FORMAL APPROACH TO RESEARCH UNITS LOCATION

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INTRODUCTION

This paper represents the results of a three month study, in which several Junior Scientists from many countries took part during the summer of 1979 at IIASA. While many of these results are not fully completed, and some represent only preliminary directions of research, we feel that the documentation of the efforts of the Junior Scientists is justified.

ABSTRACT

The formal description of the effective location of research units is given here. It is proposed for the examination of different location variations.

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One of the basic problems of research center designing is the location of effective research units satisfying a number of constraints and requirements. Usually the city planner should analyze many possible variations in order to choose the best one. This analysis is very difficult to do without a computer. The use of computers, however, demands a mathematical description of the problem and the creation of models to be used for the evaluation of different variations and the choice of the optimal one.

This paper attempts to give a formal description of the research units location problem and to use it for the evaluation of the different variations presented. The problem is considered as the space location of given research units volumes $v(l,p)$ on the area S with boundary G in order to achieve the objective function values and to satisfy the constraints. It is assumed that all units are divided into L-types $(l = 1,2,\ldots,L)$. The index p is the index of a unit of type l $(p = 1,2,\ldots,P_l)$, and the volumes, $v(l,p)$, of every unit are given. The location structure should also satisfy the given number of efficiency criteria, which will be described further in terms of the objective functions and constraints.

1. The minimization of engineering communcation cost is described by the objective function

$$J_1 = \sum_{i} \sum_{l,p} \left[c_i \, a_i(l,p) + c_i \, k_i(l,p) \bullet z(l,p) \bullet s_o(l,p) + h_i \bullet z(l,p) \right] \quad (1)$$

where

- i enumerates the supply resources (water, gas, etc.)
- c_i is the unit cost of the horizontal communication of the i-th resource;
- h_i is the unit cost of the vertical communication of the i-th resource;
- $S_o(l,p)$ is the area of the ground floor of the unit (l,p);
- $z(l,p)$ is the vertica- size of the unit (l,p);
- $a_i(l,p)$ is the distance from the origin of the i-th resource to the unit (l,p);
- $k_i(l,p)$ is the coefficient of communication density.

2. The accessibility of workers from public transportation sites to the research units is described by the objective function:

$$J_2 = \sum_{j} \sum_{l,p} L_j(l,p) \cdot \gamma(l,p) \quad (2)$$

where

j enumerates the transport station considered;

$L_j(l,p)$ is the distance from the j-th transport station to unit (l,p);

$\gamma(l,p)$ is the factor that takes into consideration the number of people working in unit (l,p).

3. Function and technological unit criterion means that all distances among units $(\tilde{l},\tilde{p})$ (from $\tilde{L},\tilde{P}$ group) should be minimized. This is described by the objective function

$$J_3 = \sum_{l',p'} \sum_{l'',p''} a(l',p';l',p') \quad , \quad \begin{array}{l} l',p', \varepsilon \tilde{L} \\ l'',p'', \varepsilon \tilde{L} \end{array} \tag{3}$$

where

$a(l',p';l'',p'')$ is the distance between units (l',p') and (l'',p'').

4. Territory use intensiveness criterion is described by the objective function:

$$J_4 = \sum_{l,p} s_0(l,p) \, , \tag{4}$$

$s_0(l,p)$ is the ground floor area of the actual research unit

These four criteria make up the set of objective functions. Other efficiency criteria will be described in terms of constraints:

1. The requirement for the simultaneous function of different technological subsystems is described by

$$\min_{l',p',p} a(l,p;l',p') \geq R_l \quad , \qquad l' \in \bar{L} \tag{5}$$

this constraint means that the distance $a(l,p;l',p')$ from the type l unit to any other units from the given set $\bar{L}$ should not be smaller than R_l.

2. The qualitative growth requirement is satisfied by the introduction of maximum radii of influence - R_l .

3. The sufficient condition to satisfy the requirement of quantitative growth possibility is

$$Z(l^*,p) = 1$$

$$|x(l^*,p) - x(l',p')| \geq \left[S^*(l^*,p)\right]^{\frac{1}{2}}$$

if

$$|y(l^*,p) - y(l',p')| \leq \left[S^*(l,p)\right]^{\frac{1}{2}}$$

(6)

and

$$|y(l^*,p) - y(l',p')| \geq \left[S^*(l^*,p)\right]^{\frac{1}{2}}$$

if

$$|x(l^*,p) - x(l',p')| \leq \left[S^*(l,p)\right]^{\frac{1}{2}}$$

where

$$|x(l^*,p) - x(l',p')| \quad \text{and} \quad |y(l^*,p) - y(l',p')|$$

are distances from unit (l^*,p) to unit (l',p') along x-axis and y-axis.

$S^*(l^*,p)$ is the area needed for the future growth of unit (l^*,p)

4. Areas which are not suitable for construction are excluded from the S territory.

5. The environmental preservation criterion says that all influence radii $R(l_I,p_I)$ should lie within the fixed area of influence S_I:

$$\min_G a(l_I; x_G, y_G) \geq R(l_I) \tag{7}$$

where

G is the boundary of S ;

$a(l_I; x_G, y_G)$ is the distance from the type l_I unit to the boundary point (x_G, y_G).

6. The unit height constraint is:

$$Z(l,p) \leq Z_{max} \quad . \tag{8}$$

7. The research unit location level is $Z(l,p)$ such that:

$$Z(l,p)_{min} \leq Z(l,p) \leq Z(l,p)_{max} \tag{9}$$

8. The "wind" requirement means that the research unit should be located on the dominating wind side:

$$a(l_w; x_{G0}, y_{G0}) \leq a(l; x_{G0}, y_{G0}) \quad , \tag{10}$$

where

l_w is the object which should be located on the dominating wind side;

$a(l; x_{G0}, y_{G0})$ is the distance from the type l unit to the tangent point of the normal to the wind direction and the boundary G.

9. The fixed location requirement means that units with fixed areas should be located first.

The formal description suggested above for the effective research unit location problem can be used for the derivation of the optimal location variations. In order to derive it, the methods for calculating the values in expressions (1) - (10) should be defined. Algorithms for calculating objective functions (1) - (4) under constraints (5) - (10) should also be defined, and, at the final stage, methods of multiobjective optimization should be used, as the presence of several objective functions comes into the vector optimization problem. This way the suggested formal approach can be used as a basis for the creation of an optimization model.

Here, we present a simpler expression of the formal approach in order to compare the variations proposed.

In order to make the comparison, all distances and expressions (1) - (4) should be measured from the geometrical centres of the units, and all units are assumed to have rectangular forms. Than,

$$J_1 = \sum_i \sum_{l,p} \left\{ c_i \left[\left(x_i - \frac{x_1(l,p) + x_2(l,p)}{2} \right)^2 + \left(y_i - \frac{y_1(l,p) + y_2(l,p)}{2} \right)^2 \right]^{\frac{1}{2}} \right.$$

$$+ h_i \frac{Z_i(l,p) + Z_2(l,p)}{2} + c_i k_i(l,p) \left[x_2(l,p) - x_1(l,p) \right] \bullet$$

$$\left. \left[y_2(l,p) - y_1(l,p) \right] \right\} \qquad (11)$$

where

$\left[x_1(l,p) ; y_1(l,p) \right] \left[u \; x_2(l,p) ; y_2(l,p) \right]$ are the boundary coordinates of unit (l,p) ,

$z_1(l,p)$ and $z_2(l,p)$ are vertical coordinates of the lowest and highest floors of the unit (l,p) ,

x_i and y_i are coordinates of the origin of i-th resource,

$$x_2 > x_1 \; ; \; y_2 > y_1 \; ; \; z_2 > z_1 \quad . \qquad (12)$$

$$J_2 = \sum_j \sum_{l,p} \left[\left(x_j - \frac{x_1(l,p) + x_2(l,p)}{2} \right)^2 \right.$$

$$\left. + \left(y_i - \frac{y_1(l,p) + y_2(l,p)}{2} \right)^2 \right]^{\frac{1}{2}} \gamma(l,p)$$

where

$(x_j \; ; \; y_j)$ are coordinates of transport station j;

$\gamma(l,p)$ is the number of people working in unit (l,p).

$$J_3 = \sum_{l'p'} \sum_{l''p''} \frac{1}{2} \cdot \left\{ \left[x_1(l',p') + x_2(l',p') - x_2(l''p'') - x_1(l'',p'') \right]^2 \right.$$

$$\left. + \left[y_1(l',p') - y_1(l'',p'') + y_2(l',p') - y_2(l'',p'') \right]^2 \right\}^{\frac{1}{2}} \tag{13}$$

$$J_4 = \sum_{l,p} \left[x_2(l,p) - x_1(l,p) \right] \left[y_2(l,p) - y_1(l,p) \right] . \tag{14}$$

In constraint 1 the value $a(l,p \; ; \; l',p')$ is the distance between the nearest points of units (l,p) and (l',p'). To calculate $a(l,p \; ; \; l',p')$ the following algorithm is suggested:

I. If conditions

$$y_1(l,p) \leq y_1(l',p') \leq y_2(l,p)$$

or (15)

$$y_1(l,p) \leq y_2(l',p') \leq y_2(l,p)$$

are satisfied, then

$$a(l,p\ ;\ l',p') = \begin{cases} x_1(l,p) - x_2(l',p') & \text{under } x_2(l',p') < x_1(l,p) \\ x_1(l',p') - x_2(l,p) & \text{under } x_2(l',p') > x_1(l,p) \end{cases} \tag{16}$$

II. If conditions

$$x_1(l,p) \le x_1(l',p') \le x_2(l,p)$$

or (17)

$$x_1(l,p) \le x_2(l',p') \le x_2(l,p)$$

are satisfied, then

$$a(l,p\ ;\ l',p') = \begin{cases} y_1(l,p) - y_2(l',p') & \text{under } y_2(l',p') < y_1(l,p) \\ y_1(l',p') - y_2(l,p) & \text{under } y_2(l',p') > y_1(l,p) \end{cases} \tag{18}$$

III. If neither (15) nor (17) are satisfied, then

$$a(l,p\ ;\ l',p') = \left\{ \left[x(l,p) - x(l',p')\right]^2 + \left[y(l,p) - y(l',p')\right]^2 \right\}^{\frac{1}{2}} \tag{19}$$

where

$$\text{A.}\quad \begin{matrix} x(l,p) = x_1(l,p) & x(l',p') = x_2(l',p') \\ y(l,p) = y_2(l,p) & y(l',p') = y_1(l',p') \end{matrix} \quad \text{under} \quad \begin{matrix} y_1(l,p) < y_1(l',p') \\ x_2(l,p) > x_1(l',p') \end{matrix}$$

$$\text{B.}\quad \begin{matrix} x(l,p) = x_2(l,p) & x(l',p') = x_1(l',p') \\ y(l,p) = y_2(l,p) & y(l',p') = y_1(l',p') \end{matrix} \quad \text{under} \quad \begin{matrix} y_1(l,p) < y_1(l',p') \\ x_1(l,p) < x_1(l',p') \end{matrix}$$

$$(20)$$

$$\text{C.}\quad \begin{matrix} x(l,p) = x_1(l,p) & x(l',p') = x_2(l',p') \\ y(l,p) = y_1(l,p) & y(l',p') = y_2(l',p') \end{matrix} \quad \text{under} \quad \begin{matrix} y_1(l,p) > y_1(l',p') \\ x_1(l,p) > x_1(l',p') \end{matrix}$$

$$\text{D.}\quad \begin{matrix} x(l,p) = x_2(l,p) & x(l',p') = x_1(l',p') \\ y(l,p) = y_1(l,p) & y(l',p') = y_2(l',p') \end{matrix} \quad \text{under} \quad \begin{matrix} y_1(l,p) > y_1(l',p') \\ x_1(l,p) < x_1(l',p') \end{matrix}$$

In requirement (6)

$$x(l^*,p) - x(l',p') = \begin{cases} x_1(l^*,p) - x_2(l',p') & \text{under } x_1(l^*,p) > x_2(l',p') \\ x_1(l',p') - x_2(l^*,p) & \text{under } x_1(l',p') > x_2(l^*,p) \end{cases} \qquad (21)$$

$$y(l^*,p) - y(l',p') = \begin{cases} y_1(l^*,p) - y_2(l',p') & \text{under } y_1(l^*,p) > y_2(l',p') \\ y_1(l',p') - y_2(l^*,p) & \text{under } y_1(l',p') > y_2(l^*,p) \end{cases} \tag{22}$$

Usually S is a convex area. In this case every value x from area S corresponds to two boundary values of y_G, and every value y from S corresponds to two boundary values x_G.

$$\begin{aligned} &x_1(l_I\ ;\ p) \text{ corresponds to } y_{11G} \text{ and } y_{12G}\ , \\ &x_2(l_I\ ;\ p) \text{ corresponds to } y_{21G} \text{ and } y_{22G} \end{aligned} \tag{23}$$

and

$$\begin{aligned} &y_1(l_I,p) \text{ corresponds to } x_{11G} \text{ and } x_{12G}\ , \\ &y_2(l_I,p) \text{ corresponds to } x_{21G} \text{ and } x_{22G} \end{aligned} \tag{24}$$

when the smaller value of the boundary coordinates in (23), (24) stands first. Then,

$$\begin{aligned} \min a(l_I\ ;\ x_G,y_G) = \min_p \Big\{ & \left[y_1(l_I,p) - y_{11G}\right], \left[y_1(l_I,p) - y_{12G}\right], \\ & \left[y_{21G} - y_2(l_I,p)\right], \left[y_{22G} - y_2(l_I,p)\right], \\ & \left[x_1(l_I,p) - x_{11G}\right], \left[x_1(l_I,p) - x_{12G}\right], \\ & \left[x_{21G} - x_2(l_I,p)\right], \left[x_{22G} - x_2(l_I,p)\right] \Big\}. \end{aligned}$$

Constraint (8) can be written as follows

$$Z_2(1,p) \leq Z_{max} \tag{25}$$

The algorithm suggested enables one to use a computer for the calculation of objective functions, (1) - (4), and for the verification of constraints, (5) - (10). Then, on the basis of the calculation of (1) - (4) the decision maker can choose the best location variation.