LEGAL TENDER AND THE VALUE OF MONEY IN FINITE ECONOMIES

Caspar G. de Vries

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FOREWORD

This Collaborative Paper is one of a series embodying the outcome of a workshop and conference on Economic Structural Change: Analytical Issues, held at IIASA in July and August 1983. The conference and workshop formed part of the continuing IIASA program on Patterns of Economic Structural Change and Industrial Adjustment.

Structural change was interpreted very broadly: the topics covered included the nature and causes of changes in different sectors of the world economy, the relationship between international markets and national economies, and issues of organization and incentives in large economic systems.

There is a general consensus that important economic structural changes are occurring in the world economy. There are, however, several alternative approaches to measuring these changes, to modeling the process, and to devising appropriate responses in terms of policy measures and institutional redesign. Other interesting questions concern the role of the international economic system in transmitting such changes, and the merits of alternative modes of economic organization in responding to structural change. All of these issues were addressed by participants in the workshop and conference, and will be the focus of the continuation of the research program's work.

Geoffrey Heal
Anatoli Smyshlyaev
Ernö Zalai

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Caspar G. de Vries

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I. Introduction

Since the end of the Bretton-Woods period, with the dollar no longer being convertible into gold, economies have used fiat currencies as a means of payment. However, to account formally for a role of fiat money in economic models turns out to be difficult; see for example Hahn [1965, 1982] and Wallace [1980]. To give an example of the problems encountered, the model has somehow to exhibit the following observation made by Samuelson [1968]:"...one must remember the crucial fact that M differs from every other good (such as tea) in that it is not really wanted for its own sake but only for the ultimate exchanges it will make possible," where M stands for cash balances. Within a Walrasian general equilibrium model, money having this purpose plays no role without some sort of additional friction. It has been widely realized in the literature that in a barter economy transaction costs, like the costs of search for double coincidence of wants, could be enormous. One way to introduce transaction costs is to assume that exchange takes time, thereby reducing the available leisure time for an individual. In a

* By Caspar G. de Vries, Erasmus Universiteit Rotterdam, Postbus 1738, 3000 DR Rotterdam, The Netherlands.

monetized economy the time spent in completing transactions can be reduced by using money. A transaction technology describes such a relation between transaction costs and cash balances; see, for example, Arrow and Hahn [1971, chapter 14] and Nagatani [1978, chapter 6]. Another way to exihibit the absence of complete markets is provided by the Overlapping Generations Models (further OGM); see Wallace [1980]. In the OGM the friction is that an individual only receives an income when young. By holding money the individual can provide for his old age. Hence, cash balances are demanded and money is used because in this way an intrinsic friction can be partially overcome. We cite an account of this way of reasoning by Clower [1969, p.13]:

"...its functions as an exchange intermediary were taken over by warehouse receipts or by socially sanctioned issues of fiat money, Giro cheques, etc. Would such 'nominal' money commodities be accepted and used? The answer is in the affirmative. The use of money in exchange transaction presupposes a certain degree of organization of trading activity. Such organization is socially beneficial because it enables individuals to channel into production or leisure labour and resources that would otherwise have to be devoted to search..."

An individual might realize the benefits of a monetized exchange economy. This, however, is not sufficient to conclude that the individual actually uses money in exchange. The fiat money being intrinsically worthless, one must be certain not to get stuck with it. It is necessary that other individuals also use and accept the fiat money as a means of payment. Even if monetized exchange is beneficial to

all agents, an absence of trust between individuals might inhibit the use of fiat money; see Gale [1982, chapters 5 and 6]. The next section provides a detailed example of this possibility. Institutional arrangements are needed to overcome this problem. In Clower's words [1969, p.13]:

"...what matters is the existence of social institutions condoned by either custom or law that enable individuals to trade efficiently if they follow certain rules - in particular, the rule that one commodity traded in every exchange should be socially sanctioned as an exchange intermediary."

In a way the argument is similar to the one for the provision of public goods. Or, as Clower [ibid., pp.14 and 15] puts it:

"Money differs from other commodities in being universally acceptable as an exchange intermediary by virtue not of individual choice but rather by virtue of social contrivance."

The specific institutional arrangement we are thinking of in this context is the notion of legal tender. The Economics Dictionary by Moffat [1976] defines legal tender as:

"Legal Tender. In the U.S., paper money and coins are designated legal tender; that is, when a debtor offers paper money and coins in settlement of a money debt, he fulfills his obligation. It is generally necessary that a nation give some form of money legal tender status

because one main requirement of a money system is that it be acceptable by parties to a trade."

If a fiat money is legal tender, then the individual does not have to worry about its intrinsic uselesnesss and its acceptability to other agents for his decision problem whether to hold cash balances or not. In our view, necessary conditions for fiat money to have positive value are both, that on the individual's level using money in exchange partially overcomes some kind of inherent friction, and that on the social level an institution like legal tender exists. This seems also to be Clower's position [ibid., p.17]:

"By virtue of these devices, practical effect is given to the institution of money; the establishment of organized markets enables individuals to channel into productive activity labour and other resources that would otherwise be devoted to search and bargaining activities. But money, as money, need not be intrinsically valuable, for what matters is not the particular commodity (or commodities) that serve as money, but rather the existence of social institutions that make monetary transactions feasible and efficient."

We are not aware of any detailed or formal treatment of legal tender in the literature. Consider, for example, the fairly recent monographs on monetary economics by Niehans [1978], Nagatani [1978], Kareken and Wallace [1980], Gale [1982] and Hahn [1982], in which the above described institutional aspect of money is treated rather implicitly. The aim of the next section is to describe how, in a model with very

specific transaction costs, legal tender arises in a natural way.

The model discussed in the next section enables us to solve a problem which has bothered economists for a while. The problem is described below.

It has been argued heuristically, that fiat money cannot have a positive price in a finite-horizon OGM under rational expectations. See for example Bryant [1980, pp.223 and 236], Starr [1980, p.261], Balasko and Shell [1981, p.113], Gale [1982, p.226], and Hahn [1982, pp.4 and 5]. Cass and Shell [1980, p.252] formulate the argument as follows:

"It is obvious (and well-known) that money cannot have a positive price - that is, cannot be a store of value - in the conventional finite-horizon model in which the 'end of the world' is known with certainty. The reason is simple. At the end of the last period, money is worthless. Therefore, in the next-to-last period, all individuals desire to dispose of money holdings in order to avoid capital losses. This drives the price of money to zero at the end of the next-to-last period. And so on. Individuals with foresight, not wanting to be stuck with the monetary 'hot potato,' thus drive the price of money to zero in each period."

As a cure for this problem, Hahn [1982, p.5] suggests to adopt a form of bounded rationality introduced by Grandmont [1982], instead of fully rational expectations. Grandmont assumes that agents always attach a positive probability to money having a positive price in the next period. This assumption implies that agents attach a positive probability to the possibility that life will continue forever. One

might stop here, because probabilities can be taken subjectively and do not have to be in accordance with the laws of physics. The latter provide a "certain" upper bound on the life of the solar system.

The model of a monetary economy with legal tender which we develop below, enables us to provide a different argument for money being valuable in finitely lived economies. The argument is cast strictly whithin the structure the OGM employ. Moreover, we allow for the possibility that agents are aware of the final date of life with absolute certainty. The argument relies on the transaction-facilitating properties of money. In monetary OGM, like in the one of Wallace [1980], money functions only explicitly in the exchange between generations. It seems to be more natural if money is used also in exchanges between members of the same generation. The model set forth in the third section gives money a function both in inter-generational trade and in intra-generational trade; see Hahn [1982,p.20]. The frictions in the two sorts of transactions derive from the structure of the OGM.

2. Legal Tender

We start out to describe a simple general equilibrium exchange model with transaction costs. Transaction costs arise due to absence of double coincidence of wants. As an example, consider the following miniature trade model known from the works by Menger [1892] and Wicksell [1934, 1935]. Suppose Sweden has timber for sale and sufficient corn for its own needs, but must buy fish. Another country, Norway, can supply fish and has sufficient timber, but must buy corn. Finally, Denmark has a

surplus of corn and sufficient fish, but lacks timber. Clearly it is advantageous to all countries if Norway exports fish to Sweden, if Sweden ships an equivalent amount of timber to Denmark, while Denmark closes the triangle by selling corn of the same value to Norway. Assume the exporting firms in the three countries do not trust the importers abroad, then there must be quid pro quo in every transaction. Rule out the possiblity of payment by indirect shipments due to freight costs. It follows that the trade plans cannot be executed. However, the situation would be quite different if there existed an internationally sanctioned IOU.

Wicksell's Problem has been formalized in a penetrating paper by Cass and Yaari [1966]. They named their model the closed-loop model, but we will refer to it shortly as the Ring Model (further RM). We turn to a precise description of the RM.

Imagine the following, admittedly specific, economy. A finite number n of economic agents is arranged in a circle. Assume that agent i desires two goods labelled i and i+1. Agent i is able to produce these two goods, though he is relatively less efficient in producing good i+1 than is his right-hand side neighbor, labelled Mr. i+1. On the other hand agent i has a comparative advantage in producing good i over his left-hand side neighbor Mr. i-1. Suppose that Mr. i+1 desires the goods i+1 and i+2. In autarky, the i's agent choice problem can be formulated as follows:

$$(1) \qquad \text{Maximize} \qquad U_i = U[x_i^i, x_i^{i+1}],$$

$$\text{subject to} \qquad y_i^i + f y_i^{i+1} \leq 1, \quad f > 1,$$

$$y_i^i \geq 0, \; y_i^{i+1} \geq 0,$$

$$y_i^i - x_i^i \geq 0,$$

$$y_i^{i+1} - x_i^{i+1} \geq 0,$$

$$\text{for } i = 1, \ldots n,$$

and where $n + 1 = 1$, U is common to all agents, and $f > 1$ indicates agent i's comparative disadvantage in producing y^{i+1}. Subscripts indicate the agent, superscripts refer to the type of commodity. Output is denoted by y, consumption is denoted by x.

It can be easily deduced that it would be advantageous for all agents in the circle, if everybody would specialize in the production of the good he has an advantage in and sell it in return for the good he has a disadvantage in. The problem is, however, a complete absence of double coincidence of wants between neighbors. (It is assumed that only neighbors can trade.) To see this, note that Mr. i likes to buy good i+1 from Mr. i+1, but he has nothing to offer him in return. One could say that the transaction costs in this model are prohibitively high.

Suppose now that money is introduced in this economy in the way discussed by Cass and Yaari [1966]. Assume agent number one offers a promissory note to his neighbor Mr. 2, as a "payment" for the goods he

"buys." This note is used by Mr. 2 to pay for his purchases, etc. It is understood that Mr. 1 can be somehow designated as the only issuer of money, i.e. Mr. 1 functions as the Central Bank. We assume that the Central Bank only issues a given fixed amount of money, and that this can be accomplished at no costs.

Agent i's, $i \neq 1$, choice problem can be formulated as follows:

$$
\begin{aligned}
(2) \qquad \text{Maximize} \quad & U_i = U[x_i^i, x_i^{i+1}], \\
\text{subject to} \quad & y_i^i + f y_i^{i+1} \leq 1, \quad f > 1, \\
& y_i^i \geq 0, \quad y_i^{i+1} \geq 0, \\
& p^i y_i^i - m_i^{i-1} - p^i x_i^i \geq 0, \\
& p^{i+1} y_i^{i+1} + m_i^i - p^{i+1} x_i^{i+1} \geq 0, \\
& m_i^{i-1} - m_i^i \geq 0, \; m_i^{i-1} \geq 0, \; m_i^i \geq 0
\end{aligned}
$$

for all $i = 2, \ldots, n$,

where the p's are commodity prices, and m stands for money. We assume that the promissory note issued by Mr. 1 is divisible and can be partially retained by the other agents. The latter will not occur if we assume local non-satiation. The superscripts attached to m indicate the agent from whom it was obtained. The restriction $p^i y_i^i - m_i^{i-1} - p^i x_i^i \geq 0$ indicates the possibility of agent i accepting cash payment in return

for his delivery of y^i. On his turn, Mr i can use m_i^i to pay for purchases of x^{i+1} as shown by $p^{i+1}y_i^{i+1} + m_i^i - p^{i+1}x_i^{i+1} \geq 0$. However, Mr. i cannot use more money than he has in stock, i.e. $m_i^{i-1} - m_i^i \geq 0$; recall that only the first agent has the possibility to issue money.

We are interested in whether a monetary equilibrium exists, whether it constitutes an improvement over the autarky situation, and if it can be achieved. Cass and Yaari [1966] formulate the monetary equilibrium as follows:

(3)
$$p^i = p^{i+1} \text{ for all } i,$$

$$y_i^{i+1} = 0 \text{ and } y_i^i = 1 \text{ for all } i,$$

$$m_i^{i-1} = m_i^i \text{ for all } i,$$

and (x_i^i, x_i^{i+1}) is chosen so as to maximize $U[x_i^i, x_i^{i+1}]$,

subject to $x_i^i + x_i^{i+1} = 1,\ x_i^i \geq 0,\ x_i^{i+1} \geq 0.$

It follows immediately that this equilibrium situation is weakly Pareto superior to the autarky situation. In game theoretic terms, the trade equilibrium constitutes the Nash solution. However, it is not immediate that this equilibrium can be attained. The process of selling ones goods in return for the promissory note and then buying goods by paying with this note, goes on until the note is offered to agent one from whom it originated. If the agent is truthful, he accepts the promissory note, but nothing in the system obliges him to do so. Put it

differently, from a game theoretic point of view there is no institution present which enables the players to make binding agreements. If agents realize this possibility of default, the note might never start to travel around the circle and function as money.

In this simple economy it is perfectly rational for the first agent not to honor the promissory note once he has been able to sell it, cf. Gale [1982, p.236]. Consider the first agent's choice problem:

$$
\begin{aligned}
(4) \qquad \text{Maximize} \quad & U_1 = U[x_1^1, x_1^2], \\
\text{subject to} \quad & y_1^1 + f y_1^2 \leq 1, \quad f > 1, \\
& y_1^1 \geq 0, \; y_1^2 \geq 0, \\
& p^2 y_1^2 + m_1^1 - p^2 x_1^2 \geq 0, \\
& p^1 y_1^1 - m_1^n - p^1 x_1^1 \geq 0 \\
& m_1^1 \geq 0, \; m_1^n \geq 0.
\end{aligned}
$$

Note that this optimization problem differs from the ones for all the other agents as described in (2), in that a constraint like $m_i^{i-1} - m_i^i \geq 0$ is absent. The reasons are that the other agents are not offered the possibility to issue promissory notes, and that they first have to obtain the note before they can use it as a means of payment. For a given price vector and a positive demand for m_1^1, Mr. 1 will always choose $m_1^n = 0$. What the economy needs is a <u>fiduciary</u> system to settle

transactions. Therefore, besides the right for agent one to issue notes, the impossiblity of default by the issuer has to be guaranteed somehow.

As a necessary condition for monetized trade in this economy with a complete absence of double coincidence of wants, given that agents have the possibility to cheat and that agents have foresight in the sense that they know how the economy works, we introduce the following institutional arrangement.

(5) Legal Tender Arrangement. All agents are offered a social contract. The contract transfers the right to enforce legal tender on any agent to an outside agency, called the governement. By legal tender we understand a form of payment which, when offered, compels a vendor to sell his commodities against posted prices, and compels a creditor to accept in settlement of a debt[1]. Moreover, the contract designates one of the agents as the central banker, which means that he has the right to issue a fixed amount of promissory notes, called money. Both the enforcement of legal tender and the provision of money are costless. The social contract only becomes effective once everybody has agreed to it.

We have constructed the Ricardian production possibilities such that complete specialization is advantageous to everybody, i.e. is weakly Pareto improving. Therefore, every agent has a strong incentive to enter the social contract. There are no disadvantages from entrance, because the institutional arrangements are assumed to be made at no costs. This can be altered fairly easily, but the no-costs assumption seems to be in

accordance with the prevailing view that the provision of money is essentially costless.

The monetized exchange process can now be described as a cooperative game in which the legal tender arrangement is the binding agreement. The complete specialization solution constitutes the Nash solution. For a more detailed treatment of the game theoretic approach to monetary economics the reader is asked to consult Dubey and Shubik [1977], Shubik [1981] and Gale [1982].

The bargaining process we envision is such that an auctioneer first establishes an equilibrium price list. Once established, each agent posts prices for the goods he is willing to sell and trade can proceed in a decentralized way. The legal tender arrangement is brought out by the following condition

$$(6) \qquad p^1 x_n^1 \geq m_1^n .$$

Condition (6) says that Mr. 1, the central banker, has to accept cash payments m_1^n by Mr. n for his sales of x^1 against the posted price p^1, because money is legal tender. Note that Mr. 1 realizes that $m_1^n = m_n^n \leq m_1^1$, because he is the only one issuing money. Therefore, (6) implies in a monetary equilibrium that Mr. 1's balance of trade is zero. The claim, $m_i^{i-1} = m_i^i$ for all i, in the characterization of the monetary equilibrium (3) is now substaniated by condition (6). We are guaranteed that when the promissory note travels around the circle, it will be accepted by the original issuer for payments of purchases by the other agents.

In most general equilibrium models some kind of auctioneer and

clearing house activities are assumed to exist. The auctioneer establishes the equilibrium price after which contracts for deliveries are finalized. The clearing house collects all the offered commodities and redistributes them according to the contracts. In this way the clearing house overcomes the problem of cheating present in the above model. In the model laid out, we implicitly assumed that the clearing house activities are more costly than the implementation of legal tender. There is no a priori reason why this should be the case, it is merely an empirical question.

The arrangement of legal tender derives its plausibility from the following two facts. First, it renders the monetary system its fidelity by overcoming the problem of individual default to which the society is liable. Second, it is compatible with individual choice . By this we mean that, depending on the transaction costs structure inherent to the economy, monetized trade only develops if it brings about a weak Pareto improvement, i.e. legal tender is not imposed. We feel that our arrangement is preferable to others which have been put forward to render fiat money valuable. Clower's [1967] constraint requires that only money can buy goods. But this assumes what should be explained, see Hahn [1982, p.21]. Clearly our RM allows for barter equilibria. The use of money is not imposed without the individual's consent. Money will be used only if it has a function, i.e. if its use overcomes some intrinsic friction. Two other devices have been put forward in the literature, being the Hahn [1971] constraint and the Grandmont [1982] assumption, see Starr [1980]. As a discussion thereof is more appropriate within a dynamic model, we postpone the discussion to the next section in which such a model is developed.

3. The Value of Money in Finite Economies

We expect that the legal tender constraint (6) is present but not apparent to the individual during normal times. Only during times of extreme social upheaval, like in the period immediately preceding a currency reform or during the last days of the world, one expects legal tender to have relevance. Therefore, we develop a dynamic model in which legal tender normally plays only a latent role, and in which it becomes active during, say, "the last period of the world." This dynamic model is then used to discuss the value of money problem in finitely lived economies.

The model we have in mind derives its dynamic structure from the OGM. Transaction costs in intra-generational trade stem form the RM structure. Before we turn to a description of the model, we review another approach. Alternative to or complementary to the transaction costs present in the RM discussed above, are the search and bargaining costs indicated by the leisure time forgone. To give money its transaction function, one imposes a relationship between money and leisure time. Money facilitates transactions by reducing the leisure time needed to accomplish exchange. Such a relationship might be taken as primitive as is the utility function in consumer theory or the production function in the theory of the firm. For the plausibility of a direct relation between money and leisure time one might appeal to the articles by Clower [1969], by Jones [1976] and by Alchian [1977]. An example of this approach can be found in Arrow and Hahn [1971, chapter 14]. It has been shown[2)] that this approach overcomes many of the unlikely properties the Monetary OGM, like the one of Wallace [1980],

exhibit. However, the proponents of the Monetary OGM (further MOGM) to study the role of money might rebuke that the money-leisure time relationship has been assumed and has to be explained. The appeal to the articles of Clower, Jones and Alchian as arguments for the way in which to describe the relationship between money and leisure time, might not convince proponents of MOGM. We could be accused of "implicit theorizing[3)]," see Kareken and Wallace [1980, p.2]. Therefore, we will model the transaction frictions explicitly. Moreover, we will use the same sort of friction as exists between periods, thereby staying within the realms of the MOGM proponents.

The Cass and Shell [1980] argument for studying infinite-horizon economies is a two-step procedure. First they argue that if money does not serve as a store of value, then it cannot serve any other useful function. After which they give the reason for studying infinite-horizon economies; see the quotation in section 1. Our model shows that even if the store of value function of money is absent in the last period, money still can have value because it accomplishes double coincidence of wants between the agents trading in the last period. To establish this, we marry the OGM of Samuelson with the RM of Cass and Yaari. The transaction frictions we propose to study are twofold. On one hand we study those stemming from the trade between generations in the well-known MOGM. On the other hand, we will assume that agents within one period also trade with their own generation members according to the RM structure, thereby introducing transaction frictions within a period. A formal desrciption of the model is as follows.

Agents of each generation live for two periods. When young, they consume three goods ${}^{t}x_{i}^{i}(t)$, ${}^{t}x_{i}^{i+1}(t)$ and ${}^{t}z_{i}^{i}(t)$. When old, the i-th

agent only desires the z^i type good. Thus, the z^i good is consumed at any time by both, one of the younger generation and one of the older generation. (One could think of these agents having a parent-child relation.) The agent maximizes his utility function

$$(7) \qquad {}^tU_i(t) = U[{}^tx_i^i(t),\ {}^tx_i^{i+1}(t),\ {}^tz_i^i(t),\ {}^tz_i^i(t+1)],$$

where:

superindices t denote the generation the agent belongs to;

superindices i and i+1 denote the type of commodity;

subscripts i denote the agent;

index t within brackets denotes the period of consumption;

${}^tx_i^i(t)$ and ${}^tx_i^{i+1}(t)$ denote the consumption levels of the perishable commodities;

${}^tz_i^i(t)$ and ${}^tz_i^i(t+1)$ denote the consumption levels of the storable commodity.

We assume that the x type goods are perishable over time, and cannot be shipped during any period by agents other than the ones who desire it. In the RM of the previous section only x type goods were present. The good z, desired in both periods, is the one common to all OGM. This good can be stored to provide for consumption in the last period, but it depreciates during storage. Agent i possesses the following production and storage capacities:

$$(8) \qquad {}^ty_i^i(t) + f\,{}^ty_i^{i+1}(t) \leq 1, \quad f > 1,$$

$$\qquad {}^tq_i^i(t) + d\,{}^tq_i^i(t+1) \leq 1, \quad d > 1,$$

where:

${}^{t}y_{i}^{i}(t)$ indicates the production of $x^{i}(t)$;

${}^{t}y_{i}^{i+1}(t)$ indicates the production of $x^{i+1}(t)$;

${}^{t}q_{i}^{i}(t)$ indicates the production of $z^{i}(t)$;

${}^{t}q_{i}^{i}(t+1)$ indicates the production of $z^{i}(t+1)$.

The coefficient f indicates agent i's comparative disadvantage in the production of the (i+1)-th good. The coefficient d represents the depreciation of z^{i} due to storage.

The outcome of the individual choice problem under autarky is described by maximizing (7) subject to (8). To enhance trade, suppose that at some date an outside agency issues, at no costs, non-redeemable notes to the elderly. The then young might want to exchange some of the storable commodity for these notes to make trade with their own generation members feasible, and to resell these notes for the storable commodity when old, to overcome depreciation. At the same time as the money is issued, all members of the two generations are offered a social contract for the enforcement of legal tender. Each period, the social contract has to be agreed upon anew. For simplicity, we assume that the young generations cannot issue their own currency. Thus, they either have to use the money supplied by the elderly or stick to barter.

Assume the following trading scheme to exist within a period, if there is any trade at all. Before trade takes place, an equilibrium price list is established by the auctioneer. We assume again that individuals are fully aware of the structure of the model. First trade between the old and the young takes place, then trade between the members of the same generation is conducted.

If agent i decides to hold cash balances and to sign the social contract, he first engages into trade with the elderly. The exchange constraint for this transaction reads

$$(9) \qquad s^i(t)\,{}^t q_i^i(t) \geq s^i(t)\,{}^t z_i^i(t) + {}^{t-1}m_i^i(t),$$

where $s^i(t)$ is the price of $z^i(t)$, and ${}^{t-1}m_i^i(t)$ is the amount of money obtained from the elderly in return for a part of the production ${}^t q_i^i(t)$. This exchange constraint describes the restriction on the demand for money by the young. However, because money is legal tender there is also a supply constraint present

$$(10) \qquad s^i(t)\,{}^{t-1}z_i^i(t) \geq {}^{t-1}m_i^i(t).$$

This latter constraint says that the young are obliged to sell $z^i(t)$ when they are offered money ${}^{t-1}m_i^i(t)$, vide our definition of legal tender in (5).

After this, agent i engages into exchange with Mr. i+1 and Mr. i-1 of his generation. From agent i+1, agent i can purchase good x^{i+1} against price p^{i+1}. The purchase restriction reads

$$(11) \qquad p^{i+1}(t)\,{}^t y_i^{i+1}(t) + {}^t m_i^i(t) \geq p^{i+1}(t)\,{}^t x_i^{i+1}(t).$$

Mr. i-1 on his turn purchases ${}^t x^i$ from Mr. i against price p^i, in return for ${}^t m^{i-1}$. Agent i's sale restriction reads

$$(12) \qquad p^i(t)\,{}^t y_i^i(t) \geq {}^t m_i^{i-1}(t) + p^i(t)\,{}^t x_i^i(t).$$

By our assumption, money is legal tender and agent i has to accept cash payments ${}^{t}m^{i-1}$ by Mr. i-1 for sales of ${}^{t}x_{i}^{i}$. Formally, legal tender can be expressed by the restriction

$$(13) \qquad p^{i}(t)\,{}^{t}x_{i-1}^{i}(t) \geq {}^{t}m_{i}^{i-1}(t).$$

Condition (13) can be formulated differently. Notice that, given the price level, conditions (11) and (12) leave open the amount of money used in intra-generational trade. This is so because the model leaves the velocity of money unspecified. If for example the price level is high and the amount of money obtained from the elderly is low, then total purchases of a commodity can be effected in more than one transaction. This possibility arises because we disregard the time needed for purchase and payment. In principle, all transactions could be effected with the same coin, cf. Wicksell [1935, p.18]. This begs the question as to why the young in "the last period of the world" are willing to hold some useless fiat money, i.e. why the currency notes will not circle around indefinitely. The reason that this does not occur is as follows. Suppose an optimal allocation has been achieved and that agent i buys some extra ${}^{t}x^{i+1}$ from Mr. i+1 in return for some ${}^{t}m^{i}$. Agent (i+1)'s position detoriates, but he can recapture part of his initial prosperity by spending the extra amount of money received on ${}^{t}x^{i+2}$. In the end, agent i's neighbor Mr. i-1 will purchase some ${}^{t}x^{i}$, thereby detoriating agent i's position with respect to the initial allocation. The upshot of this story is, that it is rational for every agent to stop purchasing once the optimal allocation has been reached, because in the

end the extra money spend will come back to the initial spender. In other words, it is rational for all agents to pursue a "Tit for Tat" strategy, see Hofstadter [1983]. To capture this idea, condition (13) is reformulated as a reaction function:

$$(14) \qquad {}^{t}m_i^{i-1}(t) = r({}^{t}m_i^{i}(t)),$$

where $dr/d{}^{t}m_i^{i}(t) = 1$. Equation (14) says that one can expect to receive an amount of money from ones neighbor i-1 equal to the amount one spends on x^{i+1} purchased from the other neighbor.

During the next period t+1, the now old agent i can exchange his currency holdings ${}^{t-1}m_i^{i}(t+1)$, which were obtained from the elderly in the previous period, with the then young in return for $z^{i}(t+1)$. His second period budget constraint reads

$$(15) \qquad s^{i}(t+1)\,{}^{t}q_i^{i}(t+1) + {}^{t-1}m_i^{i}(t+1) \geq s^{i}(t+1)\,{}^{t}z_i^{i}(t+1).$$

During normal times, when constraint (15) is operative, conditions (10) and (13) are automatically satisfied, because it is rational for the young to provide for their old age by holding cash balances. In this way storage inefficiences due to depreciation can be overcome. However, once the last period of the world has come, condition (15) is irrelevant. At this stage the legal tender restrictions (10) and (14) become active[4]. Whether agents are willing to enter the social contract in the last period depends on the transaction costs inherent to the RM structure, just like in the previous section, in combination with the costs incurred by obtaining the money from the old generation.

Agent i's choice problem under the regime of monetized exchange is to maximize his utility function (7) subject to his transformation possibilities (8), the exchange constraints (9), (11), (12), and (15), and the legal tender restrictions (10) and (14). Whether a monetary equilibrium exists if the economy is finitely lived will be discussed shortly. First, we like to settle whether money in our model can be designated as fiat money, and we want to return to the discussion at the end of the previous section.

Wallace [1980] defines fiat money as having the qualities of being intrinsically useless and of being inconvertible. If we translate intrinsic uselessness by the requirement that money is not an argument of any agent's utility function, then our model clearly satisfies this requirement. Inconvertibility means that the issuer does not promise to convert the money into anything else. From the setup of the monetary system it is clear that the notes which circulate as money are non-redeemable.

We are now in a position to come back to the discussion of the reasonableness of the legal tender arrangement in relation to other institutions which have been put forward in the literature. The Hahn [1971] constraint requires agents to pay fixed money sums to the government at the final date. In our model agents do not have to hold money for this purpose. The Hahn restriction "imposes free disposal" upon individuals, because agents have to give up something for nothing. In the model outlined above, individuals only choose to hold money if they can acquire something more valuable in return. With respect to the Grandmont [1982] assumption, legal tender, together with the RM structure, explains why agents attach positive probabilities to money

having value at each conceivable future date. A somewhat different approach is taken by Dubey and Shubik [1977] and by Shubik [1981]. They assume that individuals can obtain fiat money from an outside agency through competitive bidding with personal IOU notes. These IOU notes must be redeemed after trade, and this is guaranteed by a default penalty added to the individual's utility function. It seems that Dubey and Shubik's approach is in essence the one taken by Hahn.

Finally, we face the question whether there exists a monetary equilibrium in our model. By a monetary equilibrium we will understand the existence of a price vector which clears all markets, and which contains positive prices for money at some dates. With rational expectations the crucial question is whether money has a positive price during the last period. This depends on whether or not the young in the final period have a positive demand for cash balances.

It is intuitively clear that the young will hold cash balances if the gain in specialization in the production of the perishable goods outweighs the loss of consumption in the storable good. Money is necessary to induce agents to specialize in the production of the perishable goods. Put it differently, trade in the perishable goods is not possible without money because of the absence of double coincidence of wants. The legal tender assumption, however, guarantees that agents accept cash payments to settle debts.

Suppose that agent i's utility function takes the form

$$U_i = \ln x_i^i + \ln x_i^{i+1} + \ln z_i^i + \ln z_i^i(1),$$

where we have omitted indices if no confusion is possible. The

transformation restrictions read $1 - y_i^i - 2y_i^{i+1} > 0$, and $1 - q_i^i - 2q_i^i(1) > 0$. The exchange restrictions are as specified above. The last period's young agent maximizes $U_i = \ln x_i^i + \ln x_i^{i+1} + \ln z_i^i$. The production restrictions for these agents are $1 - y_i^i - 2y_i^{i+1} > 0$, and $1 - q_i^i > 0$. In autarky, the last period's young choose $z_i^i = 1$, $x_i^i = \frac{1}{2}$, and $x_i^{i+1} = \frac{1}{4}$. Suppose now that the elderly offer one unit of money, and suppose that the prevailing price vector is $(p^i, p^{i+1}, s^i) = (3,3,3)$. Then it follows that the young agent i has a demand vector $(x_i^i, x_i^{i+1}, z_i^i, m^i) = (\frac{1}{2}, \frac{1}{2}, \frac{2}{3}, 1)$. In this case money has a positive price and the young are better of than in autarky. It is easy to check that at this price vector all markets clear, and one can show that the demand vector maximizes the agent's utility function given the price vector. Thus, the posted price vector is an equilibrium price vector. If desired, one can continue this example by studying the period preceding the final period. By choosing the price vector $(p^i, p^{i+1}, s^i, s^i(t+1)) = (2,2,2,3)$, one can show again that this is an equilibrium price vector, and that it is in accordance with the one prevailing in the last period. Again, money is valuable and everybody is better off with respect to the autarky situation. Note the rise in prices which occurs in the last period.

Notes

1) Note that we have broadened the definition of legal tender by Moffat that was stated in section 1. The motivation for our definition is based upon how the civil code circumscribes the exchange process. To take one example, according to the Dutch law any exchange consists of two stages. First, the agreement to exchange has to be established. The second stage involves the fulfillment of the agreement. The establishment of an agreement is conditional upon the consent by both parties and some other conditions, like the legality of the subject of the agreement. The fulfillment of the agreement is achieved by delivery and payment. According to the interpretation of the Dutch law: in case a vendor posts prices it is understood that the consent for the agreement to exchange is already given by him. If a purchaser expresses the desire to purchase a given quantity against posted prices, then, according to the Dutch law, an agreement to exchange is said to be established. Hence, the obligation by both parties to fulfill the commitments of the agreement exists. The agreement is fulfilled whence delivery has taken place and payment is offered to the vendor. The vendor has to accept payment in legal tender notes in settlement of the agreement. From an economic point of view it is irrelevant to distinguish between the two stages of an agreement. In our definition of legal tender we combine the two stages by simply assuming that a vendor posts prices, established by the auctioneer, for the goods he is willing to sell, and that he is obliged to sell these goods upon an offer of money when it is designated legal tender by the social contract.

2) For reference, see Hahn [1980], McCallum [1982] and De Vries [1982].

3) We still feel that a relationship between leisure time and money might be taken as a primitive assumption.

4) The definition of the Legal Tender Arrangement (5), which was designed for the RM, has to be made fit for the dynamic model. The

legal tender notes are no longer promissory notes, they are non-redeemable notes which were issued to the elderly by an outside agency sometime in the past. In the RM the arrangement had only relevance for the exchange between the n-th and the first agent. In the dynamic model, however, it is of direct importance to every young agent in the last period. By our definition of legal tender, vendors are compelled to sell commodities against posted prices when they are offered legal tender notes. Therefore, once the social contract is agreed upon in the last period, the young ones are willing to sell their commodities and to accept in return the worthless legal tender notes. The young agents realize that they will end up with holding some of these intrinsically worthless notes; but, they realize at the same time that in this way they can capture the gains from trade.

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