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DESIGN OF AGRICULTURAL DRAINAGE UNDER UNCERTAINTY,2, A STOCHASTIC PROGRAMMING APPROACH

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ABSTRACT

The effect of soil property uncertainty on drainage system design was presented in the first of a series of papers on methods for optimal design of agricultural drains. A First Order-Second Moment (FOSM) approach was developed for the Hooghoudt steady-state drainage design equation to provide an estimate of the of the uncertainty of the dewatering zone between the drains as a function of the design variables and the uncertainty in the soil properties. In this paper , a Stochastic Programming Model for optimal design of drains under uncertainty, based upon the FOSM approach , is developed . The Stochastic Programming Model incorporates uncertainty in the objective function of the model as the expected loss in crop production as a function of uncertainty in the dewatering zone. The Stochastic Programming model is extended to included a multiple cropping situation and finally , the Chance Constraint approach ,presented in the first paper ,is compared with the Stochastic Programming Approach to drainage design and advantages of each are presented.

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DESIGN OF AGRICULTURAL DRAINAGE UNDER UNCERTAINTY,2, A STOCHASTIC PROGRAMMING APPROACH

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1. Introduction

This paper is the second in a series that presents tools for the "optimal" design of agricultural drainage. The issues of the design of agriculture drainage were presented in the first paper of this series [Strzepek, Wilson and Marks, 1982] , and it was shown that there is a need to incorporate uncertainty and economics in drain design. The approach presented in the first paper was Chance Constraint Programming which provided a drain design that meets a certain reliability on the drain performance at minimum cost. In this paper, uncertainty in drain performance is addressed not by a reliability approach ,but by "Stochastic Programming" which incorporates information of the entire probability distribution into an expected value of system performance. The Stochastic Programming approach to uncertainty in mathematical programming was developed by Dantzig [1955] and is possible if there exists a relationship between system response and system output.

It is possible to calculate an expected crop yield as a function of the drain design if a crop yield versus dewatering zone (DWZ) and a probability density function of the dewatering zone exist. It was shown in first paper that the first two moments of the dewatering zone midway between the drains ($\overline{DWZ}$ and σ_{DWZ}) can be calculated by the First Order Second Moment (FOSM) analysis of the Hooghoudt equation and that these moments defined the parameters of a probability density function of DWZ. For demonstration purposes, the normal distribution was chosen, altough other distributions will work. For the drain design problem there is empirical data relating system response, crop yield, to system output, the dewatering zone being midway between the drains. It is possible to calculate an expected crop yield as a function of drain design.

The first paper [Strzepek, et al, 1982] presents a detailed description of soil property uncertainty in soil permeability and recharge rate . An analysis of uncertainty in soil permeability revealed that this uncertainty could be decomposed into information uncertainty and spatial variability. It was shown for certain forms of the spatial structure that small scale variability can be ignored and large scale variability assumed constant for soil permeability between two drains. When this condition exists the uncertainty in the soil permeability between two drains can be described by the information uncertainty. A FOSM analysis of the Hooghoudt steady-state drainage equation [Strzepek ,et al, 1982], is presented as a method for analyzing uncertainty in drain performance due to information uncertainty in soil permeability and recharge rate .

A Chance Constraint Model which minimizes the cost of drain installation while meeting a given reliability on the dewatering zone midway between the drains was developed. This approach ,however ,leaves the designer with a number of questions to answer before an "optimal solution" can be determined: (1) Which level of the dewatering zone should be chosen as a design criterion? (2) What reliability should the design dewatering zone achieve? (3) What design criterion is used for lands where more than one crop is grown? The Chance Constraint approach can only provide an optimal design based upon the values of the dewatering zone and reliability given by the designer, but it cannot determine which values of these parameters maximize the net benefits of drainage to crop production.

The Stochastic Programming Model(SPM), examined in detail below, is a second approach to optimal drain design under uncertainty. The SPM for drain design is formulated to minimize the sum of the capital costs of drain installation and the expected value of the annual crop loss due to non-optimal soil water conditions over the life of the drains subject to certain physical constraints. The selection of an optimal drain design in the SPM formulation occurs when the marginal capital costs of providing a smaller expected crop loss equals the marginal savings from reducing the expected crop loss any further. The SPM approach resolves the problems of ,the choice of a design DWZ and design reliability , left unanswered by the Chance Constraint Model. The expected crop loss is based upon economic rather than physical criteria ,so it is possible to extend the method to a multiple crop formulation.

2. Stochastic Programming Model for Agricultural Drain Design

2.1. Formulation

The Stochastic Programming approach incorporates the expected value of system performance in the objective function of the Mathematical Programming Problem (MPP). In this way, the uncertainty in drain design is captured by an economic measure in the objective function, whereas the Chance Constraint approach accounts for uncertainty by a physical measure in the constraint set. The Stochastic Programming MPP minimizes the sum of capital costs $CC(D,L)$ and the Present Value of Expected Crop losses $EL(D,L)$, subject to constraints on drain depths, D_{max} non-negativity of D and spacing L, and the FOSM Hooghoudt definitions, $\bar{h}$ and σ_h where,

$$\bar{h}_{L/2} = f_1(L, d', \bar{N}, \bar{K}) \qquad (1)$$

$$= -d' + \left[d'^2 + \frac{\bar{N}^2}{4\bar{K}} \right]^{1/2}$$

$$d' = f_2(L, d, r)$$

$$d' = \begin{cases} \dfrac{d}{1 + \dfrac{d}{L}\left[2.55\ln(\dfrac{d}{r}) - 3.55 - 1.6(\dfrac{d}{L}) + 2(\dfrac{d}{L})^2\right]} & \text{if } 0.0 < \dfrac{d}{l} \leq 0.31 \\[2ex] \dfrac{L}{2.55\left[\ln(\dfrac{L}{r}) - 1.15\right]} & \text{if } 0.31 < \dfrac{d}{L} \end{cases} \qquad (2)$$

$$\sigma^2_{h_{L/2}} = f_3(L, d', \bar{N}, \sigma_N, \bar{K}, \sigma_K, \rho_{KN}) \qquad (3)$$

$$= \left[\frac{L^2}{8\bar{K}}\right]^2 \left[d'^2 + \frac{\bar{N}^2}{4\bar{K}}\right]^{-1} \left[\sigma_N^2 - 2\frac{\bar{N}}{\bar{K}}\rho_{KN}\sigma_{KN} + (\frac{\bar{N}}{\bar{K}})^2\sigma_K^2\right]$$

which are used to define $\overline{DWZ}$ and σ_{DWZ}.

$$DWZ = D - h \qquad (4a)$$

$$\overline{DWZ} = D - \bar{h} \qquad (4b)$$

$$\sigma^2_{DWZ} = \sigma^2_h \qquad (4c)$$

The parameters $\overline{DWZ}$ and σ_{DWZ} of the probability density function of DWZ, are used in the objective function to calculate the expected loss, EL (D,L).

The mathematical formulation of the Stochastic Program for Tile drain design is:

$$MIN \quad CapitalCost(D,L) + ExpectedLoss(D,L) \tag{5}$$

Subject to:

$$\overline{DWZ} = D - \overline{h} \tag{6a)}$$

$$\sigma_{DWZ}^2 = \sigma_h^2 \tag{6b}$$

$$\overline{h}_{L/2} = f_1(L,d',\overline{N},\overline{K}) \tag{6c}$$

$$d' = f_2(L,d,r) \tag{6d}$$

$$\sigma_{h_{L/2}}^2 = f_3(L,d',\overline{N},\sigma_N,\overline{K},\sigma_K,\rho_{KN}) \tag{6e}$$

$$D \leq D_{\max} \tag{6f}$$

$$d = Z - D \tag{6g}$$

$$D,L \geq 0.0 \tag{6h}$$

The objective function contains the same capital cost function for drain installation as used in the Chance Constraint Model. It is defined as:

$$CapitalCost(D,L) = \frac{c_1}{L}\left\{c_2 D^{c_3} + c_4\right\} \tag{7}$$

where c_1, c_2, c_3 and c_4 are coefficients that are a function of technology soil type, and regional economic costs. The expected loss function , $EL(D,L)$, is described in detail in the next section.

2.2. Expected Loss Function

The Hooghoudt equation for the dewatering zone mid-way between the drain is a steady-state model. As such, the predicted levels are assumed to be constant over the entire growing season of each crop and the same for each growing season over the life of the drains. From experimental and field data, crop yield as a function of dewatering zone can be determined. Figure 1 presents a range of crop yield functions that have been observed for steady-state field conditions in various places in the world [Visser, 1958, and Ministry of Irrigation, ARE, 1965]. The appropriate function must be determined specifically for each crop, soil condition, and climate, as well as other factors affecting crop yield. The Type I function represents the situation where there is no contribution from the subsurface water table to crop water use. Type III represents the situation where a great deal of the crop's water use comes from the subsurface water table and lowering the water table will dramatically affect yield. Neglecting effects of salinity, these two forms represent the extremes of the situation to be found in the field. These two extremes rarely occur, and Type II, which represents a combination of both effects, is widely observed [Amer,1979].

The curves shown in Figure 1 are a measure of the crops' yield as a function of the dewatering zone mid-way between the drains. These functions integrate the effects of the spatially varying dewatering zone between the drains and express this effect as a function of the dewatering zone mid-way between the drains. If this were not the case, and the function reflected a point response of the crop to a value of the dewatering zone, the approach proposed is still valid. The expected value of crop yield could be found at each point x between the drains, based upon FOSM of the Hooghoudt equation as a function of x. This spatially varying expected yield function could then be integrated over the drain spacing, L, to determine the expected yield for that drain design.

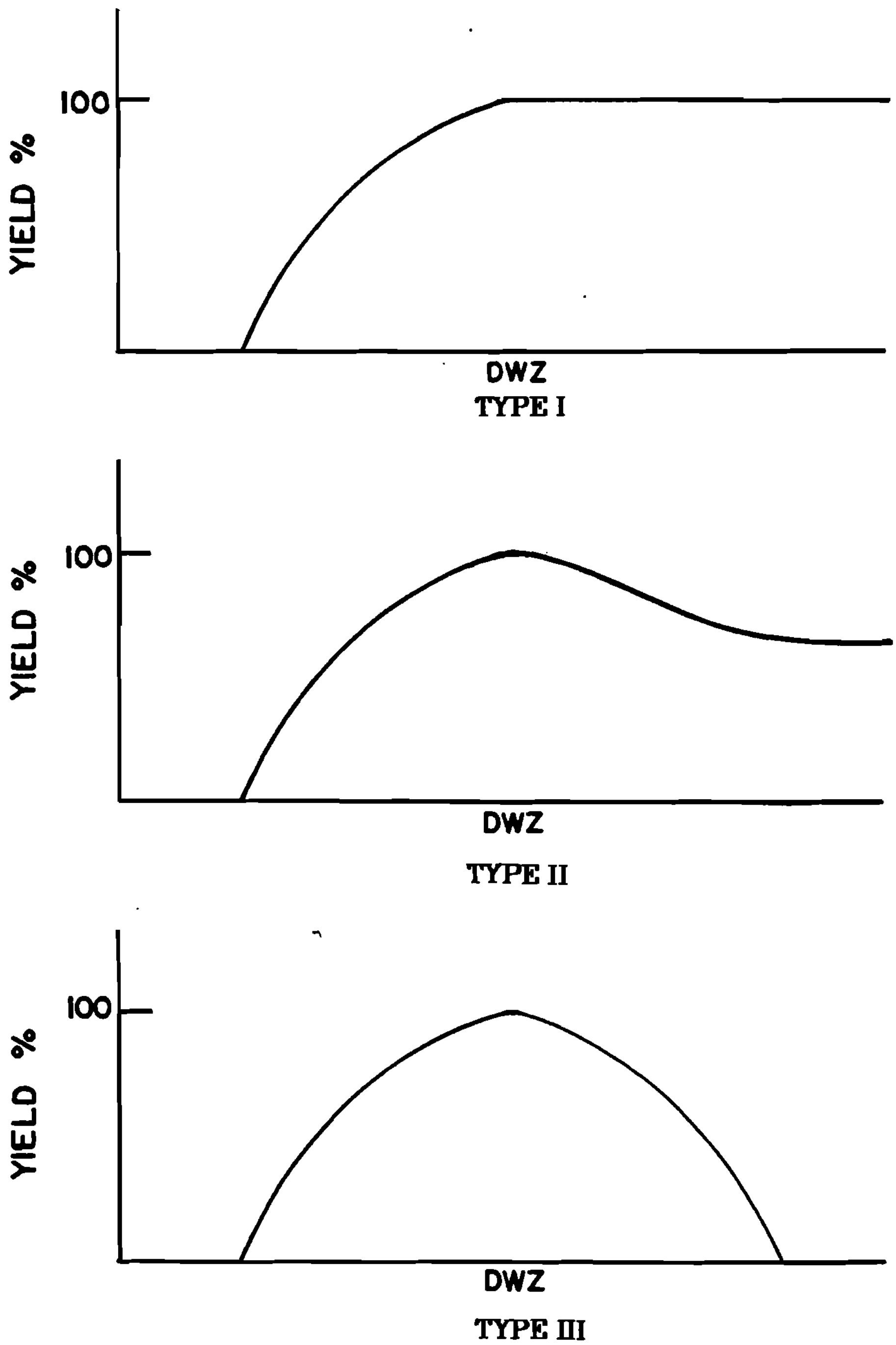

Figure 1. Crop Yield Functions

With a functional relationship between crop yield and the dewatering zone, the expected yield for any crop over the growing season can be found by integrating the product of the yield function and the probability density function of the dewatering zone over the entire range of dewatering zones. The steady-state annual expected yield for each crop can be used to generate an economic measure of drain performance. The difference between the projected yield under optimal soil water conditions , Y^* ,and the expected yield as a function of the drain design ,$E[Y]$, is defined as the annual expected yield loss. This annual yield loss is then multiplied by the price for that crop CP to obtain an annual economic loss assumed constant over the life of the drains. The present worth factor for interest rate ,i ,over the life of the drains ,t, PWF_i^t is used to determine the present value of the expected crop loss as a function of system design $EL(D,L)$ as:

$$EL(D,L) = \left[Y^* - E[Y]\right] \times CP \times PWF_i^t \tag{7}$$

The integral for determining the expected yield function cannot be evaluated analytically, but can be evaluated numerically to sufficient accuracy.

2.3. Solution Technique

The sum of capital costs and the present value of expected losses define the objective function of the Stochastic Programming MPP. From the description above, it is seen that the objective function is non-linear and the non- definition constraints are linear. (The definition constraints are actually part of the objective function, but are put in the constraint set for clarity). The drain design stochastic programming problem is a two-dimensional non-linear programming problem with a linear constraint set. To assure a global solution to the minimization problem, the constraint set must define a convex feasible region, and the objective function a unimodal or quasi-convex function.

It is necessary to show that these condition exist before we proceed with an application of the model. The Embabe Region in Egypt presented in the first paper [Strzepek,et al,1982a] will again provide the data for examining the validity of the modelling approach. A plot of the objective function using Embabe data and defined over the feasible region is shown in Figure 2. Strzepek, et al, [1982b] have shown that for the Embabe case study data and the three forms of the yield function presented in Figure 1, the objective function is quasi-convex over the feasible region.

The most widely used solution technique for this class of MPP is the gradient search approach. However, in this problem the objective function is so complex that the calculation of the gradient at each iteration is computationally burdensome. However, the objective function is unimodal in both D and L. Taking advantage of this property, a recursive algorithm is used which minimizes over D a function **G** which is the minimum over L of function **F** for each D, as follows

$$MIN_D \, G(D) \tag{8}$$

subject to

$$G(D) = MIN_L \, F(D,L) \tag{9}$$

Each one-dimensional problem was solved using the golden-section search method [Strzepek et al, 1982b]. This provides a solution accuracy well within the tolerance of drain installation.

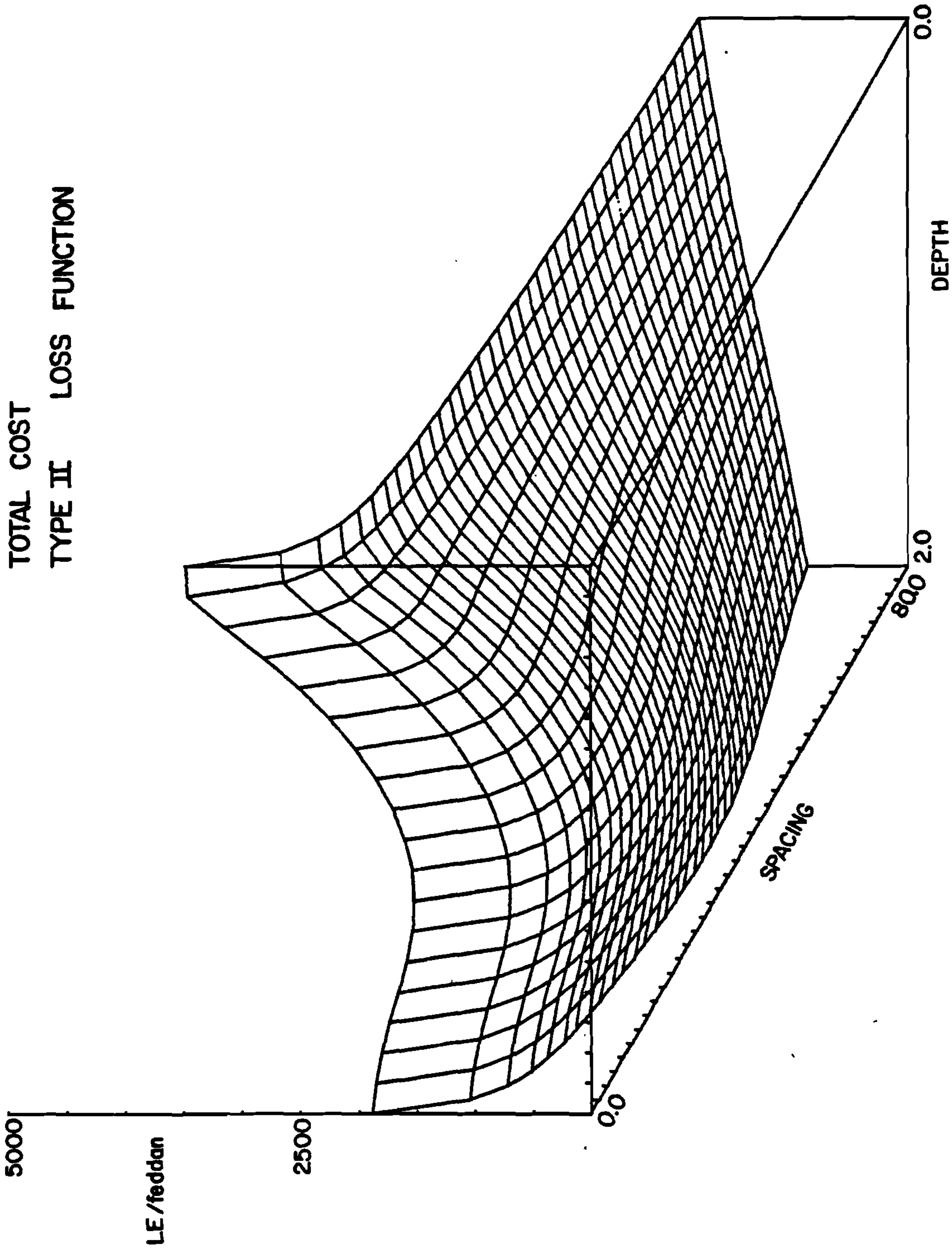

Figure 2. Objective Function for Embabe Data with Type II Yield Function

2.4. Case Study Applications

The Stochastic Programming Model for drain design has been presented together with a solution technique. However, before a solution can be generated, a number of parameters must be defined, such as projected crop yield, crop prices, interest rates, and the life of the project. Although the results may vary as these parameters vary, Strzepek, et al, [1982b], have shown that for the range of values possible for the Embabe case study, the results are stable and the most important parameter which must be determined is the type of yield function that the crop possesses. In Table 1 part A, the model parameters for the Embabe case study region in the Nile Delta are presented. Based upon these parameters, Table 1 part B presents the model solutions for the three types of yield functions presented above. The results show Type I to have an expected cost much less than, and a design much different to ,both Types II and III. This is due to the fact that for Type I, the yield remains at the optimum level for values of DWZ greater than DWZ^*. The model's goal is to find the design that minimizes the capital costs plus the losses due to reduced yields. Thus, the model will attempt to design a system such that $\overline{DWZ}$ will be close to DWZ^* and σ_{DWZ} will be as small as possible ,to concentrate the probability density at the optimum point and achieve the highest possible expected yield. However, as is seen in the model as $\overline{DWZ}$ increases and σ_{DWZ} decreases, reducing expected losses, the capital cost of the drain increases. So the model must trade-off between the reduction of expected losses and the increases of capital costs .

Table 1. Optimal Drain Design Sensitivity to Crop Yield Function

A. Model Parameters

$\underline{Z}$	$=7.0m$	D^*	$=2.0m$
$\underline{K}$	$=0.085m/day$	σ_K	$=0.0815\ m/day$
$\underline{N}$	$=0.0004m/day$	σ_N	$=0.0004m/day$
c_1	$=52.2$	c_2	$=1.646$
c_3	$=0.365$	c_4	$=55.892$
Crop	*=Clover*	*Yield*	*=200LE/feddan*
DWZ^*	$=1.0m$	t	*=50 years*
i	$=10\%$		

B. Model Results

	Yield Function		
	Type I	Type II	Type III
Drain Depth, D (m)	2.00	1.46	1.43
Drain Spacing, L(m)	43.17	21.95	20.57
Capital Cost	89.14	138.23	147.43
Expected Loss	8.93	60.09	57.21
Total Cost	98.07	198.32	204.64

Note: All cost in LE per feddan.

There is little difference in the optimal design for Types II and III because the model provides a design such that the probability density is concentrated at a point near DWZ^* and has little density in regions where the yield is low. There is a great difference between Type I and Type II and III results, because the model allows σ_{DWZ} in Type I to be large since the yield function is constant at the optimal value and the expectation will not change if the probability density is concentrated or distributed in this region. These results emphasize the necessity to obtain the best possible data on the shape of yield function for the crop for which the drains are designed.

3. Multiple Crop Stochastic Programming Model

The model presented above only partially answered the questions raised by the Chance Constraint approach. The question of design under multiple crops remains. Since the Stochastic Programming approach has transformed the measure of performance of the drains from a physical measure to an economic measure and since the economic measure can be handled in an additive way, the objective function can be extended to include the expected losses of each crop affected by the drain design. The objective function in the single crop stochastic programming model minimized total cost of capital costs and expected losses. In this manner, the approach can be extended to minimize the sum of capital costs and the expected losses of each of the crops that are grown over the year on the land drained.

The expected loss function, as defined above, can be determined for each crop, given a yield function and crop prices. The expected loss function for each crop can then be weighted by the average area cultivated in that crop A_j by the drains. The summation of the weighted expected loss functions becomes a new multiple crop loss function. The capital cost function remains the same, so that the objective function for a Multiple Crop Stochastic Programming model becomes:

$$MIN \quad CaptialCost(D,L) + \sum_{j=1}^{NC} A_j \times \left[Y_j^* - E[Y_j] \right] \times CP_j \times PWF_i^t \qquad (10)$$

where the subscript j represents each crop up to NC, the number of crops. This multiple crop objective function replaces the objective function in the single crop stochastic programming formulation ,equation (6), to provide a new Multiple Crop Stochastic Programming model.

3.1. Case Study Application

The assumption is made that for the Egyptian Delta conditions, the appropriate yield function is the form of Type II. This assumption is based upon the soil physics, irrigation practices, climate, and experimental data from the Nile Delta [Ministry of Irrigation, 1965] which show that for all crops of major importance to agriculture in the Nile Delta, the yield function follows a Type II form. Table 2 lists the data for the important crops for a non-rice area in the Nile Delta similar to the Embabe region.

Table 2. Multiple Crop Yields in the Nile Delta

	Crops				
	Cotton	Maize	Wheat	Vegetables	Berseem
Area (per feddan)	0.25	0.58	0.25	0.17	0.62
Yield (m. ton/fed.)	0.35	2.14	1.72	8.40	24.66
Price (LE/m. ton)	466.67	51.2	50.00	60.00	0.44
Total (LE/feddan)	40.83	63.55	21.50	85.68	6.73
DWZ (m)*	1.3	1.15	1.1	1.0	1.0

With this data, a multiple crop expected loss function can be defined. Figure 3 is a plot of the objective function for the multiple crop stochastic programming model using the data from Tables 1 and 2.

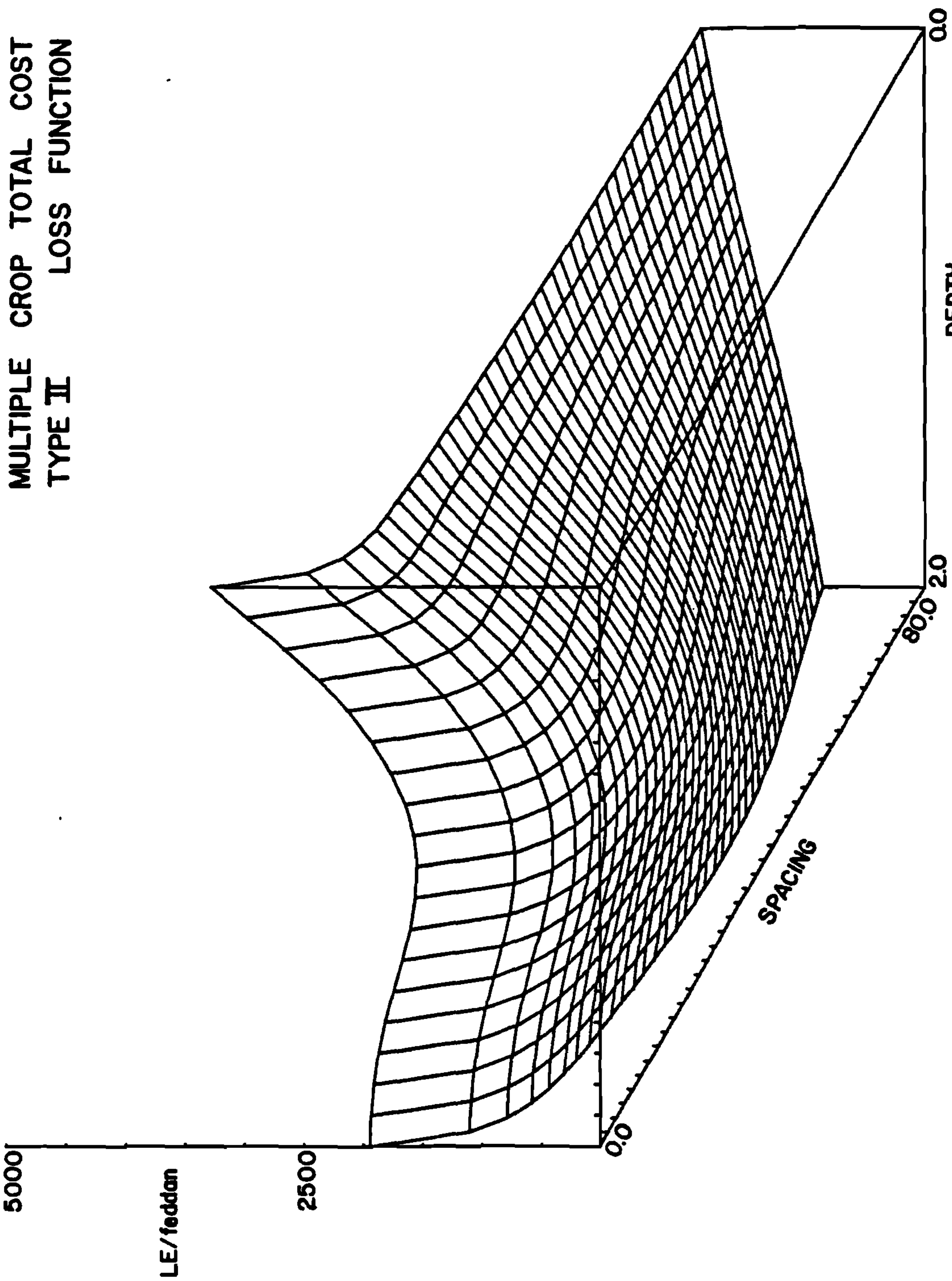

Figure 3. Multiple Crop Objective Function

Strzepek et al, [1982b], have shown that this is a quasi- convex function. The same constraint set as for the single crop model is used .The constraint set defines a convex feasible region, so a global minimum can be found using the existing solution technique. The optimal solution for this case is a drain depth $D=1.38m$, spacing $L=21.96m$. This results in a capital cost of 137.9 LE/feddan expected loss of 86.6 LE/feddan and a total cost of 224.5 LE/feddan.

4. Stochastic Programming versus Chance Constraint Approach

Thus far, in this two-paper series, chance constraint and stochastic programming have been presented as alternative methods to include uncertainty in optimal drain design. This section will examine the properties of the two approaches.

The chance constraint approach to uncertainty is a reliability approach. It requires that a system output target be met with a certain reliability. The target value for the system output is usually an optimal value of system performance. In drainage design the target value is the optimal dewatering zone for crop production. As the problem has been presented, the greater the reliability, the better the system performance. This approach assumes that if the system output surpasses the target values, the system performance will be as good, if not better, than below the target value. In other words, the system benefit function is a monotonically non-decreasing function. Figure 4 illustrates this argument. In Case (b), the target value Z^* is met with a reliability of 95% and the expected system benefits are greater than case (a), in which the target value is met with 80% reliability. This illustration shows the logic

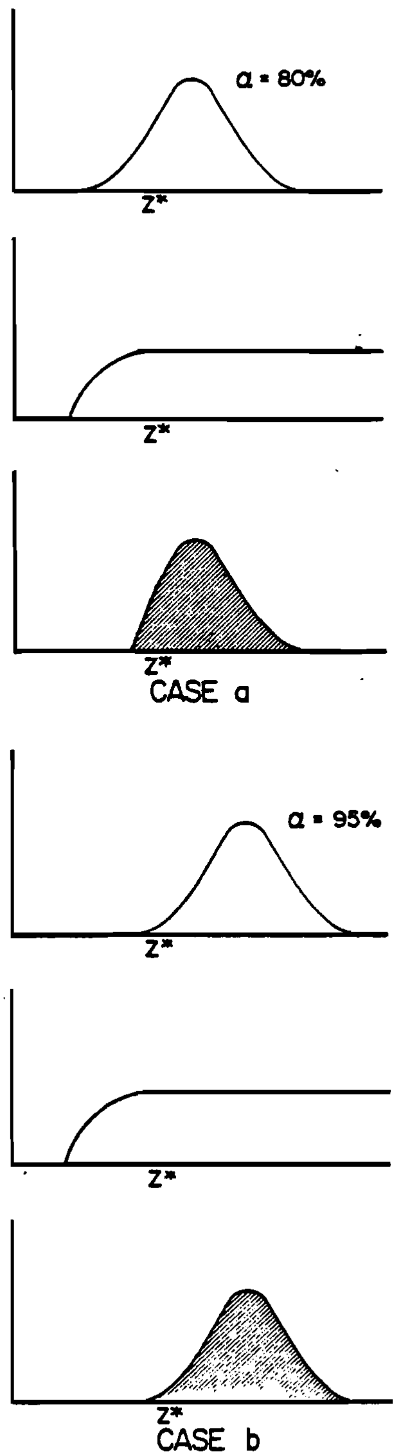

Figure 4. Monotonically Non-decreasing Yield Function

behind the chance constraint approach to uncertainty. However ,the chance constraint approach is valid only as long as the benefit function is monotonically non-decreasing or the sign of the slope of the function does not changes over the range of possible output. When this condition does not occurs the result will be that a greater reliability of the output target will produce poorer system performance than lesser reliabilities. Figure 5 illustrates this point. In Case (a), a reliability of 50% on the target value provides substantially more expected benefits than a 95% reliability on the target value in Case (b).

The implication of the above arguments for drainage design is quite clear and important. In Figure l, general forms of typical crop yield functions were shown. Three functions were presented and only one was a monotonically non-decreasing function. The two others had slopes that changed sign, making the present chance constraint approach invalid. If a monotonically non-decreasing function is assumed in a chance constraint analysis and the actual yield function is not the assumed form, there will be a "regret."

To quantify the magnitude of this regret for drainage design in the Nile Delta an experiment was performed. In Figure 1, three possible crop yield functions were illustrated. Type I is a monotonic non-decreasing function while Type II and Type III are not. An analysis was done to quantify the "regret" that would result if a drainage system was designed assuming a Type I crop yield when, in fact, the function was actually Type II or Type III. The measure of regret was the difference in expected losses as described above. The system was designed for a 98.5% reliability of a dewatering zone of 1.0 meters for clover. Table 3 is a summary of the

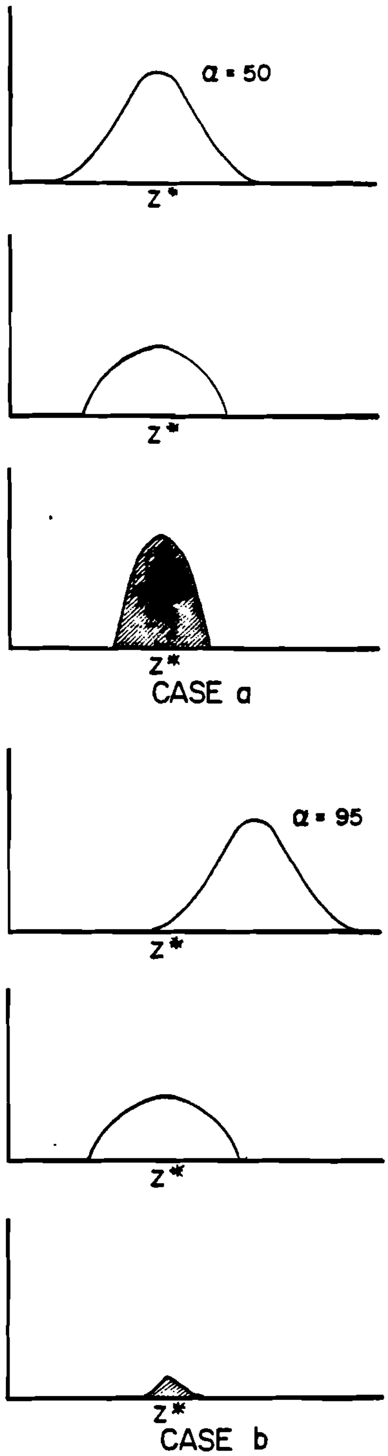

Figure 5. Non-monotonic Yield Function

results which shows that the regret can be quite substantial. This points out the necessity to carefully define the crop yield function before proceeding with a chance constraint approach.

The question that arises then is: "What is the appropriate design reliability for a crop yield function with a slope that changes sign?" To answer this question, another experiment was performed. For each type of crop yield function, a system design was found using the stochastic programming model. Then the corresponding reliability , α, on the optimal dewatering zone ,DWZ^* was found. Table 4 presents a summary of results. For Type I, the result is as expected, 98.5% reliability. For Type III, the reliability is 50%; this can be expected, since the crop yield function is symmetric around DWZ^* , so that the model will concentrate the densest portion of the probability (the mean) at the optimal yield. Although one could design for 50% reliability, this would neglect information about the variance of DWZ which has been shown above to be very important in determining expected yields. For Type II, the reliability is 54%. In this case, the crop yield function is assymetric and defining *a priori* a reliability which would reflect the optimal system performance is impossible.

5. Conclusions

These results make a strong argument for the use of stochastic programming. Chance constraint programming has been used when little or no information about the benefit function is known. This analysis has shown that this convention can lead to large losses due to the regret of assuming the wrong yield function since the chance constraint approach

Table 3. Economic Regret Due to incorrect Yield Function.

I. Drain Design based upon: Type I Yield Function

*DWZ**	= 1.0 m
Reliabiltiy, α	= 98 %
Depth, *D*	= 2.0 m
Spacing, *L*	= 34.17 m
Capital Cost	= 89.1 LE per feddan

II. Economic Regret due to actual yield function being

Type I	218.8 LE per feddan
Type II	578.5 LE per feddan

Table 4.Stochastic Programming Implications for Chance Constraint Programming.

	Crop Yield Function		
	Type I	Type II	Type III
Drain Depth,D (m)	2.00	1.46	1.43
Drain Spacing,L(m)	43.17	21.95	20.57
Capital Cost	89.14	138.23	147.43
Expected Loss	8.93	60.09	57.21
Total Cost	98.07	198.32	204.64
*DWZ**Equivalent *Reliability*[1] α %	98.5	54.0	50.0

Note: All cost in LE per feddan.
*DWZ** = 1.0 for all Types.

[1] this reliability is found by examining the probability density function produced by the stochastic programming results and determining the resulting reliability on the optimal dewatering zone *DWZ**

assumes a form to the benefit function.

An alternative approach, but still using chance constraints, is to require the system output to be greater than a lower limit and less than an upper limit with a certain reliability , thus defining a feasible range of values. However, this approach has two problems. First, to decide upon the appropriate upper and lower bounds requires almost as much information as needed to define the entire benefit function. Second, due to the irreducible uncertainty in input parameters, it may be infeasible to design a system in which the probability distribution of the ouput can meet the desired reliability for the design interval. Thus, the range of reliability would have to be changed to provide a feasible solution.

The material presented in this paper reveals that the chance-constraint approach ,outlined in Paper 1 of this series, has problems that under certain conditions cannot be overcome. The stochastic programming approach is not plagued by these problems, but requires more information and additional computation. The stochastic programming approach also provides for an explicit trade-off between economic benefits and cost of drain design and allows for analysis of multiple crop areas.

The analysis has shown that chance constraint programming is not as robust as presently perceived. Drain design using this formulation can, in certain cases, actually provide misleading results. The additional efforts needed to gather the information necessary to define the full yield function and the additional computations necessary for the stochastic programming model are well worth the effort.

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