A Gaming Experiment on Cost Allocation in Water Resources Development

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A GAMING EXPERIMENT ON COST ALLOCATION IN WATER RESOURCES DEVELOPMENT

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

This paper is oriented towards the application of systems analysis techniques to water management problems in Western Skåne, Sweden. These problems concern the allocation of scarce water and related land resources among several mutually conflicting uses, e.g., municipal, industrial, agricultural and recreational water use.

The paper is part of a collaborative study on water resources problems in Western Skåne, Sweden, pursued by IIASA in collaboration with the Swedish National Environment Protection Board and the University of Lund. The background is an earlier IIASA study concerning the regional water supply project whose viability depends on how many municipalities will participate in it. In IIASA's Working Paper WP-79-77 some methodological problems involved in allocating costs of such a joint project to provide incentives for the participants were investigated. This companion paper describes a gaming experiment undertaken for testing some of the cost allocation models developed earlier, especially those based on some game theoretic concepts. Although a single gaming experiment does not provide sufficient ground for any final conclusion concerning advantages and disadvantages of various cost allocation models, it gives a valuable insight into the problem and indicates possible way of testing some of the theoretic model developments.

Janusz Kindler
Task Leader

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ABSTRACT

The paper describes a gaming experiment concerning the allocation of costs in water planning. Six Swedish water planners participated. The aim of the experiment was to be a first test of some models of cost allocation, mainly of a game theoretic nature, presented in another IIASA working paper. The behavior of the players, partly at odds with the normative implications of some game theoretic concepts, gave ideas for future research.

CONTENTS

A GAMING EXPERIMENT ON COST ALLOCATION IN WATER RESOURCES DEVELOPMENT

Ingolf Ståhl

1. BACKGROUND

The basis for the following working paper is the IIASA working paper WP-79-77 by H.P. Young, N. Okada, and T. Hashimoto, entitled "Cost Allocation in Water Resources Development - A Case Study of Sweden".

Although the reader is advised to first read WP-79-77, a brief summary shall be made of those parts of that paper which are of particular relevance to the experiment reported in this paper, so that it can be read independently of WP-79-77.

WP-79-77 deals with the question of how costs should be allocated in a water project when different municipalities join together to develop water supplies.

The cost allocation problem arises from the fact that there are, in general, economies of scale in the construction of water facilities. For example, two municipalities can in many cases get their water demand satisfied more cheaply by building a joint facility than by building separate ones. The amount of cost savings depends on which municipalities join together, the greatest cost savings often resulting when two adjacent municipalities join together. One problem of cost allocation in the case of such a coalition is that the fixed costs of construction of the plant cannot be assigned to two municipalities obviously in any unique way.

One can only propose various principles on which such allocations should depend. Suitable principles can be found, e.g. in game theory. One principle, based on "individual rationality", is that no municipality shall pay a higher cost than it would have to pay if it were to fulfill its water needs completely on its own. If we call the costs that the municipalities A and B incur if they work completely on their own c(A) and c(B), and the payments they shall make in case of a coalition x_A and x_B, then we require according to this principle that $x_A \leq c(A)$ and $x_B \leq c(B)$.

Another principle, which we call the "full cost" principle, is that total costs should be covered, leaving no surplus and no loss to any third party. For example, the total costs of the coalition AB are called c(AB) and we require that $x_A + x_B = c(AB)$.

In this situation we are, however, not interested in cooperation between only two municipalities, but in a situation where cost savings are also obtained as a third party joins the two-party coalition and further as a fourth party joins this three-party coalition, etc. In this specific case, we study a situation in which six municipalities[1] are involved and

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[1] As a matter of fact there are six <u>groups</u> of municipalities as discussed later.

where the total cost when all six cooperate, is lower than the total cost of any other combination.

Basic rationality principles would then say that the "grand" coalition of all six municipalities should be formed, since each party can then be in a better situation than it could be under another arrangement involving higher total costs.

The cost allocation principles in the six-party case are roughly the same as in the two-party case. Individual rationality would demand that $x_i \leq c(i)$ for $i = 1,...,6$ and that $\sum_{i=1}^{6} x_i = c(N)$, where N is the "grand coalition", involving all six parties.

In this case one can add an additional demand, namely "group rationality", implying that $\sum_{i \in S} x_i \leq c(S)$ for every coalition S, which is smaller than the grand coalition. This implies that the coalition consisting of parties 1, 2, and 3 would <u>not</u> agree to paying x_1, x_2, x_3, if the payments $x_1 + x_2 + x_3$ are higher than the total costs would be to these three parties if they only formed the three-party coalition 123.

The set of all allocations satisfying the three principles stated above, (individual rationality, full cost, group rationality), are said to constitute "the core". In some cases the core might not exist; in many cases, like the one studied in this paper, it exists, but is in no way unique.

There are several ways of obtaining a unique allocation within the core. In WP-79-77, three ways are discussed: the Nucleolus, the Proportional Least Core, and the Weak Least Core. Common to all three of these concepts is that the solution is obtained by application of linear programming, where one seeks to minimize some kind of subsidy rate and that one deducts some transformation of this subsidy rate from the total cost of each coalition. The constraints of the LP-program hence imply the requirement that the sum of the payments made by the members of a specific coalition should not exceed the total costs of this coalition minus some transformation of this subsidy rate. The three ways of obtaining a core solution differ with respect to the way the subsidy rate is transformed.[2] The LP-program determines a unique value of the subsidy rate, which in turn gives a unique cost allocation.

The three core concepts also differ in regard to the extent they satisfy the so-called monotonicity principle, that if total costs go up, no party should be charged less and if total costs go down, no party should be charged more.

In WP-79-77 a fourth solution concept based on game theory is discussed: The Shapley Value. One way of representing this value is the following: The grand coalition is formed step by step; first one party joins together with another party to form a two-party coalition. Then one more party is added to form a three-party coalition, and then another party is added to form a four-party coalition, etc., until finally the grand coalition is formed. There are many (in an n-person game: n!) ways or orders in which such a procedure can take place, depending on which party "signs up" first, which party "signs up" next. For each such order, a party joining a coalition is thought only to pay the incremental costs (i.e. the difference between the cost of the new coalition and the cost of the one he joins). The Shapley value for each party is then the party's average payments, computed over all n! coalition formation orders.

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2) For further details see pp. 10-14 of WP-79-77.

To make this more concrete, let us look at a 3-person game with A,B, and C as players. There are then 6 possible orders for coalition formation with the following incremental costs for each party:

	Cost assigned to:		
	A	B	C
Coalition			
ABC	c(A)	c(AB) - c(A)	c(ABC) - c(AB)
ACB	c(A)	c(ABC) - c(AC)	c(AC) - c(A)
BAC	c(AB) - c(B)	c(B)	c(ABC) - c(AB)
BCA	c(ABC) - c(BC)	c(B)	c(BC) - c(B)
CAB	c(AC) - c(C)	c(ABC) - c(AC)	c(C)
CBA	c(ABC) - c(BC)	c(BC) - c(C)	c(C)

The Shapley value of each party is then the sum of the values in his column, divided by 6; i.e., for A it is [2c(A)+2[c(ABC) -c(BC)] + c(AC) - c(C) + c(AB) - c(B)]/6.

Finally, a fifth method is presented in WP-79-77. This is a modified version of the Separable Cost-Remaining Benefits (SCRB). This method has been developed specifically for practical use in water resource planning. We define the marginal cost for a party c'(i) as c(N) - c(N-i), i.e. the marginal cost of being the last to join the grand coalition. Next the "remaining benefit" r(i) is defined as =c(i) - c'(i), i.e., the difference between the cost if the municipality goes alone and its marginal costs. The payment made by party i is then computed as $x_i = c'(i) + [r(i) / \sum_{j=1}^{N} r(j)][c(N) - \sum_{j=1}^{N} c'(j)]$, i.e., marginal cost plus its share of the non-allocated costs, where the share is set in relation to the party's share of remaining benefits.

On the basis of a real situation in Southern Sweden (Western Skåne), a cost table for various coalitions was computed using different procedures. Although in reality there are 18 municipalities in this region, it was found practical and realistic to group these into units which might be regarded acting as independent municipalities.

The composition of these six groups is discussed in detail in WP-79-77. For the purpose of this paper it is sufficient to note that the symbols A,H, K, L, M and T denote the main municipalities in each group, namely, Ängelholm, Hälsingborg, Kävlinge, Lund, Malmö and Trelleborg.

Then the joint cost function was computed for each of the possible coalitions that these six municipality groups could form. The results are given in Appendix A, where costs are specified in millions of Swedish Crowns.

On the basis of data presented in Appendix A, the allocations were computed according to the five procedures discussed above.[3] These are given in the table below.

Method	A	H	K	L	M	T
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.60	19.21
Proportional Least Core	19.81	12.57	4.35	9.25	19.85	17.99
Weak Least Core	20.03	12.52	3.94	9.07	20.11	18.15
S.C.R.B.	19.54	13.28	5.62	10.90	16.66	17.82

Cost allocations in millions of Swedish Crowns.

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3) In WP-79-77 two other cost allocation procedures, based on specific data (population and demand) were also used. Not used in the experiment, they are omitted here.

In WP-79-77, the SCRB and the Shapley value procedures are criticized mainly because none of them satisfies the principle of group rationality. Let us look at the coalition HKL. According to the SCRB procedure, HKL shall together pay 29.81, and according to the Shapley value, 28.00. Should they not join the grand coalition, but remain satisfied with the three-party coalition HKL, they would only have to pay the cost of this coalition, or 27.70 (see Appendix A). Hence neither the SCRB nor the Shapley Value belong to the core. It should be noted that SCRB is considerably further away from the "mark" than the Shapley value.

Having criticized the SCRB and the Shapley value, the authors of WP-79-77 investigate other properties of the three core solutions. They arrive at preferring the Weak Least Core, since it is the only one of the three which always satisfies the monotonicity principle mentioned on page 4. The Nucleolus violates this principle in this specific game; the Proportional Least Core, while satisfying the monotonicity principle in this particular game, does not fulfill it in all games.

2. GAMING AND NORMATIVE GAME THEORY

It appears reasonable that before looking at the specific gaming experiment, we briefly discuss the relation between gaming and game theory, in particular when the latter has a normative purpose.

As discussed more extensively elsewhere (Ståhl, 1979) gaming and game theory can be seen as complements to each other. When a game theoretic solution has been presented in the context of application, it is reasonable to ask: Can gaming be used as a complement to the presented game theoretic solution?

This question is relevant not only if the purpose of the game theory application is to be a description or a prediction of how players will play the game, but, in accordance with our viewpoint, also if the purpose of the game theory application is said to be normative.

The relevance of gaming for a description or prediction is obvious: Do the parties play as described or predicted? If one cannot get people to play the game as predicted by the theory, in spite of a great many experiments with different institutional setups, then strong doubts are cast on the theory as far as its descriptive or predictive aim is concerned.[4]

With regard to game theoretic models presented with a normative aim, it is my view that there is only a difference in degree, not in kind. As discussed elsewhere (see e.g. Ståhl, 1972, pp. 142-144) a game theoretic model can be called normative for a decision situation only if decisionmakers, when presented with the model, will want to use it. Unless one foresees some superior norms, e.g. moral norms, based on societal or religious values, the norms concerned must be related to the norms of the decisionmakers. The important question is then: Would the decisionmakers want to use the game model if they had been properly introduced to it, and would they, after having used it, want to use it again?

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4) This is, e.g. very clearly the case of the Wicksell-Bowley bilateral monopoly model, refuted by Siegel-Fouraker and their followers and the application of the Nash equilibrium concept to iterated bi-matrix games, most notably the Prisoners' Dilemma games. (For references see Ståhl, 1980.)

One way of testing a normative model experimentally is to have the game leader instruct the players about the normative model and then to let the participants play the game to see if they would use the model. There is, however, at least one important problem with such an approach, namely the problem of conformation to authority overriding other benefits in an experiment. This implies that, if only one model is "sold", and the gaming participants, unlike decisionmakers in reality, do not stand to join or lose considerable amounts, they are likely to accept the advice of a game leader, since they do not want to appear "stupid".[5] The propensity of gaming participants for wanting to please the experiment leader, when possibly at most a few cents is lost thereby, must be taken seriously into account.

In order to avoid such effects, one could in principle do one of two things experimentally:

A) Several normative theories are presented and the gaming participants are then "free", (in the sense of being "uninfluenced by authority") to choose either one.

B) One runs the experiment in a "positivistic way" without any "normative influence" in the form of presenting the model. On the other hand, one tries to arrange the institutional setup in such a way as to increase the likelihood of the parties' behaving according to the normative theory. This, for example, could be done by supplying the decisionmakers with ample time for making the decision, supplying them with calculating equipment, by repeating the game so that learning can take place, etc.

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5) See Ståhl (1980)

Both approaches have their advantages and disadvantages:

With regard to A, the presentation of several models in a sufficiently detailed manner in one experiment might be very time-consuming and in many cases confusing. One might instead run only one model in each experiment, with a new experiment and with new subjects for each model, or one might run two models at a time, in a kind of tournament. Either of these methods would require, however, more game experiments and hence more time.

The main problem with approach B is that in many cases the normative model is based on such a sophisticated reasoning that it is completely unreasonable to expect any of the gaming participants to be able to establish the desirable solution on his own, even if the game were to be repeated and learning were allowed for, and even if the parties were given computing equipment and ample time for their decisions.

It is important in this connection to distinguish between the behavioral assumptions of the model and the deduction of the solution on the basis of these behavioral assumptions and of the assumptions regarding the institutional setup.[6]

The main contribution of the normative model is in helping the decisionmaker with the deduction (and the computation) of the solution. In line with our discussion above, it is not reasonable that the model should force behavioral assumptions on the decisionmaker. The sophisticated reasoning of the game model that must be accounted for hence refers to the deduction of the solution.

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6) Among behavioral assumptions, one should include what in many models is referred to as axioms, often concerning desirable properties of the solution , e.g., the "monotonicity principle" above. One behavioral assumption would then be a certain preference for this principle over other principles or objects.

In contrast to the descriptive and predictive testing of the model, where we want to find out whether the players really act as if they could make these deductions[7] we want in this case only to test whether the parties follow the behavioral assumptions of the model.[8]

These behavioral assumptions cannot, however, be tested "in vacuum", one by one. Presented one at a time to decisionmakers, they appear in most cases as meaningless abstractions. In this case, one reasonable way of testing would be the following: One starts with as simple a model as possible,[9] for which the deduction of the solution is no great problem. The normative test of the theory then consists of investigating whether under some sort of institutional setup, the parties will follow the behavior indicated by the theory. If this is true, it might be of interest to go on with more complicated versions of the model. If, on the other hand, parties do not follow the simple model under any institutional setup, one has every reason to raise questions regarding its value for normative applications, also for more advanced versions of the model. If players do not want to adapt to the behavioral assumptions of the simple model, it is difficult to see why they should want to do so for more

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7) They do not really have to make the deductions, but only to act as if they had made them, like, for example, Friedman's "pool shark", acting as if he could solve advanced differential equations.

8) Excluding the behavioral assumption that the parties have great computational and deductive capacity.

9) Even if the model is simple with regard to the deduction of the solution, it is, however, important that the full set of behavioral assumptions be involved also for this model.

complex models. In the simple versions of the models, the deductions are fairly easy and it is hence not the deductions that pose the problems, but rather the behavioral assumptions. Summing up the discussion, we can state our claim that a "positivistic" experiment is one way of testing the validity of a normative game theory. Furthermore, such testing of validity should be of the repeated kind, allowing for variations in the institutional setup. It should also start by testing a simpler version of the model.

3. MAIN IDEAS OF THE GAMING EXPERIMENT

Against this background it appeared reasonable to test the game theoretic models presented in the first section of this paper by a gaming experiment, even though the aim of these models was mainly of a normative nature.

The choice was between presenting several of the models with their normative implications clearly stated to the players, or just making a plain experiment of the ordinary, positivistic type. The choice was greatly influenced by resources available for conducting the experiment.

It was regarded to be of the greatest importance to involve players who would be as similar as possible to the actual decisionmakers for which the model was intended. There is ample evidence from the literature on experimental gaming that university students in many cases behave quite differently from people with practical experience in business or government. Students, in particular, seem much more likely to follow the idea of "beating the average", etc. Being more interested in the difference between what they get and what the others get, students make the games less cooperative, while businessmen and civil servants play much more cooperatively. Hence, we wanted to avoid experiments with only students, although they would have been easily obtainable. Rather we wanted to get real decisionmakers, preferably with a water planning background. The University of Lund-IIASA Workshop on Water Resources Planning and Management, November 26-28, 1979, was regarded as a unique opportunity, since it was attended by decisionmakers in water planning from the involved municipalities in Southern Sweden.

Although, as mentioned above, a great number of experiments would be needed for any kind of more serious validation of the models, for practical reasons we were forced to limit ourselves to one single gaming experiment. The main reason was that we expected at most ten people with the desired background at the workshop, and foreseeing a certain "drop-out" ratio, there would most certainly not be enough people for two experiments. The idea of letting the same individuals participate in several game runs also had to be ruled out, not only because it would be difficult to get more time from these people, but also because using the same players several times would provide an undesirable format for the repeated runs of the game.[10]

So we had to settle for a single gaming experiment. The results presented below must hence be regarded as very preliminary indications and suggestions for future research. The

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10) If one wants the same player to play in several games and allow for learning of the model, one should not let him play with the same set of players in each game. Then one does not get several games, but rather one super game, with some possibly peculiar effects.

experiment can hopefully be seen as the first one in a series of tests. Maybe it can also be used as a background for discussion of the model concepts, that is, as a stimuli for generation of new ideas and hypotheses.

Finally, it had to be decided whether one should inform the players in advance about the model, or whether the game should be played as a simple "positivistic" experiment. Several factors spoke for the <u>latter</u> approach:

1) As mentioned above, it is difficult to give several models a fair treatment in a normative experiment of this kind. It is reasonable to assume that the order in which you present the models plays a role. The participants are more likely, for example, to remember the first of the presented models in particular, and the last one.

2) The presentation of the models should take some other form than that of simply presenting the contents of WP-79-77, since there might then be considerable "authority effects".[11] Preferably one should have a "proponent" for <u>each</u> model, presenting the case for this model, like in a referendum. Such a procedure was impossible in this case, due to limitations in both time and resources.

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[11] It should be mentioned that WP-79-77 was available at the Lund workshop. However we regarded it as unlikely that the game participants would be influenced to any extent by this. This proved true, since it was found at the debriefing session that none of the game participants had studied WP-79-77.

3) The briefing of the gaming participants in a reasonably careful manner would take extra time, probably at least as much time as it would take to run the whole game. Such an extra time requirement would jeopardize the recruitment of suitable gaming participants.

4) WP-79-77 was to be presented at the Lund workshop during one hour roughly on the last day of the workshop. There was a choice as to whether the game was to be run prior to or after this session. There were rather strong reasons for running the game prior to the presentation of the paper:

   a) Some of the gaming participants might not be in attendance at this session and this would lead to special effects on the game.

   b) The presentation of the paper might have the biased "authority" effects discussed above.

   c) From the workshop point-of-view it was preferred that a very preliminary report on the gaming experiment be given in connection with this presentation. This was of particular importance, since the game, due to its limited one-shot character, might have its greatest value as a background for such a discussion.

4. HYPOTHESIS ABOUT THE OUTCOME OF THE GAME

Five hours prior to the start of the actual game, I wrote down the following conjecture about the outcome of the game:[12] "Two coalitions will be formed rapidly:

a) The 3-party coalition HKL splitting evenly, with 9.08 to 9.09 each. Possibly this will be formed after the formation of the temporary coalition HL (12.50, 12.50).

b) The 2-party coalition MT, splitting evenly 19.70, 19.71.

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12) A copy of this conjecture was transmitted to the authors of WP-79-77 for control purposes.

After this, the coalition formation will be more difficult and it will take a longer time before a new coalition is registered.

If a 4-party coalition is registered, it would most probably be AHKL with roughly 9.00 + a small amount, ε, to each of HKL and 21.95 - 3ε to A.

5-party coalitions are less likely. The most likely one would be the coalition HKLMT with 9.00 to each of HKL and around 19.50 to M and around 20.00 to T.

A grand coalition, if it comes into force, would result in roughly 8.50 to each of H, K, and L and around 19.44 to A, M, and T."

The thinking behind this conjecture was based on my readings concerning the results of a great many other gaming experiments as well as my own experience from such exercises. One important result has been that parties in experimental n-person games (with e.g. $n > 4$) very seldom seem to form the grand coalition immediately. Rather, it is formed step by step, first by forming two- or three-party coalitions and then by adding one other coalition or player at a time.

Another result has been a strong emphasis on "equity".[13] In general, parties have not been willing to accept pay-outs that are very unequal. Parties who are at a "strategic disadvantage" have been unwilling to receive considerably less than the other parties. In fact, there has often been a tendency towards equal splitting of pay-outs.

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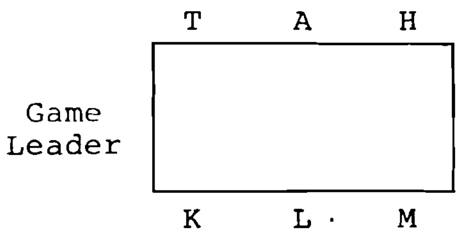
13) See, e.g. Selten (1979)

5. THE START OF THE GAMING EXPERIMENT

The experiment was carried out in the office of SSK (South-Western Skåne Municipality Confederation) on the evening of November 27,1979.

Six persons involved with municipal planning, mainly focused on water supply, participated. Most, but not all of them, had been taking part in the workshop. Five of the six players are working in Skåne, and one in another part of Sweden.

The game took place around a small table, with six seats for the players and one for the game leader. The six players' seats were randomly allotted to the six municipality roles. The configuration shown below was obtained:



Next the municipality roles were randomly allotted to the six players.

Then the game instructions were distributed to the players. A translation of these game instructions is given as Appendix B. The instructions also contained the pay-off table of Appendix A, While the figures in Appendix A referred originally to millions of Swedish Crowns, the game was only concerned with single Crowns.[14]

The players were allowed ample time to study these instructions and to think about their game strategy in silence. The actual game did not begin until everybody said that he was ready

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[14] One Swedish Crown is roughly 25 cents in U.S. currency.

to start negotiating. This occurred after thirty minutes, before which time the participants were also allowed to ask the game leader questions regarding unclear matters. Only a few questions were asked and these could all be answered briefly by referring to some point in the game instructions.

6. THE ACTUAL PLAYING OF THE GAME

After this the real game began. The first to form a coalition were T and M, who joined after only 3 minutes. Their method for dividing costs was an equal split of the total cost savings; that is, $x(M) = c(M) - (c(M) + c(T) - c(MT))/2$, giving 19.12 to M and 20.29 to T.

In the meantime H and K sought to form a coalition, leading to total costs of 22.96. Before discussing more precisely how to divide this, they found that including L as well would lead to even greater cost savings. Total costs would increase by only 4.30, while L on his own would have to pay 15.88.

When trying to form a coalition HKL, H suggested an equal split of roughly 9 to each. This was resisted by the other two, in particular by K, who would then only gain 1.91 by joining this coalition rather than remaining alone. Nor would the parties agree on the method adopted by MT, that is, of splitting the cost savings equally. This would have meant that H would have had to pay 11.54, while K would have gotten away with just 5.37.

Therefore the parties worked out an agreement based on another formula, which gave a result that can be seen as a compromise between "split total costs equally" and "split cost savings equally": the division of the total cost shall be based on the percentage obtained when each party's payment on his own is computed as a percentage of the total costs of these "one-party coalitions".[15] The table below gives a better explanation of this principle:

	cost of "one-party coalition"	percentage	payment in three-party coalition
H	17.08	39	10.53
K	10.91	25	6.82
L	15.88	36	9.91
Total	43.87	100	27.26

The using of this formula required the parties to use the available calculator. So after 8 minutes of playing, HKL registered a coalition with the division H : 10.53, K : 6.82, and L : 9.91. After this, only A was not in any coalition. Party A sought to join the coalition HKL, since he could not gain anything from joining MT. The total cost savings by forming the four-party coalition AHKL is, however, only 0.26. For the parties playing in crowns, there was hence only around six U.S. cents to divide. In the "real world" there were 260,000 Swedish Crowns (or around $60,000) to divide. The members of HKL were very reluctant to let A in. An amount of about one cent per player might be below the "psychological threshold of perception". One player also said something of the following nature: "In reality, we would never expand a three-municipality group into a four-municipality group in order to gain only around one quarter of

15) This method is in cost allocation literature known as the "justifiable expenditure method". See Eckstein (1958) and James and Lee (1971).

260,000 crowns. This is a small amount compared with the many millions we gained when we formed the three-party coalition. A larger group is always troublesome and gains must be significant in order for us to enlarge the group."

Hence HKL were reluctant to let A join. The coalition MT was interested in forming the grand coalition, but it appeared that in this specific case nothing could be done in this direction before the AHKL coalition had been formed.

According to the rules of the game, however, the game is terminated when all coalitions have been in force for 10 minutes (see p. 30 of the game instructions). Even though the game leader "stretched" the time allotment[16], no further coalitions could be formed. And so the game ended with the one two-party coalition MT and one three-party coalition HKL.

It is interesting to note that the coalition MT involves Malmö and Trelleborg, lying next to each other in the southern part of the studied region. HKL is made up of Helsingborg, Kävlinge, and Lund, adjacent to one another in the middle of the region. A, Ängelholm, situated in the very north, was left alone.

The game was followed by a discussion with the gaming participants. This focused particularly on the question as to which pay-off distributions would have resulted if the game had continued longer, e.g. not ending before a four-party or possibly a grand coalition had been formed.

As for the four-party coalition AHKL, it was agreed that one would have divided the small cost savings here equally,

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16) In reality, the 10 minutes became 15 minutes. This is not really the way a game should be played, of course, but the interest in seeing whether larger coalitions could be formed weighed heavily.

giving each of A, H, K, and L a cost reduction of around 0.05 to 0.07. This would have given 21.90 to A, 10.45 to H, 6.75 to K, and 9.85 to L, allowing for ±0.05.

The grand coalition would have been formed by a merger of AHKL and MT. It appeared that most parties seemed in favor of applying the equal division of the cost savings here as well. This would imply a cost reduction of roughly 0.75 to each of the six parties. Allowing for ±0.10 for each party, I believe that one can say with fairly great certainty, that if a grand coalition had been "enforced", i.e. by allowing for a much longer negotiation time, we would have obtained A : 21.15, H : 9.70, K : 6.00, L:9.10, M : 18.37, and T : 19.50.

The reason why these results are included in the comparison below is just to compensate in some way for the fact that only 10 minutes were allowed for in the game instructions before the coalitions became operational. By making this wider comparison, we should be able to make a more reliable comparison with the game theoretic solutions discussed in Section 1 of this paper.

7. COMPARISON OF THEORIES AND EXPERIMENTAL RESULTS

Comparisons shall be made on the basis of the table below, which summarizes both the theoretical calculations of WP-79-77 and the empirical results from the gaming experiment.

In order to see how well the theoretical allocations fit the experimental values, we shall use three measures of difference:

1) The sum of absolute differences. With T_i as the theoretical value and E_i as the experimental value for party i, the measure is: $\sum_{i=1}^{6} | T_i - E_i |$
2) The sum of the squared differences, i.e., $\sum_{i=1}^{6} (T_i - E_i)^2$. Compared to measure 1, this gives a higher relative weight to large discrepancies.

	Players					
	A	H	K	L	M	T
Theoretical Values						
Shapley Value	20.01	10.71	6.61	10.37	16.94	19.18
Nucleolus	20.35	12.06	5.00	8.61	18.60	19.21
Prop. Least Core	19.81	12.57	4.35	9.25	19.85	17.99
Weak Least Core	20.03	12.52	3.94	9.07	20.11	18.15
SCRB	19.54	13.28	5.62	10.90	16.66	17.82
Experimental Values						
Real Agreement HKL, MT	21.95	10.53	6.82	9.91	19.12	20.29
If AHKL (±0.05)	21.90	10.45	6.75	9.85	19.12	20.29
If Grand (±0.10)	21.15	9.70	6.00	9.10	18.37	19.50

3) The sum of the relative squared differences, i.e., of the squared differences after dividing each difference by the theoretical value, i.e. $\sum_{i=1}^{6} (T_i - E_i)^2 / T_i$. The idea behind this measure is that a difference is more important if it is relatively large in comparison with the "expected" value.

Computing these measures for the difference between the theoretical values and the real outcome of the game, the following table is obtained:

	Measures		
	1	2	3
Shapley Value	6.08	10.04	0.56
Nucleolus	7.85	11.29	1.25
Prop. Least Core	10.34	21.10	2.33
Weak Least Core	10.76	22.20	2.98
SCRB	12.78	27.94	1.92

We see that the result is mainly independent of the measure used. The Shapley value is by any measure the best, followed by the Nucleolus. Then there is a large step to the Proportional Least Core. At the bottom we have the Weak Least Core and SCRB. SCRB is poorest according to two measures and the Weak Least Core according to one measure.

Comparisons with the AHKL-MT case would obviously lead to almost the same results, but it might be worthwhile to examine the grand coalition. We then obtain the following table:

	Measures		
	1	2	3
Shapley Value	5.78	6.45	0.50
Nucleolus	5.17	7.59	0.73
Prop. Least Core	9.00	17.25	1.61
Weak Least Core	9.12	18.30	1.33
SCRB	10.76	24.53	1.75

The results are fairly much the same as above, the Shapley value still being on top. But by one measure, the sum of absolute differences, the Nucleolus is somewhat better. In this case SCRB is at the bottom by any measure.

8. COMPARISON WITH THE PRE-GAME HYPOTHESIS

A comparison can also be made with the hypothesis about the outcome of the game, presented in Section 4.

The hypotheses that it would be easy to form the 3-party coalition HKL and the 2-party coalition MT were correct. Furthermore, the hypothesis about the difficulty of forming a four-party coalition proved true.

The hypothesis regarding the way the costs were allotted within these two coalitions was, however, inaccurate. We had believed that the divisions would follow a simpler criterion for "equity" than they actually did. It is possible that the literature, as well as our own experience which deals mainly with experiments using students, misled us at this point. Real decisionmakers, as mentioned earlier, are less likely than students to seek simple "average" solutions.

If, however, one would regard our hypothesis as a forecast for the solution with the 2- and 3-party coalitions and with A remaining alone,[17] the forecast of payouts would not have been very bad. Using the three measures of difference presented above, we obtain: 5.69, 8.56, and 0.90, i.e., for the first two measures, our hypothesis would be even better than the Shapley value, while for the last measure, it is considerably worse.

9. IDEAS FOR FUTURE RESEARCH

As discussed above, one would have to repeat the experiment a great number of times with different institutional set-ups before entering into any serious discussions regarding

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17) Because our hypothesis was rather vague, saying "After this, the coalition formation will be more difficult", this might be a somewhat biased suggestion.

the normative validity of the models. The outcome of the experiment might, however, give some ideas about what would be the most suitable next steps of research.

The experiment only gave data to compare the five methods presented and the focus has since been on the difference between these five methods. In order to keep the experiment simple we excluded data on population and demand included in WP-79-77.[18] If one had included this data and _if_ one could still have obtained the same result, then the most visible result would have been that the five methods presented here were all much closer to the actual solution than the solutions obtained if costs had been allocated on the basis of population and demand.[19] Hence, it would be of interest to see, if the result of the experiment really will be unaffected by the introduction of this data.

If we return to the difference between the five methods presented here, the question arises as to what extent the relatively poor results of the Weak Least Core and also of the Proportional Weak Core, both fulfilling the monotonicity requirement in this game, would be improved if one in some way explicitly introduced some device which would make the players to some extent aware of this principle.

The results of the experiment might also generate some ideas for future theoretical work, in particular if they are substantiated by further experiments.

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18) See footnote 3 on page 6.

19) An allocation on the basis of population would, compared to the real agreement, had given the values 55.93, 668.14 and 54.87 for difference measures 1, 2 and 3 respectively, while an allocation on the basis of demand would have lead to 37.45, 307.24 and 20.90. These values are roughly 5 - 20 times higher than those in the first table on page 23.

Not only the favorable values of the difference measures but also the data on the gradual formation of the grand coalition, known also from many other experiments, might then focus the interest on the Shapley value. It can then be noted that the reason the Shapley value fails to be within the core of this particular game, is that the Shapley value assigns equal probability to every coalition being formed. One can envisage here a modified Shapley value, which for this game, and also many other games,[20] would lie within the core. The modification would consist of eliminating from the formula those coalitions which do not increase the sum of benefits, or in this case, do not decrease total costs.[21]

A more radical theoretical improvement would be to work completely in the extensive form, with the restriction that in the beginning, only two- and three-party coalitions would be formed and adding the constraint that new coalitions are only formed by a merger of two (or possibly three) other coalitions. The problem with applying analysis in the extensive form, using an algorithm approach similar to dynamic programming to n-person games[22] is that for $n > 4$, work in the extensive form is infeasible, due to computer memory and time requirements. If one, however, limits oneself to $n \leq 3$ in each step, extensive form analysis becomes feasible. This would also allow for greater realism, since the time path of costs can also be taken into account.

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20) But not for all games, which can be easily understood, if one has coalitions which only improve by a very small amount compared to the entering coalitions.

21) A procedure of this type is proposed in Loehman et al.,(1979).

22) See e.g. Ståhl (1977).

APPENDIX A: TOTAL COST OF EACH POSSIBLE COALITION

A	21.95	AHK	40.74	AHKL	48.95
H	17.08	AHL	43.22	AHKM	60.25
K	10.91	AHM	55.50	AHKT	62.72
L	15.88	AHT	56.67	AHLM	64.03
M	20.81	AKL	48.74	AHLT	65.20
T	21.98	AKM	53.40	AHMT	74.10
		AKT	54.85	AKLM	63.96
AH	34.69	ALM	53.05	AKLT	70.72
AK	32.86	ALT	59.81	ALMT	73.41
AL	37.83	AMT	61.36	HKLM	48.07
AM	42.76	HKL	27.26	HKLT	49.24
AT	43.93	HKM	42.55	HKMT	59.35
HK	22.96	HKT	44.94	HLMT	64.41
HL	25.00	HLM	45.81	KLMT	56.61
HM	37.89	HLT	46.98	AKMT	72.27
HT	39.06	HMT	56.49	AHKLM	69.76
KL	26.79	KLM	42.01	AHKMT	77.42
KM	31.45	KLT	48.77	AHLMT	83.00
KT	32.89	KMT	50.32	AHKLT	70.93
LM	31.10	LMT	51.46	AKLMT	73.97
LT	37.86			HKLMT	66.46
MT	39.41				
				AHKLMT	83.82

APPENDIX B: GAME INSTRUCTIONS

You have been invited to participate in a simple game on Tuesday, November 27, 1979.

The game concerns the allocation of costs in a water project, the so-called "Aqua vita" project. This project aims at bringing stimulating liquid to six municipalities. You will represent one of these. On this occasion, as the sole representative of this municipality, you will represent both the producer and the consumer side.

You will participate in this project either completely on your own, or in cooperation with one or several of the other participants in the game, who are acting as representatives for other municipalities.

All in all, representatives of six municipalities, called A, H, K, L, M, and T, participate in the game. All participants (= municipalities) must in some way take part in the water project, but their costs will depend on how they form coalitions with other participants.

Should a municipality not enter into coalition with any other municipality, it will pay that sum in the allocated table which represents what each municipality would be obligated to pay if acting alone. Prior to the start of the game, each player, i.e. each representative of a municipality, will receive this sum in cash from the game leader.

Each player can, however, by acting skillfully both during the formation of coalitions and during the allocation of the total costs within the coalition, get away with a lower payment, in some cases, a considerably lower one.

The player may keep this surplus for himself (or if he wishes to do so, may donate it at the end of the game to the Red Cross fountain drilling activity in India).

The details of the game are as follows:

By lottery, each player is assigned the role of the representative of one of the six municipalities. Next each player obtains the aforementioned sum of money corresponding to the maximum amount that he might have to pay, should he participate in the water project completely on his own. After this, the players sit down around the table and the coalition-formation negotiations can begin.

The players then must try to form coalitions and reach agreement on how much each of the participants in the formed coalition shall pay of the total cost to the whole coalition. This total cost for each possible coalition is seen in Appendix A.

As soon as the first coalition has been formed and agreement has been reached as to the allocation of the total costs of this coalition among its members, they register the coalition with the game director. He will then record the names of the coalition participants, as well as the payment each of them would make toward the total costs of the coalition. Once a coalition has been registered, its content, i.e. the participants and the cost allocation, is announced to all participants of the game.

A coalition does not come into force, however, until 10 minutes have elapsed since its registration, and then only provided that none of its members has been registered in another coalition during this period. Hence a player can leave one coalition and join another in order to decrease the amount of his payment. Furthermore, a coalition dissolves by registering a new coalition with additional members. For new coalitions, the rule still applies that it does not come into force until it has been registered unchanged for 10 minutes.

Once a coalition has come into force, each of its members pays the game leader the amount agreed upon at the time of the registration. These participants then cease to take an active part in the game, but may remain at the table if they wish to do so.

The game continues in this way until all participants are members of a coalition which has come into force (with the possible exception of a single "leftover" participant). Should the game continue more than 90 minutes from the time of its start, it will be brought to an end and those coalitions registered (but not broken) at the time will come into force.

Finally it should be stressed that the game aims at bringing out some aspects of one of the papers presented on Wednesday. Hence it is important that you try as much as possible to act as one could expect a representative for a municipality to act during such negotiations, where the economic interest of the municipality are at stake.

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