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**Interim Report IR-06-044**

# An Agent-Based Model of Endogenous Technological Change – An Extension to the Grubler-Gritsevskyi Model

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December, 2006

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## Abstract

Based on earlier, pioneering work done at IIASA, this paper presents a model of endogenous technological change under the three potential most important “stylized facts”: increasing returns to adoption, uncertainty, and heterogeneous agents following diverse technology development and adoption strategies. As an intermediary step towards the final, long-term research objective of developing a multi-agent model, this paper deals with two heterogeneous agents, a risk-taking one and a risk-aversion one. Interactions between the two agents include trade on resource and good, and technological spillover (“free-riding” and technology trade). With the two heterogeneous agents, we run optimization to minimize their aggregated costs to find out what rational behaviors are under different assumptions if the two agents are somehow cooperative. The global optimal solutions of the two-agent model are of Pareto optimality in the sense that none of the two could be made better off without the other being made worse off. The simulations show how agent heterogeneity — different risk attitudes and sizes, trade between agents and technological spillover effect influence the technological change process. Finally this paper plots and analyzes emission paths as results of different technological change process.

## Acknowledgments

I greatly acknowledge the continuous advice and numerous constructive comments and suggestions from Arnulf Grubler, Yuri Ermoliev, Marek Makowski, and Nebojsa Nakicenovic. The shortcomings remain mine.

# An Agent-Based Model of Endogenous Technological Change: An Extension to the Grubler-Gritsevskyi Model

Tieju Ma

## 1 Introduction

It has been widely recognized that the development and diffusion of new technologies is the most important source of long-run productivity and economic growth (see e.g. Metcalfe 1987; Freeman 1994). But new technologies do not fall like "manna from heaven". Technological change is costly. New technologies commonly need high investment on R&D and demonstration projects at their early stages, and with the increase of accumulated experience in new technologies, costs of using them tend to decrease. This is what we called technological learning. Historical evidences of technological learning in energy systems include reductions of investment for photovoltaic cells, gas turbines and windmills with the increase of their cumulative installed capacities (see Grubler *et al*. 1996; Nakicenovic and Rogner 1996; Nakicenovic *et al*. 1998; Watanabe 1995 and 1997). With decreased costs, new technologies can see further more adoptions of them, thus technological learning is a classical example of increasing returns (see Arthur 1983 and 1989). Technological change or technological learning is highly uncertain, which is evidenced by investment cost distributions for biomass, nuclear, and solar electricity generation from numerous engineering studies (see IIASA-WEC 1995). The importance of technological uncertainty has been recognized and explored ever since the earliest days of global environmental modeling (e.g., see Nordhaus 1973; Starr and Rudman 1973).

In most of traditional models, technological change has to date largely been treated as exogenous, i.e. technological change, typically in form of improvements in engineering and economic characteristics of individual or aggregate technologies, is a free good and also known with perfect foresight within a given scenario of technological "expectations". This is both the case for models developed within the tradition of growth theory and associated production function models (so-called "top-down" models), as well as those developed within a systems engineering perspective (e.g., detailed sectorial "bottom-up" optimization models). In both modeling traditions, technological change is either reduced to an aggregate exogenous trend parameter (the "residual" of the growth accounts), or introduced in form of numerous (exogenous) assumptions on costs and performance of future technologies. Common to both modeling traditions is that the only endogenous mechanism of technological change is that of progressive resource depletion and resulting cost increases, which also explains that the inevitable outcome of imposing additional (e.g. environmental) constraints on the model: rising costs due to the forced adoption of more costly capital vintages that remain unaffected by endogenous policy variables in the model. Such constraints which are at odds with historical experience (see Barnett and Morse, 1967) trigger both substitutions of factor inputs as well as the penetration of otherwise uneconomic technologies. These are either represented generically as aggregates in form of so-called “backstops” (see Nordhaus 1973), or through detailed assumptions on numerous technologies individually.

Traditional deterministic, social planner models have been criticized (e.g. Grubler and Messner 1998) for being overly naive and "optimistic" on the feasibility of meeting constraints, as availability and adoption of new technologies will be much slower and discontinuous due to agent heterogeneity and uncertainty than suggested in traditional policy models. However, traditional models can also be technologically too “pessimistic”, as missing out on important spillover effects and adaptive, innovative behavior that arises precisely because of agent heterogeneity and interaction.

Modeling the endogenous uncertain technological change has got increasing concerns in recent years (see Grubler, Nakicenovic and Nordhaus 2002). This paper firstly introduces Grubler and Gritsevskyi’s deliberately highly stylized model of endogenous technological change through uncertain returns on learning (see Grubler and Gritsevskyi, in press), then extends the model by considering explicit agent heterogeneity. Following the tradition of agent-based modeling (see Ma and Nakamori 2005) – studying macro-level complexities from the interactions in micro-level, which is combined here with the modeling field of optimization under uncertainty, agent heterogeneity is represented by their different risk attitudes and weights. The interaction between agents is represented via trade on resource and goods, as well as through technological spillover. With two heterogeneous agents, we run optimization to minimize their aggregated costs to find out what rational behaviors are under different assumptions if the two agents are somehow cooperative. The global optimal solutions of the two-agent model are of Pareto optimality in the sense that none of the two could be made better off without the other being made worse off.

Technological change has the potential impact on human society, and hence some social issues maybe act as drivers or resistance of technological change. This paper addresses environmental issues as possible drivers of technological change.

Mathematically, the resulting problems are non-convex stochastic optimization problems. Matlab’s Optimization Toolbox (version 3.0) was used to solve the optimization problems, which applies a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) subproblem at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula. A line search is performed using a merit function. More details of the method can be found in the user’s guide of Mathworks (See Mathworks 2004). Global optimality of solutions was checked by employing different starting points.

The model presented here is not intended to be by any means a “realistic” model in the sense of technological or sectorial detail. Rather, the main objective of the model is for exploratory modeling purposes and as a heuristic research device to examine in depth the impacts of alternative model formulations on the endogenous technology transition dynamics.

The rest of this paper is organized as follows. Section 2 introduces a stylized model of endogenous technological change through uncertain return on R&D investment with one decision agent and gives analysis on simulation results; section 3 extends the model by considering two heterogeneous agents and analyzes various simulation results with trade and technology spillover between the two agents; section 4 plots and analyzes emission paths which are the results of different technological change process; and section 5 gives concluding remarks.

# 2 Stylized Model of Endogenous Technological Change with One Decision Agent

This section introduces a stylized model of endogenous technology change through uncertain learning with one decision agent.

## 2.1 The Grubler-Gritsevskyi model

Our optimization model of technology choice is based on Grubler and Gritsevskyi's earlier work (see A. Grubler and A. Gritsevskyi, in press), and it is conceptually simple. We suppose one primary resource, whose extraction costs increase over time as a function of resource depletion. The economic system demands one homogeneous good and the exogenous demand increases over time. There are three kinds of technology, namely "Existing", "Incremental", and "Revolutionary", which can be used to produce the good. The "Existing" and "Incremental" technologies need consuming primary resource for producing the good, while the "Revolutionary" hardly need no resource input.

- The "Existing" technology is assumed to be entirely mature, its cost and efficiency do not change over time, and the emission of using it is a little bit high.
- The "Incremental" technology has a slight efficiency advantage. With a higher initial cost than that of the "Existing" technology (by a factor 2 higher than the "Existing" technology), it has potential for technological learning (we assume a mean learning rate of 10%), and its emission is lower than that of the "Existing one".
- The "Revolutionary" technology's initial cost is much higher than that of the "Incremental" one (by a factor 40 higher than the "Existing" technology), but its learning potential is also higher (we assume a mean rate of 30%). It has little emission.

Technological learning is uncertain. We represent an uncertain learning rate through an uncertainty range around the mean value adopted based on a lognormal distribution which accords with empirical data (see Messner and Strubegger 1991). The uncertainty was introduced into the model as an additional cost in the objective function. The stochastic model responds to a frequent criticism of traditional optimization models: the inappropriate assumption of a decision-making agent that operates under perfect foresight. Through endogenization of uncertainty, decision making in the model no longer operates under perfect foresight.

We address environmental issues as possible drivers of technological change. The existence, timing, and extent of possible future environmental constraints, e.g. in form of carbon taxes, are highly uncertain. Carbon taxes are introduced in the following way.

We assume that the establishment of the tax is uncertain with a given occurrence probability of 0.33. The introduction time (in case the tax would be established) is also unknown with an expected cumulative distribution function that goes from 0 in the first decision time to 99% in the final decision time.

With the homogeneous good, three different technologies, and uncertain carbon tax, we run optimization to minimize the total discounted cost of the economic system, thus the results denote optimized paths of technology development and diffusion.

Here we give the mathematic expression of the model. The demand is exogenous and it increases over time as shown in Eq. (1).

$$D^t = 100(1+\alpha)^t \tag{1}$$

where $t$ denotes time period (year), $D^t$ denotes the demand in $t$, and $\alpha$ is the annual increasing rate of demand.

Let $x_i^t$ $(i=1,2,3)$ denotes the annual production of technology $i$ at time $t$, and let $\eta_i$ denotes technology $i$'s efficiency, then the annual extraction $R^t$ is the sum of resources consumed by each technology, as shown in Eq. (2)

$$R^t = \sum_{i=1}^{3} \frac{1}{\eta_i} x_i^t. \tag{2}$$

Thus the cumulative extraction by time $t$ is:

$$\overline{R}^t = \sum_{j=1}^{t} R^j, \tag{3}$$

The extraction cost of the resource increases over time as a linear function of resource depletion, as shown in Eq. (4)

$$c_E^t = c_E^0 + k_E \overline{R}^t \tag{4}$$

where $c_E^t$ denotes the extraction cost per resource unit at time $t$, $c_E^0$ is the initial extraction cost, $\overline{R}^t$ is the total extraction by decision time $t$, and $k_E$ is a constant coefficient.

Let $y_i^t$ $(i=1,2,3)$ denotes the annual new installation of technology $i$ at time $t$, then the total installed capacity of technology $i$ at time $t$, denoted by $C_i^t$ $(i=1,2,3)$ can be calculated according to Eq. (5).

$$C_i^t = \sum_{j=t-\tau_i}^{t} y_i^j, \tag{5}$$

where $\tau_i$ denotes the plant life of technology $i$.

The cumulative installed capacity $\overline{C}_i^t$ of technology $i$ by time $t$ is calculated as:

$$\overline{C}_i^t = \sum_{j=-\infty}^{t} C_i^j = \sum_{j=1}^{t} C_i^j + \overline{C}_i^0. \tag{6}$$

Technology learning is based on experience which is quantified by the cumulative installed capacity, thus future investment cost is a function of cumulative installed capacity, as shown in Eq. (7)

$$c_{Fi}^{\,t} = c_{Fi}^{\,0} \times (\overline{C}_i^t)^{-b_i}, \tag{7}$$

where $2^{-b_i}$ is the progress ratio ($1-2^{-b_i}$ is the learning rate) of technology *i*, and $c_{Fi}^0$ is the initial cost of technology *i*.

The following intertemporal optimization will be used to minimize the total cost.

$$\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{3}\sum_{t=1}^{T}(1-\delta)^t c_{Fi}^{\,t} y_i^t + \sum_{t=1}^{T}(1-\delta)^t c_E^t R^t + \sum_{i=1}^{3}\sum_{t=1}^{T}(1-\delta)^t c_{OMi} x_i^t + \\
& \rho\left\{\mathrm{E}\left\{\sum_{t=1}^{T}\max\left\{0,\sum_{i=2}^{3}\left\{[c_{Fi}^{\,t}(\psi)-c_{Fi}^{\,t}]y_i^t\right\}\right\}\right\}\right\} + \\
& p^{tax}\left\{p^{t_0}\left\{\sum_{t=t_0}^{T}\sum_{i=1}^{3} c_C \frac{\lambda_i}{\eta_i} x_i^t + \rho\left\{\mathrm{E}\left\{\sum_{t=t_0}^{T}\max\left\{0,\sum_{i=2}^{3}\left\{[c_C(\omega)-c_C]\frac{\lambda_i}{\eta_i} x_i^t\right\}\right\}\right\}\right\}\right\}\right\}
\end{aligned} \tag{8}$$

Subject to

$$\begin{cases}
D^t \le \sum_{i=1}^{3} x_i^t & (t=1,\cdots T) & & (9)\\
x_i^t \le C_i^t & (t=1,\cdots T) & (i=1,\cdots 3) & (10)\\
x_i^t \ge 0 & (t=1,\cdots T) & (i=1,\cdots 3) & (11)\\
y_i^t \ge 0 & (t=1,\cdots T) & (i=1,\cdots 3) & (12)
\end{cases}$$

where

$T$ denotes the scale of the problem,

$\delta$ denotes the discount rate,

$c_{OMi}$ denotes the operating and maintenance (O+M) cost of technology $i$ ,

$\rho$ denotes decision maker's risk attitude (a small $\rho$ denotes a risk-taking attitude, and a big $\rho$ denotes a risk-aversion attitude),

$c_{Fi}^{\,t}(\psi)$ is a random variable with $\psi$ denoting an element from a probability space that is characterized by a lognormal distribution, and $c_{Fi}^{\,t}$ is the mean of the distribution,

$\mathrm{E}$ denotes expectation,

$p^{tax}$ is the probability that the tax will be established at all,

$p^{t_0}$ is the probability that, if established, the tax will be introduced before time $t_0$,

$c_C$ is the mean of uncertain carbon tax value,

$\lambda_i$ denotes the carbon emission of producing and consuming every unit good by technology $i$, and

$c_C(\omega)$ is a random variable with $\omega$ denoting an element from a probability space that is characterized by a Weibull distribution.

The objective function is composed of three parts. The first part is the cost with deterministic (or mean) learning rates; the second part is the expected cost resulted from overestimating larning rates; and the third part is the expected cost of paying carbon tax. The constraint function Eq. (9) denotes that total annual production of all three technologies must satisfy given demand; the constraint function Eq. (10) denotes that annual production for each technology does not exceed its total installed capacity; The constraint functions Eq. (11) and Eq. (12) denotes that decision variables can not be negative.

We assume the scale of the problem is 100-year (e.g. from 1990 to 2090) with 10-year decision inteval. The model is solved for a sufficiently large sample *N*, where the size of *N* has been determined through successive experiments. Several successive model runs with the same sample size *N* are compared. If no major changes in the solution structure and the objective function can be observed then *N* is considered sufficient large (for more detail, see Messner *et al*. 1996).

Table 1 summarizes all the initial values of parameters in the above optimization model. In the next subsection, we will introduce simulations with those initial values and sensitivity analysis of parameters.

## 2.2 Simulations and Sensitivity Analysis

### *2.2.1 Three-stage Simulations*

For showing how uncertainty in learning and the uncertain carbon tax affect technological change processes, we carried out simulations in three stages. In the first stage, simulations were carried out with deterministic learning, and without considering the carbon tax, i.e., the second and third part of the objective function (Eq. (8)) did not appear; in the second stage, uncertainty in learning was considered, but no carbon tax, i.e., the third part of Eq. (8) did not appear; and in the third stage, both uncertainty in learning and the uncertain carbon tax were considered. In each stage, we assume a basic case with those initial values in Table 1. The three basic cases for the three stages are called BC1, BC2 and BC3, respectively. Fig. 1 shows results of the three basic case simulations, from which we can see that the uncertainty in learning rate is a factor which will postpone the R&D investment on the “Revolutionary” technology, while the uncertain carbon tax will encourage earlier investment on the “Revolutionary” technology.

Table 1. Initial values of parameters.

| Parameters related to the three technologies | | | |
|---|---|---|---|
| | Existing Tech. | Incremental Tech. | Revolutionary Tech. |
| Initial cost (US$/kW(e)) | $c_{F1}^{0}=1000$ | $c_{F2}^{0}=2000$ | $c_{F3}^{0}=40000$ |
| Efficiency | $\eta_1=30\%$ | $\eta_2=40\%$ | $\eta_3=90\%$ |
| Plant life (year) | $\tau_1=30$ | $\tau_2=30$ | $\tau_3=30$ |
| Initial Total Installed Capacity (kW) | $C_1^0=100$ | $C_2^0=0$ | $C_3^0=0$ |
| Initial Cumulative Installed Capacity (kW) | $\bar{C}_1^0=1000$ | $\bar{C}_2^0=1$ | $\bar{C}_3^0=1$ |
| O+M cost (US$/kW(e)) | $c_{OM1}=30$ | $c_{OM2}=50$ | $c_{OM3}=50$ |
| Carbon emission coefficient | $\lambda_1=0.8$ | $\lambda_2=0.8$ | $\lambda_3=0.1$ |
| Mean Learning Rate of lognormal distributions[1] | $b_1=0$ $(1-2^{-b_1}=0)$ | $b_2=0.1520$ $(1-2^{-b_2}=10\%)$ | $b_3=0.5146$ $(1-2^{-b_3}=30\%)$ |

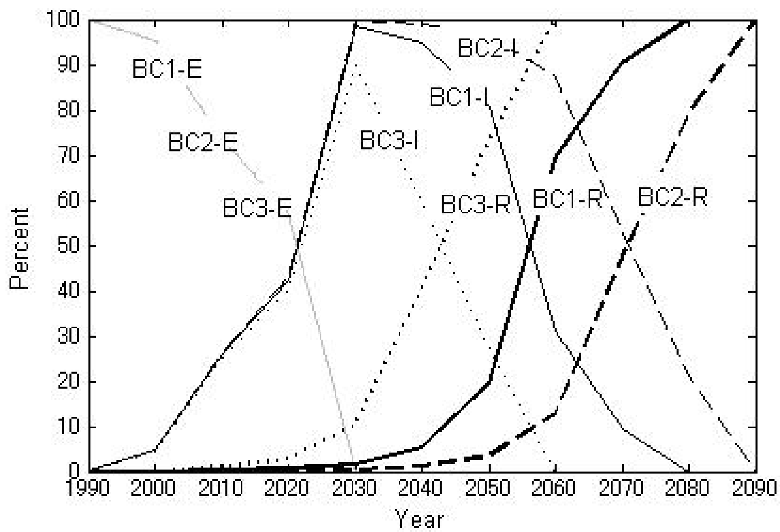
| Other Parameters | | | |
|---|---|---|---|
| Probability of carbon tax | $p^{tax}=0.33$ | Mean carbon tax of a Weibull distribution[2] (US$/t) | $c_c=75$ |
| Demand in the base year (kW(e)) | $D^0=100$ | Annual Increasing rate of demand | $\alpha=2.6\%$ |
| Initial extraction cost (US$/kW(e)) | $c_E^0=200$ | Extraction cost coefficient | $K_E^0=0.01$ |
| Scale of the problem | | $T=100$, decision interval is 10 years | |
| Discount rate | $\delta=5\%$ | Risk factor | $\rho=1$ |

[1] The lognormal PDF (probability distribution function) is $y=f(x\mid\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{-(\ln x-\mu)^2}{2\sigma^2}}$. For the learning rate of the "Incremental" technology, we set $\mu=\ln 0.1$ and $\sigma^2=0.1$; and for that of the "Revolutionary" one, we set $\mu=\ln 0.3$ and $\sigma^2=0.1$.

[2] The mathematic formulation of the Weibull distribution $y=f(x|a,b)=ba^{-b}x^{b-1}e^{-\left(\frac{x}{a}\right)^b}$ $(x\geq 0)$, where $a$ is called the scale parameter and $b$ is called the shape parameter. For the uncertain carbon tax, we set $a=75$ and $b=1$.

#### 2.2.2 Sensitivity Analysis

For studying in detail the behavior of the model, we did sensitivity analysis at both the second and the third stage. The sensitivity analysis at the second stage is for exploring model behaviors with different values of those parameters related to initial cost, uncertain learning, demand, discounted rate and extraction; and that at the third stage is for studying how different carbon tax policy influences the technological change process. In the following we first introduce the sensitivity analysis without considering carbon tax, then we briefly introduce the sensitivity analysis on the uncertain carbon tax.




*R--Revolutionary, I – Incremental, E-Existing*

Figure 1. Results of basic case simulations at three stages.

Fig. 2 shows the break-even time of the "Revolutionary" technology (i.e. when its share begins to be over 50%) with different combination of learning rate and initial cost. We can see that the break-even time of the "Revolutionary" technology was postponed with the increase of its initial cost and with the decrease of its learning rate. Due to the stochastic feature included in the model, Fig. 2 is non-smooth and non-convex. Generally, the break-even time is more sensitive to the learning rate than initial cost. But when its learning rate is small (< 20%), a small change in initial cost also resulted in great change for the break-even time of the "Revolutionary" technology.

$k_E$ in Eq. (4) denotes how extraction cost is sensitive to the total extraction. It indirectly indicates how abundant the resource is. If the resource is very abundant, then the extraction cost will not be affected very much by the total extraction, i.e., the $k_E$ is very small, with the extreme as 0. Our sensitivity analysis on $k_E$ accords with our intuition

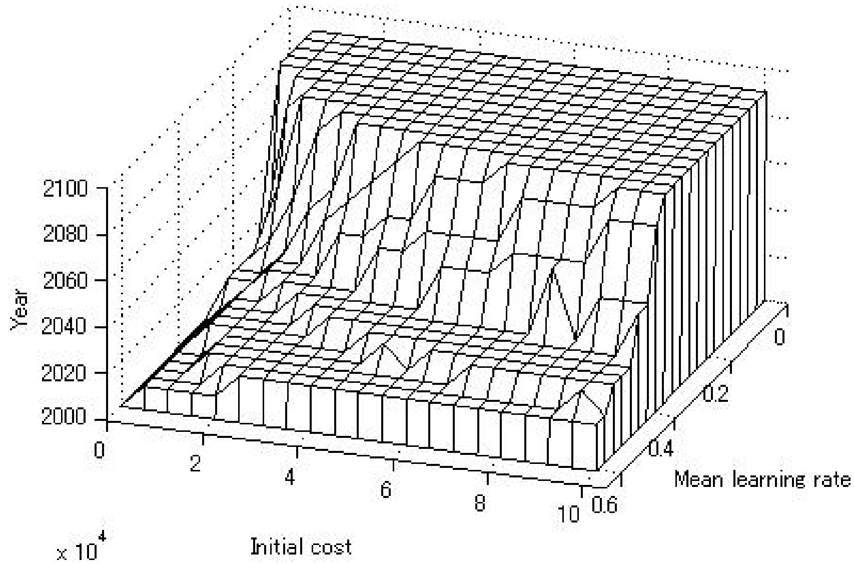
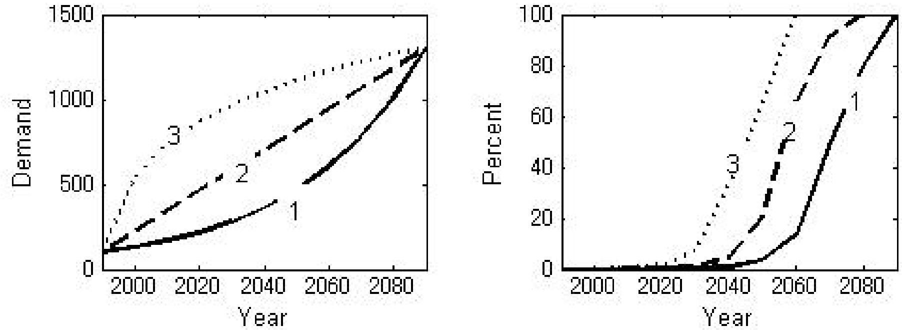



Figure 2. Break-even time of the “Revolutionary” technology with different learning rate and initial cost.

that the rarer the resource is, the earlier the “revolutionary” technology will be widely applied. With the increase of $k_E$ from 0.005 to 0.1, the break-even time of the “Revolutionary” technology was brought forward by 2 decades.

We assumed different demand scenarios by varying $\alpha$ in Eq. (1) from 1 to 2. We found high demand could bring forward the break-even time for three decades. We also assumed different demand functions. As shown in the left side of Fig. 3, we assumed “*log*”, “*linear*” and “*exponential*” demand functions. With the log demand function, the demand increases very fast at the early stage, and then becomes slower and slower, and we call it a “decelerated increase demand path”; with the linear one, the demand increases with a constant rate, and we say it is a “constant increase demand path”; and with an exponential one, demand increase faster and faster, and we call it an “accelerated increase demand path”. As shown in the right side of Fig. 3, with the same destination, different “demand increasing path” will impact the development of advanced technology. An early fast increase in demand (in log formulation) will favor the early break-even of the “Revolutionary” technology.




*1—Exponential demand function; 2—Linear demand function; 3—Log demand function*

Figure 3. Different demand functions and their corresponding diffusion paths of the "Revolutionary" technology.

We varied discount rate from 0 to 10%. Ceteris paribus, higher discount rates result in postponed break-even time of the "Revolutionary" technology. This result was to be expected considering the decisive influence of the discount rate on the objective function. Higher discount rate means much more weight is put on current capital, thus invest early is not an economic strategy.

We varied $\rho$ in Eq. (8) to see how different risk attitude affect the decision on R&D investment, we found there was a tendency that the break-even time of the "Revolutionary" technology postponed with the increasing emphasis on risk aversion. We also varied the dispersion of the lognormal distribution of the "Revolutionary" technology's and found that large dispersion, i.e., large uncertainty, can postpone the break-even time of the "Revolutionary" technology.

By simulations with different assumptions on mean value and dispersions, we studied how the uncertain carbon tax affects technological change process. We found that high mean value of the carbon tax and high uncertainty will favorite early R&D investment on the "revolutionary" technology.

## 3 Modeling with Two Heterogeneous Agents

This section extends the model introduced in Section 2 by assuming that there are two heterogeneous decision agents, agent 1 and agent 2, operating simultaneously in the technological change process. Focusing on agents' different risk attitudes to the potential learning of the "Incremental" and "Revolutionary" technology, now we assume a deterministic carbon tax for the two agents – the carbon tax will be applied from 2060 with 50$/t for carbon emission. The agents' heterogeneities considered here are agents' different risk attitudes and weights. We use $\rho_1$ and $\rho_2$ to denote the risk factors for agent 1 and agent 2, respectively. We assume agent 1 is a risk-taking one and $\rho_1 = 0.1$, and agent 2 is a risk-aversion one and $\rho_2 = 1$. With smaller risk factor, agent 1 is a pioneer to develop and adopt new technology, while agent 2 is a follower. Agents' weights denote their sizes or their share in the total system. The weight for the agent 1 is $w_1 \in (0,1)$, and the weight for agent 2 is $w_2 \in (0,1)$. The two weights satisfy the formulation: $w_1 + w_2 = 1$.

The interaction between the two agents includes trade on resource and good and technology spillover. Trade on resource and good means that one agent can buy resource and good from the other. In terms of minimize the aggregated costs of the two agents, the model does not treat the price of resource and good, instead it includes the cost of the trade. This cost can be viewed as cost for transportation, distributions and any other additional cost caused by moving and using resource and good from the other agent. We assume $\theta_1$ and $\theta_2$ are the unit costs for trade of resource and good, respectively. The quantity of trade flow at each time step is treated as decision variables.

We distinguish two kinds of technology spillover effects: technological "free-riding" and technology trade. Technological free-riding means that one agent can benefit from the other's learning effect without cost, but most of time with some delay. There are no additional decision variables for free-riding. Technology trade means that one agent can benefit from the other's experience (quantified by cumulative installed capacity) with some cost, and we assume $\theta_3$ as the unit cost of buying experience. Technology trade is different from resource trade and good trade in sense that the bargainer agent does not lose the experience, unlike in the case of resource and good trade. It just shares the experience with the purchaser agent. Again, here we do not consider the price of technology. And we let the quantity of technology trade at each time step be decision variables.

The objective function of the optimization can be simply denoted as

$$\begin{aligned} \min \quad & A^1 + A^2 \\ & + \sum_{t=1}^{T} \left[ (1-\delta)^t \left( \theta_1 \left| r^t \right| + \theta_2 \left| g^t \right| \right) \right], \\ & + \sum_{t=1}^{T} \left[ (1-\delta)^t \left( \theta_3 \left| s^t \right| \right) \right] \end{aligned} \tag{13}$$

where

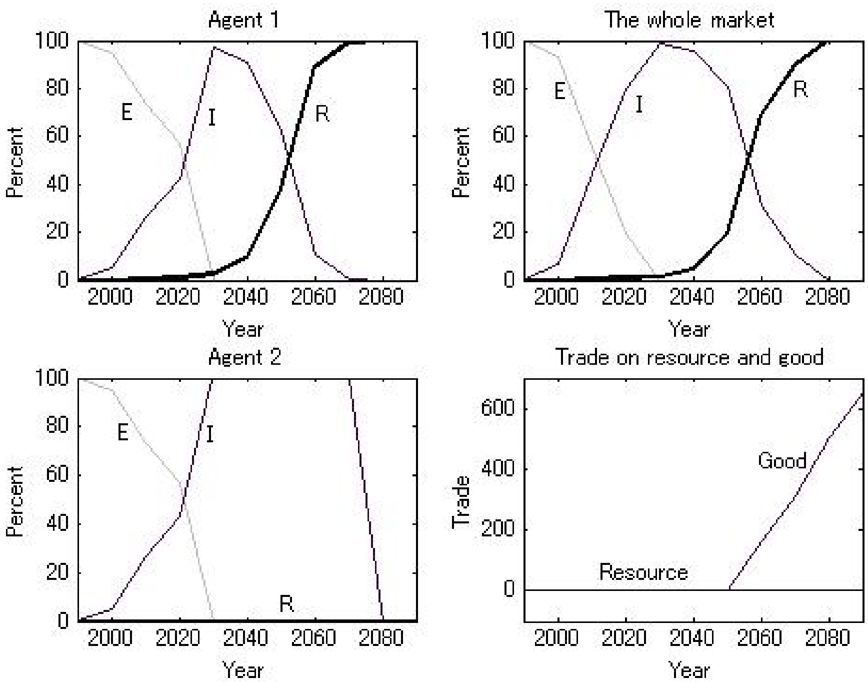
> $A^1$ and $A^2$ denotes agent 1's and agent 2's costs, respectively, introduced in Eq. (8), but with a deterministic carbon tax,
>
> $T$ denotes the scale of the problem,
>
> $\delta$ denotes the discount rate,
>
> $\theta_1$, $\theta_2$ and $\theta_3$ denote the unit costs of trade on resource, good and technology, respectively,
>
> $r^t$, $g^t$ and $s^t$ denotes trade quantity of resource, good and technology at time $t$, respectively.

In Eq. (13), the first part includes all cost mentioned in Section 2, but with a deterministic carbon tax, for both agent 1 and agent 2; the second part is the cost of trade on resource and good; and the third part is the cost of technology trade. The two agents' weights do not appear in the objective function, instead they appear in constrains related to demand. Suppose $D^t$ is the demand in whole market at time step *t*, then agent 1's demand at time step *t* is $D_1^t = w_1 D^t$, and agent 2's demand at time step *t* is $D_2^t = w_2 D^t = (1 - w_1) D^t$. The $r^t$, $g^t$ and $s^t$ can be negative, depending on the direction of the trade, and we assume the flow from agent 1 to agent 2 is positive.

Obviously, we can generate infinite future scenarios and stories with different combinations of those parameters. And also with some specification value, the model can be used for some practical analysis. But before that, we would show the behaviors of the model, and which is the main purpose of this paper.

### 3.1 Optimization Without Technology Spillover

Firstly we run a simulation called BC4 with $w_1 = 0.5$, $w_2 = 0.5$, $\rho_1 = 0.1$, $\rho_2 = 1$, $\theta_1 = \theta_2 = 140$ and without technology spillover effect. Fig. 4 shows the result of BC4, we can see that agent 2 develops no "Revolutionary" technology, and it imports good from agent 1 from 2050. We varied the trade costs of resource and good to see how it would influence the two agents' decision, and we found:




*R--Revolutionary, I – Incremental, E-Existing*

Figure 4. Simulation result of BC4.

- When the trade cost is small ($\theta_1 = \theta_2 < 80$), agent 2 develops neither the "Incremental" technology, nor the "Revolutionary" one. It exports its resource to agent 1 and imports good from agent 1.
- With the increasing of the trade costs, there is a general tendency that the trade appears later and later and the quantity of trade becomes smaller and smaller, which means both agents operate more and more locally, and this results in delay of the development of the "Revolutionary" technology. For example,

when the trade costs of resource and good increase from 40 to 200, the break-even of the "Revolutionary" technology in agent 1's market is delayed for one decade, and it is delayed for 2 decades in the whole market. When the trade costs are high enough, for example $\theta_1 = \theta_2 = 300$, there is neither trade on resource nor that on good. Both agent 1 and agent 2 operate on their local market and based on their local resource. And both agents develop the "Incremental" technology without developing the "Revolutionary" one during the 100 years.

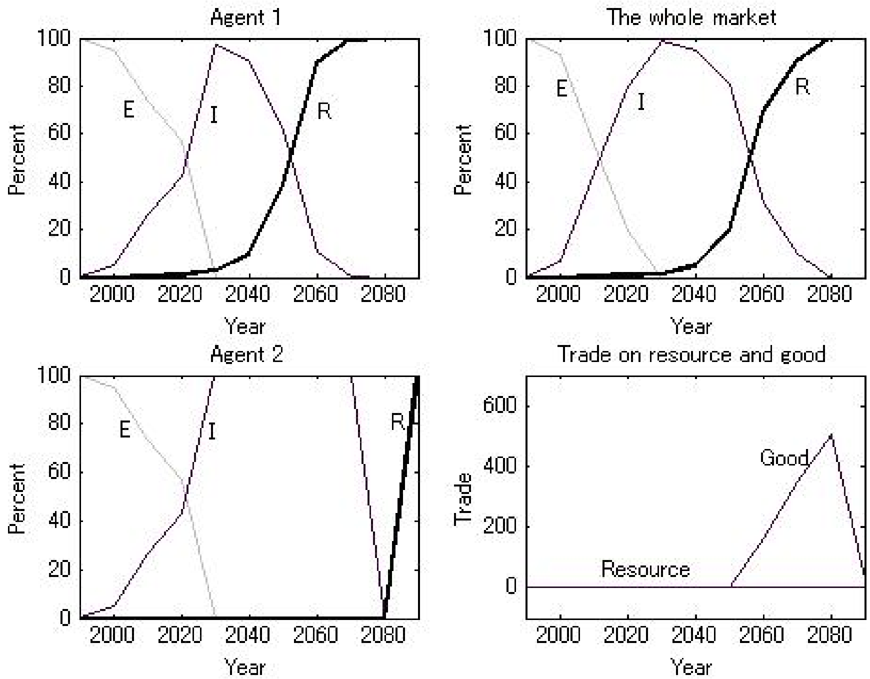
From the above simulations, we can learn that the interaction between agents really influences the technological change process, both in global and in local level. Globalization maybe acts as a driving force for the development of advanced technologies because development of advanced technologies commonly needs huge investment which probably requires a very large potential market to reimburse it.

### 3.2 Optimization with Technological "Free-Riding"

Now we consider the situation that there is free-riding between agents. That is to say, although agent 2 does not have R&D investment on the "Revolutionary" technology, it can benefit from agent 1's learning effect. We assume that agent 2's future investment cost on the "Revolutionary" technology relies on agent 1's cumulative installed capacity, but with one-decade delay.

With the parameter values set in BC4, we found the "free-riding" made agent 2 develop the "Revolutionary" technology from 2080, and the diffusion time of it was very short. As shown in Fig. 5, agent 2 starts to import good from agent 1 from 2050, then after making a successful "free-riding" from 2080, it begins to produce good for itself and decreases the import from agent 1. We found with low trade costs, i.e., $\theta_1 = \theta_2 = 40$, the "free-riding" did not show its effect at all, because with low trade cost, it is more economic for the whole system if agent 2 exports resource to agent 1 and imports good from agent 1. In the rest of this paper, we call the simulation with the parameter values set in BC4 and plus "free-riding" the BC5.

Based on BC5, we varied the two agents' weights to see how different weight influences agents' decision behaviors. We found when agent 1's weight is small, i.e., $w_1 < 0.2$, agent 1 will jump to the "Revolutionary" technology, without developing the "Incremental" one. This is because small weight (thus a small local market) will make it reluctant to develop new technologies, but the global market will encourage it to develop new technologies, and agent 2's "free-riding" on the "Revolutionary" technology will encourage agent 1 to develop the "Revolutionary" technology since it can reduce the total system's cost. Fig. 6 shows the trade on good with different size of the two agents, from which we can see that with the decrease of agent 1's weight (or the increase of agent 2's weight) agent 2 imports more good from agent 1 during the period from 2040 to 2090. With a small $w_1$, agent 2 exports some good to agent 1 during the period from 2020 to 2040 because during that period agent 1 is doing R&D on the "Revolutionary" technology, while agent 2 builds a bigger capacity of the "Incremental" technology. With a big $w_1$, i.e., $w_1 > 0.5$, from 2080 to 2090, the import from agent 1 to agent 2 decreases, this is because agent 2's local market is small and its production can satisfy its own market after making "free-riding".




*R--Revolutionary, I – Incremental, E-Existing*

Figure 5. Result of BC4+free-riding (or BC5).

### 3.3 Optimization with Technology Trade Instead of "Free-Riding"

In the above, "free-riding" means one agent can benefit from the other's learning effect without any cost, but with some delay (eg. one decade). In terms of technology trade, we allow an agent to decide whether it need buy technology, or more precisely the experience in a new technology, from the other and when to buy. Technology trade is different from resource trade and good trade in the sense that the bargainer agent does not lose the experience, unlike in the case of resource and good trade. It just shares the experience with the purchaser agent. In our simulations, the bought experience does not be calculated when calculating the cumulative installed capacity of the next term. In the following simulation, which we call BC6, we assume that based on BC4, agent 2 will buy the "Revolutionary" technology from agent 1 with the trade cost $\theta_3 = 10$ for each unit experience (or cumulative installed capacity). Fig. 7 shows the result of BC6, from which we can see that agent 2 buys the "Revolutionary" technology in 2060, and the diffusion of the "Revolutionary" technology in agent 2 is shorter than that in agent 1. We varied the technology trade cost and found that with a small one, e.g., $\theta_3 < 6$, the quantity of trading is higher, but the trading time remains the same – in 2060 – which makes the break-even time of the "Revolutionary" technology in agent 2 slightly earlier; and with a high technology trade cost, e.g., $\theta_3 > 12$, it becomes uneconomic for agent 2 to import technology from agent 1, and agent 2 keeps using the "Incremental" technology, without developing the "Revolutionary" one during the 100 years.

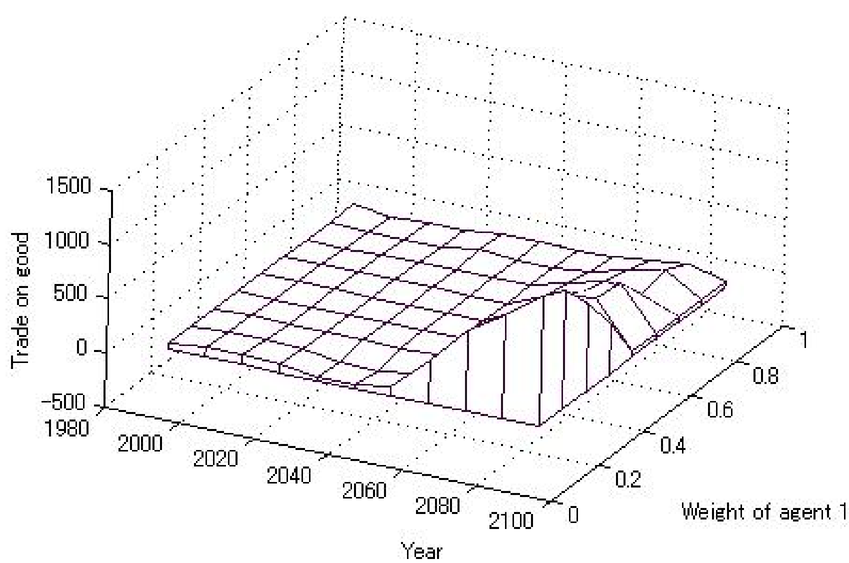
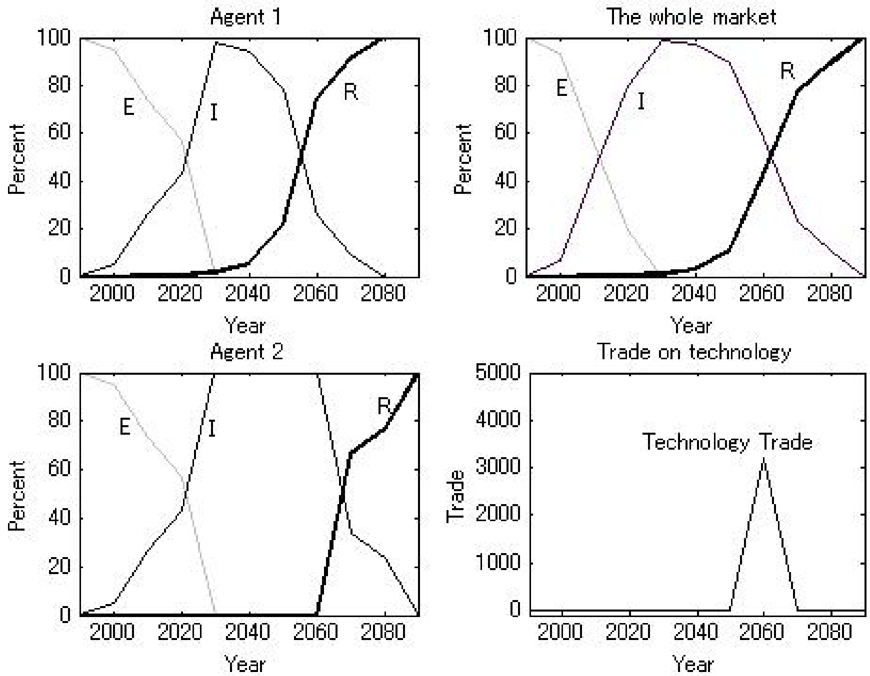



Figure 6. Trade on good with different weights of agents.




*R--Revolutionary, I – Incremental, E-Existing*

Figure 7. Result of BC6.

In our simulations, with different risk attitude on the future cost of advanced technology and with technology spillover effect between them, agent 1 and agent 2 act as a pioneer and a follower, respectively, and the diffusion time of the "Revolutionary" is shorter for the follower than that for the pioneer, which accords with historical observation that the later developer of a new technology can obtain a shorter diffusion period (see Grubler, A., Nakicenovic, N., and D.G. Victor 1999).

### 3.4 Pareto Optimality of the Solutions

Simply speaking, Pareto Optimality is the "best that could be achieved without disadvantaging at least one group" (see A. Schick 1970). Here we mathematically prove that the global optimal solutions of the two-agent model are of Pareto optimality, in the sense that none of the two agents could be made better off without the other being made worse off. The mathematic symbols used here are independent form those used above.

Suppose the two agents' objective functions are $f_1(x)$ and $f_2(x)$, and $x^* \in \Omega$ ($\Omega$ is the feasible set) is a global optimal solution for the problem:

$$\min f(x) = f_1(x) + f_2(x) . \tag{14}$$

$x^* \in \Omega$ is proved to be of Pareto optimality by using the following reduction to absurdity.

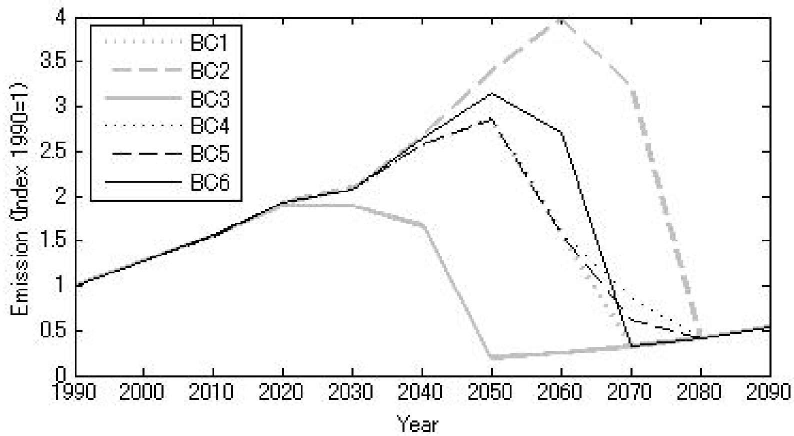



Figure 8. Different carbon emission paths.

Suppose there exist a $x \in \Omega$ such that $f_i(x) \le f_i(x^*)$ for all $i \in \{1,2\}$, with at least one strict inequality, then $f(x) < f(x^*)$ is true, i.e., $x^*$ is not a global solution, which is not true. So there is no $x \in \Omega$ such that $f_i(x) \le f_i(x^*)$ for all $i \in \{1,2\}$, with at least one strict inequality, which means the global optimal solution is Pareto efficient. It is also easy to prove that if there are more than two agents, global optimal solutions are also of Pareto optimality.

## 4 Carbon Emission Paths as Results of Different Technological Change Process

There are two important factors contributing to emission paths: the demand (or consumption) and the technologies used to satisfy the demand. Fig. 8 shows different emission paths of different simulations. BC3 shows the strongest carbon abatement in all the six simulations, while BC2 is the weakest one. The following are the main discoveries related to carbon emission paths in our simulations.

- Our model demonstrates an endogenous learning mechanism for the advanced technology to replace the existing one. The simulations show that even without carbon tax, the carbon emission could be reduced by the wide application of advanced technology.
- The uncertainty in learning rate will delay the development of the "Revolutionary" technology, thus results in delayed and weaker carbon abatement.
- Carbon tax, especially the uncertainty in carbon tax, is a driving force for the earlier development of the "Revolutionary" technology. That's the reason why BC3 shows the strongest carbon abatement. BC4, BC5 and BC6 include a deterministic carbon tax, not a uncertain one, so the carbon abatement is weaker than that in BC3.

- Although technological learning can lead to the reduction of carbon emission, a carbon tax is still important in the following two senses:
  1. It can control the maximal annual emission. As show in Fig. 8, without carbon tax, the maximal annual emission is relatively high in BC2. While with the uncertain carbon tax, the maximal annual emission is low in BC3.
  2. It can bring forward the low-emission time. In Fig. 8, with the uncertain carbon tax, in BC3, the carbon emission starts to decrease from 2030; while without carbon tax, i.e., in BC2, the carbon emission starts to decrease from 2060. In some special situations, carbon tax will become extremely important. For example, in some urban cities with high density of population and rapid increase in energy consumption demand, without carbon tax, it is possible that the emission reduction caused only by technology improvement will be too late for maintaining the ecosystem in those urban cities.
- People maybe have the intuition that technological spillover should be helpful for carbon abatement. But this is not always true. Sometimes the existing of technological spillover would weaken carbon abatement in a certain period. As shown in Fig. 8, with technology trade, the carbon emission of BC6 is higher than that of BC4 during the period from 2040 to 2065. This is because with technological spillover effect, agent 2 imports less good from agent 1 during that period which results in two consequences; the first one is that agent 1 develops the "Revolutionary" technology slightly late since its market is smaller, and the second one is, the two agents, especially agent 2, consume more good produced by the "Incremental" technology rather than by the "Revolutionary" one during that period which results in weaker carbon abatement. Another story which we learnt from the simulations about why technological spillover could weaken carbon abatement during a certain period is that when the trade on good is little because of high trade cost, in a short or middle-term, it is possible that agent 2, knowing the technological spillover effort, will rely more on the "Existing" technology and develop less "Incremental" one, waiting for the "Revolutionary" technology to be developed by the pioneer agent.
- The emission path is not necessary convex. For example, for BC4, by 2050, the global energy system has completely shifted to the "Revolutionary" technology, and the carbon emission reaches its bottom in 2050, then it increases again because of the increasing consumption.

# 5 Concluding Remarks

Based on earlier and pioneering work done at IIASA, this paper presented a model of endogenous technological change with increasing return, uncertainty and heterogeneous agents. Although the model and simulations are highly stylized, they can enhance people's imagination about how the three stylized facts impact technological change processes. Here we summarize what we learnt from the modeling and simulations introduced in this paper.

- The model and simulations demonstrates an endogenous learning mechanism for the advanced technology to replace the existing one. The S-shape diffusion pattern of new technologies in our simulations accords with historical observations.
- Facing uncertainty in technological learning, decision makers would prefer late R&D on advanced technologies. Of course, decision makers' different risk-attitudes will play an important role in their decisions. A risk-taking decision maker would prefer earlier R&D on advanced technologies than a risk-aversion one.
- The factors which can contribute to the early R&D investment on an advanced technology and its wide application include
  - high learning rate of the new advanced technology,
  - lower initial cost of the advanced technology,
  - high resource extraction cost, or that the resource is becoming rare,
  - low discount rate,
  - low uncertainty in learning rate,
  - low-sensitivity to risk, or the decision agent is adventuring,
  - high carbon tax,
  - and high uncertainty in carbon tax.
- Globalization maybe acts as a driving force for the development of advanced technologies because development of advanced technologies commonly needs huge investment which probably requires a very large potential market to reimburse it.
- Technological spillover could also slow or delay the wide application of advanced technologies and thus weaken carbon abatement in a certain period, mainly in a short or middle term period.

In terms of minimizing their aggregated costs, the two heterogeneous agents are assumed to be cooperative. In real world, it is possible that some decision makers do not accept the optimization result, because they want to maximize their profit. For example, a technology pioneer develops an advanced technology earlier than others, it is possible that it would apply a very high pricing strategy for its products and technology, and this will delay the wide adoption of the new technology than what Pareto optimization suggests. Other factors prevent decision makers from following Pareto optimization include security issues. For example, in some situations, Pareto optimization suggests an agent with small local market to import good such as gasoline from others, instead of building its own capacities, but the agent thinks the good is very important for it and so it refuses to completely depend on import since it does not want its fate to be controlled by others.

Matlab Optimization Toolbox was used to solve the optimization problems, and global optimality of solutions was checked by employing different starting points. In the future work, global optimization software or solvers, such as TomLab (see http://www.tomlab.biz) and BARON (see Sahinidis 2000), will be applied for global optimization, and it is also important to develop a specific global search algorithm that essentially utilizes the features of the general model. And the stability of Pareto optimal solutions should be explored when the model is used for real applications.

We started from understanding of the three important stylized facts that were summarized from historical observations, then included them into equation-based models, and generated some patterns according with other historical observations, which makes the equation-based model more reliable. On the other hand, the equation-based model can help us to get better understanding of history and future. History (story)-based study and equation-based models can be and should be complementary to each other in the research of social issues.

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