WORKSHOP ON THE VISTULA AND TISZA RIVER BASINS 11-13 FEBRUARY 1975

ANDRAS SZÖLLÖSI-NAGY, EDITOR

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International Institute for Applied Systems Analysis
2361 Laxenburg, Austria

PREFACE

This is the written version of presentations that were delivered in February 1975 at the IIASA Workshop on the Vistula and Tisza River Basins. The contributions reproduced here were prepared by IIASA scholars and by our guests, scientists from Hungarian and Polish research institutions.

In accordance with the results of the IIASA Planning Conference in June 1973, the Water Project has focused its attention on specific universal methodological problems of water resources development and optimal operation. Large-scale demonstrations of problems and methods of river basin management represent an important part of this research program. The Tisza and Vistula case studies provide an excellent framework for discussing the practical significance of methodological developments.

A number of our IIASA colleagues have helped in getting these Proceedings into print. To all, my sincere thanks.

Zdzislaw Kaczmarek

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Schloss Laxenburg

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E. Wood

The Vistula River Project

Aleksander Laski and Janusz Kindler

1. INTRODUCTION

The control and utilization of water has a long history in Poland. Compared with many other European countries, nature has bestowed on Poland limited water resources and great variability of their occurrence. This has forced the population to learn early the importance of rational water management. Initially, water resources management was concerned predominantly with flood control, river navigation, and hydroelectric power production. The period following World War II, however, saw a distinct shift toward multiple-purpose water projects designed to meet the needs of growing population, industrial development and urbanization.

Poland is a socialist country where planning is the basic instrument for management of the economy. National planning implies that central governmental bodies draw up guidelines for all aspects of the social and economic development of the country as a whole. The starting point for an overall plan is a projection of the principal macroeconomic categories such as GNP, national income, amount of foreign trade, individual and collective consumption, and output of the basic economic sectors. During planning, there is a continuous two-way exchange of information between the lower organizational levels and the central planning agencies. This is a dynamic process leading by successive approximations to the formulation of the overall plan for social, regional and economic development. It is important that the reader recognize at the outset that the water planning and management activities discussed in this paper should be seen in the broad context of the national planning process.

The first long-term national water resources development plan was drafted by the Polish Academy of Sciences in the years 1953-1956 (time horizon of 1975). The plan was then twice revised in the early 60s by the National Water Authority, and the time horizon extended to 1985. By 1968, it became clear that the water situation, particularly in the Vistula River Basin which covers about 54% of the country's area, required special attention. Preliminary long-term projections developed by the Planning Commission and the Polish Academy of Sciences indicated that the state of water availability in the basin was not compatible with future demands. In 1968, comprehensive studies were initiated with the assistance of the United Nations Development Program and the United Nations itself, under the name of the "Vistula River Project" ("Planning Comprehensive

Development of the Vistula River System"). On the Polish side, "Hydroprojekt," a firm of consulting engineers, was charged with preparation of the project and its coordination with numerous cooperating agencies.

The goal of the project was to formulate a water resources development (investment) program capable of meeting demands projected to the years 1985 and 2000. It was assumed that the project would make use of all possible improvements in the methodology of designing and operating large-scale and complex water resource systems (application of mathematical techniques, computer simulation, and the like). Continuous revision and verification of plans is unavoidable in a rapidly expanding economy; the value of an operational tool for quick evaluation of the consequences to water management of some new development concepts and alternatives cannot be exaggerated.

2. DESCRIPTION OF THE BASIN

The Vistula River Basin has a total area of 194,000 km^2, of which 168,000 km^2 lies within Poland (see Figure 1). On the south is the Carpathian mountain range with its highest peak at 2663 m. The average altitude of the basin is 270 m, and most of the area (55%) lies between 100 and 200 m above the Baltic Sea level. The topography and geology of the basin account for the fact that all the more significant potential storage sites are in the Carpathian region. This region is also marked by a comparatively high precipitation of about 800 mm (mean annual value). The average annual precipitation in the basin amounts to about 600 mm, while the lowest in Central Poland is less than 500 mm. Mean values of evaporation fluctuate from 390 to 520 mm annually. The relative humidity varies from 76% in the south to 84% in the north.

The climatic conditions are characterized by frosty winters (December-March), when the temperature may drop to -30° C, and sunny summers (June-August), with temperatures ranging from 15° C to 30° C. Spring and autumn are transition periods typical for the continental climate. The atmospheric conditions, however, show great variability when the same seasons or even months of different years are compared.

In 1969, the population of the Vistula River Basin was about 19.5 million, giving a density of about 112 inhabitants per km^2. The rural population then accounted for about 53%, but this percentage has diminished each year. Of the population employed in the basin, about 35% were engaged in agriculture, 25% in industry, 10% in construction, and 30% in transportation, trade, finance, education, health service, public administration and other services.

Figure 1. Vistula River Basin.

Although about 68% of the total national industrial production originates in the Vistula Basin, there is a distinct concentration of industry in the southern region of Katowice and Cracow, and in the region of Warsaw and Lodz. The first one in particular is a huge industrial conglomeration which grew up around the Upper Silesia Coal Basin. The basin ranks among the richest in the world and its coal reserves are estimated at 70 billion tons. Other resources of importance in the basin are sulphur, zinc, natural gas, gypsum and various building materials.

The area under cultivation in 1969 was 65% of the total basin, and the medium fertile soils of post glacial origin are dominant. The principal crops are grain (58.9%), potatoes (19%), fodder (13.7%) and sugar beets (4%). Irrigation practices are at present limited to the grass lands producing hay, which is the principal cattle fodder. The forests amount to about 26% of the basin area. Coal-based thermal power constitutes approximately 97% of Poland's total generation of electricity and about 94% of its total installed capacity.

On the whole, the Vistula River Basin (see Figure 1) is a region of rapidly developing economy with a large number and variety of problems confronting water management. At present, the most important of them are water supply for the population, industry and agriculture; water pollution control, and flood control.

3. THE RIVERS AND WATER RESOURCES

The Vistula River may be divided into three main reaches. The Upper Vistula stretches from the Sola River 280 km north along the main stem to its confluence with the San River. Although the so-called Carpathian tributaries are characterized by great variability of flow, and the flood hazard is very high here (summer floods induced by rainfall), they are most important as a source of good-quality water for the highly industrialized southern part of the basin. The catchment area of the Upper Vistula amounts to about 27% of the entire basin, but the mean annual flow in the Vistula below the outlet of the San River approximates 40% of the mean annual flow discharged to the Baltic Sea. Implementation of a long-term program of reservoir construction on the Carpathian tributaries has been going on for several years. The program also includes a system of water transfer installations. On the main stem of the river, upstream from Cracow, there are three low-head barrages provided with navigation locks. They are the first elements of a future waterway intended primarily for transportation of coal and various bulk commodities to the industrial plants (among them, coal-burning thermal power plants) located in the lower reaches of the Vistula. The problem of water pollution is severe in the Upper Vistula.

The Middle Vistula is a 270-km reach between the San and Bug Rivers. It is a virtually undeveloped river, except for some river training works and flood levees. The river channel is typical for large lowland rivers, with numerous branches, islands and shoals which practically exclude its use as a navigation waterway. The Middle Vistula, however, has already attracted a considerable number of large industrial plants and high-capacity thermal power plants. During the low-flow periods, which sometimes last for several weeks, water supply and pollution control problems are acute. In contrast to the Carpathian tributaries, most serious floods occur here in the early spring as a consequence of rapid snow-melt. The flooding phenomena are further aggravated by ice jams damming up the river. The catchment area of the Middle Vistula occupies about 21% of the entire basin.

The Lower Vistula extends 391 km from the Narew River confluence to the Bay of Gdansk. Its resources are considerably augmented by the waters of the Narew and the Bug tributaries, whose catchment area (predominantly agricultural) amounts to about 32% of the entire basin. The relatively deep valley of the Lower Vistula makes possible the construction of a cascade of multiple-purpose barrages; the first of them has already been constructed at Wloclawek. Although the barrage heads are comparatively small (less than 10 m), streamflow rates are sufficiently high for economically attractive development of the hydropower potential of the river. Among other important goals are the development of navigation and stabilization of water levels for numerous municipal and industrial intakes. The flood phenomena have characteristics similar to those of the Middle Vistula.

An important factor is that the total potential reservoir storage capacity within the Vistula Basin is less than about 15% of the mean annual runoff. Opportunities are limited, therefore, to within-the-year storage and flow regulation. At present, total conservation reservoir volume in the basin amounts to 1200 million m^3 (about 4% of the mean annual runoff).

Some of the characteristic Vistula River flows are presented in the following table.

Profile	Catchment area (km^2)	km	Minimum 1% flow NQ (m^3/s)	Mean annual flow (m^3/s)	Maximum 1% flow WQ	Irregularity rate WQ NQ
Krakow	8021	76.5	14.1	84.0	3400.0	243
Sandomierz	31781	265.0	44.7	274.0	6120.0	139
Warszawa	85176	513.8	110.0	541.0	7820.0	71
Tozow	193170	909.0	229.0	961.0	9130.0	40

Any analysis of potential development and operating alternatives in the river basin requires estimates of the future uncontrolled flows at numerous gauging stations and control sites. These uncontrolled flows are the resources to be regulated and allocated to various water users. In the "Vistula River Project", the quantity and the distribution of future flows were assumed to equal mean monthly flows recorded at over 100 gauging stations for the fifteen-year period from 1951 to 1965. While there are gauges that have records over more than 100 years, inaccurate or incomplete information throughout the basin precluded the adoption of a longer historical record. The potential limitations of using only the relatively short historical record are at least partially offset by the fact that the 1951-1965 period was a relatively dry one. This was ascertained in a special study, taking advantage of much longer historical traces that exist for some gauging stations throughout the basin.

To comply with "Vistula River Project" requirements, an assessment of available ground-water resources included (1) a regional ground-water study of the entire basin, and (2) a study of 47 localities where water supply shortages existed or would exist in the near future. These localities were selected on the basis of previous water management planning studies. The regional study estimated the available ground-water resources in the basin at approximately 50 million m^3/day. According to the hydrogeological conditions prevailing in the individual parts of the basin, water supply yield per one km^2 varies from 40 to 1000 m^3/day (the mean for the entire basin is 273 m^3/day).

The quality of water resources should be viewed in light of the official regulations in force in Poland. At the time of Project preparation, there were four acceptable water-quality standards, each pertaining to a general-use category, which defined the maximum permissible concentration for a large number of water quality parameters. The reference flows for all water-quality analyses are the statistical means from all recorded minimum annual flows. To illustrate the magnitude of the water-quality problems: in 1969 about 80% of the main stem of the Vistula and about 40% of the total length of the major tributaries were below the lowest acceptable water-quality standard. It is evident that the target quality levels established for each reach of the Vistula River System were not being met then, and cannot be met now or in the future without additional investments in waste-water control facilities.

4. PLANNING STRATEGY

Optimization of an investment policy in multiple-purpose and multi-reservoir water control systems is an intricate and complex problem. Since conventional water resources planning procedures cannot provide a comparison of a large number of alternative investment programs, it was decided at the outset that maximum possible use would be made of mathematical techniques and digital computer facilities.

Major difficulties in the Vistula River Basin are caused by:

- The numerous interrelated water requirements of virtually all branches of the national economy;

- Social demands and environmental protection requirements exerting a distinct influence on the objectives of water use;

- The different formal procedures by which water resources are distributed to various users, expressed by a variety of technical and administrative arrangements for water management;

- The special character of water resources development investments (high capital outlays and time-consuming design and construction);

- Delays in implementing a water resources development program not compatible with the overall social and economic development of the country.

A starting point for methodological studies was a proposal made by the Institute of Environmental Engineering, Warsaw Technical University, for a spatial and problem-oriented decomposition of the system. Such a decomposition was justified by the exceptional size of the Vistula River Basin, the large number of users, the complicated system structure, and the limited computer facilities available at that time. It was decided, therefore, to decompose the basin spatially into 13 subsystems (see Figure 2). Each of these represents an area whose economic structure is as uniform as possible, which is of homogeneous hydrological nature, and which creates similar hydraulic engineering problems.

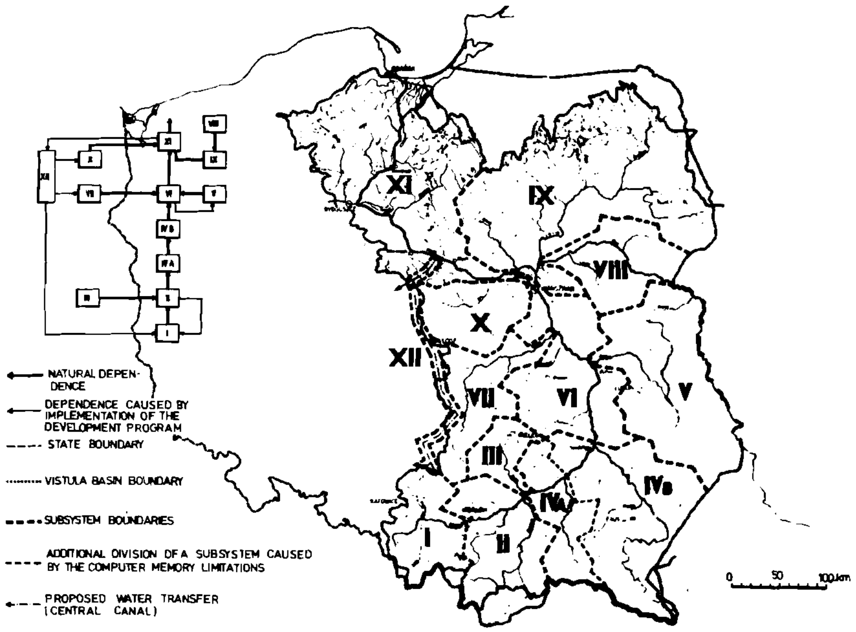


Figure 2. Topology of subsystems.

With regard to problem-oriented decomposition, the proposal--in conformity with the special character of water management in Poland--was directed mainly toward the problem of water supply for the population, agriculture and industry; toward water pollution control; and toward independent investigation of the most rational solutions for flood control.

The list of water control objectives identified in the "Vistula River Project" included:

1) Water supply to the population, agriculture and industry;

2) Maintenance of the minimum acceptable flows (established via a detailed study of the environmental effects of various minimum flows);

3) Water pollution control;

4) Flood control;

5) Development of recreational facilities;

6) Development of hydropower production and inland navigation, taking into consideration the effectiveness of alternative power production and transport modes.

The target values of all water control objectives have been established by the specialized agencies (14 ministries in collaboration) for two levels of future development, 1985 and 2000. The common base for all projections has been the national long-term development plan. Final compilation, critical evaluation and preparation of these data has been assigned to the National Water Authority and its agencies, especially the "Hydroprojekt" previously mentioned. The total water requirements are shown in Figure 3.

Awareness of the fact that some control objectives will not be covered by mathematical modelling led to the necessity for post-optimization analyses and formulation of program proposals based on interpretation of the results of all Project investigations.

The Project was elaborated in the years 1968-1972 in the following steps:

- Formulation of the so-called "program card" and delineation of major methodological assumptions;

- Preparation of basic information and its transformation into project data;

- Development of mathematical models;

- Preparation of input data for these models;

- Simulation of water resources allocation in each subsystem, and analysis of investment alternatives;

- Evaluation of simulation results and choice of acceptable solutions;
- Definition of the most rational solution and formulation of the investment program for the Vistula River Basin.

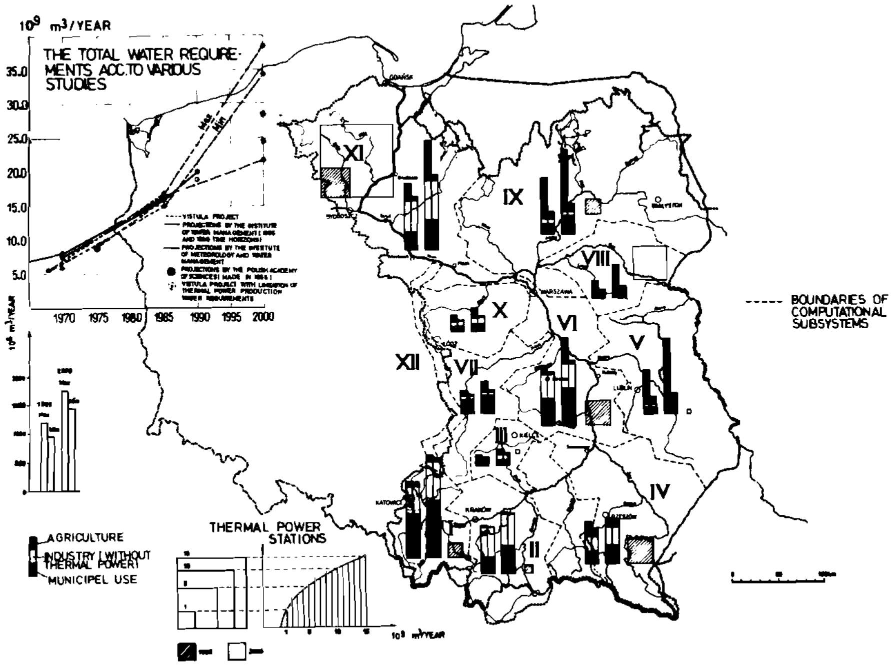


Figure 3. Total water requirements.

The fundamental assumption which underlies the planning strategy is that the investment program alternatives--including reservoir sites, water transfer routes, diversions and sizes of all hydraulic engineering undertakings--are reduced to a finite set Dij by subsystems i and variants j. Different sizes at the same locations have been analysed via separate variants. Potential limitations of this approach are, in the case of the Vistula Basin, almost completely offset by the fact that development opportunities are unfortunately quite restricted, owing to topographic and geological conditions.

The primary thrust of modelling activities was directed toward development of the so-called Water Resources Management Model. Once each investment variant was defined for a given subsystem, its performance was evaluated by a simulation-optimization procedure (WRM Model) to allocate each of the 180 monthly flows on record to each water use (target values for 1985 and 2000 corresponding to control objectives (1) and (2) and hydropower production). When an allocation fell short of the target, it was defined as a deficit allocation. Each deficit was weighted by a penalty factor reflecting the relative priority of each water use over all other users.

At the beginning of the Vistula study, the intention was to estimate these penalty factors in monetary terms. The question to be answered was how high the economic losses would be if some of the water demands were not satisfied or at least not fully satisfied. Capital outlays and operation costs for a given investment alternative were to be compared with losses due to possible water shortages and those benefits which could be evaluated in monetary terms. Evaluation of every investment alternative was to be made according to the formula:

$$(J + K) + S - E ,$$

where

J = capital outlays for water resources investment

K = operation costs

S = losses due to water deficits

E = measurable economic benefits (e.g. hydropower production, averted flood losses, overall economic promotion of a given region).

Unfortunately, the preliminary studies showed that for many water users there is no way of assessing losses due to water deficit in comparable units. Moreover, at least some of them could not be expressed strictly in monetary terms and their comparison with costs (J and K) and benefits (E) proved to be infeasible. This fact forced the Vistula planners to modify the approach originally contemplated.

An attempt was made, therefore, to define the hierarchy of water users by a system of weights to be used in the simulation analysis of water resources allocation in each investment variant contemplated. Analysis of the J, K and E factors was consequently shifted to the next phase of post-optimization analysis.

The system of weights finally adopted is based on the relationship between the unit production costs of a given commodity and the volume of water needed per production unit. Unit production costs were evaluated on the basis of economic studies carried out by each branch of the national economy. Projections of technological changes were taken into account in evaluating the indispensable volume of water. In general, the weights were computed according to the formula $W = A/Q$, where W is the weight expressed in million zlotys per 1 m^3/sec (mean monthly flow). For each branch of the economy a number of weights were computed, $W_1 = A_1/Q_1$, W_2, $= A_2/Q_2, \ldots, W_n = A_n/Q_n$, and the arithmetic mean weight was adopted as representative for this branch. The branches included heavy industry, light industry, etc.; thermal power production, taking into account various cooling systems; water transportation; and agriculture (irrigation of pasture lands, arable lands and fishpond water requirements). For agriculture, mean world prices for agricultural products were taken, rather than unit production costs.

The procedure used was not suitable for evaluating weights for such requirements as water supply to the population and maintenance of minimum acceptable flows. For these control objectives the weights were adopted arbitrarily; water supply to the population was taken to have the highest priority with the highest possible weight, while the weights of the minimum acceptable flows varied depending on some additional studies.

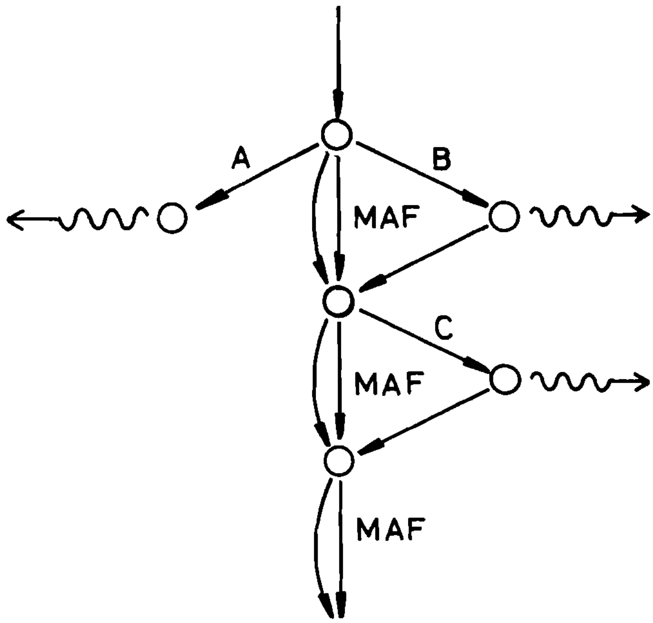
In one of the Vistula subsystems, a special study was made to assess the impact of different weight patterns on water management.

The general conclusion was that the water resources allocation process depends primarily on the proportion of weights and not on their absolute value. This statement is illustrated by the simple example below.

In the case of water user A, all water withdrawn from the river is being consumed (supply = consumption losses). Users B and C have very small consumption losses, and high percentages of water withdrawn if it is discharged back to the river. It is obvious that the decision on allocation of water to A or B depends not only on the relative value of their weights, but also on the relation between the weight of A and the aggregate weight of B, C and the minimum acceptable flow (MAF).

It seems that the system of weights could be considerably simplified. Instead of a search for more accurate values, future studies should concentrate on evaluation of arbitrary weights and their proportions.

The model ensures minimization of the sum of weighted deficit allocations. This set of allocations was defined as the optimum for a given investment variant. Because of its basic importance to the "Vistula River Project", the WRM Model is discussed in more detail in the next section.



It should be noted that the WRM Model has been used to evaluate the potential investment variants within each separate subsystem. Since each subsystem affects the flows available for use in downstream subsystems, and is subjected to regulated flows from upstream subsystems, some means had to be found for incorporating within each subsystem some consideration of the neighboring subsystems. The problem was solved by analysing all subsystems sequentially, starting with the one farthest upstream. To allow for downstream needs, it was judged that it would be sufficient to establish a monthly target of the mean annual flow at the boundary with the neighboring downstream subsystem. Deficits in flows at this location were weighted and included in an objective function to be minimized. Before proceeding to the next subsystem, the previously analysed investment variants were carefully inspected, and those incapable of meeting high-weighted target demands within 5% tolerance were excluded from further analysis. Regulated monthly outflows from the upper subsystem were next used as an input to the lower subsystem. Again it must be stressed that potential limitations of this approach are, to a large extent, offset by a finite number of intensively studied development (investment) opportunities. Altogether, 148 investment variants were analysed in this way--46 for the time horizon of 1985 and 102 for the year 2000.

In parallel with the WRM Model, four mathematical models were developed to assess the characteristics and the outputs of the hydro-electric power plants planned in the basin. The first POWDYN Model (dynamic programming) was developed to determine the optimal operating policy, optimum firm power, installed capacity and optimum monthly releases from the storage reservoirs when the latter are operated only for energy production. The releases (outputs of this model) were afterwards taken as target values for the hydro-electric water used in the WRM Model. The second POWREC Model (simulation) computes the energy outputs for each of the 15 years of the simulation period, using for each reservoir the optimum releases derived from the WRM Model. As a result of the simulation, a new firm power value, as well as a newly installed capacity of each storage power plant, was determined. These figures were next used for evaluating the economic effects of the hydro-electric power plants planned at the various storage reservoirs in each development alternative. Two other hydro-power simulation models have been developed for optimizing the installed capacities and energy outputs of the low-head hydro-power plants on the Lower Vistula, due consideration being given to their thermal alternatives.

The analytical phase of the work concerned with flood control has concentrated on developing a mathematical model to simulate flood wave propagation in order to assess the value of flood damage reduction for the various investment alternatives. The assessment was based on the cumulative frequency distribution curves of the losses with and without the planned storage reservoirs and flood protective levees. In the first attempt, a flood routing model based on the Saint Venant equations of motion was developed, but the difficulties encountered in adjusting this model to the hydrographs of the reference waves forbade its general application. Finally, the flood propagation computations were carried out by using a simplified model, a modification of the SSARR Model (Rockwood, 1968).

In the analysis of water quality regulation, the project study has focused on estimating the cost of required treatment for the future augmented stream flows at various control profiles throughout the river system analysed. The results have shown that flow augmentation, resulting from implementation of the contemplated investment program, does not appreciably alter the required wastewater treatment costs. This permitted separating the analysis of water quality regulation from that of water quantity control (except for the minimum acceptable flows which were explicitly incorporated in the WRM Model). The studies on water quality have started with a survey of all pollution sources currently discharging more wastewater per day than 1000 m^3 (with a 500 m^3/day rate for the chemical and food industries). Wastewater discharge projections have taken into account changes in industrial technologies, as far as these can be anticipated. Analysis of wastewater treatment alternatives included determination of the stream's assimilative capacity and assessment of various treatment facilities, as well as mechanical, chemical and biological treatment installations, combined regional treatment plants, effluent storage reservoirs, and the like.

It has already been mentioned that except for some local traffic and the Lower Vistula waterway, inland navigation in the basin is largely undeveloped. Following evaluation of the potential demand for water transport in the years 1985 and 2000 (respectively 44 and 79 million tons per year), numerous alternatives of waterway development were analysed. They included river training works in parallel with river canalization by a cascade of barrages. Alternatives took into account not only different engineering facilities, but also the type of traffic which moves only locally, as distinct from traffic involving longer distances. Depending on the type of traffic, trucking and rail transport served as the alternative transport modes considered for evaluation of the relative effectiveness of water transport. The additional costs of extra warehouse and standby rail facilities, for use during the winter when the Vistula is frozen, were also taken into account in the economic analysis.

The last water control objective to be briefly mentioned is the development of water-based tourism and recreation. Overall social and economic development, as well as the increase of per capita income in Poland, fosters continuous growth of demand for recreational facilities. Evaluation of the recreational value of the contemplated development (investment) alternatives involved a checklist of site characteristics, such as the local physical factors and the distance of the site from centres of population. Each item on the checklist was rated in points according to its value as a recreational feature. The points for all items were then totalled and the site was ranked in one of four classes, from "special" through "very good", "poor" to "inferior".

It can be seen from the foregoing that the potential development (investment) alternatives have been subjected to the screening process predominantly with the aid of the WRM Model. The next phase of the so-called "post-optimization analysis" was restricted to investment alternatives which survived the screening process, at the same time taking account of the results of all other studies carried out within the framework of the Project. At this point in the investigation, final recommendations on flood control storage capacities were formulated. All the basin-wide investment alternatives had also been assessed for their economic feasibility. The economic analysis (net present value approach) had taken into consideration (1) capital requirements, with due regard for the cost of capital immobilization during construction; (2) annual charges, including operation and maintenance, replacement and capital repair costs, interest on capital invested and amortization;

(3) measurable benefits, including primarily power production, flood control, navigation, and promotion of regional development; and (4) agricultural losses due to elimination of permanently inundated land (storage reservoirs).

Final formulation of the basin-wide investment program has been carried out taking account of the different possible levels of capital expenditure. Therefore, two alternative programs have been formulated for the target year of 1985, two for 2000 and two for the ultimate development of the basin's water resources. The programs were so arranged that the highest reviewing authority, the Planning Commission of the Council of Ministers, could make the final choice before presenting the program for Government approval. Some of the key elements of the proposed investment programs are shown in Figure 1. Figure 4 shows the projected water deficits before and after implementation of the program.

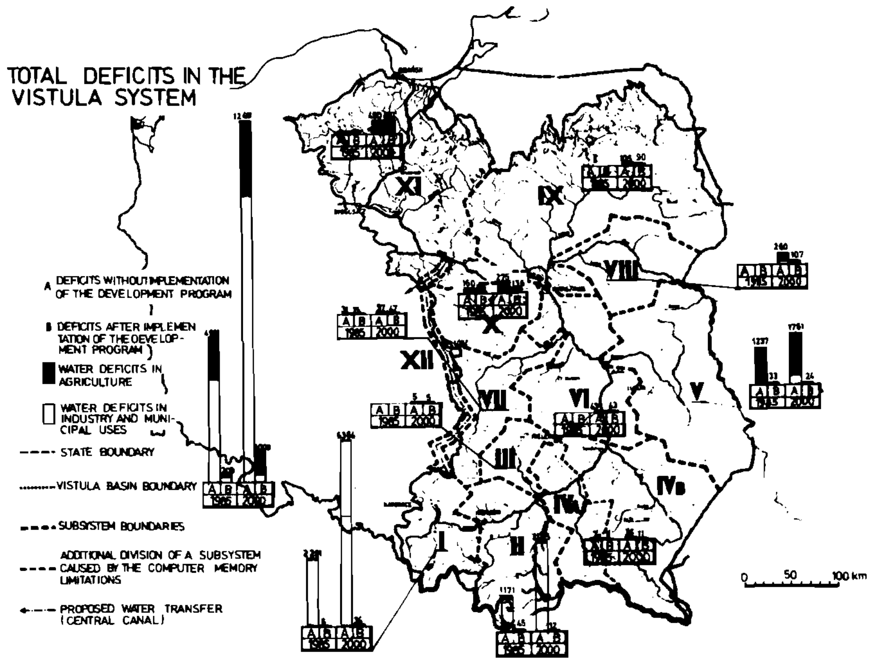


Figure 4. Total deficits in the Vistula system.

Further work in 1972-1974 and current studies have led to improved methodology, increased precision of analysis, and an enlarged research field.

The Vistula Basin was again examined in 1973-1974 for alternative technical solutions. The work was based on modified rules placing more emphasis on alimentation of the Vistula River flow. Data on water needs were updated according to changes in overall development plans. The characteristics of available water resources were based on mean 10-day flow data instead of monthly values. The number of subsystems in the Vistula Basin was limited to three instead of the earlier 14. This was achieved by the use of more powerful computers and organization of data storage in disc files with direct access to particular records.

Similar work is being carried out for the Odra River Basin. It is expected that the general water development plan for the whole country will be prepared before the end of 1975, based on system analysis methodology.

On the basis of the Vistula Project, the examination of smaller river basins or country regions was begun, with more detailed data on resources and water demands. The investment program, fixed in basic studies, gives general directions for regional analyses. The work already undertaken must define optimal parameters of regional development elements; Figure 5 gives examples of three regional systems being studied at the present time.

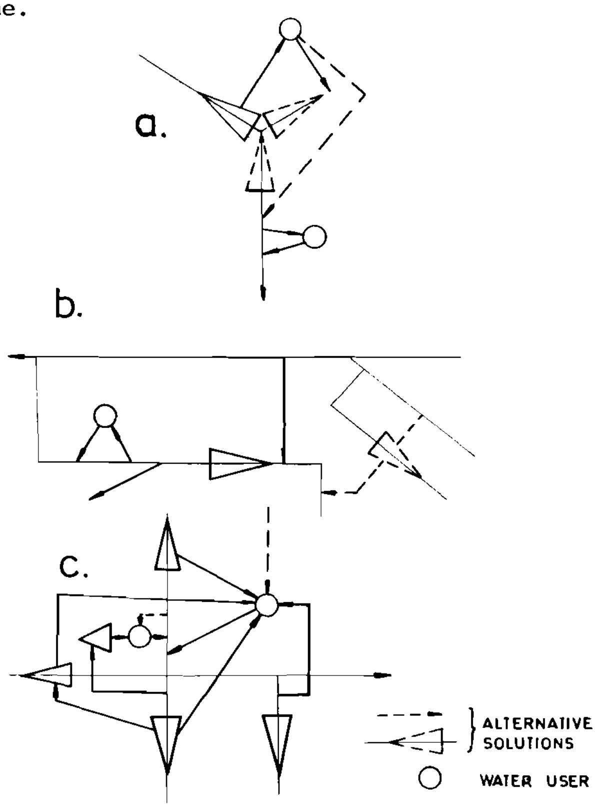


Figure 5. Examples of regional water resources systems.

5. WATER RESOURCES MANAGEMENT MODEL

It has already been mentioned that conventional techniques are inadequate for planning and formulating complex water resource systems such as the Vistula River System. Unfortunately, it may never be possible to take all the many variables, inputs and outputs fully into account in a systematic manner. Assumptions and simplifications will continue to be necessary. Nonetheless, application of the system approach provides water resource planners with a much better set of tools than were available 10 to 15 years ago.

The work described in this section was an attempt to develop and apply a simulation (optimization) methodology directed primarily at the allocation of available water to various uses. It was the product of joint efforts by a mathematical modelling group comprising experts from Poland and from Water Resources Engineers, Inc.[1]

The methodological work was first organized around a basic scheme proposed by the Institute of Environmental Engineering of Warsaw Technical University. The so-called Three-Step Method is composed of three computer programs which are applied sequentially in order to: (1) determine a set of target releases for individual reservoirs in the system, (2) develop operating rules for the reservoirs given the inflow hydrology and the target outflows, and (3) determine the optimal allocation of available water to all water uses considered in the model, given the operating rules from (2). Steps (1) and (3) were based on the Out-of-Kilter Algorithm, which is a special-purpose linear programming method derived from network flow theory. Step (2) was based on the method developed by Kornatowski (1969), employing stochastic-dynamic programming. Details of the Three-Step Method are described by Kaczmarek et al. (1971). The programs were made operational on the Polish-made Odra 1204 and 1304 computers; however, they could not be combined into a single program because of the limited capacity of the machines available at that time. Under the circumstances, implementation of the method was rather difficult and attention was focused on the development of the so-called Single-Step Method (referred to as the WRM Model). That method utilizes the Out-of-Kilter Algorithm to solve water resource allocation problems in a complex multi-reservoir system. First, the physical system is represented by a node-arc network in which arcs are inserted for all river reaches, demands, supplies, return flows, etc. An arc is, therefore, defined as any link connecting two nodes. Figure 6 illustrates an example of such a network derived from Figure 6a. Nodes portray the reservoirs, non-storage junction, control and balance profiles. Since the

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[1]Water Resources Engineers, Inc., Walnut Creek, California, USA, sub-contracted by the UN for mathematical modelling assistance to the Vistula River Project.

network shown in Figure 6 is only a spatial representation of the problem, it must be expanded to include time considerations. This is accomplished by introducing the storage arcs. For multi-period analysis the final storage from one period becomes the initial storage for the next period and so on. The one-period network may be thought of as being expanded into the third dimension with interconnection of time planes by the storage arcs.

In the case of the Vistula WRM Model, the "best" allocation in each month has been determined, hedging against future requirements in the three-dimensional network by covering a total of six layers (1+5 months). Calculations have shown that under specific Vistula Basin conditions (rather low storage capacities in comparison to the mean annual flows), there is no need to cover more than six time periods. In other words, a more distant future exerts negligible influence on the reservoir releases in a month actually taken for analysis. Therefore, the model is first used to solve the six-month allocation problem at the beginning of the simulation period; considering the first-month solution as valid, it moves on to span the second through seventh month, and so on. The final result of the simulation consists in the aggregation of all the first-month solutions. In order to make solutions more realistic, only the first-month water supplies to the system were used as historical streamflow rates. The median monthly flows from the entire period on record were taken as water supplies in the remaining five months.

One of the main advantages of the WRM Model is that reservoir operating policies need not be specified a priori, and complex methods to determine such policies are avoided. The reservoir releases are generated dynamically according to the demand situation and available inflow predictions. This factor takes on increasing significance in multi-reservoir systems. Moreover, incorporation of new reservoirs or demands during simulation present no difficulties as operating rules need not be modified.

It is also important that the nonlinear penalty functions can be represented in the model by a number of parallel arcs, with each arc denoting a linearized segment of the nonlinear curve. Similarly, canals with pumping costs may be incorporated.

As input, the WRM Model requires a network as shown in Figure 6b in which, for each arc, the following information must be specified: (1) source node number, (2) sink node number, (3) lower bound for flow in the arc, (4) upper bound for flow in the arc, and (5) penalty cost associated with unit flow deficit in the arc. To make the network continuous over time and space, and extra node to accommodate net balance is added. This is the so-called balance node to and from which all demands (or consumption losses) and water supplies are routed. Identification of arc types and definitions of their lower and upper bounds may be found in the Final Report on the

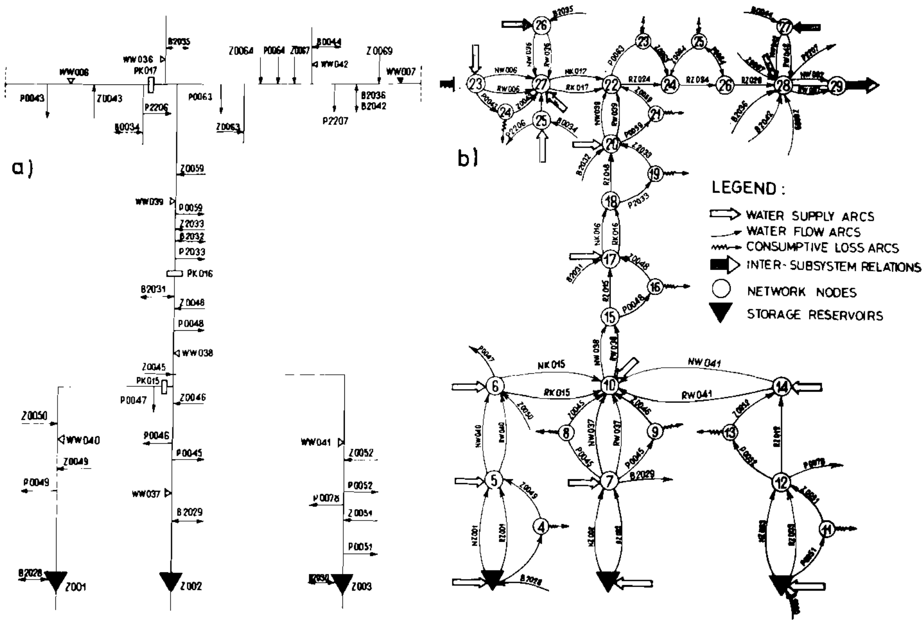

Figure 6. Node-arc network.

Vistula River Project (1972). Besides the objective function, which is to minimize the sum of penalties associated with not meeting the demands and minimum flow requirements in the river, the mathematical structure of the WRM Model comprises three sets of constraint equations. One set requires that continuity be satisfied at all nodes in the network. The remaining two sets describe the lower and upper bounds on flow in all arcs in the network. Thus in principle there is one equation for each node and two equations for each arc. For the finer points of problem formulation, the reader is referred to the Final Report on the Vistula River Project (1972) and to the paper by King et al. (1971). Details of the Out-of-Kilter Algorithm have been presented by Fulkerson (1961), Ford and Fulkerson (1962), and Durbin and Kroenke (1967). It should be noted that after completion of the UN/UNDP Vistula River Project, the WRM Model was considerably improved by the "Hydroprojekt" previously mentioned. The computer programs are now operational on the IBM 360/50 System, and they are being used for a similar study on optimal development of water resources in the Polish part of the Odra River Basin.

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Systems Analysis Basis of Water Management in the Tisza River Valley

István Bogárdi, Béla Csodó, and Géza Hankó

1. INTRODUCTION

The Tisza is the second-greatest river in Hungary. It drains half of the land in the country. More broadly considered, the catchment at present is shared by five countries, namely Czechoslovakia, the Soviet Union, Rumania, Hungary and Yugoslavia.

Water management in the Tisza Valley is of great influence in the economic and social development of Hungary. Furthermore, problems due to both excessive abundance and shortage of water are aggravated by pollution.

Conventional engineering or economic methods are virtually ineffective for describing in quantitative terms the behavior of a complex natural, economic and social system such as the Tisza River Basin. The methods of systems analysis, on the other hand, appear suited for solving the fundamental problems of water management in the Tisza Valley, for providing optimal control of the present system, and for delineating the strategy of water development. This paper has been compiled in preparation for solutions to water management problems and water development strategy.

Chapter 2 describes the natural, socio-economic and water management conditions prevailing in the Tisza River System. In Chapter 3 a review is presented of the systems analyses prepared thus far on the Hungarian part of the Tisza Valley. The results are not only of theoretical interest, but have been applied in practice. Based on the information compiled in the preceeding two chapters and on international experiences related to systems analysis, Chapter 4 suggests some system models that approach the expectations of practice. Further studies are needed to perfect the models.

2. DESCRIPTION OF THE SYSTEM

2.1 Natural conditions in the Tisza River Basin

The waters from the eastern part of the Carpathian Basin are collected and conveyed to the Danube by the Tisza River, the largest left-hand tributary of the Danube. The catchment area of the almost 1,000-km-long river is around 157,000 km^2 (see Figure 1). The catchment can be divided into two parts; on the one side there is a mountain range bounding the catchment and rising to elevations of 2,000 m. Further the slopes of the block of mountains at the center of the catchment have elevations up to 1,800 m. These two form the mountain area. On the other side there are the Great Plains of about 80,000 km^2 with elevations below 200 m. The latter part comprises 51% of the catchment. Aerial distribution of the catchment according to countries is shown in Table 1.

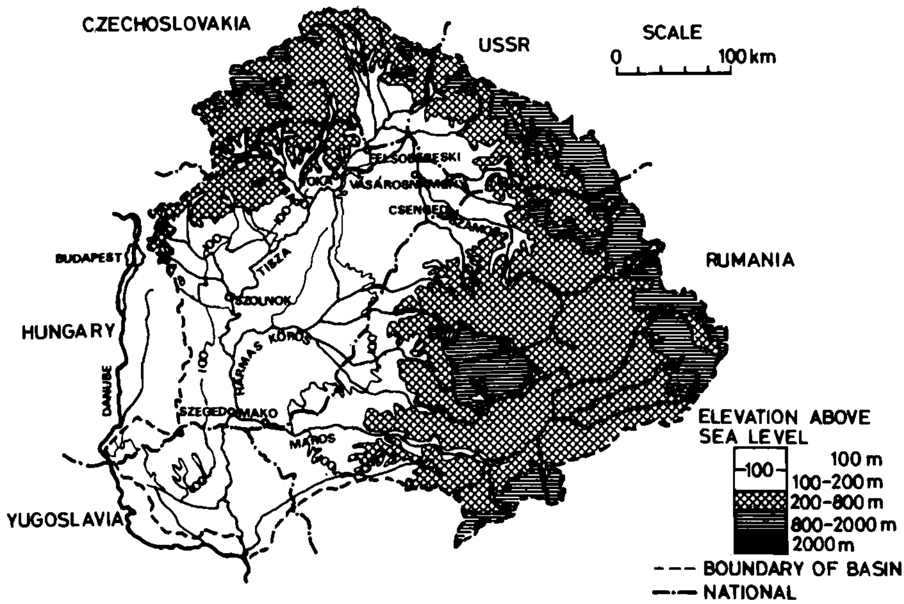

Figure 1. Morphology of the Tisza River Basin.

Table 1. Territorial distribution of the Tisza River catchment by countries.

Country	Catchment area thousands of km^2	Percentage from total part of the catchment	Percentage from plains part of the catchment	Percentage from the whole area of the country
Hungary	46.2	29.6	56	50
Rumania	72.0	46.0	24	30
USSR	12.7	8.1	4	insignificant
Czechoslovakia	15.5	9.9	4	12
Yugoslavia	10.0	6.4	12	4
Total	156.4	100.0	100	-

Around 30% of the entire catchment, and 56% of the Great Plains is situated in the territory of Hungary. More than one-half of the territory of Hungary lies in the Tisza River Basin. The climate of the basin is basically continental in character, but at times is influenced by air currents of Mediterranean and Atlantic origin. It is owing to this periodic influence that a regular alternation is observable between the wet and dry years, every decade having two or three arid and two or three wet years.

The average precipitation in the Great Plains is less than 600 mm, and even less than 500 mm in a large part of the area; the minimum observed was 458 mm. The average rainfall in the growing season ranges from 300 mm to 350 mm. The distribution of precipitation over the year is characterized by a peak in June and a low in January.

Typical of the continental character of the climate in the Great Plains is the wide range of temperature fluctuations between $-30^{\circ}C$ and $+40^{\circ}C$. The average number of sunshine hours is in excess of 2,000 over a large part of the plains. Three-quarters of the sunshine hours occur in the summer half-year. Taking an average over 50 years, the difference between potential evaporation and precipitation is 175 mm in the central region of the plains, while the aridity factor, namely the ratio of potential evaporation and precipitation, which is of fundamental importance to agricultural production, ranges from 1.2 to 1.4.

The Hungarian part of the Tisza River Basin receives the least precipitation and the greatest number of sunshine hours in the entire catchment. High summer temperatures together with extended dry periods frequently result in droughts. Also, the area is often exposed to inundation by floods and undrained runoff water. In the Tisza River Basin, owing to climate, topography, and further to water budget conditions, the average runoff volume is 25.5 km^3/year, while the specific runoff per km^2 is 16.2 lit/sec.

The main water course of the river system is the Tisza River which originates in the North-Eastern Carpathians and discharges at Titel into the Danube. The major left-hand tributaries are the Szamos, the Maros and the Körös Rivers, while the right-hand tributaries are the Bodrog, the Hernád, the Sajó and the Zagyva Rivers (see Figure 2).

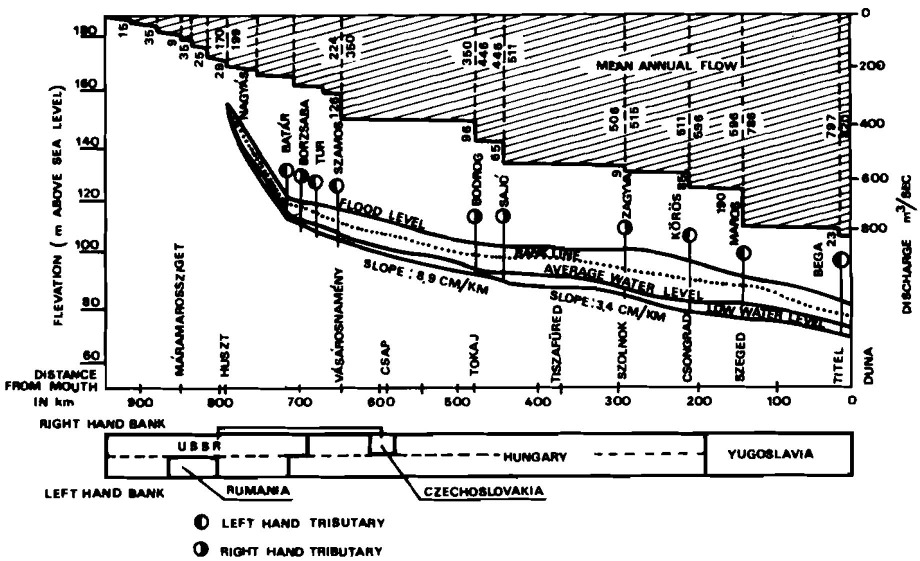

Figure 2. Profile of the Tisza River.

After the steep headwater sections in the mountains, the slope of the rivers decreases abruptly on the plains, remaining generally below 0.1% and even below 0.02% over long sections. As a consequence, flood waves rushing down the mountains overtake each other and are superimposed in the plains. Characteristic data on the flow regimes have been compiled in Table 2.

Table 2. Characteristic data of rivers in the Tisza Basin.

River	Section	Range of stages cm	High	Mean discharges m^3/sec	Low	Ratio of excess flows	Mean annual runoff 10^9 m^3/year
Tisza	Tokaj	1,056	4,000	464	53	76	14.6
	Szolnok	1,141	3,820	546	65	59	17.2
	Szeged	1,240	4,700	810	95	49	25.1
Szamos	Csenger	839	1,350	120	15	90	3.8
Bodrog	Felsöberecki	651	1,300	120	4	325	3.7
Sajó	Felsözsolca	486	520	32	2	217	1.0
Hármas-Körös	Torkolat	1,181	1,330	67	4	333	2.1
Maros	Makó	658	1,800	160	22	82	5.0

The difference between the highest and lowest stages, that is, the range of stage fluctuations, is from 10 m to 12 m over the Hungarian reach of the Tisza River. Even along the plain reaches of the tributaries, this range is as wide as 7 m to 8 m. It is interesting to note that as a consequence of human activities during the past 150 years, the range of stage fluctuations has widened considerably, the rise in stages over the Szolnok-Szeged section of the Tisza River being around 5 m to 6 m. Over the upper reaches of the Tisza River the flood discharge may be taken as around 100 times as large as the dry weather flow, whereas on the lower reach this ratio is around 50. On the tributaries, the ratio may attain the magnitude of several hundred.

The water regime in the river system is usually controlled by precipitation of Atlantic origin, as well as by the water accumulated in the snow-pack during the winter. The distribution of monthly runoff values over the year (see Figure 3) indicates at the same time the characteristic annual stage hydrograph. Normally the stages start to rise in March when the river is still frozen over. Under the combined effect of melting snow and ice, and spring rains, stages in the rivers rise rapidly. In the Tisza River Basin the highest annual stages occur usually around the middle of March, but may be delayed frequently to the end of April or the beginning of May. At Szeged, in the down-stream part of the catchment, the greatest annual runoff in April may attain 14% of the annual average. High stages are usually followed by a rapid recession. From May to October the discharge in the Tisza River and its tributaries will usually decrease continuously, causing frequent difficulties in meeting the water demands. In September and October extremely low discharges are common.

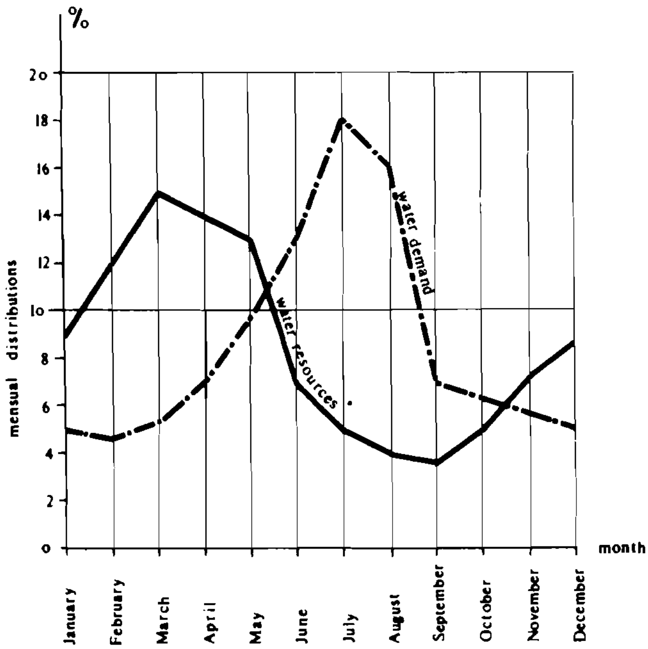

Figure 3. Average distribution of water resources and water demands of the central part of the Tisza basin.

Characteristic values of the water regime over the Hungarian section of the Tisza River are illustrated in Figure 4. As can be perceived, flood discharges in the Tisza River are more or less the same in magnitude over the upper and lower Hungarian reaches, in contrast to the mean and dry weather discharges, which increase gradually downstream.

The sediment concentration in the Tisza River is in general three to five times as great as in the Danube and, consequently, regardless of the considerably smaller discharges, the annual average sediment transport is of the same order of magnitude (20-22 million tons/year) as in the Danube. The bed load transport, on the other hand, is no more than 30% to 50% of that in the Danube.

Arriving at the plains, the flow velocity and tractive force in the tributaries is greatly reduced and thus the coarse sediment is deposited so that the tributaries are of practically no influence on the sediment composition in the Tisza River. The Sajó and the Maros Rivers are exceptions to this rule. Their slopes at their confluences to the Tisza River are steep enough to build long gravel and coarse sand bars in the Tisza channel.

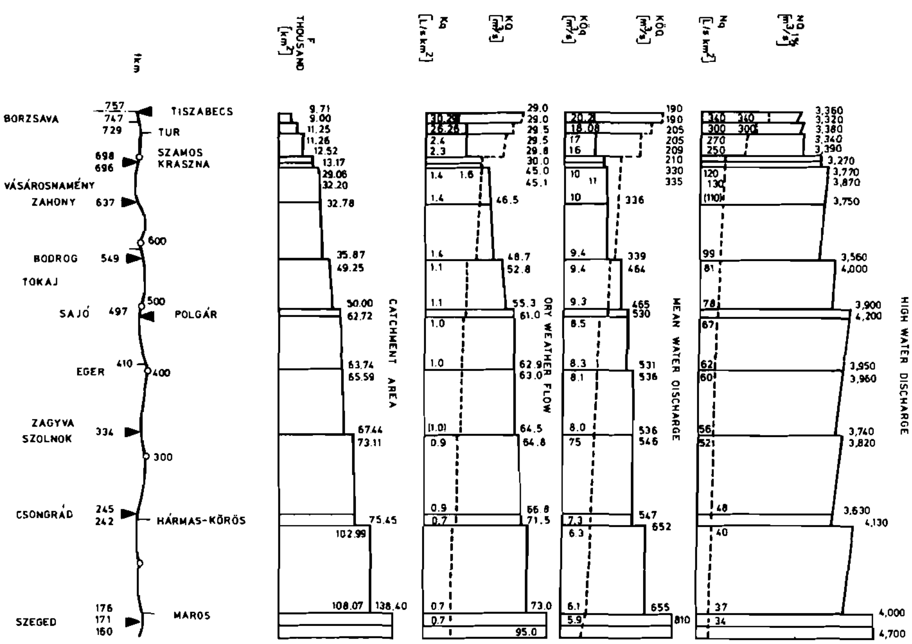

Figure 4. The hydrological profile of the Tisza River within the borders of Hungary.

The extreme natural conditions in the Great Plains demonstrate already the necessity of deliberate water management in this region. The generally impervious soils and the virtually horizontal terrain call for the artificial drainage of rainwater, whereas water shortages owing to the arid character of the climate emphasize the importance of irrigation. The necessity of controlling damages by floods rushing down from the surrounding mountains, of pollution control and of conserving the limited water supplies has focused attention on comprehensive water management extending to the entire Tisza River Basin.

2.2 Economic Conditions

Water management in the Tisza River Basin has been of fundamental influence on the economic conditions not only in the Tisza Region itself, but also on the national economy as a whole in Hungary. Under the socialist system, the significance of water management in the economic life in the country has further increased.

The importance of the region stems from a combination of several factors. This region accommodates 40% of the 10 million inhabitants of Hungary. This is the region where 42% of the national property is concentrated. The region comprises one-third of the arable lands of the country, producing more than 50% of the total agricultural product. The conditions for agricultural production are among the most favourable within the Carpathian Basin. Animal husbandry is well developed but the fodder supply is still inadequate. The share of population employed in agriculture (30%) is above the average for the country.

A dense network of railways and roads handles large volumes of east-west transportation. Further economic development depends, however, to a considerable extent on the gradual development of a waterway network and the connection to inland and transit water traffic.

Geographically, the Hungarian part of the Tisza River Basin can be subdivided into several districts, which will be described subsequently, proceeding from the North to the South. The Upper Tisza Region is largely coincident with the catchment above the Tokaj section. The economy in this district is predominantly agricultural in character, cattle and hog raising being typical of the animal husbandry. Gardens and orchards represent an outstandingly high proportion in the district, occupying about 10% of the arable area. Industry has developed first of all in the major towns and is engaged mainly in food processing. In the southern parts of the district the natural gas produced is of considerable importance to the national economy and has contributed to the modernization of some thermal power stations.

The Northern Industrial Region comprises the mountain and hilly areas extending from the eastern foothills of the Cserhát range to the valley of the Tisza River. This area accommodates 15% of the industry of Hungary. (The industrial centers are concentrated in the coal basins and around producing industries.) The brown-coal mines and open-cast lignite extraction provide the fuel for the network of thermal stations in the area. Iron and steel metallurgy and the machine industry are already important, while the chemical industry is in a stage of rapid development. In the Northern Industrial Region, agriculture plays a secondary role, expect for the wine region of Tokaj which is also of historical interest.

The Central Tisza Region lies on the right-hand side of the central Tisza reach and extends down to the mouth of the Körös River. The district is basically agricultural in character with developing industry. Besides farming, cattle and hog raising are of great economic importance. Regardless of the large scale development during the past decade, the role of industry is still inferior to that of agriculture. Along with heavy industry, the development of the chemical industry in this region is related to the natural gas supplies found here.

The South-Eastern Great Plains are comprised of the valley of the Körös Rivers and areas to the South. The district is predominatly agricultural in character, the share of ploughlands being above the average for the country. Besides cereals and maize, the production of industrial crops is significant. Vegetable production in hot houses supplied with natural thermal waters is of outstanding importance. Animal husbandry in the district consists mainly of hog raising. In accordance with the agricultural character, the foodstuffs industry is developed vigorously. The natural gas and petroleum resources found in the area are of great industrial significance.

The eastern half of the area between the Danube and the Tisza River belongs to the catchment of the latter. The area is of primarily agricultural character with considerable fruit and grape production. Some light industry has settled in the area and the extraction of petroleum is under way.

Regarding this background the special importance of deliberate water management in the Tisza River Basin will be readily appreciated as essential to the development of the national economy, as well as for the improvement of the living standard of the population. The avoidance of water related damages, provisions for adequate supplies to meet the growing water demands of industry, of agriculture and of the settlements, as well as the protection of the population from pollution are impossible, unless this particular branch of the national economy is developed at a rate commensurate with the general development of the economy.

2.3 Description of Water Management

2.3.1 Damage Control

Of the 93,000 km^2 area of Hungary, 25% is protected from inundation by levees of 4,200 km in aggregate length. The greater part of this area, namely 18,000 km^2, is located in the Tisza Valley.

Comprehensive flood control development in the Great Plains was started along the Tisza River in 1846. To appreciate the volume of work involved, it should be noted that in Hungary the 18,000 km^2 of flood plains in the Tisza Valley are protected by a system of levees extending over 2,900 km length (Figure 5). The extent of levee construction and related flood water regulation on the Tisza River is shown in Table 3.

As a consequence of flood water regulation performed in the interest of accelerating the passage of floods, the total length of the river was reduced from 1,419 to 966 km. The approximately 1,000 km length of the Hungarian reach was shortened to 597 km. By cutting 114 overdeveloped bends, new channel sections were excavated that are over 136 km in total length. As a result of regulation work, the slope of the river became steeper, and as a consequence the travel time of floods was reduced. Diking was of considerable influence on water-conveying conditions. The waters flowing earlier over the flood plains have been confined to the flood bed between the levees. The conditions of sediment transport and channel development were modified as reflected by extensive scouring and deepening of the channel. Low water stages decreased by 2 m on the average, while flood stages display a generally increasing trend (Table 4).

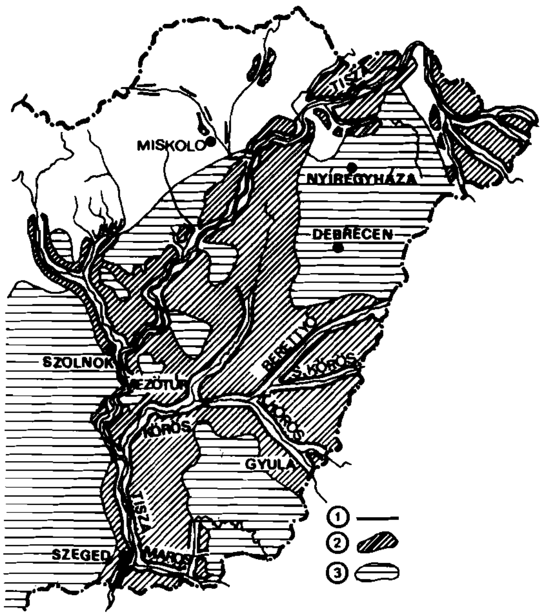

Figure 5. Reclaimed areas in the Tisza Valley.

After exploring the causes of these phenomena we have concluded that the meteorological elements triggering the floods have remained essentially unchanged in the Tisza River catchment over the period under study, but the factors controlling runoff have suffered appreciable changes. In the upstream mountain part of the catchment, conditions have also altered considerably, a consequence of which is that the times of concentration have become shorter along the headwater reaches. Owing to the hydrometeorologically nonuniform character of the upstream subcatchments, the areal development of the flood waves is highly variable and the flood contribution of the area drained directly by the main recipient is inadequate to compensate for these effects. The result of these intricate relationships has been termed the flood exposure, defined as the product of the height of overbank stages and their duration, that is, cm × number of days. The variation of this characteristic along the river is indicative of the adverse evolution of flood exposures. It will be observed that whereas the flood exposures tend to decrease along the upper Hungarian section of the Tisza River, they have been increasing considerably down-stream of the Zagyva River.

The present flood safety created by a tremendous effort in the Tisza Valley has been thus successively reduced by the environmental changes accompanying economic evolution in the catchment. This deterioration must be offset by advanced, complex water management and environment conservation measures.

Table 3. Characteristic data of river training.

River	Length of river before river draining km	Length of river after river draining km	Lengths of cuts km	Number of bending cuts	Lengths of cuts km	Average slope before river draining cm/km	Average slope after river draining cm/km
Tisza[2]	1,419	966	136	114	589	3.7	6
Maros[1]	86	50	-	13	-	14	24
Hármas-Körös[2]	234	91	34	39	177	2	5.7
Kettös-Körös[2]	84	37	25	15	70	4	8
Fehér-Körös[3]	126	67	25	81	84	-	-
Fekete-Körös[3]	166	90	26	61	102	-	-
Sebes-Körös[3]	162	86	53	24	129	-	-
Berettyó[3]	269	91	51	46	229	-	-
Szamos[3]	187	108	-	36	-	-	-
Bodrog[1]	84	50	-	8	-	3.5	-

[1]Over the Hungarian reach

[2]Over the total length

[3]Over the regulated reach.

Table 4. Area protected from floods and peak stages.

Year	Protected area in the percentage of the whole flood-plain	Peak stage in Szeged cm
1855	13.0	687
1860	39.5	670
1867	52.2	722
1876	63.5	786
1879	65.9	806
1881	68.2	845
1888	77.5	847
1895	87.0	884
1919	87.5	916
1932	87.5	923
1970	90.0	961

About 45% of the territory of Hungary, including the flood plain, is exposed to inundation by undrained runoff and by rising groundwater. Almost 80% of this area is located in the catchment of the Tisza River. At present, a canal network of over 25,000 km total length and pumping stations with an aggregate discharging capacity of close to 500 m^3/sec are available for providing drainage to these waters. Although this is the region with the dryest climate in the country, the soil and topography are adverse enough to cause minor or major inundation damages almost every year.

Although the flood and drainage control systems in the Tisza Valley have been developed at accelerating rates during the past decades, the extent and control capacity thereof has still not attained the level commensurate to that of the socio-economic development. This has been demonstrated strikingly by the flood in 1970. Experiences gained during this flood and the advances in techno-economic conditions have been taken into consideration in delineating the trends of further development.

In the course of improvement work on the flood control system envisaged up to 1985, the levees will be reinforced by about 100 million m^3 of earth to offer reliable protection against the floods of 1% probability. Along areas of high socio-economic value (urban areas, industrial plants) the 150-years flood and, exceptionally, the 1000-years flood, has been adopted as the design criterion, while along less valuable areas the 50-to-80-year flood has been adopted as the design criterion.

In the drainage systems, the modernization of existing structures will be accompanied by increasing the density of the canal network with special emphasis to the minor canals in the farming operations. Along with these a variety of controlled drainage methods is envisaged. These comprise the creation of around 400 million m^3 storage volume for the runoff waters which will be made available for subsequent uses.

2.3.2 Water Uses

Under the extreme natural conditions drought disasters are not uncommon in the Tisza Valley with damages extending normally to the entire plain part of the catchment. To illustrate the extent of aridity, it is of interest to note that in the period from May to August the number of rainless days exceeds 12 in every third year and 20 in every tenth year. Irrigation development in the Tisza Valley was started in 1950 following the large-scale reorganization of agriculture and by 1973 260,000 hectares had been equipped for irrigation.

Water supply has become increasingly the factor controlling the method of production in the agricultural operations. Besides the large scale use of mechanical equipment, increasing emphasis is placed on the application of coordinated machine fleets for the complex methods of production. Fertilizer is applied at the average rate of 590 kg/hectare. In the development programs of agriculture, the introduction of irrigation is considered preferable over at least 18% of the arable lands up to 1985. Along with irrigation, the construction of fish ponds is also envisaged, especially in saline areas supporting poor crops. The present pond area of 16,000 hectares will be increased to 26,000 to 28,000 hectares.

A large number of inhabitants grouped in a loose housing pattern is characteristic of the settlements in the Great Plains. In these, rural water supply is provided to 56% of the population. The situation in the field of sewerage and waste treatment is less favorable, and at present no more than 16% of the area inhabited is served by sewers. In prospective development programs of the national economy, it is envisioned that by 1985 the rural water supply will be extended to 85% of the population and sewer facilities to almost 60%.

The annual fresh water demand of industry in the Tisza Valley is currently around 1×10^9 m^3, and it is expected to double by 1985. The rate of recycling is 57%. Owing to the gradual exhaustion and insufficiency of local supplies, the successively growing rural, urban and industrial water demands will be met by the construction of regional water works.

Over the Hungarian section of the Tisza Valley, the slope of the rivers is inadequate to make water power development attractive. The power station incorporated in the Kisköre Barrage has an installed capacity of 45 mw and produces about 180 million kwh of hydroelectric power in an average year.

The uncanalized sections of the Tisza River and its tributaries are navigable only at certain times of the year. In the river system, the waterway, suited for navigation by vessels up to about 1,500 tons, is 300 km long, but it is little used owing primarily to the absence of a connection to the Danube. Waterway development will be served by the canalization of the Tisza River and by the construction of a Danube-Tisza Canal. The latter will provide a link between the waterways in the Tisza Valley and the Trans-European Danube-Main-Rhine waterway.

2.3.3 Water Resources Management, Flow Augumentation

The average runoff volume carried by the Tisza River past the Szeged gauge is $25 \cdot 10^9$ m^3, more than 95% of which originates in the upstream countries. The annual runoff ranges from 11 to $50 \cdot 10^9$ m^3. In an average year 52% of the total runoff is conveyed during the winter half year, but in extreme cases this may increase to as much as 75%.

One of the gravest problems facing water management in Hungary is that the bulk, almost three-quarters, of the water demand arises in the Tisza Valley, whereas in the critical August period the available supplies in this region are only one-fifth of the surface water resources of the country.

The character and peak value of the annual distribution of water demands in this region are governed in general by irrigation uses. In this respect, knowledge of the runoff pattern in the growing season is of paramount importance (Figure 6). Actual water uses are influenced also by more or less random effects, so that both water demands, and the supplies are justified to be regarded as random variables. Consequently an investigation has been started on the development of stochastic water balance methods.

In the period from 1970 to 1985 the overall annual water demands are expected to increase 2 to 2.5 times, the rate of increase in the growing season being 1.5 to 2-fold. From the study on the areal and temporal distribution of water supplies and demands we concluded that aside from the growing season the natural water supplies are at present generally sufficient to meet demands. During the growing season, however, the equilibrium of the water balance can be maintained even under the present conditions by means of the existing 200 million m^3 storage volume. The water shortage reflected by the water balance for the growing season compiled for 1975 on the basis of data correlated with the upstream countries is composed of the monthly data for the individual subcatchments (Table 5). By around 1985 it will become necessary to augment the present supplies by about 1×10^9 m^3 to increase the supplies during the growing season in drought years.

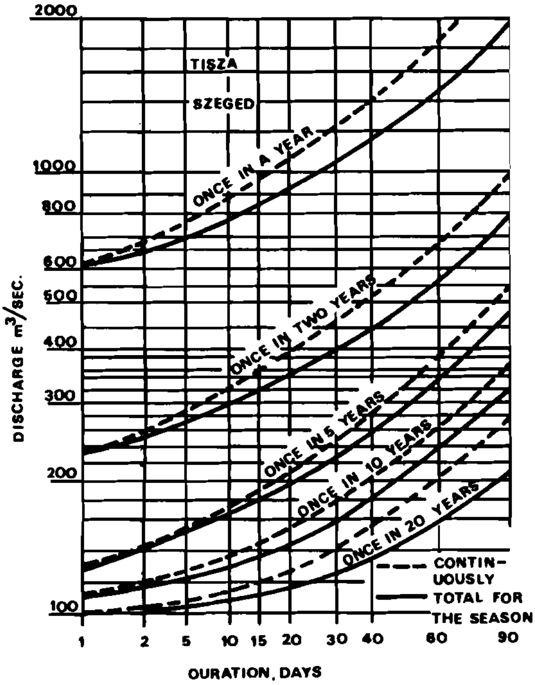

Figure 6. Dry weather flows of different duration and frequency of recurrence in the Tisza River during the irrigation season at Szeged.

Table 5. Water balance in the Szeged section of the Tisza River, May-September, 1975.

	million m^3/5months
Water resources of 80% duration	4,810
Water left in the channel	900
Net water demand	2,640
Storage	660
Available	2,330
Water Deficiency	400

Remark: Values refer to the whole catchment upstream of Szeged.

The major projects to be realized under the Tisza River canalization programme (Figure 7) and the more efficient operation of the water supply systems will make additional water resources available for use. The first barrage on the Tisza River was inaugurated in 1954 at Tiszalök. By means of this project, diversion at the rate of 60 m^3/sec was made to irrigate 150,000 hectares, for supplying fish ponds and for transferring water to the Körös Valley; moreover 55 million kwh of hydroelectric power have become available annually at 14 mw peak capacity and a 130 km long river section was opened to navigation. At the same time the possibility was created for meeting growing water demands in the area of Debrecen town.

An important step in the canalization of the Hungarian Tisza River is the Kisköre Barrage, where operation was started in 1973. In the ultimate stage of development, 400 million m^3 of water will be stored between the flood levees and this storage contributes to balanced runoff conditions in the river. The discharge available for diversion from the river will be increased by 175 m^3/sec. This will be used for irrigating 300,000 hectares, for supplying 12,000 hectares of fish ponds, and to meet considerable industrial water demands. At the power station, 100 million kwh of electric power are produced annually. The barrage will make permanent navigation possible over a 120 km long river reach and the reservoir will create favorable conditions for recreation and water sports.

The Csongrád Barrage, on which design work is under way, the Upper-Tisza Barrage, to be realized by the turn of the century, and the Novi-Betshey Barrage, under construction in Yugoslavia, will complete the canalization of the Tisza River. In the interest of runoff control, a number of additional storage reservoirs are necessary in plain and hilly areas alike, and water transfer from the Danube will become essential (Figure 7). In the Tisza Valley flow augmentation system thus developed, the projects of water utilization and damage control will be combined organically, creating thus the possibilities for optimal flow regime control to meet variable, complex demands.

Changes in water quality are taken also into consideration along with the quantity characteristics of water resources in the Tisza Valley. The water in the Tisza River upstream of the Bodrog and downstream of the Zagyva River is qualified "clean" (Class I). Of the tributaries the water in some sections of the Tur Creek and the Körös Rivers is clean, the rest being more or less polluted. The Sajó, the Hernád, the Bodrog and the Szamos Rivers are greatly polluted. In Hungary legal regulations and economic incentives serving the interest of pollution control have resulted in retarding the fast rate of pollution in the recipients. For stopping further pollution and for improving water quality in the recipients further effective measures are considered necessary.

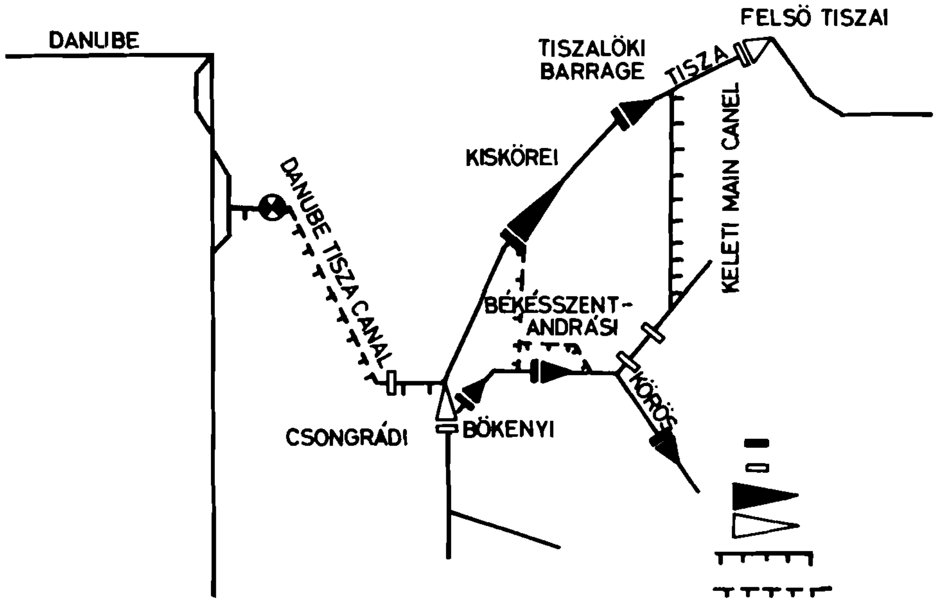

Figure 7. Schematical drawing of water management projects contemplated in the Tisza Valley.

3. EARLIER SYSTEMS ANALYSES

3.1 Necessity of Systems Analyses

In the Tisza Valley the systems approach is warranted by the necessity of solving the following problems of major importance:

1) The natural flow regime in the system is unbalanced in space and time alike. Water shortages occur at a number of points, while simultaneously water is available in abundance at other places. The supplies are lowest at times of greatest demands.

2) In the Tisza Valley, floods with inundation potential occur every third year on the average.

3) The undrained rainwater in the plains causes considerable damage in the spring season to both agriculture and buildings.

4) Considering the entire river basin, human interference with the natural flow regime and regional development

have brought about the following consequences in the Hungarian part of the Tisza Valley:

- higher floods occur with greater frequency,
- the flow available in the rivers decreases, and
- the waters are increasingly polluted.

5) The value of the areas protected from floods and from undrained rainwater has increased in the wake of economic growth.

6) The demand for domestic, industrial and irrigation water, as well as for recreation, has increased rapidly.

The above conflicting interests are stimulated by complex natural and economic processes which occur often with a random character in space and time alike. It is already considered desirable to control the system as far as possible in a manner to minimize the adverse social-economic consequences of these conflicts. The same principle applies to future development as well.

For adjusting the water supplies in space and time to the demands of the water users, the key factors of a water management system, namely the major water projects were realized during the past decades in the Tisza Valley. The provision and distribution of water for domestic, industrial and irrigation uses, and the avoidance of water damages (from flood control, drainage) have been identified as their primary purposes.

With the construction of the second barrage and the realization of the connecting network of irrigation canals (Figure 7), the system has developed into a complex one which a few years ago called for the introduction of systems analyses for the guidance, supervision and control of the water volumes handled.

3.2 Control Model of the System

3.2.1 Water Management

The water distribution model of the Tisza Valley Water Management System (see Dávid and Szigyártó, 1973) has been developed with the aim of meeting the demands of agricultural water uses by the most efficient water distribution decisions. The objective is to adjust the water supplies which are variable in time, space, volume and quality to the similarly variable user demands in the system, and to ensure the fastest drainage under the prevailing requirements relying for these purposes on the appropriate mathematical models. The units of the model system developed thus far do not comprise the entire existing physical system but extend only to the irrigation sections involved in the Tiszalök irrigation project.

Depending on the conditions of water distribution, two basic modes are distinguished in the operation of the water management systems to be simulated in the model. These modes are referred to as normal and restrictive operation. Normal operation is practiced when the supplies available and the conveying capacity of the system are adequate for meeting the demands. In the restrictive mode of operation diverse constraints must be introduced because the supplies and/or the conveying capacity is inadequate to meet the prevailing demand. The normal mode of operation is prevalent virtually over the entire length of the operation period. In the relatively rare restrictive mode of operation the optimal use of water supplies assumes special significance in water management.

The first component of the model system comprises the set of programs of water distribution in upstream controlled irrigation sections, by which the intake gate is controlled at a rate, at intervals not shorter than 30 minutes, to meet the irrigation demand in the section without undue water losses.

The second component of the model system serves to control water distribution in the Tiszalök Irrigation Project. For both modes of operation the process of water conveyance is described by the distribution of water volumes required or conveyed during discrete, independent intervals, disregarding variations in discharge within these periods of time.

The entire water conveyance system is resolved into sections (lines) with each section-principal canal or reach forming a line along which conveyance can be controlled. In the model this system is represented by an oriented network for the mathematical description requiring the following constants:

- the maximum capacity of the individual lines at the entrance and departure points thereof,
- the water losses along the lines,
- the functions $t_{xy}(V)$ representing the conveyance time in the individual lines, where (x,y) is the symbol of the line, V is the water volume conveyed during a particular period of time, t is the conveyance time.

This latter quantity is needed for the normal model only.

In the normal model the objective function and the three constraints represent the following restrictions concerning the water volumes to be conveyed along the individual lines:

- Even the highest value of the conveyance times along the individual lines should be the lowest possible.
- The conditions represented by the capacity limits should be observed.

- The continuity criteria at the junction points should be satisfied.

- The demand along the individual lines should be met.

The result of the computation--the "potential flow" obtained before the last step of successive approximation--yields for the individual lines of the network the discharges to be introduced and to be removed during a particular period of time.

The constraints in the restrictive model represent substantially the first three requirements of the normal model and an additional restriction is imposed to ensure the observation of a specified priority sequence in the computation of the "potential flows" permitting the demands to be met (by an out-of-kilter algorithm) by their superimposition along the individual lines.

At present, the operating instructions are determined for 12 hour periods (daytime and night operation) by the models. The data and results are transmitted between the irrigation sections and the computer center by teletype. Development work is under way on the model with the objective of controlling flows in the Tisza and Körös Valleys simultaneously. A multi-stage, stochastic programming model has been compiled for the optimum design and control of a system consisting of multi-purpose, series-connected storage reservoirs (see Prekopa and Ijjas, 1974).

Three reservoirs are considered in this model, two of which, the Tiszalök and Kisköre, have fixed capacities, while the capacity of the third, the Csongrád, is a decision variable. The different kinds of water demand are introduced according to utilization as water volumes available at the "taps" of the individual reservoirs. In the model the control period is resolved into discrete intervals; at the beginning of the optimization process full reservoirs are assumed, and the control period is terminated at the next high water period. The variation in reservoir states thus substantially follows a regeneration process. In each reservoir and in each period, the variations of storage volumes are restricted by imposing lower and upper limits on the water levels (permitting a fluctuation of about 30 cm). The constraints on the deterministic basic model of decomposed structure represent the hydrologic and capacity restrictions, as well as the criteria specified for the storage volumes. The decision variables comprise the water volumes released from the individual reservoirs for diverse purposes. The objective function of the linear model represents the desirability of maximum utilization of the water volumes released over the entire control period, reducing the benefit attained by the construction costs of the reservoir (of unknown capacity) to be realized. The multi-stage character of the stochastic programming model compiled on the foregoing basis ensures the dynamic nature of process control. In accordance with the principle fundamental to the stochastic model, the control policy is not decided upon firmly at the beginning of the control stage of the process, although the decision is adopted in anticipation of the entire control period.

A further model has been formulated for the optimal distribution of the water resources influenced by the Tiszalök and Kisköre Barrages (see Prekopa and Ijjas, 1974). The objective of the model is to determine a way of distributing water volumes arriving in, and stored above, Kisköre from which the highest benefits will accrue to the national economy, taking into consideration the multi-purpose uses of water, each of which may result in different economic benefits. Concerning its character, this model is a deterministic one with mixed (linear and dynamic) programming. The distribution of supplies available within a single time period--the determination of the optimal distribution of the supplies over several time intervals within the full control period is computed by a dynamic programming model. The linear programming model is used for determining the optimal subpolicy, the variables of which are the water volumes classified according to location and purpose of use. These latter include agriculture, communal and industrial supplies, transfers to other systems, peak-load generation and power production in off-peak periods. The constraints in the model represent the following restrictions:

- the hydrological restrictions on the water supplies that can be allocated,
- the technical (capacity) restrictions on water uses (diversions),
- the actual uses must not exceed the demands.

The objective function of the linear model is:

$$\max \left[\sum_{i=1}^{k} B_i V_i \right]$$

where

k is the number of variables;
V_i is the water volume used for a particular purpose;
B_i is the net economic benefit per unit volume used for a particular purpose. This is given by the slope of the linear section of the linearized benefit functions.

The optimal operation strategy of the barrages is determined in the model by dynamic programming, assuming that storage in the reservoir can assume certain discrete values only. The model is operated to produce the maximum net economic benefit while starting from any actual condition the water level in the reservoir proceeds to a particular condition within the control period of the model.

The state variable of the dynamic programming model is thus the storage volume, whereas the decision variable is the change thereof. For any particular change in storage volume, the optimal distribution is determined by the linear model. For operating the dynamic programming model the inflow discharges and the water demands must be predicted for the entire control period, but the decision concerning the optimal strategy can be perfected step by step in accordance with the situation actually developed, and with occasional modifications in the water demand and discharge forecasts. By using the set of models information is obtained on the optimal variation in storage, as well as on the water volumes available for utilization in the various regions.

3.2.2 Damage Aversion

The drainage of rainwater as a component of water damages can be described by a conceptual-physical model whose stages include sheet flow, depression storage, the development of pond-chains and eventually channel flow. The actual physical phenomena in the individual stages are described by mathematical relations in accordance with the character of flow. This set of mathematical equations, where the expressions describing the individual stages are interrelated, forms the mathematical model of the process. The concentration of surface runoff is represented by two models. The first, named the EXPRE model (see Kienitz, 1972), yields the volume of runoff from rainfall, whereas the second, the DRAINAGE model (see Kienitz, 1969), has been conceived to transform undrained rainwater into a runoff hydrograph.

3.3 Development Models of the System

3.3.1 Water Management

A dynamic model has been compiled for planning long-term development of water resources management (see Dávid et al., 1974). Socio-economic evolution is accompanied by an increase in the responsibilities of water management. A growing share of resources must be expended for establishing and maintaining the balance between the natural supply and the social demands for water. The potential uses of these supplies, however, in general are limited. The dynamic model conceived for planning long-term development of water management is not intended to promote the most efficient use of resources over long periods in socio-economic systems with limited resources but in particular stages of development. In any particular period and in any given catchment, such as the Tisza-Valley, or subcatchment (economic district) the demands for fresh water or for damage control resulting from the socio-economic evolution envisaged can be met by the fundamental water management functions, such as runoff control, water production and supply, waste water treatment. For these activities the resources (funds, manpower, energy and water supplies) are made available to water management by the society and by nature, respectively. For the justified development of the basic water management activities considered, the foregoing resources must be combined in the appropriate manner. Where the resources are adequate as regards both their quantity and their distribution among the basic activities, for realizing the basic water management activities required, there are no obstacles to the realization of the socio-economic evolution contemplated. On the other hand, where the amount of resources is less than required, the basic activities of water management will be incapable of meeting the demands of society. In such cases a loss, whose magnitude depends on the extent of water shortage, will accrue which results, with a certain time lag, in slowing down the rate of economic evolution and population growth. The magnitude of the loss depends on the quantity of resources available and their distribution among the basic activities. The allocation of these resources comprises the development strategy. In the dynamic development model of water management it is assumed that the resources needed for ideal development are not fully available. Actual development is controlled by the resources actually available and by the development strategy adopted. The losses in income and in population growth represented by the difference between the ideal and the actual development are the model output for a potential alternative, that is, a particular set of resources in combination with a particular development strategy. Each development alternative consisting of a set of resoruces and of a development strategy involves a loss value, indicating the extent of failure in realizing the development contemplated. The optimal alternative is the one resulting in the smallest loss. The model can be perfected to find the optimum solution from a very great number of alternatives which are autogenerated. Stochastic parameters can also be included. An algorithm can be incorporated which makes it possible to return to the ideal condition once any loss situation is obtained, by the large scale expenditure of resources. At the same time, hydro-economic analyses are advisable for the as exact as possible formulation of the interrelations between social-economic evolution and water management development.

A study has been performed on the significance of runoff control in the Tisza catchment (see Szász, 1974), with the objective of estimating the regulating impact of reservoirs, storage opportunities and water transfers both in Hungary and abroad, on the evolution of dry weather flows, with allowance for the growth in demands accompanying socio-economic evolution. The variations in the dry weather supplies are examined on the Tisza River and its tributaries along their Hungarian section. With controls adopted in the border cross-sections and downstream of the confluence of tributaries, the changes of supplies are estimated over an annual cycle by monthly intervals, considering also the modifying influences along the upstream sections. The growth of water demands is specified as a boundary condition and with allowance to storage opportunities, as well as to water transfers within the catchment it is desired to find the potential highest level of meeting water demands from supplies available within the catchment.

Related to the investigations on the water management system in the Tisza Valley, a sectorial development model has been elaborated with the title: "Integrated water management and economic parameters for describing the activities in water management" (see Salamin and Barna, 1972). For particular catchments, for a single catchment, or several subcatchments the activities of water management are analyzed for their effectiveness in providing supplies for various demands, protecting human life and property from water damages and for the effect of water management interferences performed, in the interest of the foregoing, (on other branches of the economy). The relevant parameters and relationships are analyzed in building a set of models by which the diverse processes are described in both space and time.

3.3.2 Flood Control

The objectives of a system model compiled for determining the optimal allocation of flood control development expenditures (see Meszéna et al., 1972) is to determine the extent of improving the protection of levees along the individual sections. These may include several sections along the same water course or individual sections along several water courses. The flood plain parts protected by the individual levee sections may become inundated if the flood stage on a particular river section is higher than the value critical for control. Higher stages involve greater areas exposed to inundation and as a rule longer inundation periods. The damages to property in the area and the losses in production (operation) increase parallel with the area inundated and with the duration of inundation. The economic losses comprise also the costs of flood fighting and rescue. The objective of the model is to indicate the extent of levee reinforcement along the individual sections considered and over the period analyzed in a manner to result in a total cost comparable to the available resources and to maximize the potential net benefit of flood control over the period studied.

Further system models have been applied in several regions of the Hungarian part of the Tisza valley as tools in making flood control development decisions. Examples for these are the flood control studies on the Upper-Tisza River (see VIKÖZ, 1974a) and in the Körös River System (see VIKÖZ, 1974b). The objective of these studies was to find a least-cost solution by which the degree of protection specified for the area (1% and 0.5%) can be realized. In these systems models, allowance has been made for other possible engineering methods besides levee improvement, including retention storage beyond the boundary, emergency storage along the rivers, etc.

4. MODELS PROPOSED FOR THE TISZA VALLEY WATER MANAGEMENT SYSTEM

Chapter 2 described the present water management system in the Tisza Valley. It concluded that both excessive abundance and scarcity of water present problems in the operation thereof.

Attention has to be focused on the solution of two fundamental problems:

a) how to control in the best possible way the present system, and

b) how to develop water management in the system in order to comply with the development of the national economy envisaged or predicted.

Chapter 3 pointed out that several initiatives have been taken in Hungary towards the application of systems planning. Useful information has been gained further from international experiences on the modelling of water management systems (see Hall and Dracup, 1970; James and Lee, 1971; Buras, 1972; de Neufville, 1970; de Neufville and Stafford, 1971). The results attained thus far under the joint research programme of the Hungarian National Water Authority and the University of Airzona have also contributed to a realistic and at the same time scientifically founded analysis of the Tisza Valley System (see Fogel and Bogárdi, 1974; VIKÖZ, 1974c).

The 'soundness' of a systems model is believed to depend on its answers to the following three points:

1) Does it represent the objectives of the actual system, does it describe truly the natural, economic, etc. conditions under which the system is to be operated?

2) Does it take into account the uncertainties owing to natural conditions (floods, droughts, etc.) and to sampling practices (shortness of records) related to the system?

3) Is a solution possible?

In the present chapter an attempt has been made at the proposal of models which meet more or less the above three points. The actual problem has invariably been adopted as the starting point of the analysis and the optimum possible method has been sought for its solution. These methods, however, are not always complete and call for considerable research work owing to the complexity of the problem.

In the preceeding chapters it was demonstrated in detail that in the Hungarian Tisza Valley System the flood control subsystem consists predominantly of the flood levees. Consequently, in this particular system flood control and the other branches of water management can be considered separately as regards both control and development. It should be obvious that this separate treatment is not possible if the entire basin of the Tisza River, or a subcatchment thereof is analyzed, where multipurpose valley storage can be realized.

With the foregoing considerations in mind, the systems models proposed will be introduced in the following grouping: The optimum control of the present system will be treated first by outlining the models for the optimal operation of water management (excluding flood control) and water damage aversion. In the second part the development model of the system will be considered dealing again with the models of water management and water damage aversion.

It should be noted that the models will be presented in a simplified form, omitting the mathematical details. None of the models has been applied yet to the entire system, and so far only calculations have been performed. A number of theoretical problems await to be solved in connection with each of the four models before their practical application becomes possible.

4.1 Optimum Control Model of the Present System

4.1.1 Optimum control of the water management system (excluding flood control)

The purpose of the model is to establish a mode of operation for the water management system situated in the Hungarian part of the Tisza River catchment.

Symbols and Definitions

The principal elements in the water management system model are the reservoirs kR, the river sections i, as well as the related water supply and irrigation districts, and the drainage sections. The following model is operated on a monthly time scale. It should be noted that the reservoirs involved in the system are mostly channel inpoundments, whose storage volumes are in general significantly smaller than the monthly runoff volume. Therefore it is considered desirable to operate these on a ten-day time interval basis, for which purpose the development of a ten-day time scale model would be necessary.

Reservoirs

Operation and control of the reservoirs is possible on the basis of each of the four decision variables:

Release: $x(kR,t)$: the water volume to be released from the reservoire kR in the month t,

Water Transfer: $y(kR,i,t)$: the water volume to be transferred from the reservoir kR to the river section i in the month t, Industrial Withdrawal: v(kR,t): the water volume diverted from the reservoir kR in the month t,

Irrigation withdrawal: z(kR,t): the water volume diverted for irrigation from the reservoir kR in the month t.

River Sections

The river sections can be controlled on the basis of three decision alternatives:

Water Transfer: y(i,j,t): the water volume transferred from river section i to river section j in the month t,

Industrial Withdrawal: v(i,t): the water volume diverted for industrial purposes along river section i in the month t,

Irrigation Withdrawal: z(i,t): the water volume diverted for irrigation along river section i in the month t.

The state variable is the natural inflow to the reservoir kR (q(kR,t)), or to the river section i (q(i,t)) in the month t.

It should be noted that state variables are random variables correlated in time and space alike.

Water Demands

Domestic and industrial water demands in the month t are considered deterministic. Thus domestic water demand in the month t in the district connected to the river section will be denoted by w(i,t), whereas in the case of the reservoir kR will be w(kR,t); similarly, industrial water demand in the reservoir kR will be u(kR,t).

The irrigation water demand in the month t is a time-dependent random variable. Thus the demand for irrigation water in the month t along the river section i will be denoted by r(i,t) and in the case of reservoir kR will be r(kR,t).

Losses

It is appreciated that water shortage, undrained rainwater and water pollution may not only cause economic losses to which a monetary value can be assigned, but also indirect losses which are difficult to make tangible. The social losses owing to these water related problems are also important and intangible in character. In the present model the tangible economic losses only will be considered. Further investigation is needed before these intangible losses can be included in the system model. For this analysis economic losses will be grouped as follows:

a) The economic losses resulting from <u>inadequate domestic water supply</u> will be considered of infinite magnitude.

b) The economic losses resulting from <u>inadequate industrial water supply</u> are as follows:

LP (i,t): the economic loss owing to inadequate industrial supply in the region partaining to the river section i in the month t, and

LP (kR,t): the economic loss owing to inadequate industrial supply from the reservoir kR in the month t.

Moreover, for each river section i:

$$LP(i,t) = 0 \quad \text{if } v(i,t) - u(i,t)$$

$$LP(i,t) = LP\left[u(i,t) - v(i,t)\right]; \quad \text{if } v(i,t) < u(i,t) \quad ,$$

Likewise, in the case of the reservior kr:

$$LP(kR,t) = 0 \quad \text{if } v(kR,t) - u(kR,t)$$

$$LP(kR,t) = LP\left[u(kR,t) - v(kR,t)\right]; \quad \text{if } v(kR,t) < u(kR,t) \quad .$$

c) The economic losses owing to <u>undrained rainwater</u> are:

LD(i,t): the economic loss owing to undrained rainwater in the area pertaining to the river section i in the month t,

LD [undrained (i,t); q(i,t)]: depends on the occurrence of undrained rainwater, as an event and on the discharge in the river.

d) The economic losses owing to <u>inadequate irrigation water</u> are defined as follows:

LI(i,t): the economic loss in the area pertaining to river section i in the month t and

LI(kR,t): the same in the case of the reservoir kR in the month t. Moreover, for each river section:

$$LI(i,t) = 0 \quad \text{if } r(i,t) \leq z(i,t)$$

$$LI(i,t) = LI[r(i,t) - z(i,t)]; \quad \text{if } r(i,t) > z(i,t) \quad .$$

Likewise in the case of the reservoir kR:

$$LI(kR,t) = 0 \quad \text{if } r(kR,t) - z(kR,t)$$

$$LI(i,t) = LI[r(kR,t) - z(i,t)]; \quad \text{if } r = (kR,t) > z(kR,t).$$

e) <u>The environmental losses</u>, such as water pollution, deterioration of recreation opportunity, landscape beauty, are regarded as tangible losses, with monetary terms assigned as follows:

LR(i,t): the environmental loss along river section i in the month t,

LR(kR,t): the same along reservoir kR in the month t.

Accordingly for a river section:

$$LR(i,t) = 0 \quad \text{if } \frac{1}{2}\,[q(i,t) + q(i+1,t)] \geq q\,crit(i,t)$$

$$LR(i,t) = LR\left\{\frac{1}{2}\,[q(i,t) + q(i+1,\, t)]\right\} \quad \text{otherwise.}$$

In the case of a reservoir

$$LR(kR,t) = 0 \quad \text{if } S(kR,t) \geq Scrit(kR,t) \quad ,$$

$$LR(kR,t) = LR[S(kR,t)] \quad \text{otherwise,}$$

where

S(kR,t) is the water volume stored in the month t.

The losses mentioned under c), d) and e) are random variables because of the random character of precipitation occurrence, water demands and water resources. The symbols are indicated schematically in Figure 8.

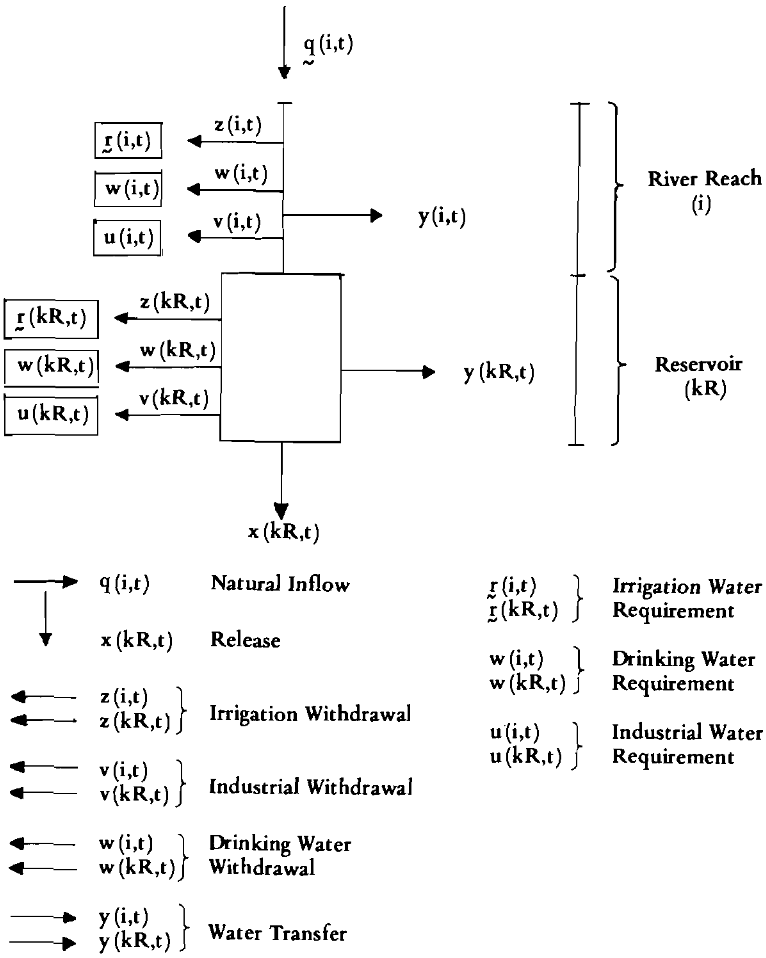

Figure 8. Notations for the system model of optimum control.

The Objective Function

The objective function minimizes the expected value of the sum of economic losses owing to inadequate industrial and irrigation water supply, to undrained rainwater and to environmental damages for all reservoirs, for all river sections and for all districts related thereto, over the entire year.

Since it is permissible to assume that all reservoirs can be filled completely by each March, a period of finite length, namely T = 12 months, can be taken into consideration. This implies that any decision concerning any month affects the potential economic losses only up to the next March. The demands for domestic water supply must be met in each district and in each period, since an economic loss of infinite magnitude has been assigned thereto. The production of hydroelectric power is of secondary importance in the system and has therefore been omitted in the objective function. Accordingly the objective function takes the following form:

$$L(T) = \sum_{i=1}^{m} \sum_{t=T}^{12} E[LP(i,t) + LD(i,t) + LI(i,t) + LR(i,t)] \rightarrow \min$$

where

m is the total number of reservoirs and river sections within the system,

T is the time of optimization, $1 \leq T \leq 11$,

E is the symbol of the expected value in accordance with the water demands r and undrained rainwater.

The solution of the objective function yields the vector of optimal control: opt x(t = T + 1), opt y(t = T + 1), opt z(t = T + 1) and opt v(t = T + 1). Optimization is started in the month of March and in the first step it is possible to find the optimal policy for April.

Starting in turn from the actual situation in April, optimization is again performed and a new policy is found for May. In other words, in the course of optimization the result related to the first month must only be realized in each case. In the first step, no hydrological forecast is assumed in the optimization model. In the second step, however, it is possible to include a monthly forecast in the model, using the Bayes method.

<u>Constraints</u>

1) The constraints for the river sections are essentially the continuity equations:

$$q(i,t) = q(i - 1,t) - w(i,t) - z(i,t) - v(i,t)$$
$$- y(i,t) + y(k,i,t) \quad .$$

2) The storage equation for all reservoirs is:

$$S(kR,t) = q(kR, t - 1) - z(kR,t) - y(kR,j,t)$$
$$- w(kR,t) - v(kR,t) - x(kR,t) \quad .$$

3) The constraints concerning reservoir size are:

$$S(kR,t) > \min S(kR,t) \quad ,$$

$$S(kR,t) < \max S(kR,t) \quad .$$

4) The constraint on water releasing capacity is:

$$x(kR,t) < \max x(kR) \quad .$$

5) The constraint on water transfer capacity is:

$$x(i \text{ or } kR, t) < \max y(i \text{ or } kR) \quad .$$

6) The constraints on withdrawal capacities are:

$$z(i \text{ or } kR,t) < \max z(i \text{ or } kR) \quad ,$$

$$v(i \text{ or } kR,t) < \max v(i \text{ or } kR) \quad .$$

7) The lower limit of dry weather flow (the obligatory discharge to be retained in the channel for sanitary or other purposes) is

$$q(i,t) > \min q(i,t) \quad .$$

8) International obligations:

q(at the downstream boundary,

t) $\geq$ min q(at the downstream boundary.

The Necessary Data, Parameters and Functions

In the Hungarian Tisza Valley system considered, the total number of reservoirs and river sections included is around 30, that is $m \cong 30$. Of this total number, five are reservoirs and 25 are river sections. The number of water transfers is four to five, the number of diversions 30, the number of state variables is about 15.

Decision Variables

Discrete possible values of the decision variables x,y,v and z must be specified for each reservoir and river section (for all m values), for example, in million m^3/month.

State Variables

For each state variable q(i,t), the type of distribution (probably log-normal) must be specified together with the parameters thereof, with the first-order serial correlation coefficients for the successive months, as well as with the cross-correlation matrix (spatial dependence at the boundary of the system).

Water Demands

The demand for domestic supply w(i,t), industrial supply u(i,t) and irrigation water r(i,t) must be specified for all districts.

The irrigation water demands can be expressed presumably in the form of first-order Markov chains. According to the investigations thus far irrigation water demand follows normal distribution in each month. Consequently, the parameters of the distributions and the correlation coefficients $\rho(i, i + 1)$ are needed.

Loss Functions

To be specified are for each month and district:

- losses owing to inadequate industrial supply for the river sections LP(i,t), for the reservoirs LP(kR,t); typical LP loss functions are shown in Figure 9.

- Losses LI(i,t) owing to inadequate irrigation water; a typical LI loss function is illustrated in Figure 10.
- Losses owing to undrained rainwater for each river section LD(i,t) and for each reservoir LD(kR,t); typical LD loss functions are shown in Figure 11.
- environmental losses for each river section LR(i,t) and for each reservoir LR(kR,t); typical LR loss functions are shown in Figure 12.

It is conceivable that linear loss functions can be produced for the individual sections.

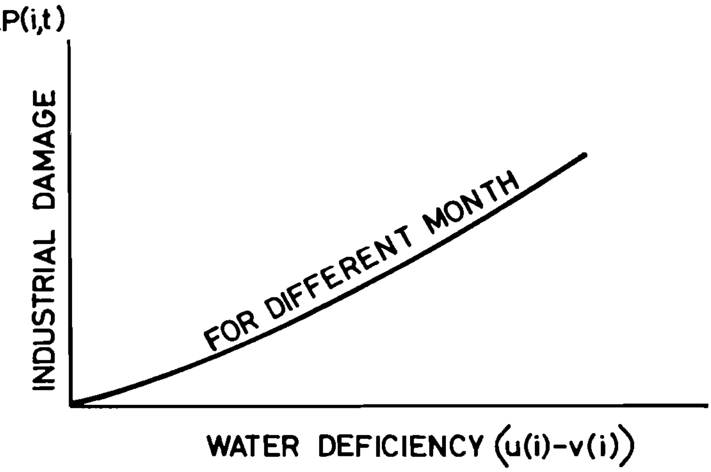

Figure 9. Typical loss function owing to industrial water shortage.

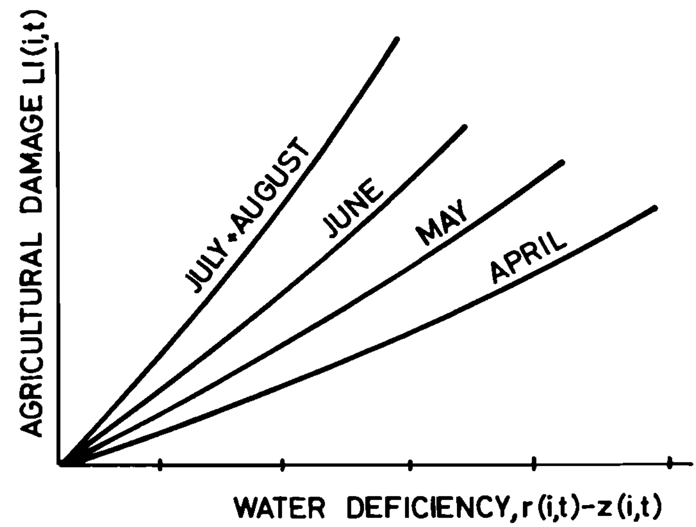

Figure 10. Typical irrigation loss function.

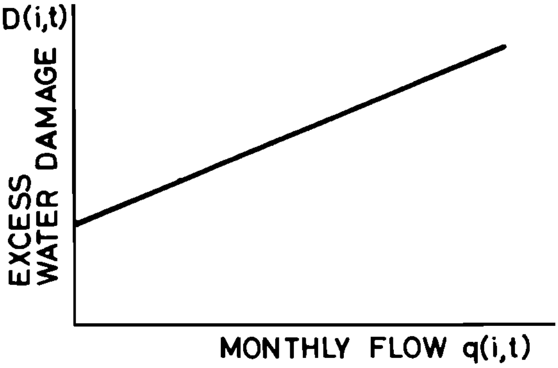

Figure 11. Typical loss function owing to undrained precipitation.

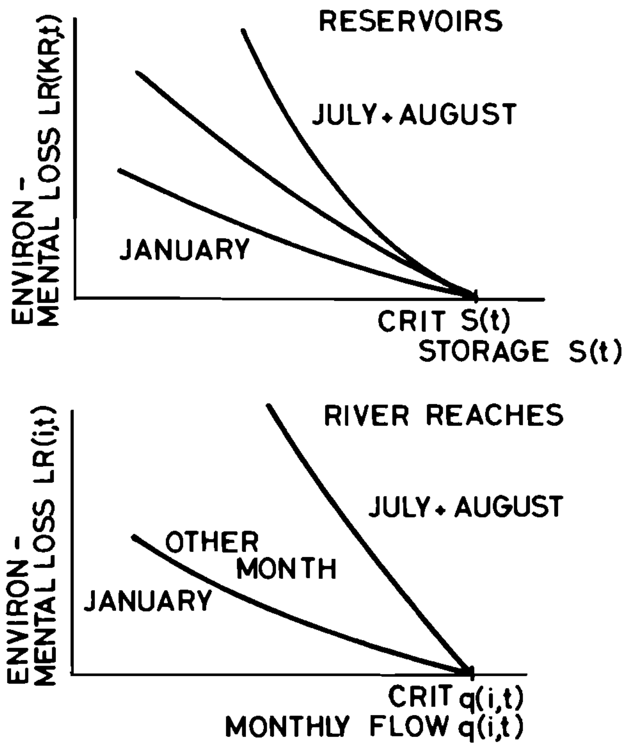

Figure 12. Typical environmental loss functions.

Constraints

In accordance with the constraints mentioned above, the following constraints must be specified for each river section and/or reservoir:

Storage limits:	min S(kR,t)	max S(kR,t)
Water release:		max x(kR,t)
Water transfer:		max y(i or kR)
Irrigation withdrawal:		max z(i or kR)
Industrial withdrawal:		max v(i or kR)
Obligatory flow:		min q(i,t)
International:	min q(at the downstream boundary).	

4.1.2 Optimum Damage Control

One of the most important functions of the Tisza Valley System is to minimize as far as possible the losses caused by undrained rainwater and by flood inundations. For reducing the damages by undrained rainwater, drainage sections have been established, where the excess rainwater is collected by a system of drainage canals and discharged either by gravity or by pumping stations into the recipients. It is therefore important to determine the optimal control for the operation of the drainage systems. In this domain models have been applied thus far to the minor physical system involved only.

At times of floods on the Tisza River the operation of the system of flood levees, that is, flood fighting, is often a technico-economic activity involving considerable responsibility and mobilizing which is influenced by social and political considerations as well (see The 1965 Flood...). Successful flood fighting operations on the Tisza River may avert flood damages amounting to billions of Hungarian forints. In the course of flood fighting operations, temporary measures are implemented to increase the protection of the levees, the areas endangered are evacuated and in the case of inundations rescue work is organized.

Actual flood fighting work is founded on flood forecasts. A forecasting system of varying degrees of accuracy is available for all flood levee sections in the country. The earliest forecast is evidently the least accurate, but the flood fighting center (national or regional) must regard this already in deciding on the optimal distribution of flood fighting materials (for example, sand bags), equipment (for example, earth machines) and labour, and on the means of evacuation among the levee sections. It is of interest to note that along the lower reaches of the Tisza River the first forecast is issued around ten days before the flood event. It should be realized that the protection of the flood levees can be improved considerably in ten days if the resources are employed appropriately for attaining the highest degree of damage reduction.

Forecasts covering a gradually shorter lead time with a higher accuracy are received continuously (for example, every second day) at the flood fighting center. The earlier decisions may be confirmed or modified accordingly. The objective of the systems model is to determine with due allowance to the forecasts, the sequence of optimal decisions concerning the distribution of flood fighting and evacuation resources. In the case of absolutely reliable forecasts the model is a conventional problem of operations research; but actually this condition is rarely satisfied. Consequently, owing to the inevitable uncertainties of flood forecasts during a particular flood, the objective function has the purpose of minimizing the difference between the expected flood losses which can be averted by flood fighting and evacuation on the one hand, and the costs of flood fighting and evacuation work, on the other. The model can be conceived as an algorithm of stochastic, dynamic programming. In the case of a few, for example one or two levee sections and four to five forecasts, or, for example a number of levee sections and a single forecast, the dynamic programming model can be operated with an acceptable computer time requirement. But in more complex cases (several levee sections and four to five forecasts for each) the time requirement increases sharply.

4.2 Development Model of the System

4.2.1 Development Model of the Water Management System (Excluding Flood Damage Control)

In contrast to the model concerned with controlling the present system the following development model is related to a specific part of the Hungarian Tisza Valley System. This subsystem comprises an economic district of about 8,000 to 10,000 km^2 in area in the northern part of Hungary. Owing to the complexity of the whole Tisza Valley System, no attempt has been made at extending this development model to the entire area. The objective of the study is, however, to provide planning data for the development of the complete system. It is suggested to attain this final objective along two approaches:

- development of models for the individual subsystems and subsequently combining them to study the complete system,

- the perfection of development models representing the complete system, which in their present form are applicable to determining the principal long-range development strategies (see Dávid et al., 1974).

The model suggested here may consequently be regarded as the first step along the first approach.

The system is bounded on the upstream side by the national boundary, while on the downstream side by the economic district. In the area considered both industrial (metallurgy, mining, chemical industry, etc.) and agricultural activities are intensive. The population density is higher than the average for

Hungary. To the mountain parts of the system important environmental interests are attached. Both excess abundance and scarcity of water present problems in the system for the following two main reasons:

a) Regional development is accompanied by growing water demands and property values in the protected flood plains increase steadily.

b) As a consequence of factors uncontrollable by the system, an adverse change has occurred in the hydrological input (inflow). As a result, major floods occur at shorter intervals and the dry-weather flow has decreased. The water entering across the upstream boundary is more polluted than before.

These adverse factors are expected to prevail in the future as well. Consequently, the need arises for a long-term water management development strategy in which regional development is harmonized with the necessary water management development. Although the flood control and the water management strategies are interrelated, it again will be presumed that these are independent, except for the reservoirs which serve flood retention purposes as well. The objective of the system model is, consequently, the determination of the water resources management strategy.

Regional Development in the System

1) Planned development in the sectors considered (settlement, industry, agriculture and environment protection): The development objectives must be entered by regions in the first step as input data distributed over time. Consequently, the expected level of development must be defined for each sector. Starting from these regional development input data we want to find the optimal development strategy of water management which may act as feedback on regional development. It is also considered desirable to include development trends other than the originally planned ones in the course of a sensitivity analysis.

2) Water demands: Starting from the development data regarded before as input quantities, water demands must be predicted for the various sectors and regions. For this purpose multivariate regression models appear applicable. These models must reflect among others the effects of potential changes in technology (for example, surface instead of sprinkling irrigation, the introduction of a water saving cooling system, etc.) and the effects of water prices. As a first step, forecasts based on present technologies (at least as far as water is concerned) and on the present water price system are likely to be used in the model.

3) Wastewater production: As in the preceeding paragraph, forecasting models must be developed concerning the volumes of wastewater produced in the individual sectors and regions. Here again allowance must be made for changes in technology and the effects of economic incentives (pollution charges and fines).

4) Possible development in water management: The analyses must be extended to the following major alternatives: surface flow, reservoirs, ground water, water transfer and water reuse. The possible distribution in space of these sources of supply must be described in terms of quantitative and quality data.

Construction of the Model

In constructing the model it is suggested to observe the following sequence of steps:

a) The objective of the model is to meet the requirements of regional development for water management at the minimum possible cost. It should be noted that another objective function is theoretically also applicable. It is conceivable that by minimizing the sum of economic losses owing to water shortage and the development cost, a more exact definition of the objective can be formulated, but a model of this kind is likely to become too complex to be solved in this particular system. It is not intended to restrict studies to the solution of the objective function defined above, provided we succeed in formulating the actual objective of system development by a model which is more exact yet still accessible to solution.

b) The unit cost is represented by the following sum of the annual costs

$$C_1 + C_2 + C_3$$

where

C_1 are the costs of water acquisition and conveyance (investment and operation),
C_2 are the costs of wastewater treatment,
C_3 are the indirect costs of production (for example, the introduction of a water saving technology results in higher costs of production).

c) The input to the system model

The input vector is composed of the following parts:

- the hydrological input along the boundary of the area considered,
- the regional development objectives, as input data,
- unit cost data.

d) Formulation of the possible alternatives

A network of nodal points must be drawn up, with nodal points representing regions of water demands and supplies. In general the water demand in a particular region can be met from several points of supply.

e) The output of the system model

- the development strategy of water management,
- the hydrological output, thus the volume and quality of water leaving the system.

f) Evaluation of the possible alternatives

In order to formulate the optimal water management strategy of the system operating under complex physical, economic and social conditions it is suggested to solve the problem in two steps, using two different models.

Model I

Theoretically this should be a relatively simple model, comparable, for example, to a linear programming model. Accordingly, let C_{ij} be the unit cost $(C_1 + C_2 + C_3)$ of meeting the water demand at point j from the supply point i. The problem then is to minimize the following objective function:

$$Z = \sum_{i}^{m} \sum_{j}^{n} C_{ij} x_{ij} \quad ,$$

where

m is the number of supply points,
n is the number of demand points,
x_{ij} is the water volume conveyed from supply point i to demand point j.

Although a number of assumptions are involved in the above model, it has been applied successfully in several cases for planning regional water management systems. One of the most important basic assumptions is the use of linear cost functions. In its fundamental form this model operates, as will be recalled, with the following two sets of constraints:

1) $\sum_{j}^{m} x_{ij} = d_j$

that is, the total water demand d_j must be met at the point j, and

2) $\sum_{i}^{n} x_{ij} \leq S_i$

that is, the supply (S_i) available at point i cannot be overdrawn.

However, owing to the nature of the problem, further constraints must also be taken into consideration:

3) Water quality constraints.

4) Hydraulic constraints, for example, those arising from the continuity equation (since the supply points are not independent, but are related hydrologically, for example, river sections).

Besides the above four kinds of constraints the stochastic character of the system must also be allowed for in the model. In fact the water supplies S_i and the water demand are both random variables of given distribution. In view of this it can be seen that a rather complicated stochastic linear programming algorithm could be used for solving model I.

Considering different time horizons, for example, 1990, 2000, 2010, solutions which can be realized by the lowest cost can be determined on the basis of the expected value. In the model the annual data on water supplies and demands are used and optimal development is also expressed in terms of annual data. Model I is regarded a preliminary, informative, so-called "screening" model, the solution of which is checked and improved by means of a further, more exact model II.

Model II

The detailed description and method of solution of model II are not yet available. However, this model is believed to be necessarily a dynamic one, which uses a monthly time scale, analyzes a finite time horizon and yields the optimal strategy within a time horizon. Model II starts from the results of model I, using these as input data and seeks solutions which can be realized at the lowest cost discounted to present value. Owing to its complexity, model II is expected to be a simulation model.

4.2.2 The Development Model of Damage Control

In the Tisza Valley, damage control implies the combination of drainage and flood control activities. Nevertheless the following system model is concerned exclusively with the development of flood control. For drainage development a model formulated on similar considerations but of a different character is needed.

From Chapters 1 and 2 it will be realized that in the Hungarian part of the Tisza Valley the potential development of flood control consists mainly of the improvement of the levee system. The other engineering and economic methods of flood control, such as storage, channel regulation, etc. and flood plain zoning, respectively, are of potential interest outside of the system, in the territory of the upstream countries. We wish, however, to include the impact of these measures in the development model of the Hungarian flood control system. As will be demonstrated this can be accomplished by the appropriate changing of the hydrological input.

Basic Criteria (see Bógardi, 1972a, 1972b)

The system under consideration should be divided into sections and each of them can be represented by a particular gage concerning both the peak flood stage h and the flood exposure w (duration). This implies that the peak stage, or the stage and discharge hydrographs along any point of the section can be transferred unanimously to the gauging section. The protection of each levee section to flood stage H and duration W is assumed to be known.

Determination of Transition Functions

The transition functions relate to each other, the successive levee sections proceeding downstream. Accordingly, to the stage or exposure on the gauge of any j-th section a stage or exposure can be assigned on the j + 1-th gauge.

$$h_{j+1} = f_1(h_j, w_j, HJ_{oj}, HB_{oj}, WJ_{oj}, WB_{oj}) \qquad (1)$$

$$w_{j+1} = f_2(h_k, w_j, HJ_{oj}, HB_{oj}, WJ_{oj}, WB_{oj}) \qquad (2)$$

where

h_j, w_j are the stage and exposure (duration), respectively on the j-th gauge,

HJ_{oj}, HB_{oj} are the minimal protections against flood level of the right-hand and left-hand levee sections, respectively along the j-th river section,

WJ_{oj}, WB_{oj} are the minimal protections against exposure (duration) of the right-hand and left-hand levee sections, respectively, along the j-th river section.

In connection with Eqs. (1) and (2), two fundamental cases are distinguished, depending upon whether a breach in the levee occurs or not on the j-th levee section. In the absence of a levee breach on the j-th section the stage and exposure on the j+1-th section are controlled alone by the hydraulic conditions of the flood bed between the levees. In such cases both stages and exposures are determined by simple gauge relationships.

If there is a breach on the j-th levee section, then lower stages and loads will evidently occur along the downstream section j + 1, than could be expected from the simple gauge relationship. Allowance must be made for the water volume conveyed as a consequence of the breach in the flood plain rather than between the levees. Under a given set of flood bed and flood plain conditions, at a given flood hydrograph, hydraulic considerations in combination with the known protection of the section will yield the time t_o of the breach, further the hydrograph of the discharge $Q_k(t)$ entering the flood plain. Deducting from the streamflow before the breach the discharge across the latter, the streamflow and the hydrograph $Q_f(t)$ thereof conveyed downstream in the channel are obtained. Once the discharge hydrographs $Q_f(t)$ remaining in the channel over the section j is known, the peak stage and exposure along the j+1-th section can be determined from gauge relations.

Eqs. (1) and (2) are therefore written for further two cases each

$$h_{j+1} = \begin{cases} f_{1n}(h_j) \text{ if } (h_j < \min\{HJ_{oj}, HB_{oj}\}) \cap (w_j < \min\{WJ_{oj}, WB_{oj}\}) & (3) \\ f_{1s}(h_j) \quad \text{otherwise} & (4) \end{cases}$$

$$w_{j+1} = \begin{cases} f_{2n}(w_j) \quad \text{as with } f_{1n} & (5) \\ f_{2s}(w_j) \quad \text{as with } f_{1s} & (6) \end{cases}$$

where in addition to the symbols explained before, f_{1n} and f_{2n} are the transition functions in the absence of a breach, while f_{1s} and f_{2s} are the transition functions in the case of a breach.

The Determination of the Loss Functions

In connection with the transition functions it has been stated already that for a given set of conditions it is possible to determine the hydrograph $Q_k(t)$ of discharges entering the flood plain through a breach. For a particular hydrograph $Q_k(t)$, the size of the area inundated will be controlled by the topography in the flood plain. The value of the area inundated during the flood wave (or the damages caused by inundation) are related to the parameters resulting in the breach as

$$\ell_j = f_3(h_j, w_j, HJ_j, HB_j, WJ_j, WB_j) \tag{7}$$

where ℓ_j is the loss caused along the j-th section by a single inundation at the flood wave and protection parameters involved in the relationship. The flood wave parameters h_j and w_j are random variables and with the relevant records available it is possible to find their joint distribution:

$$p_j = f_4(h_j, w_j) \tag{8}$$

where f_4 is the joint density function of h_j and w_j.

The magnitude of the expected annual damage along the j-th section is obtained by summing the products of the corresponding p_j and ℓ_j values:

$$L_j = \int_0^1 (\ell_j p_j) dp_j \quad . \tag{9}$$

As in the case of the transition functions, Eq. (7) is written for the two cases distinguished above. In the absence of a breach there is evidently no damage, so that in Equ. (9) the lower limit of the integral is practically the probability value p_{jo} at which such values of h_j, w_j are expected which cause no failure, that is, damage on the section. Consequently, in this range the integral (9) assumes 0 value.

In the case of a breach, as mentioned before, the magnitude of the damage is determined by examining the area inundated. The description of the relevant investigation would exceed the scope of the present paper and it will be noted only that the losses can be estimated on the basis of a 1:10,000 scale map by considering unsteady flow. The relevant computations can be performed on a computer.

The Determination of Cost Functions

With the parameters mentioned in connection with the transition and loss functions the development costs of each section are given by the cost functions. No costs arise unless it is desired to improve the protection of the levees over the present values H_o and W_o. The cost of development c_j over the j-th section is thus expressed by the following function

$$c_j = f_5(H_{oj}, W_{oj}, H_j, W_j) \tag{10}$$

$$H_j > H_{oj}; \quad W_j > W_{oj} \quad .$$

The costs obtained by Eq. (10), which are nonrecurring costs, are multiplied by the discount factor d for comparing them with the expected annual losses

$$C_j + dc_j \tag{11}$$

where C_j is the annual cost.

Determination of the Objective Function

The objective function refers to the determination of the system development at which the sum of the expected annual losses and of the development costs is minimum. For each section j of the system a development alternative is assumed $(H_1, W_1, \ldots, H_j, W_j, \ldots, H_n, W_n)$ and thus a development alternative is obtained for the system. For the most upstream section the expected annual loss L_1 ks found from Eqs. (7) and (9), together with the annual cost C_1 according to Eqs. (10) and (11).

With the help of the transition functions, Eqs. (3) to (6), the computation is repeated for the successive downstream sections 2 to n. The L_j, C_j values thus obtained are summed:

$$K_i + \sum_{j=1}^{n} (L_j + C_j)_i \tag{12}$$

where K_i is the value of the objective function pertaining to the i-th development alternative of the system. The computation is then repeated for additional development alternatives. If the number of development alternatives by sections is b and the number of sections is n, then the computation must be repeated b^n-times for the system.

The objective function becomes accordingly

$$I(H_{ji}, W_{ji}) = \min K_i \tag{13}$$

where

I is the symbol of the optimal development alternative for the system,

H_{ji}, W_{ji} are the degrees of development over the j-th section pertaining to the i-th system-development alternative.

The computerized solution of the problem described can be performed by stochastic dynamic programming.

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River Basin Network Modelling and Its Application to the Tisza River Basin

Pavel Koryavov and Igor Belyaev

1. INTRODUCTION

Computer simulation of complex dynamic systems combined with decision-making methods is becoming one of the main techniques for systems design, and is an important goal of the IIASA Water Resources Project. In this paper, possible variants of regional development under the condition of a water deficit are analyzed.

An essential block of a simulation system is that which represents the river basin and its development perspectives. Models of this block should begin by selecting reasonable water-use alternatives. By using the model presented in this paper, it is possible to consider the problem of regional water resources use in aggregated indexes. If the exogenous factors, i.e. weather prediction and runoff in a river basin, are given, planning experts can study the consequences of different measures. The model can also be used to solve optimization problems; it has been applied to a concrete situation, that of the Hungarian part of the Tisza River Basin.

2. NETWORK MODEL OF A RIVER BASIN

The entire river basin system may be illustrated by an oriented graph (without cycles) or network (see Figure 1). This graph is not a tree because of the artificial channels and water lines that allow the removal of water from one point of the river and its return to another.

The network nodes are points denoting the following:

- Different runoff areas corresponding to each tributary considered;

- Locations where tributaries enter, and those of reservoirs and other hydrotechnical constructions on the river and its tributaries;

- Location of water intake by and return from the users.

One of the conditions for using the model is that the length of the river reach (i.e. the distance between adjacent nodes) be less than the distance the river flows during the time interval considered. This criterion restricts the number of nodes and their locations.

Directed arcs connect nodes and indicate the direction of the water flow (Figure 2). Some value of F_{ij} is assigned to each of the arcs connecting nodes i and j; F_{ij} is equal to the volume of the water flow from node i to node j during time interval t. Nodes may be divided into those with both ingoing and outgoing arcs (for which a balance equation can be written), and fictitions nodes with either ingoing or outgoing arcs.

Nodes with ingoing arcs denote the water outflow from the river system (i.e. non-return losses of water and transfer of water to another region). F_{io} denotes the outflow from the system throught node i. Nodes with outgoing arcs indicate the water inflow to the river system, which could be from parts of the river basin, tributaries, channels or surfaces, or from an underground inflow. Inflow to the system through node i is denoted by F_{oi}.

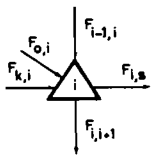


FIGURE 1

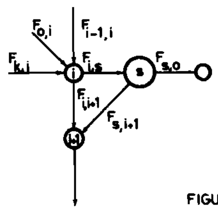


FIGURE 2

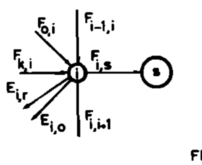


FIGURE 3

Figures 1–3. Network flow diagram of the storage process.

Volumes of water stored in the network nodes W_i (t) will be considered phase variables of the system, where i = number of nodes and t = number of time intervals. For each node we will have the following balance equation:

$$W_i(t + 1) = W_i(t) + \sum_{K_i} F_{k_i,i}(t) - \sum_{M_i} F_{i,r_i}(t) \quad , \qquad (1)$$

where

$F_{K_i,i}$ = volume of water inflow to the river system through the arcs entering node i,

F_{i,r_i} = volume of water outflow from the river system through the out-going arcs of node i,

during time interval t.

The phase variables W_i are subject to the following constraints:

$$W_{i\ min} \le W_i(t) \le W_{i\ max} \quad . \qquad (2)$$

For the nodes denoting reservoir locations,

$W_{i\ max}$ = maximum storage capacity,

$W_{i\ min}$ = minimum volume of water to be stored.

$W_{i\ max}$ is defined by the storage capacity of the reservoir, and $W_{i\ min}$ by such factors as transportation, ecology and recreation.

For all other network nodes, $W_{i\ min} = W_{i\ max} = 0$, and the dynamic equation (1) can be reduced to a balance equation that describes conservation of water flow in a node; that is,

$$\sum F_{k_i,i}(t) = \sum F_{i,r_i}(t) = 0 \quad . \qquad (3)$$

Values $F_{ij}(i,j \neq 0)$ are the control variables that represent either the amount of water supplied to users or the flow along the river, tributary, or channel. When the node is not a storage node, these variables may be found by using the flow balance equation (3). Values F_{io} can be found automatically since they are either functions of the ingoing losses, or given values (if they represent the water transferred form one region to another). Also, F_{io} can be a control variable if the consumer makes use of the total amount of water obtained from the river basin. Values $F_{ij}(t)$ are subject to the following constraints:

$$d_{ij}^{-} \leq F_{ij} \leq d_{ij}^{+} \tag{4}$$

where

d_{ij}^{+} = city or irrigation area (CIA) demand or maximum capacity of the river reach, tributary, or channel;

d_{ij}^{-} = minimum amount of water needed for users' industrial activity, for transportation along the river, tributary or channel, or for prescribed sanitary and ecological standards.

Let us consider a few examples (Figures 1,2 and 3). Figure 1 shows the storage node. Areas shown may be defined as follows:

$F_{i-1,i}$ = volume of water coming from the upper flow of node i-1;

$F_{i,i+1}$ = volume of water flowing to downstream node i+1 along the river bed;

$F_{k,i}$ = volume of inflow from tributary node i;

$F_{i,s}$ = volume of water supplied from node i to the users (cities and irrigation systems) in node S;

F_{oi} = volume of surface and underground inflow from which the volume of evaporated water is subtracted; F_{oi} should be given on the basis of water forecast.

These definitions may also be applied when i is a storage node, because when full storage capacity is reached, this node will work as a node that transfers water.

3. USE OF THE NETWORK MODEL

We see that the functioning of the river can be illustrated in the form of a dynamic system with discrete time. The description of our model is based on integral characteristics that are valid independent of time and space scales. To find the integral characteristics of the water flow, we should solve corresponding hydrodynamic problems. In models of this type, it is possible to use an average value of the river reach capacity and to neglect some transfer processes such as, for example, flood propagation along the river. It is therefore necessary to consider a long time interval, that is, one that is (at least) longer than the time needed for water to flow from one node to the next.

Let us consider our model with the given control variables $F_{ji}(t)$ that allow us to solve the following boundary value problem: find all phase variables and their functions entering in balance equations (as non-return losses) at any time interval t for given $F_{oi}(t)$. Thus, the model can be used as a simulation system.

By changing in some way the control variables $F_{ji}(t)$ and the constraints (i.e. the distribution of water among users and the conditions of basin functioning) we can estimate the results of the decisions made. An important feature of the application of our model is that it permits investigation of critical situations. For this, we will use a procedure that resembles a game in which one of the players is Nature.

The values of $F_{oi}(t)$ (i.e. the "nature factor") pose difficulties. Those responsible for planning water resources development should consider this factor. Through the process of man-machine analysis, we can identify new storage sites and their volumes. The planners have at their disposal normative costs of hydroconstructions as well as estimates of the economic effects of water use on the city and irrigation areas and industry. They can thus use our model to obtain various types of information that would allow a comparison of decision variants for the water resources system.

Note that our model is rough and can be used only for first-stage analysis to eliminate non-compatible variants. Nevertheless, it has a few advantages. Since the time and space intervals can be altered over a broad range, it is easy to match the model with those of industry, cities and irrigation systems, using the same time interval.

The model could be used for operational control. When meteorologic and hydrologic forecasts change, the model makes it possible to estimate easily the required changes in the structure of water allocation. It is thus an essential block of the general simulation system.

The model can also be used for solving optimization problems. An interesting problem is that of finding the control variables that minimize damage from overflow and damage to users from water shortage. The objective function then will have the form:

$$\min = \sum_{t=1}^{N} \left\{ \sum_{i=1}^{M} \left(P_{1i}^{t}\, d_{ki}^{+}(t) - F_{ki}(t) \right) + \sum_{i-1}^{L} P_{2i}^{t}(E_{ir}(t)) \right\} , \qquad (5)$$

where

P_{1i}^{t} = function of losses due to shortage of water supply for users,

P_{2i}^{t} = function of losses caused by overflow,

M = number of users,

L = number of river reaches,

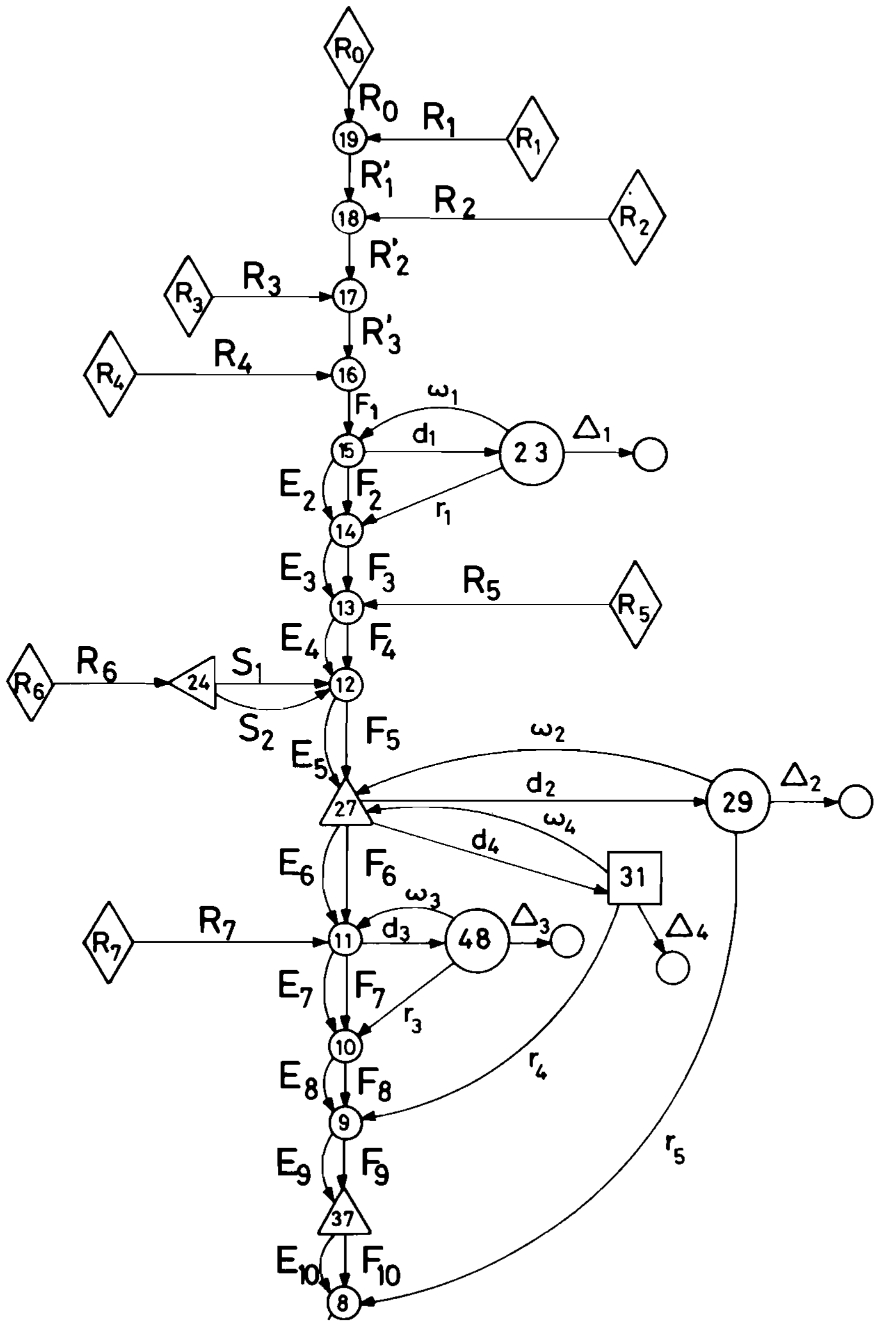
N = number of time intervals.

Solutions to optimization problems should be given the form of the functions P_{1i}^{t} and P_{2i}^{t}, and values $F_{oi}(t)$ and $d_{ki}^{t}(t)$.

4. CHARACTERISTICS OF THE TISZA RIVER BASIN

The Tisza River Basin has been selected as a case study for the application of the network flow approach. The Tisza River is a main left-side tributary of the Danube. The basin is 976 km long, and has a total area of 156,000 km^2 as well as a complex relief. About half of the river is in flat territory that is bounded on the north, the east and the south-east by the Carpathian Mountain range. The climate in the basin is temperate continental; the average yearly temperature ranges from 3^o to 9^o C in the northern part, and from 8^o to 11^o C in the southern part. The vegetative period varies from 166 to 236 days. The basin's total annual precipitation is 1200-1400 mm in the hilly parts and 550-580 on the plains.

A network flow diagram for the Hungarian part of the Tisza River Basin is shown in Figure 4. Nodes and arcs in the figure have the following meanings:


R0
R0
R1
R1
19
R'1
R 2
R2
18
R'2
R3
R3
17
R'3
R4
R4
16
F1
ω1
15
d1
23
Δ1
E2
F2
r1
14
E3
F3
R5
R5
13
E4
F4
R6
R6
S1
24
12
S2
F5
ω2
E5
d2
29
Δ2
27
ω4
d4
E6
F6
31
ω3
R7
R7
11
d3
48
Δ3
Δ4
E7
F7
r3
10
r4
E8
F8
9
r5
E9
F9
37
E10
F10
8

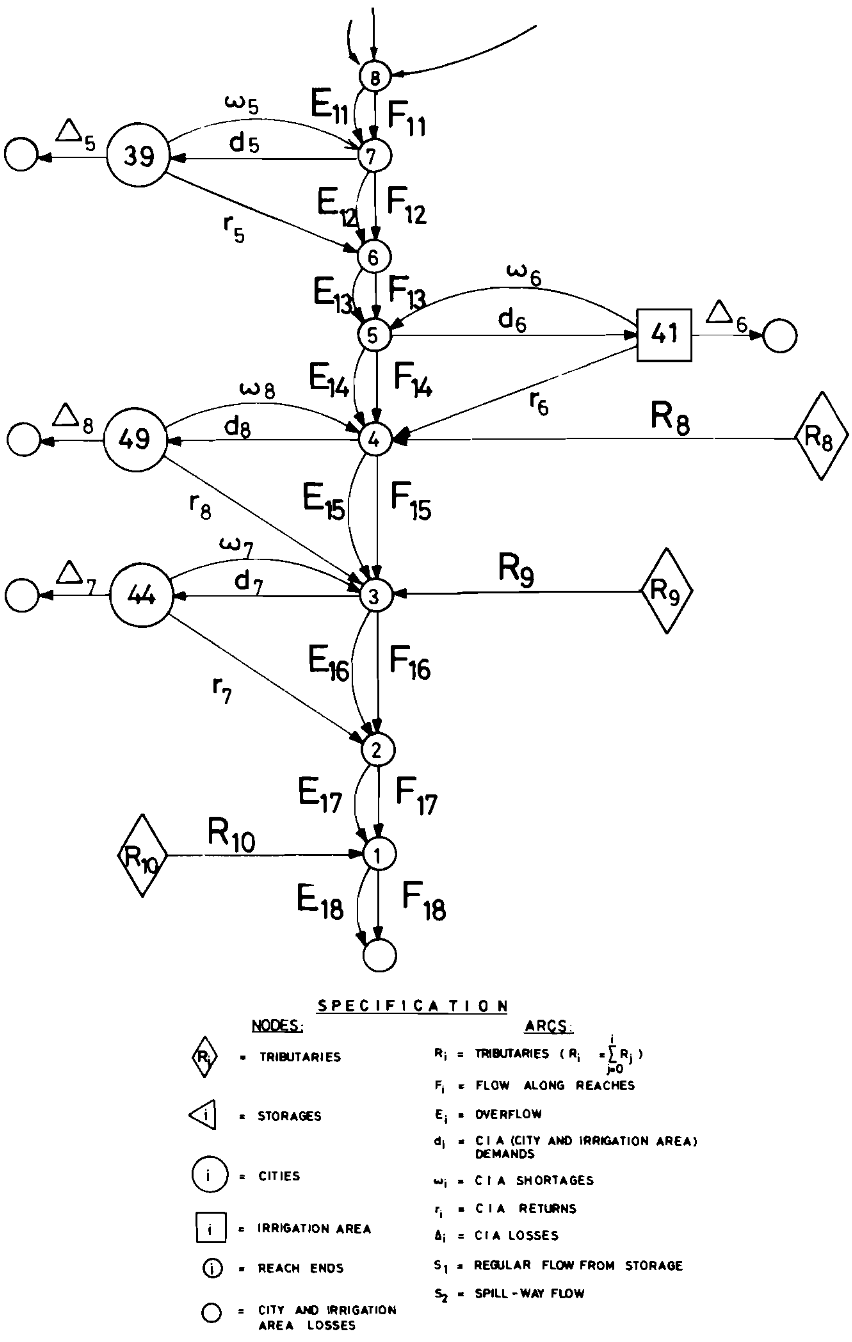


Figure 4. Network flow diagram of the Tisza River Basin.

Nodes: ◇ R_i = tributaries (i = 0,1,...,10),

R_o = Tisza source,

R_1 = Viseu,

R_2 = Iza,

R_3 = Teresva,

R_4 = Borzava,

R_5 = Szamos,

R_6 = Bodrog,

R_7 = Sajó,

R_8 = Körös,

R_9 = Maros

R_{10} = Danube

◁ i : storages (i = 24, 27, 37),

24 = Besha,

27 = Tiszalök,

37 = Kisköre;

(i): cities (i = 23, 39, 44, 48, 49),

23 = Fehérgyarmet,

29 = Debrecen

39 = Szolnok,

44 = Szeged,

48 = Polgár,

49 = Csongrád;

[i]: irrigation areas (i = 31, 41),

31 = Tiszalök,

41 = Körös system;

(i): reach ends (i = 0,1,2,...,19) (ends of reaches are located at specific points where either the tributaries enter or the users take or return the water);

○ : CIA losses (these nodes are not marked by numbers because they are used only to indicate the ends of the arcs);

Arcs :

R_i = tributary inflows (i = 1,2,...,10),

$R'_i = \sum_{j=0}^{i} R_j$ (i = 1,2,3),

F_i = flows along reaches (i = 1,2,...,18),
E_i = overflows (i = 1,2,...,18),
d_i = CIA demands (i = 1,2,...,8),
ω_i = CIA shortages (i = 1,2,...,8),
r_i = CIA returns (i = 1,2,...,8),
Δ_i = CIA losses (i = 1,2,...,8),
S_1 = regular flow from storage,
S_2 = spillway flow from storage.

The distribution of water resources in the Tisza River Basin is extremely nonuniform in time and space. This causes floods (in the spring period) and droughts (in the summer-autumn period). The seasonal runoff for the various tributaries and rivers during a mean of the wet years is given in Table 1.

Table 1. Runoff in the Tisza River Basin for the average wet year (km^3).

Rivers and Tributaries	Identifiers in Fig. 4	Months					
		XII-I	II-III	IV-V	VI-VII	VIII-IX	X-XI
Tisza Source	R0	.05	.1	.42	.21	.16	.1
Viseu	R1	.2	.42	2.5	.83	.47	.31
Iza	R2	.1	.26	.42	.42	.16	.11
Teresva	R3	.22	.36	1.3	.83	.94	.62
Boržava	R4	.06	.13	.50	.29	.15	.13
Szamos	R5	.43	1.3	1.3	.64	.22	.43
Bodrog	R6	.35	1.0	1.0	.57	.10	.35
Czisul Alb	R7	.16	.5	.43	.24	.08	.15
Körös	R8	.02	.03	.04	.08	.01	.02
Maros	R9	.55	.83	1.1	2.2	.27	.55
Danube	R10	4.0	12.0	28.0	18.0	10.0	8.0

There are many hydrotechnic constructions in the Tisza River Basin; the two main types are reservoirs and dikes (or levees). The total reservoir volume is o.6 km^3, and the total length of the dikes about 4,000 km. Additional reservoirs with a total capacity of about 1 km^3 are, of necessity, under construction, and the dike system is being improved. Large-scale investments in development and construction of water resources units are related to growth plans for industry, agriculture and cities in all countries located in the Tisza River Basin, since growth leads to increasing amounts of pollutants in the basin and thus presents problems of water quality.

There are several reasons for our analysis of the Tisza River Basin. These are:

- The basin's physical structure (geology, topography, hydrology) is known;
- Three main problems of water resources exist in the basin: shortages of water during the dry period, floods produced by rainfall during the spring and the summer, and water quality;
- The data are available (it appears that most of the necessary data could be supplied to IIASA by the Hungarian National Water Authority);
- The river basin is international (Czechoslovakia, Hungary, Romania, USSR and Yugoslavia have parts of their territories within the basin).

5. OPTIMIZATION PROBLEM FOR THE TISZA RIVER BASIN

This paper considers the following two aspects of the problem of water resources control in the Tisza River Basin:

- Water distribution among consumers at different time intervals (seasons) during a one-year period;
- Prevention or reduction of overflow during a flood period (i.e. a decrease in flood damage).

Water quality, navigation, and recreation are not considered, although these three factors could easily be taken into account by introducing restrictions in the form of:

$$F_i^t \geq F_i^t \min \tag{6}$$

$$W_i^t \geq W_i^t \min \quad , \tag{7}$$

where

$F^t_{i\ min}$ = minimum flow permitted in a reach,

$w^t_{i\ min}$ = minimum volume of water permitted in a storage reservoir.

Estimates of the values $F^t_{i\ min}$ and $W^t_{i\ min}$ can be made from submodels for water quality, navigation and recreation.

To minimize both water shortage and overflow, we need an objective function written in the form:

$$\min P = \sum_{t=1}^{N} \left[\sum_{i=1}^{8} P^t_{1i}(\omega^t_i) + \sum_{j=1}^{18} P^t_{2j}(E^t_j) \right] , \qquad (8)$$

where

$P^t_{1i}(\omega^t_i)$ = function of losses due to shortage of water for users,

$P^t_{2j}(E_j)$ = function of losses caused by overflow.

Defining these functions is a problem not considered in this paper. We shall assume that these functions are given as convex functions of their arguments, and can be approximated by a broken line, as shown in Figure 5. The objective function to be minimized will then take the form:

$$\min P = \sum_{t=1}^{N} \left[\sum_{i=1}^{8} (\beta^t_{i1}\ \omega^t_{i1} + \beta^t_{i2}\ \omega^t_{i2}) + \sum_{j=1}^{18} (\gamma^t_{j1}\ E^t_{j1}\ \gamma^t_{j2}\ E_{j2}) \right] , \qquad (9)$$

where

$$\beta^t_{i1},\ \beta^t_{i2},\ \gamma^t_{j1},\ \gamma^t_{j2},\ (1 = 1,\ldots,8\ ;$$

$$j = 1,\ldots,18\ ; \qquad t = 1,\ldots,N)$$

are given coefficients. This minimization is subject to constraints related to the phase variables shown in Table 2. When necessary, the number of terms in the broken-line approximations can be increased.

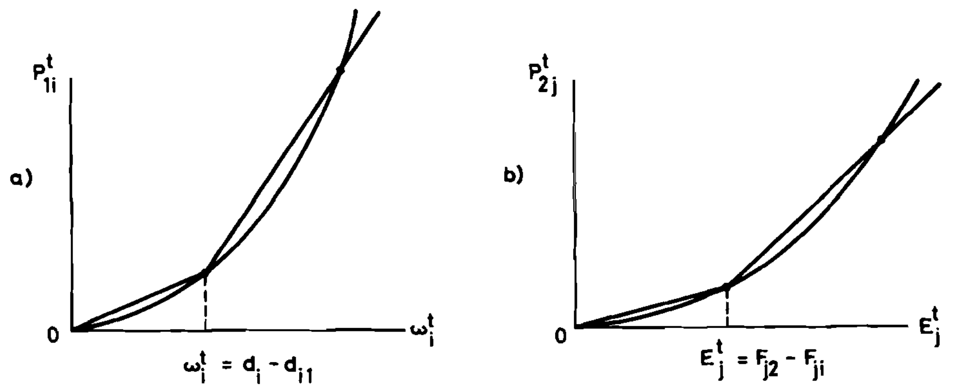


Figure 5. Function of losses due to (a) water shortage and (b) overflow.

Table 2. Constraints of the minimization.

$$F_1^t = R_0^t + R_1^t + R_2^t + R_3^t + R_4^t - E_1^t \quad ,$$

$$F_2^t = F_1^t + \omega_1^t - d_1^t + E_1^t - E_2^t \quad ,$$

$$F_3^t = F_2^t + r_1^t + E_2^t - E_3^t \quad ,$$

$$F_4^t = F_3^t + R_5^t + E_3^t - E_4^t \quad ,$$

$$F_5^t = F_4^t + S_1^t + S_2^t + E_4^t - E_5^t \quad ,$$

$$F_6^t = F_5^t + \omega_2^t - d_2^t + \omega_4^t - d_4^t + E_5^t - E_6^t - I_{27}^t + M_{27}^t \quad ,$$

$$F_7^t = F_6^t + R_7^t + \omega_3^t - d_3^t + E_6^t - E_7^t \quad ,$$

$$F_8^t = F_7^t + r_3^t + E_7^t - E_8^t \quad ,$$

$$F_9^t = F_8^t + r_4^t + E_8^t - E_9^t \quad ,$$

$$F_{10}^t = F_9^t + E^t - E_{10}^t - I_{37}^t + M_{37}^t \quad ,$$

$$F_{11}^t = F_{10}^t + r_2^t + E_{10} - E_{11}^t \quad ,$$

$$F_{12}^t = F_{11}^t + \omega_5^t - d_5^t + E_{11}^t - E_{12}^t \quad ,$$

$$F_{13}^t = F_{12}^t + r_5^t + E_{12}^t - E_{13} \quad ,$$

$$F_{14}^t = F_{13}^t + \omega_6^t - d_6^t + E_{13} - E_{14}^t \quad ,$$

$$F_{15}^t = F_{14} + R_8^t + r_6^t + \omega_8^t - d_8^t + E_{14}^t - E_{15} \quad ,$$

$$F_{16}^t = F_{15} - R_9^t + r_8^t + \omega_7^t - d_7^t + E_{15}^t - E_{16}^t \quad ,$$

$$F_{17} = F_{16} + r_7^t + r_7^t + E_{16}^t - E_{17}^t \quad ,$$

$$F_{18} = F_{17} + R_{10} + E_{17} - E_{18} \quad ,$$

$$d_i^t = \omega_i^t + r_i^t + \Delta_i^t \quad , \qquad (i = 1,2,\ldots,8) \quad ,$$

$$S_1^t = R_6^t - I_{25}^t + M_{25}^t - S_2^t \quad ,$$

$$W_i^t = W_i^{t-1} + I_i^t - M_i^t \quad , \qquad (i = 25,27,37) \quad ,$$

$$W_i^O = W_i^N \ (i = 25,27,37) \quad ,$$

$$E_i^t = E_{i1}^t + E_{i2} \quad , \qquad (i = 1,2,\ldots,18) \quad ,$$

$$\omega_i^t = \omega_{i1}^t + \omega_{i2}^t \quad , \qquad (i = 1,2,\ldots,8) \quad ,$$

$$\Delta_i^t = \alpha_{i0}^t \cdot d_i^t \quad , \qquad (i = 1,2,\ldots,8) \quad ,$$

$$F_i^t \geq F_{i\ min}^t \qquad (i = 1,2,\ldots,18)$$

$$E_{i1}^t, E_{i2}^t \geq 0 \qquad (i = 1,2,\ldots,18) \quad ,$$

$$\omega_{i1}, \omega_{i2} \geq 0 \qquad (i = 1,2,\ldots,18) \quad ,$$

$$d_i^t \geq \omega_i^t \geq 0 \qquad (i = 1,2,\ldots,8) \quad ,$$

$$r_i^t \geq 0 \qquad (i = 1,2,\ldots,8)$$

$$W_{i\ max} \geq W_i^t \geq W_{i\ min}^t \qquad (i = 25,27,37)$$

$$S_{1\ max}^t \geq S_1^t \geq 0 \quad ,$$

$$S_2^t \geq 0 \quad ,$$

$$I_{i\ max}^t \geq I_i^t \geq 0 \qquad (i = 25,27,37) \quad ,$$

$$M_{i\ max}^t \geq M_i^t \geq 0 \qquad (i = 25,27,37) \quad ,$$

Table 3. Water demand for cities and irrigation areas in the Tisza River Basin (km^3).

Cities and Irrigation Areas	Population of cities and irrigation areas (ha)	Number of Node	Months					
			XII-I	II-III	IV-V	VI-VII	VIII-IX	X-XI
Fehérgyarmat	16,000	23	3,825	4,600	6,875	7,650	9,550	5,725
Debrecen	155,000	29	21,675	20,800	31,220	43,350	54,200	32,625
Polgár	15,000	48	6,625	7,650	11,475	12,750	15,950	9,550
Szolnok	450,000	39	8,300	9,950	14,925	16,575	20,725	12,425
Csongrád	323,000	49	40,675	48,800	73,200	81,350	101,675	61,000
Szeged	119,000	44	16,700	20,050	12,030	33,400	41,750	25,050
Tiszalök Irrigation System	150,000	31	10,000	40,000	80,000	103,680	103,680	60,000
Körös Irrigation System	15,000	41	10,000	40,000	50,000	68,120	69,120	40,000

Table 4. Water losses for cities and irrigation areas in the Tisza River Basin as a proportion of demand.

Cities and Irrigation Areas	Number of Node	Months					
		XII-I	II-III	IV-V	VI-VII	VIII-IX	X-XI
Fehérgyarmat	23	.2	.3	.3	.8	.6	.2
Debrecen	29	.2	.3	.3	.8	.6	.2
Polgár	48	.2	.3	.3	.7	.7	.2
Szolnok	39	.2	.3	.3	.7	.7	.2
Csongrád	49	.2	.3	.3	.6	.8	.2
Szeged	44	.2	.3	.3	.6	.8	.2
Tiszalök Irrigation System	31	.1	.6	.6	.8	.9	.2
Körös Irrigation System	41	.1	.6	.6	.8	.9	.2

For all the above constraints, $t = 1,2,\ldots,N$, the given parameters of the problem (presented in Tables 1,3, and 4) are as follows:

$$R_i^t \quad (i = 0,1,\ldots,10 \quad ; \qquad t = 1,\ldots,6)$$

$$d_i^t \quad (i = 1,\ldots,8 \quad ; \qquad t = 1,\ldots,6)$$

$$\alpha_{io}^t \quad (i = 1,\ldots,8 \quad ; \qquad t = 1,\ldots,6)$$

Some additional constraints to which the variables are subjected are the following:

$$F_{i\ min}^t,\ F_{i1}^t,\ F_{i2}^t \qquad (i = 1,\ldots,18 \quad ; \quad t = 1,\ldots,N) \quad ,$$

$$W_{i\ max},W_{i\ min}^t,\ I_{i\ max}^t,\ M_{i\ max}^t \qquad (i = 25,27,37 \quad ; \quad t = 1,\ldots,6)$$

$$d_{i1}^t \quad (i = 1,\ldots,8 \quad ; \qquad t = 1,\ldots,N) \quad ,$$

$$S_{1\ max}^t \quad (t = 1,\ldots,N) \quad .$$

6. SOME RESULTS OF CALCULATIONS

To solve the problem presented in the previous section, a Matrix Generator (MAGEN) and Linear Programming Package (APEX) were used.[1]

Calculations were done for a one-year horizon with time intervals of two months; that is, $\Delta t = 2$ months and $N = 6$.

Each program run on the CDC-6600 computer (located in Frankfurt) took about 3/4 sec of central processing unit time and 12 sec for input and output. Some results of the calculations are shown in Figures 6 and 7. Figure 6 also shows

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[1]Program prepared by Dr. D. Bell, Methodology Project, IIASA.

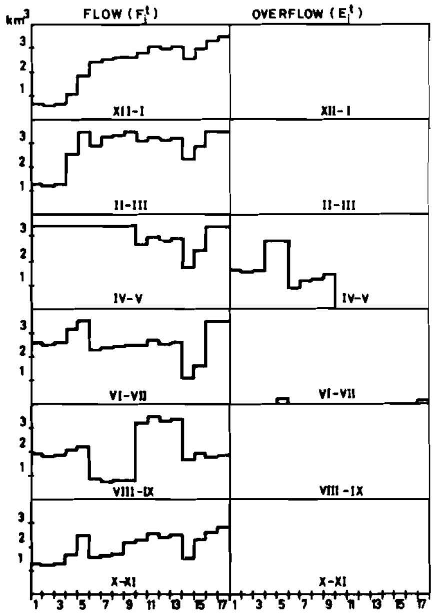

Figure 6. Flows and overflows along the Tisza River.

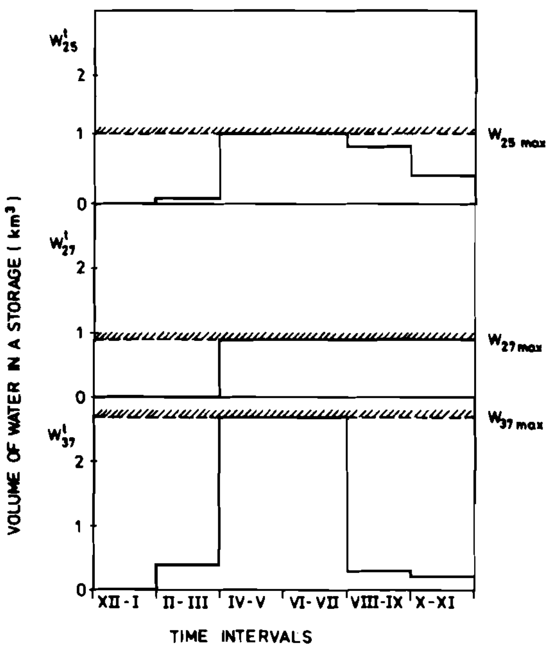

Figure 7. Variation with time of the volume of stored water.

the flows and overflows along the Tisza River for six time intervals. For example, it can be seen that storage in node 37 (between arcs 9 and 10) accumulates overflow in the third time interval (April-May), and can release water downstream during the fifth time interval (August-September) to cover water supply.

Figure 7 shows that the volume of water in the reservoirs varies in a reasonable way: reservoirs were empty before the flood which began in the second time interval (February-March). At that time, flood water accumulated in the reservoirs (see Figure 7) and there was no overflow. In the next time interval (April-May), the reservoirs in nodes 25 and 27 could not accumulate flood water coming from upstream, and overflow occurred. The reservoir in node 37 was able to accumulate the water coming from upstream and there was no overflow downstream from this node.

Two observations on Figures 6 and 7 can be made. First, data used in the calculations are for a wet year; thus, there were no shortages in water supply for the users. Second, it became clear from the calculations that the maximum capacities of reservoirs $W_{j\ max}$ were too small to influence flow distribution along the river, or to offer protection from overflows during the one-year period. Physically, the second fact can be explained as follows. The arc that represents flow in a reach at time interval t of duration Δt could be defined as an integral

$$F_i^t = \int_{\Delta t} \tilde{F}_i^t(\tau)d\tau \quad , \tag{10}$$

where

$\tilde{F}_i^t$ is the discharge in a reach.

We could also introduce an average discharge at time interval t. That is,

$$\bar{F}_i^t = \frac{1}{\Delta t} \int_{\Delta} \tilde{F}_i^t(\tau)d\tau \quad , \tag{11}$$

and hence,

$$F_i^t = \bar{F}_i^t \cdot \Delta t \quad . \tag{12}$$

Let us introduce the ratio

$$\varepsilon_{ij} = \frac{W_{j\ max}}{F_i^t\ \Delta t} \quad . \tag{13}$$

According to our calculations, control of the flow in a river basin by reservoirs can be done if:

$$\varepsilon_{ij} = 0(1) \quad ; \tag{14}$$

it cannot be done if:

$$\varepsilon_{ij} >> 1 \quad . \tag{15}$$

7. INITIAL DATA FOR THE MODEL

The geographic, hydrologic and economic information needed for some mathematical models rarely exist in a useful form in which it can be applied. Thus preliminary analysis and handling of initial data are essential; but preliminary analysis often cannot be done with certain formalized methods.

To build a model, it is necessary to know the geometric structure of the network, and to choose the proper graphic points that represent a particular water economy system. Geographic information, and especially geographic maps, are the basis for the network diagram in Figure 4. To construct our model of the Tisza River Basin water resources, large-scale maps were used [1], and all the limitations imposed by natural conditions and human activity were considered.

Input data for our model are volumes of runoff of the Tisza River (R_o) and its tributaries ($R_1, R_2, \ldots, R_{10}$) according to two-month time periods starting with December ($t_1, t_2, \ldots, t_n$). In the highlands of the basin (comprising 19 percent of the territory), there is a great influence of flow from small water courses. There are no hydrologic measurements for this part of the river basin. We relied mostly on the concrete results of measurements, excluding those of the highlands.

Data from [2] were used as a basis for the calculation of runoff. The procedure used for these calculations is as follows:

- Those hydrologic stations on the tributaries were selected where it was possible to describe runoff into discharge-gauging stations close to the Tisza River, and to observe for a sufficiently long period.

- The normal annual discharge was determined using a mean annual discharge for every station.

- Those years were selected in which the annual runoff was equal to (or greater than) that for the average total observation time.

- We constructed a mean monthly flow for each station for a number of wet years.

- Data were analyzed, taking into consideration some of the conditions for runoff generation for each year.

Tables 4 and 5 give data for 17 hydrological stations on the tributaries of the Tisza River, and for one station on the Tisza River itself. We have made some corrections to the data for discharge-gauging stations situated far from the river bed; for example, we have not taken into consideration the runoff of the Zagyva and Eger Rivers.

One of the most complicated problems we encountered is the determination of values F_{i1} and F_{i2}. Network descriptions of the river sources were simplified; we assumed that there is a limit to the number of crossings of volumes of water.

Based on our exercise, the following procedure for calculating F_{i1} and F_{12} can be used.

1) Using navigational and topographic maps, determine the minimal (over the part-length x) cross-sections of the river bed (A_r) and flood lands within the boundaries of the artificial dikes (A_d) for each part of the river. These elements may not have uniform locations.

2) Determine the inclinations of the water surface ($\Delta h_2/\Delta x$ and $\Delta hd/\Delta x$), on the assumption that water entirely fills the river bed and flood lands. These inclinations are the result of the morphometric peculiarities of the river bed and flood lands.

3) The following operations can be made, based on water level and discharge measurements:

   - Check whether the calculated water surface inclinations are maximal over all data observed. As a general

Table 5. Characteristics of the main Tisza River Basin hydrological stations used in the model.

River Tributary	Location of inflow	Hydro station	Distance from the mouth (km)	Watershed area (km^2)	General flow (km^3)
Tisza	Danube	Delovol	946	1,190	1.03
Viseu	Tisza	Bistro	8.5	1,150	1.05
Teresva	"	Dibovol	34	(1,221)	0.756
Tereblya	"	Kolchair	59	(755)	0.365
Rika	"	Chust	1.2	1,145	0.788
Iza	"	Vad	12.0	1,128	0.510
Borzava	"	Dibrosel	10.4	1,405	0.870
Túr	"	Turulung	2.0	944	0.321
Szamos	"	Csenger	47.6	15,282	3.65
Krasznó	"	Ágerdömajor	44.9	1,974	0.178
Bodrog	"	Felsöberecki	46.9	12,381	3.02
Sajó	"	Banréve	127.3	3,239	0.624
Hernád	Sajó	Hidasnémeti	93.2	4,515	0.923
Eger	Tisza	Borsodivanka	13.7	1,237	-
Zagyva	"	Jásztelek	60.4	4,207	0.006
Berettyó	Sebes Körös	Berettyóújfalu	45.0	3,712	0.306
Sebes Körös	Körös	Körösszakál	54.6	2,498	0.816
Fehér Körös	"	Gyula	4.7	4,251	0.743
Fekete Koros	"	Sarkad	14.9	4,302	0.986
Maros	Tisza	Makó	237	30,149	5.49
Danube	Black Sea	Mohács	1447	209,064	(750)

rule, observed inclinations are quantitatively less than those obtained on the basis of morphometric data;

- Check whether water surface inclinations (as calculated and/or corrected) are acceptable on the basis of the actual information. Calculate maximum possible stream velocities V_2, V_d, and water discharges Q_2, Q_d, taking into account the roughness of the flood lands and the river bed;
- On the basis of available observations, compare measured values V_2, V_d, Q_2 and Q_d with possible maximum values in the corresponding ranges;
- Check whether values Q_2, Q_d (as calculated and/or corrected) are acceptable.

4) Consider hydrographs for the values Q_2 and Q_d. Actual volumes of water (E_{i1}, E_{i2}) were determined for all seasons and for each part of the river by direct calculation of daily water discharges.

5) Construct the relationships of E_{i1} and E_{i2}.

6) Identify the values F_{i1} and F_{i2} for each time interval by extrapolating the functions $E_{i1} = f(F_i)$ and $E_{i2} = g(F_i)$.

There are indirect techniques for F_{i2} evaluation that are based on historical data of overflow and water balance investigations. Essential difficulties arise in determining water resource data, and in particular in obtaining values d_i^t and Δ_i^t that characterize water balance among the various users. If the locations of network nodes do not provide the information, the search for the users of the Tisza River water is a complex problem. Water demands in the Tisza River Basin are due to urban activities, industry, agriculture, navigation, fishing, and so on. From statistical data for the period 1930 to 1970 it is known that water from the Szeged range of the Tisza River characterizes the lower reaches of the Maros river.

All data for the Hungarian part of the Tisza River Basin refer to the Tiszabetch range. Final allocation of the network nodes corresponding to water consumers in the network diagram (Figure 4) are based on [3].

Thus, data on annual and in some cases monthly water amounts are known for the Hungarian part of the Tisza River Basin.

To arrive at rational allocation among water users, we used a simplified procedure:

- Annual water consumption was divided in proportion to the population in 1970, agricultural areas in 1970 and industrial output (based on COMECON standards);

- Monthly consumption was calculated using data from [4]. Note that initial characteristics for large units (cities, irrigation areas, etc.) were obtained by using aggregate data of small units (about 40 towns).

Only towns such as Szeged, Szolnok and Debrecen could be considered large industrial centers; all other towns have been considered civil consumption units.

The data used refer to the water ranges and characterize only annual water consumption. In our model we considered water consumption only for the Tisza River; data were recalculated by transforming annual into monthly water consumption figures.

To complete the second part of the exercise, we investigated available data for several water ranges: Záhony, which includes mouth regions of the Túr, Szamos and Kraszna Rivers: Taskony, which includes the Hungarian part of the Sajó and the Hernád River basins; and Midszent, which characterizes the Körös system tributaries.

Data were generalized in order to obtain information for water ranges.

For our model, we used information on the Tiszalök and Kisköre reservoir capacities and on discharges of various channels [4]. Using data on overfloods and the resulting damages, we obtained approximate damages for each river part.

Water resource planning will certainly improve as more detailed and precise data become available

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A Review of Conflict Resolution Models in Water Resources Management

Anthony R. Ostrom

1. INTRODUCTION

In problems concerned with the allocation of water resources along a river basin, certain characteristics are often present. These include multiple objectives, multiple interests, uncertainty, and a great deal of subjective information. The term multiple interests refers to the public or private interest groups upon which a decision impacts. These could be local authorities, environmental groups, an industry, or a regional government, each with its own preferences for various water management strategies. We would like to include the subjective preferences of these groups explicitly in our analysis. Since we are often faced with conflicting objectives (e.g., national economy versus regional development) when planning water resources systems, we would also like to include in the analysis an explicit statement of these objectives (see Mera, 1967). By uncertainty we mean not only hydrologic uncertainty, but also the strategic uncertainty related to planning in an environment where there are opposing or competing interest groups whose development plans are unknown to the other interest groups.

Several approaches have been proposed for looking at problems of conflict resolution. Game theory (von Neuman and Morgenstern, 1947) and bargaining (Kahan and Rapoport, 1974) are often suggested, and Rogers (1969) has used a game theoretic analysis for the planning of water resources development on the border of India and Bangladesh. Although multiple interests and several objectives were included in Rogers' formulation, uncertainty was not considered explicitly. Other approaches fall into the loose category of multi-objective programming[1] (see Cohon and Marks, 1973; Major, 1969; and Monarchi et al., 1973). These authors have attempted to generate a process for constructing the tradeoff function between multiple objectives. Other work (Vermuri, 1974) has been directed towards finding a scalar value of a vector objective function. All the above models ignore the underlying preferences of the decision maker(s) and the subjective information available from relevant interest groups.

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[1] See the illustrative list of references, and, in particular, a review article on multiobjective programming by Cohon and Marks (1975).

Looking at water resources from a higher-level political viewpoint, Haefele (1971, 1973) has proposed 'vote trading' as a mechanism for analyzing relative preferences for several competing issues. One such issue might be air quality, and Haefele shows how air and water quality might be traded off in a so-called democratic voting session (see also Russel, Spofford and Haefele, 1972). Metagame analysis, as outlined by Howard (1968), allows us to deal with relative preferences for outcomes of a single issue where there are several competing interests. The purpose of this analysis is to predict a stable outcome where preferences depend upon perceptions of opponents' possible counter-strategies. (We will discuss this approach in more detail below.) None of the above techniques handle uncertainty in a rigorous fashion. Decision analysis (Pratt, Raiffa and Schlaifer, 1965), however, provides us with a framework to consider all the characteristics of a conflict problem (multiple objectives, multiple interests, subjective information and uncertainty) within one rational framework.

It is the last two techniques mentioned above, metagame analysis and decision analysis, that will be discussed and compared in this paper. For both techniques a simple example of water quality management will be illustrated. In addition, for the metagame approach, we will examine the problem of development on an international river basin where two countries have opposing interests.

2. METAGAME ANALYSIS

Consider the simple conceptual model of waste water treatment in Figure 1. Three cities are located near a river and each has two options for treating its municipal waste waters. They can either treat or not treat their waste effluents.[2] (In practice this is rather unlikely; the cities would have the choice of treating at various degrees of efficiency, and so these levels would, in reality, need to be represented by a much larger set of discrete options.) Let us represent the two discrete options by 0 and 1: 1 means that the wastes are treated, and 0 that they are not treated. We now specify all possible outcomes depending on the action of each city.

OUTCOMES

CITY 1	0	1	0	0	1	0	1	1
CITY 2	0	0	1	0	1	1	0	1
CITY 3	0	0	0	1	0	1	1	1

[2]The example is adapted from Hipel et al., 1974.

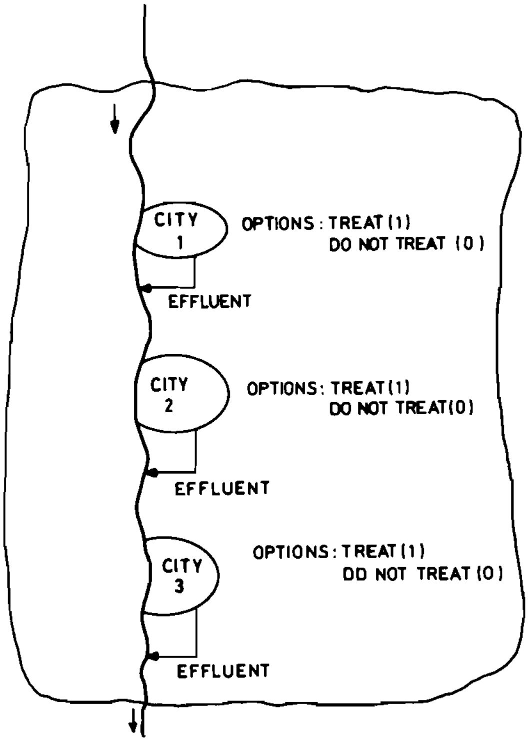

Figure 1. Conceptual model of municipal wastewater treatment along a river basin: metagame approach.

Let us examine the preferences of each city for any arbitrary outcome. For example, does city 1, not treating, prefer an outcome where at least one of the other cities treats to an outcome where all cities treat? We call any outcome that we wish to examine a 'particular outcome'; an example of the relative preferences of city 1 for the particular outcome (1,1,1) is shown in Figure 2. City 1 prefers outcomes (0,1,1) and (0,1,0) to the particular outcome (1,1,1), and does not prefer the remaining outcomes. In other words, these preferences indicate that city 1 wants at least one city to clean up the river, but under the following two conditions: a) city 1 or 3 should not 'go it alone, and b) city 2 should do at least as much as city 1 (if not more).

Such preferences (which could be obtained by questioning city officials) are not altogether unrealistic. City 1 does not want to leave city 3 (a smaller neighbor) to clean up the river alone but would prefer city 2 (a larger competitor for external to help. At the same time city 1 wants to avoid as far as possible paying the high costs of waste water treatment. In a similar fashion, preference structures could be examined for cities 2 and 3 in relation to the 'particular outcome' we have chosen.

CITY 1 PREFERENCES

CITY	OUTCOMES PREFERRED BY CITY 1		PARTICULAR OUTCOME	OUTCOMES NOT PREFERRED BY CITY 1				
1	0	0	1	0	1	1	0	1
2	1	1	1	0	0	0	0	1
3	1	0	1	0	0	1	1	0

UNILATERAL IMPROVE- MENT BY CITY 1

INESCAPABLE SANCTIONS BY CITIES 2 AND/OR 3

Figure 2. Outcomes preferred and not preferred by city 1 in relation to the particular outcome (1,1,1).

Let us concentrate, however, on city 1 preferences as shown in Figure 2. If city 1 chooses to not treat (i.e., moving to the preferred outcome (0,1,1), we term this a 'unilateral improvement' by city 1. (Recall that we are not referring here to an improvement in water quality, but to the occurrence of a move to an outcome that city 1 prefers relative to the previous outcome.) We now ask the question: is this new outcome 'stable', i.e., is one of the other cities likely to act so as to move to a less preferred outcome for city 1? In the case (0,1,1), cities 2 and 3 could take 'inescapable sanctions' against city 1 by moving to either (0,0,0) or (0,0,1), both of which are less preferred by city 1 than (1,1,1). The sanctions refer to a simple refusal to treat their wastes. So, in our example, a unilateral improvement by city 1 does not result in a stable outcome. This shows that as far as city 1 is concerned, outcome (1,1,1) is stable: any move by city 1 is likely to produce a less preferred outcome. Hence, it is in city 1's interest to make no further improvement.

The analysis for the outcome (1,1,1) is repeated for cities 2 and 3 and again for all possible 'particular outcomes.' The purpose of the approach is to predict the stability of each outcome in relation to the preferences of all interest groups simultaneously. The 'stable equilibrium' that we have learned from game theory is extended to the concept of a 'metaequilibrium,' i.e., an overall equilibrium depending upon what each city perceives the other cities will do in the light of the other cities' preferences.

A second example of metagame theory applied to water resources management has been outlined in Hipel et al. (1974). The approach they adopted in analyzing conflicts on an international river basin (illustrated in Figure 3) was as follows:

1. Construct an objective function for each interest group (in Hipel's example, Canada and the USA);

2. Generate two particular outcomes by integer programming;

3. Test each particular outcome for stability using metagame analysis;

4. If the outcome is unstable, include an increment of side payment in the objective function;

5. Repeat integer program, stability analysis and step 4 until a stable outcome occurs.

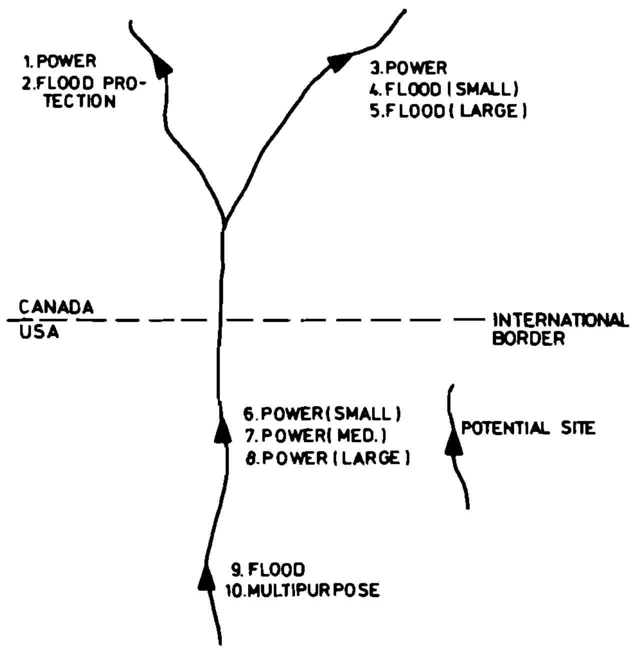

Figure 3. Alternatives for reservoir development along an international river basin.

The application was able to predict exactly how much side payment was required to produce a mutually preferred outcome for the USA and Canada in developing an international river basin lying on the border of the two countries.

The analysis in both examples has been based on three simplifications: the use of a discrete decision rule (i.e., treat or not treat), the ignoring of hydrologic uncertainty, and the consideration of only one time period. To add realism to the water quality example, such an analysis should be extended to include preferences for various treatment efficiencies, and for changing treatment levels over time. There are other general difficulties with the approach. The preferences of each interest group may be such that no outcome is stable and a quantitative definition of relative stability is needed to compare 'almost stable' outcomes. Furthermore, there may be computational problems when we search for a stable solution where there are a large number of outcomes for consideration.

3. Decision Analytic Model

Consider again a simple conceptual model of water quality management along a river basin (see Figure 4). As with the metagame model discussed previously, we will look at preferences of each city for strategies for waste water treatment. The differences from the previous approach are that:

- The treatment level for each city is continuous;
- Hydrologic uncertainty is considered explicitly;
- The risk characteristics of the decision makers are incorporated in the model;
- Multiple objectives are handled in a straightforward fashion.

The interest groups in the example are the two cities and the regional authority. Each city i is concerned about the quality of water (x_i) flowing past and the cost (z_i) of waste water treatment, depending upon efficiencies (y_i) of treatment plants. The regional authority is concerned not only about quality of water at the downstream political border, total treatment costs (which it may have to subsidize), and equity of treatment by both cities.

Decision analysis provides a framework for including all these considerations within one conceptual model. The unifying device is the single-attribute utility function, (Fishburn, 1968), a monotonic function of peoples' preferences with the property that its expected value is a guide for decision making (von Neuman and Morgenstern, 1947).

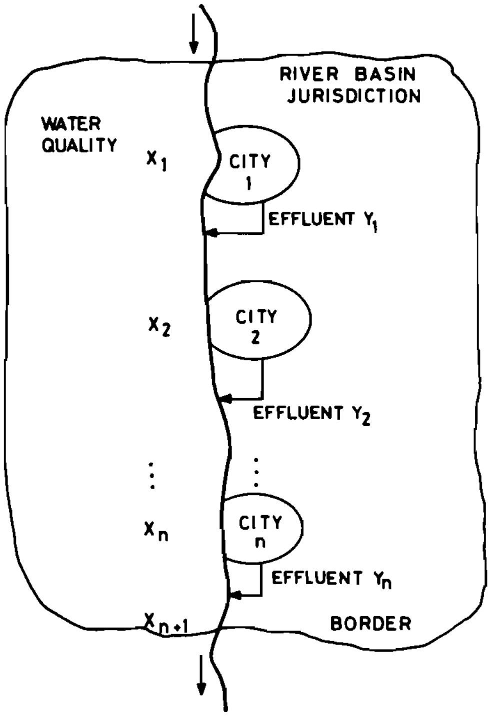

Figure 4. Conceptual model of municipal wastewater treatment along a river basin: decision analysis approach.

The assessment and verification of utility functions will not be discussed here as it is well treated in the literature (see Keeney, 1969, and Meyer and Pratt, 1968). An example of utility assessment is presented in Gros and Ostrom (1975) who consider a 'risk-averse' exponential function for both water quality (BOD) and operational waste water treatment costs (refer to Figure 5). Both single-attribute utility functions u were of the form

$$u = 1 - \exp[a(z - z_{max})] \quad ,$$

where

a = risk-aversion parameter,

z = attribute (e.g., cost of waste treatment),

z_{max} = maximum conceivable value of the attribute z.

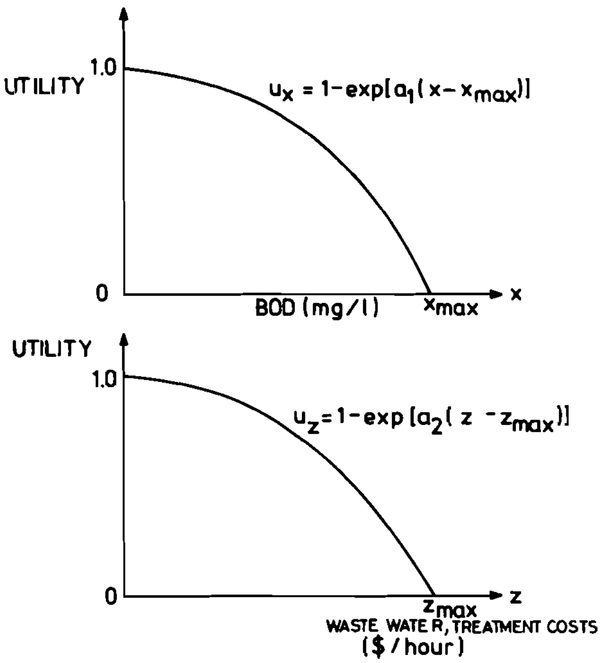

Figure 5. Risk-averse utility functions for water quality and wastewater treatment costs.

The multiattribute utility function $u_i(x_i,z_i)$ is a weighted sum (or product) of a single-attribute function--the weights are derived from the same assessment process that determines the value of the risk aversion parameter a.

The problem was to find the set of Pareto-admissible solutions, i.e., pollution control strategies that made it impossible to meet the preferences of one city without failing to meet those of another. This 'Paretian' approach has been widely discussed in the water resources literature (see Dorfman and Jacoby, 1972; Ostrom, 1974; and Wood, 1974). Such applications, while considering impacts of various decisions on several interest groups, did not, however, treat the question of uncertainty in a rigorous fashion.

Using optimal control theory, Gros and Ostrom (1975) found optimal trajectories of waste water treatment for a number of cities on a river basin by maximizing expected utility expressed as an integral,

$$\int u(x,z)\ p(q)dq \quad ,$$

where u(x,z) is the multiattribute utilities function over quality x and costs z at each city, and p(q) is the probability distribution of the random streamflow, q. The optimization is subject to the technological constraints representing BOD decay and transportation phenomena. Minimum quality levels were not specified; rather they were included implicitly as a part of the objective function by incorporating them in the utility assessment process.

The purpose of this type of analysis is twofold: descriptively it generates a set of non-inferior alternatives and shows the impacts on each interest group. It then remains for the decision maker to choose among non-inferior alternatives. If many of the alternatives are politically infeasible then the analysis can predict the outcome of a decision process. There are obvious difficulties in this approach, particularly with regard to selection of representative interest groups, assessment of utility functions, evaluation of scaling weights and risk-aversion parameters, and construction of a multiattribute utility function (see Keeney, 1969). Nevertheless, the decision analysis is one of the few frameworks available for combining the large amount of subjective information from many sources that we often wish to include in our decision process.

4. SUMMARY

We have considered only two examples from a growing family of conflict-resolution models.[3] The metagame approach allows us to deal with conflicting preferences for discrete strategies that are subject to 'unilateral improvements' and 'inescapable' sanctions by opposing interests. The analysis gives us the means for assessing a stable outcome, but only if we can realistically express the preferences for strategies and counter-strategies in an uncertain environment. The model cannot be used alone: it must be applied in conjunction with an optimization model whose purpose is to generate outcomes for the preference (metagame) analysis. Although the metagame analysis searches for a 'metaequilibrium' by analyzing static preferences of opposing interests, it does not deal with repeated conflicts over time.

The decision analytic approach is an improvement over the metagame analysis because it allows the modeller to include a great deal more subjective information in the model: preferences, risk aversion, tradeoffs between objectives and uncertainty can all be treated in a rigorous fashion. The usefulness of the approach depends largely on the accuracy of the assessment process and the degree of detail needed to specify the technological relations. More work, however, is needed on the assessment process and the development of multiattribute utility functions.

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[3]See the references for an illustrative list of such models.

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An Optimal Adaptive Prediction Algorithm for Real-Time Forecasting of Hydrologic Time Series

Andras Szöllösi-Nagy

1. INTRODUCTION

In order to achieve effective control of water resource systems, one must know the future behaviour of the inputs to the particular systems. This is the ancient challenging task of the human being, because man's encounter with the prediction problem is as old as civilization itself.

The first successful scientific attack dates back to the early 1940's when Wiener and Kolmogorov solved the problem independently for the case of linear dependent stationary processes, which requires solution of the Wiener-Hopf equation. In 1966, Eagleson et al. [4] were the first to apply this technique for runoff prediction. Their paper initiated the avalanche of articles dealing with the various tricky modifications of Wiener's procedure for practical hydrologic forecasting problems (see e.g. [6], [22] and [24]). The classical Wiener-Hopf technique, however, has some serious drawbacks: it can be applied only for strictly linear and time invariant systems with stationary input/output processes. Some efforts had been made to extend the theory (see e.g. [7]); due to the computational burden, however, they practically failed. Additional difficulties were raised by the use of spectral factorization, and from a practical point of view, by the necessity of using a relatively large computer to store all the data.

Obviously, for real-time operation of water resource systems, small computers are preferable. Hence, our prediction algorithms must be suited for these small computers. But how? The answer is simple: using recursive prediction algorithms in which there is no need to store all the past measurements for the purpose of predicting the future behaviour of the time series in question. Moreover, these algorithms would offer the following advantages:

(1) The treatment of the information of each measurement in a sequential manner allows for on-line implementation (e.g. by means of data acquisition by automatic measurement devices connected in real-time mode with a central processor); and

(2) Time variable parameters and different types of disturbances can easily be treated.

Hence, a suitable prediction scheme should preferably satisfy the following requirements:

(1) It should be mathematically tractable;

(2) It should be easily implemented for relatively small computers;

(3) It should be generally applicable;

(4) It should yield an 'optimum' prediction;

(5) It should be adaptable to varying environmental conditions;

(6) It should yield an acceptable convergence.

The hydrologic prediction schemes used nowadays unfortunately generally fail to meet one or more of these requirements.

In the early 1960's R.E. Kalman [10] developed an optimal sequential estimation technique, usually referred to as the Kalman filter, which has proved extremely useful in dealing with the description of stochastically excited dynamic systems.[1] The Kalman filtering technique is based on the state space, time domain formulation of the processes involved, and with slight modifications offers a procedure as a candidate for satisfying the above requirements of a suitable hydrologic prediction scheme.

In this paper we briefly outline the basis of the Kalman filtering technique and propose a simple state-space-based model for the recursive adaptive estimation of the impulse response of a hydrologic system. Discrete time models are considered. The proposed algorithms can be applied to slightly non-linear and time varying systems using a proper moving data window. Having obtained the optimal time varying impulse response(s), the well-known techniques can be used for predicting the output process(es).

2. STATE-SPACE REPRESENTATION OF HYDROLOGIC PROCESSES

Consider a water resource system (Figure 1), the behaviour which, evolving on the discrete-time set $T = \{t_k: k = 0, 1,2,...\}$, can be described by

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[1] In this respect, the reader is referred to the extremely rich literature, e.g. [1], [16], [18] and [20].

$$\underline{x}(t_{k+1}) = \mathscr{A}[\underline{x}(t_k), \underline{u}(t_k), \underline{w}(t_k)] \qquad (1)$$

$$\underline{z}(t_k) = T[\underline{x}(t_k), \underline{v}(t_k)] \qquad (2)$$

where $\underline{x}(t_k)$ is the n-vector of the states of the system at the discrete time $t_k \in T$; $\underline{u}(t_k)$ is the s-vector of control variables or known system inputs; $\underline{w}(t_k)$ is the r-vector of uncertain disturbances 'driving' the system; $\underline{z}(t_k)$ is the m-vector of measurements on the system; $\underline{v}(t_k)$ is the m-vector of uncertain disturbances corrupting the observations; and $\mathscr{A}$ and T are certain functionals characterizing the properties of that particular system. Eq. (1) is called the state equation, and Eq. (2) the measurement equation (as the measurement noise $\underline{v}(\cdot)$ is sometimes referred to as measurement uncertainty, while some components of $\underline{w}(\cdot)$, or the entire $\underline{w}(\cdot)$ itself, might be referred to as model uncertainty). Considering the simple example of a reservoir system consisting of n reservoirs, $\underline{x}(t_k)$ might be sought as a vector composed of the values of the amount of stored water of each reservoir at time t_k; $\underline{u}(t_k)$ as a vector of water releases (control variables); $\underline{w}(t_k)$ as the vector of natural (uncontrolled stochastic) inflows to the reservoirs, and $\underline{z}(t_k)$ as the vector of measured outflows from the reservoirs.

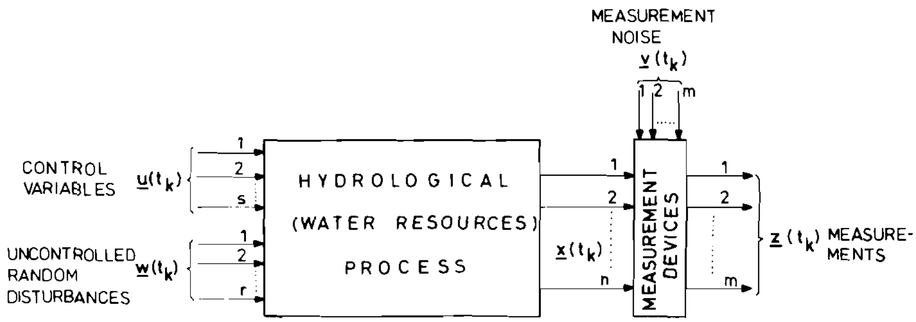

Figure 1. Hydrologic process identification.

In this case, the state vector $\underline{x}(\cdot)$ refers to actual physical states, namely to the amount of stored water in the system; but, as will be shown later, it is not at all necessary to associate the state vector with "physical" states. In other words, one can choose amongst different types of state variables to describe the same process.

Dealing with the above type of models, one must determine the structure of the system, in other words, the functionals $\mathscr{A}$ and T. This is the problem of system identification [18]. Having identified the system, the next step is to find the 'best' prediction of the state vector (which may sometimes contain the output process depending upon the choice of the state variables) $\ell > 0$ time periods ahead, based upon knowledge of the measurement on the system at t_k

$$\hat{\underline{x}}(t_{k+\ell} | t_k) = \Omega[\underline{x}(t_k), \underline{z}(t_k)] \quad , \tag{3}$$

where ℓ is the lead time of the prediction, Ω denotes the prediction algorithms, and the circumflex refers to the predicted (estimated) value. Obviously, the goodness of prediction must be evaluated through a given loss (cost) function, $L(\cdot)$. Now, the prediction problem can be formulated as follows: given the set of measurements $\underset{\sim}{Z}_k = \{\underline{z}(t_i): i = 1,2,\ldots,k\}$, find and estimate $\hat{\underline{x}}(t_{k+\ell} | Z_k)$ of $\underline{x}(t_{k+\ell})$, $\ell > 0$, subject to the condition that this estimation (prediction) minimizes the chosen loss function.

We mention in advance that the identification and prediction algorithms will be imbedded here into the same general adaptive algorithms.

In this paper we consider linear lumped parameter water resource systems where $\mathscr{A}$ and T are linear functionals. In other words, the processes are assumed to be represented by the linear vector difference equation

$$\underline{x}(t_{k+1}) = \underline{\Phi}_k \underline{x}(t_k) + \underline{\Gamma}_k \underline{w}(t_k) + \underline{\Lambda}_k \underline{u}(t_k) \tag{4}$$

where, beyond the variables already defined, $\underline{\Phi}_k \hat{=} \underline{\Phi}(t_{k+1}, t_k)$ is the n x n nonsingular state transition matrix which, in the case of an unforced system, maps the state vector from time t_k to the state vector at time t_{k+1}; $\underline{\Gamma}_k \hat{=} \underline{\Gamma}(t_k)$ is the n x r system noise coefficient matrix, and $\underline{\Lambda}_k \hat{=} \underline{\Lambda}(t_k)$ is the n x s control matrix. Note that in general these matrices are time varying.

As for the stochastic model uncertainty $w(t_k)$, without loss of generality it is assumed to be a Gaussian white noise sequence with zero mean

$$\mathcal{E}\{\underline{w}(t_k)\} = \underline{O}$$

and covariance matrix

$$\mathcal{E}\{\underline{w}(t_k)\ \underline{w}^T(t_j)\} = \underline{Q}_k\ \delta_{kj}$$

where $\mathcal{E}\{\cdot\}$ denotes the expected value operator, T the matrix transposition, δ_{kj} the Kronecker delta, and $\underline{Q}_k \hat{=} \underline{Q}(t_{k+1}, t_k)$ the r x r noise covariance matrix, i.e. $\underline{w}(t_k) \sim N(\underline{O}, \underline{Q}_k)$. Also, it is assumed that the measurement equation (cf. Eq. (2)) is linear and has the form

$$\underline{z}(t_k) = \underline{H}_k\ \underline{x}(t_k) + \underline{v}(t_k) \quad . \tag{5}$$

Here $\underline{H}_k \hat{=} \underline{H}(t_k)$ is the m x n measurement matrix, and the measurement uncertainty $\underline{v}(t_k)$ is also assumed to be a Gaussian white sequence with zero mean

$$\mathcal{E}\{\underline{v}(t_k)\} = \underline{O}$$

and covariance matrix

$$\mathcal{E}\{\underline{v}(t_k)\ \underline{v}^T(t_j)\} = \underline{R}_k\ \delta_{kj}$$

where the m x m noise covariance matrix $\underline{R}_k \hat{=} \underline{R}(t_k)$ is assumed to be positive-definite. That is, $\underline{v}(t_k) \sim N(\underline{O}, \underline{R}_k)$. Moreover, it is assumed that the noise processes are uncorrelated with one another, i.e.

$$\mathcal{E}\{\underline{w}(t_k)\ \underline{v}^T(t_j)\} = \underline{O} \quad , \qquad \forall\ k,j \quad .$$

Further, we will utilize the separation theorem (see e.g. Bryson and Ho [2]) which states that, for linear systems with quadratic cost functions and subject to additive white Gaussian noise inputs, the optimum stochastic controller is realized by cascading an optimal estimator (predictor) with a deterministic optimum controller (Figure 2). According to this principle, the optimal stochastic control of a water resource system can be decoupled into two parts. Now, we concentrate on the first problem, the state estimation/prediction problem. Therefore, the terms in Eq. (1) and (4) consisting of the control function $\underline{u}(\cdot)$ will be omitted from now on.

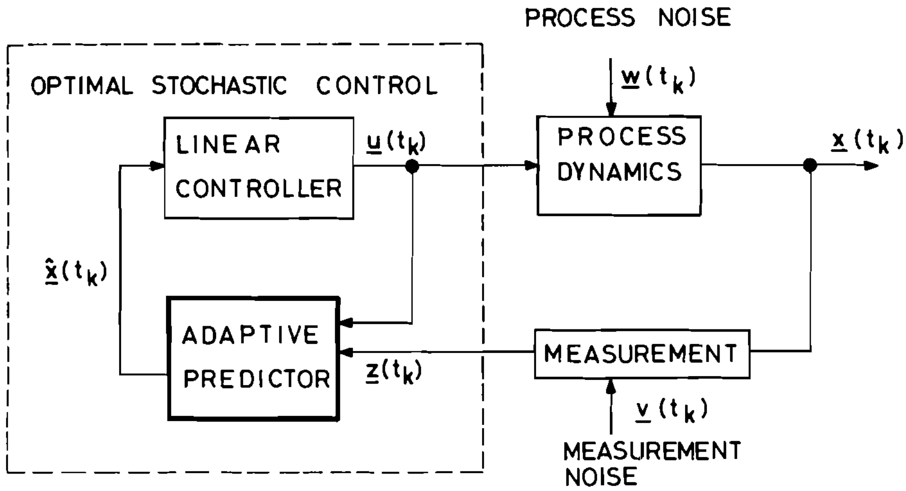

Figure 2. Separation principle.

One can argue about the basic assumptions of the noise processes being Gaussian white sequences with known covariance matrices. In particular, it is hard to say that the latter values are known in dealing with hydrologic time series. To overcome this difficulty, an adaptive noise covariance matrix algorithm will be introduced. As for handling 'colored' noises, if the state vector might be properly augmented with the dependent part of the processes, the resulting residual is a white sequence (for details see Porebski [15]).

It is a well-known fact that the autoregressive (AR) models and moving-average (MA) models, or their combinations, the ARMA and ARIMA models, have found a fruitful application area in describing the behaviour of hydrologic time series. There are tremendous amounts of literature to prove this; however, almost each paper offers a different approach for handling the models. It can be shown that all those time series models could be included, as special cases, under the umbrella of the general state space model. As an example, consider the m^{th} dimensional discrete-time autoregressive model p^{th} order, $AR_m(p)$:

$$\underline{y}(t_{k+1}) + \sum_{j=1}^{p} \tilde{\underline{\Phi}}_j(t_k)\, \underline{y}(t_{k-j+1}) = \tilde{\underline{\Theta}}(t_k)\, \underline{w}(t_k)$$

$$\underline{z}(t_k) = \underline{y}(t_k)$$

where, beyond the known notations, the matrices $\tilde{\underline{\Phi}}_j(\cdot)$ and $\tilde{\underline{\Theta}}(\cdot)$ contain the AR parameters, and the vector $\underline{y}(\cdot) = [y_1(\cdot), y_2(\cdot), \ldots, y_m(\cdot)]^T$ represents m (possibly correlated) water resource processes such as runoff, soil moisture content, water use, BOD, DO, toxic materials in the water, etc., depending upon the objective of the study. For the sake of notational simplicity initial conditions are ignored and the p = 2 case is considered here. For the state space representation of the $AR_m(2)$ process we define the state vector as

$$\underline{x}(t_k) = \begin{bmatrix} \underline{x}_1(t_k) \\ \underline{x}_2(t_k) \end{bmatrix} = \begin{bmatrix} \underline{y}(t_{k-1}) \\ \underline{y}(t_k) \end{bmatrix}$$

and, using the AR parameter matrices, define the following matrices:

$$\underline{\Phi}_k = \left[\begin{array}{c:c} \underline{O} & \underline{I} \\ \hdashline -\tilde{\underline{\Phi}}_2(t_k) & -\tilde{\underline{\Phi}}_1(t_k) \end{array}\right]$$

$$\underline{\Gamma}_k = \left[\begin{array}{c} \underline{O} \\ \hdashline \tilde{\underline{\Theta}}(t_k) \end{array}\right] \qquad \underline{H} = \left[\begin{array}{c:c} \underline{O} & \underline{I} \end{array}\right] .$$

Then the state space model

$$\underline{x}(t_{k+1}) = \underline{\Phi}_k \underline{x}(t_k) + \underline{\Gamma}_k \underline{w}(t_k)$$

$$\underline{z}(t_k) = \underline{H} \underline{x}(t_k)$$

is completely equivalent to the $AR_m(2)$ model. That this is really a special case is seen when the above state space model is compared with Eq. (4) and (5). A similar formulation can be obtained for MA,ARMA, processes. It should be noted again, however, that the above state space formulation of an AR process is not unique, in the sense that if another form is chosen for the state vector the matrices $\underline{\Phi}$, $\underline{\Gamma}$, $\underline{H}$ will change but the input-output behaviour of the system will not. In other words, the choice of a particular set ($\underline{\Phi}$, $\underline{\Gamma}$, $\underline{H}$) corresponds to the choice of a coordinate system [20]. However, the proper choice of the state vector has great significance from the point of view of practical computations on the one hand, and of system controlability and observability on the other [21].

3. THE ADAPTIVE SEQUENTIAL PREDICTION ALGORITHMS

Assume that a prior estimate $\hat{\underline{x}}(t_k|t_{k-1})$ of the system state $x(t_k)$ is given at time t_k which is based on previous measurements up to t_{k-1}. Then we seek an updated estimate $\hat{\underline{x}}(t_k|t_k)$ which takes into account the new measurement $\underline{z}(t_k)$ at time t_k. (For the notations and timing see Figure 3.) Consider the updated estimation as being the linear combination of the previous state and the new (noisy) measurement

$$\hat{\underline{x}}(t_k|t_k) = \tilde{\underline{K}}_k \hat{\underline{x}}(t_k|t_{k-1}) + \underline{K}_k \underline{z}(t_k) \quad , \tag{6}$$

where $\tilde{\underline{K}}_k \hat{=} \tilde{\underline{K}}(t_k)$ and $\underline{K}_k \hat{=} \underline{K}(t_k)$ are time varying weighting matrices as yet unspecified. As a matter of fact, we wish to minimize, in a certain sense, the prediction error

$$\tilde{\underline{x}}(t_k|t_k) = \hat{\underline{x}}(t_k|t_k) - \underline{x}(t_k) \quad . \tag{7}$$

Substituting Eq. (5) into Eq. (6) and utilizing the properties of the noise process, it can readily be seen that Eq. (6) will be an unbiased estimation only if $\tilde{\underline{K}} = \underline{I} - \underline{K}_k \underline{H}_k$. Hence, the state estimation $\hat{x}(t_k|t_k)$, using the new measurement $\underline{z}(t_k)$, is

$$\hat{\underline{x}}(t_k|t_k) = \hat{\underline{x}}(t_k|t_{k-1}) + \underline{K}_k[\underline{z}(t_k) - \underline{H}_k \hat{\underline{x}}(t_k|t_{k-1})] \quad (8)$$

where $\underline{K}_k$ is still unspecified, and the initial condition at $t = t_o$ for the state estimation is given by

$$\hat{\underline{x}}(t_o|t_o) = \mathcal{E}\{\underline{x}(t_o)\} = \hat{\underline{x}}(t_o) \quad .$$

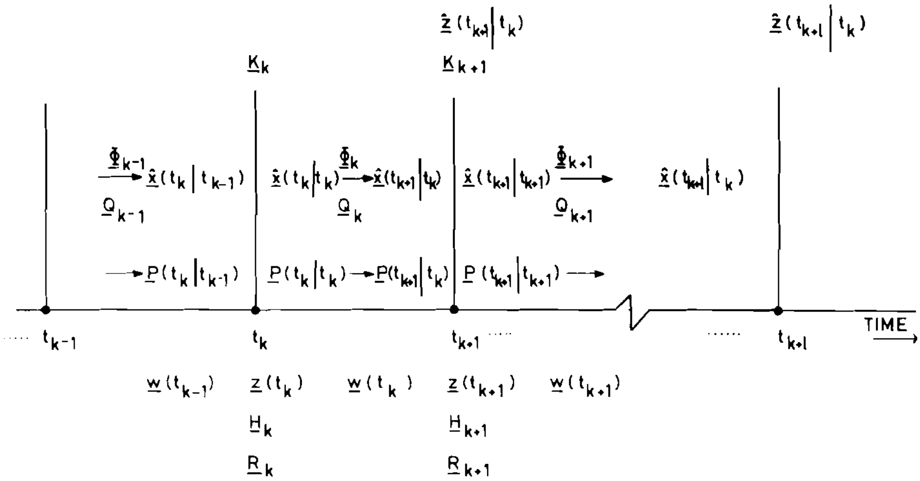

Figure 3. Discrete timing diagram.

As a measure of the goodness of the estimation, we use the n x n covariance matrix $\underline{P}(\cdot)$ of the prediction error defined as

$$\underline{P}(t_k|t_k) = \mathcal{E}\{\tilde{\underline{x}}(t_k|t_k)\ \tilde{\underline{x}}^T(t_k|t_k)\} \quad , \quad (9)$$

which is obviously symmetric, and its trace is the mean square length of the vector $\tilde{\underline{x}}(\cdot)$. Its initial condition is given by

$$\underline{P}(t_o|t_o) = \mathcal{E}\{(\underline{x}(t_o) - \hat{\underline{x}}(t_o))\ (\underline{x}(t_o) - \hat{\underline{x}}(t_o))^T\}$$

$$= \text{var}\ \{\underline{x}(t_o)\} = \underline{P}(t_o) \quad .$$

It can also easily be seen that the covariance matrix of $\tilde{\underline{x}}(t_k|t_k)$ can be projected from that of $\tilde{\underline{x}}(t_k|t_{k-1})$ as

$$\underline{P}(t_k|t_k) = (\underline{I} - \underline{K}_k \underline{H}_k)\, \underline{P}(t_k|t_{k-1})\, (\underline{I} - \underline{K}_k \underline{H}_k)^T + \underline{K}_k \underline{R}_k \underline{K}_k^{\ T} \quad . \tag{10}$$

Now, we define the loss function as the following quadratic form:

$$L_k \triangleq L\,(\tilde{\underline{x}}(t_k|t_k)) = \tilde{\underline{x}}^T(t_k|t_k)\, \underline{S}\, \tilde{\underline{x}}(t_k|t_k) \quad ,$$

where $\underline{S}$ is any positive semi-definite matrix; for the sake of simplicity let $\underline{S} = \underline{I}$, the identity matrix. Having defined the loss function we seek that estimate $\hat{\underline{x}}(t_k|t_k)$ of $\underline{x}(t_k)$--in other words, that form of the yet unspecified $\underline{K}_k$--which minimizes the expected loss (or Bayesian risk)

$$B_k = \mathcal{E}\{L(\tilde{\underline{x}}(t_k|t_k))\} \quad .$$

Since B_k is the trace of the error covariance matrix (cf. Eq. (9)) the problem is to minimize the Euclidean norm $||\underline{P}(t_k|t_k)||$ of $\underline{P}(t_k|t_k)$, i.e. the length of the estimation error vector. Using the properties of matrix derivatives, it can be seen that the weighting matrix $\underline{K}_k$ can be obtained from

$$\frac{\partial}{\partial \underline{K}_k}\; ||\underline{P}(t_k|t_k)|| = \underline{0}$$

as

$$\underline{K}_k = \underline{P}(t_k|t_{k-1})\, \underline{H}_k^T\, [\underline{H}_k\, \underline{P}(t_k|t_{k-1})\, \underline{H}_k^T + \underline{R}_k]^{-1} \quad , \tag{11}$$

which is referred to as the Kalman gain matrix. Now, the next step is the extrapolation of the state variable. Consider the one-step-ahead case, when $\ell = 1$. In the process model, Eq. (4), $\underline{w}(\cdot)$ is a white noise sequence, so no more information on it is contained in $\underline{z}(\cdot)$; thus the best prediction of $\underline{w}(\cdot)$ that can be made from $\underline{z}(\cdot)$ is its mean value, i.e. 0. Consequently, the one-step-ahead prediction of the state vector, given observations up to t_k, is

$$\hat{\underline{x}}(t_{k+1}|t_k) = \underline{\Phi}_k \hat{\underline{x}}(t_k|t_k) \quad . \tag{12}$$

The propagation of prediction errors, i.e. $\underline{P}(t_k|t_k) \to \underline{P}(t_{k+1}|t_k)$, can be determined by computing the predicted error covariance matrix as

$$\underline{P}(t_{k+1}\ t_k) = \mathcal{E}\{\tilde{\underline{x}}(t_{k+1}|t_k)\ \tilde{\underline{x}}^T(t_{k+1}|t_k)\} \quad .$$

Using Eq. (12) and (4) and utilizing the fact that the prediction error and model error are independent of each other, we obtain

$$\underline{P}(t_{k+1}|t_k) = \underline{\Phi}_k \underline{P}(t_k|t_k) \underline{\Phi}_k^T + \underline{\Gamma}_k \underline{Q}_k \underline{\Gamma}_k^T \quad . \tag{13}$$

Using the formulas in the order of Eq. (12), (13), then with k: = k + 1 in (11), (8) and (10) the celebrated Kalman filter is obtained. The algorithms should be used sequentially, k = 1,2,..., starting with the given initial conditions at time t_o.[2] The complete algorithms, together with the initial conditions, are summarized in Table 1. Kalman has shown that the algorithms are convergent and stable [11].

Up to this point, we assumed that the noise covariance matrices $\underline{Q}_k$, $\underline{R}_k$ at time t_k are known in the estimation algorithms. But in dealing with water resource time series, this is far from being true; it is necessary to predict $\underline{Q}_k$ and $\underline{R}_k$ based upon measurements at the previous stage. Hence, to take into account the changing structure of uncertainties, an adaptive algorithm should be constructed for estimating the noise covariances, starting with arbitrary initial guesses. Since the noise covariance matrix $\underline{R}(t_k)$ is assumed to be independent of time, the one-step-ahead prediction of it is

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[2]It might be mentioned that the same algorithms are obtained by maximizing the a posteriori probability $P(\underset{\sim}{X}_k|\underset{\sim}{Z}_k)$ where $\underset{\sim}{X}_k = \{\underline{x}(t_i) : i = 1,2,\ldots,k\}$ and $\underset{\sim}{Z}_k$ is as before. For a detailed discussion, consult Sage [16].

$$\underline{\hat{R}}(t_k | t_{k-1}) = \underline{\hat{R}}(t_{k-1} | t_{k-1}) \quad . \tag{14}$$

System Model	$\underline{x}(t_{k+1}) = \underline{\Phi}_k \, \underline{x}(t_k) + \underline{\Gamma}_k \, \underline{w}(t_k) \quad , \quad \underline{w}(t_k) \sim N(\underline{o}, \underline{Q}_k)$
Measurement Model	$\underline{z}(t_k) = \underline{H}_k \, \underline{x}(t_k) + \underline{v}(t_k) \quad , \quad \underline{v}(t_k) \sim N(\underline{o}, \underline{R}_k)$
Initial Conditions	$\mathcal{E}\{\underline{x}(t_o)\} = \underline{\hat{x}}(t_o) = \underline{\hat{x}}(t_o \| t_o)$
	$\mathcal{E}\{(\underline{x}(t_o) - \underline{\hat{x}}(t_o))(\underline{x}(t_o) - \underline{\hat{x}}(t_o))^T\} = \mathrm{var}\,\{\underline{x}(t_o)\} = \underline{P}(t_o) = \underline{P}(t_o \| t_o)$
Other Assumption	$\mathcal{E}\{\underline{w}(t_k) \, \underline{v}^T(t_j)\} = \underline{o}, \quad \forall \, k,j$
State Prediction	$\underline{\hat{x}}(t_{k+1} \| t_k) = \underline{\Phi}_k \, \underline{\hat{x}}(t_k \| t_k)$
Predicted Error Covariance Matrix	$\underline{P}(t_{k+1} \| t_k) = \underline{\Phi}_k \, \underline{P}(t_k \| t_k) \, \underline{\Phi}_k^T + \underline{\Gamma}_k \, \underline{Q}_k \, \underline{\Gamma}_k^T$
Predictor Gain Algorithm	$\underline{K}_{k+1} = \underline{P}(t_{k+1} \| t_k) \, \underline{H}_{k+1}^T [\underline{H}_{k+1} \, \underline{P}(t_{k+1} \| t_k) \, \underline{H}_{k+1}^T + \underline{R}_{k+1}]^{-1}$
State Estimation Using the New Measurement	$\underline{\hat{x}}(t_{k+1} \| t_{k+1}) = \underline{\hat{x}}(t_{k+1} \| t_k) + \underline{K}_{k+1} [\underline{z}(t_{k+1}) - \underline{H}_{k+1} \, \underline{\hat{x}}(t_{k+1} \| t_k)]$
Error Covariance Matrix Algorithm	$\underline{P}(t_{k+1} \| t_{k+1}) = (\underline{I} - \underline{K}_{k+1} \, \underline{H}_{k+1}) \underline{P}(t_{k+1} \| t_k)(\underline{I} - \underline{K}_{k+1} \, \underline{H}_{k+1})^T + \underline{K}_{k+1} \, \underline{R}_{k+1} \cdot \underline{K}_{k+1}^T$

Table 1. Discrete Kalman filter algorithms.

For the sequential estimation of $\underline{\hat{R}}(\cdot)$ Sage and Husa [17] developed a suboptimal adaptive estimation algorithm:

$$\underline{\hat{R}}(t_k | t_k) = \frac{1}{t_k} [t_{k-1} \, \underline{\hat{R}}(t_{k-1} | t_{k-1}) + \underline{\nu}(t_k) \, \underline{\nu}^T(t_k) - \underline{H}_k \, \underline{P}(t_k | t_{k-1}) \, \underline{H}_k^T] \quad , \tag{15}$$

where

$$\underline{\nu}(t_k) = \underline{z}(t_k) - \underline{H}_k \, \underline{\hat{x}}(t_k | t_{k-1}) \tag{16}$$

is known as the 'innovation sequence' (Kailath [9] for the sub-optimal estimator. The innovation process $\underline{\nu}(\cdot)$ is a white noise sequence; i.e. heuristically there is no information left in $\underline{\nu}(\cdot)$ if $\hat{\underline{x}}(\cdot)$ is an optimal estimation (Mehra [12]). A similar expression can be obtained for the adaptive estimator of the model noise covariance. Sage and Husa have also shown in their paper cited that the suboptimal estimation rapidly converges to the optimal one when t_k is increasing. It should be mentioned that there are numerous adaptive algorithms ([18, 12]) on the market, but for our purpose the above seems to be the most effective, at least from a computational point of view.

4. ADAPTIVE PREDICTION OF LINEAR HYDROLOGIC SYSTEMS

It is well known (see e.g. Dooge [3]) that a fairly large class of hydrologic systems (e.g. rainfall excess/surface run-off, runoff/runoff transformations of flood routing, etc.) can be described by a convolution type of model

$$y(t) = h(t) * u(t)$$

where $u(t)$ is the input of the system (either controllable or not), $h(t)$ is the impulse response of the system and $y(t)$ is the output process; the asterisk denotes the convolution. In practice, however, we have only noise corrupted measurements

$$z(t) = y(t) + v(t)$$

where $v(t)$ is an unknown noise process (Figure 4). Hence for linear time invariant lumped systems

$$z(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau + v(t) \quad , \qquad (17)$$

where in case of physically realizable systems the upper bound of the integration is t. Note that although the system was assumed linear, in case of slight non-linearities, the noise process $v(\cdot)$ might be sought as a term including those 'small' non-linear disturbances.

Considering discrete-time systems with finite memory q, Eq. (17) can be written as

$$z(t_k) = \sum_{j=0}^{q} h(t_j) u(t_{k-j}) + v(t_k) \quad , \qquad (18)$$

and by defining the vectors

$$\underline{H}_k = [u(t_k),\ u(t_{k-1}),\ldots,u(t_{k-q})] \quad ,$$

$$\underline{x}(t_k) = [h(t_o),\ h(t_1),\ldots,h(t_q)]^T$$

Eq. (18) becomes

$$z(t_k) = \underline{H}_k\, \underline{x}(t_k) + v(t_k) \quad . \tag{19}$$

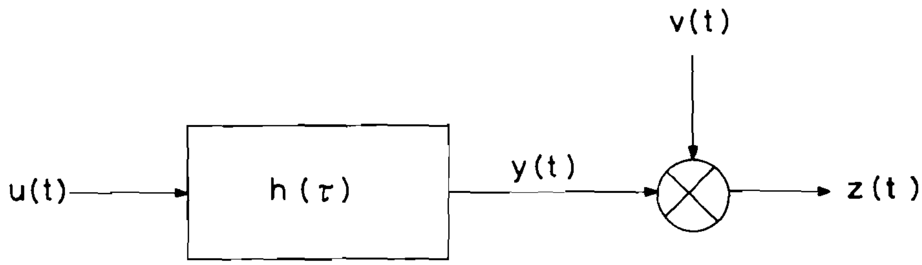

Figure 4. Linear system model.

This equation can be looked upon as a measurement equation for the above-defined state vector $\underline{x}(\cdot)$; cf. Eq. (5). The missing state equation can also be introduced without much difficulty. It was assumed that the system is time invariant; i.e. its impulse response $h(\cdot)$ does not change with time. Using the state vector defined above, this statement can be formulated as

$$\underline{x}(t_{k+1}) = \underline{x}(t_k) \tag{20}$$

which plays the role of the state equation.

Although it was assumed that the system is truly time variant, it should be stressed that the above formulation can be used for describing slightly time variant systems which, due to seasonal changes, are most common in hydrology. This concept is illustrated in Figure 5 where the system behaviour is considered to be time invariant within a well defined 'data window.' This data window, of course, is of a moving type. As to the length of the moving data window, it is essentially equal to the memory of the system and might be estimated from cross-correlation analysis of the input/output processes. The moving data window creates the basis of the sequential prediction.

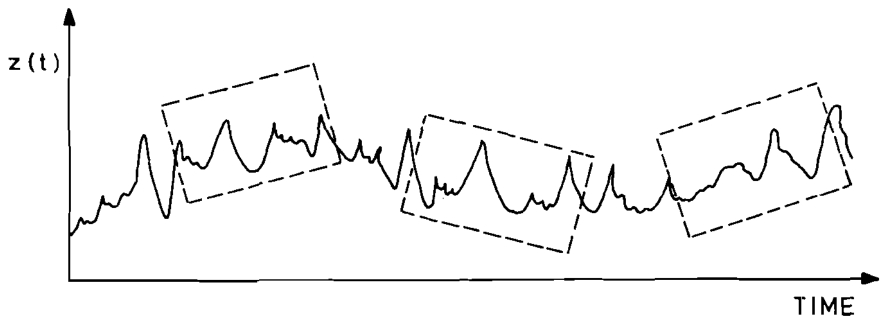

Figure 5. Time varying measurement sequence with moving data window.

If we assume that the noise sequence $v(t_k)$ is Gaussian white with $v(t_k) \sim N(0,R_k)$, then it is still an open question how to determine its variance. This can be done by the adaptive algorithm of Saga and Husa previously discussed, or even more easily because of the special structure of the state space model. The specialities are $\underline{\Phi}_k = \underline{I}$, $\underline{\Gamma}_k = \underline{0}$, and hence the state prediction is

$$\hat{\underline{x}}(t_{k+1}|t_k) = \hat{\underline{x}}(t_k|t_k) \quad ,$$

and the predicted error covariance matrix is in the form of

$$\underline{P}(t_{k+1}|t_k) = \underline{P}(t_k|t_k) \quad .$$

Since v(•) is a zero mean white noise sequence, the optimal one-step-ahead Bayes (minimum variance) prediction of the output process, based upon observations up to t_k, is

$$\hat{z}(t_{k+1}|t_k) = \underline{H}_k \hat{\underline{x}}(t_k|t_k) \quad .$$

The complete sequential prediction algorithms are summarized in Table 2. Note that to use the recursive algorithms, the initial conditions $\hat{\underline{x}}(t_o)$, var $\{\underline{x}(t_o)\}$ and R_o must be specified (or rather assumed).

Process Model	$\underline{x}(t_{k+1}) = \underline{x}(t_k)$
Measurement Model	$z(t_k) = \underline{H}_k \underline{x}(t_k) + v(t_k) \quad , \quad v(t_k) \sim N(o, R_k)$
Initial Conditions	$\hat{\underline{x}}(t_o) = \mathcal{E}\{\underline{x}(t_o)\} \quad , \quad \underline{P}(t_o) = \text{var}\{\underline{x}(t_o)\}$
State Prediction	$\hat{\underline{x}}(t_{k+1}\|t_k) = \hat{\underline{x}}(t_k\|t_k)$
Output Prediction	$\hat{z}(t_{k+1}\|t_k) = \underline{H}_k \hat{\underline{x}}(t_k\|t_k)$
Predicted Error Covariance Matrix	$\underline{P}(t_{k+1}\|t_k) = \underline{P}(t_k\|t_k)$
Predictor Gain Algorithm	$\underline{K}_{k+1} = \underline{P}(t_k\|t_k) \underline{H}^T_{k+1} [\underline{H}_{k+1} \underline{P}(t_k\|t_k) \underline{H}^T_{k+1} + \hat{R}_k]^{-1}$
State Estimation Using the New Measurement	$\hat{\underline{x}}(t_{k+1}\|t_{k+1}) = \hat{\underline{x}}(t_k\|t_k) + \underline{K}_{k+1}[z(t_{k+1}) - \underline{H}_{k+1} \hat{\underline{x}}(t_k\|t_k)]$
Error Covariance Matrix Algorithm	$\underline{P}(t_{k+1}\|t_{k+1}) = (\underline{I} - \underline{K}_{k+1} \underline{H}_{k+1})\underline{P}(t_k\|t_k)(\underline{I} - \underline{K}_{k+1} \underline{H}_{k+1})^T + \underline{K}_{k+1} \hat{R}_{k+1} \cdot \underline{K}^T_{k+1}$
Measurement Noise Variance Algorithm	$\hat{R}_{k+1} = \frac{1}{t_{k+1}} [t_k R_k + \nu(t_{k+1}) \nu^T(t_{k+1}) - \underline{H}_{k+1} \underline{P}(t_k\|t_k)\underline{H}^T_{k+1}]$
Innovations Algorithm	$\nu(t_{k+1}) = z(t_{k+1}) - \underline{H}_{k+1} \hat{\underline{x}}(t_k\|t_k)$

Table 2. Discrete suboptimal adaptive prediction algorithms.

In order to illustrate the utility of the proposed algorithms, a simulation exercise was elaborated. A given impulse response was assumed, and using that and an arbitrary input sequence, the output process was calculated through the simple discrete convolution. Then a Gaussian white noise sequence was generated with zero mean and variance 0.1. This sequence was then added to the output process; the resulting noise corrupted sequence and the original input sequence were further analyzed to see whether the algorithm does or does not give 'back' the impulse response assumed. As an example Figure 6 shows the situation concerning a particular ordinate of the impulse response. The constant line (a) means the 'true' third ordinate of the impulse response, h_3, while curve (b) shows its estimated values using the prior knowledge (if it is available) of the variance; curve (c) shows how its estimated values evolve when there is no prior knowledge, i.e. an initial guess for the variance had been considered and the adaptive noise variance estimation technique was used. It is clear from the figure that whatever the initial guess is, the estimation procedure is convergent as the number of measurement data increases. The history of the adaptive sequential noise variance estimation is depicted in Figure 7. In fact, the same conclusion might be drawn.

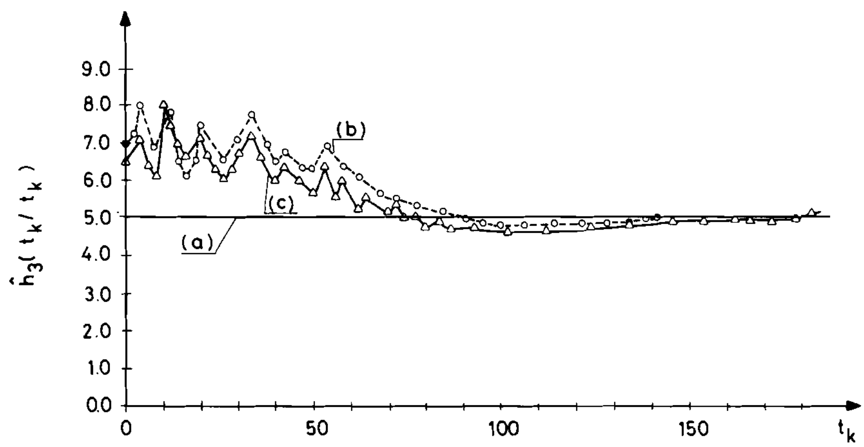

Figure 6. Sequential estimation of the 3rd impulse response ordinate, h_3.

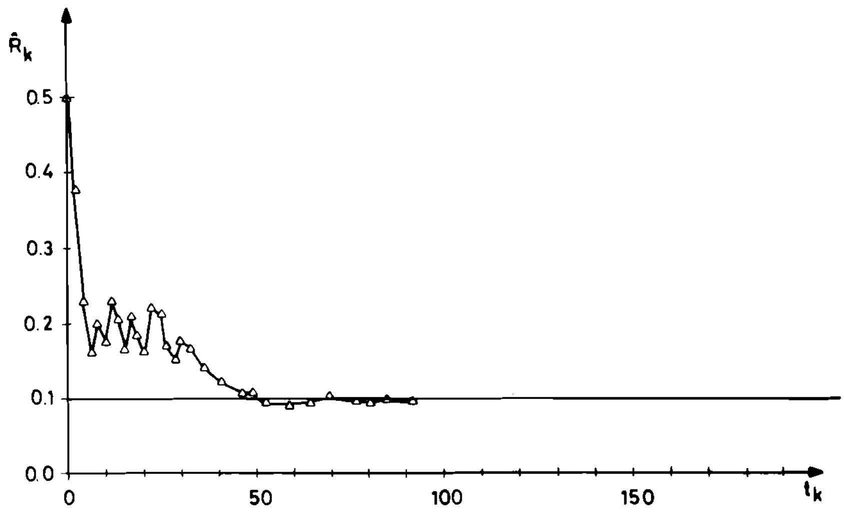

Figure 7. Adaptive suboptimal sequential estimation of the noise variance.

5. SUMMARY AND CONCLUSIONS

The paper outlined the state space formulation of hydrologic/water resource systems. Prediction algorithms have been proposed which satisfy the requirements of the suitable prediction scheme laid down in the introduction since:

(1) Using time domain formulation, the usual frequency-domain-based computations can be avoided on the one hand and the problem becomes mathematically tractable on the other;

(2) Due to the recursiveness of the algorithms the scheme can easily be implemented even for small computers and be applicable for real-time on-line forecasting, always taking into consideration the newest information gathered;

(3) Due to the state space formulation, it is generally applicable to most general hydrologic time series (water quantity and/or quality); thus the joint handling/prediction of multidimensional time series (which might include some economic data) becomes feasible even in the presence of different kinds of uncertainties;

(4) The algorithms give optimal prediction in Bayes' sense (Bayesian minimum variance estimators);

(5) The requirement of adaptivity to changing environmental conditions is fulfilled, as through a moving data window slight modifications in the model parameters are allowed;

(6) The algorithms are convergent and stable under very general conditions.

To illustrate the above properties, an example was presented using simulated data. The results obtained indicate the practical applicability of the proposed procedure.

As a final remark, it might be mentioned that the procedure can be extended to include the identification/prediction of stochastic non-linear hydrologic system. This could be done, for example, by augmenting the state vector with the ordinates of the higher-order impulse responses and then taking advantage of the non-linear filtering techniques. But a lot of effort still remains to be made in the future towards the solution of these problems.

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Algorithms for the Stochastic Inflow-Nonlinear Objective Water Reservoir Control Problem

John Casti

1. INTRODUCTION

In an earlier IIASA internal paper (Casti, 1974) and private communication to the author (Dantzig, 1975), algorithms to determine the optimal control of a water reservoir network with stochastic inflows and nonlinear utilities have been proposed. Both studies utilize a dynamic-programming type approach, coupled with approximations of one kind or another, in order to yield a computational algorithm in which the bulk of the calculation is carried out by efficient (and rapid) network flow algorithms. The purpose of this note is to present a synthesis of the work in these papers and to spell out the precise steps of an algorithm in sufficient detail to enable a computer program to be constructed.

2. BASIC PROBLEM

Before considering our algorithm, let us briefly review the basic water reservoir control problem. We are given m reservoirs connected in some type of network configurations by various branches (rivers, tributaries, etc.). At the beginning of each time period t, reservoir i contains an amount of water u_i, $i = 1,2,\ldots,m$, to be released from each reservoir in order that some measure of utility of the water released is maximized, subject to various constraints. For a single stage process, this problem is not <u>too</u> difficult; however, the real problem is complicated by being a multistage process with the added feature of having stochastic inflow at each reservoir at each time due to rainfall and underground water run-off. In addition, the various utility functions for each reservoir are often nonlinear, thereby precluding any direct application of linear programming procedures. Consequently, other approaches are required.

In order to formulate our problem in mathematical terms, let

$s_i(t)$ = amount of water available in reservoir i at time t,

$u_i(t)$ = amount of water released from reservoir i at time t,

$r_i(t)$ = external inflow to reservoir i at time t (stochastic quantity) $i = 1,2,\dots,m$,

$t = 0,1,\dots,T$.

Clearly, the dynamics of each reservoir are described by the equation

$$s_i(t+1) = s_i(t) - u_i(t) + r_i(t) + \sum_{j\in I_i} \beta_j u_j(t) \qquad (1)$$

where I_i is the subset of $\{1,2,\dots,m\}$ consisting of those reservoirs which input water to reservoir i, and β_j represents the fraction of water released from reservoir j which is not absorbed by the network before it reaches reservoir i, $0 \le \beta_j < 1$, $i = 1,2,\dots,m$.

Let us assume that there is a certain cost associated with releasing an amount of water u_i available at reservoir i. Following Casti (1974), we assume this cost is expressible by the convex function $\phi_i(u_i)$, $i = 1,2,\dots,m$. i.e. The total objective function is

$$J(s_1,s_2,\dots,s_m) = \sum_{k=1}^{m} \phi_k(u_k) \quad . \qquad (2)$$

Since the quantities $r_i(t)$ in (1) are random variables with distribution function $dG_i(r)$, our optimization problem may be formulated as

$$\min \mathcal{E}[J]$$

over all control sequences $\{u_1(t),\dots,u_m(t),\ t = 0,1,\dots,T-1\}$, where s(t) and u(t) are related by (1) and the constraints

$$\mu_i(t) \le u_i(t) \le s_i(t) \quad , \tag{3}$$

are satisfied. Here $\mathcal{E}$ denotes the expected value relative to the distribution function $dG_i(r)$, while the quantities $\mu_i(t)$ represent certain minimal demands for water which must be met by release from reservoir i, i = 1,2,...,m.

We tackle this problem by dynamic programming. Let

$f_t(s_1,\dots,s_m)$ = expected value of J when the process has T-t time periods remaining, is in state $(s_1,\dots,s_m)$ and an optimal policy is pursued, t = 0,1,...,T.

Then it is an easy application of the principle of optimality to see that f_t satisfies the functional equation

$$f_t(s_1,s_2,\dots,s_m) =$$

$$\min_{\substack{\mu_i(t) \le u_i(t) \le s_i(t) \\ i = 1,2,\dots,m}} \left\{ \int \left[\sum_{k=1}^{m} \phi_k(u_k) + f_{t+1}\Big(s_1 - u_1 + r_1 + \sum_{j \varepsilon I_1} \beta_j u_j \right. \right. ,$$

$$\left. \left. s_2 - u_2 + r_2 + \sum_{j \varepsilon I_2} \beta_j u_j, \dots, \Big) \right] dG(r) \right\} ,$$

$$t = 0,1,\dots,T-1 \quad , \tag{4}$$

$$f_T(s_1,s_2,\dots,s_m) = \sum_{k=1}^{m} \left[\phi_k(0) + \psi_k(s_k) \right] \quad , \tag{5}$$

where $\psi_k(s_k)$ represents the cost of terminating the process with water level s_k in reservoir k.

Our next objective is to make approximations in Eq. (4) so that it will be possible to utilize network flow algorithms to effect the minimization over the u's for fixed values of $s_1,\ldots,s_m$, $r_1,\ldots,r_m$. This means that both the individual reservoir costs $\phi_i(u_i)$ and the "next stage" return $f_{t+1}(\alpha_1,\alpha_2,\ldots,\alpha_m)$ must be judiciously approximated. The heart of our methods is in the selection of approximations for these quantities that not only preserve accuracy, but also enable us to apply network flow techniques for solution of the minimization over the u's.

The first approximation is to replace the individual reservoir costs by piecewise linear functions. Since we have assumed each ϕ_i is a convex function of its argument with $\phi_i(0) = 0$, we have

$$\phi_i(u_i) = \begin{cases} m_{i1}\, u_i & , \; 0 \le u_i \le u_i^{(1)} \\ m_{i2}\, u_i + u_i^{(1)}(m_{i1} - m_{i2}) & , \; u_i^{(1)} < u_i \le u_i^{(2)} \; . \\ \vdots & \end{cases} \qquad (6)$$

Hence, in each segment of the form $u_i^{(j)} \le u_i \le u_i^{(j+1)}$, $j=0,1,\ldots,$ the function ϕ_i is linear.

Our second approximation is in "policy space", i.e. we guess an operating policy $u^o(s_1,\ldots,s_m;\ t)$ and use this policy to determine a return function from the relations (4) and (5). This is a type of approximation well-suited to taking advantage of experience and "seat-of-the-pants" operating rules for reservoir systems. In addition, it can be shown that the algorithm described below will monotonically improve (in the sense of the criterion (2)) the current policy as the iteration procedure progresses. Thus, we have a systematic method for improving any existing policy.

Having fixed an approximation to the policy u, our last approximation is to the optimal value function

$$f_{t+1}\Big(s_1 - u_1(s_1,\dots,s_m;t) + r_1 + \sum_{j\varepsilon I_1} \beta_j u_j(s_1,\dots,s_m;t),\dots,$$
$$s_m - u_m(s_1,\dots,s_m;t) + r_m + \sum_{j\varepsilon I_m} \beta_j u_j(s_1,\dots,s_m;t)\Big).$$

By virtue of the criterion (2) and the structure of the ϕ_i, it is not difficult to see that the function $f_{t+1}(\cdot,\dots,\cdot)$ is separable in its arguments, i.e.

$$f_{t+1}(\alpha_1,\dots,\alpha_m) = \gamma_1(\alpha_1) + \gamma_2(\alpha_2) +\dots+ \gamma_m(\alpha_m) \qquad (7)$$

where the functions γ_i will be convex relative to the variable u_i (here we use $\alpha_i = s_i - u_i(s_1,\dots,s_m;t) + r_i + j$ $\sum \beta_j u_j(s_1,\dots,s_m;t)$. Again we may approximate the γ's by piecewise linear functions, thereby giving $f_{t+1}(\cdot,\dots,\cdot)$ the desired linear structure. Clearly, the previous approximation to the ϕ_i will be used to approximate $f_T(\alpha_1,\dots,\alpha_m)$, while for $t < T - 1$, approximation algorithms in the DYGAM computer program may be employed.

3. THE ALGORITHMS

We shall present two alternative algorithms in this section. The first will be based directly upon the policy space idea presented above, while the second is based upon ideas introduced in Dantzig (1975). In both cases, the primary objective is to reduce the calculation to a level at which almost all the work is done by the efficient network flow algorithms.

3.1 Alternative I (Policy Space Iteration)

The steps in this algorithm are the following:

1. Approximate the functions $\phi_i(u_i)$ by piecewise linear functions as in (6);
2. Guess an initial policy $u^o(s_1,\ldots,s_m;t)$ for all $s_1,\ldots,s_m$, $t = 0,1,\ldots,T-1$;
3. Determine the approximate optimal value functions $f_t^o(s_1,\ldots,s_m)$ by iterating the relation (4) for $t = 0,1,\ldots,T-1$, using the initial function (5);
4. Approximate each function $f_t^o(\alpha_1,\ldots,\alpha_m)$ by piecewise linear functions of $\alpha_1,\ldots,\alpha_m$ as in (7);
5. Determine the up-dated policy estimate $u^1(s_1,\ldots,s_m;t)$ as that function which minimizes

$$\int \left[\sum_{k=1}^{m} \phi_k(u_k) + f_{t+1}^{(o)} \left(s_1 - u_1 + r_1 + \sum_{j\varepsilon I_1} \beta_j u_j, \ldots, s_m - u_m + r_m + \sum_{j\varepsilon I_m} \beta_j u_j\right) \right] dG(r) \quad .$$

Notice that for each <u>fixed</u> set of values for $s_1,\ldots,s_m$, $r_1,\ldots,r_m$, t, this is a network flow problem since the ϕ_k are convex. This step is carried out for all $t = 0,1,\ldots,T-1$, and all $s_1,s_2,\ldots,s_m$. (Remark: for computational purposes, it may be better to let the s_i vary only over the regions $(s_1,0,\ldots,0)$, $(0,s_2,0,\ldots,0),\ldots,$ $(0,0,\ldots,0,s_m)$ and then interpolate the values of $u^1(s_1,\ldots,s_m)$ for non-lattice points. Having obtained the next policy u^1, return to step 3 and continue until convergence.

3.2 Alternative II

In this approach (which follows Dantzig, 1975) we note that the optimal release policy $\{u_i^*(t),\ t = 0,1,\ldots,T-1,\ T = 1,2,\ldots,m\}$ is equivalent to knowledge of the optimal levels $\{s_i^*(t)\}$. Hence, we reformulate the problem in terms of water levels only. That is, the function $f_t(s_1,\ldots,s_m)$ satisfies

$$f_t\left(s_1(t),\ldots,s_m(t)\right) = \int\left[\sum_{k=1}^{m} \psi_k\left(s_k(t)\right) f_{t+1}\left(s_1(t+1),\ldots,s_m(t+1)\right)\right] dG(r)$$

when the optimal policy is used at time t, or, equivalently, when we have optimal water levels at time t+1 (here the random quantities r_i are implicitly included in the term $s_i(t+1)$) and the functions ψ_k are determined from the ϕ_k. The problem, of course, is that the optimal levels $s_i(t+1)$ (for fixed r_i) are not known and must be determined. To accomplish this task, the following algorithm is proposed:

0. Let t = t-1 and approximate $f_{t+1}\left(s_1(t+1),\ldots,s_m(t+1)\right) = \sum_{i=1}^{m} \psi_i\left(s_i(t+1)\right)$ as in (6);

1. Fix a value of the water levels, say $\bar{s}_1(t),\ldots,\bar{s}_m(t)$;

2. Fix a value of the random parameters $r_i(t)$;

3. Solve the network-flow problem of minimizing

$$\sum_{i=1}^{m} \psi_i\left(\bar{s}_i(t)\right) + f_{t+1}\left(s_1(t+1),\ldots,s_m(t+1)\right)$$

   over all $\mu_i(t) \leq u_i(t) \leq \bar{s}_i(t)$, where $s_i(t+1)$ is given by (1);

4. Change the random variables to new levels and repeat steps 2 to 4, forming expected values according to the probability distribution dG(r);

5. Change to a new set of water levels $\bar{s}_i(t)$ and repeat step 3 until all levels have been considered;

6. Approximate the function $f_t(s_1,\ldots,s_m)$ by a piecewise multilinear form (using DYGAM subpackage), let $t \rightarrow t - 1$, and return to step 1.

3.3 Remarks

a) At step 1, in view of the separable form of the objective function, it will again probably be best to use only water levels of the form $(s_1,0,\ldots,0)$, $(0,s_2,\ldots,0)$, etc. This will save on computing time by cutting down the number of cases, while still yielding sufficient information to make the approximation in step 6 accurate if the s_i grid is fine enough.

b) In the approximation of step 6, some experimentation will probably be necessary to determine how many pieces should be taken in the "piecewise" multilinear form. The usual trade-off between fewer pieces and high-order terms versus more pieces and lower order approximations needs to be examined. Generally speaking, however, it is preferable to take several low-order pieces.

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