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A Social Security Forecasting and Simulation Model

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Abstract

This paper presents and validates a multiregional neoclassical economic-demographic growth model developed by the IIASA Social Security Reform Project. This model is intended to study linkages between population age distribution, the macroeconomy, the nature of pension arrangements, the intergenerational distribution of income and wealth, and international capital flows. In this paper, we concentrate on the model in single-region form, showing that

i. reasonable exogenous assumptions give rise to a reasonable long-run model solution;

ii. when exogenous assumptions or model parameters are changed, the model performs sensibly on a baseline-vs.-alternative basis; and

iii. model projection results are reasonably robust to selection of demographic scenario and exogenous assumptions regarding household saving and labor supply; they are sensitive, however, to the selection of the parameters of the core production function.

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1. Introduction

With population aging underway, the future of social security regimes -- the broad spectrum of institutions, public and private, which provide for income in old age -- is being examined (OECD, 1998; World Bank, 1994).[1] Long-term projections and simulations of pension system revenues, expenditures, and assets play a large role in the policy debate. In this paper, we report on a model designed for such applications.

1.1. The nature of social security

The main purpose of formal social security systems is to provide income in old age, and three main institutional mechanisms have arisen:

- One, which dates from the end of the 19th century, is the public defined-benefit pension system, which in most countries is funded on a Pay As You Go (PAYG) basis. "Defined benefit (DB)" means that the pension entitlement is determined relative to an earnings benchmark, sometimes the worker's last salary, sometimes average lifetime earnings, etc. PAYG means that today's workers are taxed to fund today's pensioners, the implicit contract being that tomorrow's workers will similarly make transfers to tomorrow's retirees. In a classic defined-benefit PAYG pension system (such as Germany's), system liabilities are determined on an annual or quarterly basis as a function of the number of beneficiaries and their pension entitlements, then the payroll tax rate required to balance contributions and expenditures is calculated. In other countries (such as the United States) the PAYG public pension system is "partially funded," meaning that a capital reserve capable of meeting part of the system's liabilities is maintained.[2]
- The second institutional arrangement is the private DB pension system, in which capital reserves adequate to pay projected future pension liabilities (according to prevailing actuarial rules) are maintained. Traditionally, most large corporate pension plans have been run on a DB basis.

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[1] We use "social security system" in its American sense to denote the public pension system. When we wish to make it explicit that we are including private pensions as well, we use the term "social security regime."

[2] If the accumulated reserve simply relaxed the budget constraint and allowed Government to spend more money in other areas, the net impact on national savings would be zero. Recent (October 1999) legislation in the United States has segregated the U.S. Social Security Trust Fund from the rest of government spending

- In recent years, however, there has been increasing reliance on a third arrangement, namely private "defined contribution (DC)" plans. "DC" means that the worker gets out what s/he put in (including employer contributions on the worker's behalf, but these may be regarded as forced savings out of total wage income, plus accumulated capital returns. A central element of public pension reforms being implemented in many countries is the replacement or supplementing of downsized public PAYG systems with public DC pension schemes (James, 1998) and the putting in place of institutional structures to encourage private retirement saving. The Chilean system is the most commonly cited example, but there are many variants on the approach.

Individual retirement saving accounts, which are popular in some countries, are conceptually identical to private defined-contribution pension schemes described above. Intrafamily transfers, which provide old-age support for much of the population in less developed countries, are effectively a variant on the PAYG theme (or, more precisely, public PAYG schemes mandate intergenerational transfers which might or might not take place on an intrafamily basis).

As evidenced by the current policy debate, for example, the failure of the U.S. Advisory Commission on Social Security to reach a consensus on necessary steps, social security is an ideologically charged field. The debate is made more intense by the fact that each of the major institutional arrangements offers its own set of advantages and disadvantages. For example, defined-contribution systems may lead to increased national savings (although the point is hotly disputed), while defined-benefit systems may be more suited to achieving targeted income redistribution. The stakes are high. The social security regime is the main determinant of the economic status of the elderly, whose relative weight in the total population, as measured by the dependency ratio (population over 60 divided by population 15-59) is projected to double over the next 50 years in OECD countries. The social security regime also affects economic growth, in at least two ways. First, saving and dissaving related to retirement (including the balance in government programs for the elderly) affect capital formation; second, incentives built into the social security regime affect labor supply over the entire life span.

The models applied to analyze questions related to social security have tended to be either actuarial models with few economic linkages (e.g., Roseveare et al. 1996) or economic models with little detail on the mechanics of pension and health systems (e.g., Auerbach and Kotlikoff 1987). Among the few exceptions to this generalization are the models applied by Leimer and Petri (1981) and Warshawsky (1999). As the debate increasingly takes account of linkages between social security and the economy, practitioners and policy makers will need models containing both economic behavior and structural detail on pensions and health. The IIASA research project, which has developed a model to study long-run capital accumulation and economic growth as functions of the evolving age distribution of the population and the nature of the social security regime is, in part, an effort to meet this need.

1.2 Purpose and organization of this paper

The purpose of this paper is to present the structure and simulation properties of the IIASA model. Section 2 describes the model briefly, and presents some of its deterministic solution properties. Section 3 reports the results of some model robustness tests. Conclusions are in Section 4. In Annex 1, the full algebraic structure of the model is set forth, and in Annex 2 the initialization and baseline exogenous assumptions are described.

2. The model

2.1 Model structure

The IIASA model, which is also described in MacKellar and Ermolieva (1999), extends work originally presented by Blanchet and Kessler (1992). It is a neoclassical two-factor multiregional economic-demographic model with a focus on population age structure and pensions.[3] The main research application of the model is an ongoing analysis of the global macroeconomic impacts of population aging, where different regions are characterized by widely varying demographic dynamics and initial economic conditions. When collapsed to its single region form (as for purposes of the presentation in Annex 1) the model is sufficiently aggregated to be generalizable, i.e. it can be easily initialized for a new country and used to produce illustrative scenarios. While these scenarios are not detailed enough to serve as a basis for social security system management or planning, they can nonetheless improve the policy dialogue by illustrating economic and demographic tendencies and the macroeconomic linkages between the two (see, for example Dobronogov and Mayhew 2000 for an analysis of pension reform in Ukraine).

Compared to other models which have been used to analyze population aging at the global level, the comparative advantage of the IIASA model lies in the fine-grained accounting treatment of age-specific stocks and flows, while its comparative *dis*advantage lies in the limited amount of endogenous economic behavior built into the model. In particular, the IIASA model is a one-sector model (i.e., there are no relative prices) and age-specific saving/consumption propensities are exogenous assumptions, as are age-specific labor force participation rates. Regarding the first limitation, general equilibrium overlapping generations (OLG) models combine demographic detail with market-clearing behavior, but there is no functioning global multiregional OLG model (Bryant and McKibben, 1999). Regarding the second limitation, simulations with the IIASA model can be seen as a complement to other analyses (e.g., Börsch-Supan, 1996, Cutler *et al.*, 1990) in which there is little age-detail but more economic behavior is endogenized. Closely related to these are global macroeconomic model-based analyses (e.g., Turner *et al.*, 1998; Masson and Tryon, 1990), in which indices of age structure such as the dependency ratio are incorporated into the major macroeconomic functions. However, none of these models incorporates, as we do, an explicit pension system or tracks the intergenerational distribution of income and wealth.

2.2 Initialization, baseline scenario, and simulation properties

The purpose of this section is not to propose a forecast for any specific country or region, but rather to demonstrate that the unadjusted model gives rise to a reasonable projection of trends and that it has sensible marginal simulation properties and uncertainty characteristics.

Details of initialization and baseline scenario assumptions are given in Annex 2. The model has been implemented in two-region form in MATLAB and solved for the period 1995-2000. The baseline assumptions correspond to those used in MacKellar et al. (1999) and MacKellar and Reisen (1998) to analyze a two-region world consisting of "Fast Aging

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[3] An earlier model version which lacked full demographic dynamics is described by MacKellar and Reisen (1998). A more recent version, not yet documented, adds health spending to the picture.

Countries" (essentially, industrial countries) and "Slow Aging Countries" (essentially, developing countries). The present paper presents only results pertaining to the Fast Aging Countries region, and Annex 1 describes the model in single-region form for simplicity and economy of space. In Annex 2, we describe baseline assumptions for Fast Aging Countries in detail and give key driving assumptions for the Slow Aging Countries region. Here, we briefly summarize assumptions for the Fast Aging Countries region.

The baseline demographic scenario corresponds to the Central scenario of the most recent IIASA global population projection (Lutz 1996). Among the major driving assumptions for the Fast Aging Countries region are that the capital coefficient in the two-factor Cobb-Douglas production function is 0.333 and that the productivity and depreciation rates are 1 and 4 percent per year, respectively. Labor force participation rates for both sexes combined average about 75 percent between ages 15 and 59 and average about 5 percent above age 60 (assumed to be the pension eligibility age). The model was initialized on initial per capita income of approximately $25,000 and a capital-output ratio of 3.1.

All workers and entrepreneurs are assumed to participate in the public PAYG pension system. The contribution rate is assumed to be 15 percent and two-thirds of the wage bill (as well as profits from private unincorporated enterprises) are assumed to be subject to taxation; thus, the implicit contribution rate is 10 percent of gross income. The public pension system replacement ratio (pension entitlement upon retirement divided by wage at the time of retirement) is assumed to be 80 percent. After retirement, public pensions are indexed to rise each year by half the growth rate of the real wage. One quarter of all workers and entrepreneurs is assumed to participate in the private DB pension scheme, contributing 4 percent of total gross income. The initial pension entitlement is assumed to be 25 percent of final salary; on the assumption that most participants contribute for about 25 years, this corresponds to an accrual rate (pension upon retirement divided by final salary) on the order of 1 percent of final salary per year of system participation. Private DB pensions are not indexed to increases in the real wage rate. One quarter of workers and entrepreneurs is also assumed to participate in the private DC pension system, again contributing 4 percent of total gross income. Private DC pension system assets are annuitized upon retirement, as are all other capital assets. Any assets remaining upon death being bequeathed to the next generation.

For further assumptions, including consumption rates, the allocation of savings among asset classes, etc., see Annex 2.

Baseline scenario

Some of the major Fast Aging Countries demographic and macroeconomic variables from the baseline scenario are shown in Table 1. The aggregate population growth rate, which is presently decelerating, reaches zero between 2020 and 2030 and is slightly negative thereafter. The dependency ratio (ratio of population 60+ to population 15-59, expressed as a percentage) more than doubles from its present level (43.6 percent) by mid-century. Labor force growth, currently about 0.5 percent per year, turns negative between 2000 and 2010 and remains on the order of -0.5 percent per year for the remainder of the simulation.

The economic consequences of these demographic trends might be described best as "macroeconomic stagnation." The aggregate saving rate (net national savings over GDP, expressed as a percentage) declines from 7.8 percent in 1995 to 2.0 percent at the end of the solution period. Economic growth per capita decelerates from 2.1 percent per year in 1995 to 1.4 percent per year in 2010 and roughly 1 percent per year in later decades (for reference, the Slow Aging Country region grows at an average annual rate of 2.9 percent per capita over the solution period). The capital-output ratio rises through 2040, then begins to decline as cumulative effects of the decrease in the aggregate saving rate become pronounced. The rate of return to capital given in Table 1 is gross, i.e. it includes depreciation and indirect taxes.[4] Given our assumptions regarding the depreciation rate and indirect tax rate, the decline in the gross rate of return to capital in Table 1 corresponds to a decline in the net rate of return from 4.6 percent per annum in 1995 to 3.9 percent in 2050.

Table 1: Baseline scenario: demographic, macroeconomic, and distributional variables

	1995	2000	2010	2020	2030	2040	2050
Demographic variables							
Population (annual % change)	0.2	0.2	0.2	0.1	0.0	-0.1	-0.2
Labor force (annual % change)	0.5	0.5	-0.2	-0.5	-0.5	-0.5	-0.4
Pop. 60+ : Pop. 15-59 (%)	42.5	43.6	48.5	60.2	72.9	83.1	90.5
Macroeconomic variables							
GDP per capita (annual % change)	2.1	2.1	1.4	1.1	1.1	1.1	1.2
Capital-output ratio	3.14	3.12	3.19	3.32	3.41	3.43	3.40
Rate of return to capital (% per annum)	10.5	10.6	10.4	9.9	9.7	9.6	9.7
Aggregate saving rate (%)	7.8	7.5	7.1	5.4	3.7	2.7	2.0
Ratios related to intergenerational distribution							
Disposable income per capita, pop. aged 60+ : pop. aged 15-59 (%)	87.9	83.8	76.5	70.7	64.0	59.9	58.6
Assets per capita, pop. aged 60+ : pop. aged 15-59 (%)	308.1	330.3	294.1	250.6	235.1	235.1	229.6
Consumption per capita, pop. aged 60+ : pop. aged 15-59 (%)	109.8	108.9	99.8	90.6	82.2	77.4	75.7

At the bottom of Table 1, we show some indices of intergenerational distribution. Disposable income (per capita) of the elderly population (aged 60+) falls steadily over the entire solution period relative to disposable income of the non-elderly population (aged 15-59). Because the main source of income for the elderly is the public pension system, this decline reflects mostly the fact that public pensions are less than fully indexed to the wage rate, which increases in response to slow labor force growth. In addition, private pension

[4] The net rate of return to capital is equal to the gross rate minus the indirect tax rate (relative to GDP) divided by the capital-output ratio, minus the depreciation rate.

benefits are not indexed to wages at all. Consumption of the elderly, which reflects decumulation of assets (i.e., consumption out of annuity income) as well as consumption out of disposable income, starts off slightly higher than that of the non-elderly but declines, by the end of the solution period, to about three-quarters that of the non-elderly. Note that consumption implicitly includes the consumption of health care. The ratio of assets (per capita) of the elderly to assets of the non-elderly rises from 308 percent at the beginning of the period to a peak of 330 percent in 2000 and declines thereafter, ending the solution period at 229.6 percent.

The decrease in the aggregate saving rate is due to two factors. One is population aging, i.e. the increase in the proportion of persons who are consuming in excess of current income. The second, and closely related cause, is deterioration in the balances of all three components of the pension system (see Table 2).[5] The public pension system, which starts off with a surplus of contributions over benefits equal to 1.4 percent of *GDP*, moves continuously toward deficit as the population ages.[6] The system crosses into deficit between 2010 and 2020; by 2050, the deficit is over 2 percent of *GDP*. The same broad trend applies to the private DB pension system. Assets rise until 2030, then begin to decline. Assets of the private DC private pension system, by contrast, continue to grow throughout the simulation period, although the balance steadily declines as a proportion of *GDP*. If the two components of the private system are added together, the private pension system as a whole ceases to be a net contributor to capital formation shortly before 2040. This replicates, about fifteen years later, the result found for the United States by Schieber and Shoven (1997). To reiterate, our results are not meant to be interpreted as a forecast of the future of OECD pension systems, but just as an indication that, given reasonable exogenous assumptions, the model solution tracks a significant trend identified by other researchers. When balances of private and public pension systems are added together, the net shift is from a pensions-related surplus of 3.0 percent of *GDP* in 1995 to a deficit of 2.5 percent of *GDP* in 2050, with the switchover from financial source to sink occurring between 2030 and 2040.

<u>Deterministic simulation properties</u>

Some of the basic deterministic simulation properties of the model are reported in Table 3 (macroeconomic variables) and Table 4 (variables related to intergenerational distribution and the pension system). Shocks are administered (to the baseline solution described above) in the initial year of the simulation and maintained throughout the solution period. Results for three years are given, Year 0 representing the near term, Year 10 representing the medium term, and Year 55 representing the long term. All differences, whether measured in terms of percentage change or absolute percentage point change, are calculated by comparing the shocked alternative solution to the baseline solution. The first two shocks relate to population, and establish the basic neoclassical properties of model with

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[5] Balance of the public pension system is equal to contributions minus benefits. As this balance is assumed to be blended with the overall government balance, no explicit accounting is made for assets of the public pension system. Balance of the private DC pension system is equal to contributions minus benefits plus investment income minus bequests (the paying out of assets to the heirs of claimants who die) minus withdrawals associated with job change. The same identity applies to the private DB pension system, however, we assume that both early withdrawals and bequests are zero. Regarding the first, most private DB pension plans and tax regimes make it highly advantageous for a job-switcher to roll over his / her accumulated pension rights into another plan. Regarding the second, the question of whether the assets of DB pension plans belong to the employee or to the firm is a contentious issue. In the event, we have adopted the formulation that, in the event of the death of the participant, his / her pension assets revert to the plan.

[6] No attempt is made to account explicitly for public health system contributions and benefits, which are often (for example, in Germany and France, although not in the United States) included under the social security umbrella.

respect to its treatment of demography. The remaining shocks relate to the main behavioral assumptions in the model (apart from productivity growth), namely age-specific consumption rates and labor force participation rates.

Increase in aggregate population. In the first shock, population in all age groups is increased by 10 percent (see first row of Tables 3 and 4). As assured by the neoclassical nature of the model, the result is an immediate decline in *GDP per capita* (*vis à vis* the baseline scenario; see first column of Table 3). Since the shock is equiproportional across the age spectrum, change in labor force is proportionally the same as change in population, as a result of which the wage rate experiences the same percentage decline as *GDP per capita*. The capital-output ratio is reduced, reflecting substitution toward more abundant labor, and the rate of return to capital rises accordingly. The aggregate saving rate increases *vis à vis* the baseline, fairly sharply in the near term and less so in later years. This is because, when population and, as a result, labor force increases, total income and consumption out of income rise *pari passu*; however, total assets of the elderly and consumption financed by their decumulation do not rise. The increase in income is thus greater than the increase in consumption, and the saving rate rises. Over the long term, this effect is dissipated as assets of the elderly increase to reflect saving out of the higher wage bill.

Disposable income of the non-elderly consists essentially of wages; that of the elderly consists mostly of pension benefits, the bulk of which come from the public pension system. In the initial year, the impact of the population shock is to increase relative disposable income of the elderly by 0.7 percentage points because wages are lower (see first column of Table 4). In the medium term (Year 10), however, pensions are lower because the ranks of retirees have been filled with persons whose pension entitlements from the public pension system reflect the low wage rate at the beginning of the simulation period. The wage rate, by contrast, has by Year 10 recovered a significant proportion of ground lost due to the initial shock. The combined effect of changes in public pensions and the wage rate is to reduce the disposable income of the elderly relative to that of the non-elderly.

Disposable income of the elderly also includes profits on non-pension assets, of which the most significant are imputed rents to owner-occupied housing. The initial impact of the population shock is to dilute assets per capita across the board, with no significant impact on relative wealth (see second column of Table 4). By Year 10, assets of the elderly have not recovered from the initial dilution; by contrast, assets of the 15-59 year-old population include the wealth of persons in their twenties and thirties which was not affected by the initial dilution effect because it was, at the time, effectively zero. This explains why wealth of the elderly, expressed as a percentage of wealth of the non-elderly, is significantly (35.9 percentage points in Year 10) lower in the high-population alternative scenario than it is in the baseline.

Thus, changes in the age-distribution of wealth, translated into changes in income therefrom, reinforce the relative income trends set in motion by changes in public pension system benefits and the wage rate. In Year 10 the ratio of disposable income of the elderly to disposable income of the non-elderly is 1.7 percentage points lower in the alternative scenario than in the baseline. By the end of the solution period, however, the difference between the two scenarios is insignificant: elderly persons' pensions reflect a wage rate that has nearly regained its baseline level, while the asset dilution effect has been dissipated as young persons unaffected by the initial dilution move up the age ladder.

Note that pension system balances are not significantly affected by change in total population. This makes sense since the ratio of contributors to beneficiaries is unchanged by an across-the-board change in population.

Table 2: Baseline scenario, pension systems

	Benefits (bill. 1995 US $)	Contributions (bill. 1995 US $)	Investment income (bill. 1995 US $)	Bequests (bill. 1995 US $)	With-drawals (bill. 1995 US $)	Balance (% of GDP)	Private Pension System Assets (bill. 1995 US $)	Change in Private Pension System Assets (bill. 1995 US $)
Public								
1995	1739	2165	-	-	-	1.4	-	-
2000	1952	2413	-	-	-	1.3	-	-
2010	2511	2912	-	-	-	1.0	-	-
2020	3589	3338	-	-	-	-0.5	-	-
2030	4526	3736	-	-	-	-1.1	-	-
2040	5262	4134	-	-	-	-1.9	-	-
2050	6090	4410	-	-	-	-2.3	-	-
Private DC								
1995	29	428	21	43	55	1.0	3081	323
2000	35	478	33	54	97	0.9	4695	325
2010	48	576	52	74	180	0.8	7967	326
2020	75	659	66	92	243	0.7	11190	315
2030	115	736	79	117	286	0.6	14255	297
2040	161	814	93	156	321	0.5	17118	270
2050	205	892	109	196	352	0.4	19700	249
Private DB								
1995	283	433	32	-	-	0.6	4621	182
2000	322	483	39	-	-	0.6	5567	200
2010	404	582	51	-	-	0.5	7776	230
2020	561	668	62	-	-	0.3	9805	141
2030	764	747	70	-	-	-0.2	10669	-115
2040	968	827	81	-	-	-0.5	9591	-325
2050	1162	906	93	-	-	-0.6	5858	-549
Total								
1995	2051	3026	53	43	55	3.0	7702	505
2000	2309	3374	72	54	97	2.8	10262	525
2010	2963	4070	103	74	180	2.3	15743	556
2020	4225	4665	128	92	243	0.5	20995	456
2030	5405	5219	149	117	286	-0.7	24924	182
2040	6394	5775	174	156	321	-1.9	26709	-55
2050	7457	6208	202	196	352	-2.5	25558	-300

Table 3. Sensitivity of macroeconomic variables to changes in baseline exogenous assumptions

	GDP per capita (% change)	Capital-output ratio (absolute change)	Rate of return to capital (absolute % point change)	Wage rate (% change)	Aggregate saving rate (absolute % point change)
Population increased 10% (age structure unchanged)					
Year 0 (1995)	-3.1	-0.19	0.7	-3.1	0.9
Year 10 (2005)	-2.5	-0.16	0.6	-2.5	0.9
Year 55 (2050)	-0.4	-0.03	0.1	-0.4	0.1
Older age structure (total population unchanged)					
Year 0	0.0	0.00	0.0	0.0	0.0
Year 10	0.8	-0.01	0.1	-0.2	0.2
Year 55	-8.4	-0.02	0.1	-0.3	-3.1
Labor force increased 10% (age structure unchanged)					
Year 0	6.6	-0.19	0.7	-3.1	1.0
Year 10	7.9	-0.12	0.4	-1.9	0.6
Year 55	9.7	-0.02	0.1	-0.3	0.1
Age-specific consumption rates increased 10%					
Year 0	0.0	0.00	0.0	0.0	-6.1
Year 10	-5.3	-0.33	1.2	-5.3	-5.3
Year 55	-16.3	-1.04	4.2	-16.3	-1.7

Table 4. Sensitivity of intergenerational distribution and pension-related variables to changes in baseline exogenous assumptions

	Disposable income per capita, pop. aged 60+ : pop. aged 15-59 (absolute % point change)	Assets per capita, pop. aged 60+ : pop. aged 15-59 (absolute % point change)	Balance of public defined benefit, PAYG financed pension system : GDP (absolute % point change)	Balance of private defined benefit, partially funded pension system : GDP (absolute % point change)	Balance of private defined contribution, fully funded pension system : GDP (absolute % point change)
Population increased 10% (age structure unchanged)					
Year 0 (1995)	0.7	1.5	-0.1	0.0	-0.8
Year 10 (2005)	-1.7	-35.9	0.1	0.0	0.1
Year 55 (2050)	0.2	3.2	-0.2	-0.1	0.0
Older age structure (total population unchanged)					
Year 0	0.0	0.0	0.0	0.0	0.0
Year 10	-2.4	-1.8	-0.1	-0.0	0.1
Year 55	-9.0	-22.0	-2.0	-0.7	0.0
Labor force participation rates increased 10% (age structure unchanged)					
Year 0	-2.2	0.0	0.5	0.1	0.1
Year 10	-0.8	-6.6	0.3	0.1	0.1
Year 55	-0.2	0.1	0.0	-0.1	0.0
Age-specific propensities to consume increased 10%					
Year 0	0.0	0.0	0.0	0.0	0.0
Year 10	7.2	56.5	-0.2	-0.0	-0.1
Year 55	5.9	56.8	0.2	0.1	-0.1

Older age structure. In the second simulation (see second row of Tables 3 and 4), the level of population was left unchanged, but its age structure was shifted to reflect IIASA's Maximum Population Aging demographic scenario.[7] Note that, from the beginning of the solution period through 2025, labor force in the Maximum Aging scenario is *higher* than it is in the Central scenario, not decreased as might be intuitively thought. This is because lower mortality immediately increases the number of elderly workers whereas lower fertility, which reduces the number of young workers, does not make its impact felt for roughly 20 years. Thus, results for Year 10 in Tables 3 and 4, which might appear counterintuitive in the context of a rapid aging scenario, must be interpreted keeping in mind that labor force is initially higher, not lower, in the alternative scenario.[8] Because labor force is higher and capital stock is, roughly speaking, unchanged (having not yet adjusted to the new saving rate), the Year 10 capital-output ratio and wage rate are (slightly) lower in the alternative scenario than in the baseline. The long-term story is the reverse, because after 2025, labor force is lower in the alternative than in the baseline scenario.

Given fixed age-specific propensities to consume, the compositional effect of the age structure change is to reduce the household saving rate and, through changes in the ratio of public pension system beneficiaries to contributors, the government balance. These translate into a substantial long-term reduction in the aggregate saving rate, which in Year 55 is 3 percentage points lower in the alternative than in the baseline scenario.

In this model, the long run impact of the lower aggregate saving rate will be reduction in the capital-output ratio and hence *GDP per capita*. The wage rate will decline to reflect lower capital per worker while the rate of return to capital will increase to reflect greater scarcity of capital. As illustrated in the second column of Table 3, when cumulated over the 55-year simulation period, lower saving rates result in reduction of the capital-output ratio by 0.02 in absolute terms (baseline vs. alternative basis), corresponding to approximately 0.8 percent in proportional terms. Per capita output is reduced by 8.4 percent *vis à vis* the baseline, meaning that *GDP* itself is reduced by this amount, since total population is the same. The implied "elasticity" of *GDP* with respect to the dependency ratio is roughly -0.2 (i.e., a 50 percent increase in the dependency ratio gives rise to a 10 percent decline in *GDP* per capita). The wage rate, which declines in the same proportion as the ratio of *GDP* to labor force, is reduced by 0.3 percent.

The older age structure significantly reduces disposable income of the elderly population relative to that of the non-elderly population throughout the simulation

[7] In IIASA's population projections (Lutz 1996), a panel of experts was instructed to give high-low-central forecasts of fertility, mortality, and migration rates so that the each high-low range would correspond to a subjective 95 percent confidence interval. The "Maximum Aging Scenario" (our term, not that used by Lutz) is the projection resulting when Low Fertility, Low Mortality, and Low Net Migration assumptions are combined. The "Minimum Aging Scenario" incorporates High Fertility, High Mortality, and High Net Migration assumptions. While these two extreme scenarios do not, strictly speaking, represent 95 percent confidence bounds around the Central Scenario population projection, they represent something conceptually similar. Note that it is not valid to construe empirical estimates of the impact of population aging from a comparison of the Maximum Population Aging scenario with the Central scenario, because the latter, baseline, projection already includes a substantial aging component.

[8] Note also that in the initial year of the simulation, the age structure of population is identical in the baseline and alternative scenarios; this explains the row of zeros in Tables 3 and 4.

period, by 2.4 percentage points in Year 10 and by 9 percentage points by Year 55 (see Table 4). The long-term reduction reflects lower income from assets (including private pensions), which in turn results from lower accumulation (see the saving rate in Table 3). The reduction in aggregate saving is in significant part due to the widened deficit in the public social security system (2.0 percentage points in Year 55, see third column of Table 4). The older age structure also causes the balance of the private DB pension systems to move 0.7 percentage points towards deficit in Year 55 (see fourth column of Table 4). The balance of the private DC pension system is unaffected by the age structure change. Note that the ratio of assets per capita of the population 60+ to assets per capita of the population 15-59 (see the second column of table 4) can be expressed as the ratio of age-specific assets divided by the dependency ratio. The fact that distribution of wealth as measured by this index shifts significantly against the elderly (by 22 percentage points in Year 55) is largely due to the direct "denominator effect" of the increase in the dependency ratio.

Higher age-specific labor force participation rates. In this simulation, we increase all age-specific labor force participation rates by 10 percent (i.e., multiply by 1.1), leading to an increase in output (see third row of Tables 3 and 4). The wage rate falls to reflect greater abundance of labor, the capital-output ratio is reduced, and the rate of return to capital rises, all of which reflect the neoclassical nature of the model. The aggregate saving rate increases, in large part because of an increase in the balance of the pension system, especially its public component (see Table 4). These pension balance effects diminish over time, and along with them the impact on the aggregate saving rate, as the increased number of workers is eventually translated into an increased number of pension beneficiaries. The initial impact of higher labor supply is to reduce disposable income of the elderly by 2.2 percentage points relative to disposable income of the non-elderly (wage income of the non-elderly population increases, whereas the pensions of the elderly do not). However, this impact disappears over time as higher pension income enters the picture. The increase in labor force does nothing to affect assets in Year 0, hence there is no initial-year change in assets of the elderly relative to assets of the non-elderly. In the medium term (Year 10), the distribution of assets is shifted against the elderly, because a substantial proportion of the assets of the elderly in Year 10 already existed in Year 0 and was hence unaffected by the labor force shock. Put differently, in Year 10, only those elderly under 70 will have experienced higher labor force participation, and hence greater asset accumulation in the alternative scenario. In the longer term, however, assets of the elderly population gradually reflect higher labor force participation while young, as a result of which the age-distribution of assets returns to its baseline pattern.

Higher age-specific propensities to consume. In this simulation, all age specific consumption rates were increased by 10 percent (i.e., were multiplied by 1.1).[9] As there are no demand-side multiplier effects in the model, there is no impact on *GDP* in the initial year (see last row of Table 3). By the end of the simulation, however, the cumulative effect of lower savings is to reduce *GDP* by 16.3 percent *vis à vis* the baseline (see last row of Table 3). The capital-output ratio is reduced by 1.04 in absolute terms, the rate of return to capital is increased by 420 basis points (i.e., 4.2 percentage

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[9] Among the exogenous variables are the proportion of inherited wealth converted to consumption and the proportion of annuity income consumed, the complement of the latter being the proportion of wealth which the elderly wish to bequeath. These "propensities to consume" were also multiplied by 1.1.

points), and the wage rate falls in step with *GDP*. The ratio of disposable income of the elderly to that of the non-elderly is higher in the alternative than in the baseline scenario for two reasons. First, the wage rate is lower while the rate of return to capital is higher (see third and fourth columns of Table 3). Second, the age-distribution of wealth is shifted strongly towards the elderly (see second column of Table 4). The difference in the age-distribution of wealth observed in Year 10 has to do mostly with the inertia of capital accumulation. Assets of the elderly population in Year 10 reflect primarily pre-shock accumulation behavior, that is, relatively high baseline propensities to save combined with relatively high baseline wage rates. Assets of the non-elderly in Year 10, by contrast, reflect in significant degree post-shock accumulation behavior, that is, relatively low alternative-scenario propensities to save and relatively low alternative-scenario wage rates. Because the wage rate continues to fall and the rate of return to capital to rise between Years 10 and 55, the shift in the age distribution of wealth persists in the long run.

Conclusion

In this section, we have seen, first, that initialization of the model with reasonable parameters and baseline assumptions gives rise to a reasonable long run solution path. Second, we have seen that changes in baseline assumptions give rise to alternative solutions whose deviations from the baseline have plausible economic interpretations. Subject to its restrictive neoclassical assumptions and the fact that several important classes of parameters (age-specific propensities to consume and labor force participation rates) are exogenous, the model appears appropriate for long-run simulations related to demography, pension arrangements, and macroeconomic performance. In the next section, we extend our analysis of the model to test its robustness characteristics.

3. Uncertainty analysis

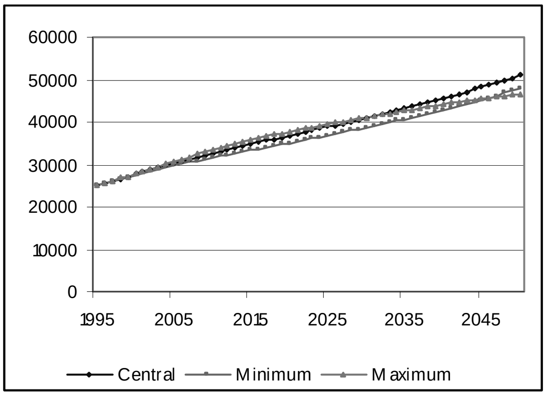
In view of the long-term nature of questions involving social security, special attention should be given to uncertainty and model robustness. One subjective definition of robustness might be that, when model parameters and/or exogenous assumptions are randomized and the model is solved in Monte Carlo fashion,

- the mean and median of the stochastic forecasts lie "reasonably close" to the deterministic baseline forecast throughout the simulation period and
- uncertainty bands are not "too wide."

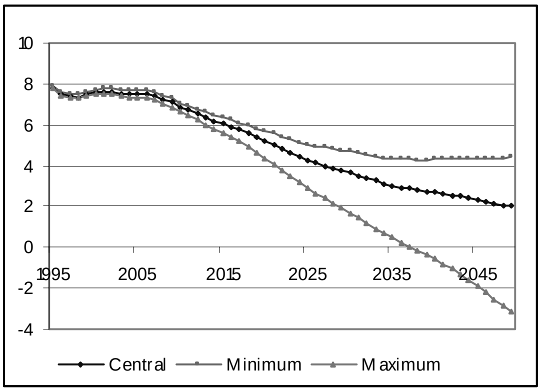
Robustness does not mean that results are little changed when exogenous assumptions or model parameters are subject to massive fluctuations (Hackl and Westlund 1991). It means, rather, that reasonable stochasticity in model input gives rise to reasonable stochasticity in model results.

Perhaps the best understood uncertainties in the social security field are those arising from stochastic mortality, fertility, and immigration rates (Lee and Tuljapurkar, 1994). As we discussed in Footnote 7 above, the Maximum and Minimum Population Aging scenarios from IIASA's population projections do not represent strict confidence intervals around the Central scenario, but they are conceptually close. In Figure 1, we display paths of selected variables under the three demographic scenarios.

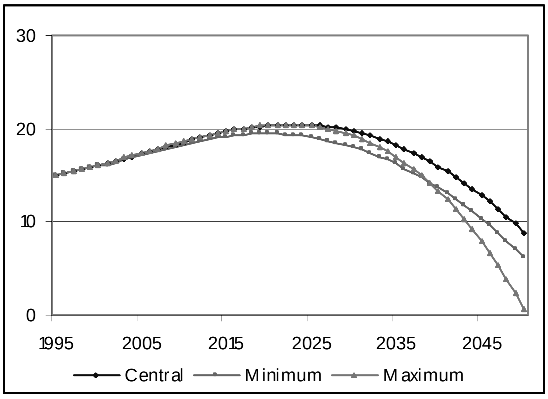
Figure 1. Three demographic scenarios (Central, Minimum Aging, Maximum Aging).



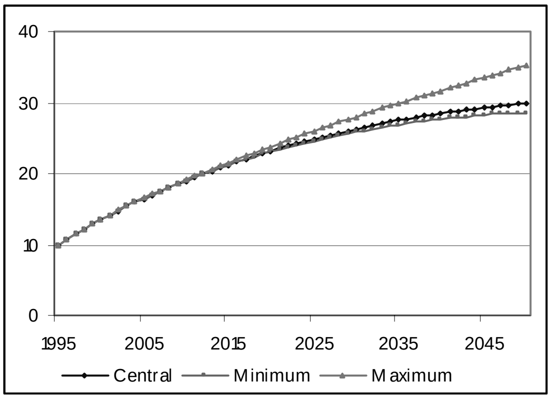
A. *GDP per capita ($US)*



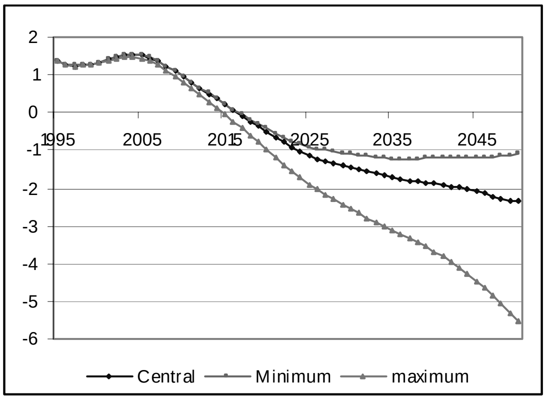
B. *Aggregate saving rate (percent)*



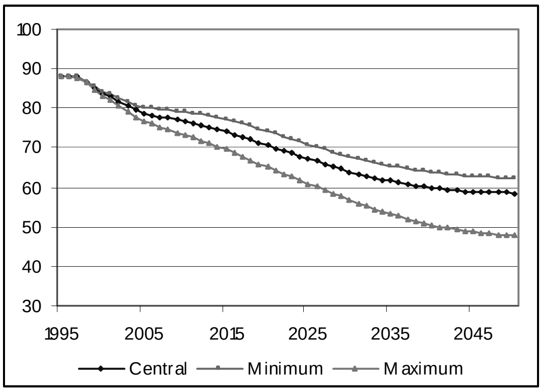
C. *Assets, DB private pension system (percent of GDP)*



D. *Assets, DC private pension system (percent of GDP)*



E. *Balance, public pension system (percent of GDP)*



F. *Disposable income per capita, population aged 60+ : population aged 15-59 (percent)*

These selected variables are *GDP* per capita, the aggregate saving rate, assets of the private DC and private DB pension systems (as a percentage of *GDP*), balance of the public pension system (as a percentage of *GDP*), and the ratio of disposable income per capita of the elderly population relative to that of the working-age population (expressed as a percentage). The conclusion to be drawn from examining Figure 1 is that, while the uncertainty band naturally "fans out" in the more remote years of the solution, overall trends in these selected variables are robust to the choice of demographic scenario. Under all but the most extreme assumptions regarding fertility, mortality and migration rates, these results tell us, the aggregate saving rate is likely to decline, the balance of the public pension system will decline as a share of GDP, assets of the private DB pension system will decline as a share of GDP and disposable income per capita of the elderly population will fall relative to that of the working-age population.

In the rest of this section, we illustrate model behavior when a key model parameter (β) and the main exogenous assumptions (age-specific saving and labor force participation rates) are assumed to follow an ARCH-M process, in which the mean of the variable depends on its own conditional variance.[10] Taking β as an example, the process is modeled as $\beta(t) = \mu(t) + \varepsilon(t)$, where $\mu(t) = \beta^{*} + \delta\, h(t),\ \ \delta > 0$,

$h(t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon^2(t-i)$, $\beta^* = 0.33$ is the baseline parameter and we

assume $\varepsilon(t) \sim N(0{,}0.05\beta^*)$. We model $\beta(t)$ as ARCH(1), when the conditional forecast of $\beta(t)$ is based on one time period lagged error term, i.e., $q=1$. In empirical analysis, the parameters δ, α_0, and α_i must be econometrically estimated. For our analysis we assumed $\delta = 1, \alpha_0 = 0$, $\alpha_1 = 0.65$ (see, for example, Enders 1995).

In addition to being interested in the "drift" of the mean parameter value due to conditional volatility, we are concerned with the possibility of unforeseen changes in parameters. In order to incorporate such effects, we define a shock function that gradually shifts the selected parameter by 10 percent over a 10-year period. During the remainder of the simulation, the parameter remains at its shifted value.

We illustrate these numerical experiments with the parameter $\beta(t)$. At $t=1$, the baseline parameter $\beta^* = 0.33$ is shocked by a random variable $\varepsilon(t) \sim N(0{,}0.05\beta^*)$, i.e., $\beta(1) = \beta^* + \varepsilon(t)$. For $1 < t \le 20$, that is, from 1995 to 2014, the parameter is simulated according to the process $\beta(t) = \mu(t) + \varepsilon(t)$, $\mu(t) = \beta^* + h(t)$, $h(t) = 0.65\varepsilon^2(t-1)$. We then impose a shift in the parameter, as follows. Starting in 2015, β^* is assumed to decline by 1 percent of its baseline value each year. That is, for $20 < t \le 30$ (i.e., 2015 through 2024), we define $\beta^{*i}(t) = (1-0.01i)\beta^*$, $i = 1, \ldots\ldots, 10$ and set $\mu(t) = \beta^{*i}(t) + h(t)$ with $\varepsilon(t) \sim N\left(0{,}0.05\beta^{*i}\right)$. For $t > 30$, i.e., 2025-55, and letting $\beta^{**} = 0.9\,\beta^*$, we have $\mu(t) = \beta^{**} + h(t)$ with $\varepsilon(t) \sim N\left(0{,}0.05\beta^{**}\right)$.

[10] The uncertainty analysis presented here summarizes more extensive analyses presented by Ermolieva et al. (2000) and Westlund et al. (2000).

The variables whose behavior we examine are the same as above. Results are shown in Figure 2 (uncertainty in β), Figure 3 (uncertainty in age-specific labor force participation rates) and Figure 4 (uncertainty in age-specific average propensities to consume). Unlike in Figure 1, where the uncertainty bands have only an informal interpretation as confidence intervals, the uncertainty bands in Figures 3-5 were calculated based on Monte Carlo runs and can be strictly interpreted as confidence intervals. Note however, that shocks which are the same in proportional terms (across variables) cannot be compared in the real world. For example, a 10 percent shock (or uncertainty) which increases a consumption rate from 90 to 99 percent is not comparable to a 10 percent shock which shifts $\beta = 0.33$ to $\beta = 0.363$.

Based on the two-fold definition of robustness given above, most observers would conclude from the results in Figure 2 that, with the exception of the relative income variable, model projections are *not* robust to uncertainty in β. Putting this differently, *GDP*, the saving rate, and the state of the various components of the pension system are sensitive to uncertainty in β; however, the projected steady deterioration in the income of the aged relative to the income of the young is not. Based on the graphs in Figure 3, all variables appear to be robust to uncertainty in labor force participation rates. In Figure 4, all variables with the obvious exception of the aggregate saving rate (which is simply the complement of the consumption rate) are robust to uncertainty in age-specific propensities to consume.

4. Conclusions

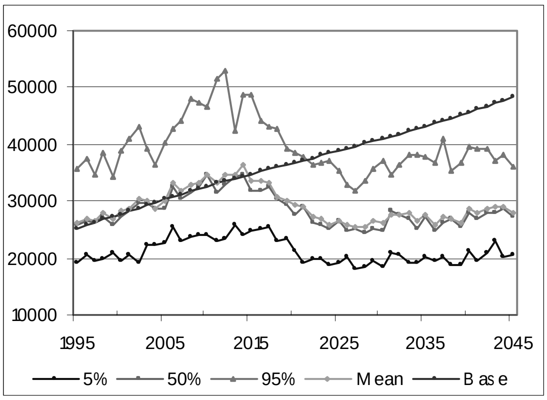
The model reported on here is designed to study the evolution of pension systems and the age-distribution of income and wealth in a consistent macroeconomic framework as a function of population age distribution and exogenous assumptions about consumption rates, labor force participation rates, and the nature of pension arrangements. In this paper we have shown that

i. reasonable exogenous assumptions give rise to a reasonable long-run model solution;

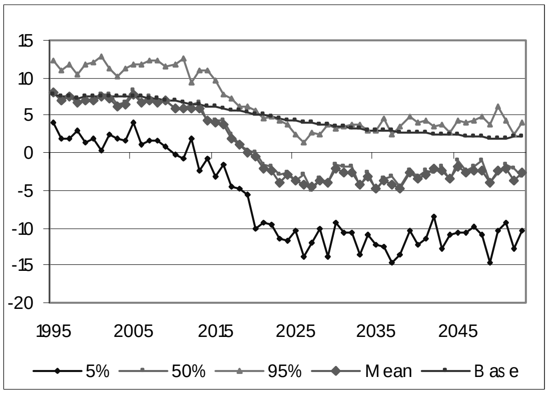
ii. when exogenous assumptions or model parameters are changed, the model performs sensibly on a baseline-vs.-alternative basis; and

iii. model projection results are reasonably robust to selection of demographic scenario and exogenous assumptions regarding household saving and labor supply; they are sensitive, however, to the selection of the parameters of the core production function.

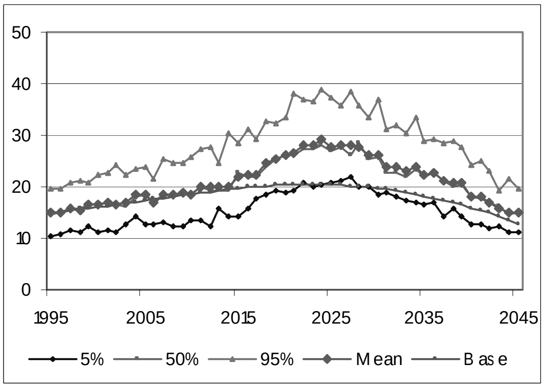
Figure 2. Uncertainty in β



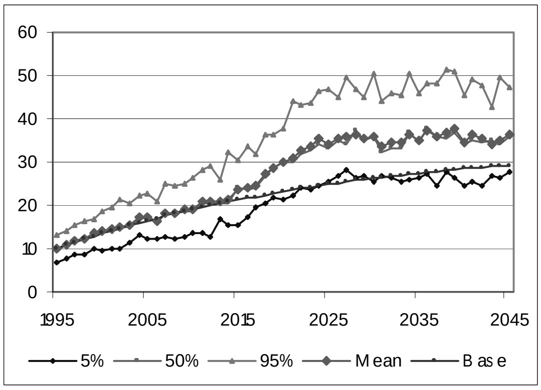
A. *GDP per capita ($US)*



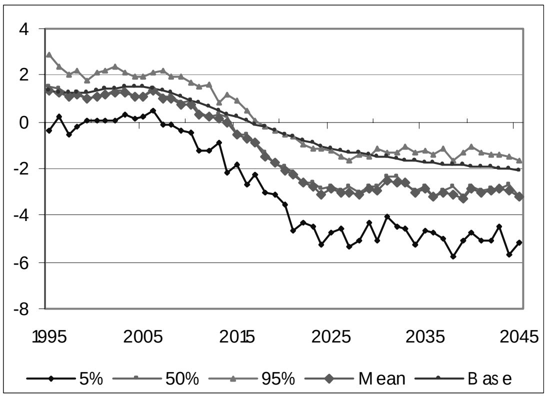
B. *Aggregate saving rate (percent)*



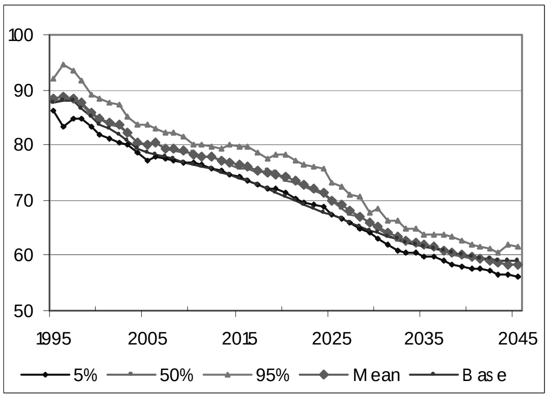
C. *Assets, DB private pension system (percent of GDP)*



D. *Assets, DC private pension system (percent of GDP)*

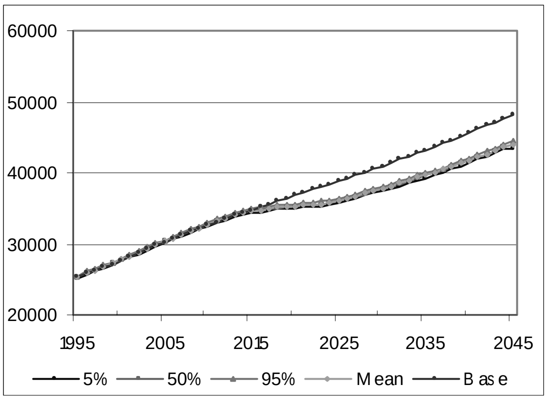


E. *Balance, public pension system (percent of GDP)*

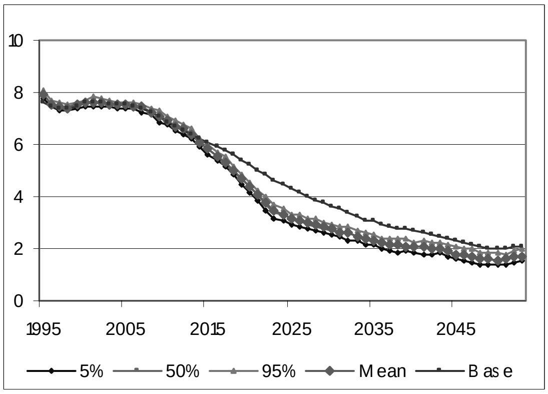


F. *Disposable income per capita, population aged 60+ : population aged 15-59 (percent)*

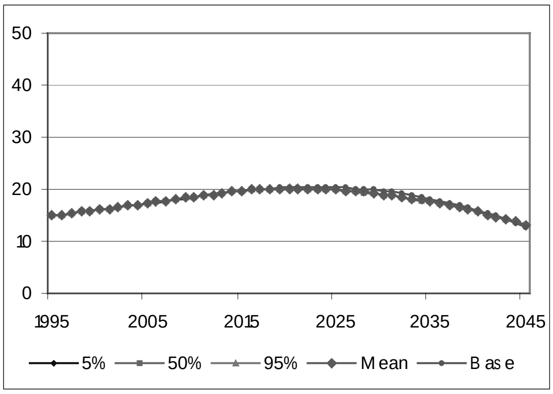
Figure 3. Uncertainty in age-specific labor force participation rates



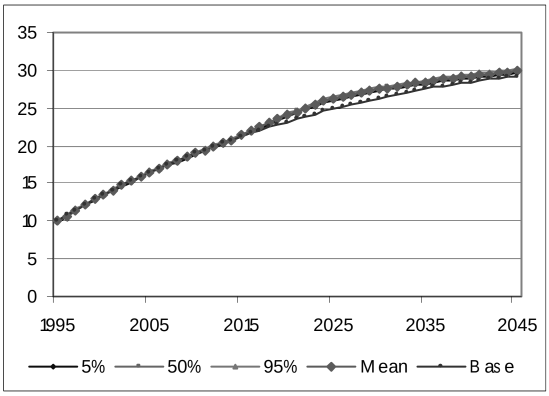
A. *GDP per capita ($US)*



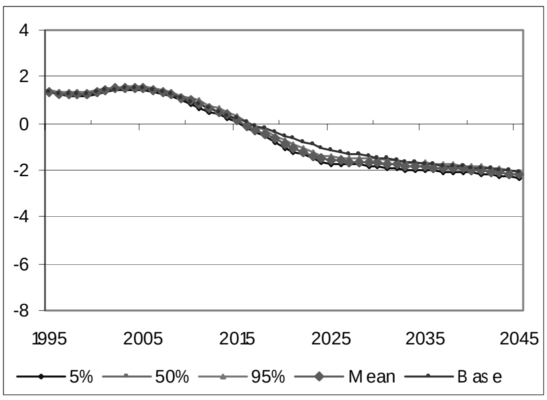
B. *Aggregate saving rate (percent)*



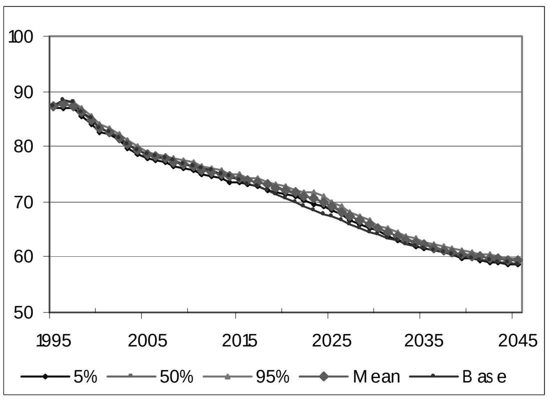
C. *Assets, DB private pension system (percent of GDP)*



D. *Assets, DC private pension system (percent of GDP)*

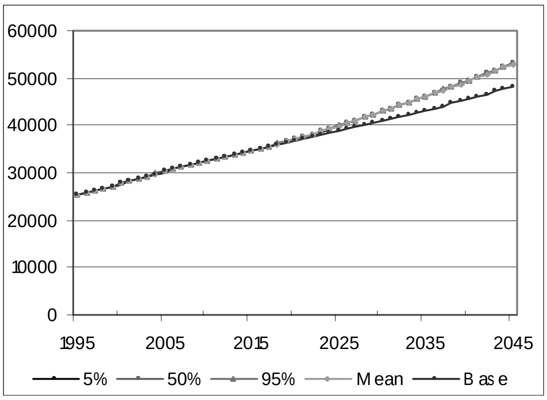


E. *Balance, public pension system (percent of GDP)*

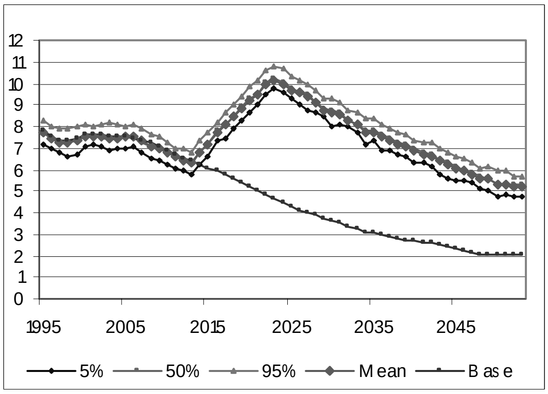


F. *Disposable income per capita, population aged 60+ : population aged 15-59 (percent)*

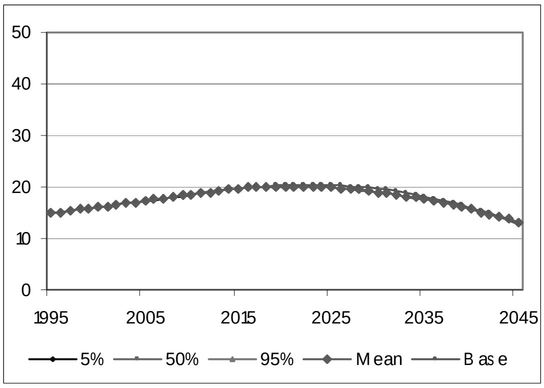
Figure 4. Uncertainty in age-specific average propensities to consume



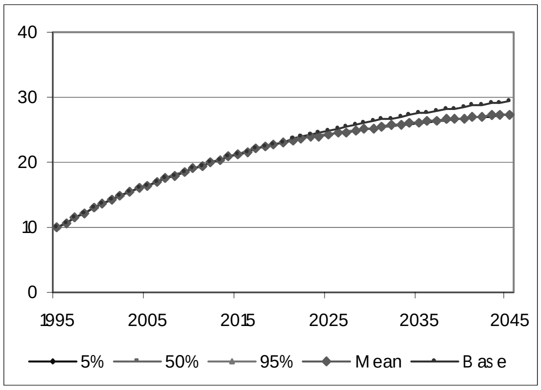
A. *GDP per capita ($US)*



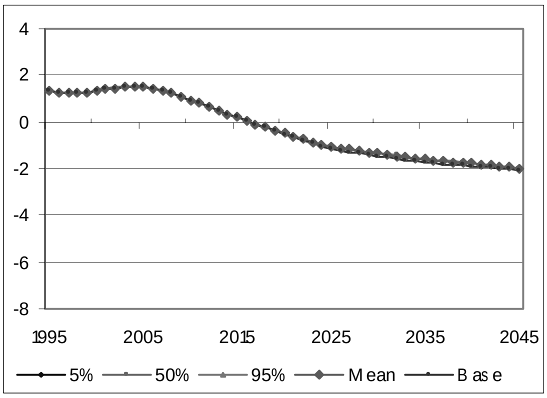
B. *Aggregate saving rate (percent)*



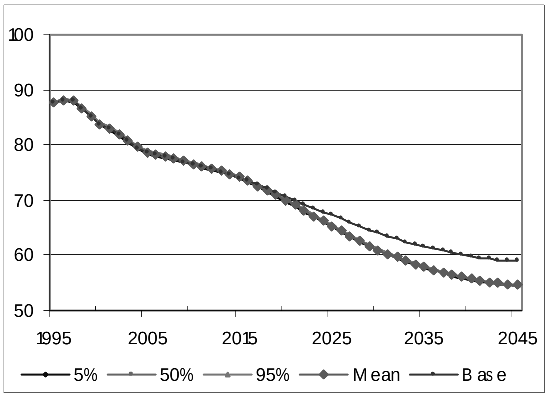
C. *Assets, DB private pension system (percent of GDP)*



D. *Assets, DC private pension system (percent of GDP)*



E. *Balance, public pension system (percent of GDP)*



F. *Disposable income per capita, population aged 60+ : population aged 15-59 (percent)*

Annex 1. The IIASA model: algebraic structure

In the following pages, the algebraic structure of the IIASA model is presented and its economic logic is described. For simplicity and economy of space, a single-region version of the model is described. MacKellar and Ermolieva (1999) present an earlier version of the model in two-region form, and a subsequent paper will document the latest model version in multiregional form.

1. Population, labor force, and employment

1.1 Population

Population is divided into age groups $age = \overline{0, MaxAge}$ where *MaxAge* is the beginning year of the terminal age category (for example, $MaxAge = 85$ if the terminal age category is 85+). There are four demographic model solution options. In the simplest of these, which we implement in this paper, a single deterministic demographic scenario consisting of population by single-year age groups is loaded from another source. In the second, stochastic population scenarios produced outside the model are loaded. In the third, a deterministic population scenario is produced within the model using standard cohort-component projection methodology. In this case, for $age > 0$

$$Pop(sex, t, age) = Pop(sex, t-1, age-1)\left[1 - MortRate(sex, t-1, age-1)\right] + NetMigr(sex, t, age)$$

where *Pop* is population by age and sex, *MortRate* is the age-specific mortality rate and *NetMigr* is the number of net migrants. The mortality rate is calculated on the basis of life expectancy at birth, an exogenous assumption, using a model life table. Similarly, the number of net migrants by age is calculated by sharing down total net migration, an exogenous assumption, using a Castro-Rogers model migration schedule. For $age = 0$,

$$Pop(0, t) = \sum_{age=15}^{49} Pop(female, t, age)\ FertRate(t, age)$$

$$Pop(male, t, 0) = SexRatio\ Pop(t, 0)$$

$$Pop(female, t, 0) = (1 - SexRatio)\, Pop(t, 0)$$

where the age-specific fertility rate *FertRate* is calculated on the basis of an exogenous total fertility rate and a model fertility schedule. A final solution option is to define mortality and fertility rates (and, if desired, number of net migrants as well) as random variables and produce a stochastic population projection within the model.

Total population is the sum over age groups

$$Pop(t) = \sum_{age=0}^{MaxAge} Pop(male, t, age) + \sum_{age=0}^{MaxAge} Pop(female, t, age)$$

1.2 Labor force and employment

Total labor force is the sum over age groups:

$$LabForce(t) = \sum_{age=15}^{MaxAge} LabForce(t, age)$$

where

$$LabForce(t) = Pop(t, age)\ LabForcePartRate(t, age)$$

Age-specific labor force participation rates are exogenous assumptions, as are unemployment rates:

$$Emp(t) = \sum_{age=15}^{MaxAge} Emp(t, age)$$

$$Emp(t) = LabForce(t, age)\left[1 - UnempRate(t, age)\right]$$

Inclusion of the unemployment rate leaves the way open for applications in which structural unemployment is taken as a function of the long-term trend growth rate of GDP.

2. Capital and the nature of claims

There are four types of capital: residential capital (*KRsdntial*), capital operated by private unincorporated enterprises (*KPvtUnincorpEnt*), capital operated by incorporated enterprises and held on households' behalf by the private pension system (*KPvtPenSys*), and capital operated by incorporated enterprises and held on households' behalf by other financial institutions (*KOthFinIns*).[11] Also implicitly assigned to *OthFinIns* are households themselves to the extent that they individually hold claims on capital operated by firms (i.e., claims that are disintermediated). Firms operate capital, either distributing or reinvesting earnings; they do not own shares in other firms. Financial claims on the capital operated by firms are held on behalf of households by institutions (*PvtPenSys* and *OthFinIns*) which collect and distribute dividends. No distinction is made between equity and debt claims.

The private pension system is divided into two components, one of which is a partially funded defined benefit (DB) system (*PvtPenSysDB*) and the other of which is a fully funded defined contribution (DC) system (*PvtPenSysDC*). Voluntary retirement savings accumulated outside pension funds are implicitly assigned to *PvtPenSysDC*. Corresponding to each of the four types of capital is an age-specific capital accumulation equation, which tracks the accumulation of assets for each cohort as it ages. There is a structural difference between the dynamics of *KPvtPensSys* and the dynamics of the other three asset classes. Funds flow into *PvtPenSys* only through payroll deductions (including deductions from entrepreneurial income) on behalf of system participants. Dividends earned on assets held by the *PvtPenSys* remain within the system. By contrast, savings of all origins, not just captive retirement-related savings, are invested in *KOthFinIns, KRsdntial, and KPvtUnincorpEnt.* Dividends earned on assets held by *OthFinIns* accrue to households, instead of being retained by the intermediary, and may be allocated to consumption at any point during the life cycle, as may profits accruing to *KPvtUnincorpEnt.* Implicit rents on *KRsdntial* are oonsumed, by assumption, in their entirety; equivalently, all housing is assumed to be owner-occupied. If saved, dividends earned on assets held by *OthFinIns* may remain within *OthFinIns*, or be allocated to residential investment or investment in capital operated by *PvtUnincorpEnt.*

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[11] From here on, we use "firms" to designated "incorporated enterprises."

All capital ultimately belongs to households. As described in Section 8, each single-year age-cohort is tracked as it accumulates capital during its working life and draws it down during retirement. Total assets of a cohort in a given year are

$$\begin{aligned}
&KTot(t, age) = \\
&\quad KRsdntial(t, age) + KPvtUnincorpEnt(t, age) \\
&\qquad + KPvtPenSysDC(t, age) + KPvtPenSysDB(t, age) \\
&\qquad + KOthFinIns(t, age)
\end{aligned}$$

or, aggregating together the two components of the private pension system,

$$KTot(t, age) = KPvtPenSys(t, age) + KRsdntial(t, age) + KPvtUnincorpEnt(t, age) \ + KOthFinIns(t,.age)$$

which expresses a cohort's wealth as the sum of pension- and non-pension wealth.

3. Output and rates of return to factors

3.1 GDP, wage rate, and rate of return to capital

Gross domestic product (*GDP*) is given by a Cobb-Douglas production function

$$GDP(t) = \alpha \left(1 + \mathrm{g}\right)^{t} KTot(t)^{\beta}\, Emp(t)^{1-\beta}$$

where g, the rate of total factor productivity growth, is exogenous. Rates of return to factors are neoclassical:

$$R(t) = \beta \left[\frac{GDP(t)}{KTot(t)} \right]$$

$$\overline{WageRate}(t) = \left(1 - \beta\right) \left[\frac{GDP(t)}{Emp(t)} \right]$$

where R is the gross profit rate, including depreciation and indirect taxes net of subsidies; and $\overline{WageRate}$ is average (over age groups) employee compensation, including social insurance contributions (contributions to public and private pension schemes).

In order to net depreciation and indirect taxes out of the rate of return to capital, we define

$$r(t) = R(t) - \frac{IndTaxRate(t)\, GDP(t)}{KTot(t)} - DeprRate(t)$$

where *IndTaxRate* is defined with respect to *GDP* and *DeprRate* is the depreciation rate. The advantage of netting out depreciation and indirect taxes immediately is that we can henceforth ignore them in calculating income, outlay, and net savings.

3.2 Age-specific wage rates

In a model with age-structure detail, we require a procedure to ensure that the average wage rate calculated from the marginal productivity condition above equals the average wage rate calculated by summing across age groups. In practice, this means that age-specific wage rates must be adjusted to be consistent with the overall average wage rate. We have approached this problem by defining a scale factor $\sigma(t, age)$ and then calculating age-specific wage rates as

$$WageRate(t, age) = \sigma(t, age)\ \overline{WageRate}(t)$$

$\sigma(t, age)$, a proxy for human capital, is specified to be logarithmic in age and paramaterized so that wages rise rapidly in the twenties and thirties, the average wage over the life cycle is earned at approximately age 45, and there is little increase after 55.

The required consistency condition is

$$\overline{WageRate}(t) = \frac{\sum_{age=15}^{MaxAge} Mult(t, age)\ \sigma(t, age)\ \overline{WageRate}(t) Emp(t, age)}{\sum_{age=15}^{MaxAge} Emp(t, age)}$$

To simplify the problem, let $Mult(t, age)$ to be the age-invariant

$$Mult(t) = \frac{\overline{WageRate}(t) \sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\ \overline{WageRate}(t)\ Emp(t, age)} = \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\ Emp(t, age)}$$

This variable can be interpreted as total "nominal" employment relative to total "effective," i.e. human-capital adjusted employment. Then the identity required for consistency may be rewritten

$$\overline{WageRate}(t) = \frac{\sum_{age=15}^{MaxAge} \left\{ \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\ Emp(t, age)}\ \sigma(t, age)\ \overline{WageRate}(t) Emp(t, age) \right\}}{\sum_{age=15}^{MaxAge} Emp(t, age)}$$

Moving age-invariant terms outside the braces,

$$\overline{WageRate}(t) = \frac{\dfrac{\sum_{age=15}^{MaxAge} Emp(t,age)}{\sum_{age=15}^{MaxAge} \sigma(t,age) Emp(t,age)} \overline{WageRate}(t) \sum_{age=15}^{MaxAge} \sigma(t,age) Emp(t,age)}{\sum_{age=15}^{MaxAge} Emp(t,age)}$$

which will clearly always hold true. Therefore, we calculate

$$WageRate(t,age) = Mult(t)\, \sigma(t,age)\ \overline{WageRate}(t)$$

where $Mult(t)$ is as defined above.

4. Income, capital transfers, outlay, and net saving of households

The articulation of income flows elaborated below has two main purposes. The first is to disaggregate income and consumption by age. The second is to make explicit the role of the private pension system in saving and the allocation of capital.

4.1 Income

The sources of household income are wages, imputed rents from residential capital, profits which accrue to capital operated by unincorporated enterprises, dividends distributed from earnings on capital operated by firms, public social security system benefits, and private pension benefits.

4.1.1 A note on taxation

The taxation of factor incomes in this model follows three simplifying assumptions. First, factor income is taxed once and only once, when it is earned. Thus, dividend income is not taxed because profits have already been taxed at the level of the firm; similarly, there is no capital gains tax when assets are sold because capital gains reflect profits which have already been taxed. Second, no distinction is made from a taxation point of view between different types of capital: profits on capital operated by firms, capital operated by private unincorporated enterprises, and the imputed services of residential housing are all assumed to be taxed at the same rate. Third, tax rates are not indexed by income or age.

4.1.2 Wage income

Disposable wage income is equal to gross wages minus direct taxes minus social insurance contributions to the public PAYG and private pension systems:

$$WageY(t,age) = WageRate(t,age)\ \ Emp(t,age)$$

$$DispWageY(t, age) = WageY(t, age) - DirTaxWageY(t, age) - ContPubPenSysWageY(t, age, - ContPvtPenSysWageY(t, age)$$

where $ContPvtPenSysWageY(t, age)$ consists of the sum of contributions to the DB and DC components of the private pension system. Calculation of direct taxes and pension system contributions are described in Sections 6 and 7, respectively. Note that, even though *PvtPenSys* contributions really represent the acquisition of a financial asset, rather than a current expenditure flow, the System of National Accounts (SNA) nonetheless counts such transactions as a debit in the calculation of disposable income. However, an adjustment is made (see Section 4.1.9) to ensure that the savings associated with such flows are credited to households.

4.1.3 Rental income

Imputed rental income is assumed to be taxed like any other form of income; however, social contributions are assumed to be zero:

$$RntlY(t, age) = r(t)\, KRsdntial(t, age)$$

$$DispRntlY(t, age) = RntlY(t, age) - DirTaxRntlY(t, age)$$

Recall, from Section 3.1, that capital returns are already net of depreciation and indirect taxes.

4.1.4 Entrepreneurial income

Profits from capital operated by unincorporated enterprises are treated the same as wages:

$$EntrY(t, age) = r(t)\, KPvtUnincorpEnt(t, age)$$

$$DispEntrY(t, age) = EntrY(t, age) - DirTaxEntrY(t, age) - ContPubPenSysEntrY(t, age) - ContPvtPenSysEntrY(t, age)$$

4.1.5 Dividend income

The assets held on households' behalf by *PvtPenSys* and *OthFinIns* earn dividends. However, in the first case, dividends are not considered by the SNA to be part of household income; rather, they are considered to represent the acquisition of a financial asset. The adjustment described in Section 4.1.9 will add these dividend earnings captured by the private pension system to household income. Unadjusted household disposable income includes only dividends on assets held by *OthFinIns*:

$$DividY(t, age) = DividDistErngsFirmsKOthFinIns(t, age)$$

The calculation of distributed earnings is given below in Section 5.2.2. Having already been taxed when earned, dividend earnings are not taxed when received by households. Disposable dividend income is thus simply

$$DispDividY(t, age) = DividY(t, age)$$

4.1.6 Pensions income

Pension income comes from three sources: the public PAYG pension system, the private DC pension system, and the private DB pension system. All three systems provide benefits, which are current transfers in the first case and, while representing sales of capital assets in the second two cases, are nonetheless considered for accounting purposes to represent income. The calculation of pension benefits is described in Section 7.

In addition, as described in Section 7.2.2, in any year, some persons will change jobs and a given proportion of these will choose to withdraw their assets from the private pension system rather than rolling them over into new plans. These withdrawals are also treated as income. We assume that withdrawals occur only from the DC pension system. While this point is debatable, however, most countries have in place measures that strongly encourage job-switchers to transfer their DB pension assets into another plan.[12] Combining the two components of the private pension system,

$$PensionY(t,age) = BenPubPenSys(t,age) + BenPvtPenSys(t,age) + WthdrwlKPvtPenSys(t,age)$$

Public pension income is subject to taxation because it is a current transfer. Private pension income and early withdrawals from the DC pension system are not taxed because these represent the sale of capital assets whose returns were taxed at the level of the firm. Disposable pension income is therefore

$$DispPensionY(t,age) = \\ [1 - DirTaxRate(t)]\, BenPubPenSys(t,age) + BenPvtPenSys(t,age) + WthdrwlKPvtPenSys(t,age)$$

Since private pension system benefits represent the drawing-down of a capital asset, they are included in the adjustment to disposable income described in Section 4.1.9. Early withdrawals (usually in consequence of job change) from the DC private pension system are described in Section 4.2.3; these are also included in the adjustment to disposable income described below.

4.1.7 Total income

Total income of households is equal to the sum over all income sources:

$$TotYHH(t,age) = WageY(t,age) + RntlY(t,age) + EntrY(t,age) + DividY(t,age) + PensionY(t,age)$$

[12] The logic behind this is that, on retirement, the benefit entitlement from the private DB pension scheme is calculated on the basis of years of participation and earnings. If we were to allow withdrawal of assets, it would be necessary to "restart the clock" every time assets were withdrawn, or to link benefits with accumulated assets (as in the DC private pension system) rather than earnings.

4.1.8 Disposable income

Disposable income is analogous:

$$DispYHH(t,age) = DispWageY(t,age) + DispRntlY(t,age) + DispEntrY(t,age) + DispDividY(\mathrm{t},age) + DispPensionY(t,age)$$

4.1.9 Adjusted disposable income.

Adjusted disposable income is equal to disposable income

- plus contributions to *PvtPenSys*,
- plus dividends earned on assets held by *PvtPenSys*,
- minus benefits received from *PvtPenSys*.
- minus early withdrawals from the defiend contribution private pension system (see Section 4.2.3)

Thus,

$$AdjDispYHH(t,age) = DispYHH(t,age) + ContPvtPenSys(t,age) + DividKPvtPenSys(t,age) - BenPvtPenSys(t,age) - WthdrwlKPvtPenSysDC(t,age)$$

where the third term represents the sum over DB and DC components of the private pension systems of dividends paid out by firms (given in Sections 7.2.1 and 7.3.1, respectively).

Adjusted disposable income is close to, but not exactly the same as, disposable income plus change in pension wealth. The latter would be equal to

$$\Delta KPvtPenSys(t, age) = ContPvtPenSys(t, age) + DividKPvtPenSys(t, age) - BenPvtPenSys(t, age) - BeqKPvtPenSys(t, age) - WthdrwlKPvtPenSysDC(t, age)$$

where $BeqKPvtPenSys(t, age)$ reflects the fact that upon the death of the claimant, accumulated pension assets are paid out to heirs.[13] There is no accounting for inheritance of pension wealth because paying-out is assumed to take the form of cash allocated either to consumption or to the acquisition of non-pension capital assets. Stated differently, there is no explicit modeling of survivors' benefits, which amount to reassigning title to existing pension assets. In conclusion, we could also write

$$AdjDispYHH(t,age) = DispYHH(t,age) + \Delta KPvtPenSys(t,age) + BeqPvtPenSys(t,age)$$

---

[13] From the standpoint of calculating individual wealth, the bequest term is irrelevant, because the individual must die in order to bequeath. In calculating cohort wealth, however, bequests must be taken into account.

4.2 Capital transfers

Resources available for household consumption take the form of disposable income and the proceeds of transferring claims to capital assets. In this section, the second of these is elaborated.

4.2.1 Annuitization of assets in old age

Starting at the pension eligibility age, households are assumed to divest themselves of non-pension assets in a way calculated to exhaust assets at age 105. This "annuitization" process -- which we model for simplicity's sake just as a program of asset sales -- is assumed to begin whether households are still in the labor force or not. For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$, annuity income is:

$$AnnYK[\bullet](age,t) = \frac{K[\bullet](t,age)}{105-age} \quad , age \geq \; EligAge$$

If the propensity to consume out of the proceeds of asset sales is unity, there is no bequest motive; if, for example, the propensity is 0.95, elderly households aim to die with 5 percent of their wealth intact, etc. Note that it is assumed that no assets are sold prior to retirement, apart from the special case of assets received via inheritance and the withdrawal of DC pension assets associated with job change, which we discuss in the next sections.

4.2.2. Bequests / inheritance

In all asset classes, age-specific bequests are equal to assets times the proportion of persons in the age group dying. For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns, PvtPenSysDC, PvtPenSysDB$:

$$BeqK[\bullet](t,age) = K[\bullet](t,age)\left[\frac{Deaths(t,age)}{Pop(t\text{-}1,age\text{-}1)}\right]$$

Without question, DC pension system assets, like non-pension wealth, belong to the individual and are heritable. As we discuss in Section 8.2, the case of assets of the DB pension system is more debatable.

Bequests are received, in the form of inheritance, by the surviving population. For simplicity, we estimate age-specific inheritance simply by dividing total bequests by population age shares. We exclude the population under age 15. Total bequests are

$$BeqK[\bullet](t) = \sum_{age=15}^{MaxAge} BeqK[\bullet](t,age)$$

and inheritance (for age groups over 15) is

$$InhK[\bullet](t,t) = \left[\frac{Pop(t,age)}{\sum_{age=15}^{MaxAge-1} Pop(t,age)}\right] \sum_{age=15}^{MaxAge} BeqK[\bullet](t,age)$$

Summing over age groups,

$$InhK[\bullet](t) = \sum_{age=15}^{MaxAge} InhK[\bullet](t, age)$$

This simplification admittedly exaggerates the number of "backwards" bequests (elderly persons inheriting wealth from middle-aged persons, who are in fact more likely to bequeath assets to their children than to their parents). [14]

The assumption is made that, when wealth is inherited, it is converted to cash, some of which is allocated to consumption and the remainder of which is allocated among *ΔKOthFinIns*, *ΔKRsdntial*, and *ΔPvtUnincorpEnt* using the same share coefficients applied to household net saving (see Section 8.3). Note, however, that the portion not consumed does not comprise new household savings; it represents the acquisition of a claim formed as the result of past saving.

Under these assumptions, sales of inherited assets are

$$SaleInhK[\bullet](t, age) = InhK[\bullet](t, age)$$

Consumption out of the proceeds of such sales is described in Section 4.3.4.

4.3 Outlay

4.3.1 Direct taxes.

These are described in Section 6.1.

4.3.2 Social insurance contributions

These are described in Section 7.

4.3.3. Consumption out of income

Average propensities to consume (*AvgPropCons*) out of disposable income streams are exogenous assumptions:

$$ConsWageY(t, age) = DispWageY(t, age)\ Avg\Pr opConsWageY(t, age)$$

$$ConsEntrY(t, age) = DispEntrY(t, age)\ Avg\Pr opConsEntrY(t, age)$$

$$ConsDividY(t, age) = DispDividY(t, age)\ Avg\Pr opConsDividY(t, age)$$

$$ConsBenPubPenSys(t, age) = BenPubPenSys(t, age)\ Avg\Pr opConsBenPubPenSys(t, age)$$

$$ConsBenPvtPenSysDC(t, age) = BenPvtPenSysDC(t, age)\ Avg\Pr opConsBenPvtPenSysDC(t, age)$$

$$ConsBenPvtPenSysDB(t, age) = BenPvtPenSysDB(t, age)\ Avg\Pr opConsBenPvtPenSysDB(t, age)$$

[14] One expedient way to solve this problem is to assume that only persons under some age, say 65, but this runs the danger of failing to account for significant spousal bequests. Ultimately, a vector of age-specific share coefficients should be applied to allocate bequests from persons of a given age group over heirs by age group.

It is assumed that all imputed housing services are consumed:

$$ConsRntlY(t, age) = DispRntlY(t, age)$$

4.3.4 Consumption out of the proceeds of asset sales.

It is assumed that consumption out of the proceeds of asset sales takes place in the year of the sale, i.e., households do not hold liquid balances.

4.3.4.1 Consumption out of the proceeds of selling inherited assets

For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns, PvtPenSysDC, PvtPenSysDB$ consumption out of the sales of inherited assets is

$$ConsSaleInhK[\bullet](t, age) = SaleInhK[\bullet](t, age)\, ConsShareSaleInhK[\bullet](t, age)$$

and the sharing-out of what is not consumed between *ΔKOthFinIns*, *ΔKRsdntial*, and *ΔKPvtUnincorpEnt* is described in Section 8.3 below. We use a mnemonic corresponding to "consumption share" instead of *AvgPropCons* because average propensity to consume is properly considered with reference to income.

4.3.4.2 Consumption out of retirement annuity income.

Consumption in old age financed by the sale of assets accumulated during working life is treated in the same way. Because private pension system benefits are classified for purposes of the SNA as income, rather than capital transfers, this component has already been described above. For the remaining components $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$:

$$ConsAnnYK[\bullet](t, age) = AnnYK[\bullet](t, age)\, ConsShareAnnYK[\bullet](t, age)$$

As mentioned above in discussion private pension system benefits, if there is no bequest motive, the consumption shares are assumed to be unity. However, this assumption can be generalized to allow for bequests. In this case, the complement of the consumption share is simply the proportion of wealth upon retirement which households wish to bequeath.

4.3.4.3 Consumption out of the proceeds of selling DC pension assets withdrawn in consequence of job change.

The final component of consumption is:

$$ConsWthdrwlKPvtPenSysDC(t, age) = WthdrwlKPvtPenSysDC(t, age)\, ConsShareWthdrwlKPvtPenSys(t, age)$$

Early withdrawals from the privare defined-benefit pension system are assumed to be zero.

4.4 Net savings of households

Recapitulating, disposable and adjusted disposable household incomes are

$$
\begin{aligned}
&DispYHH(t,age) = \\
&\quad DispWageY(t,age) + DispRntlY(t,age) + DispEntrY(t,age) + DispDividY(\mathrm{t},age) + DispPensionY(t,age)
\end{aligned}
$$

$$AdjDispYHH(t,age) = DispYHH(t,age) + ContPvtPenSys(t,age) + DividPvtPenSys(t,age) - BenPvtPenSys(t,age)$$

and total consumption is

$$
\begin{aligned}
&PvtCons(t,age) = \\
&\quad ConsDispWageY(t,age) + ConsDispRntlY(t,age) + ConsDispEntrY(t,age) \\
&\quad\; + ConsDispDvidY(\mathrm{t},age) + ConsDispPensionY(t,age) \\
&\quad\; + ConsSaleInhKRsdntial(t,age) + ConsSaleInhKPvtUnincorpEnt(t,age) \\
&\quad\; + ConsSaleInhKOthFinIns(t,age) + ConsSaleInhKPvtPenSysDC(t,age) \\
&\quad\; + ConsAnnYKRsdntial(t,age) + ConsAnnKYPvtUnincorpEnt(t,age) + ConsAnnYKOthFinIns(t,age) \\
&\quad\; + ConsWthdrlKPvtPenSysDC(t,age)
\end{aligned}
$$

The first two lines on the right-hand side give consumption out of income (including pension income), the second two lines give consumption financed by the sale of inherited assets, the fifth line gives consumption out of annuity income, and the sixth line covers consumption which occurs when a worker changes jobs and elects to withdraw DC pension assets.

Household net saving is the difference between disposable income and consumption:

$$NetSvngHH(t,age) = DispYHH(t,age) - PvtCons(t,age)$$

and adjusted net savings includes savings captured by the private pension system:

$$AdjNetSvngHH(t,age) = AdjDispYHH(t,age) - PvtCons(t,age)$$

or, expressing in terms of unadjusted disposable income and change in pension wealth (see Section 4.1.9),

$$AdjNetSvngHH(t,age) = DispYHH(t,age) + \Delta KPvtPenSys(t,age) + BeqKPvtPenSys(t,age) - PvtCons(t,age)$$

In performing the consistency check in Sectio 9 below, we will use this identitiy in the form

$$NetSvngHH(t,age) = AdjNetSvngHH(t,age) - \Delta KPvtPenSys(t,age) + BeqKPvtPenSys(t,age)$$

5. Income, outlay, and net savings of firms

Firms operate capital, earn profits and pay out direct taxes and dividends.

5.1 Income

Earnings of firms consist of earnings on capital owned by the three institutional claimants $[\bullet\ ] = PvtPenSysDC, PvtPenSysDB, OthFinIns$:

$$ErngsFirmsK[\bullet\ ](t, age) = r(t)\, K[\bullet\ ](t, age)$$

$$ErngsFirmsK[\bullet](age) = \sum_{\text{age}=0}^{\text{MaxAge}} ErngsFirmsK[\bullet](t, age)$$

Recall that depreciation and indirect taxes have already been netted out.

5.2 Outlay

5.2.1 *Direct taxes*

Taxes on profits are described in Section 6.1.

5.2.2 *Dividends*

Dividend distributions are made out of pre-tax earnings, and the proportion of earnings distributed is independent of the claimant by assumption. For the three claimants $[\bullet] = PvtPenSysDC, PvtPenSysDB, OthFinIns$:

$$DividDistErngsFirmsK[\bullet](t, age) = DividDistShare(t)\ ErngsFirmsK\ [\bullet](t, age)$$

$$DividDistErngsFirmsK[\bullet](t) = \sum_{age=0}^{MaxAge} DividDistErngsFirmsK[\bullet](t, age)$$

where the share of earnings distributed as dividends is an exogenous variable.

5.3 Net savings of firms

Net savings (retained earnings) of firms are

$$\begin{aligned} NetSvngErngsFirmsK[\bullet](t, age) &= ErngsFirms\ K[\bullet](t, age) \\ &- DirTaxErngsFirmsK[\bullet](t, age) - DividDistErngsFirmsK[\bullet](t, age) \end{aligned}$$

The sum over claimants is total net savings of firms:

$$NetSvngFirms(t, age) = \sum_{[\bullet]} NetSvngErngsFirmsK[\bullet](t)$$

and the sum over age groups gives total corporate savings:

$$NetSvngFirms(t) = \sum_{\mathrm{age}=0}^{\mathrm{MaxAge}} NetSvngFirms(t, age)$$

6. Income, outlay, and net savings of government

The government sector is rudimentary. Government consumes an exogenous share of GDP, collects taxes and social security contributions and pays social security benefits.

6.1 Income

Government revenues are

$$GovRvn(t) = IndTax(t) + DirTax(t) + ContPubPenSyst(t)$$

where

$$IndTax(t) = IndTaxRate(t)\, GDP(t)$$

$$\begin{aligned} DirTax(t) = {} & \\ & \sum_{age=15}^{MaxAge} DirTaxWageY(t, age) + \sum_{age=15}^{MaxAge} DirTaxEntrY(t, age) + \sum_{age=15}^{MaxAge} DirTaxRntlY(t, age) \\ & + \sum_{age=EligAge}^{MaxAge} DirTaxBenPubPenSys(t, age) \\ & + \sum_{age=0}^{\mathrm{MaxAge}} \sum_{[\bullet]} DirTaxErngsFirmsK[\bullet](t, age) \end{aligned}$$

where $[\bullet] = PvtPenSysDC, PvtPenSysDB, OthFinIns$ and the direct tax streams are

$$DirTaxWageY(t, age) = DirTaxRate(t)\, WageY(t, age)$$

$$DirTaxRntlY(t, age) = DirTaxRate(t)\, RntlY(t, age)$$

$$DirTaxEntrY(t, age) = DirTaxRate(t)\, EntrY(t, age)$$

$$DirTaxBenPubPenSys(t, age) = DirTaxRate(t)\, BenPubPenSys(t, age)$$

$$DirTaxErngsFirmsK[\bullet](t, age) = DirTaxRate(t)\, ErngsFirmsK[\bullet](t)$$

Contributions to the public pension system are described in Section 7.1.1.

6.2 Outlay

Government expenditure is:

$$GovExp(t) = GovCons(t) + BenPubPenSys(t)$$

where government consumption is taken simply as a fixed share of GDP:

$$GovCons(t) = GovConsShare(t)\, GDP(t)$$

and benefits paid out by the public pension system are described in Section 7.1.2.[15]

6.3 Net savings of government

Government net savings are

$$NetSvngGov(t) = GovRvn(t) - GovExp(t)$$

Net savings of government are allocated across age groups using shares drawn from the age-distribution of wealth:

$$NetSvngGov(t, age) \;=\; \frac{KTot(t, age)}{\sum_{\hat{a}ge=0}^{MaxAge} KTot(t, age)}\, NetSvngGov(t)$$

7. Pension system

7.1 Public DB PAYG pension system

7.1.1 Income

Contributions to the public pension system out of wages are

$$ContPubPenSysWageY(t, age) \;=\; \left[PartShareWorkPubPenSys\;\; ContRatePubPenSys(t)\;\; TaxableWageYShare\right]\; WageY(t, age)$$

[15] The most recent version of the model, not yet dosumented, includes public debt dynaics, in cluding the calculation of interest payments which are imputed to households as income.

where *PartShareWorkPubPenSys* is the proportion of the workforce participating in the public pension system and *TaxableWageYShare* is the proportion of the wage bill which is subject to social security taxation. For simplicity, both of these parameters are assumed to be age-independent and time-invariant. The social security contribution rate *ContRatePubPenSys(t)* is assumed to be age-independent. No distinction is made between employees' and employers' contributions. Social security contributions out of entrepreneurial income are calculated similarly:

$$ContPubPenSysEntrY(t, age) = \\ \left[PartShareEntrPubPenSys\ \ ContRatePubPenSys(t)\ \ TaxableShareEntrY\right]\ EntrY(t, age).$$

Contribution rates out of wage and entrepreneurial income are assumed to be the same.

Total social security system revenues out of each income stream are

$$ContPubPenSysWageY(t) = \sum_{age=15}^{MaxAge} ContPubPenSys\,WageY(t, age)$$

$$ContPubPenSysEntrY(t) = \sum_{age=15}^{MaxAge} ContPubPenSysEntrY(t, age)$$

and the system total is:

$$ContPubPenSys(t) = ContPubPenSysWageY(t) + ContPubPenSysEntrY(t)$$

7.1.2 Outlay

The public pension system is assumed to be a DB system financed on a Pay As You Go (PAYG) basis. Let $BenEntPubPenSys(t, age, RtrmntDuration)$ be the social security benefit entitlement for the average person aged age who retired $RtrmntDuration$ years ago, where we assume that $BenEntPubPenSys(t, age, 0) = 0$. The pension for persons entering retirement is computed according to the formula:

$$BenEntPubPenSys(t, age, 1) = \\ \left[PartSharePubPenSys\ \ \ RplcmntRatioPubPenSys(t)\right] \\ \frac{\left(\sum_{j=1}^{age-14} LabForcePartRate(t-j, age-j)\right)}{age-14} \frac{\left(\sum_{k=1}^{RfrncePeriod} WageRate(t-k, age-k)\right)}{RfrncePeriod}$$

$RplcmntRatioPubPenSys(t)$ is a coefficient of proportionality which translates earnings into an initial pension entitlement. This replacement ratio is applied to the average labor force participation rate between age 15 and age (assuming $age \geq EligAge$) times average annual earnings during the $RfrncePeriod$ years prior to retirement.

Once persons have retired, their pension is indexed to the average wage rate. For people who were already retired at *(t-1)*, the pension is

$$BenEntPubPenSys(t, age, RtrmntDuration) =$$
$$BenEntPubPenSys(t-1, age-1, RtrmntDuration-1)\left[1 + IndexRate(t)\,\frac{\overline{Wage}(t) - \overline{Wage}(t-1)}{\overline{Wage}(t-1)}\right]$$

where $IndexRate$ is the rate of indexation of pensions to the average wage rate $\overline{Wage}(t)$. When $IndexRate = 1$ pensions are fully indexed to wages, when $IndexRate = 0$ there is no indexation.

Social security system benefits paid out by age group of recipient are equal to the age- and retirement-duration specific entitlement times the number of recipients:

$$BenPubPenSys(t, age) = \sum_{RtrmntDuration=0}^{MaxAge-EligAge+1} BenEntPubPenSys(t, age, RtrmntDuration)\; Pop(t, age, RtrmntDuration)$$

where, making the simplifying assumption that once retired, persons stay retired,

$$Pop(t, age, RtrmntDuration) =$$
$$Pop(t, age)\begin{bmatrix} LabForcePartRate(t - RtrmntDuration, age - RtrmntDuration) \\ - LabForcePartRate(t - RtrmntDuration + 1, age - RtrmntDuration + 1) \end{bmatrix}$$

for $age = \overline{1, age - RtrmntAge + 1}$. System-wide expenditures are equal to the summation over age groups

$$BenPubPenSys(t) = \sum_{age=EligAge}^{MaxAge} BenPubPenSys(t, age)\; Pop(t, age)$$

7.1.3 System balance

In a classic PAYG system (for example, the German system), total contributions equal total benefits; there is neither accumulation of a return-generating surplus nor a deficit to be financed out of general government revenue. The default model solution option is one in which the required contribution rate is calculated by setting contributions equal to expenditures. However, there are cases (for example, the United States), where nominally PAYG public pension systems are currently running large surpluses in an effort to pre-finance the retirement of the baby boom generation. In other cases (for example, Italy), deficits in the PAYG pension system are financed by transfers from general tax revenue. To cover such cases, an alternative solution option is to set the contribution rate independent of benefits, in which case the model solves for the implied surplus or deficit. In this case, the balance of the social security system is:

$$BalPubPenSys(t) = ContPubPenSys(t) - BenPubPensSys(t)$$

In the event of an unreasonable deficit (or surplus) in the public pension system, assumptions on retirement age, the contribution rate, and benefit calculation must be examined.

7.2 Private DC pension system

7.2.1 Revenue.

Income of the private DB pension system is comprised of (1) current contributions (zero for persons who have retired), and (2) receipt of dividends. (1) is the sum over contributions out of wage and entrepreneurial income, each consisting of the share of the workforce participating times the proportion of total income contributed:

$$ContPvtPenSysDCWageY(t, age) = \\ \left[PartSharePvtPenSaysDC \;\; ContRatePvtPenSysDCWageY(t, age)\right] \; WageY(t, age)$$

$$ContPvtPenSysDCEntrY(t, age) = \\ \left[PartSharePvtPenSaysDC \;\; ContRatePvtPenSysDCEntrY(t, age)\right] \; EntrY(t, age)$$

In the case of the private DC pension system (and the DB pension system as well) there is no term analogous to $TaxableWageYShare$. Total contributions and dividend earnings are

$$ContPvtPenSysDC(t, age) = ContPvtPenSysDCWageY(t, age) + ContPvtPenSysDCEntrY(t, age)$$

and

$$DividPvtPenSysDC(t) = \sum_{age=15}^{MaxAge} DivDistErngsFirmsKPvtPenSysDC(t, age)$$

where the paying-out of dividends was described in Section 5.2.2.

7.2.2. Expenditure

Expenditures of the private DC pension are (1) benefits paid out (zero for persons still in the labor force), (2) payout to heirs of the pension assets of system participants who die, and (3) withdrawal of assets by job-switchers who choose not to roll over their pension wealth into another plan. (1) is analogous to the "annuitization" of non-pension capital assets described in Section 4.2.1 above, with the difference that only those who have left the labor force receive pension benefits:

$$BenPvtPenSysDC(t, age) = \left[1 - LabForcePartRate(t, age)\right] \frac{KPvtPenSysDC(t, age)}{100 - age}$$

where $age \geq EligAge$ and 105 is the maximum age to which a person expects to live. (2) was described above in Section 4.2.2. (3) is calculated using an exogenously assumed withdrawal rate reflecting both the number of job-changes and the proportion who choose not to roll over their assets:

$$WthdrwlPvtPenSysDC(t, age) = WthdrwlRatePvtPenSysDC\ KPvtPenSysDC(t, age)$$

If, for example, 10 percent of system participants change jobs every year and half choose to withdraw their assets, we would have $WthdrwlRatePvtPenSysDC = 0.05$.

7.3 Private DB pension system

7.3.1 Revenue

This is analogous to the public pension system:

$$ContPvtPenSysDBWageY(t, age) = \\ \left[PartSharePvtPenSysDB\ \ ContRatePvtPenSysDB(t, age)\right]\ \ WageY(t, age)$$

$$ContPvtPenSysDBEntrY(t, age) = \\ \left[PartSharePvtPenSysDB\ \ ContRatePvtPenSysDB(t, age)\right]\ \ EntrY(t, age)$$

and total contributions are the sum over the two sources:

$$ContPvtPenSysDB(t, age) = ContPvtPenSysDBWageY(t, age) + ContPvtPenSysDBEntrY(t, age)$$

Age-specific dividends are

$$DividKPvtPenSysDB(t, age) = DividDistErngsFirmsKPvtPenSysDB(t, age)$$

$$DividKPvtPenSysDC(t, age) = DividDistErngsFirmsKPvtPenSysDC(t, age)$$

where the paying-out of dividends by firms is described in Section 5.2.2. The total over age groups is

$$DividPvtPenSysDB(t) = \sum_{age=15}^{MaxAge} DividKPvtPenSysDB(t, age)$$

7.3.2 Expenditure

The average private DB pension entitlement for a newly-retired person is calculated similarly to the average initial public pension entitlement:

$$BenEntPvtPenSysDB(t, age, 1) = \\ \left[PartSharePvtPenSysDB\ RplcmntRatioPvtPenSysDB(t)\ \right] \\ \frac{\left(\sum_{j=1}^{age-14} LabForcePartRate(t-j, age-j)\right)}{age\text{-}14} \frac{\left(\sum_{k=1}^{RfrncePeriod} WageRate(t-k, age-k)\right)}{RfrncePeriod(t)}$$

The reference period is assumed to be the same for both public and private systems. Unlike in the public pension system, the pensions of already-retired persons are not indexed to wages. Benefits are

$$BenPvtPenSysDB(t, age) = \sum_{\mathrm{Re}tDur=0}^{MaxAge-EligAge+1} BenEntPvtPenSysDB(t, age, RtrmntDuration)\ Pop(t, age, RtrmntDuration)$$

We assume that $RtrmntDuration$ is the same for the public PAYG and private DB pension systems. The total benefits paid out are

$$BenPvtPenSysDB(t) = \sum_{age=EligAge}^{MaxAge} BenPvtPenSysDB(t, age)\ Pop(t, age)$$

Bequests of DB pension system assets were described in Section 4.2.2. Recall that withdrawals from the DB pension system associated with job-change were assumed to be zero.

7.4 Total private pension system contributions and benefits

Private pension system totals are

$$ContPvtPenSysWageY(t) = ContPvtPenSysDCWageY(t) + ContPvtPenSysDBWageY(t)$$

$$ContPvtPenSysEntrY(t) = ContPvtPenSysDCEntrY(t) + ContPvtPenSysDBEntrY(t)$$

$$DividKPvtPenSys(t) = DividKPvtPenSysDC(t) + DividKPvtPenSysDB(t)$$

$$BenPvtPenSys(t) = BenPvtPenSysDB(t) + BenPvtPenSysDC(t)$$

8. The life-cycle dynamics of capital accumulation

Corresponding to each of the types of capital *KPvtPenSysDC*, *KPvtPenSysDB*, *KRsdntial*, *KPvtUnincorpEnt* and *KOthFinIns* is an age-specific capital accumulation identity.

8.1 DC private pension system

Change in age-specific private DC pension wealth is

$$\Delta KPvtPenSysDC(t, age) = ContPvtPenSysDC(t, age) + DividKPvtPenSysDC(t, age) -$$
$$- BenPvtPenSysDC(t, age) - BeqKPvtPenSysDC(t, age) - WthdrwlKPvtPenSysDC(t, age)$$

The most important characteristic of the private DC pension system is that there is a fixed relationship between the amount a cohort pays in during its working life and the amount it receives after retirement. For an individual cohort born in year $t = 0$ whose last members die out in year $t = 105$ lifetime pension contributions plus lifetime earnings on pension assets minus lifetime pension benefits received equals bequest of pension wealth. Expressing this differently,

$$\sum_{t=0}^{105}\sum_{age=15}^{105}\Delta KPvtPenSysDC(t,age) =$$

$$\sum_{t=0}^{105}\sum_{age=15}^{105}\begin{bmatrix} ContPvtPenSysDC(t,age) \\ \quad + DividPvtPenSysDC(t,age) \\ \quad - BenPvtPenSysDC(t,age) - BeqKPvtPenSysDC(t,age) \\ \quad - WthdrwlKPvtPenSysDC(t,age) \end{bmatrix} = 0$$

8.2 DB private pension system

Contributions to the private DC pension system purchase an asset, which is owned by the system participant who made the contribution. Contributions into the private DB pension system, by contrast, purchase a claim on a future pension to be paid by the firm, which is in turn bycked by an asset acquired by the firm. The question of whether assets backing a DB pension scheme belong to system participants or to the firm is a complicated one, and legal regimes differ from country to country. For accounting purposes, however, we treat assets of the DB pension system the same way we treat assets of the DC pension scheme.

$$\begin{aligned}&\Delta KPvtPenSysDB(t, age) = \\ &\quad ContPvtPenSysDB(t, age) + DividPvtPenSysDB(t, age) - BenPvtPenSysDB(t, age) \\ &\quad\quad - BeqKPvtPenSysDB(t, age)\end{aligned}$$

For very aged cohorts, unlike in the case of the DC pension scheme, DB pension wealth can be negative. In this case, there is a negative "inheritance" upon death; however, with reasonable parameterization. the number of persons in cohorts characterized by negative DB pension wealth will be small.[16]

8.3 Other assets

For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$, the age-specific accumulation equations are

$$\begin{aligned}&\Delta K[\bullet](t,age) = \\ &\quad K[\bullet]Share(t)\,[NetSvngHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)] \\ &\quad\quad - AnnYK[\bullet](t,age) + K[\cdot]Share(t)\sum_{[\bullet]} AnnYK[\bullet] \\ &\quad\quad - BeqK[\bullet](t,age) + InhK[\bullet](t,age) - SaleInhK[\bullet](t,age) + K[\bullet]Share(t)\sum_{[\bullet]} SaleInhK[\bullet] \\ &\quad\quad + K[\bullet]Share(t)[SaleInhKPvtPenSysDC(t,age) + SaleInhKPvtPenSysDB(t,age)]\end{aligned}$$

The components of change are, in order:

---

[16] Let a hypothetical individual contribute 5 percent of annual wages, set equal to 1, from age 20 to 60. Then upon retirement at age 60, assuming an annual rate of return of 6 percent, the individual will have assets equal to 7.7. Now let the individual receive a benefit equal to 0.5 (i.e., the replacement rate is 50 percent) while continuing to earn 6 percent per year on remaining assets. Than at age 100, there are still assets of 1.8 remaining. At the level of the cohort, proportion in the labor force and proportion of workers participating must be taken into account, but as both contributions and benefits are adjusted equally, the same qualitative insight applies.

- In the first line on the right-hand side of the identity, a share variable $K[\bullet]Share(t)$ summing to unity across the three forms of non-pension wealth is used to apportion unadjusted household net savings plus the imputed age-specific savings of firms and government between $\Delta KRsdntial$, $\Delta KPvtUnincorpEnt$, and $\Delta KOthFinIns$.[17]

- The second line on the right-hand side is of relevance only for elderly households. The first term subtracts dissaving in the form of annuitization of assets, as described in Section 4.2.1. The second term, when combined with the consumption from annuity income which is implicit in net household savings in the first line, has the effect of distributing savings from annuity income between the non-pension asset classes.

- The third line on the right-hand side subtracts net bequests (the first two terms) and, analogously to the second line, distributes that portion of inherited wealth not converted into consumption among asset classes. Consumption financed by the sale of inherited assets is not accounted for here because, like consumption from annuity income, it has already been subtracted off in calculating net household savings in the first line.

- The fourth line on the right-hand side distributes inheritance of pension assets between the non-pension asset classes.[18] Again, associated consumption has already been accounted for when net household savings in the first line is calculated.

9. Macroeconomic identities

9.1 Gross domestic product

Gross domestic product (GDP) is the sum of wages, net profits, indirect taxes, and depreciation:

$$\begin{aligned}
&GDP(t) = \\
&\quad WageY(t) + RntlY(t) + EntrY(t) \\
&\qquad + ErngsFirmsKPvtPenSysDB(t) + ErngsFirmsKPvtPenSysDC(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad + \left[\frac{IndTaxRate(t)\, GDP(t)}{KTot(t)} + DeprRate(t) \right] KTot(t)
\end{aligned}$$

Since

$$\begin{aligned}
&KTot(t) = \\
&\quad KRsdntial(t) + KPvtUnincorpEnt(t) \\
&\qquad + KPvtPenSysDB(t) + KPvtPenSysDC(t) \\
&\qquad + KOthFinIns(t)
\end{aligned}$$

it is clear without further checking that GDP thus expressed will be equal to GDP calculated using the production function in Section 3.1.

---

[17] Allocation shares are not indexed by age for computational simplicity when the model is solved in stochastic mode, however, shares may be age-indexed in non-stochastic model applications.

[18] Note that early withdrawals from the private DC pension system, as well as consumption financed by such withdrawals, are included in net houshold savings in the first line.

9.2 National disposable income

National disposable income is GDP adjusted for depreciation:

$$NatDispY(t) = GDP(t) - DeprRate(t)KTot(t)$$

9.3 Net national savings

Net national savings are equal to national disposable income minus consumption:

$$NetNatSvng(t) = NatDispY(t) - PvtCons(t) - GovCons(t)$$

We show in the next section that net national savings thus calculated are equal to the sum of net savings of households, firms, and government.

10. Accounting consistency checks

We apply two accounting consistency checks, first to confirm that the sum of net savings over age groups equals total capital formation, and second to confirm that net savings calculated by summing across households, firms and government equal net savings calculated by subtracting consumption from GDP.

10.1 Net savings equals capital formation

In order to stress the consistency linkage between the pension system and the macroeconomic framework we show here that total net capital formation is equal to total net savings. First, adding across the accumulation equations given in Section 8.3 for the three non-pension forms of wealth, and remembering that the $K[\bullet]Share(t)$ variables sum to unity across the three non-pension asset classes,

$$\begin{aligned}
&\Delta KRsdntial(t,age) + \Delta KPvtUnincorpEnt(t,age) + \Delta KOthFinIns(t,age) = \\
&\quad [NetSvngHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)] \\
&\qquad - AnnYKRsdntial(t,age) - AnnYKPvtUnincorpEnt(t,age) - AnnYKOthFinIns(t,age) \\
&\qquad + AnnYKRsdntial(t,age) + AnnYKPvtUnincorpEnt(t,age) + AnnYKOthFinIns(t,age) \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) \\
&\qquad - SaleInhKRsdntial(t,age) - SaleInhKPvtUnincorpEnt(t,age) - SaleInhKOthFinIns(t,age) \\
&\qquad + SaleInhKRsdntial(t,age) + SaleInhKPvtUnincorpEnt(t,age) + SaleInhKOthFinIns(t,age) \\
&\qquad + SaleInhKPvtPenSys(t,age)
\end{aligned}$$

Making cancellations and remembering that

$$InhKPvtPenSys(t, age) = SaleKPvtPenSys(t, age)$$

we arrive at

$$
\begin{aligned}
&\Delta KRsdntial(t,age) + \Delta KPvtUnincorpEnt(t,age) + \Delta KOthFinIns(t,age) = \\
&\quad \left[NetSvngHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)\right] \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) \\
&\qquad + InhKPvtPenSys(t,age)
\end{aligned}
$$

Adding pension wealth, change in total wealth is

$$
\begin{aligned}
&\Delta KTot(t,age) = \\
&\quad \Delta KPvtPenSys(t,age) + \left[NetSvngHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)\right] \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) \\
&\qquad + InhKPvtPenSys(t,age,)
\end{aligned}
$$

Based on the definition of adjusted net household savings given above in Section 4.4,

$$
NetSvngHH(t,age) = AdjNetSvngHH(t,age) - \Delta KPvtPenSys(t,age) - BeqKPvtPenSys(t,age)
$$

so

$$
\begin{aligned}
&\Delta KTot(t,age) = \\
&\quad \Delta KPvtPenSys(t,age) \\
&\qquad + \begin{bmatrix} AdjNetSvgHH(t,age) - \Delta KPvtPenSys(t,age) - BeqKPvtPenSys(t,age) \\ \quad + NetSvngFirms(t,age) + NetSvngGovt(t,age) \end{bmatrix} \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) \\
&\qquad + InhKPvtPenSys(t,age)
\end{aligned}
$$

$\Delta KPvtPenSys(age,t)$ cancels, leaving the result

$$
\begin{aligned}
&\Delta KTot(t,age) = \\
&\quad \left[AdjNetSvgHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)\right] \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKPvtPenSys(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) + InhKPvtPenSys(t,age)
\end{aligned}
$$

In other words, change in wealth for members of a given cohort is equal to

- their net saving, adjusted to include include net saving through the private pension system,
- plus their imputed share of the net savings of firms and government,
- plus the sum across all asset classes (pension- and non-pension alike) of inheritance minus bequests.

Summing over age groups, inheritance and bequests cancel out, leaving

$$\Delta KTot(t) = AdjNetSvngHH(t) + NetSvngFirms(t) + NetSvngGovt(t)$$

which is the desired result.

10.2 Net savings calculated "bottom up" equal net savings calculated "top down"

In this section, we wish to confirm that net national savings calculated “top down” as national disposable income minus consumption is equal to net national savings calculated "bottom up" by summing net savings across households, firms, and government.

10.2.1 "Bottom up"

We start by summing across sectors:

10.2.1.1 Households

From Section 4.4,

$$\begin{aligned} &AdjDispYHH(t) = \\ &\quad DispWageY(t) + DispRntlY(t) + DispEntrY(t) + DispDividY(t) + DispPensionY \\ &\quad + ContPvtPenSys(t) - BenPvtPenSys(t) + DividKPvtPenSys(t) - WthdrwlKPvtPenSysDC(t, age) \end{aligned}$$

Expanding this expression and subtracting off consumption, we obtain adjusted net saving:

$$\begin{aligned} &AdjNetSvngHH(t) = \\ &\quad WageY(t) - DirTaxWageY(t) - ContPubPenSysWageY(t) - ContPvtPenSysWageY(t) \\ &\quad + RntlY(t) - DirTaxRntlY(t) \\ &\quad + EntrY(t) - DirTaxEntrY(t) - ContPubPenSysEntrY(t) - ContPvtPenSysEntrY(t) \\ &\quad + DividY(t) \\ &\quad + BenPubPenSys(t) - DirTaxBenPubPenSys(t) + BenPvtPenSys(t, age) + WthdrwlKPvtPenSysDC(t, age) \\ &\quad + ContPvtPenSys(t) + DividKPvtPenSys(t) - BenPvtPenSys(t,age) - WthdrwlKPvtPenSys(t,age) \\ &\quad - PvtCons(t) \end{aligned}$$

Making cancellations and substituting dividends paid out by firms for dividends received by households, the expression is written as

$$
\begin{aligned}
&AdjNetSvngHH(t) = \\
&\quad WageY(t) - DirTaxWageY(t) - ContPubPenSysWageY(t) \\
&\quad + RntlY(t) - DirTaxRntlY(t) \\
&\quad + EntrY(t) - DirTaxEntY(t) - ContPubPenSysEntrY(t) \\
&\quad + DividDistErngsFirmsKOthFinIns(t) \\
&\quad + BenPubPenSys(t) - DirTaxBenPubPenSys(t) \\
&\quad + DividDistErngsFirmsKPvtPenSys(t) \\
&\quad - PvtCons(t)
\end{aligned}
$$

10.2.1.2 Firms

$$NetSvngFirms(t) = NetSvngErngsFirmsKPvtPenSys(t) + NetSvngErngsFirmsKOthFinIns(t)$$

which expands to

$$
\begin{aligned}
&NetSvngFirms(t) = \\
&\quad ErngsFirms\ KPvtPenSys(t) - DirTaxErngsFirmsKPvtPenSys(t) - DividDistErngsFirmsKPvtPenSys(t) \\
&\quad + ErngsFirmsKOthFinIns(t) - DirTaxErngsFirmsKOthFinIns(t) - DividDistErngsFirmsKOthFinIns(t)
\end{aligned}
$$

10.2.1.3 Government

$$NetSvngGov(t) =$$

$$
\begin{aligned}
&IndTax(t) + DirTax(t) + ContPubPenSys(t) \\
&- GovCons(t) - BenPubPenSys(t)
\end{aligned}
$$

which expands to

$$
\begin{aligned}
&NetSvngGov(t) = \\
&\quad IndTax(t) \\
&\quad + DirTaxWageY(age,t) + DirTaxEntrY(age,t) + DirTaxRntlY(age,t) + DirTaxBenPubPenSys(t) \\
&\quad + DirTaxErngsFirmsKPvtPenSys(t) + DirTaxErngsFirmsKOthFinIns(t) \\
&\quad + ContPubPenSysWageY(t) + ContPubPenSysEntrY(t) \\
&\quad - GovCons(t) - BenPubPenSys(t)
\end{aligned}
$$

10.2.1.4 Total

Adding across sectors and making cancellations, net national savings are

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\quad WageY(t) + EntrY(t) + \mathrm{Re}\,ntalY(t) \\
&\qquad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad + IndTax(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

10.2.2. "Top down"

Net national savings are given by the expression

$$NetNatSvng(t) = NatDispY(t) - PvtCons(t) - GovCons(t)$$

First, we express *NatDispY* in terms of *GDP*

$$NatDispY(t) = GDP(t) - DeprRate(t)\ KTot(t)$$

and then expand GDP using the expression from Section 9.1 above:

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\quad WageY(t) + EntrY(t) + \mathrm{Re}\,ntalY(t) \\
&\qquad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad + IndTax(t) + DeprRate(t)\ KTot \\
&\qquad - DeprRate(t)\ KTot(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

Depreciation cancels, leaving

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\quad WageY(t) + EntrY(t) + RntlY(t) \\
&\qquad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad + IndTax(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

This is the same as the expression at the end of Section 10.2.1.

Annex 2: Initialization, solution, and baseline assumptions

The model has been implemented in two-region form in MATLAB. The baseline assumptions described in this Annex correspond to those used in MacKellar et al. (1999) and MacKellar and Reisen (1998) to analyze a two-region world consisting of "Fast Aging Countries" (essentially, industrial countries) and "Slow Aging Countries" (essentially, developing countries). The present paper has presented only results pertaining to the Fast Aging Countries region, and Annex 1 has described the model in single-region form for simplicity and economy of space. In this Annex we describe assumptions for Fast Aging Countries in detail and give key driving assumptions for the Slow Aging Countries region. For more details on the latter, please refer to MacKellar et al. (1999).

Demography and labor markets

Demographic assumptions are taken from the IIASA Central Scenario population projection (Lutz, 1996) and correspond to the “industrial country" region.

Labor force participation rate estimates for both sexes combined are based on the International Labour Office`s *Economically Active Population* series (International Labour Office 1986) and correspond to the "developed region" aggregate. These rise from 38 percent at age 15 to 84 percent at age 24, declining to 75 percent at age 50 and 57 percent at age 59, averaging 73 percent over the age span 15-59. The labor force participation rate at age 60 (the pension eligibility age) was assumed to be 8 percent, declining to zero at age 69 and above. Labor force participation rates are assumed to be time-invariant.

The production function

The ß coefficient in the Cobb-Douglas production function is assumed to be 0.33. The rate of total factor productivity growth is assumed to be 1 percent per year. The model is initialized on 1995 per capita GDP of approximately $25,000. The depreciation rate is assumed to be 4 percent per year.

Social insurance contribution rates

We assume that all workers and entrepreneurs participate in the public pension system, that two-thirds of wages and entrepreneurial income are subject to social security taxation, and that the payroll contribution rate is 15 percent. In combination, these assumptions mean that 10 percent of wages and entrepreneurial income is contributed to the public pension system.

One-quarter of all workers and entrepreneurs is assumed to participate in the private DB pension scheme, contributing 8 percent of pre-tax income, and the same assumptions are made for the DC private pension system. Therefore, 4 percent of wage and entrepreneurial income is contributed to the private pension system, with contributions split equally between the DB and DC components. For comparison, in the U.S. case Scheiber and Shoven (1997, p. 122) assumed that 4.5 percent of total payroll was contributed to pension plans, with the bulk going to DC plans. Our assumption on the number of entrepreneurs participating in the DB pension system is an overestimate, but entrepreneurial income is small relative to wage income, so the overall impact on the model solution is slight. As pointed out in a footnote above, contributions and expenditures related to health are implicitly excluded from the social security system.

Initial pension entitlement calculation, pension indexation, and early pension withdrawal

The public pension system replacement ratio (initial pension relative to average wage during the 3 years prior to retirement) is assumed to be 80 percent. The same parameter for the private DB pension system is assumed to be 25 percent, corresponding to average length of pension plan participation of 25 years and an accrual rate of 1 percent of final salary per year of participation.

Public pensions are assumed to be indexed to wages with a multiplier of 0.5 (i.e., retired persons' rise every year at half the rate as real wages). Private DB pension system benefits are not indexed.

In each year, 10 percent of the workforce is assumed to change jobs and 40 percent of job-changers are assumed to withdraw their assets accumulated in the private DC pension system. 40 percent of these withdrawals are assumed to finance consumption, 40 percent to finance investment in residential capital, 10 percent to finance investment in capital operated by private unincorporated enterprises, and 10 percent to finance investment in capital operated by firms and held by non-pension financial institutions. It is assumed that there are no withdrawals from the DB private pension system.

Assets of the defined contribution pension system are assumed to be heritable on death of the participant, assets of the DB pension system revert to the system on death of the participant.

Consumption / saving rates

We assume for persons of all ages that the average propensities to consume out of disposable wage, entrepreneurial, and dividend income are 90, 70 and 50 percent, respectively. All imputed rental income is assumed to be consumed. The propensity to consume out of all pension income (public, private DC, and private DB) is assumed to be 90 percent, as is the propensity to consume out of annuity income, i.e., the proceeds of decumulating assets accumulated during the working years. Thus, households are assumed to target bequests equal to 10 percent of their wealth upon retirement. Heirs receiving bequests are assumed to convert 30 percent of their inheritance into consumption and to "roll over" the remaining 70 percent into savings.

Taxes and government consumption

The direct tax rate (relative to wages and profits) is assumed to be 14.5 percent. The indirect tax rate (relative to GDP) is set at 6 percent. Government consumption, including interest payments, is assumed to equal 20 percent of GDP.

Dividends

Firms are assumed to pay out 15 percent of pre-tax profits to holders of claims.

Initializing capital stocks and claims

The total initial capital stock is estimated based on the assumed per capita GDP level given above and an assumed capital-output ratio of approximately 3. Total assets of the private pension system are assumed to be approximately $8,000 billion based on data from InterSec Research Company (see Table 1.2 in Reisen and Fischer 1994). Based on shares from Shoven and Scheiber (1997), these assets are divided into approximately $5,000 billion belonging to the DB pension system and $3,000 billion belonging to the DC pension system. Residential capital and capital operated by private unincorporated enterprises are each assumed to account for 33 percent of the total capital stock. This leaves capital operated by corporate firms and held by other financial institutions to be calculated as a residual. All capital assets are distributed across the population by age of claimant using shares for the U.S. inferred from Figure 7.7, "Mean wealth by age of head of household," in Hoynes and McFadden (1997).

Allocation of adjusted net saving across asset classes

The shares of net domestic unadjusted saving allocated to residential investment, investment in capital operated by private unincorporated enterprises, and investment in corporate assets held by non-pension financial institutions are assumed to be 35, 30, and 35 percent, respectively. The share of change in pension wealth invested in the home region (i.e., in Fast Aging Countries) is assumed to decline from 99 percent in 1995 to 90 percent in 2004, after which it remains constant. This share is assumed to apply equally to the DB and DC components of the private pension system. The home share of investment in corporate assets held by non-pension financial institutions is assumed to be 90 percent from 1995 to 2015 and 85 percent thereafter. For detail on the breakdown into portfolio and foreign direct investment, see MacKellar et al. 1999.

Foreign region: driving assumptions

The Slow Aging Country region corresponds to "developing countries" in the IIASA population projections (Lutz) 1996. As in the Fast Aging Country Region, the capital coefficient in the two-factor Cobb-Douglas production function is 0.333. The rate of productivity growth is assumed to be 1 percent per year and the depreciation rate is assumed to be 5 percent per year. The model was initialized on initial per capita income of approximately $1,500 and a capital-output ratio of 2.5. The home share of pension-system investment is assumed to decline from 95 percent in 1995 to 95 percent in 2009, and then to 90 percent in 2014, remaining fixed thereafter. The home share of of investment in corporate assets held by non-pension financial institutions is assumed to decline from 99 percent in 1995 to 90 percent in 2004 and remain steady for the remainder of the solution period.

Quick fixes, ad hoc adjustments, simplifications

Any application of a complicated, evolving model such as this involves some degree of *ad hoc* adjustment. The adjustments and *errata* associated with this exercise are as follows:

1. It was assumed that all inheritance goes to persons aged 15-65. The advantages and disadvantages of this were discussed above.
2. It was assumed that persons over 60 consume all entrerpreneurial income. In hindsight, this "special case" serves no useful purpose, but its impact is minimal.
3. It was assumed that participants in the private DC pension system begin to receive benefits at age 60 regardless of whether they are still in the labor force. Since labor force participation drops off rapidly after 60, the practical impact is insignificant.
4. Contributions to the DB pension system over age 60 were assumed to be zero even for persons still in the labor force. The same observation applies.
5. Depending on the asset being annuitized, terminal age groups used for the calculation of annuities varied between 100 and 110.

In work subsequent to reaching closure on this manuscript, these special cases and *ad hoc* adjustments have all been "zeroed out." None of these changes have affected the model's solution or marginal simulation properties significantly.

References

Auerbach, A., L. Kotlikoff, 1987, *Dynamic Fiscal Policy*. Cambridge: Cambridge University Press.

Blanchet, D. and D. Kessler, 1992, Pension Systems in Transition Economies: Perspectives and Choices Ahead, in P. Pestieau (ed.), *Public Finance in a World of Transition*, Supplement to Public Finances/Finances Publiques, Vol. 47.

Börsch-Supan, A., 1996, The Impact of Population Aging on Savings, Investment and Growth in the OECD Area, in OECD (ed.), *Future Global Capital Shortages: Real Threat or Pure Fiction?,* OECD, Paris.

Cutler, D., J. Poterba, L. Sheiner and L. Summers, 1990, An Aging Society: Opportunity or Challenge?, *Brookings Papers on Economic Activity*, 1990:1.

Dobronogov, A., and L. Mayhew, 2000, Pension Reform in a Highly Informalized Post-Soviet Economy. IIASA *Interim Report* IR-00-41. Available on-line at www.iiasa.ac.at.

Enders, W., 1995, *Applied Econometric Time Series*, John Wiley & Sons, Inc., US.

Ermolieva, T., L. MacKellar, and A. Westlund, 1999, Robustness to stochastic shocks of alternative old-age pension arrangements: macroeconomic stability. IIASA *Interim Report* IR-99-24. Available on-line at www.iiasa.ac.at.

Ermolieva, T., L. MacKellar, and A. Westlund, 2000, Robustness to stochastic shocks of alternative old-age pension arrangements: macroeconomic stability. Forthcoming in Journal of Quantity and Quality.

Hackl, P. and A.H. Westlund, 1991, *Economic Structural Change Analysis and Forecasting*. Berlin: Springer.

Hoynes, H. and D. McFadden, 1997. The impact of demographics on housing and non-housing wealth. In *The Economic Effects of Aging in the United States and Japan*, eds. M. Hurd and Y. Naohiro, pp. 153-194. Chicago: University of Chicago Press.

James, E., 1998, New models for old-age security: experiments, evidence, and unanswered questions, *World Bank Research Observer* 13(2):271-301.

Lee, R.D. and S. Tuljapurkar, 1994, Stochastic Population Forecasts for the U.S.: Beyond High, Medium and Low, *Journal of the American Statistical Association*, Vol. 89, No. 428, pp. 1175-1189.

Lutz, W. (ed.), 1996, The Future Population of the World: What Can We Assume Today? London: Earthscan.

MacKellar, L. and T. Ermolieva, 1999, The IIASA Multiregional Economic-Demographic Model: Algebraic Structure, IIASA *Interim Report* IR-99-007. Available online at www.iiasa.ac.at.

MacKellar, L. and H. Reisen, 1998, A simulated model of global pension investment, *Ageing Working Paper* AWP 1.5. Paris: OECD. Available online at www.oecd.org.

MacKellar, L., T. Ermolieva and H. Reisen, 1999, Globalization, social security, and intergenerational transfer. IIASA Interim Report IR-99-056/October. Available online at www.iiasa.ac.at.

Masson, P. and R. Tryon, 1990, Macroeconomic effects of projected population aging in industrial countries. *IMF Staff Papers* 37(3): 453-85.

OECD, 1998, *Maintaining prosperity in an ageing society.* Paris: OECD.

Reisen, H. and B. Fischer 1994. Pension fund investment: from ageing to emerging markets. *OECD Development Centre Policy Brief*, No. 9. Paris: OECD Development Centre.

Roseveare, D., W. Leibfritz, D. Fore and E. Wurzel, 1996, Ageing Populations, Pension Systems and Government Budgets: Simulations for 20 OECD Countries, Economics Department Working Paper No. 168. Paris: OECD.

Schieber, S. and J. Shoven, 1997, The Consequences of Population Aging on Private Pension Fund Saving and Asset Markets. In M. Hurd and N. Yashiro, eds., The Economic Effects of Aging in the United States and Japan, pp. 111-30. Chicago: University of Chicago Press.

Turner, D., C. Giorno, A. de Serres, A Vourc'h and P. Richardson, 1998, The macroeconomic implications of ageing in a global context. *Ageing Working Paper* AWP 1.5. Paris: OECD. Available online at www.oecd.org.

Warshawsky, M. 1999. An enhanced macroeconomic approach to long-range projections of health care and social security expenditures as a share of GDP. *Journal of Policy Modeling* 21(4):413-26.

Westlund, A, T. Ermolieva, L. MacKellar, 1999, Analysis and Forecasting of Social Security: A study of Robustness, IIASA *Interim Report* IR-99-4. Available online at www.iiasa.ac.at.

Westlund, A., Ermolieva, T., MacKellar, L., 2000, Analyses and Forecasting of Social Security: A Study of Robustness, in: M. Bonilla, T. Casasus, R. Sala (Eds.), *Financial Modelling*, Physica-Verlag Heidelberg, Germany.

World Bank, 1994, Averting the old age crisis, World Bank, Washington, D.C.