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PREFACE

Work on multicriteria analysis has long been an important part of IIASA's research agenda. Multicriteria problems arise in virtually all of the real systems studied at IIASA, and methods for handling various aspects of these problems are still being developed.

One important aspect of multicriteria techniques lies in the overall evaluation or ranking of alternatives in terms of a number of possibly conflicting criteria. Virtually all of the existing methods assume that the analyst has access to quantitative information on criteria weights and the performance of each option with respect to individual criteria, although this is very rarely the case. In general the analyst is presented with a mixture of quantitative and qualitative data from which he must derive his conclusions.

In this paper, Henk Voogd from the University of Delft develops a new multicriteria evaluation approach capable of handling mixed quantitative and qualitative data, and illustrates how it may be applied to an urban planning problem.

Andrzej Wierzbicki
Chairman
System and Decision Sciences

ABSTRACT

This paper is concerned with the development of a new multicriteria evaluation approach capable of handling mixed quantitative and qualitative data. First, a brief overview is given of the state of the art of multicriteria evaluation in urban and regional planning. A mixed data evaluation approach which includes three techniques based on different interpretations of basic assumptions is also discussed. Next, it is shown that there are some practical ways of dealing with the weighting problem that arises in evaluation. Finally, an illustration of the mixed data approach is provided by means of an empirical application to a housing allocation problem.

MULTICRITERIA EVALUATION WITH MIXED QUALITATIVE AND QUANTITATIVE DATA

Henk Voogd

1. INTRODUCTION

An urban or regional planner is often confronted with the need to list, classify, organize, and analyze the available information concerning a number of possible options. These options may be alternative plans, but could also be alternative construction sites, urban renewal schemes, implementation procedures, and so on. One attractive way of dealing with this kind of problem is to use multicriteria techniques, which are designed to evaluate a discrete number of alternatives by means of explicitly formulated criteria. These techniques can be used for various planning purposes, for instance, for regional disparity analysis (Van Setten and Voogd, 1979), for location analysis (Miller, 1980), for plan generation (Van Delft and Nijkamp, 1977), for plan implementation (Ball, 1977), or for process monitoringb (Voogd, 1981a, 1982).

The *basic principle* of a multicriteria evaluation approach is very simple (see also Voogd, 1982). Firstly, a matrix should be constructed such that its elements reflect the characteristics of a given set of options as determined by a given set of criteria. Evaluation matrices of this type are quite common in daily life; many people will be familiar with, for instance, the use of such matrices to present the results of a consumer survey. To obtain an impression of the quality of the various alternatives by means of an evaluation matrix, it is necessary to have some information about the relative importance attached to each criterion. It often happens that some of the criteria conflict, which makes a straightforward interpretation of the matrix almost impossible. Consequently, the criteria have to be assigned individual weights or priorities. As will be shown in the next section, there are many techniques employing priority statements which could be used to condense the information from the evaluation matrix. Most of these procedures are based on the availability of *quantitative* information. However, in practice the use of quantitative multicriteria techniques is often limited by the lack of reliable metric data on both weights and evaluation scores. This has led to the introduction of several techniques capable of analyzing *qualitative* information. However, this approach has also been criticised on the grounds that in this case any available quantitative information will be only partially used, since only its ordinal characteristics are required (see, for instance, Stunet, 1979). Evidently, multicriteria evaluations will in practice often be based on criteria which may be assessed partially on a quantitative (cardinal) measurement scale and partially on a qualitative (ordinal) measurement scale. Evaluation techniques able to deal with such "*mixed*" evaluation matrices in a theoretically consistent way would therefore be very valuable additions to the existing repertoire of quantitative and qualitative techniques.

In this paper attention is focussed on the development of a new multicriteria evaluation approach for mixed "quantitative-qualitative" data. Section 2 gives a brief overview of the literature dealing with multicriteria evaluation. A mixed data evaluation approach is discussed in Section 3: this involves three techniques based on different interpretations of basic assumptions. Section 4 is devoted to the treatment of criterion weights or priorities; it is shown that there are some practical ways of dealing with this problem. The mixed data approach is illustrated in Section 5 by means of an empirical application to a housing allocation problem. This paper concludes with some final remarks.

2. BRIEF OVERVIEW OF MULTICRITERIA EVALUATION APPROACHES

In the past, multicriteria evaluation has been largely based on quantitative methods. These techniques were apparently introduced into urban and regional planning in the USA in the sixties (Boyce *et al.*, 1970): there were a number of leading studies during this period in which some kind of multicriteria technique was used. Articles describing these techniques generally appeared a few years later (see, *inter alia*, Hill, 1967,1968; Schimpeler and Grecco, 1968; Schlager, 1968). Many of these methods have their roots in traffic and transportation research, and it is remarkable that they are *all* based on the principle of weighted summation. Much European research in this field has also followed the same general line (see, e.g., Strassert, 1973; Stanley, 1974; De Goede, 1974). Many empirical applications could be mentioned here, the goals-achievement approach based on weighted summation enjoying particular popularity in planning practice in the 1970s. Several variants have also been developed, such as the extension by Mackie and King (1974), which involves the use of sensitivity analysis, or the approach taken by Saaty (1977,1978), which mainly focuses on the determination of scores and weights by means of a pairwise comparison technique. However, in both cases the weighted summation rule is used for evaluation.

The weighted summation rule has some important limitations; for example, the outcomes depend very strongly on the (usually arbitrary) origins taken for the various measurement scales used. Consequently, much attention has been paid to the development of other multicriteria evaluation techniques. The concept of *multicriteria decision aid* promulgated by the French school (see, *inter alia*, Benayoun *et al.*, 1966; Buffet *et al.*, 1967; Guigou, 1971; Roy, 1972), has been taken up elsewhere, especially in The Netherlands, where much research has concentrated on finding better quantitative evaluation techniques (see, e.g., Van der Meer and Opschoor, 1973; Nijkamp, 1974,1977; Voogd, 1975,1976; Van Delft and Nijkamp, 1977). Most of these techniques involve some kind of pairwise comparison of alternatives, such as, for instance, the concordance analysis approach.

The application of the quantitative techniques described above to empirical problems soon led to a realisation of the qualitative (i.e., non-metric) nature of many decision and classification problems. In practical applications this problem is often solved by using qualitative data (i.e., rankings) as if they were metric quantities (see, for example, Schlager, 1968; Hill, 1968; Bernard and Besson, 1971), thus destroying their qualitative characteristics. A theoretically consistent treatment of qualitative data is given by Holmes (1972), who proposed a lexicographic evaluation method. The "best" alternative in Holmes's approach is that alternative which has the best evaluation score for the most important criterion, irrespective of the evaluation scores for the other criteria. The reactions to this approach make it clear that this simple definition of the "best" was not accepted by everyone (see Kettle and Whitbread, 1973; Nowlan, 1975).

A more sophisticated approach to qualitative evaluation problems has been developed by Paelinck (1977,1978). His permutation technique assesses all possible (final) rankings of the alternatives in an attempt to find the best "final ranking". A limitation of this approach is that it can only be used for problems involving a few alternatives, due to the number of possible permutations, unless one accepts a more heuristic extension to deal with many alternatives. Another recent qualitative evaluation approach involves the use of geometric or multidimensional scaling models based on the so-called *ideal point* concept (Coombs, 1950): see Nijkamp and Voogd (1979, 1981), Nijkamp (1979), and Voogd (1980, 1981b). This approach may be used to draw quantitative inferences from qualitative information in a theoretically consistent way, i.e., without violating the ordinal character of the input data. A limitation of present geometric scaling models is that the evaluation problem should have sufficiently many degrees of freedom to allow geometric scaling (see also Voogd, 1982). This implies that not all multicriteria evaluation problems can be solved using this approach.

The articles cited above all deal with deterministic evaluation approaches. Stochastic methods have received little attention in the urban and regional planning literature, although a few exceptions are described by Nijkamp (1977, 1979) and Voogd (1980, 1982).

As already mentioned in the preceding section, not much work has yet been done on mixed data multicriteria techniques. There is an approach developed by Jacquet-Lagrèze (1969) which is capable of treating qualitative evaluation scores and quantitative weights, and Kolfoort and Nijkamp (1977) have devised a number of such methods based on qualitative weights and quantitative evaluation scores. However, there are very few techniques that could cope with an evaluation matrix similar to Table 1.

An approach capable of analyzing such a mixed data evaluation matrix by means of a geometric evaluation model has recently been developed: see

Table 1. *A mixed data evaluation matrix.*

	Alternatives					
	A	B	C	D	E	....
C 1	*	**	*	***	**	
r 2	29	89	63	12	74	
i 3	***	***	**	*	*	
t 4	**	*	*	***	***	
e 5	.26	.48	.96	.37	.92	
r .						
i .						
a .						
.						

Nijkamp and Voogd (1981). However, this approach has the same limitation as the geometric scaling models mentioned earlier: because of the need for a certain number of degrees of freedom, there will always be some mixed data evaluation problems which cannot be tackled with such a model. The next section outlines an approach which can be used to define a number of distinct techniques for evaluating alternatives in problems with mixed data.

The brief overview given in this section reveals that multicriteria evaluation in planning research is still a relatively young field. Most publications still deal with "technical" matters, showing that there are many methodological areas yet to be explored. Promising directions for future research are suggested, *inter alia*, by Nijkamp (1980), Rietveld (1980),Kmietowicz and Pearman (1981), and Voogd (1982). Mention should also be made of developments in the field of multiobjective optimization. Although these approaches are sometimes also referred to in the literature as multicriteria evaluation techniques, they are intrinsically different - since they involve continuous rather than discrete alternatives and make use of optimization algorithms - from the approaches discussed in this section. Readers interested in the state of the art in this field are referred to, *inter alia*, Keeney and Raiffa (1976), Starr and Zeleny (1977), Wierzbicki (1979), and Spronk (1981).

3. MIXED DATA EVALUATION

3.1. The Basic Principles

The starting point is an evaluation matrix, such as that presented in Table 1, containing elements e_{ji}, where i ($i,i'=1,2,...,I$) represents an alternative and j ($j=1,2,...,J$) a criterion. The set of criteria can be divided into two subsets denoted as O and C, where:

$$O = \{ j \mid j = \text{ordinal} \} \tag{3.1}$$

$$C = \{ j \mid j = \text{cardinal} \} \tag{3.2}$$

It is postulated that the differences between the options can be summarized by means of two dominance measures: one based on the qualitative criteria and the other on the quantitative (cardinal) criteria. Both measures are standardized in such a way that they may be compared with each other. By weighting these standardized dominance measures using the aggregated weights of the constituent criteria a new overall dominance score can be created, which represents the degree to which an alternative is better (or worse) than another alternative. In addition, an appraisal score for each option can be calculated on the basis of this overall measure.

This procedure can be summarized formally as follows: first we calculate a dominance score $\alpha_{ii'}$ for the ordinal criteria and a dominance score $a_{ii'}$ for the cardinal criteria. These scores reflect the degree to which alternative i dominates alternative i' and they have the following structure:

$$\alpha_{ii'} = f \ (e_{ji}, e_{ji'}, w_j) \qquad (\forall j \in O) \tag{3.3}$$

$$a_{ii'} = g \ (e_{ji}, e_{ji'}, w_j) \qquad (\forall j \in C) \tag{3.4}$$

where e_{ji} shows how alternative i scores under criterion j and w_j represents the weight attached to criterion j. Clearly, the functions f and g will differ because in (3.3) only the ordinal characteristics of the e_{ji} are taken into account, while in (3.4) their metric properties are also used. Since $\alpha_{ii'}$ and $a_{ii'}$ will be measured in different units, it is necessary to convert them into the same unit to allow comparison between the outcomes of (3.3) and (3.4). The ***standardized dominance measures*** can be written:

$$\delta_{ii'} = h \ (\alpha_{ii'}) \tag{3.5}$$

$$d_{ii'} = h \ (a_{ii'}) \tag{3.6}$$

where h represents a standardization function. Let us assume for the moment that the weights w_j have quantitative properties; the treatment of qualitative weights is discussed in Section 4. It is then possible to express the weight w_O of the qualitative criterion set O as:

$$w_O = \sum_{j \in O} w_j \tag{3.7}$$

The weight of the quantitative criterion set can be found in a similar way:

$$w_C = \sum_{j \in C} w_j \tag{3.8}$$

We are now able to determine an ***overall dominance measure*** $m_{ii'}$ for each pair of alternatives (i, i'):

$$m_{ii'} = w_O \ \delta_{ii'} + w_C \ d_{ii'} \tag{3.9}$$

This overall dominance score gives the degree to which alternative i dominates alternative i'. On the other hand, $m_{ii'}$ may also be considered as a function k of the appraisal scores s_i and $s_{i'}$:

$$m_{ii'} = k \ (s_i, s_{i'}) \tag{3.10}$$

Equation (3.10) describes a well-known paired comparison problem (see, *inter alia*, Davidson and Farquhar, 1976). Given k, the appraisal scores can be calculated.

The preceding formulae represent the - in essence very simple - structure of an analytical technique by which multicriteria mixed data sets can be summarized in a straightforward manner. The most important assumptions behind this approach are connected with the definition of the various functions. It will be shown that it is possible to distinguish at least three different techniques, which are based on different definitions, particularly of equations (3.5), (3.6), and (3.10). These techniques are the subtractive summation technique, the subtractive shifted interval technique, and the additive interval technique.

3.2. The Subtractive Summation Technique

In order to construct the cardinal dominance score $a_{ii'}$ (see equation (3.4)) it is necessary to convert the quantitative evaluation scores e_{ji} $(j \in C)$ to a standard unit. There are many ways in which this could be done (see, e.g., Voogd, 1982). However, since the dominance measure involves a pairwise comparison which requires only the interval characteristics of the standardized scores, the following attractive standardization procedure may be used (Cain and Harrison, 1958):

$$\hat{e}_{ji} = \frac{e_{ji} - e_j^-}{e_j^+ - e_j^-} \qquad (0 \leq e_{ji} \leq 1) \tag{3.11}$$

where:

e_j^- is the lowest e_{ji} value for criterion j in the problem at hand

e_j^+ is the highest e_{ji} value for criterion j in the problem at hand

$\hat{e}_{ji}$ is the standardized evaluation score of alternative i with respect to criterion j

Evidently, all standardized scores should have the same directional sense, i.e., a 'higher' score should (for instance) imply a 'better' score. The scores of those criteria for which 'lower' means 'better' should therefore be transformed, for example by subtracting them from 1. Note that the rankings e_{ji} $(j \in O)$ of the qualitative criteria should also follow the principle 'the higher, the better'. In order to simplify the notation we will further assume that all e_{ji} scores $(j \in C)$ are standardized so that we can drop the circumflex (^).

For each set of criterion weights w_j we are now able to define the qualitative dominance measure $\alpha_{ii'}$ (see equation (3.3)) as follows:

$$\alpha_{ii'} = \left\{\sum_{j \in O} [w_j \cdot \mathrm{sgn}\,(e_{ji} - e_{ji'})]^{\gamma}\right\}^{\frac{1}{\gamma}} \quad (\gamma=1,3,5,..) \tag{3.12}$$

where:

$$\mathrm{sgn}\,(e_{ji} - e_{ji'}) = \begin{cases} +1 & \text{if } e_{ji} > e_{ji'} \\ 0 & \text{if } e_{ji} = e_{ji'} \\ -1 & \text{if } e_{ji} < e_{ji'} \end{cases} \tag{3.13}$$

The symbol γ denotes an arbitrary scaling parameter which may take any positive odd value. Clearly, even values cannot be allowed because this would distort the various signs. Equation (3.12) can be set up in various ways. The larger the value of γ, the less influence the minor criteria will have on the value of the qualitative dominance measure $\alpha_{ii'}$.

It is evident that the assumption $\gamma = 1$ would be very reasonable if the criterion weights were fairly reliable. If this is not the case, however, a higher value for γ may be assumed. If γ approaches infinity, equation (3.12) becomes:

$$\alpha_{ii'} = \lim_{\gamma \to \infty} \left\{ \sum_{j \in O} [\, w_j \cdot \mathrm{sgn}\,(e_{ji} - e_{ji'})]^{\gamma} \right\}^{\frac{1}{\gamma}} \tag{3.14}$$

$$= \max_{j \in O} (\, w_{ji} \cdot \xi_{ji'} \,)$$

where:

$$\xi_{ii'} = \prod_{j \in \Omega} \mathrm{sgn}\,(e_{ji} - e_{ji'}) \tag{3.15}$$

$$\Omega = \left\{ j \mid \max_{j \in O} w_j \right\} \tag{3.16}$$

A quantitative dominance measure $a_{ii'}$ can be defined for the cardinal criteria in a similar way:

$$a_{ii'} = \left[\sum_{j \in C} (w_j \mid e_{ji} - e_{ji'} \mid)^{\gamma} \right]^{\frac{1}{\gamma}} \tag{3.17}$$

Obviously, in order to be consistent, the scaling parameter should take the same value in both (3.12) and (3.17). If γ approaches infinity, equation (3.17) reduces to:

$$a_{ii'} = \text{sgn}\ (\zeta_{ii'}) \cdot \max_{j \in C} (w_j \mid e_{ji} - e_{ji'} \mid) \tag{3.18}$$

where:

$$\zeta_{ii'} = \sum_{j \in \Psi_{ii'}} w_j \ (e_{ji} - e_{ji'}) \tag{3.19}$$

$$\Psi_{ii'} = \left\{ j \mid \max_{j \in C} (w_j \mid e_{ji} - e_{ji'} \mid) \right\} \tag{3.20}$$

The general functions f and g in (3.3) and (3.4) have thus been given specific form in equations (3.12) and (3.17). The next step is to equalize the dimensions of $\alpha_{ii'}$ and $a_{ii'}$ in order to make both scores comparable (see equations (3.5) and (3.6)). This may be done in several ways, depending on the form of the relationship assumed for (3.10).

The subtractive summation technique is based on the assumption that (3.10) has the following form:

$$m_{ii'} = s_i - s_{i'} \tag{3.21}$$

which implies that the standardization functions of (3.5) and (3.6) should be such that $m_{ii'} = -m_{i'i}$. We may thus define the following standardized measures:

$$\delta_{ii'} = \alpha_{ii'} \;/\; \left[\sum_i \sum_{i'} | \; \alpha_{ii'} \; |\right] \tag{3.22}$$

$$d_{ii'} = a_{ii'} \;/\; \left[\sum_i \sum_{i'} | \; a_{ii'} \; |\right] \tag{3.23}$$

Obviously, since $\alpha_{ii'} = -\alpha_{i'i}$ we have $d_{ii'} = -d_{i'i}$, and for similar reasons $\delta_{ii'} = -\delta_{i'i}$. Because equation (3.9) holds it can be concluded that this standardization is consistent with assumption (3.21). The appraisal score can now be found by summing the left- and right-hand sides of (3.21) over i' :

$$\sum_{i'} m_{ii'} = \sum_{i'} (s_i - s_{i'}) = Is_i - \sum_{i'} s_{i'} \tag{3.24}$$

which means that s_i can be expressed as:

$$s_i = \frac{1}{I}\sum_{i'} m_{ii'} + \frac{1}{I}\sum_{i'} s_{i'} \tag{3.25}$$

By assuming that the mean of the appraisal scores s_i is zero (or any other constant if only the ranking characteristics of the s_i scores are used), i.e.,

$$\frac{1}{I}\sum_{i'} s_{i'} = 0 \tag{3.26}$$

the appraisal scores can be expressed as:

$$s_i = \frac{1}{I}\sum_{i'} m_{ii'} \tag{3.27}$$

The higher the value of s_i, the better alternative i will appear for the given weight set w_j.

3.3. The Subtractive Shifted Interval Technique

This technique differs from the previous one by its standardization procedure, which is defined here as:

$$\delta_{ii'} = [(\alpha_{ii'} - \alpha^-) \;/\; (\alpha^+ - \alpha^-)] - 0.5 \tag{3.28}$$

$$d_{ii'} = [(a_{ii'} - a^-) \;/\; (a^+ - a^-)] - 0.5 \tag{3.29}$$

where:

α^- is the lowest qualitative dominance score for any pair of alternatives (i, i')

a^- is the lowest quantitative dominance score for any pair of alternatives (i, i')

α^+ is the highest qualitative dominance score for any pair of alternatives (i, i')

a^+ is the highest quantitative dominance score for any pair of alternatives (i, i')

This standardization function is quite similar to that given in equation (3.11) except for the subtraction of 0.5, which is necessary to guarantee that $d_{ii'}$ and $\delta_{ii'}$ are equal to $-d_{i'i}$ and $-\delta_{i'i}$, respectively. The appraisal score s_i for option i can then be calculated using an equation similar to (3.27).

3.4. The Additive Interval Technique

This differs from the subtractive summation technique in its formulation of (3.10), which in this case is as follows:

$$m_{ii'} = s_{i'} \ / \ (s_{i'} + s_i) \tag{3.30}$$

This implies that $m_{ii'} + m_{i'i} = 1$. In order to obtain overall dominance measures with this property, the following standardization procedure is used (see (3.5) and (3.6)):

$$\delta_{ii'} = (\alpha_{ii'} - \alpha^-) \ / \ (\alpha^+ - \alpha^-) \tag{3.31}$$

$$d_{ii'} = (a_{ii'} - a^-) \ / \ (a^+ - a^-) \tag{3.32}$$

Since $a_{ii'} = -a_{i'i}$ it can be concluded that $a^+ = -a^-$ (or, in a similar way, that $\alpha^+ = -\alpha^-$). Consequently, if i equals i' the dominance measures (3.31) and (3.32) take the value 0.5. It is then easy to see that

$$\delta_{ii'} + \delta_{i'i} = 1 \tag{3.33}$$

$$d_{ii'} + d_{i'i} = 1 \tag{3.34}$$

Substitution of (3.31) and (3.32) into (3.9) yields overall dominance scores $m_{ii'}$. It can easily be seen that the additivity condition also holds for these scores, i.e., by multiplying the elements of (3.33) by w_O and the elements of (3.34) by w_C and adding (3.33) to (3.34) leads to the following expression:

$$w_O\delta_{ii'} + w_C d_{ii'} + w_O\delta_{i'i} + w_C d_{i'i} = w_O + w_C \tag{3.35}$$

By postulating that the weights add up to unity, i.e., that

$$\sum_{j=1}^{J} w_j = 1 \tag{3.36}$$

equation (3.35) can be rewritten as:

$$m_{ii'} + m_{i'i} = 1 \tag{3.37}$$

In other words, the standardization procedure described by (3.31) and (3.32) is consistent with relationship (3.30). The appraisal score s_i for alternative i can now be found by rearranging the elements of (3.30):

$$m_{ii'}s_i + m_{ii'}s_{i'} = s_{i'} \tag{3.38}$$

or

$$\frac{s_{i'}}{s_i} = \frac{m_{ii'}}{(1 - m_{ii'})} \tag{3.39}$$

Summing (3.39) over i' leads to the following expression:

$$\frac{1}{s_i}\sum_{i'} s_{i'} = \sum_{i'}\left[\frac{m_{ii'}}{1 - m_{ii'}}\right] \tag{3.40}$$

Assuming that the appraisal scores s_i add up to unity, i.e., that

$$\sum_{i'} s_{i'} = 1 \tag{3.41}$$

and using relationship (3.37), we obtain the following expression for the appraisal score:

$$s_i = 1 / \sum_{i'} \frac{m_{ii'}}{m_{i'i}} \tag{3.42}$$

4. THE CRITERION WEIGHTS

The mixed data approach outlined in the preceding section assumes quantitative criterion weights w_j $(j=1,2,..,J)$. Although there are circumstances in which this information is directly available (see also Voogd, 1982), usually only qualitative expressions of priority can be given. In this latter case there are two possible approaches: (a) the ***extreme weights*** approach, and (b) the ***random weights*** approach. Both are illustrated in the flow chart given in Figure 1 and are briefly explained below.

(a) *Extreme Weights*

Paelinck (1976) has shown that the underlying cardinal weight vector $\mathbf{w}$ of an ordinal weight vector ω with rankings ω_j $(j=1,2,..,J)$ can be approximated by extreme weight vectors which limit the values available to the metric weights. This may be illustrated by means of a simple example.

Suppose that we have the following qualitative priorities: $\omega_1 \leq \omega_2 \leq \omega_3$, where a lower ranking implies a higher priority. If condition (3.36) holds, i.e., if

$$\sum_{j=1}^{J} w_j = 1 \tag{4.1}$$

then we have the following extreme weight vectors w_l $(l=1,2,3)$:

$w_1 = (1,0,0)$ (i.e., criterion 1 receives maximum priority)

$w_2 = (\frac{1}{2},\frac{1}{2},0)$ (i.e., criteria 1 and 2 receive maximum priority)

$w_3 = (\frac{1}{3},\frac{1}{3},\frac{1}{3})$ (i.e., criteria 1, 2 and 3 receive equal priority)

Each (quantitative) extreme weight vector w_l can thus be used in the formulae constituting the mixed data evaluation approach instead of the 'real' weights **w**. The final outcome of the evaluation will be an appraisal score s_{il}, which allows the analyst to make a very detailed interpretation of the results (see, for example, Nijkamp and Voogd, 1979).

It is evident that the maximum number of extreme weight sets L ($l=1,2,...,L$) depends on the number of ordinal 'levels' in the set of qualitative weights considered. The preceding example has three 'levels'; however, if we assume that $\omega_1 = \omega_2$ only two 'levels' - and consequently two extreme weight sets (w_2 and w_3) - remain. In general, for each quantitative weight vector the number of extreme weight sets L is given by:

$$L = J - \xi \tag{4.2}$$

where ξ is the number of strict equalities in the qualitative weight vector ω. Evidently, if L is large (in general larger than 3), the extreme weight approach becomes rather cumbersome and the resulting s_{il} scores may then be very difficult to interpret.

(b) *Random Weights*

A qualitative weight vector may also be treated using a stochastic approach. This implies that quantitative criterion weights are selected at random from an area defined by the extreme weight sets. These random weights (denoted as v_j) must fulfill the following conditions:

$$\begin{cases} \omega_j \leq \omega_{j'} \rightarrow v_j \geq v_{j'} \\ \sum_j v_j = 1 \end{cases} \tag{4.3}$$

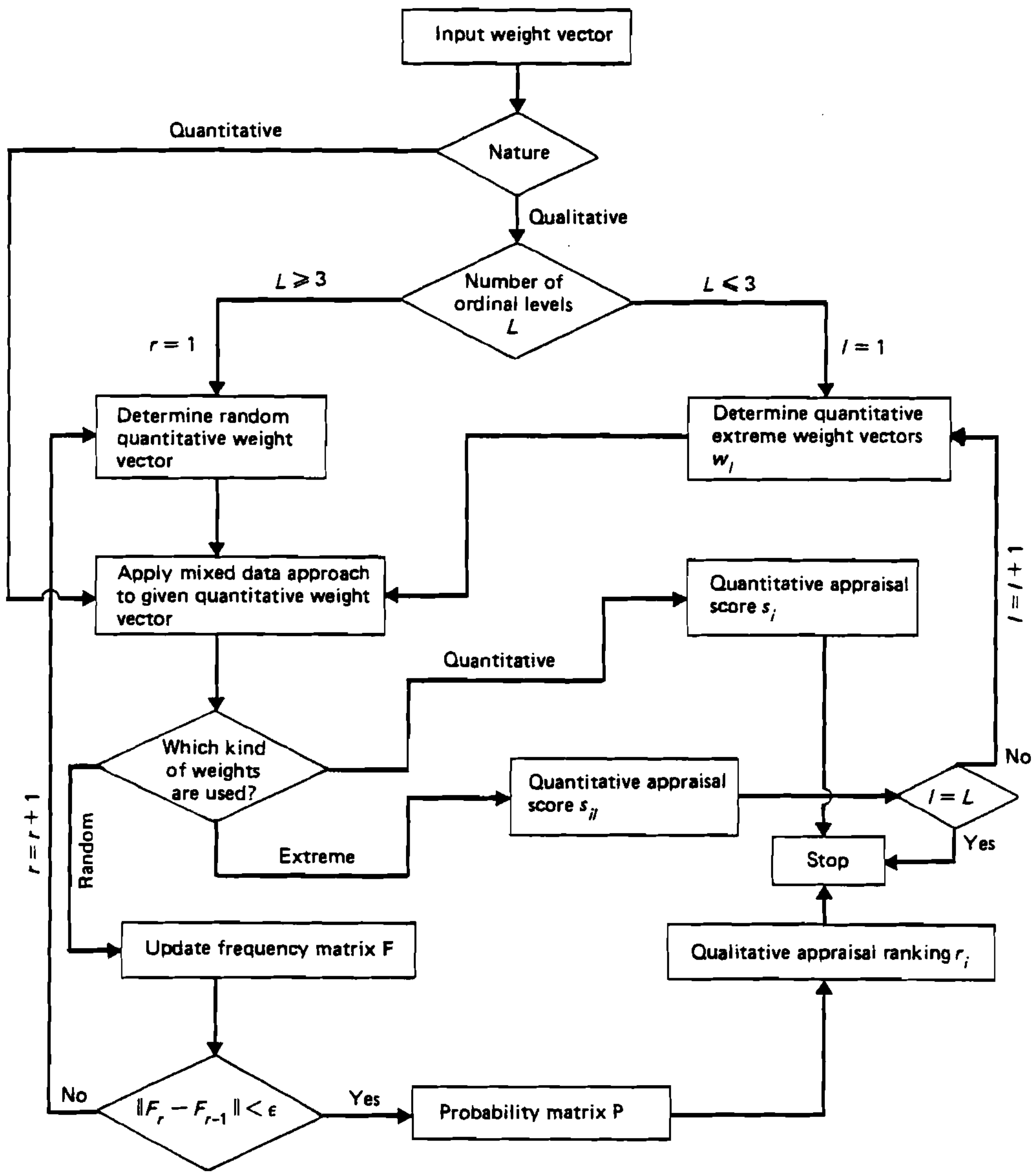

Figure 1. *Simple flow chart of approaches for treating criterion weights.*

A set of appraisal scores can be determined for each set of metric weights v_j generated during one run of the random generator. By repeating this procedure many times it is possible to construct a *frequency matrix* **F** of order $R \times I$ with elements f_{ri} which represent the number of times alternative i was ranked at position r by the evaluation technique under consideration. The runs with the random generator are discontinued if **F** shows only marginal changes (i.e., less than ε). A probability matrix **P** of order $R \times I$ with elements p_{ri} can then be constructed, where:

$$p_{ri} = \frac{f_{ri}}{\sum_i f_{ri}} \tag{4.4}$$

i.e., p_{ri} represents the probability that i is ranked at position r.

Obviously, the user who wants to gain some insight into the consequences of the assumptions implicit in the various techniques (i.e., the so-called *method uncertainty*; see Voogd, 1982) for his particular evaluation problem will be confused if he or she has to compare probability tables for all techniques. Therefore, it may be necessary to condense the information given in matrix **P** by determining a final ranking of the alternatives (denoted as r_i) in the following way:

$r_i = 1$ if p_{1i} is maximal

$r_{i'} = 2$ if $p_{1i'} + p_{2i'}$ is maximal and $i' \neq i$

$r_{i''} = 3$ if $p_{1i''} + p_{2i''} + p_{3i''}$ is maximal and $i'' \neq i' \neq i$ and so forth.

Hence, for each evaluation technique a ranking r_i can be obtained which makes it possible to compare the results of the various evaluation techniques very easily.

5. AN EMPIRICAL ILLUSTRATION

The use of the three mixed data evaluation techniques described in this paper can be illustrated by means of an application to a housing allocation problem in South-East IJsselmonde, an area near Rotterdam in The Netherlands. A large part of this area is covered by urban developments, which are to a large extent the result of recent growth in the urban areas. As a consequence, the region has many new residential areas and a relative lack of high-level services. Many people living in this area are, therefore, still dependent on Rotterdam for cultural or educational facilities. Continuation of the current policy might cause massive dislocation in this region. The increasing number of commuters will cause tremendous traffic problems when certain capacity limits are reached; even now approximately 8000 people living in the study area are employed elsewhere. Limiting the urban sprawl would also be desirable because the open areas left between the cities could then be turned into industrial parks. This sort of approach is essential if a balanced urban structure is to be attained.

The above comments suggest that a step-wise development of the remaining open area would be desirable. This area is therefore divided into eleven distinct zones, which vary in size from about 20 hectares to 68 hectares. Each zone may contain one single urban (i.e., housing or industrial) function. The purpose of the evaluation is to classify these zones with respect to their suitability for industrial or housing development. This suitability is determined using the criteria summarized in Table 2 and described in more detail elsewhere (Voogd, 1982). Table 2 also presents the evaluation matrix, which shows that some of the criteria are assessed on a cardinal scale, while others are measured on an ordinal scale. If the criterion is such that a high score or large number of crosses represents a favorable outcome, there is a (+) in the final

Table 2. *The evaluation matrix.*

Criterion	Zones 1	2	3	4	5	6	7	8	9	10	11	Direction
1. SOIL CONDITION												
1.1 Depth of drainage	20	20	10	20	10	0	0	20	20	30	10	(-)
1.2 Level of ground water	++	++	++	+++	++	+++	+++	++	++	++	+	(+)
1.3 Type of soil	++	+	+	++	++	++	+	++	++	++	++	(+)
1.4 Depth of Pleistocene sand	+	+	+	+	++	+	+	++	++	++	++	(+)
2. HOUSING ENVIRONMENT												
2.1 Existing landscape	++++	+	+++	+	++	+	+	+	+	+	++	(+)
2.2 a-Biotic diversity	24.0	37.5	22.5	13.1	64.0	14.2	13.3	26.7	48.2	55.5	64.0	(+)
2.3 Noise nuisance	0	0	5	0	32	77	86	17	0	0	0	(-)
3. RECREATION FACILITIES												
3.1 Full day recreation	+	+	+	+	+++	+	+	+	+++	++	+++	(+)
3.2 One hour recreation	50	50	50	67	403	73	25	25	0	0	7.	(+)
4. AGRICULTURAL SITUATION												
4.1 Soil suitability	.67	.52	.52	.08	.58	.33	.06	.47	.38	.38	.52	(-)
4.2 Horticulture under glass	0	1.5	0	9.8	5.2	68.8	12.7	30.7	20.3	20.9	10.2	(-)
4.3 Orchards	17.6	0	16.9	0	0	0	0	0	0	0	4.3	(-)
5. ACCESSIBILITY												
5.1 Railway station	+	+	+	+	+	+	++	+++	++	+++	+	(+)
5.2 Highways	++	+++	+++	+++	+++	+++	+++	++	++	+	+	(+)
5.3 Rotterdam centre	+++	+++	+++	++	+++	++	++	++	++	++	+	(+)
5.4 Dordrecht centre	++	++	++	+	++	++	++	+++	+++	+++	++	(+)
5.5 Nearest city centre	+++	++	++	+	+	+	++	+++	+++	+++	+	(+)

column; the converse is indicated by a (−).

Table 2 does not immediately suggest any obvious conclusions. None of the zones fully dominates the others, which implies that a 'final ranking' can only be obtained if priorities are assigned. Because there is a different number of subcriteria in each main criteria category it is necessary to adopt a so-called "sector evaluation" approach (see Voogd, 1982). This means that all zones are first evaluated with respect to each main criteria category separately. A few qualitative sets of weights are given in Table 3. Only two 'ordinal' levels are distinguished; the information available does not justify a more detailed priority structure.

The symbol ≥ means 'is more or equally preferred to'. The labels used to describe the alternative priority rankings are quite arbitrary: the term 'economic view' is used when the criteria emphasized are important from a financial or broader "cost-benefit" point of view; the term 'social view' is used if the criteria stressed are of some general social importance. Because this

Table 3. *Priority rankings within criteria categories.*

Criteria category	Priority ranking	Label
1. Soil condition	1.1=1.4≥1.2=1.3	Economic view
	1.2≥1.1=1.3=1.4	Social view
2. Housing environment	2.3≥2.1=2.2	Economic view
	2.2=2.3≥2.1	Social view
3. Recreation facilities	3.1≥3.2	Economic view
	3.2≥3.1	Social view
4. Agricultural situation	4.2=4.3≥4.1	Economic view
	4.2≥4.3=4.1	Social view
5. Accessibility	5.1=5.2≥5.3=5.4=5.5	Economic view
	5.3=5.4≥5.1=5.2=5.5	Social view

application is being described purely to illustrate the use of the various techniques, the rankings themselves will not be explained here.

The evaluation scores of Table 2 and the priorities of Table 3 were then analyzed using the three mixed data techniques (with scaling parameter γ = 1), yielding a number of 'aggregated evaluation matrices'. This simply means that an ordinal appraisal score for each zone was calculated for each priority ranking and technique, using the random weights procedure (see Section 4). Two aggregated evaluation matrices are possible: a matrix based on economic priorities and a matrix based on social priorities. These matrices are given in Tables 4 and 5: the elements of the matrices represent rankings, where the lower the value the better the ranking.

Two conclusions may be drawn from these condensed evaluation matrices. The first is that they are remarkably similar: the same zones come out 'first' in each criteria category in both tables. The second conclusion is that the various criteria categories yield rather conflicting rankings for some zones. For example, zone 11 is a relatively poor housing location with respect to accessibility and agricultural situation, although it is a relatively good location from the point of view of recreation and the housing environment.

Table 4. *The aggregated evaluation matrix emphasizing economic criteria.*

Criteria category	Zones 1	2	3	4	5	6	7	8	9	10	11
1. Soil condition	7	8	6	4	2	1	5	3	3	2	3
2. Housing environment	2	6	3	10	2	8	9	7	5	4	1
3. Recreation facilities	7	7	7	6	1	5	8	8	3	4	2
4. Agricultural situation	10	4	6	7	9	8	1	5	2	3	11
5. Accessibility	6	4	4	8	7	9	3	1	5	2	10

Table 5. *The aggregated evaluation matrix emphasizing social criteria.*

Criteria category	Zones 1	2	3	4	5	6	7	8	9	10	11
1. Soil condition	7	9	8	3	2	1	4	5	5	2	6
2. Housing environment	3	5	4	9	2	7	8	6	4	4	1
3. Recreation facilities	5	5	5	4	1	4	6	6	3	4	2
4. Agricultural situation	7	4	6	9	10	8	1	5	2	3	11
5. Accessibility	3	2	2	7	5	6	4	1	2	2	8

It is also possible to draw more straightforward conclusions from Tables 4 and 5 by once again treating these evaluation matrices with the three evaluation techniques mentioned earlier. The priority sets used in this evaluation are given in Table 6.

Table 6. *Priorities assigned to each criteria category. The criteria categories are denoted by the numbers introduced in Tables 4 and 5, i.e., 1 represents soil condition, 2 housing environment, and so on.*

Label	Priority ranking
Industrial view I	$5 \geq 4 \geq 1 \geq 2 \geq 3$
Industrial view II	$5 \geq 4 = 1 \geq 2 \geq 3$
Housing view I	$2 \geq 3 \geq 1 \geq 5 \geq 4$
Housing view II	$2 \geq 3 = 1 \geq 5 = 4$

The priorities labelled 'industrial views' are used with Table 4 while the priorities labelled 'housing views' are combined with Table 5. It is found that the three mixed data techniques yield different probability matrices **P** but exactly the same final ranking of the zones (for further information on the probabilities, see Voogd, 1982). These results are illustrated in Figure 2. The first four columns give the rankings of the zones for the four priority sets listed in Table 6; the last two columns show the aggregated final rankings when priority is given to industry or housing, respectively.

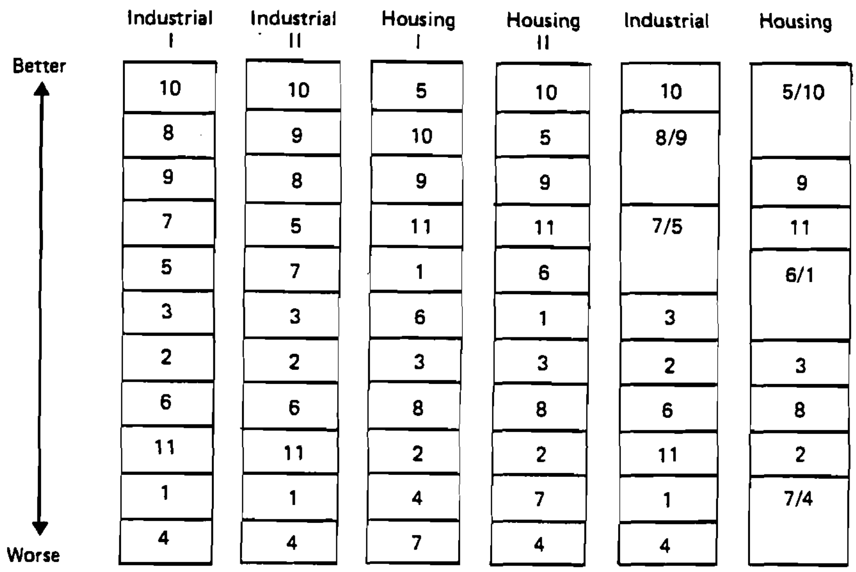

Figure 2. *The final rankings resulting from the evaluation of the aggregated evaluation matrices.*

Figure 2 reveals that the differences in priorities between the two industrial views and between the two housing views have little effect on the final ranking of zones. Zone 10 is undoubtedly the most appropriate location from an industrial point of view, given the priorities listed in Table 6. From the housing viewpoint, both zone 10 and zone 5 are very attractive, and for this reason no distinction is made between these zones in the last column of Figure 2. Zone

1 is undoubtedly much more suitable for housing than for industrial development, while the opposite is true of zones 7 and 8. The other zones have similar rankings for both functions, and could be regarded as equally suited to either. However, Figure 2 shows that zone 4 is not very attractive for either housing or industrial development, and another use for this piece of land should perhaps be considered.

6. FINAL REMARKS

This paper presents a new approach which is capable of evaluating a discrete number of options using 'mixed' qualitative and quantitative criteria. The three analytical techniques considered differ mainly in their interpretation of the relationship between the overall dominance measure $m_{ii'}$ and the appraisal scores s_i and $s_{i'}$. It should be clear that a different specification in terms of a multiplicative relation (i.e., $s_i \times s_{i'}$) or a summation relationship (i.e., $s_i + s_{i'}$) would be less useful than those given here, because in both cases it would be assumed that any available information on the 'direction' of the discrepancy between alternatives would not be used further. It therefore seems reasonable to exclude these possibilities from further consideration.

Despite the detailed and mathematical nature of this presentation, the underlying analytical approach is essentially very simple and straightforward. Much attention has been paid to alternative specifications of parts of the analytical framework outlined in Section 3, since the underlying assumptions may influence the final results. This is very important, because there is some empirical evidence to show that the users of these results (such as politicians, civil servants, and so forth) concentrate less on the technique itself if it is shown what influence its assumptions will have on the final outcome. These and other problems related to the application of multicriteria techniques are discussed extensively elsewhere (Voogd, 1982).

REFERENCES

Ball, R.B.(1977), Priority Ranking of Schemes within the TRP, *PTRC Summer Annual Meeting*, Paper J.16; Warwick.

Benayoun, R., B. Roy and N. Sussman (1966), Manual de Reference du Programme Electre, *Note de Synthese et Formation*, 25, Direction Scientifique SEMA, Paris.

Bernard, G. and M.L. Besson (1971), Douze Methodes d'Analyse Multi-Critere. *Revue Francaise d'Informatique et de Recherche Operationelle*, 5(2), 19-66.

Boyce, D.E., M.D. Day and C. McDonalds (1970), Metropolitan Plan Making; an Analysis of Experience with the Preparation and Evaluation of Alternative Land Use and Transportation Plans, *Monograph Series*, Regional Science Research Institute, Philadelphia.

Buffet, P., J.P. Greeny, M. Marc and B. Sussman (1967), Peut-on Choisir en Tenant Compte de Critères Multiples, *Metra*, 6(2), 283-316.

Cain, A.J. and G.A. Harrison (1958), An Analysis of the Taxonomist's Judgement of Affinity, *Proceedings of the Zoological Society*, 131, 85-98.

Coombs, R.H. (1950), *Psychological Review*, 57, 145-158.

Davidson, R.R. and P.H. Farquhar (1976), A Bibliography on the Method of Paired Comparisons, *Biometrics*, 32, 241-252.

De Goede, C.G. (1974), Alternatieven voor een spoorlijntrace door Lelystad, *Flevobericht*, No. 94, Rijksdienst voor de IJsselmeerpolders, Lelystad.

Guigou, J.L. (1971), On French Location Models for Production Units. *Regional and Urban Economics*, 1, 107-138 and 2, 289-316.

Hill, M. (1967), A Method for Evaluation of Transportation Plans, *Highway Research Record*, 180, 145-167.

Hill, M. (1968), A Goals-Achievement Matrix for Evaluating Alternative Plans, *Journal of the American Institute of Planners*, 34(1), 19-29.

Holmes, J.C. (1972), An Ordinal Method of Evaluation, *Urban Studies*, 9(1), 179-191.

Jacquet-Lagrèze, E. (1969), L'Aggregation des Opinions Individuelles, *Informatiques en Sciences Humaines*, 4.

Keeney, R.L. and H. Raiffa (1976), *Decisions with Multiple Objectives; Preferences and Value Trade Offs*, Wiley, New York.

Kettle, P. and M. Whitbread (1973), An Ordinal Method of Evaluation: A Comment, *Urban Studies*, 10, 43-46.

Kmietowicz, Z.W. and A.D. Pearman (1981), *Decision Theory and Incomplete Knowledge*, Gower, Aldershot.

Kolfoort, F. and P. Nijkamp (1977), Multi-Criteria Interview Methoden in de Ruimtelijke Planning, *Research Memorandum*, No. 60, Economic Faculty, Free University, Amsterdam.

Mackie, S. and A. King (1974), Managing Uncertainty in Plan Evaluation, *The Planner*, 60(9), 37-40.

Miller, D.H. (1980), Project Location Analysis Using the Goals Achievement Method of Evaluation, *Journal of the American Planning Association*, April, 195-208.

Nijkamp, P. (1974), A Multicriteria Analysis for Project Evaluation, *Papers of the Regional Science Association*, 35, 88-111.

Nijkamp, P. (1977), Stochastic Quantitative and Qualitative Multicriteria Analysis for Environmental Design, *Papers of the Regional Science Association*, 39, 175-199.

Nijkamp, P. (1979), *Multidimensional Spatial Data and Decision Analysis*, Wiley, Chichester.

Nijkamp, P. (1980), *Environmental Policy Analysis*, Wiley, Chichester.

Nijkamp, P. and H. Voogd (1979), The Use of Psychometric Techniques in Evaluation Procedures, *Papers of the Regional Science Association*, 42, 119-138.

Nijkamp, P. and H. Voogd (1981), New Multicriteria Methods for Physical Planning by Means of Multidimensional Scaling Techniques. In Y. Haimes and J. Kindler (Eds.), *Water and Related Land Resource Systems*, Pergamon Press, Oxford, 19-30.

Nowlan, D.M. (1975), The Use of Criterion Weights in Rank Ordering Techniques of Project Evaluation, *Urban Studies*, 12, 169-176.

Paelinck, J.H.P. (1976), Qualitative Multiple Criteria Analysis, Environmental Protection and Multiregional Development, *Papers of the Regional Science Association*, 36, 56-59.

Paelinck, J.H.P. (1977), Qualitative Multiple Criteria Analysis, an Application to

Airport Location, *Environment and Planning*, 9, 883-895.

Paelinck, J.H.P. (1978), Qualiflex, a Flexible Multiple Criteria Method, *Economic Letters*, 3, 193-197.

Rietveld, P. (1980), *Multiple Objective Decision Methods and Regional Planning*, North-Holland, Amsterdam.

Roy, B. (1972), Decisions avec Critères Multiples, *Metra*, 11, 121-151.

Saaty, T.L. (1977), A Scaling Method for Priorities in Hierarchical Structures, *Journal of Mathematical Psychology*, 15, 234-281.

Saaty, T.L. (1978), Exploring the Interface between Hierarchies, Multiple Objectives and Sets, *Fuzzy Sets and Systems*, 1, 57-68.

Schimpeler, C.C. and W.L. Grecco (1968), The Community Systems Evaluation; an Approach Based on Community Structure and Values, *Highway Research Record*, 238, 153-158.

Schlager, K. (1968), The Rank-Based Expected Value Method of Plan-Evaluation, *Highway Research Record*, 238, 123-152.

Spronk, J.(1981), *Interactive Multiple Goal Programming for Capital Budgeting and Financial Planning*, Martinus Nijhoff, Boston.

Stanley, J.K.A. (1974), A Cardinal Utility Approach for Project Evaluation, *Socio-Economic Planning Sciences*, 8, 329-338.

Starr, M.K. and M. Zeleny (Eds.) (1977), *Multiple Criteria Decision Making*, North-Holland, Amsterdam.

Strassert, G. (1973), Nutzwertanalyse, *Veroffentlichungen der Akademie fur Raumforschung und Landes Planung*, Hannover, Band 87, 54-68.

Stunet (1979), Een evaluatie van de Stunet-studie, Interdepartmental Committee on the Stunet Project, The Hague.

Van Delft, A. and P. Nijkamp (1977), *Multi-Criteria Analysis and Regional*

Decision-Making, Martinus Nijhoff, The Hague.

Van der Meer, G.J. and J.B. Opschoor (1973), Multicriteria Analyse: de Electra Methode, *Working Paper*, No. 17, Instituut voor Milieuvraagstukken, Amsterdam.

Van Setten, A. and H. Voogd (1979), The Determination of Regional Disparities in Western Europe, *Planologisch Memorandum*, 79-3, Delft University of Technology, Dept. of Urban and Regional Planning.

Voogd, H. (1975), Planevaluatie; een noodzakelijke voorwaarde bij het werken met alternatieven, *Stedebouw en Volkshuisvesting*, 6/7, 37-46.

Voogd, H. (1976), Concordance Analysis: Some Alternative Approaches, *Research Paper*, No. 2, Research Centre for Physical Planning PSC-TNO, Delft.

Voogd, H. (1980), Multicriteria Evaluation by Means of the Ideal Point Concept. In G. Hirsch (Ed.), *Aide a la Decision Multicritères; La Theorie face a la Pratique*, Cahiers d'Etudes et de Recherches, Vol. 10, Paris, 46-68.

Voogd, H. (Ed.) (1981a), *Strategic Planning in a Dynamic Society*, Delftsche Uitgeversmaatschappij, Delft.

Voogd, H. (1981b), Qualitative Multicriteria Evaluation Methods for Development Planning, *The Canadian Journal of Regional Science*, IV, 1, 73-87.

Voogd, H. (1982), *Multicriteria Evaluation for Urban and Regional Planning*, Pion, London.

Wierzbicki, A.P. (1979), A Methodological Guide to Multiobjective Optimization, *Working Paper*, WP-79-122, International Institute for Applied Systems Analysis, Laxenburg, Austria.