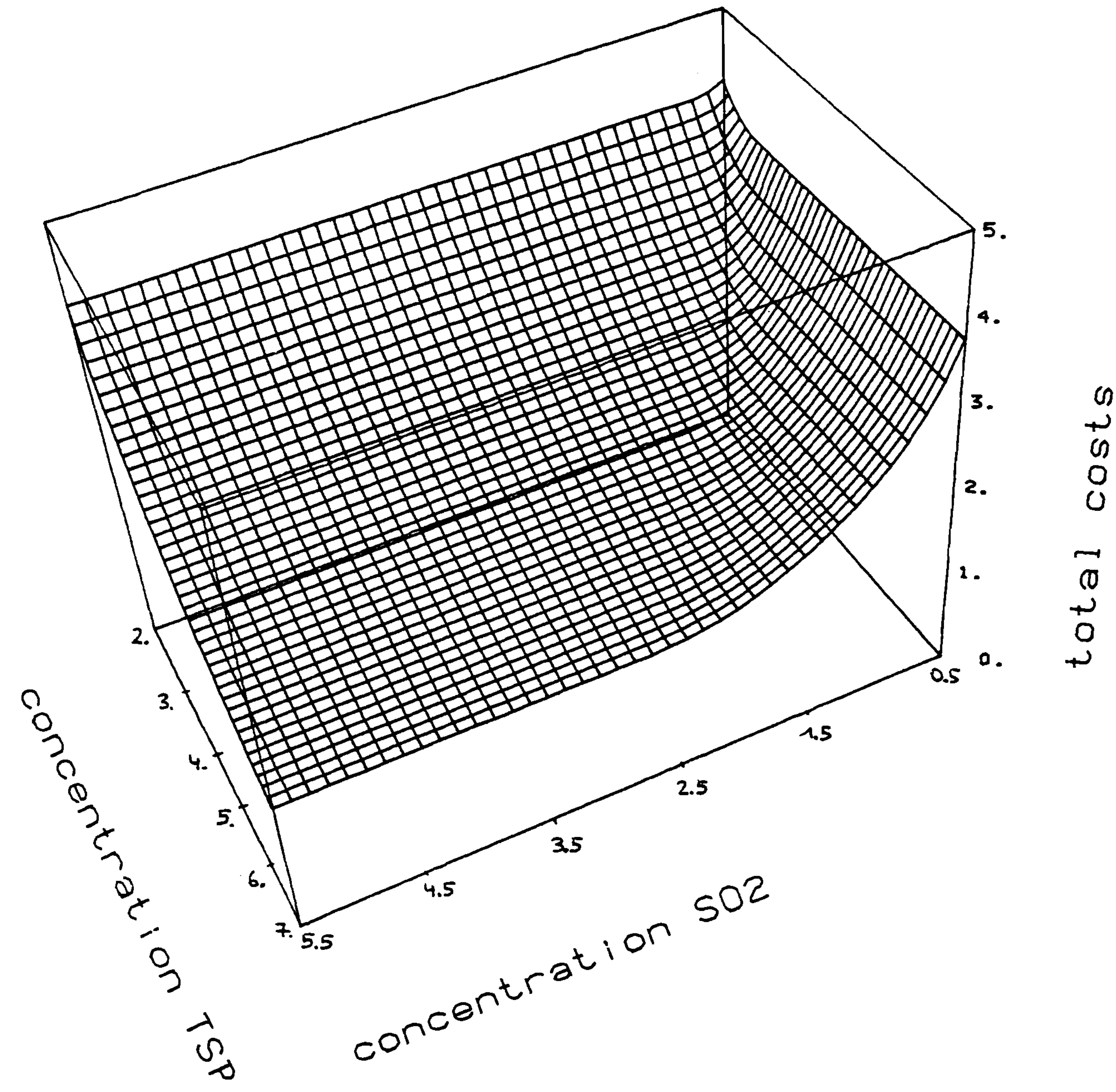

Figure 2. Minimum cost function Γ_I (in arbitrary unit) for the considered case (see text)

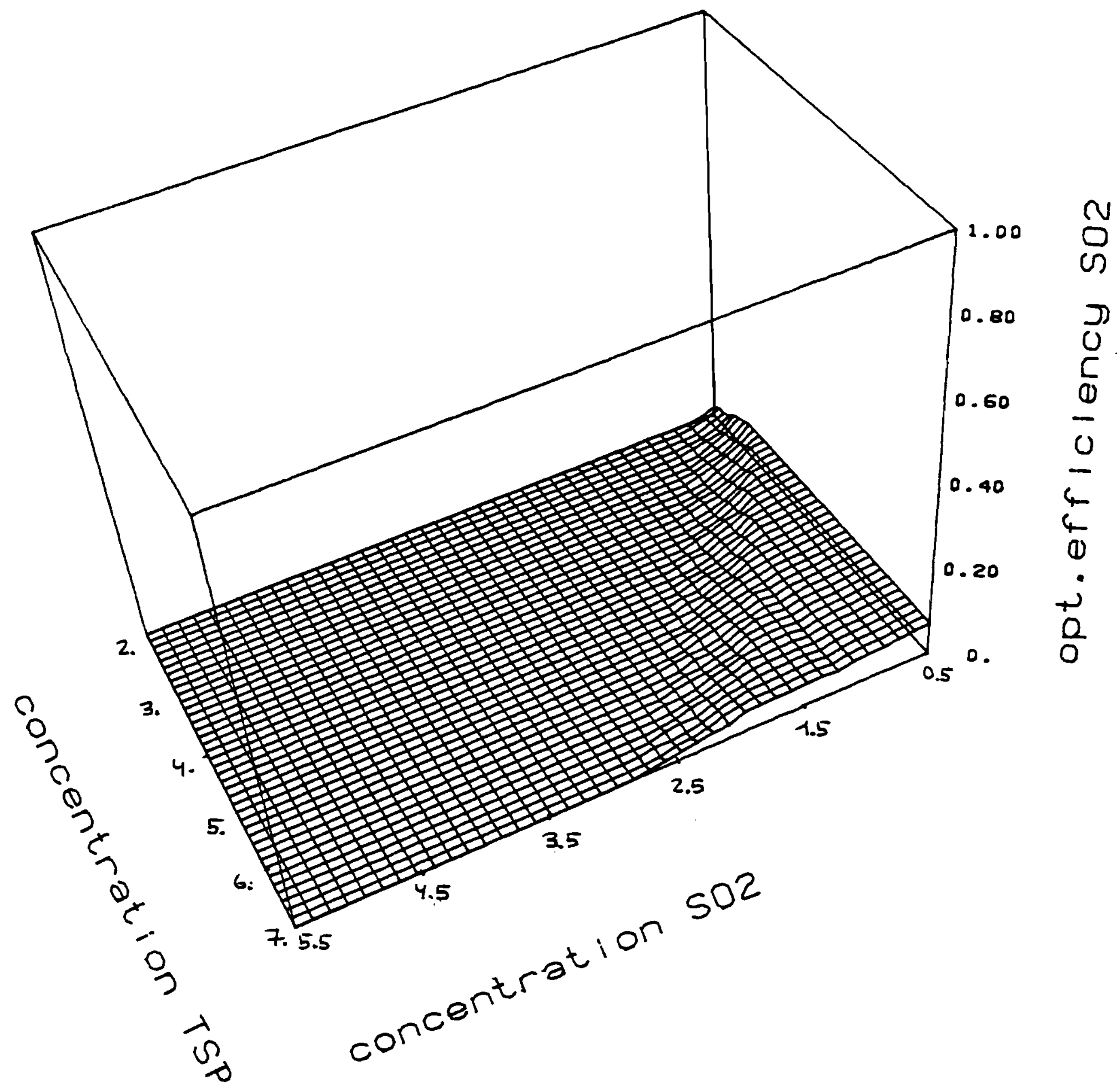

Figure 3. Optimal percent reduction (efficiency of the abatement technique) of the emission rate of the gaseous pollutant, assumed to be sulfur dioxide.
TSP: total suspended particle.

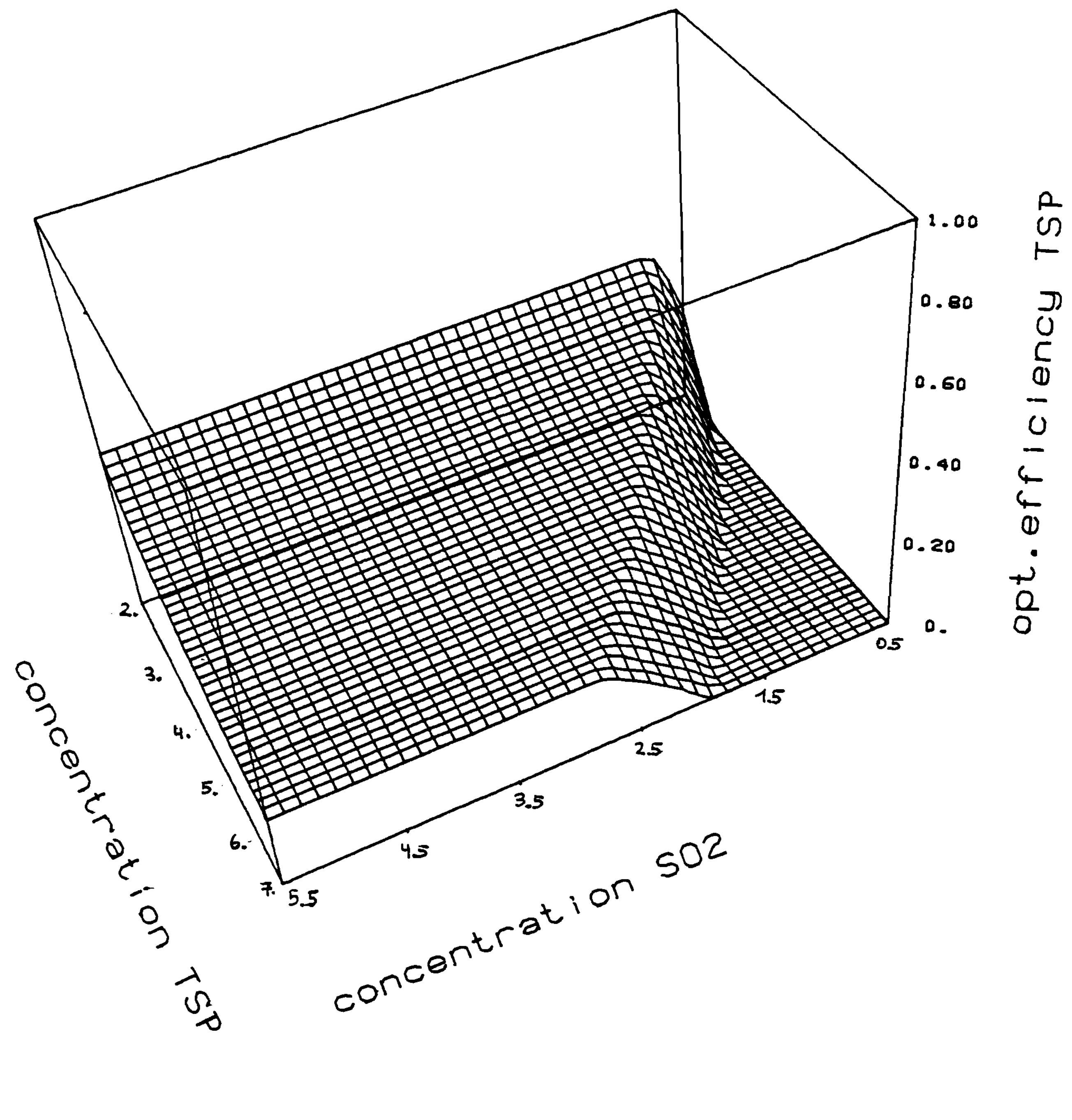

Figure 4. Optimal percent reduction (efficiency of the abatement technique) of the emission rate of the particulate matter

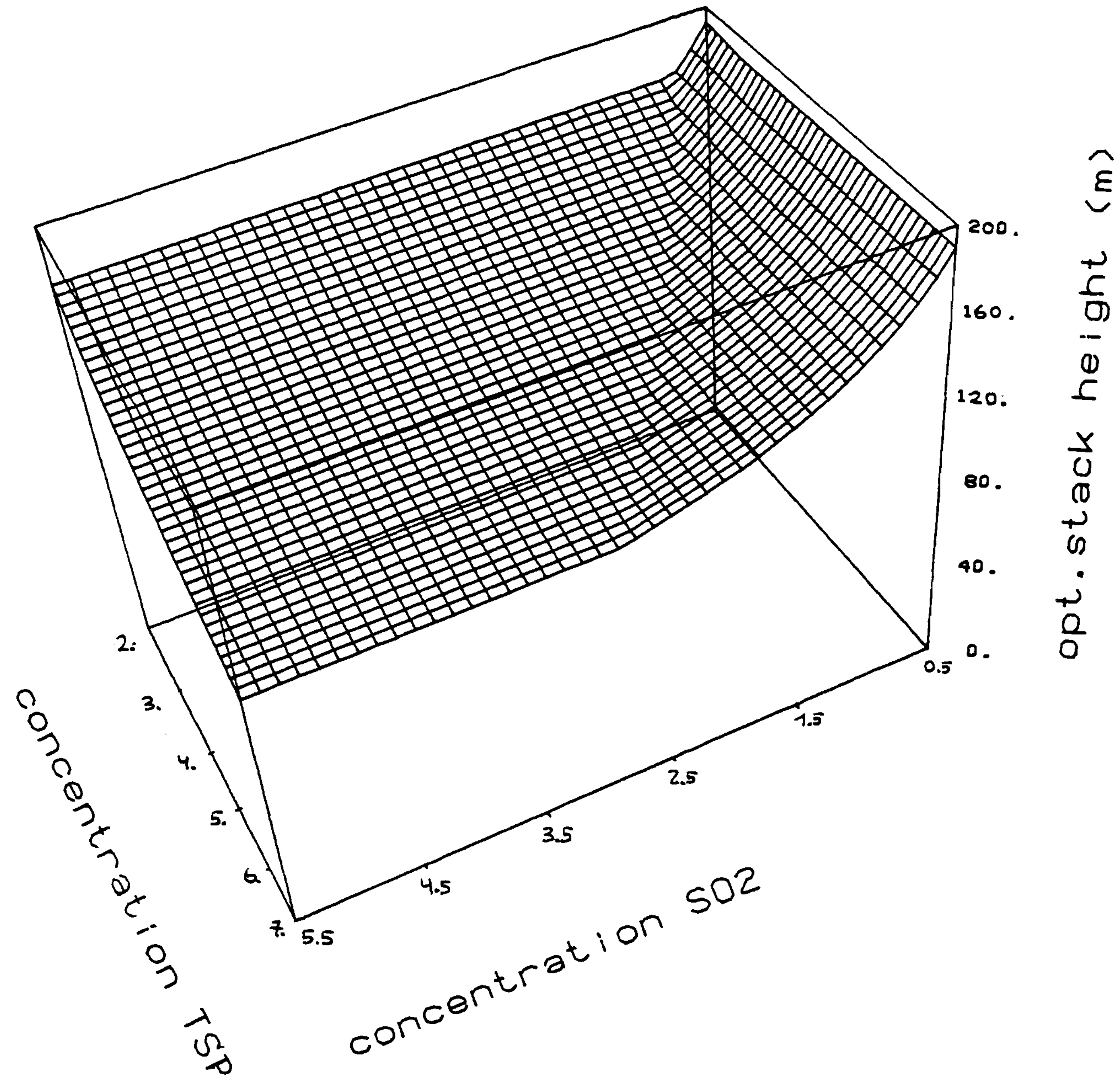

Figure 5. Optimal stack height corresponding to Γ_I of Figure 2

Graphics of Figures 2-5 provide a practical way to analyze alternative control strategies. The mathematical optimization program has been conceived in a modular way and can be used interactively. Cost functions are an input to the program. Analysis on temporal and spatial scales different from the seasonal and local scales treated by the diffusion model used in this study can be done by implementing a diffusion model suitable to the point of interest.

Urban Centralized Heating System

Centralization of the heat supply in a densely populated area provides a powerful, although very costly, tool to reduce pollution. We discuss in the following identification of minimum cost networks of heat conveyance necessary to supply a set of city subregions chosen in such a way that a given percent reduction of the existing pollution is achieved at a specified location.

Let S_i, $i = 1,\ldots,N$ be the number of subregions which can be potentially supplied with heat, and let us assume that they do not intersect. Furthermore, we consider these subregions small enough to be represented by their geographical barycenter in the chosen reference frame, and we indicate by l_{ij} $(i,j = 0,1,\ldots,N)$ the length of the network connecting S_i with S_j ($l_{ij} = l_{ji}$, $l_{ii} = 0$; the index 0 refers to the power plant). In addition we call E_i the heat needed by S_i and E_T the total maximum heat produced by the central heat plant.

Let A be the location where pollution should be reduced, then the optimization problem to be solved can be formalized as follows:

$$\min \sum_{i=0}^{N} \sum_{j=0}^{N} \varepsilon_{ij}\, \gamma_{ij}(\ell_{ij}) \qquad (25)$$

subject to:

$$\sum_{i=1}^{N} \varepsilon_i E_i \leq E_T \qquad (26a)$$

$$\sum_{i=1}^{N} \varepsilon_i C_i^A \geq pC^A \qquad (26b)$$

where C^A is the existing concentration in A; C_i^A is the contribution from S_i to A; $p(0 \le p \le 1)$ is the percent reduction of C^A:

$$\varepsilon_i = \begin{cases} 1 & \text{if } S_i \text{ is supplied} \\ 0 & \text{otherwise;} \end{cases}$$

$$\varepsilon_{ij} = \begin{cases} 1 & \text{if } S_i \text{ is connected to } S_j \text{ or to the central heat plant} \\ 0 & \text{otherwise;} \end{cases}$$

and γ_{ij} are the costs of construction of ℓ_{ij}.

The straightforward approach to the solution of this combinatorial-minimization problem is to identify out of the 2^N possible combinations of the N subregions those, which verify constraints (26a) and (26b) and then to select the one which gives the minimum cost network. This approach is not implementable even on a large computer because of the rapid growth with N of the number of possible combinations.

To overcome this difficulty it is necessary to reduce a priori the possible choices by means of practical considerations. For example, it does not make too much sense to analyze cases in which the selected subregions are far from each other. On this basis, as an alternative to the combinatorial-minimization approach the following algorithm has been adopted:

(1) Identify the subregion S_{io} which contributes most to A, that is $C_{io}^A = \max_{i=1,\ldots,N} C_i^A$, (we call it the "core");
(2) Check (26b), if verified go to step (5), otherwise
(3) Identify the subregions surrounding the "core" (we call it the "belt") and select from it the subregion which contributes most to A. Remove this subregion from the "belt" and add it to the "core".
(4) Check (26b), if verified to step (5), otherwise go to (3).
(5) Check (26a), if verified, a feasible solution to (25)-(26) has been found.

To minimize the cost of the network, steps (1) to (5) are repeated N times taking at each time a new subregion as initial "core", and out of the identified feasible solutions the one for which the cost of the heat conveyance network is minimal is selected. For each feasible solution the network of minimum cost is computed by an algorithm built on the determination of the shortest spanning subtree of a graph (Kruskal, 1956).

Of course with the above procedure generally the global minimum is not achieved, however, it provides a "common sense based" method to identify a solution with constraints (26a) and (26b) and to select out of all these possible solutions the one of minimum cost. Also in the construction of the network the following factors which can be included in the adopted procedure must be taken into account:

(a) In an urban area the heat conveyance network can only follow the existing network of streets. Thus, the minimum cost network must be identified within this given network.

(b) A heat conveyance network might already exist, and can be extended. To include this in the above procedure the length of the network between S_i and the central heat plant must be replaced by the length between subregion S_i and the existing network.

(c) There are parts of the urban area which cannot be crossed by the pipeline. This reduces the number of feasible solutions.

The procedure above outlined has been applied to the city of Vienna as illustrated below.

First, the model specified in the "Atmospheric System" section has been used to simulate the field of sulfur dioxide concentration averaged over the period, October 1977-April 1978, for which the emission data were available. Isolines on concentration computed over a grid of 30x24 points, spaced 1 km apart, are displayed in Figure 6. As expected, due to averaging over the whole heating period, the concentration distribution is relatively smooth with a maximum in the center of the city.

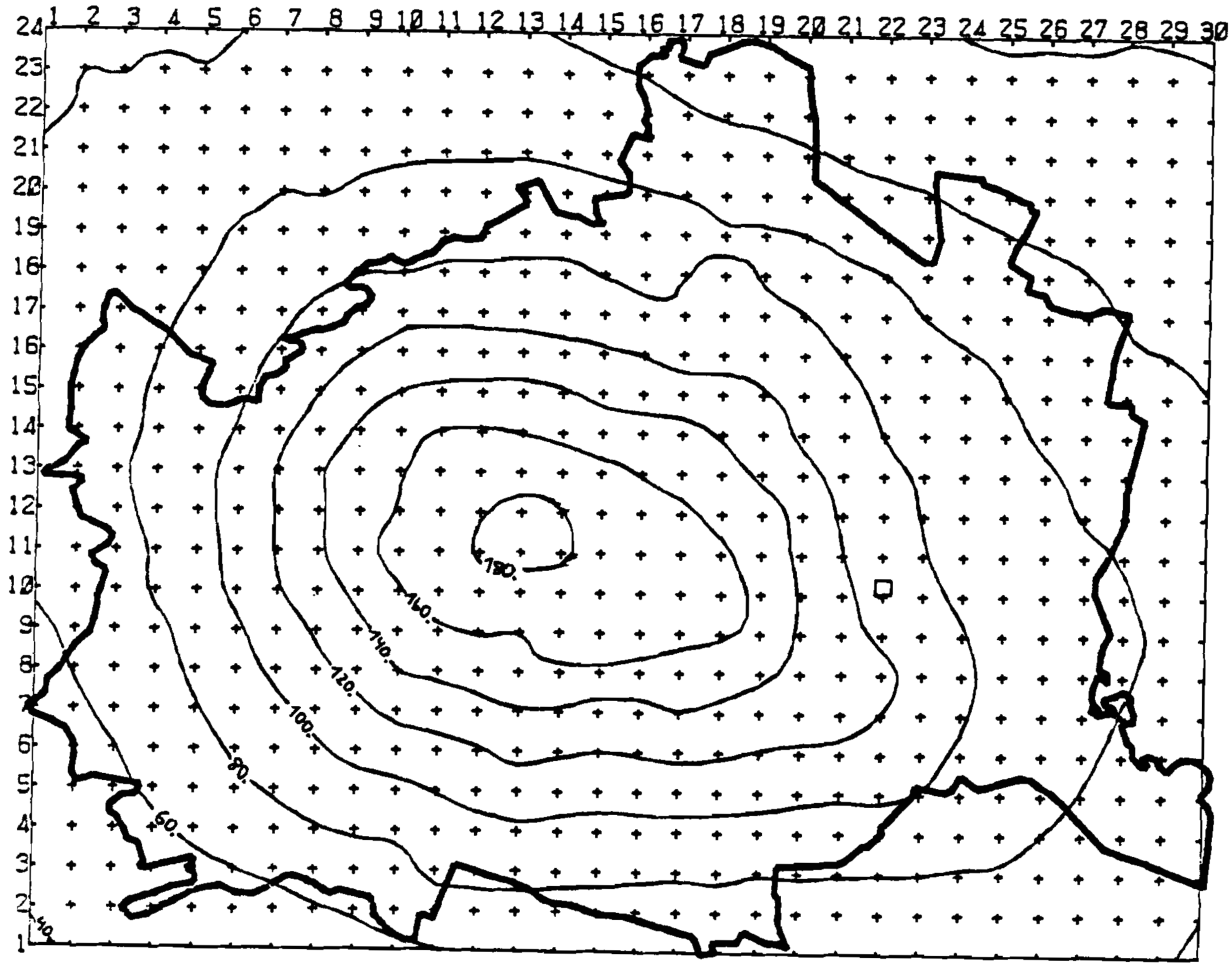

Figure 6. Isolines of the heating period - SO_2 average concentration in the city of Vienna. Values reported are in ($\mu g/m^3$). The thick line represents the city border, and the square the assumed location of the central heat plant

Then, taking the location of the central heat plant in the area indicated by the square in Figure 6, the procedure outlined above has been applied to identify an "optomal" heat conveyance network which could ensure a 30% reduction of pollution in the center of the city. Assuming that only household emissions would be connected to the network, the resulting optimal network is shown in Figure 7. Figure 7 also display the concentration distribution as it would be if the network would be in operation.

CONCLUSION

The selection of control strategies to achieve the given environmental goals cannot be done on a purely "good sense" basis, due to the impossibility to perceive completely the "intrigued" relationship between the impact on environmental system and each of the possible alternative solutions.

With reference to the atmospheric system we have shown that information on atmosphere as well as control strategies can be integrated in a mathematical program which determines under given constraints, an optimal configuration of a selected control strategy. Specifically this was done in two cases: a) the control of air pollution from a large point source (thermal power plant) and b) reduction of pollution in an urban area by means of a centralized heating system.

Although simplifications have been introduced in the description of both the atmospheric system and adopted control strategy, numerical experiments conducted with emission and meteorological data of the city of Vienna provided results which especially through their representation in graphical form, appeared to be a more valuable information basis (with respect to the input information) on which to evaluate alternative control strategies.

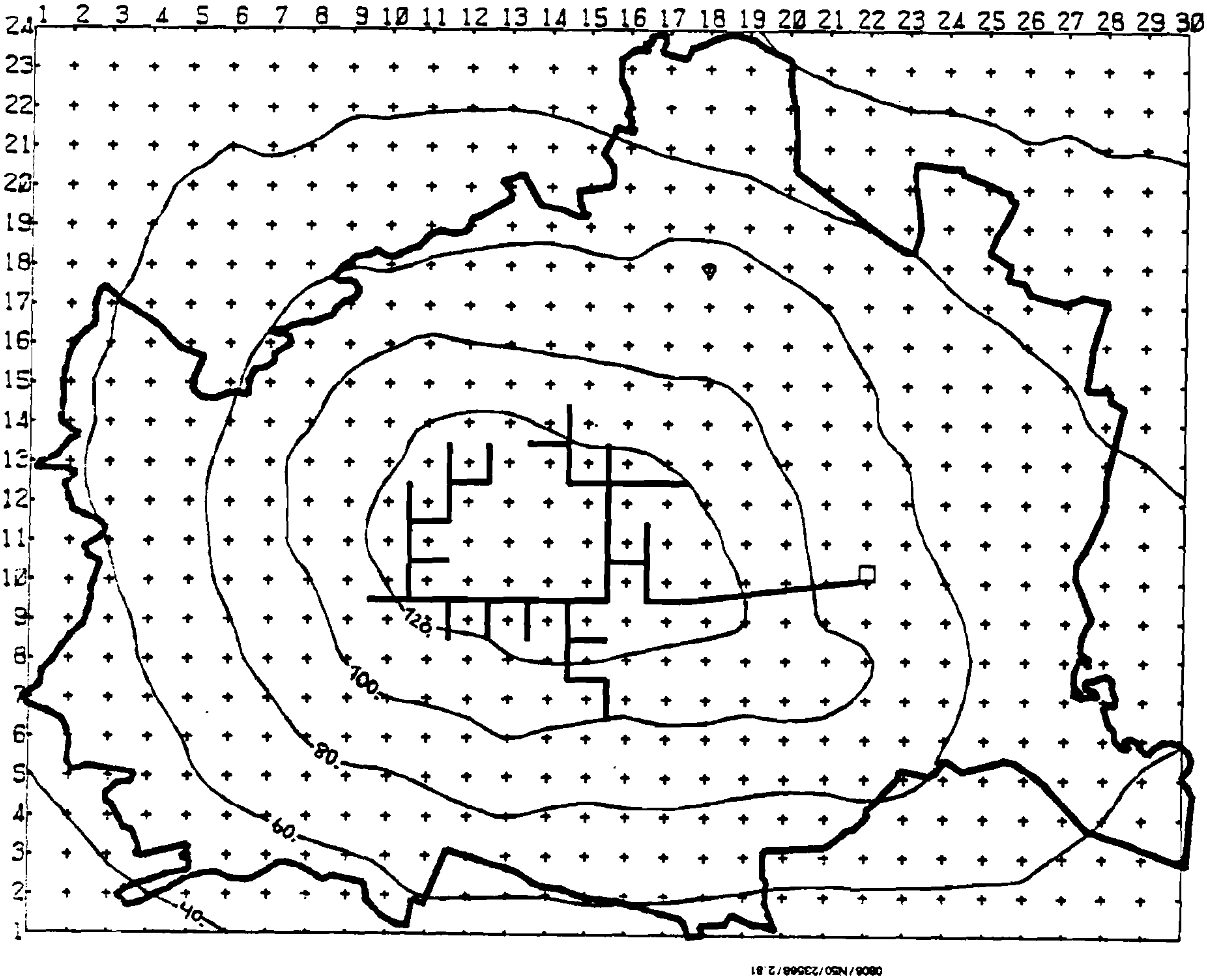

Figure 7. Heat conveyance network supplying household emissions determined under the condition that pollution in the center of the city is reduced by 30%. Isolines refer to the concentration distribution which would result with the network in operation

APPENDIX A

For both unstable and neutral atmosphere the plume rise is given by:

$$\Delta h(x) = \left[\frac{3F_m x'}{\beta_j^2 \bar{u}^2} + \frac{3Fx'^2}{2\beta_1^2 \bar{u}^3}\right]^{1/3}$$

where

$$F_m = \frac{T_a}{T_s} V_s^2 \left(\frac{d}{2}\right)^2$$

$$F = \begin{cases} F' = gV_s \left(\frac{d}{2}\right)^2 \left(1 - \frac{T_a}{T_s}\right) & \text{if } F' > F_c \\ 0 & \text{if } F' \leq F_c \end{cases}$$

$$F_c = \begin{cases} 0.0727 \ (V_s d)^{4/3} & , \text{ if } \quad F' \leq 55 \ m^4/s^3 \\ 0.0141 \ (V_s d)^{5/3} & , \text{ if } \quad F' > 55 \ m^4/s^3 \end{cases}$$

$$\beta_j = \frac{1}{3} + \frac{\bar{u}}{V_s}$$

$$x' = \begin{cases} x & ; \quad x < 3.5x^* \quad \text{and } F > 0 \\ x & ; \quad x < \dfrac{4d(V_s+3\bar{u})^2}{V_s\bar{u}} \quad \text{and } F = 0 \\ 3.5x^* & ; \quad x \geq 3.5x^* \quad \text{and } F > 0 \\ \dfrac{4d(V_s+3\bar{u})^2}{V_s\bar{u}} & ; \quad x \geq \dfrac{4d(V_s+3\bar{u})^2}{V_s\bar{u}} \quad \text{and } F = 0 \end{cases}$$

$$x^* = \begin{cases} 14F^{5/8} & ; \quad F \leq 55 \ m^4/s^3 \\ 34F^{2/5} & ; \quad F > 55 \ m^4/s^3 \end{cases} .$$

The symbols used in the above equations are:

- T_a : ambient air temperature (K)
- T_s : stack exit temperature (K)
- V_s : stack exit velocity (m/s)
- d : stack inner diameter (m)
- $\bar{u}$: mean wind speed (m/s)
- g : gravity acceleration (9.81 m/s^2)
- F_m : momentum flux term
- F : buoyancy flux term
- F_c : buoyancy flux below which plume rise is due momentum only
- β_j : jet entrainment coefficient
- β_1 : buoyancy entrainment coefficient (assumed = 0.6)
- x : downwind distance

For stable situations Δh is given by:

$$\Delta h(x) = \left[\frac{3F_m}{\beta_j^2 \bar{u} \sqrt{S}} \sin \left(\sqrt{S}\, \frac{x'}{\bar{u}}\right) + \frac{3F}{\beta_2^2 \bar{u} S} \left(1 - \cos \left(\sqrt{S}\frac{x'}{\bar{u}}\right)\right) \right]^{1/3}$$

where $S = \dfrac{g}{T_a} \dfrac{\partial \theta}{\partial z}$

$$x' = \begin{cases} x & ; \; x < \frac{\pi\bar{u}}{\sqrt{S}} \text{ and } F > 0 \\ x & ; \; x < \frac{\pi\bar{u}}{2\sqrt{S}} \text{ and } F = 0 \\ \frac{\pi\bar{u}}{\sqrt{S}} & ; \; x \geq \frac{\pi\bar{u}}{\sqrt{S}} \text{ and } F > 0 \\ \frac{\pi\bar{u}}{2\sqrt{S}} & ; \; x \geq \frac{\pi\bar{u}}{2\sqrt{S}} \text{ and } F = 0 \end{cases}$$

β_2 : buoyancy entrainment coefficient for stable condition also assumed = 0.6, and

$\frac{\partial\theta}{\partial z}$: vertical potential temperature gradient (K/m); potential temperature is the temperature which an air parcel originally at an arbitrary height would assume if it were compressed or expanded adiabatically to the pressure of 1000 mb (Note that $\frac{\partial\theta}{\partial z} < 0$; $\frac{\partial\theta}{\partial z} = 0$; and $\frac{\partial\theta}{\partial z} > 0$ corresponds to unstable, neutral and stable atmosphere, respectively).

APPENDIX B

Let us consider the solution of the following optimization problem:

$$\min_{x_1,\ldots,x_n} \sum_{i=1}^{n} K_i(x_i) \qquad (1)$$

subject to

$$\prod_{i=1}^{n} (1-x_i) \leq A \qquad (2a)$$

$$0 \leq x_i^{min} \leq x_i \leq x_i^{max} \leq 1, \qquad i=1,\ldots,n \qquad (2b)$$

First we see immediately that a solution is feasible only if $\Pi(1-x_i^{max}) \leq A$; and assuming that the functions K_i are monotically increasing (which is a reasonable assumption for cost-efficiency functions) the problem is nontrivial only if $\Pi(1-x_i^{min}) > A$.

Second we reformulate (2a) as follows:

$$\prod_{i=1}^{n} \left(\frac{1 - x_i}{1 - x_i^{min}}\right) \leq \frac{A}{\Pi(1-x_i^{min})} = A' \qquad (3)$$

The problem is nontrivial if $A' < 1$.

We introduce the following transformation:

$$y_i = -\log \frac{1 - x_i}{1 - x_i^{min}}, \qquad i = 1,\ldots,n$$

equivalent to $x_i = 1 - (1 - x_i^{min})e^{-y_i}$.

Defining $F_i(y_i) = K_i(1 - (1-x_i^{min})e^{-y_i})$,

$$\alpha = -\log A'$$

problem (1)-(3) becomes:

$$\min_{y_1,\ldots,y_n} \sum_{i=1}^{n} F_i(y_i) \tag{4}$$

$$\sum_{i=1}^{n} y_i - \alpha \geq 0 \tag{5a}$$

$$0 \leq y_i \leq y_i^{max}, \quad i=1,\ldots,n \tag{5b}$$

where $y_i^{max} = -\log \dfrac{1 - x_i^{max}}{1 - x_i^{min}}$ (Note: if $x_i^{max} = 1$, then $y_i^{max} = +\infty$). Note that by the adopted transformation, constraints have become linear.

The Lagrangian of (4)-(5) reads:

$$L(\underline{y},\underline{\lambda},\underline{\mu},\underline{\upsilon}) = \sum_{i=1}^{n} F_i(y_i) - \lambda(\sum_{i=1}^{n} y_i - \alpha) + \sum_{i=1}^{n} \mu_i y_i + \sum_{i=1}^{n} \upsilon_i(y_i^{max} - y_i) \tag{6}$$

The necessary Kuhn-Tucker (see, e.g., Wismer and Chattergy, 1978) conditions for a minimum point are

$$\frac{\partial F_j}{\partial y_j}(y_j) - \lambda + \mu_j - \nu_j = 0 \qquad j = 1,\ldots,n \tag{7}$$

$$\lambda(\sum_{i=1}^{n} y_i - \alpha) = 0, \quad \lambda \geq 0, \quad \sum_{i=1}^{n} y_i - \alpha \geq 0 \tag{8}$$

$$\mu_j y_j = 0, \quad \mu_j \geq 0, \quad y_j \geq 0, \quad j=1,\ldots,n \tag{9}$$

$$\upsilon_j (y_j^{max} - y_j) = 0, \quad \upsilon_j \geq 0, \quad y_j^{max} - y_j \geq 0, \; j=1,\ldots,n \tag{10}$$

(In case of $y_j^{max} = +\infty$, omit the corresponding υ_j in (6) and (10)).

First a solution to (7)-(10) in the interior of the $(\underline{y},\lambda)$ space is searched. The system (7)-(10) reduces to ($F_j{}' = \partial F_j/\partial y_j$)

$$F_j{}'(y_j) - \lambda = 0 \qquad j = 1,\ldots,n \tag{11}$$

$$\sum_{i=1}^{n} y_i - \alpha = 0 \tag{12}$$

for $\lambda \geq 0$; $0 \leq y_j \leq y_j^{max}$, $j = 1,\ldots,n$ or more formally, $\underline{f}(\underline{z}) = \underline{0}$ were $\underline{z} = (\underline{y},\lambda)$, and the function $\underline{f}$ is defined by Equations (11) and (12).

To solve (13) we use Newton-iteration:

$$\underline{z}^{(m+1)} = \underline{z}^{(m)} - [\underline{f}'(\underline{z}^{(m)})]^{-1} f(\underline{z}^{(m)}), \tag{14}$$

where $\underline{f}'$, the Jacobian of $\underline{f}$ is given by (assuming that the F_j's are twice differentiable)

$$\underline{f}'(\underline{z}) = \begin{bmatrix} F_1''(y_1) & 0 & 0 & \cdots & 0 & -1 \\ 0 & F_2''(y_2) & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & F_n''(y_n) & -1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \tag{15}$$

and its inverse is given by

$$[\underline{f}'(\underline{z})]^{-1} = \frac{1}{\sum\limits_{i=1}^{n} \frac{1}{F_i''(y_i)}} \begin{bmatrix} \ddots & \frac{1}{F_\ell''(y_p)} \sum\limits_{i=1}^{n} \frac{1}{F_i''(y_i)} & & \vdots \\ \cdots \frac{-1}{F_k''(y_k)F_\ell''(y_\ell)} & & \ddots & \frac{1}{F_k''(y_k)} \\ \cdots \frac{-1}{F_\ell''(y_\ell)} \cdots & \cdots & \cdots & 1 \end{bmatrix} \qquad (16)$$

as can be proved by induction.

The iteration scheme (14) reads then:

$$z_j^{(m+1)} = \frac{1}{F_j''(z_j^{(n)}) \sum\limits_{i=1}^{n} \frac{1}{F_i''(z_i^{(m)})}} \left\{ \sum_{i=1}^{n} z_i^{(m)} - \alpha - \sum_{i=1}^{n} \frac{F_i'(z_i^{(m)}) - F_j'(z_j^{(m)})}{F_i''(z_i^{(m)})} \right\} \qquad (17)$$

$$j = 1,\ldots,n+1 \quad ,$$

where we have introduced formally

$$F_{n+1}'(z_{n+1}^{(m)}) = \lambda^{(m)}, \quad F_{n+1}''(z_{n+1}^{(m)}) = 1.$$

Next the solutions at the boundary of the $(\underline{y},\lambda)$ space are searched. An arbitrary hyper-surface of the boundary of the considered domain is characterized uniquely by a certain set of variables $\{y_j\}$, which take the value 0 or y_j^{max}.

Without loss of generality we can assume that:

$$y_j = 0, \quad j = 1,\dots,n_1$$

and

$$y_j = y_j^{max}, \; j = n_1+1,\dots,n_2<n \quad .$$

The system (7)-(10) can then be put in the form:

$$F_j'(0) - \lambda + \mu_j = 0, \quad j=1,\dots,n_1 \tag{18}$$

$$F_j'(y_j^{max}) - \lambda - \upsilon_j = 0, \quad j = n_1+1,\dots,n_2 \tag{19}$$

$$F_j'(y_j) - \lambda = 0, \; j = n_2+1,\dots,n \tag{20}$$

$$\sum_{j=n_2+1}^{n} y_j = \alpha - \sum_{j=n_1+1}^{n_2} y_j^{max} \tag{21}$$

Putting $\alpha - \sum_{j=n_1+1}^{n_2} y_j^{max} = \alpha'$ we see that (20)-(21) is equivalent to (11)-(12). They can therefore be solved by the previously described Newton-iteration method. Once λ is computed from (20)-(21), μ_j and υ_j are obtained by inserting from (18)-(19).

The total number of systems of the type (11)-(12) and (20)-(21) respectively inside and on the boundary of the (y,λ) space, to be solved is $(3^n - 2^n)$. The global optima is selected from the $(3^n - 2^n)$ local optima. The number of systems to be solved reduces, if y_j^{max} is infinite for some j.

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