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A DISAGGREGATED HEALTH CARE RESOURCE ALLOCATION MODEL

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Preface

The aim of the IIASA Modeling Health Care Systems Task is to build a National Health Care System model and apply it in collaboration with national research centers as an aid to Health Service planners. The modeling work is proceeding along the lines proposed in earlier papers by Venedictov (1) among others. It involves the construction of linked sub-models dealing with population, disease prevalence, resource need, resource supply, and resource allocation.

This paper examines how a National Health Care System model can be applied to the planning of health services, considers the role of the resource allocation sub-model in particular, and proposes a disaggregated sub-model to perform in this role. Following the recommendation of an earlier paper (2), in which the literature was reviewed, the resource allocation sub-model proposed in this paper is of the behaviour simulation type.

Recent related publications of the IIASA Modeling Health Care Systems Task are listed on the back page of this Memorandum.

Evgenii N. Shigan
December 1977

Abstract

The planning of health services can be viewed as occurring in two stages--the estimation of the amounts of health care resources that would be needed if the Health Care System (HCS) were to test all sick individuals at clinically desirable standards and the downward revision of these estimates in order to comply with economic constraints. To assist in the second stage a model is proposed which includes sub-models for population, disease prevalence, resource supply, and resource allocation and which could be used interactively by the planner to explore resource options. The role of the resource allocation sub-model in this design is to simulate how the HCS allocates limited resources between competing demands. To perform this role a sub-model is proposed which derives from a resource allocation model which is being used in health service planning in the UK. The sub-model as proposed here can be applied to only one sector of the HCS at a time whereas the UK model can be applied to several sectors simultaneously. However it is more easy to use than the UK model and its computational requirements are considerably lighter. The sub-model is described in terms of its application to the hospital in-patient sector and its performance is illustrated by a hypothetical application to the South Western Region of England.

A Disaggregated Health Care Resource Allocation Model

1. PLANNING THE FUTURE PROVISIONS OF HEALTH SERVICE RESOURCES

One of the main tasks in the strategic planning of health services is to determine the provision of an appropriate mix of resources (hospital beds, physicians, etc.) for the future. The way in which this is done (and the extent to which it is done through a central planning agency) vary from one country to another. However in most countries the strategic planning of resource provision can be seen to occur in two stages. We shall refer to these two stages here as the *unconstrained stage* and the *constrained stage* of planning. In the unconstrained stage planners attempt to estimate future resource needs in terms of what would be needed to meet all anticipated demands for health care at clinically perceived ideal standards. Following this, in the constrained stage, account has to be taken of certain constraints, particularly economic constraints, that limit the total amounts of resources which a country can afford to devote to health care.

A fully developed form of the unconstrained stage of Health Care System (HCS) planning is depicted in Figure 1. In this scheme the estimation of HCS resource needs for a given future year proceeds as follows. A forecast of population is combined with a forecast of disease prevalence rate to yield an estimate of future morbidity--the amount of sickness in the population as a whole; this estimate can be an aggregate figure--*general morbidity*--or it can be disaggregated, e.g. by age, sex, and disease type. The pattern of future morbidity is to a large extent a product of nature and therefore the estimation of it is amenable to scientific forecasting. By contrast the other factors involved, ideal standards* and policies for treatment and prevention (which determine the modes of care), are the products of the intervention of the HCS and are therefore subject to HCS policy (see Figure 1). As an example let us consider the treatment of pneumonia. In one country or region it might be considered, on clinical grounds, that 90% of pneumonia cases need the hospital in-patient mode of care and that their average length of stay should be 20 days. However in another country or region, where

*In this paper the term *standard* denotes the average amount of a HCS resource consumed per patient for a given treatment (e.g. 20 days for a hospitalised case of pneumonia). In some of the literature this term is used instead to denote the aggregate amount of resource provision (e.g. 1.9 surgical beds per 1000 population); this latter type of quantity is covered in this paper by the terms *resource needs* and *resource supply*.

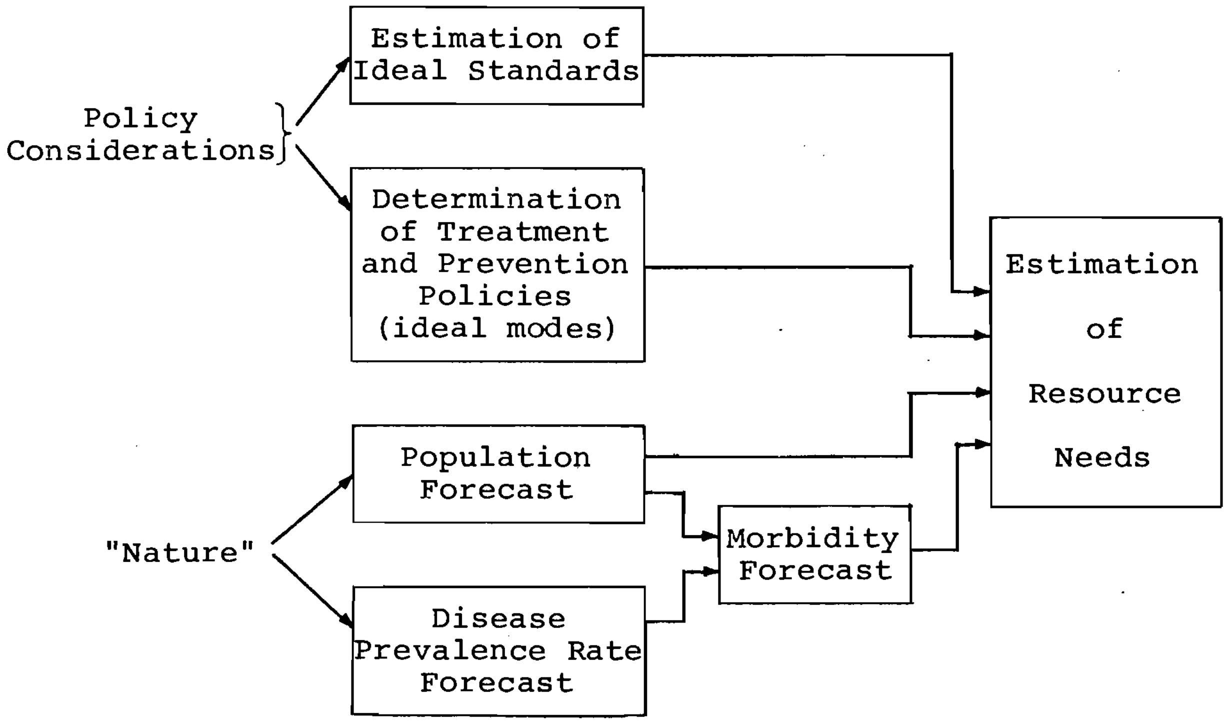

Figure 1. Planning the provision of HCS resources: unconstrained stage.

perhaps the quality of housing is better and the availability of domiciliary services higher, a lower hospitalisation rate and/or a shorter average length of stay might be considered appropriate. From the quantities described above--population, morbidity, standards and policies--resource needs for the HCS can now be calculated. For example the number of hospital beds required for treating pneumonia can be calculated from the following equations:

$$(\text{MORBIDITY}) = (\text{POPULATION}) \times (\text{PREVALENCE RATE}) \quad ,$$

$$(\text{HOSPITAL BEDS}) = (\text{MORBIDITY}) \times (\text{HOSPITALISATION RATE}) \times \left(\frac{\text{AV. LENGTH OF STAY}}{\text{OCCUPANCY}}\right) \quad ,$$

where (OCCUPANCY) = the average number of days per year for which a bed can be occupied. This calculation can be performed for each type of disease and by summation the need for each resource can be computed.

The unconstrained stage in the HCS strategic planning of certain countries has been documented. For example Popov [3] describes how it is conducted in the USSR. Disease prevalence rates are estimated by combining routine disaggregated data on sick persons contacting the HCS--registered prevalence--with more aggregate data, from sample surveys of the general population (see also Shigan [4]). Prospective standards and treatment policies are determined by a combination of statistical analysis of current activity in the HCS and expert opinions on ways in which the current performance of the HCS should be improved.

In a similar way resource needs are estimated for the National Health Service and Personal Social Services in the UK, although there is less quantification than in the USSR of morbidity and ideal standards at a disaggregated level. These resource needs (sometimes termed *planning norms*) are published by the central authority, the Department of Health and Social Security. As an example some of the published figures on services for the mentally handicapped are displayed in Table 1 (derived from reference [5]).

Table 1. Planning figures for services for the adult mentally handicapped compared with existing provision--from UK Government White Paper [5].

	Places Required		Places Provided
	Per 100,000 total population	Total England and Wales 1969	Total England and Wales 1969
Occupation and training:			
In the community	150	73,500	24,600
In hospitals:			
for in-patients	35	17,200	30,000
for day-patients	10	4,900	200
Residential care in the community:			
Residential homes	60	29,400	4,300
Foster homes, lodgings	15	7,400	550
Hospital treatment:			
For in-patients	55	27,000	52,100
For day-patients	10	4,900	500

The figures in column 1 of Table 1 are multiplied by population figures (for 1969) to give the estimates of resource requirements (needs) in column 2. Two main points should be noted. Firstly the figures are based on a change in treatment policy which involves a shift from hospital based care to care based much more on residential homes coupled with educational and training services. Secondly the amounts of resources needed, column 2, are, except for hospital in-patient resources, much in excess of the figures on current provision, column 3.

The planning of HCS resource provision would be complete after the unconstrained stage were it not for the fact that no country appears to be able to afford to provide HCS resources at the levels of estimated resource needs. Because of economic and other constraints the actual provisions of HCS resources have to be at lower levels. Thus planners have to embark on a second process, the constrained stage of planning, in order to determine a more modest set of resource provisions which (1) comply with the economic constraints and yet (2) enable the actual performance of the HCS to come as close as possible to achieving the ideals defined in the unconstrained stage. This situation is depicted in Figure 2.

This constrained stage of planning has also been documented in certain countries. For example Popov [3] provides comparisons between figures for the long-term HCS plan (resource needs) and figures from current plans, which are based on an understanding of the economic constraints. Some figures relating to the provision of hospital beds are shown in Table 2; note that the constrained planning figures are, in general, significantly less than the long-term, *need*, figures. Similarly in the UK, the DHSS has published a document [6] which proposes a set of resource provisions for the Health and Personal Social Services in a future year which are less than the previously published resource needs but which are consistent with the anticipated HCS budget. In the document an attempt is made, for each resource, to assess the relative priority of achieving a level of provision equal to the need figure; for those resources where the assessed priority is higher the document proposes a greater rate of increase in provision, i.e. a faster approach towards the level of estimated need.

The dichotomy between the unconstrained and constrained stages of HCS planning has been exaggerated here, to ease the exposition. In real life the two stages are probably merged to some extent; in assessing resource needs planners are bound to take some account of what levels might be afforded in the not-too-remote future. On the other hand it would be undesirable for all planning investigations to be dominated by the constraint of what can be afforded in the immediate future since planning might then degenerate into a process of *disjointed incrementalism*, described by Lindblom [7] and others, in which there is little incentive for examining the HCS as a whole and for considering structural, rather than marginal, change. Accordingly the IIASA

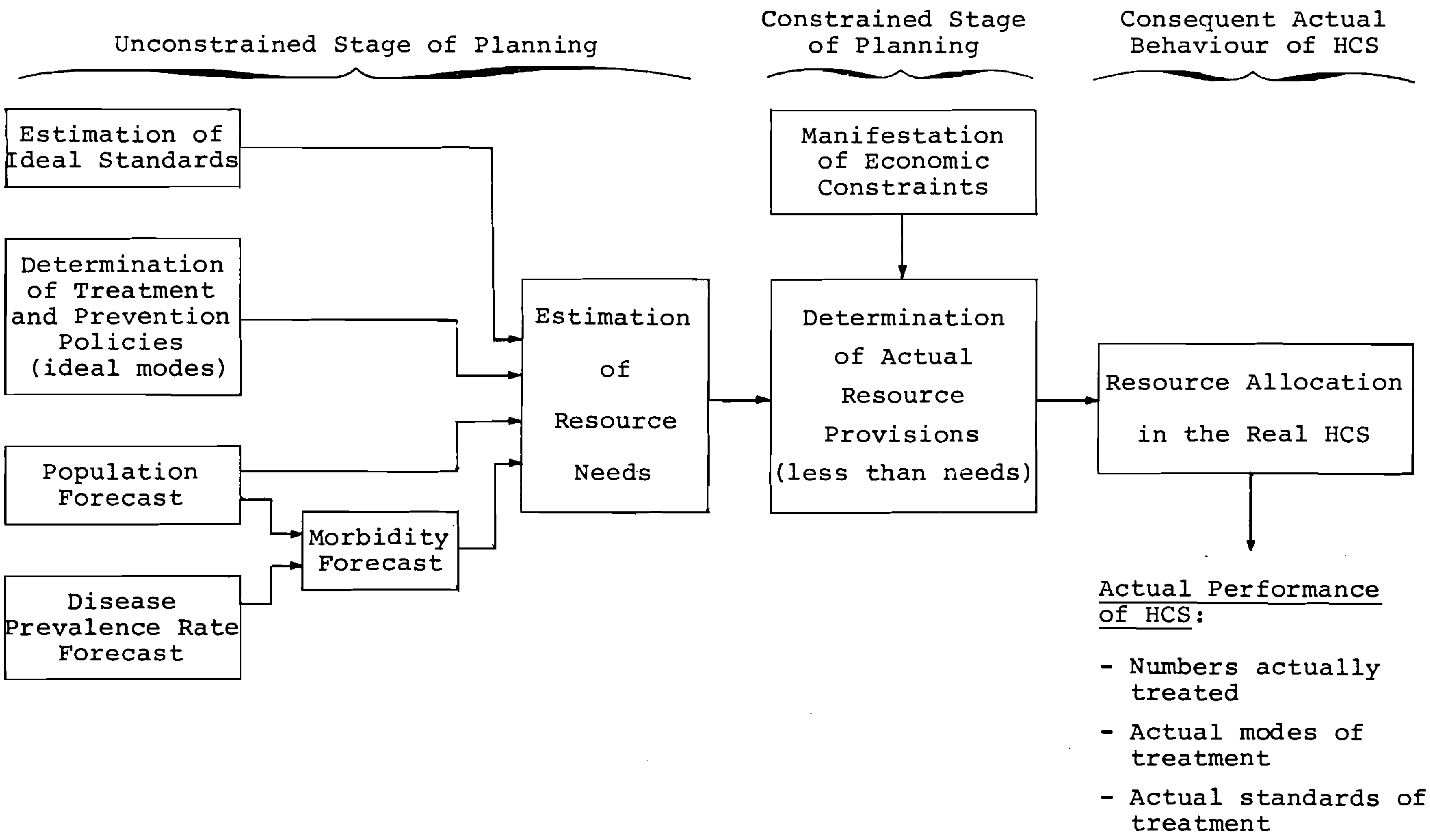

Figure 2. The context of the constrained stage in planning the provision of HCS resources.

Table 2. Requirements of urban population for hospital beds in various specialties (per 1000 population) for the USSR, from Popov [3].

Specialty	Plan for 1970	Draft Plan for 1975	Long-term Plan
Internal medicine	2.59	2.69	3.4
Paediatrics (excluding communicable diseases).	1.32	1.38	1.2
Obstetrics	0.80	0.80	0.8
Gynaecology	0.67	0.75	0.8
Surgery	1.67	1.81	1.9
Neurology	0.30	0.37	0.4
Phthisiology	1.17	1.12	0.8
Dermatovenereology	0.22	0.25	0.35
Ophthalmology	0.18	0.23	0.35
Otorhinolaryngology	0.18	0.23	0.3
Communicable diseases (adults and children).	0.79	0.80	0.7
Total	9.89	10.43	11.0
Psychiatry	1.08	1.27	2.5
Total	10.97	11.70	13.5

HCS Modeling Task, to which we now turn, is concerned with assisting planners in both the unconstrained and constrained stages of planning and helping to preserve the dichotomy.

2. THE IIASA HCS MODEL AND ITS APPLICATION TO HCS PLANNING

The aim of the IIASA HCS Modeling Task is to produce a model, or more precisely a suite of sub-models, to be used by HCS planners. The long-term aim, as set out in earlier papers by Venedictov [1] and Kiselev [8], envisages the construction of a mathematical simulation model relating activities both within the HCS and between the HCS and other interacting systems (e.g. the population, environment, and socio-economic systems). The purpose of the simulation model is to illuminate the future consequences of alternative policies both for the HCS and the interacting systems and thus assist planners to examine strategic options. Within this framework the current short-term plan for the IIASA HCS Modeling Task, as set out in the IIASA Research Plan 1977 [9] and in a recent paper by Shigan [10], is to concentrate effort initially on modeling the HCS itself and its interaction with one external system--the population system.

To assist planners in the *unconstrained* stage of planning a model is required which estimates resource needs and which draws upon sub-models for estimating population and disease prevalence. An aggregate version of such a model, *AMER* (Aggregate Model for Estimating Resource Requirements), has been built by the IIASA team and is described in a separate paper by Klementiev and Shigan [11].

To assist planners in the *constrained* stage of planning a different model is required which combines sub-models for population and disease prevalence with sub-models of resource allocation and resource supply. Such a model can be built at more than one level of sophistication. Let us start by considering a level of model sophistication that is relatively simple, in concept if not in practice. We will call this the *Mark 1* model.

The design and operation of the Mark 1 HCS Model is shown in Figure 3. In this design a key role is played by the resource allocation sub-model which simulates how the real HCS allocates scarce resources between competing demands. On the demand side it receives the following inputs:

- disease prevalence, from the prevalence sub-model,
- policies for treatment and prevention (modes of care), and } from the user of the model
- ideal standards. } from the user of the model

This is the same set of inputs as used in the unconstrained stage of planning (see Figure 1) in calculating resource needs. The difference here is that the resource allocation sub-model receives an additional input on resource supply, from the resource supply sub-model. This input consists of a set of resource provisions that are in general less than the corresponding resource needs but which are consistent with the given economic constraints. The resource allocation sub-model then simulates how the real HCS would actually allocate these resources between competing demands. The outputs of the sub-model would include the following indicators of the expected performance of the HCS, which the user can compare with the corresponding ideal quantities:

- the actual numbers of patients treated, which can be compared with the morbidity figures,
- the actual modes of care (percent hospitalisation, etc.), which can be compared with the treatment and prevention policies, and
- the actual standards, which can be compared with the ideal ones.

From the statement of its inputs and outputs it follows that in simulating the behaviour of the real HCS the resource allocation sub-model will have to represent at least three main processes:

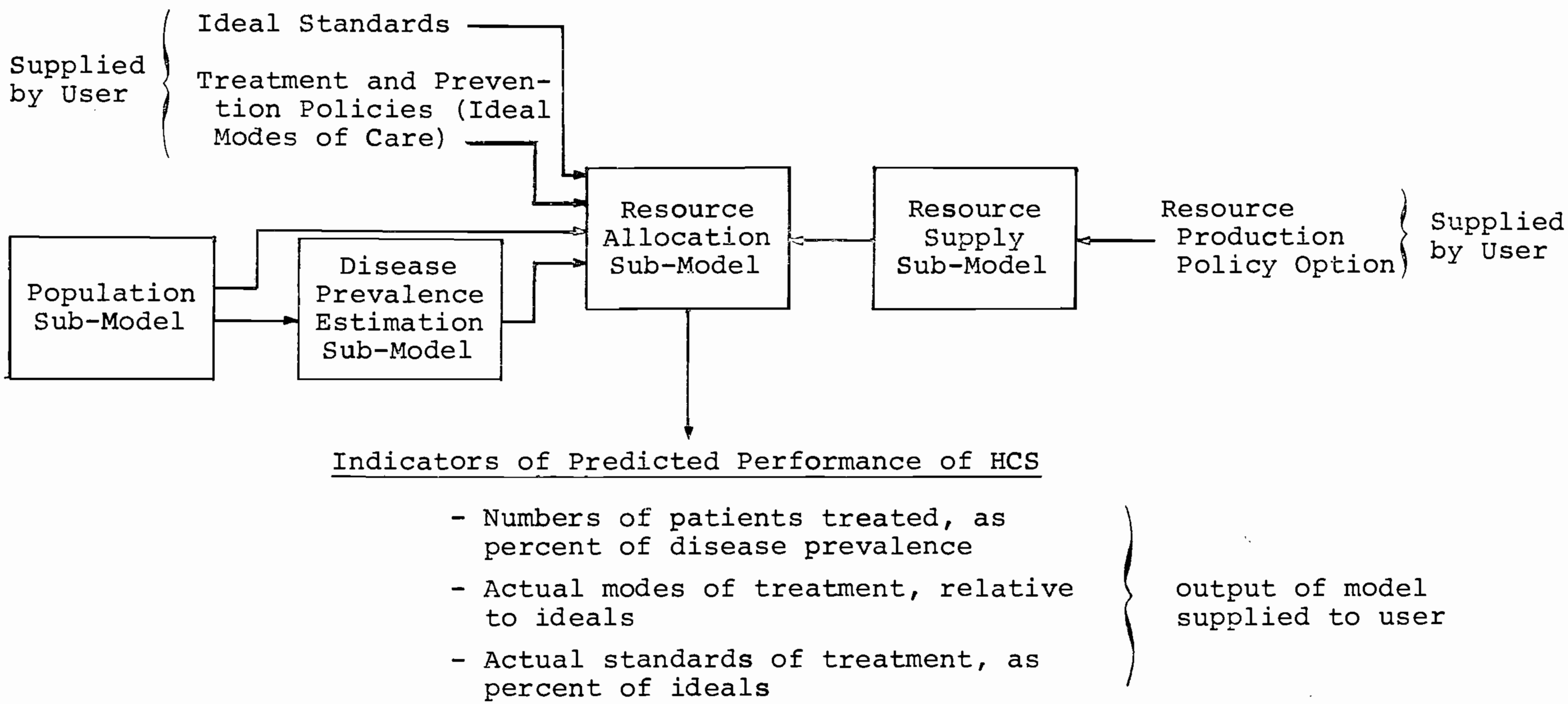

Figure 3. Mark 1 IIASA model for the constrained stage of HCS planning.

- patient selection, the process which determines *who* receives treatment and prevention activities,

- mode selection, the process which determines which *type* of treatment or prevention activity an individual receives, and

- standard attainment, the process which determines *how much* of a treatment or prevention activity is received.

The Mark 1 HCS model can be used in the following way (see Figure 3). The user can suggest an option for resource production (in terms of building programmes, physician training, etc.) which can be submitted to the resource allocation sub-model via the resource supply model. He can then inspect the output of the model, compare this with the ideal HCS parameters, submit a revised resource production option, and so on until, in his view, a satisfactory output is achieved.

Now at this stage the reader might be critical of the implication that an HCS planner should evaluate a resource production option in terms of what might be called the *intermediate* outputs of the HCS--the numbers of people treated, the modes and standards of treatment. Surely it would be better for him to evaluate policies in terms of their likely impact upon the future health of the population--the final outputs of the HCS? To answer this criticism and to extend the model so that it can predict final HCS outputs we need to add a fifth sub-model, of prognosis, as shown in Figure 4. We will denote the model, so extended, the *Mark 2* HCS model. The function of the prognosis sub-model is to take as input the allocations of treatments to patients (from the resource allocation sub-model) and predict the outcomes of these treatments in terms of the numbers of individuals who recover from sickness and the durations of their sickness, the numbers who remain sick and the numbers who die. The ability of the Mark 2 model to estimate these final HCS outputs is not only useful to planners, it is also desirable from a scientific point of view because only through these final HCS outputs can we examine the feedback effects from the resource allocation process to the future structure of population and morbidity. To illustrate the possible importance of such feedbacks let us imagine that one of the resource production options being considered by the HCS planners has a major effect, via the resource allocation process, on mortality for a certain disease. This could significantly affect the population structure for a future year. In this case the use of the *Mark 1* version of the model (which takes no account of such feedbacks) for planning HCS resource provisions for this future year could lead to error since the model run would be based on an erroneous estimate of population for the year in question.

Thus both from the user's point of view and from the scientific point of view it is desirable to progress from the Mark 1 to the Mark 2 version of the HCS model. Unfortunately there is a major technical difficulty in incorporating a model of prognosis. For many of the clinical procedures undertaken in the

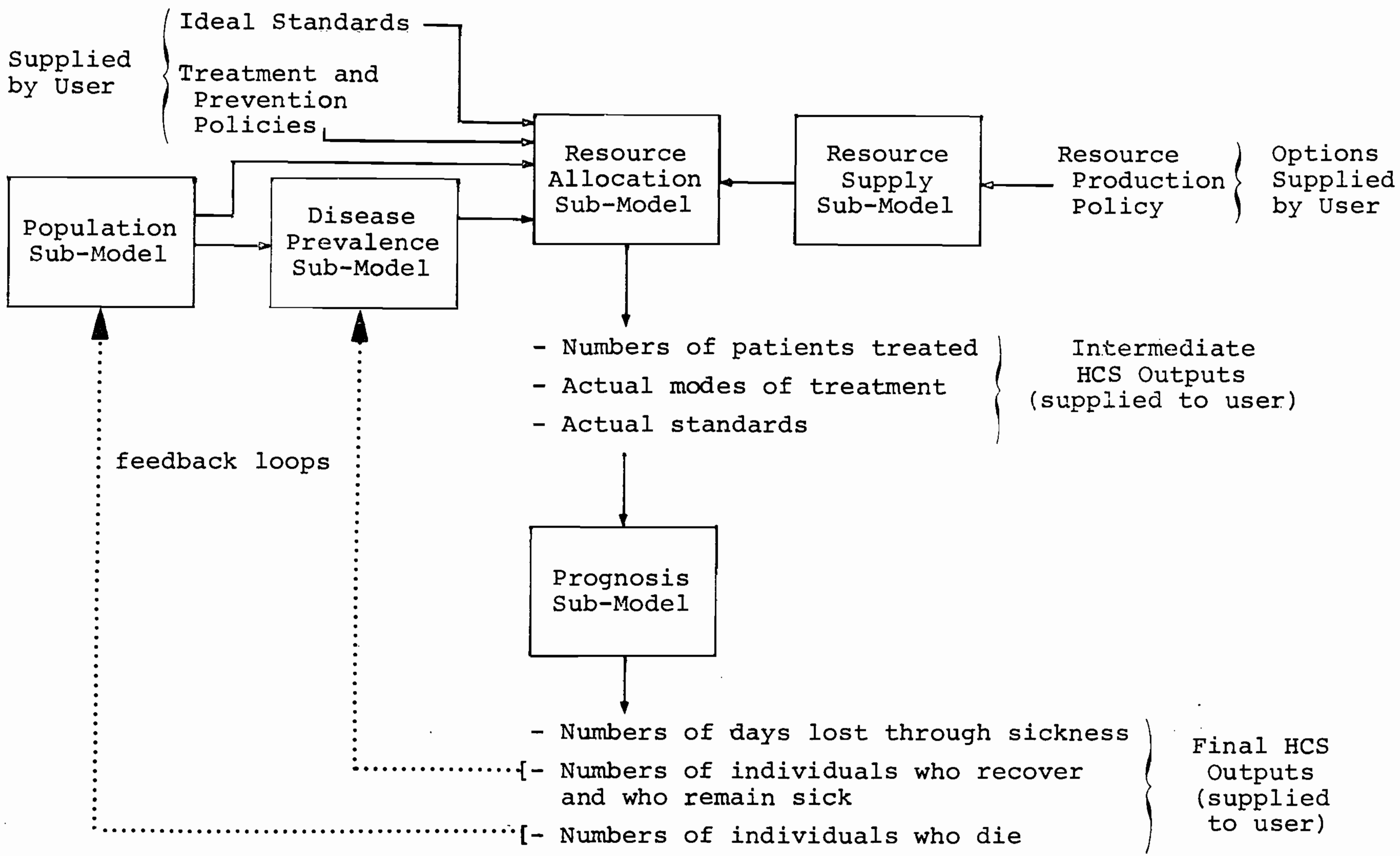

Figure 4. Mark 2 IIASA model for the constrained stage of HCS planning.

HCS very little is known in a systematic way about prognosis, especially in the longer term. One only has to peruse the medical journals to find that eminent physicians disagree, sometimes quite diametrically, about the likely prognosis of many clinical procedures. Given this lack of data it may not be fruitful to attempt to build a formal disaggregated sub-model for prognosis. It may be more fruitful to seek expert opinion on the likely major effects, at an aggregate level, of the resource allocation process on mortality and morbidity and to use these, in place of a formal prognosis model, in the *Mark 2* design to perform some simple experiments in order to discover whether there are any significant feedback effects.

Whatever the viability and desirability of the Mark 2 model it is clearly necessary that the Mark 1 version be constructed first. Accordingly we will now turn our attention to the building of the Mark 1 version. In particular, since earlier papers by Shigan [4], Kaihara et al. [12], and Klementiev and Shigan [11] have dealt with the work on the population and disease prevalence sub-models and with an integrated aggregate model, this paper will now describe an approach to building a disaggregated resource allocation sub-model.

3. THE RESOURCE ALLOCATION SUB-MODEL--GENERAL STRUCTURE

An earlier paper [2] reviewed the literature on HCS resource allocation models and concluded that the type of model appropriate to the IIASA Task was the behaviour simulation type rather than one of the classical econometric or optimisating types. In particular it was concluded that the IIASA model should take into account the preferences and priorities being used by the actors in the HCS at the point of delivery of health care. It should draw upon the models of McDonald et al. in the UK [14] and Rousseau in Canada [15] and represent the actors in the HCS striving to attain some ideal pattern of behaviour within resource constraints. In this view these resource constraints are the main means through which the planner can affect the behaviour of the HCS.

Accordingly the model proposed here is a simplification of the model of McDonald et al [14]. Of the three main mechanisms of the HCS resource allocation process--patient selection, treatment mode selection, and standard attainment--which were described above (p. 9) and which are included in the McDonald model, the initial version of this model includes only two--patient selection and standard attainment. Thus it can be applied to only one sector (mode) of the HCS at a time (although it may prove possible, after further study, to extend the model to cover more than one sector). However one of the advantages of the model is that the computing requirements are relatively light so that it can be readily implemented on different computers without using elaborate software and so could be relatively easily applied in different countries; (by contrast the

McDonald model, in the current form, requires relatively sophisticated software and a large computer in order to solve the non-linear programming formulation). Being more simple this model is also more transparent. Keyfitz [16], among others, has argued persuasively that with a transparent model the user can gain an insight into the workings of the model and is then more likely to have confidence in its results than with a "black box" model.

The model is particularly relevant to the acute hospital in-patient sector and it will be presented here with this application in mind. But the essence of the model is the concept of the HCS achieving an equilibrium by balancing the desirability of treating more patients of one type against treating more of other types and against the desirability of treating each type patient at a higher average standard. It is likely that this concept could be readily applied to other sectors of the HCS.

The way in which the HCS achieves such an equilibrium has been extensively researched. One finding, which has been so frequently obtained (e.g. [17,18,19]) that the accumulated evidence for it is by now overwhelming, is that for a wide range of clinical conditions and specialties, both the number of admissions and the average length of in-patient stay are elastic to the supply of beds, that is to say the greater the supply of beds the greater are both the numbers admitted and their length of stay. Furthermore it appears that in *none* of the places studied has the supply of beds reached the level at which in-patient care is given to all individuals who need it, at the ideal average length of stay. It seems, as Rousseau [15] has observed, that the demands for in-patient (and other) care cannot be saturated, at least within the constraint of the amounts of services that society can afford to supply.

A good example of a study of the equilibrium between the demand and supply of in-patient care is that of Feldstein [19] for England and Wales in 1960. Some of his results are shown in Table 3 below. These results come from a cross-section study of acute hospitals and show for a number of clinical conditions the elasticities of both numbers of patients admitted and their length of stay with respect to aggregate bed supply. For example the interpretation of the result for haemorrhoids is that a 1% increase in total bed supply is associated with a 0.70% increase in the number of haemorrhoid patients treated and a 0.44% increase in their average length of stay.

The model presented below attempts to represent how the HCS achieves an equilibrium between numbers of patients and lengths of stay on the one hand and bed supply on the other. The model is of the behaviour simulation kind, in the sense defined in a previous paper [2]. Thus, *if its underlying hypothesis is sound,* it can not merely describe past equilibria, as can econometric models such as Feldstein's, but it can also, unlike classical econometric models, predict how the equilibrium is likely to

Table 3. Elasticities of hospital admissions and lengths of stay with respect to total bed supply* for England, 1960, for certain diseases (from Feldstein [19]).

Disease	Elasticity of:	
	Admissions*	Average Stay
Varicose Veins	0.78	0.62
Haemorrhoids	0.70	0.44
Ischaemic Heart**	1.14	1.08
Pneumonia	0.71	0.23
Bronchitis	1.13	-0.23
Appendicitis	-0.16	0.31

*Per thousand population.
**Excluding acute myocardial infection.

change in the future as a result of changes in factors such as clinical standards, disease prevalence, and the preferences and priorities operating in the HCS. The model has been named *DRAM* --Disaggregated Resource Allocation Model.

4. FORMULATION OF THE MODEL *DRAM*

Definitions

Subscript

i = Patient category (e.g. by disease type).

Variables

x_i = Number of patients of type i admitted to hospital.

u_i = Average length of stay for patients of type i who are admitted (days).

Data

X_i = Ideal, maximum, number of patients of type i who need hospital treatment.

U_i = Ideal average length of stay (days).

B = Total number of bed-days available for occupation ($< \sum_i X_i U_i$).

α_i, β_i are strictly positive constants $\forall$ i.

Hypothesis

The HCS chooses the x_i, u_i so as to maximise a utility function, Z, where:

$$Z = \sum_i g_i(x_i) + \sum_i x_i h_i(u_i) \quad ,$$

$$g_i(x_i) = -\frac{U_i X_i}{\alpha_i}\left(\frac{x_i}{X_i}\right)^{-\alpha_i} \quad ,$$

and

$$h_i(u_i) = \frac{U_i}{\beta_i}\left\{1 - \left(\frac{u_i}{U_i}\right)^{-\beta_i}\right\} \quad ,$$

subject to the constraint

$$\sum_i x_i u_i = B \quad .$$

Solution

It can be shown (see Appendix) that the solution to the maximisation problem is:

$$u_i = U_i \lambda^{-1/(\beta_i+1)} \quad ,$$

$$x_i = X_i\left\{\frac{1}{\beta_i}\left[(\beta_i + 1)\ \lambda^{\beta_i/(\beta_i+1)} - 1\right]\right\}^{-1/(\alpha_i+1)} \quad ,$$

where λ is a Lagrange Multiplier whose value can be found by numerical methods, as described in the Appendix.

5. POSSIBLE APPLICATIONS OF THE MODEL

The model can be used to predict the actual performance of the HCS, in terms of numbers of patients treated and their length of stay, under different options for bed supply, B. It can readily be applied to countries where the HCS is centrally planned since, in many of these countries, norms and standards are defined for ideal quantities such as the X_i and U_i. The quantities X_i and U_i are not so readily available in other countries. However the X_i could be regarded as the prevalence of disease i times a hospitalisation factor; prevalence could be estimated using techniques such as the IIASA morbidity estimation submodels [4,12,13] and the hospitalisation factor by expert judgement. Expert judgement could also be used to estimate the U_i [20]. Indeed a paper by McDonald and Gibbs [20] describes how ideal standards were estimated in this way for the model of McDonald et al.

The other model parameters are the α_i and β_i; these reflect the relative priorities, for different types of patient, of attaining the ideals X_i and U_i. Appropriate values of these parameters can be derived from estimates of the actual elasticities of the admissions and lengths of stay to total bed supply such as those obtained by Feldstein displayed in Table 3; the method of derivation is described in the Appendix. The model can then be used to simulate the performance of the HCS, under *the prevailing priorities operating in the HCS*. If however a planner was interested in exploring how the HCS would behave under a different set of priorities, *and had reason to believe that such different priorities could be implemented in the real HCS*, then he could adjust the parameters α_i and β_i accordingly.

In order to reflect different views about the future pattern of morbidity and/or different policies with regard to ideal hospitalisation rates, the model can be run using alternative values of the parameters X_i. Similarly, if one was considering alternative scenarios for the progress of medical technology and its impact on lengths of stay, then it would be appropriate to adjust the values of the parameters U_i accordingly.

In the next phase of the IIASA project it is intended to apply the model, in the form presented here, to the hospital in-patient sector using data from the UK, the USSR, and, later on, other countries. An illustrative, hypothetical example of such an application is given below.

6. ILLUSTRATIVE EXAMPLE OF AN APPLICATION OF THE MODEL

To illustrate how the model can be used we shall examine a hypothetical example of an HCS resource allocation problem. Consider the allocation of acute hospital bed-days in the South Western Region of England in 1968 between patients suffering from six diseases:

- varicose veins,
- haemorrhoids,
- ischaemic heart disease, excluding acute myocardial infarction,
- pneumonia,
- bronchitis, and
- appendicitis.

The first task in applying the model to this problem is to obtain suitable values for the parameters of the models. Values for the power parameters, the α_i and β_i can be derived from the empirical estimates of elasticities obtained by Feldstein and described in Section 3 (see Table 3) using equations given in the Appendix. These values are listed in Table 4. The parameters X_i and U_i, representing total numbers of patients needing hospital care and ideal lengths of stay, are more difficult to estimate. In a real application of the model their estimation would involve expert clinical opinions and estimates of morbidity. However for this purely illustrative run of the model proxy measures of the parameters were used. These were obtained using data [21] for the 15 regions of England and Wales in 1968; for each individual parameter the highest figure from the 15 regions was selected. For example the largest figure for pneumonia admissions per million population is 12.8, from the North West Metropolitan Region, and this figure was used for the parameter value, X_i, for pneumonia admissions. The full list of these parameter values is given in Table 4.

In the first illustrative run of the model the aggregate bed supply, B, was set equal to the actual number, 1094, of bed days used, per million population, in the South Western Region in 1968 for the six diseases taken together. The output of the model--admissions and average stay for each disease--can then be compared with data [21] for these quantities for the actual situation in the Region in 1968. These figures are shown in Table 5. The degree of agreement between the model outputs and the data on the actual situation can be expressed in terms of their percentage differences: for the six admissions figures the average difference is 12% whilst for the six stay figures

Table 4. Illustrative run of model: parameter values for England and Wales, 1968.

Disease (i)	Power Parameters*:		Ideal Levels**:	
	Admissions (α_i)	Average Stay (β_i)	Admissions per Million Population (X_i)	Average Stay (days) (U_i)
1. Varicose Veins	1.64	3.03	12.8	15.4
2. Haemorrhoids	2.11	4.68	7.7	13.1
3. Ischaemic Heart	0.54	1.31	10.4	52.1
4. Pneumonia	2.28	9.87	21.0	19.7
5. Bronchitis	1.18	49.0	21.3	34.2
6. Appendicitis	44.4	7.06	24.8	10.1

*Derived from elasticities estimated by Feldstein, shown in Table 3.
**The maximum levels found among the 15 regions of England and Wales for 1968.

Table 5. Illustrative run of model for South Western Region of England, 1968: model output for current bed supply (1094 bed-days per million population) compared with actual situation.

Disease (i)	Model Output for Current Bed Supply		Actual Situation	
	Admissions per Million Population (x_i)	Average Stay (days) (u_i)	Admissions per Million Population (x_i)	Average Stay (days) (u_i)
1. Varicose Veins	7.9	10.8	6.3	11.3
2. Haemorrhoids	5.1	10.2	4.1	13.1
3. Ischaemic Heart	4.9	28.3	4.6	40.2
4. Pneumonia	13.9	17.3	12.3	14.7
5. Bronchitis	11.2	33.2	11.8	27.4
6. Appendicitis	24.1	8.5	24.8	11.3

the average difference is 20%. This degree of agreement is reasonable enough in an illustrative run with proxy values for some of the parameters, but in a real application with suitably estimated parameters one would expect, and probably require, a closer degree of agreement.

The application of the model to exploring policy options for bed supply is illustrated by two further model runs in which the bed supply figure is set at two different values, one larger and one smaller than the current figure, 1094 bed-days, which was used in the initial run. The results of these two runs are displayed in Table 6.

Table 6. Illustrative run of model for South Western Region of England, 1968: model output for alternative bed supply situations.

Disease (i)	Low Bed Supply: 800 bed-days		High Bed Supply: 1400 bed-days	
	Admissions per Million Population (x_i)	Average Stay (days) (u_i)	Admissions per Million Population (x_i)	Average Stay (days) (u_i)
1. Varicose Veins	6.4	9.2	9.4	12.4
2. Haemorrhoids	4.1	9.0	5.9	11.2
3. Ischaemic Heart	3.6	20.7	6.3	35.6
4. Pneumonia	11.3	16.1	16.2	18.1
5. Bronchitis	8.1	32.8	14.2	33.6
6. Appendicitis	23.7	7.7	24.3	9.1

It is worth noting how the outputs of the three model runs can be understood in terms of the corresponding parameter values. For example the admission rate for appendicitis has a large power co-efficient, 44.4 (see Table 4), which in turn derives from a low value of estimated elasticity, -0.16 (see Table 3). This means that, in the model, the admission rate for appendicitis is relatively insensitive, or inelastic, to bed supply. This explains why the output for appendicitis admissions varies relatively little between the three runs (see Tables 5 and 6) and is relatively close to ideal level (see Table 4). Similar remarks apply to the average stay for bronchitis. By contrast the admission rates for ischaemic heart disease and bronchitis are relatively elastic to bed supply; thus the corresponding output figures vary much more between runs and are relatively far from the ideal levels.

7. FUTURE DEVELOPMENT OF THE MODEL

Although the model is presented here with many simplifying assumptions, it can be generalised in certain ways. For example we can add constraints of the form:

$$x_i = X_i \qquad \text{all } i \in S_x \quad ,$$

and

$$u_i = U_i \qquad \text{all } i \in S_u \quad ,$$

to represent situations where patients in some categories S_x have such priority urgency that *all* are admitted, even if patients of other categories have to be discharged early, and other categories S_u where the lengths of stay must be equal to the ideal ones.

We can also disaggregate the single resource, bed-days, and consider a number of hospital resources such as physicians, nurses, X-ray equipment, etc. The single constraint in the basic version is replaced by:

$$\sum_i x_i u_{ik} = B_k \qquad \text{for each resource } k \quad ,$$

and terms of the form $\sum_i x_i h_{ik}(u_{ik})$ replace the term $\sum_i x_i h_i(u_i)$ in the objective function. This extended version can still be solved using Lagrange Multipliers, though not so easily as the basic version. However this version has the advantage that it represents both how different types of patient make different demands on each resource and how some resources have a greater effect on admissions or length of stay than others. For example Feldstein [19] and Prevett [22] have shown that lengths of stay are much more elastic to the availability of doctors than to that of nurses. Thus this version of the model can be used to examine the consequences of changing the *mix* of resources within the hospital service, not merely the total number of beds.

Finally we would like to extend the model to cover more than one sector of the HCS at a time; e.g. to examine in-patient and out-patient specialist modes of care together. The difficulty here will be to retain sufficient simplicity in the formulation so as to allow efficient solution by Lagrange Multipliers (and so avoid being forced to use large and highly specialised computer programmes) while at the same time capturing the essence of the problem of the balance between alternative modes of care. Only further study will reveal whether this difficulty can be overcome.

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Appendix

Model Solution by Lagrange Multiplier Techniques

Problem

$$\text{Max } Z = \sum_i g_i(x_i) + \sum_i x_i h_i(u_i) \tag{1}$$

subject to

$$\sum_i x_i u_i = B \tag{2}$$

where

$$g_i(x_i) = -\frac{U_i X_i}{\alpha_i}\left(\frac{x_i}{X_i}\right)^{-\alpha_i} \tag{3}$$

and

$$h_i(u_i) = \frac{U_i}{\beta_i}\left[1 - \left(\frac{u_i}{U_i}\right)^{-\beta_i}\right] . \tag{4}$$

Let

$$L = \sum_i g_i(x_i) + \sum_i x_i h_i(u_i) + \lambda(B - \sum_i x_i u_i) \quad , \tag{5}$$

where λ is the usual Lagrange Multiplier.

Solution

$$\frac{\partial L}{\partial x_i} = g_i'(x_i) + h_i(u_i) - \lambda u_i \qquad \forall\ i \quad , \tag{6}$$

and

$$\frac{\partial L}{\partial u_i} = x_i h_i'(u_i) - \lambda x_i \qquad \forall\, i \quad , \tag{7}$$

at optimum

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial u_i} = 0 \qquad \forall\, i \quad .$$

From (7):

$$x_i h_i'(u_i) - \lambda x_i = 0 \quad ,$$

$$h_i'(u_i) = \lambda \qquad \text{since } x_i > 0 \quad ,$$

$$\left(\frac{u_i}{U_i}\right)^{-(\beta_i+1)} = \lambda \quad , \tag{8}$$

$$u_i = U_i \lambda^{-1/(\beta_i+1)} \quad . \tag{9}$$

From (6):

$$g_i'(x_i) = \lambda u_i - h_i(u_i) \quad ,$$

$$U_i\left(\frac{x_i}{X_i}\right)^{-(\alpha_i+1)} = \lambda U_i \lambda^{-1/(\beta_i+1)} - \frac{U_i}{\beta_i} + \frac{U_i}{\beta_i}\lambda^{\beta_i/(\beta_i+1)} \quad ,$$

$$\left(\frac{x_i}{X_i}\right)^{-(\alpha_i+1)} = \frac{1}{\beta_i}\left[(\beta_i + 1)\,\lambda^{\beta_i/(\beta_i+1)} - 1\right] \quad ,$$

$$x_i = X_i \left\{ \frac{1}{\beta_i} \left[(\beta_i + 1) \lambda^{\beta_i/(\beta_i+1)} - 1 \right] \right\}^{-1/(\alpha_i+1)} . \quad (10)$$

λ is obtained from substituting (9) and (10) in (2), which gives $f(\lambda) = 0$ where

$$f(\lambda) = -B + \sum_i X_i U_i \left\{ \frac{1}{\beta_i} \right.$$

$$\left. \times \left[(\beta_i + 1) \lambda^{(\alpha_i+\beta_i+1)/(\beta_i+1)} - \lambda^{(\alpha_i+1)/(\beta_i+1)} \right] \right\}^{-1/(\alpha_i+1)} . \quad (11)$$

From (11) an analytic expression for $f'(\lambda)$ can be obtained.

In general $f(\lambda) = 0$ has several roots. However we can demonstrate that there is only one root relevant to this problem. First, since $B < \sum_i X_i U_i$ (see main text, p. 13) it follows that, in general, we are searching for values of the x_i and u_i in the ranges given by

$$0 < x_i < X_i \quad , \qquad 0 < u_i < U_i \quad .$$

From inspecting equations (9) and (10) it can be seen that these ranges imply $\lambda > 1$. Second, it can be shown that $f'(\lambda) < 0$ for $\lambda \geq 1$. It can also be shown that if $a_i \geq U_i$ for all i, then $f(1) > 0$. Therefore, provided $a_i \geq U_i$ for all i, $f(\lambda) = 0$ has only one real root ≥ 1. Also $f'(\lambda) \to 0$ as $\lambda \to \infty$. Thus the function $f(\lambda)$ is well behaved in this region.

Thus the required value, λ^*, of λ can be found by solving $f(\lambda) = 0$ by gradient methods such as the Newton-Raphson procedure. We are also interested in computing the values of the elasticities, γ_i and η_i, of the variables x_i and u_i respectively, with respect to bed-days available, B. These elasticities are defined thus:

$$\gamma_i = \frac{d(\log x_i)}{d(\log B)} \quad , \quad (12)$$

$$\eta_i = \frac{d(\log u_i)}{d(\log B)} \quad . \tag{13}$$

From (9):

$$\log u_i = \log U_i - \frac{1}{\beta_i + 1} \log \lambda \quad ;$$

therefore

$$\frac{d(\log u_i)}{d\lambda} = - \frac{1}{(\beta_i + 1)\lambda} \quad . \tag{14}$$

Similarly from (10):

$$\frac{d(\log x_i)}{d\lambda} = - \frac{\beta_i \lambda^{-1/(\beta_i+1)}}{\alpha_i + 1} \Bigg/ \left[(\beta_i + 1)\,\lambda^{\beta_i/(\beta_i+1)} - 1\right] \quad . \tag{15}$$

To find the γ_i and η_i we need, in addition, to find $\frac{d\lambda}{d(\log B)}$. Since $f(\lambda) = 0$, from (11) we have

$$B = F(\lambda) \quad , \tag{16}$$

where

$$F(\lambda) = \sum_i X_i U_i \left\{ \frac{1}{\beta_i} \times \left[(\beta_i + 1)\,\lambda^{(\alpha_i+\beta_i+1)/(\beta_i+1)} - \lambda^{(\alpha_i+1)/(\beta_i+1)} \right] \right\}^{-1/(\alpha_i+1)} \quad ;$$

therefore

$$\frac{dB}{d\lambda} = F'(\lambda) \quad .$$

But from comparing (11) and (16) it can be seen that

$$F'(\lambda) = f'(\lambda) \quad ; \tag{17}$$

therefore

$$\frac{d(\log B)}{d\lambda} = \frac{1}{B}\frac{dB}{d\lambda} = f'(\lambda)/B \quad ,$$

and

$$\frac{d\lambda}{d(\log B)} = B/f'(\lambda) \quad . \tag{18}$$

We can now derive the required expressions for the elasticities γ_i, η_i. From (12), (15) and (18) we have

$$\gamma_i = -\frac{\beta_i}{\alpha_i + 1} \cdot \frac{B\lambda^{-1/(\beta_i+1)}}{\left[(\beta_i + 1)\,\lambda^{\beta_i/(\beta_i+1)} - 1\right] f'(\lambda)}$$

$$= -\frac{\beta_i}{\alpha_i + 1} \cdot \frac{B}{\left[(\beta_i + 1)\,\lambda - \lambda^{1/(\beta_i+1)}\right] f'(\lambda)} \quad , \tag{19}$$

and from (13), (14) and (18)

$$\eta_i = -\frac{1}{(\beta_i + 1)} \cdot \frac{B}{\lambda\, f'(\lambda)} \quad . \tag{20}$$

Since λ and $f'(\lambda)$ can readily be computed in the same algorithm, the γ_i and η_i can be determined, using (19) and (20), for any given values of B and the α_i and β_i.

Equations (19) and (20) are particularly important in estimating the values of the model parameters α_i and β_i. They were used in this way for the illustrative model run described in Section 6 of the main text, using the empirical estimates of elasticities obtained by Feldstein and listed in Table 3.

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