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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
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Foreword

Water temperature is a very important parameter of water quality. In the natural conditions water temperature is determined by hydrological and meteorological factors, in particular by the heat budget of river or lake. For the few decades, however, due to the many new power and industrial developments, anthropogenic factors have greater influence on river and lake water temperatures. The temperature of water affects biological and chemical processes in aquatic environment and can play an important role in shaping ice phenomena.

In this paper a simple, one-dimensional thermal model is presented. It allows to determine the average water temperature in the cross-sections of the river downstream of the heated water discharge point. The model has been developed in the Department of Water Physics at the Institute of Meteorology and Water Management, Warsaw, Poland. It can be useful for the design of water management systems in taking decisions on hydraulic structures or on location of large conventional and nuclear power plants.

The work has been done within the framework of cooperation between the Polish scientific institutions and the International Institute for Applied Systems Analysis in Laxenburg, Austria.

Professor B.R. Döös
Leader
Environment Program

Simple Model of Thermal Pollution in Rivers

D. Jurak and A. Wisniewski

Institute of Meteorology and Water Management
Warsaw, Poland

1. Introduction

The work on the *Decision Support Systems (DSS) for Large International Rivers* (LIR) was started at IIASA in 1986. The main objective of the project is the methodological aid for national and international institutions in solving the conflicts in international river basins. Cooperation with some national science institutions includes the Institute Geophysics of the Polish Academy of Sciences, and the Institute of Meteorology and Water Management in Poland. This cooperation depends on working out the individual parts of the main system by the national institutions.

In the paper, the main methodological assumptions and the computer program for one part of the system is presented. This paper is concerned with the distribution of water temperature in the river influenced by thermal discharges from a power plant or other industrial developments.

Work on this problem was undertaken in Poland towards the end of the 1960's in connection with the cooling systems designed for the group of power plants in the Konin Lakes region, and continued as part of the planning studies concerned with location and design of large thermal power stations along the Vistula river and other rivers in Poland.

Several thermal models were developed within the framework of these studies (4.6), and one of them is presented in this paper. This model is particularly useful in decision making.

Development of model is based on the following assumptions:

1. The model should be simple enough to be useful for a rapid analysis of different locations of thermal power stations and for estimation of the impact of heated water discharges and related changes in the aquatic environment.
2. Computations should be based on such hydrological and meteorological data, majority of which is easily accessible from standard measurement network.

3. Computer program should be operational on IBM-PC/AT microcomputers which are generally available in the developing countries.

A one-dimensional thermal model that satisfies these assumptions is chosen. This model enables the calculation of water temperature changes in river cross-sections along the river for different locations of thermal power plants (conventional and nuclear), for different parameters of their operation, and for different hydrological and meteorological conditions.

The role of thermal models in the decision making may be very important. Discharges of heated waters into polluted rivers have negative impacts on aquatic life and water quality. Thus, one of the important elements of any aquatic ecological model is water temperature, which for the existing thermal conditions can be measured and for the planning of future situations must be simulated. In some climatic zones heated water discharges may also influence significantly the physical processes associated with ice phenomena, their spatial distribution, and duration of ice cover. Therefore, heated water discharges may also have an impact on the navigation conditions of the river and on the operation of hydrotechnical installations during the winter period. For all these reasons the model described hereby may be a useful tool for taking decisions related to water resources planning and operation.

2. Basic Assumptions and Equations

Let us take one sector of the river (Figure 1), where the existing or planned industrial plant is discharging heated water into the river. Depending on topographic conditions, character of tributaries, etc., this sector may be divided into several segments. For each section, the flow and water temperature at the final cross-section are the boundary conditions for the next segment.

Taking into account the first segment downstream of the outlet of the heated water, natural conditions of the river flow may be described by flow rate and water temperature. The following notation is introduced:

q [m^3/s] - flow rate in river segment,

T_{ox} [deg] - water temperature in the cross-section situated x meters down the river from the beginning cross-section,

T_{op} [deg] - water temperature at the beginning cross-section $x = 0$.

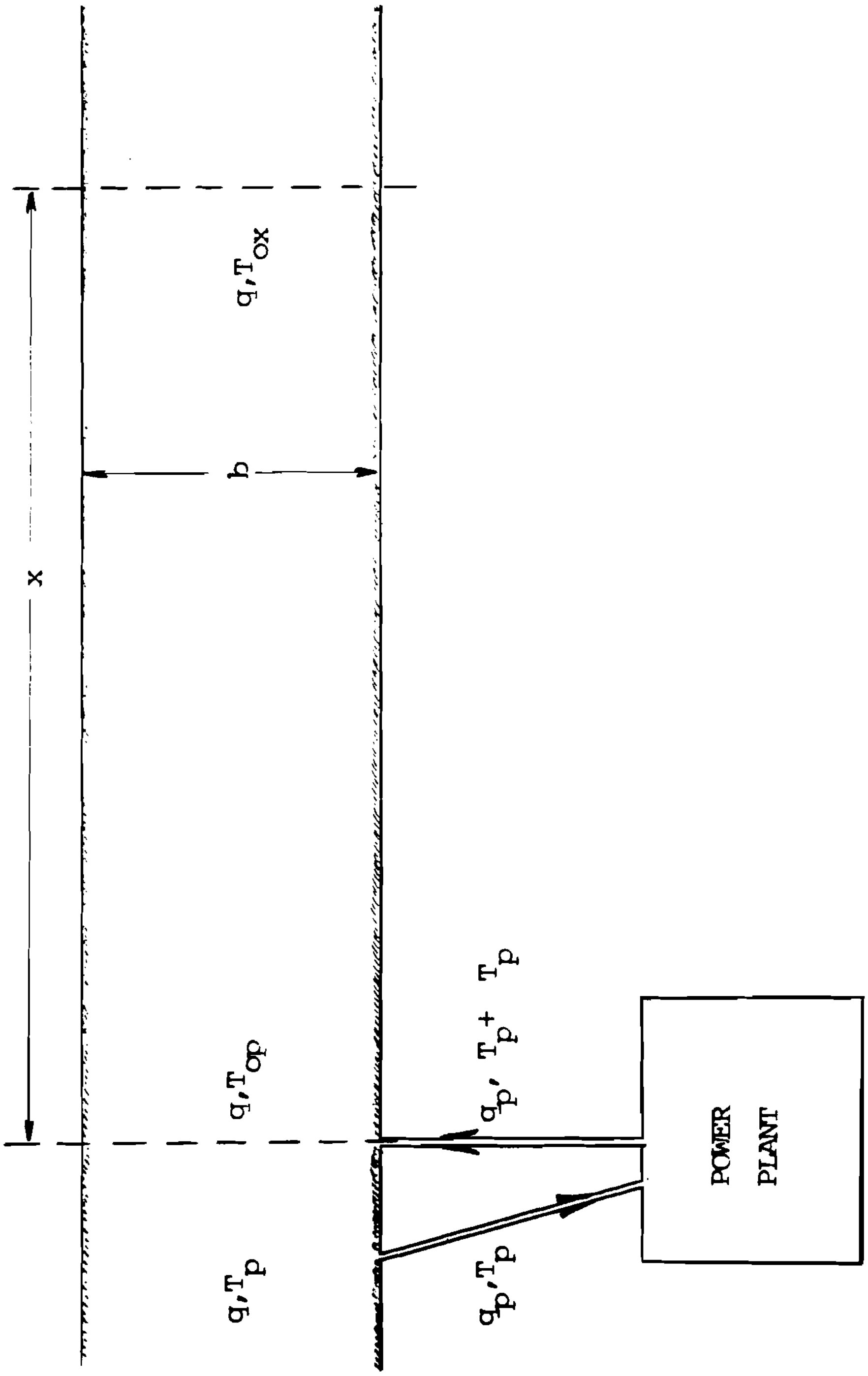


Figure 1. River segment with discharge of the heated water.

Cooling process and the water temperature depend on meteorological conditions. Water temperature simulations changes can be done either on the basis of hydrological and meteorological data taken from measurements in the chosen period of time or on the basis of computed characteristic parameters, such as river flows for a given probability of recurrence or for some meteorological conditions.

Because of the dependence between hydrological and meteorological parameters, the first of these two approaches is recommended. It means that computations should be based on continuous historical data (e.g. all years between 1951 and 1980), or on periods that are critical from the meteorological and hydrological viewpoints, characterized by high air temperature and low flow in the river.

It should be underlined, that in the segment situated directly below the heated water discharge, water temperature in the beginning cross-section is calculated as a mixing temperature from equation:

$$T_{op} = (1/q)[T_p(q-q_p) + (T_p+\Delta T_p)q_p)] \qquad (1)$$

where

T_p - water temperature measured above the heated water discharge,

ΔT_p the difference between discharged and natural water temperature,

q_p the flow of heated water in cooling system,

q river flow above the water intake.

In the conventional power plants ΔT_p is 8–10 [deg], and the flow rate in the cooling system q_p–45 [m^3/s] per 1000 MW of installed capacity.

For development of the basic equation, only the first segment below the heated water discharge is considered. Calculations for other sections may be carried out on the basis of the same equation changing meteorological and hydrological data only.

The river sector does not have to be divided into segments if there are no changes in meteorological and hydrological data along full length. It means that if there is a tributary, the sector has to be divided into segments up and down from the tributary. For the cross-section situated just below the tributary, the value of water temperature has to be calculated as the waged average of the temperatures of the main river and the tributary. It is always necessary to make careful analysis of hydrological and meteorological conditions before simulation of water temperature is made.

The one-dimensional thermal process in the river below the heated water discharge may be described as:

$$\rho c_w (q/b)(dT_o/dx) = Q_c + Q_b \tag{2}$$

where

ρ	1000 [kg/m^3] – water density,
c_w	4187 [J/kg/deg] – specific heat of water,
$b[m]$	average width of river bed,
$x[m]$	distance from the beginning of the segment,
$Q_c[W/m^2]$	total heat exchange between water surface and atmosphere,
$Q_b[W/m^2]$	total heat exchange between water and river bed.

Precise evaluation of Q_b is difficult and requires complex measurements. Moreover, the values of Q_b are small compared to other factors in equation (2), and this is why they may be taken from Table 1 based on measurements published in the guide book (10).

The most important factor in equation (2) is the value of Q_c describing energy exchange between water and atmosphere. It depends on water temperature as well as on meteorological and solar radiation elements. Dependence on water temperature may be expressed by the following equation:

$$dT_o/dx = f(T_o) \tag{3}$$

where function $f(T_o)$ is rather complex and depends on the value of Q_c. Many authors tried to simplify this formula by linear function, but it was shown that this is not enough precise for solving equation (2) or (3). The second order approximation was chosen as the following in Jurak [1]:

$$Q_c = \alpha + \beta T_o + \gamma (T_o)^2 \tag{4}$$

This enables solution of equation (3) with satisfactory accuracy. In equation (3) α,β and $\gamma[W/m^2]$ are heat exchange coefficients. The methods to be used for evaluation of these coefficients are described in the next section of this paper.

Table 1. Approximate values of Q_b $[\frac{W}{m^2}]$. If average depth of water is more than 50 m, $Q_b = 0$.

North latitude	Average depth of water body [m] 0–5	10	15	20	North latitude	Average depth of water body [m] 0–5	10	15	20
	January					July			
30	13	12	9	8	30	-11	-9	-8	-7
40	11	9	8	8	40	-11	-9	-8	-7
50	7	6	6	5	50	-12	-11	-9	-8
60	5	5	3	3	60	-12	-11	-9	-8
70	3	3	2	2	70	-12	-12	-11	-8
	February					August			
30	8	8	6	5	30	-5	-5	-3	-3
40	6	6	5	3	40	-5	-5	-3	-3
50	5	3	3	2	50	-5	-3	-3	-2
60	3	2	2	2	60	-3	-3	-2	-2
70	2	2	2	1	70	-3	-3	-2	-2
	March					September			
30	-3	-3	-2	-2	30	2	2	2	1
40	1	1	1	0	40	3	2	2	2
50	3	3	2	2	50	5	3	3	2
60	2	2	2	2	60	5	5	5	3
					70	6	6	5	5
	April					October			
30	-19	-17	-15	-13	30	14	13	12	9
40	-14	-13	-12	-9	40	14	12	11	8
50	-8	-6	-6	-5	50	12	11	9	8
60	0	0	0	0	60	60	12	9	8
	May					November			
30	-16	-14	-13	-11	30	16	14	13	11
40	-16	-14	-13	-11	40	15	13	13	11
50	-15	-14	-12	-11	50	13	12	11	8
60	-14	-13	-12	-9	60	12	11	9	8
70	-9	-9	-8	-6	70	9	9	8	6
	June					December			
30	-15	-14	-12	-11	30	17	15	14	12
40	-16	-14	-13	-11	40	14	12	11	9
50	-16	-14	-13	-12	50	11	9	8	7
60	-16	-14	-13	-12	60	7	6	6	5
70	-17	-15	-13	-12	70	5	3	3	2

3. Energy Exchange Coefficients for an Open Water Surface

It is generally acknowledged that the formula for calculating total heat exchange between water surface and atmosphere is of the form:

$$Q_c = Q_{sr} - Q_{lr} - Q_e - Q_h \tag{5}$$

where

$Q_{sr}[W/m^2]$	short wave radiation balance of water surface,
$Q_{lr}[W/m^2]$	long wave radiation balance of water surface,
$Q_e[W/m^2]$	energy utilized by evaporation,
$Q_h[W/m^2]$	energy carried away from the water surface as sensible heat.

The last three factors of the right side of equation (5) are nonlinear functions of water temperature presented in Jurak (1).

Development of formula (5) into a power sequence

$$Q_c = Q_c(a) + [dQ_c(T_o)/dT_o]_{T_o=a}(T_o-a) + \tag{6}$$
$$[d^2Q_c(T_o)/dT_o^2]_{T_o=a}(T_o-a)^2$$

enables transformation of equation (5) into the form of equation (4) where coefficients α,β and τ can be expressed as:

$$\alpha = Q_{sr} + 4.536 - 524.6(t_p+273.16)^4(0.41-0.05\sqrt{e})(1-0.05n_o)10^{-10} + \tag{7}$$
$$+ 2.373\,t_p - 697.9\,B_1(\mu)u_1[(e_o - ae_o' - e - 0.61\,t_p)\ (1 - \frac{0.00075\,a}{B_1(\mu)u_1^2} + 0.5\,a^2e_o'']$$

$$\beta = -2.373 - 697.9\,B_1(\mu)u_1[0.61 + \frac{0.00075}{B_1(\mu)u_1^2}\,[e_o - e - 2ae_o' - 0.61(a+t_p)] + e_o' - ae_o''] \tag{8}$$

$$\gamma = -697.9\,B_1(\mu)u_1\left[\frac{0.00075}{B_1(\mu)u_1^2}\,(e_o' + 0.61) + 0.5e_o''\right] \tag{9}$$

where

t_p [deg] air temperature,
$e[hPa]$ vapour pressure of the air,
$u_1[m/s]$ wind velocity,
n_o total cloudiness (in 1÷10 degrees scale).

The values of meteorological parameters should be taken from the nearby meteorological station. Appropriate selection of parameter a is of a special significance. It is the water temperature that is used for transformation of equation (5) into a sequence. Its value should be as near as possible to the predicted water temperature.

For practical purposes it may be assumed that $a = T_{op}$, i.e. it is equal to the water temperature in the beginning cross-section calculated according to equation (1). The other parameters should be calculated as shown by Jurak (2):

$$e_o = 6.11\,(10^{7.63a/(242+a)}) \tag{10}$$

$$e_o' = \frac{4252}{(242.0+a)^2}\, e_o \tag{11}$$

$$e_o'' = \frac{4252}{(242.0+a)^3}\left[\frac{4252}{(242.0+a)} - 2\right] e_o \tag{12}$$

$$B_1(\mu) = \frac{2.80(k_1/u_1)^{1-\mu}}{(1-\mu)(1-2\mu)^{1-2\mu}\Gamma(\mu)X^{\mu}} \tag{13}$$

where $X[m]$ is the mean length of air mass transformation above water surface. With the satisfactory accuracy for practical use, for rivers one may assume $x = 2b$ (b is the mean river width). Other coefficients should be calculated as shown by Jurak (1, 2), Laichtman (7,8), and Ognieva (9):

$$\mu = \frac{\frac{1}{\pi}\,\mathrm{arc}\;ctg(2.057 + 0.238\frac{\Delta t}{u_1^2})}{1+\frac{2}{\pi}\,\mathrm{arc}\;ctg(2.057 + 0.238\frac{\Delta t}{u_1^2})} \tag{14}$$

$$\epsilon = 0.167(27\mu^2 + 15\mu - 2) \tag{15}$$

$$\frac{k_1}{u_1} = 0.144\frac{\epsilon z_o^{2\epsilon}}{(1-1.75\epsilon)^2(1-z_o^{\epsilon})} \tag{16}$$

where

$\Delta t =$ $a - t_p$

$z_o =$ 0.0005 [m] is roughness coefficient of water surface.

Equations (7) to (16) enable calculation of coefficients for equation (4) on the basis of:

- known value of meteorological elements t_p, u_1, e, n_o, and Q_{sr},
- assumed value of the coefficient $a = T_{op}$.

It should be underlined that the above-mentioned meteorological coefficients have to be calculated for each period of time (e.g. as average for a day, five days, 10 days, etc.). The length of the period depends on the objectives of the calculations and the access to measurement data. Calculations for the time period of less than one day are not recommended.

The above equations have been verified many times on the basis of experimental data. It has been shown by Jurak (4, 6) that the accuracy of calculated water temperature and heat balance elements is good enough for practical purposes.

4. Distribution of the Temperature Along the River Stretch

Substituting formula (4) to the differential equation (2) the following equation is obtained:

$$\rho c_w (q/b)(dT_o/dx) = \alpha + \beta T_o + \gamma (T_o)^2 + Q_b \tag{17}$$

Solving equation (17) for the boundary condition $T_0 = T_{op}$, and $x = 0$, the formula for calculation of water temperature T_{ox} at the x distance from the initial point (e.g. discharge cross-section) is obtained in the following form:

$$T_{ox} = -(\beta/2\gamma) + \{[\beta^2 - 4(\alpha + Q_b)\gamma]^{1/2}\}/2\tau \cdot (1 + \Phi) \tag{18}$$

where:

$$\Phi = \{[\beta + 2\gamma T_{op} - (\beta^2 - 4(\alpha + Q_b)\gamma)^{1/2}]/[\beta + 2\gamma T_{op} + (\beta^2 - 4(\alpha + Q_b)\gamma)^{1/2}]\} \cdot \tag{19}$$
$$\cdot \exp [\frac{bx}{q}(\beta^2 - 4(\alpha + Q_b)\gamma)^{1/2}]$$

All symbols in equations (17), (18) and (19) were discussed earlier. It should be noted that T_{ox} is a nonlinear function of a distance from the discharge point.

For $x \longrightarrow \infty$, equation (18) takes the form:

$$T_{o\infty} = -(\beta/2\gamma) - \{[\beta^2 - 4(\alpha + Q_b)\gamma]^{1/2}\}/2\gamma \tag{20}$$

The above equation is for heat exchange equilibrium condition ($Q_c + Q_b = 0$). T_{ox} calculated from formula (18) is the mean water temperature at the cross-section x meters from the initial point.

If the evaluation of the mean water temperature in river segment from $x = O$ to $x = l$ is required, then the value of T_{ox} should be integrated along the river segment.

$$\bar{T}_o = \int_0^l T_{ox} dx \tag{21}$$

Following transformations:

$$\bar{T}_o = -(\beta/2\gamma) - [q/(2\gamma bl)]\{ln[(1-\Phi)^2/\Phi] - \ln[(1-\phi)^2/\phi] \tag{22}$$

where

$$\phi = \Phi/\{\exp[(bl/q) \cdot (\beta^2 - 4(\alpha + Q_b)\gamma)^{1/2}]\} \tag{23}$$

For the solution of practical problems related to the distribution of water temperature along the river, formula (22) is less useful than the fundamental equation (18). The latter enables to watch the water temperature changes along the river together with all heat water discharges and natural tributaries.

It should be stressed that in the discharge zone there are significant changes in the cross-sectional water temperature. As the laboratory and field experiments indicate, the distribution of water temperature in this zone depends on the ratio of discharged flow volume to river flow volume and on the ratio of discharge flow velocity to river flow velocity. Assuming constant value of a discharge, the mixing process is more intensive for the case of low flow in the river and distribution of temperature for such cases is more uniform.

As mentioned above, the influence of the heated waters on the river ecology is usually investigated for such periods when at the same time appear the hot weather, high water temperature and low flow in the river appear simultaneously. In such conditions the mixing process in the river is relatively quick, and application of equation (18) for the computation of water temperature in the vicinity of the discharge zone does not lead to significant errors.

In the other cases, when mixing heated water in river is not so quick, the use of two and three-dimensional models is needed. Otherwise, the point of uniform water temperature in cross-section should be established on the basis of laboratory experiments, and then equation (18) should be used.

Application of two and three-dimensional models is very complex. From the practical point of view, the complex thermal models are unnecessary for studying location of power plants along the river.

The above relationships can be used only if adequate observational data originating from the standard network of hydrological and climatological stations are available. For each river segment under consideration one should select the representative meteorological station. Selection of such a station is not an easy task. It requires good understanding of the local conditions and considerable professional experience of the team charged with such a task. For calculation of water temperature the following parameters are needed:

a) hydrological data:
   - reliable flow for a given river segment (q),
   - mean width of the river (b),
   - water temperature immediately below the thermal water discharge for evaluation of the initial water temperature (T_{op}) with the use of formula (1);

b) meteorological data:
   - short wave radiation balance Q_{sr} (measured or calculated),
   - cloudiness (n_0),
   - air temperature (t_p),
   - wind speed (u_1), and
   - water vapour pressure (e) measured at the height of 2.0 m at the chosen meteorological station.

In most cases, the above data are easily accessible. When in the segment of the river being investigated no water measurements are carried out, one may calculate the data with the satisfactory accuracy assuming that $T_p = T_o$, where T_o is calculated for the condition of equilibrium according to equation (20).

The sequence of calculations depends on the character of a given task. In the case of simulation of water temperature below the heated water discharge, it is done as follows:

a) the river stretch is divided into segments considering location of heated water discharge points, more significant river tributaries, etc.

b) for each river segment the reliable meteorological station is chosen and values of hydrological data (q,b and T_p for the first cross-section above the heated water discharge), and meteorological data (Q_{sr}, t_p, e, u_1, n_o) are defined.

c) the values of alpha, beta, gamma coefficients are determined according to equations (7) to (9) and values Q_b according to Table 1;

d) water temperature distribution along the river channel is calculated with the use of equations (18) to (19), and in the justified cases calculated are also mean temperatures for individual segments with the use of equations (22) to (23).

These calculations may be carried out either for the selected critical periods, i.e. for the cases of particularly low flows, or they may be repeated over and over for multiannual historical flow series.

The relationships presented above are then the tool for simulation of thermal conditions for various conditions of the power plant operation.

5. Description of the Computer Program

The computer program is called TEMPERAT, which enables simulation of water temperature for the river stretch under consideration. First some information about preparing data for river temperature simulation.

5.1. General principles of input data selection.

When starting calculations of water temperature at longitudinal profiles of a river, the following tasks have to be tackled:

1. Fixing computational cross-sections on the river.
2. Selecting meteorological and actinometric stations data that will be used for calculations of heat exchange coefficients.
3. Assigning computational river segments to meteorological and actinometric stations.

When fixing computational profiles one has to take into account major tributaries and discharges of heated water.

Assignment of computational segments to the selected meteorological stations, is carried out on the basis of the climatic diversity of the respective region.

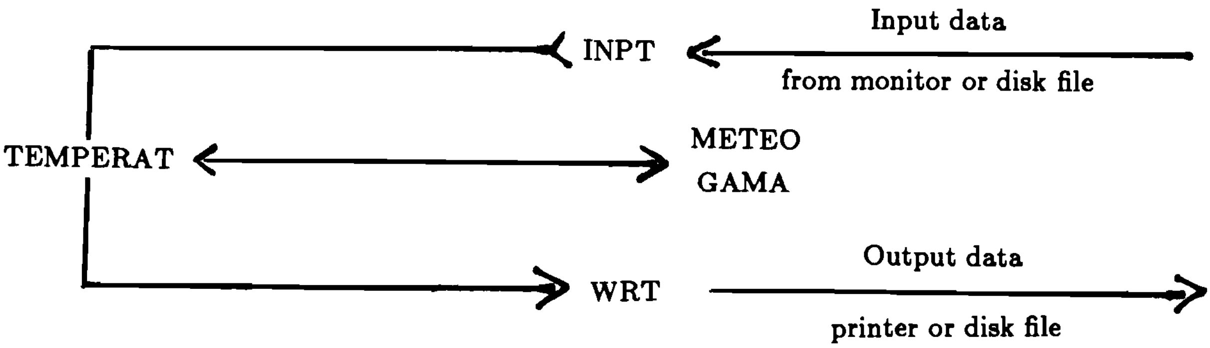
5.2. Main principles of program TEMPERAT operation

Program and input data structure

The program TEMPERAT includes the following elements:

1. Subroutine INPT which reads the data
2. Main Program TEMPERAT
3. Subroutine METEO
4. Subroutine GAMA
5. Subroutine WRT which produces outputs

The program structure can be presented as follows:



Input data are read by subroutine INPT called by the main program TEMPERAT. The subroutine reads data from display monitor (keyboard) or optionally from disk files.

The program uses the following groups of data:

1. Control data
2. Meteorological data
3. Geometrical data
4. Hydrological data

Selection of main river segments and assigning numbers to them is an important element for preparation of input data and subsequent calculations. The calculations are made for successive segments. River flow between first and last cross-section of a given river segment is assumed to be constant. Water temperature changes at longitudinal profiles at each river segment are the result of heat exchange between water and environment (atmosphere and river bottom). This is why their boundaries are determined by tributaries, water intakes or discharges of heated water, e.g. from power plants, which affect significantly heat balance of a river. As boundary cross-section we assume any cross-section of known flow intensity and water temperature.

For each main segment we have to know flow intensity and water temperature at its beginning, and data set from the nearest meteorological station.

After reading input data programme TEMPERAT selects river segments according to their numbers. The numbers assigned should ensure that calculations are made in the first place for those segments in which water temperature at its last cross-section is used for calculations in the next segment. If for a current segment there exists a tributary, then the main program calculates the initial temperature, after mixing, according to the formula.

$$T_{op} = \frac{T_{om} q_m + T_{ot} q_t}{q_m + q_t} \tag{24}$$

where m relates to the main river and t to the tributary.

Each river segment is for computational reasons divided into a number of sub-segments. In the program it is assumed that their length is about 5 000 m and test calculation indicate that this ensures required accuracy of simulation. The parameter X in formula (13) is assumed automatically as a double river width.

A single set of meteorological data is assumed for a given segment, then successive sub-segments are called, beginning at the first until the last cross-section of the river segment. For a given segment subroutine METEO is called once, which with the help of subroutine GAMA calculates for each subsequent heat exchange coefficients ALFA, BETA, GAMMA and then calculates water temperature, which becomes the initial temperature for the next sub-segment.

Finally after calculating in the same way the water temperature for each segment, it is stored in two-dimensional array TEND for each time period (day, month or 10-day period).

The calculated water temperature distributions, for the whole river stretch under consideration, found for successive computational time periods, are then written to OUTPUT.DAT file in external store (with the help of subroutine WRT). From there the data can be sent to printer or used for further calculations.

5.3. Additional information about program

Programme TEMPERAT has been written for personal computer compatible with IBM PC/AT and coded in FORTRAN F77L. The programme includes a vast number of remarks and comments which should make it easier to understand its operation. Its listing includes detailed description of all variables both global in COMMON blocks and local in each subroutine.

Global variables are placed and described in DATA.INC file, which is automatically added to all subroutines by the use of INCLUDE command. The program is written in a way that ensures saving internal store, therefore during calculations data and results are stored in external store files. These files should be stored on virtual or hard disk together with the executive program in order to save computer time.

The only arrays kept in internal store are the following two-dimensional arrays (in COMMON block [TAB]):

1. Water temperature at the end of main river segments

   TEND (segment number, time period number)

2. Water temperature at the beginning of main segment

   TBEG (segment number, time period number)

3. Flows for main river segments

   QODC (segment number, time period number)

Subroutine INPT reads input data. Subroutine WRT provides output data and data for control tests. The executive program is in TEMPERAT.EXE file.

5.4. Test Calculations

In order to test the computer programme the 310.1 km long stretch of the River Vistula was used. The boundary cross-sections (first at 194.1 km and lastly 504.2 km) were selected at the points where measurements of water level, water temperatur and flow are carried out. Along this river stretch we distinguished twelve main computational cross-sections, four of which are identical with the existing measurement cross-sections.

Taking into account the climatic regions of Poland (12), meteorological and actinometric stations located in the Vistula valley were assigned to the computational river segments for calculation of water temperature at cross-sections 1–4 meteorological data from Sandomierz station, and actinometric data from Pulawy station were used. For other cross-sections data from meteorological and actinometric station Warszawa-Bielany have been used.

As at the actinometric station in Poland total short wave radiation Q_{cr} is measured, for the test calculation values of Q_{sr} in formulae (5) and (7) were calculated using value of Q_{cr} from the following relationship:

$$Q_{sr} = Q_{cr}(1-r) \tag{25}$$

where r is the albedo of water surface (per cent) measured or determined from special tables (10).

Water temperature calculations were made for monthly time periods using input data for the period April 1963–October 1963. Methodology of input data preparation is described in (2)–(4). The operation of the thermal model and the associated computer programme was tested on a computer compatible with IBM PC/AT. Calculated mean monthly values of mean water temperature in individual cross-sections were compared with analogous water temperatures found on the basis of measurements.

The comparison of calculated and measured water temperatures was carried out for four cross-sections of the selected segments of the Vistula river, namely, Szczucin, Sandomierz, Pulawy and Warsaw. The results are presented in Table 2 where it is shown that the calculated and measured water temperature profiles are similar. The differences between calculated and measured water temperatures have different signs with the maximum of 1.1 °C. In more than 84% of the cases, differences are less or equal to 0.5 °C, thus indicating considerable accuracy of simulation.

In conclusion, we can state that the calculation results obtained indicate successful operation of the computer program and sufficient accuracy of the developed thermal model.

6. An Example

Possible application of the proposed methodology will be illustrated on a 131.7 km long segment of the Vistula river between the gauging stations Pulawy and Warsaw. Along this river stretch two important tributaries – the Wieprz river and the Pilica river – are entering Vistula. In the Kozienice town (Figure 2) a thermal power plant has been situated with the following characteristics:

Table 2. Comparison of calculated T_{oz} and measured T_o monthly average values of the water temperature at the control cross-section of the Vistula river.

Cross-section		Month						
		IV	V	VI	VII	VIII	IX	X
Sandomierz	T_{oz}	8.5	15.8	20.1	23.2	21.3	17.7	11.0
	T_0	9.0	16.2	19.7	22.6	21.0	17.8	10.7
	ΔT	-0.5	-0.4	0.4	0.6	0.3	-0.1	0.3
Pulawy	T_{oz}	8.9	16.7	19.7	22.8	20.8	16.8	10.2
	T_o	9.2	17.3	20.0	22.8	21.1	17.6	10.7
	ΔT	-0.3	-0.6	-0.3	0	-0.3	0.8	-0.5
Warsaw	T_{oz}	9.2	17.2	19.4	22.5	20.5	16.1	9.4
	T_o	8.7	17.7	19.8	22.1	20.8	17.0	9.5
	ΔT	0.5	-0.5	-0.4	0.4	-0.3	-0.9	0.1

$$N = 2400\ \mathrm{MW}$$
$$q_p = 108\ \mathrm{m^3/sec}$$
$$\Delta T_p = 8.0\ \mathrm{deg\ C}$$

where ΔT_p is the difference between water temperature discharged from the power plant and the natural water temperature.

The described segment of the Vistula river has been divided into four sectors:

1. Pulawy–Wieprz river (19.3 km long, b = 225 m)
2. Wieprz river–Kozienice (34.2 km, b = 276 m)
3. Kozienice–Pilica river (31.0 km long, b = 280 m)
4. Pilica river–Warsaw (47.2 km long, b = 282 m)

The one-dimensional longitudinal profile of the Vistula water temperature has been calculated for the period IV–X 1963 with the monthly time intervals.

In Table 3, necessary hydrological and meteorological data are given based on observations done by the State Institute for Meteorology and Water Management and its local observing stations. It should be noted that both hydrological and meteorological conditions are changing substantially during the year. Using formulae (1), (18) and (22) the following temperature values have been calculated for each of the four sectors of the Vistula river (all values in deg C):

- the initial temperature T_{op},

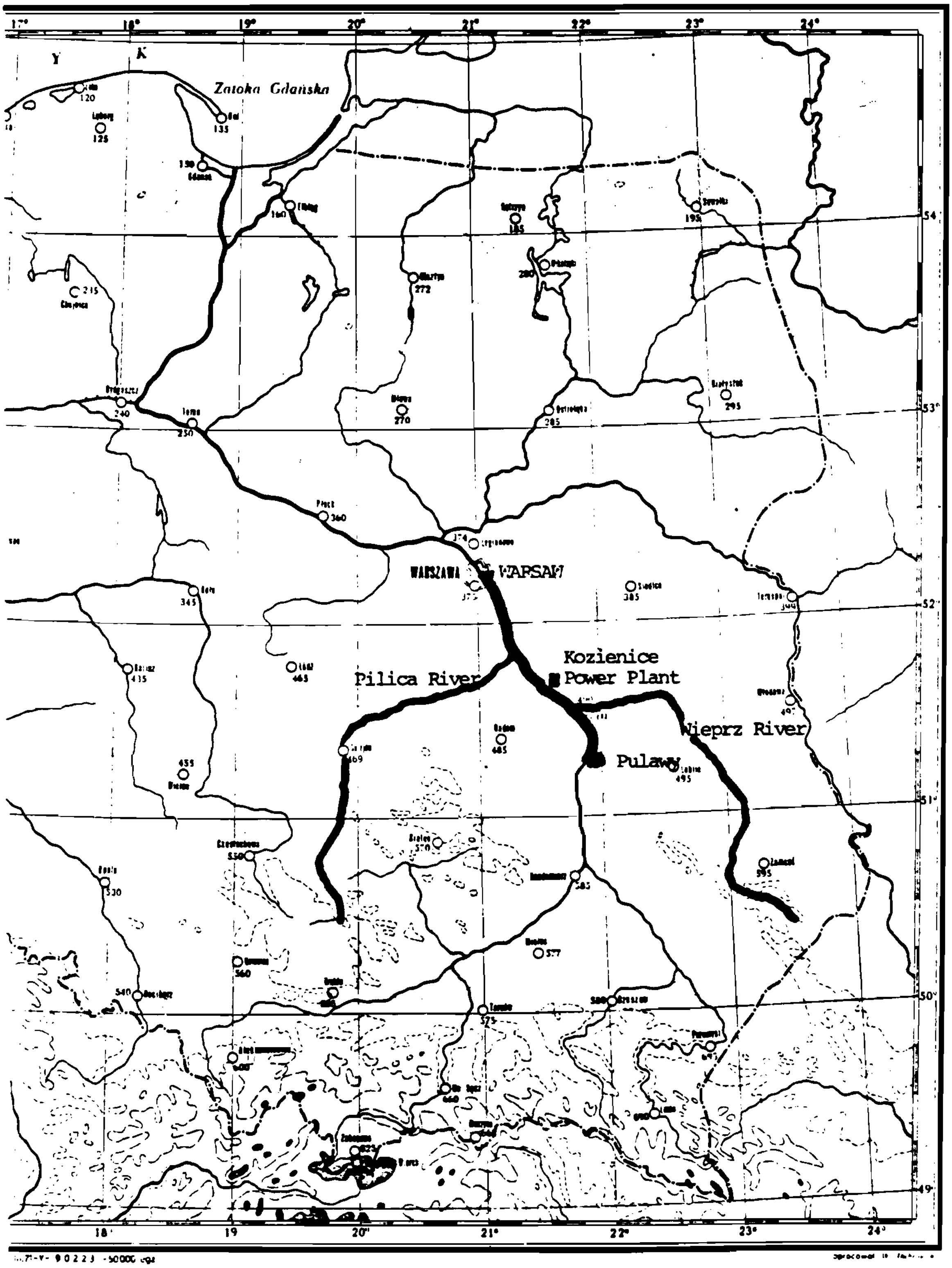


Figure 2. Vistula stretch with the Kozienice Power Plant.

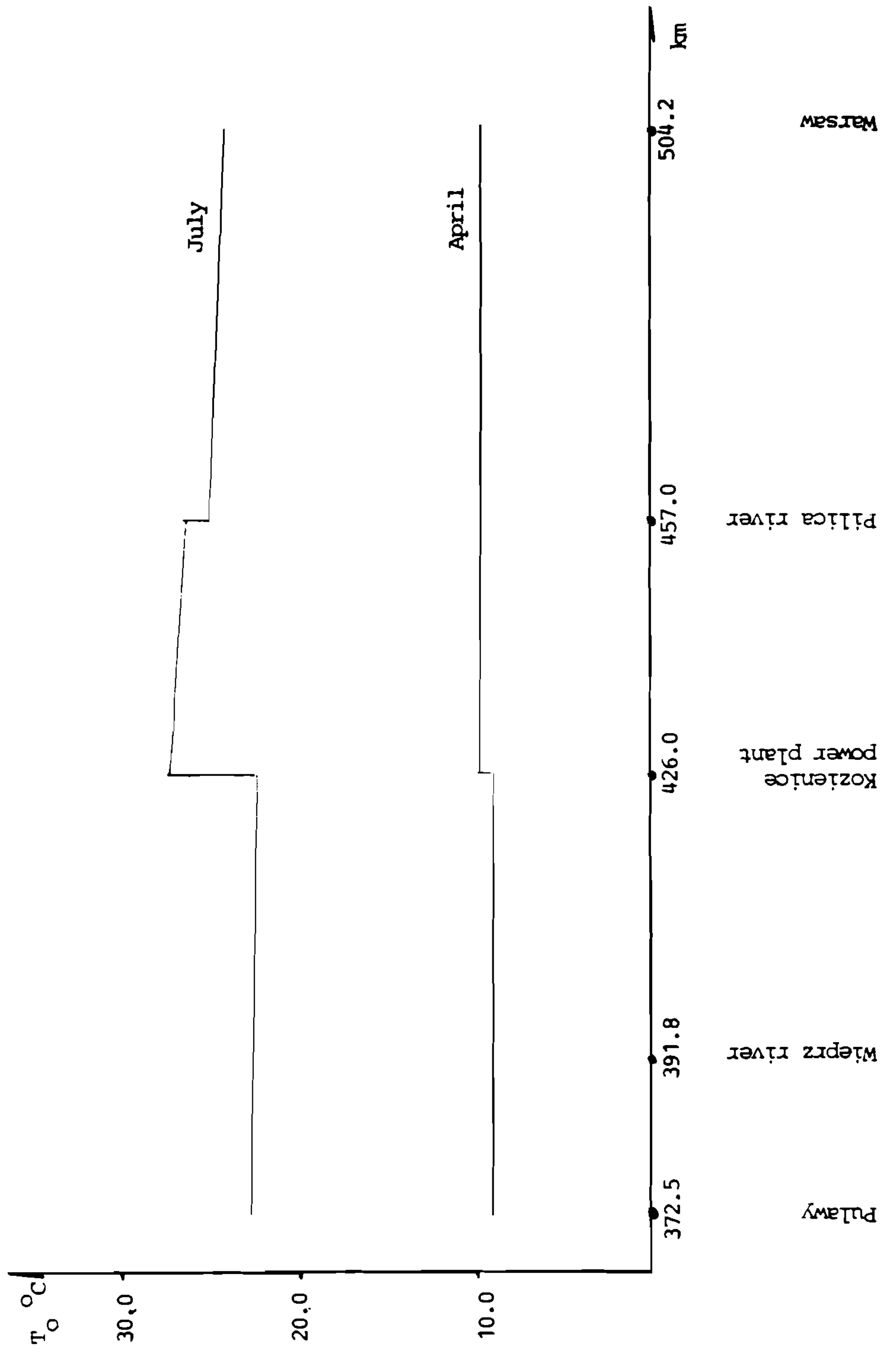


Figure 3. Temperature profile for Vistula river (year 1963).

Table 3. Input data for calculating temperature profile for the segment of Vistula river between gauging stations Pulawy and Warsaw.

Parameter	Month						
	IV	V	VI	VII	VIII	IX	X
Q_{cr} [W/m^2	152	204	227	259	175	114	60
r	0.08	0.07	0.07	0.07	0.07	0.08	0.10
t_p [deg C]	8.2	15.7	17.1	20.9	19.3	14.6	8.2
e [hPa]	7.9	12.9	12.9	14.1	15.8	14.0	9.6
u_1 [m/sec]	2.7	2.4	2.8	2.6	2.7	2.7	2.8
n_0 [1 ... 10]	5.9	5.8	5.8	4.6	6.0	5.5	7.6
Q_b [W/m^2]	-8	-15	-16	-12	-5	5	12
T_0 [deg C] Vistula-Pulawy	8.9	16.6	19.7	22.8	20.9	16.8	10.3
T_0 [deg C] Wieprz river	8.9	17.6	19.6	21.0	20.1	16.4	9.5
T_0 [deg C] Pilica river	8.4	14.9	18.0	19.6	18.6	15.5	9.0
q [m^3/sec] Vistula-stretch 1	873	515	295	163	162	205	345
q [m^3/sec] Vistula-stretch 2	945	549	313	176	175	218	367
q [m^3/sec] Vistula-stretch 3	890	526	301	180	165	209	352
q [m^3/sec] Vistula-stretch 4	1053	592	338	190	183	227	354
q [m^3/sec] Wieprz river	70	32	17	11	12	14	21
q [m^3/sec] Pilica river	58	56	29	18	18	34	42

- the temperature T_{oz} at the end of a given sector, and
- the average temperature $\bar{T}_0$ for each sector,

The above characteristics are given in Table 4 and are presented in Figure 3. It may be observed that after heating the river in Kozienice by the power plant, the cooling process is going rather slowly. Even in Warsaw at the distance of 88.2 km from Kozienice power plant, the water temperature is still substantially higher than above the heated water discharge.

Table 4. Calculated water temperature profile for the segment of Vistula river between gauging stations Pulawy and Warsaw (year 1963).

River Stretch n	Parameter [deg C]	Month						
		IV	V	VI	VII	VIII	IX	X
1	T_{op}	8.9	16.6	19.7	22.8	20.9	16.8	10.3
	T_{oz}	8.9	16.6	19.6	22.7	20.7	16.6	10.2
	$\bar{T}_o$	8.9	16.6	19.7	22.7	20.8	16.7	10.2
2	T_{op}	8.9	16.6	19.6	22.6	20.7	16.6	10.1
	T_{oz}	9.0	16.6	19.5	22.5	20.5	16.4	9.9
	$\bar{T}_o$	8.9	16.6	19.6	22.5	20.6	16.5	10.0
3	T_{op}	9.9	18.3	22.4	27.3	25.8	20.5	12.4
	T_{oz}	9.9	18.1	21.7	25.5	23.8	19.3	11.9
	$\bar{T}_o$	9.9	18.2	22.0	26.3	24.7	19.9	12.1
4	T_{op}	9.9	17.8	21.4	25.0	23.3	18.8	11.6
	T_{oz}	9.9	17.7	20.8	23.9	22.1	17.9	11.1
	$\bar{T}_o$	9.9	17.7	21.1	24.4	22.7	18.3	11.3

REFERENCES

1. Jurak, D. (1978) Heat and mass exchange coefficients for open water surfaces. Proceedings of the Baden Symposium, September 1978, *IAHS-AISH Publ.*, No. 125.
2. Jurak, D. (1976) Heat exchange coefficients for cooling lakes. *J. Hydrol. Sc.*, Vol. 3, No. 3–4.
3. Jurak, D. (1975) Estimation of intensity of evaporation from natural and heated water reservoirs. *J. Hydrol. Sc.*, Vol. 2, No. 3–4.
4. Jurak, D. (1985) Thermal regime forecast for the Vistula river and its application for the needs of power and transport industry. PR-7.01.04.09, Task 05, Final Report, Archive IMWM Library (in Polish).
5. Jurak, D., Wisniewski, A. (1987) Model of thermal pollution propagation in river (Phase I). Theme IIASA (LIR) Phase I. Unpublished (in English), IMWM Library, Warsaw.
6. Jurak, D. (xxx) Models for water temperature forecasting in unstratified natural lakes and the ones loaded with thermal water discharges. Reports of the Institute of Meteorology and Water Management.
7. Laykhtman, D.L. (1947) O profile vetra v prizemnom sloe atmosfery pri stacionarnych uslovyach. *Trudy NIUGUMS*, 1, 39.
8. Laykhtman, D.L. (1970) Fizika pogranichnenego sloya atmosfery, eningrad.
9. Ogneva, T.A. (1956) O raspredelenii meteoelementov nad vodoemami. *Trudy G.G.O.*, 59.
10. Rukovodstvo po gidrologičeskim rascetam pri projektirovanii vodokhranilišč (1983). Leningrad, Gidromieteoizdat.
11. Wisniewski, A. The application of finite element method to prediction of water temperature in rivers and reservoirs. Ph.D. Thesis, IMWM Library (in Polish), Warsaw.

12. Wiszniewski, W., Chelchowski, W. (1975) Climatic characteristics and regions in Poland (in Polish). Publishing House of Transport and Communication, Warsaw.

Notation

q	volume of flow in the river $[m^3/s]$
q_p	volume of water intake from the river equal to water discharge q_d $[m^3/s]$
T_{ox}	computed water temperature in x distance from the beginning cross-section [deg C]
T_{op}	initial water temperature for the beginning cross-section [deg C]
T_p	water intake temperature [deg C]
ρ	water density $[kg/m^3]$
c_w	specific heat of water [J/kg/deg]
b	river width [m]
x	distance from the beginning to the computational cross-section [m]
Q_c	heat exchange between water surface and the atmosphere $[W/m^2]$
Q_b	heat exchange between mass of water in the river and river bed $[W/m^2]$
Q_{sr}	short-wave radiation balance of water surface $[W/m^2]$
Q_{lr}	long-wave radiation balance of water surface $[W/m^2]$
Q_e	heat loss from the water surface by evaporation $[W/m^2]$
Q_h	energy exchange between water surface and the atmosphere as sensible heat $[W/m^2]$
α,β,γ	heat exchange coefficients $[W/m^2]$
a	assumed water temperature [°g C]
r	albedo of water surface (ratio of the incoming to the reflected short-wave radiation [per cent]
e	water vapour pressure of the air at 2.0 m above the land surface $[hP_a]$
e_o	saturation vapour pressure corresponding to the temperature of water surface $[hP_a]$
t_p	air temperature at 2.0 m above the land surface [deg C]
u_1	wind velocity at 1.0 m above the water surface [m/s]
z_o	roughness coefficient [m]

n_o	cloudiness on a 10 degree scale
e_o' , e_o''	first and second derivatives of $e_0(a)$
$B_1(\mu)$	exchange coefficient in the evaporation formulae
μ,ϵ	parameters characterizing the equilibrium state of the air layer adjacent to the water surface
k_1	turbulent exchange coefficient at 1.0 m level computed by Laykhman's formula [m/s]
$\bar{X}$	mean length of air mass transformation for the section of the river [m]
l	length of the river section [m]