ESTIMATION OF MIGRATION FLOWS: A VALIDATION OF ENTROPY SOLUTIONS

Per Forslund
Juergen Schoettner

WP-79-59

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

FOREWORD

Interest in human settlement systems and policies has been a central part of urban-related work at IIASA since its inception. From 1975 through 1978 this interest was manifested in the work of the Migration and Settlement Task, which was formally concluded in November 1978. Since then, attention has turned to dissemination of the Task's results and to the conclusion of its comparative study which, under the leadership of Frans Willekens, is carrying out a comparative quantitative assessment of recent migration patterns and spatial population dynamics in all of IIASA's 17 NMO countries.

As part of its work on migration and settlement, IIASA has concluded research on entropy maximization and bi- and multiproportional adjustment techniques to infer detailed migration flows from aggregate data. This paper reports on some of this research, which was carried out by Per Forslund and Juergen Schoettner working under the direction of Frans Willekens in the IIASA Summer Students' Program in 1978. Mr. Forslund is from the University of Stockholm, Sweden, and Mr. Schoettner is from the University of Mannheim, Federal Republic of Germany. The paper focuses on ways to test the validity of entropy maximization as an estimation procedure in migration studies and illustrates the results using data from Sweden and Austria.

Selected papers summarizing previous work on migration and settlement at IIASA are listed at the back of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

ABSTRACT

The collection of disaggregated data is in most economic areas an expensive as well as a time-consuming procedure. If real data could be replaced by estimations from data on a highly aggregated level, much effort could be saved.

The entropy maximizing method can be used to estimate interregional migration flow matrices for the whole population or subgroups of the population, when the available data are in an aggregated form. This means estimating the elements of matrices in which individuals are classified according to two or more discrete variables. Matrices of this form are called contingency tables.

In this paper we present the entropy-maximizing method and test its validity for different levels of data aggregation. The tests are carried out by means of information theory and the chi-square distribution. For the tests we have used data from two of the countries that produce disaggregated data, Sweden and Austria.

ACKNOWLEDGEMENTS

The idea for this paper, as well as all the data used, was provided by Dr. F. Willekens.

We are also indebted to A. Por for his help with the statistical part of the paper.

Table of Contents

Estimation of Migration Flows: A Validation of Entropy Solutions

1. SPATIAL POPULATION ANALYSIS

1.1 Introduction

In neoclassical economic growth theory, population is entered as an exogenous variable affecting economic growth through the labor supply, but is itself unaffected by changes in economic conditions. A few economists have endogenized population in their models by relating it to per capita income or a similar economic index. In recent years there has been an increasing interest in the dynamics of spatial demographic growth. Models for multiregional population growth have been developed to describe the growth process and to analyze its impact on future population characteristics [Rogers, 1].

The dynamics of a multiregional population system are governed by fertility, mortality and migration. Age-specific rates of fertility, mortality and migration are the fundamental components of demographic analysis [Rogers and Willekens, 2]. They determine not only the growth of the population, but also its age composition and spatial distribution.

In this paper we are only concerned with age-specific migration, which may be represented in a three-dimensional space (Figure 1).

If we let i denote the area of origin, j the area of destination, and k the age group, then we represent the number of people in age group k migrating from i to j by x_{ijk}, and the whole matrix by $X_{r,c} = \left(x_{ijk}\right)_{i,j,k=1}^{r,c,d}$. With this notation the face

$F_1 = \left(\sum_{k=1}^{d} x_{ijk}\right)_{i,j=1}$ will contain the origin-destination flow

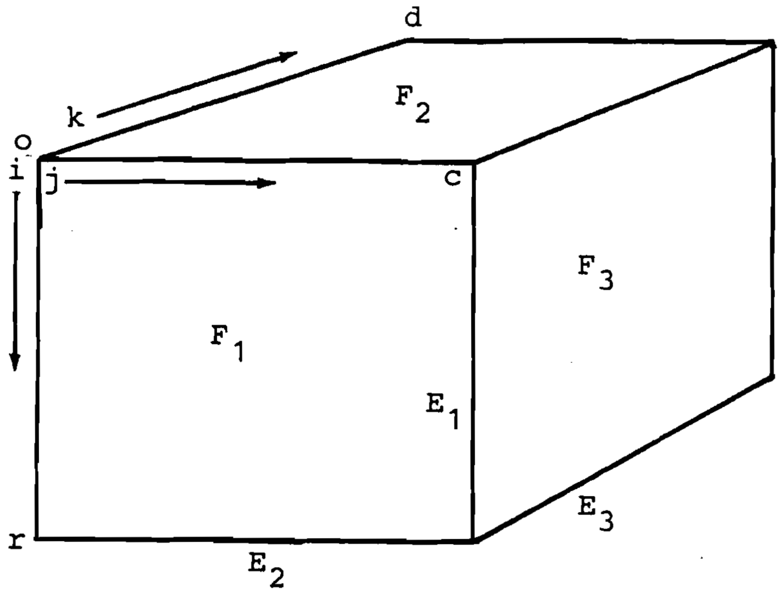

Figure 1. Age-specific migration represented in a three-dimensional space.

matrix of the total population, $F_2 = \left(\sum_{i=1}^{r} x_{ijk}\right)_{j,k=1}^{c,d}$ the age distribution of arrivals and $F_3 = \left(\sum_{j=1}^{c} x_{ijk}\right)_{i,k=1}^{r,d}$ the age distribution of departures. The edge $E_1 = \left(\sum_{j=1}^{c} \sum_{k=1}^{d} x_{ijk}\right)_{i=1}^{r}$ will give the total distribution of departures, $E_2 = \left(\sum_{i=1}^{r} \sum_{k=1}^{d} x_{ijk}\right)_{j=1}^{c}$ the total distribution of arrivals and $E_3 = \left(\sum_{i=1}^{r} \sum_{j=1}^{c} x_{ijk}\right)_{k=1}^{d}$ the total age distribution.

1.2 Available Statistical Data

The lack of statistical data is a great problem for regional analysis. The major deficiency is interregional migration data, while the situation is generally better as far as data on population, births and deaths are concerned.

Among IIASA's National Member Organization countries, complete age-specific origin-destination flows are available for nine of the 17. These are:

- Austria,
- Canada,
- The Federal Republic of Germany,
- Finland,
- Hungary,
- Japan,
- Sweden,
- the United Kingdom,
- the United States of America.

However, the procedures of collecting data vary. The data for Austria, Canada, Japan, the UK and the USA are based on population census questions, such as where people were living one or five years ago. The Swedish and Finnish data, on the other hand, are derived from population registers and relate to migration during a single year.

Some countries do not publish the complete interregional migration flow matrices by age, but only the flow matrix of the total population and the age-structure of arrivals and departures by region, e.g., the Netherlands and Poland.

For the following statistical analysis we will use data from Austria and Sweden (see Appendix), two of the countries providing complete data. The Austrian data contain four regions and 18 age groups, while the Swedish contain eight regions and 18 age groups.

2. THE ENTROPY MAXIMIZING METHOD

The entropy maximizing concept can be justified in different ways. We will concentrate on the information theoretical explanation, but first we present a more descriptive explanation, using an analogy from thermodynamics. For a more comprehensive explanation of the method, see Willekens [3].

2.1 A Thermodynamic Analogy

The second law of thermodynamics tells us that the most likely arrangement of a system is one which maximizes the entropy. Consider a three-dimensional migration matrix with probabilities p_{ijk}, for a person of age k to migrate from i to j. The entropy, or degree of disorder, can be expressed as

$$-c \cdot \sum_i \sum_j \sum_k p_{ijk} \ln p_{ijk}$$

where c is a constant [see Wilson, 4].

In analogy with the second law of thermodynamics we estimate the migration matrix, which is the most likely one, i.e., the one having the maximum entropy. However, we usually have some a priori information, consisting of marginal distributions. To use this information, we enforce it on the estimation by giving it as constraints.

The problem may now be formulated as

$$\max - \sum_i \sum_j \sum_k p_{ijk} \ln p_{ijk}$$

subject to marginal constraints.

2.2 An Information Theoretical Explanation

The entropy model can be justified in more detail by using information theoretical tools. This can also provide us with a better understanding of the basic statistical assumptions of this approach. If H_i is the hypothesis that $\hat{x}$ (where $\hat{x}$ is the stochastic variable and x a specific value of $\hat{x}$) is from the statistical population with density function $f_i(x)$, then it follows from Bayes' theorem for conditional probabilities that:

$$P(H_i|x) = \frac{P(H_i) \cdot f_i(x)}{P(H_1) \cdot f_1(x) + P(H_0) \cdot f_0(x)} , \qquad i = 0,1$$

from which we obtain

$$\ln \frac{f_1(x)}{f_0(x)} = \ln \frac{P(H_1|x)}{P(H_0|x)} - \ln \frac{P(H_1)}{P(H_0)}$$

where $P(H_i)$ is the prior probability of H_i and $P(H_i|x)$ is the posterior probability of H_i, given the observation $\hat{x} = x$.

The right-hand side of this equation is a measure of the difference between the logarithm of the odds in favor of H_1 after and before the observation of $\hat{x} = x$. This difference may be considered as the information resulting from the observation $\hat{x} = x$. The logarithm of the likelihood ratio, $\ln[f_1(x)/f_0(x)]$, is defined as the information in $\hat{x} = x$ for discrimination in favor of H_1 against H_0.

Consider the hypotheses

$$H_0: \quad p(x) = p_0(x) = p_0(x_1, x_2, \ldots, x_c) = \frac{N!}{x_1! \ \ldots \ x_c!} \cdot p_{01}^{x_1} \cdot p_{02}^{x_2} \ldots p_{0c}^{x_2}$$

$$H_1: \quad p(x) = p_1(x) = p_1(x_1, x_2, \ldots, x_c) = \frac{N!}{x_1! \ \ldots \ x_c!} \cdot p_{11}^{x_1} \cdot p_{12}^{x_2} \ldots p_{1c}^{x_c}$$

where $p_0(x)$ and $p_1(x)$ are two different N-total multinomial distributions on a c-valued population. The mean information for discrimination in favor of H_1 against H_0 per observation from $p_1(x)$ is Kullback [5].

$$I(H_1, H_0) = \sum_{x_1+\ldots+x_c=N} p_1(x) \ln \frac{p_1(x)}{p_0(x)}$$

$$= \sum_{x_1+\ldots+x_c=N} \frac{N!}{x_1! \ldots x_c!} \cdot p_{11}^{x_1} \cdot p_{12}^{x_2} \ldots p_{1c}^{x_c} \ln \left(\frac{p_{11}}{p_{01}}\right)^{x_1} \cdot \ldots \left(\frac{p_{1c}}{p_{0c}}\right)^{x_c}$$

$$= \sum_{x_1+\ldots+x_c=N} \frac{N!}{x_1! \ldots x_c!} \cdot p_{11}^{x_1} \ldots p_{1c}^{x_c} \sum_{i=1}^{c} x_i \ln \frac{p_{1i}}{p_{0i}}$$

$$= \sum_{i=1}^{c} \ln \frac{p_{1i}}{p_{0i}} \sum_{x_1+\ldots+x_c=N} \frac{N!}{x_1! \ldots x_c!} \cdot p_{11}^{x_1}$$

$$\ldots p_{1c}^{x_c} \cdot x_i$$

$$= \sum_{i=1}^{c} \ln \frac{p_{1i}}{p_{0i}} \sum_{x_1+\ldots+x_c=N} p_1(x) \cdot x_i$$

$$= \sum_{i=1}^{c} N \cdot p_{1i} \ln \frac{p_{1i}}{p_{0i}} \qquad (1)$$

In the migration matrix with only marginals given, nothing is known about the distribution of the migration. An obvious a priori assumption is then a uniform distribution, which maximizes the entropy. The effort is now to find the matrix which is "nearest to" or most closely resembles the uniform a priori distribution and also fulfills the constraints given in the marginals. To measure the discrimination in favor of the adjusted distribution we use the mean information for discrimination in favor of the adjusted distribution against the uniform distribution (1), which in the three-dimensional case is

$$N \cdot \sum_i \sum_j \sum_k p_{ijk} \ln \frac{p_{ijk}}{u_{ijk}}$$

where p_{ijk} are the adjusted probabilities, u_{ijk} are the uniformly distributed probabilities, and N is the total amount of migrations. We call the distribution "optimal" if this discrimination is the smallest. The problem can be formulated as

$$\min \sum_i \sum_j \sum_k p_{ijk} \ln p_{ijk}$$

subject to the marginal constraints.

3. FORMULATION OF THE HYPOTHESES

To test the validity of the solutions provided by the entropy model, we will formulate four hypotheses referring to four different levels of data aggregation.

We deal with three-dimensional migration matrices where origin-destinations (i,j) are described for age-specific subgroups (k).

The first hypothesis to be tested refers to the situation where only three edges in the matrix are known. This means that the information consists of the total number of people leaving every region, the total number of people arriving in every region and the total number of people of every age-group migrating.

The second hypothesis refers to the case where one face and one edge of the matrix are known. This can obviously be given in three different ways. The given face, for example, can show the origin-destination flow for the aggregated population and the edge can show the distribution of migration among the age groups. We will only formulate the hypothesis for this case, but tests will be carried out for all three cases.

The third hypothesis is applied when two faces of the matrix are known. Even this can be given in three different ways. We will formulate the hypothesis for the case when one face shows the number of people of each age group leaving each region, and the other the number of people of each age group arriving in every region. Tests will, however, be carried out for all three cases.

The fourth hypothesis is for the case where all three faces are known. This means that the flow matrix of the total population and the age structure of the departures as well as arrivals is known.

We first formulate the basic hypothesis from which the hypotheses for the different levels of aggregation can be derived. We also describe the test procedure here.

3.1 The Basic Hypothesis

The validity of the entropy solution with respect to the observed matrix can be tested by examining the discrimination in favor of the observed matrix against the entropy solution. We want to test the hypothesis:

$$H_0 : X^{obs} = X^{est} ,$$

$$H_1 : X^{obs} \neq X^{est} ,$$

where

$$X^{obs} = \left(x_{ijk}^{obs}\right)_{i,j,k=1}^{r,c,d}$$

is the observed matrix, and

$$X^{est} = \left(x_{ijk}^{est}\right)_{i,j,k=1}^{r,c,d}$$

is the matrix estimated with the entropy method.

When in Section 2.2 we explained the entropy approach by means of information theory, we used the mean information for discrimination in favor of H_1 against H_0(1) as a measure of the discrepancy between the two distributions. Accordingly, we will consider (1) as an appropriate measure of the discrepancy in our tests.

As a statistic for testing the hypotheses we use the minimum discrimination information multiplied by two [see (1)]:

$$2I(H_1, H_0) = 2 \cdot \sum_i \sum_j \sum_k x_{ijk}^{obs} \ln \frac{x_{ijk}^{obs}}{x_{ijk}^{est}} , \qquad (2)$$

which is asymptotically distributed as χ^2, with degrees of freedom (DF) depending on the form of the entropy solution [Kullback, 5]. In the tests we will compute the value (v) of the test variable, $2I(H_1, H_0)$. We then get the highest level of significance α, for which H_0 can be accepted in a χ^2-test, as $\chi^2(v) = 1 - \alpha$, where

$\chi^2(v)$ is the value of the χ^2-distribution function at v with DF degrees of freedom. We will obviously consider the tested solution acceptable for reasonably high values of $\alpha (\alpha \geq 0.05)$ and unacceptable for too small values of α.

3.2 The Hypotheses to be Tested

3.2.1 Three Edges Given (3E)

With three edges known, the entropy problem to be solved is:

$$\min \sum_i \sum_j \sum_k x_{ijk} \ln x_{ijk}$$

subject to

$$\sum_j \sum_k x_{ijk} = x_{i..}$$

$$\sum_i \sum_k x_{ijk} = x_{.j.}$$

$$\sum_i \sum_j x_{ijk} = x_{..k}$$

where $x_{i..}$, $x_{.j.}$ and $x_{..k}$ are the given edges.

It can be proved [Csizár, 6, 7, 8; Ireland and Kullback, 9; Sinkhorn, 10] that this has the solution:

$$x_{ijk}^{est} = x_{i..}^{obs} \cdot x_{.j.}^{obs} \cdot x_{..k}^{obs} \Big/ \left(x_{...}^{obs}\right)^2$$

where

$$x_{...}^{obs} = \sum_i \sum_j \sum_k x_{ijk}^{obs} \quad .$$

The solution implies the assumption of independence between all three classifications. We examine the validity of the assumption and the solution by testing with the statistic (2), which here has [rcd - r - c - d + 2] degrees of freedom. The degrees of freedom are derived by reducing the unconstrained degrees of freedom (rcd-1) with the number of degrees of freedom locked by the constraints. In this case the given edges represent respectively r-1, c-1 and d-1 degrees of freedom.

3.2.2 One Face and One Edge Given (FE)

When one face and one edge are known, e.g., the j,k-face and i-edge, the entropy problem can be formulated as

$$\min \sum_i \sum_j \sum_k x_{ijk} \cdot \ln x_{ijk}$$

subject to

$$\sum_j \sum_k x_{ijk} = x_{i\cdot\cdot}$$

$$\sum_i x_{ijk} = x_{\cdot jk}$$

where

$x_{i\cdot\cdot}$ is the given edge, and $x_{\cdot jk}$ is the given face.

It can be proved (see Section 3.2.1) that the solution is

$$x_{ijk}^{est} = x_{i\cdot\cdot}^{obs} \quad x_{\cdot jk}^{obs} / x_{\cdot\cdot\cdot}^{obs} \quad .$$

This implies the assumption of independence between the i classifications and the other classifications, which can be tested by the test statistic (2). In this case we have [(r - 1) · (cd - 1)] degrees of freedom.

3.2.3 Two Faces Given (2F)

Knowing two faces, e.g., the i,k- and the j,k-face, we formulate the following entropy problem:

$$\min \sum_i \sum_j \sum_k x_{ijk} \cdot \ln x_{ijk}$$

subject to

$$\sum_j x_{ijk} = x_{i\cdot k}$$

$$\sum_i x_{ijk} = x_{\cdot jk}$$

where $x_{i\cdot k}$ and $x_{\cdot jk}$ are the given faces.

It can be proved (see Section 3.2.1) that this problem has the solution:

$$x_{ijk}^{est} = x_{i\cdot k}^{obs} \cdot x_{\cdot jk}^{obs} / x_{\cdot\cdot k}^{obs} .$$

The probabilities $x_{i.k}^{obs} / x_{\cdot\cdot k}^{obs}$ and $x_{\cdot jk}^{obs} / x_{\cdot\cdot k}^{obs}$ are conditional, and the solution obviously implies the assumption of conditional independence between the i and j classifications for every k. To test the assumption and the solution we will use the test statistic (2) with $[d \cdot (r - 1) \cdot (c - 1)]$ degrees of freedom.

3.2.4 Three Faces Given (3F)

When the a priori information consists of three faces, the entropy problem is

$$\min \sum_i \sum_j \sum_k x_{ijk} \ln x_{ijk}$$

subject to

$$\sum_j x_{ijk} = x_{i\cdot k}$$

$$\sum_k x_{ijk} = x_{ij\cdot}$$

$$\sum_i x_{ijk} = x_{\cdot jk} \quad .$$

It can be proved [Willekens, 3] that the solution is of the form

$$x_{ijk}^{est} = s_{ij\cdot} \cdot s_{\cdot jk} \cdot s_{i\cdot k} \quad .$$

We test the validity of the solution with the test statistic with $[(r - 1) \cdot (c - 1) \cdot (d - 1)]$ degrees of freedom.

The entropy solutions for this case and for Sweden and Austria are shown in the Appendix.

4. TESTS OF THE HYPOTHESES

As mentioned earlier the tests will be carried out with an asymptotically χ^2-distributed information statistic as a test variable. The test variable measures the discrepancy between the real migration array and the array estimated by the entropy method. We will from that compute the highest value of the level of significance for which the estimated matrix can be accepted. The dimensions of the input arrays leave a high degree of freedom which makes the test very decisive.

As input data for the analysis, we use migration data from Sweden and Austria. Both Sweden and Austria are among the few countries that publish detailed migration statistics. For the purpose of this study, age-specific data are used.* The available

*The data were provided by Dr. A. Arvidsson of Sweden, and by Dr. M. Sauberer of Austria and were collected as part of the Comparative Migration and Settlement Study at IIASA.

statistics on origin-destination migration flows by age group were first aggregated to obtain the marginal totals for the various levels of aggregation mentioned in Section 3.2. For instance, the aggregate data for the three-face problem consists of: (i) the number of arrivals and departures by region and age group, and (ii) the flow matrix of the total population. The aggregate information is then used to estimate the migration flows by age group. Finally, the estimates are compared with the observed detailed flow data to investigate the validity of entropy maximization as an estimation method. The results for the three-face problem are given in the Appendix.

4.1 Computation of the Highest Level of Significance

To compute the highest level of significance we used a computer program that has been developed at IIASA. In the version used, the program is written in FORTRAN and consists of one main program and seven subroutines. The main program reads the observed migration array and computes the face and edge sums, which are needed for calculating the entropy method estimations. In the three edges, the one face/one edge, and two-face cases, the entropy solution is derived explicitly from the face and edge sums, while in the three-face case the solution is calculated with an iterative algorithm [Willekens, 3]. It is done by calling the subroutine RAS, which in turn calls the subroutine COUN. The subroutine CHE is called from COUN to check the estimation.

The subroutine CDTR is called from the main program to compute the probability for the test variable to be less than or equal to its calculated value. This is done with the help of the subroutines NDTR and DLGAM. Finally, the main program computes the level of significance α, and prints the results.

4.2 Results

The results contain the value of the test variable (CHI) and the degrees of freedom (DF) for each case. For better understanding of the results we present the value of CHI/DF and its theoretical value for $\alpha = 0.05$, instead of the calculated value of the highest level of significance.

4.2.1 Three Edges Given

As mentioned earlier, the a priori information here consists of the total distributions of people leaving and arriving in regions and of their age structure.

Result

	Austria	Sweden
CHI	55090	67480
DF	264	1120
CHI/DF	208.7	60.29
$CHI/DF \mid \alpha = 0.05$	1.15	1.07

The result shows that the hypothesis could not have been accepted on the 5% level ($\alpha = 0.05$) in a χ^2-test. We must therefore reject this hypothesis and the assumption of independence between all three classifications.

4.2.2 One Face and One Edge Given

In this case the information can be given in three different ways:

i. The face gives the age structure of arrivals and the edge gives the total distribution of departures.

Result

	Austria	Sweden
CHI	53490	65130
DF	213	1001
CHI/DF	251.1	65.06
$CHI/DF \mid \alpha = 0.05$	1.17	1.07

The hypothesis cannot be accepted on the 5% level. This implies that the underlying assumption of independence between departures and the other classifications is invalid.

ii. The face gives the age structure of departures and the edge gives the total distribution of arrivals.

Result

	Austria	Sweden
CHI	52540	64780
DF	213	1001
CHI/DF	246.7	64.72
CHI/DF | α = 0.05	1.17	1.07

The hypothesis cannot be accepted on the 5% level, which implies that the underlying assumption of independence between arrivals and the other classifications is invalid.

iii. The face gives the total origin-destination flow matrix and the edge gives the age distribution.

Result

	Austria	Sweden
CHI	3616	5764
DF	255	1071
CHI/DF	14.18	5.38
CHI/DF | α = 0.05	1.15	1.07

The hypothesis also cannot be accepted on the 5% level in this case. However, the result is here much better than in the previous cases, i and ii. We accordingly conclude that the assumption of independence between age and the other classifications is more adequate, though not sufficiently adequate.

4.2.3 Two Faces Given

Even in this case the information can be given in three different ways:

i. The faces give the total origin-destination flow matrix and the age structure of the arrivals.

Result

	Austria	Sweden
CHI	2024	3480
DF	204	952
CHI/DF	9.92	3.65
CHI/DF $\mid \alpha = 0.05$	1.17	1.07

The hypothesis must be rejected on the 5% level. Thus we must even consider the underlying assumption of conditional independence between age and origin as invalid.

ii. The faces give the total origin-destination flow matrix and the age structure of departures.

Result

	Austria	Sweden
CHI	1069	3056
DF	204	952
CHI/DF	5.24	3.21
CHI/DF $\mid \alpha = 0.05$	1.17	1.07

Even here the result shows that the hypothesis cannot be accepted, and the underlying assumption of conditional independence between age and departures cannot be accepted as sufficiently adequate.

iii. The faces give the age structure of departures and arrivals.

Result

	Austria	Sweden
CHI	50950	62420
DF	162	882
CHI/DF	314.5	70.77
CHI/DF $\mid \alpha = 0.05$	1.19	1.08

The result clearly implies the hypothesis to be rejected. A comparison with i and ii reveals the assumptions of independence there, though not sufficiently adequate, seems to be much more valid than the assumption of conditional independence between arrivals and departures in this case.

4.2.4 Three Faces Given

Here we know all three faces, giving us the total origin-destination flow matrix, the age structure of the departures and the age structure of the arrivals.

Result

	Austria	Sweden
CHI	272.6	1288
DF	153	833
$\mathrm{CHI}/_{\mathrm{DF}}$	1.782	1.546
$\mathrm{CHI}/_{\mathrm{DF}} \mid \alpha = 0.05$	1.19	1.09

Even here the values of $\mathrm{CHI}/_{\mathrm{DF}}$ are too high to accept the hypothesis on the 5% level.

5. CONCLUSIONS

The calculations demonstrate that the values of the highest level of significance of the hypothesis $X^{obs} = X^{est}$ must be rejected in all cases.

Nevertheless, it is often necessary to use estimated data in the absence of real data, or in the absence of sufficiently disaggregated data. Some conclusions can be drawn from the results presented, concerning the kind of aggregated data providing the most accurate estimations.

A comparison between the different values of CHI and $\mathrm{CHI}/_{\mathrm{DF}}$ reveals, as could be expected, that the estimated matrices are significantly more accurate when three faces are known, than in the other cases.

Comparing the other cases we find that two known faces will provide better estimations in two of the three cases, the third case being the one with known age structures of departures and arrivals. In this case the estimations are made assuming conditional independence between the origin and the destination. The inaccuracy in these estimations implies that the assumption is inadequate, while the assumptions of conditional independence between, respectively, origin and age in the other cases seem to be more valid.

Among the different cases with one face and one edge known, better accuracy is found when the face gives the total origin-destination flow matrix and the edge gives the age distribution. Here the estimation is made with the assumption of independence between age and the other classifications. The other two solutions, made with the assumption of independence between, respectively, departures and the other classifications and arrivals and the other classifications, are substantially worse, implying the inadequacy of these assumptions.

When three edges are known the solution is derived under the assumption of independence between all three classifications. The inaccuracy of the estimated matrices implies the rejection of this assumption.

To summarize, it is obvious that the only kind of assumption that seems reasonable is one between age and the other classifications. If disaggregated data are not available this means that, to provide an acceptable estimation, we definitely need a total origin-destination flow matrix and an age structure. The additional knowledge of the age structure of departures or arrivals does not make a very big difference. This result indicates the potential applications of model migration schedules in estimation procedures.

The entropy approach is based on a uniform a priori distribution (see Section 2.2). One way of improving the entropy maximizing estimation is to use some other a priori distribution. We present here two suggestions for further analysis.

i. If disaggregated migration data are available for some previous year these can be used as an a priori distribution.

ii. If common patterns can be found between all countries providing disaggregated migration data, or categories among them, these can be used to estimate relatively good a priori distributions for other countries.

REFERENCES

[1] Rogers, A., *Introduction to Multiregional Mathematical Demography*, New York: John Wiley, 1975.

[2] Rogers, A. and F. Willekens, Spatial Population Dynamics, RR-75-24, in *Papers Regional Science Association*, Vol. 36, 1976, pp. 3-34.

[3] Willekens, F., *The Recovery of Detailed Migration Patterns from Aggregate Data: An Entropy Maximizing Approach*, RM-77-58, International Institute for Applied Systems Analysis: Laxenburg, Austria, 1977.

[4] Wilson, A.G., *Entropy in Urban and Regional Modeling*, London, 1970.

[5] Kullback, S., *Information Theory and Statistics*, New York-London, 1959.

[6] Csizár, J., Information-Type Measures of Difference of Probability Distributions and Minimisation Problems, in *Studia Sci. Math. Hungar.*, Vol. 4, 1969, pp. 401-419.

[7] Csizár, J., I-divergence Geometry of Probability Distributions and Minimization Problems, in *Studia Sci. Math. Hungar*.

[8] Csizár, J. On Generalized Entropy, in *Studia Sci. Math. Hungar.*, Vol. 4, 1969, pp. 401-419.

[9] Ireland, C.T. and S. Kullback, Contingency Tables with Given Marginals in *Biometrica*, Vol. 55, 19, pp. 179-188, 1968.

[10] Sinkhorn, R., Diagonal Equivalence to Matrices with Prescribed Row and Column Sums, in *The American Mathematical Monthly*, Vol. 74, 1967, pp. 402-405.

APPENDIX

Observed and (3F) estimated migration flows by 5-year age groups for Austria (1971, 4 regions), and Sweden (1974, 8 regions).*

*The age groups are 0-4, 5-9, 10-14, ... 80-84, 85+.

AUSTRIA

Age group: 1

To	1	2	3	4
From				
1	0 (0)	670 (675)	877 (874)	236 (234)
2	853 (881)	0 (0)	575 (557)	481 (471)
3	814 (764)	363 (402)	0 (0)	268 (280)
4	229 (251)	395 (352)	281 (302)	0 (0)

Age group: 2

To	1	2	3	4
From				
1	0 (0)	328 (328)	468 (482)	134 (120)
2	530 (493)	0 (0)	329 (348)	256 (274)
3	448 (483)	282 (251)	0 (0)	187 (184)
4	108 (111)	124 (154)	184 (150)	0 (0)

Age group: 3

To	1	2	3	4
From				
1	0 (0)	537 (482)	828 (820)	232 (294)
2	1342 (1373)	0 (0)	1044 (1072)	1276 (1216)
3	771 (746)	344 (370)	0 (0)	453 (452)
4	151 (149)	171 (197)	246 (222)	0 (0)

Age group: 4

To	1	2	3	4
From				
1	0 (0)	1280 (1351)	2192 (2026)	700 (795)
2	3760 (3804)	0 (0)	1950 (2018)	2613 (2501)
3	2892 (2892)	1123 (1103)	0 (0)	1282 (1302)
4	772 (739)	809 (753)	659 (749)	0 (0)

Age group: 5

To	1	2	3	4
From				
1	0 (0)	1289 (1337)	2231 (2246)	707 (644)
2	2081 (2095)	0 (0)	1381 (1327)	1163 (1203)
3	1861 (1806)	701 (734)	0 (0)	688 (710)
4	640 (678)	816 (736)	779 (821)	0 (0)

Age group: 6

To	1	2	3	4
From				
1	0 (0)	910 (939)	1464 (1477)	433 (391)
2	1225 (1185)	0 (0)	800 (734)	600 (656)
3	998 (1028)	482 (467)	0 (0)	405 (390)
4	389 (395)	491 (479)	493 (499)	0 (0)

Age group: 7

To / From	1	2	3	4
1	0 (0)	368 (378)	588 (596)	167 (149)
2	464 (465)	0 (0)	346 (334)	252 (263)
3	485 (492)	255 (241)	0 (0)	184 (191)
4	173 (165)	212 (216)	221 (226)	0 (0)

Age group: 8

To / From	1	2	3	4
1	0 (0)	293 (295)	480 (492)	142 (127)
2	408 (391)	0 (0)	276 (282)	219 (230)
3	379 (402)	213 (187)	0 (0)	158 (162)
4	119 (113)	116 (140)	173 (155)	0 (0)

Age group: 9

To / From	1	2	3	4
1	0 (0)	312 (281)	438 (474)	121 (116)
2	411 (409)	0 (0)	240 (224)	156 (174)
3	411 (419)	141 (147)	0 (0)	136 (122)
4	128 (121)	87 (113)	146 (127)	0 (0)

Age group: 10

To	1	2	3	4
From				
1	0 (0)	222 (204)	289 (306)	68 (69)
2	269 (276)	0 (0)	149 (141)	99 (100)
3	275 (277)	102 (102)	0 (0)	70 (68)
4	81 (70)	50 (69)	77 (69)	0 (0)

Age group: 11

To	1	2	3	4
From				
1	0 (0)	241 (229)	312 (314)	65 (75)
2	263 (255)	0 (0)	134 (148)	117 (111)
3	273 (272)	119 (124)	0 (0)	86 (81)
4	65 (73)	81 (88)	95 (80)	0 (0)

Age group: 12

To	1	2	3	4
From				
1	0 (0)	280 (262)	331 (347)	78 (81)
2	296 (288)	0 (0)	132 (134)	93 (99)
3	295 (303)	114 (115)	0 (0)	80 (71)
4	84 (82)	64 (83)	89 (73)	0 (0)

Age group: 13

To	1	2	3	4
From				
1	0 (0)	269 (259)	357 (374)	74 (67)
2	253 (244)	0 (0)	137 (135)	65 (76)
3	263 (270)	114 (111)	0 (0)	62 (58)
4	60 (59)	51 (65)	74 (62)	0 (0)

Age group: 14

To	1	2	3	4
From				
1	0 (0)	210 (203)	280 (290)	53 (50)
2	190 (184)	0 (0)	102 (101)	48 (55)
3	198 (203)	84 (84)	0 (0)	46 (42)
4	43 (42)	37 (46)	51 (44)	0 (0)

Age group: 15

To	1	2	3	4
From				
1	0 (0)	138 (132)	182 (189)	33 (32)
2	121 (117)	0 (0)	65 (65)	31 (35)
3	125 (129)	54 (54)	0 (0)	30 (26)
4	28 (26)	21 (29)	33 (27)	0 (0)

Age group: 16

To	1	2	3	4
From				
1	0 (0)	74 (71)	101 (105)	19 (18)
2	65 (63)	0 (0)	36 (36)	17 (20)
3	66 (67)	27 (28)	0 (0)	16 (14)
4	14 (14)	13 (16)	19 (16)	0 (0)

Age group: 17

To	1	2	3	4
From				
1	0 (0)	26 (24)	35 (37)	7 (6)
2	22 (21)	0 (0)	12 (11)	5 (6)
3	22 (22)	8 (8)	0 (0)	5 (4)
4	5 (5)	3 (5)	6 (5)	0 (0)

Age group: 18

To	1	2	3	4
From				
1	0 (0)	13 (13)	18 (19)	3 (3)
2	11 (11)	0 (0)	7 (7)	3 (3)
3	11 (11)	6 (6)	0 (0)	2 (2)
4	2 (2)	2 (3)	3 (2)	0 (0)

SWEDEN

Age group: 1

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	1356 (1289)	330 (339)	343 (362)	382 (396)	657 (653)	356 (359)	325 (351)
2	814 (814)	0 (0)	371 (372)	331 (302)	524 (528)	679 (705)	228 (220)	275 (283)
3	200 (196)	323 (348)	0 (0)	350 (359)	425 (401)	96 (95)	36 (31)	56 (56)
4	274 (253)	247 (259)	372 (385)	0 (0)	393 (387)	123 (114)	37 (48)	73 (72)
5	314 (299)	433 (437)	359 (364)	374 (386)	0 (0)	332 (339)	120 (111)	156 (152)
6	237 (256)	449 (471)	76 (67)	67 (63)	256 (262)	0 (0)	135 (129)	135 (107)
7	149 (149)	133 (133)	26 (27)	44 (36)	62 (72)	115 (112)	0 (0)	142 (142)
8	134 (155)	167 (171)	56 (37)	37 (38)	92 (87)	101 (86)	119 (133)	0 (0)

Age group: 2

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	920 (884)	261 (247)	267 (261)	257 (263)	397 (412)	230 (243)	211 (233)
2	613 (614)	0 (0)	264 (279)	238 (224)	357 (361)	440 (458)	161 (153)	209 (193)
3	152 (144)	246 (240)	0 (0)	228 (260)	282 (267)	76 (60)	10 (21)	33 (37)
4	199 (177)	160 (169)	261 (268)	0 (0)	232 (245)	73 (69)	34 (31)	44 (45)
5	229 (219)	284 (299)	265 (265)	274 (278)	0 (0)	221 (213)	84 (75)	94 (100)
6	175 (195)	320 (337)	66 (51)	59 (48)	187 (181)	0 (0)	85 (91)	85 (74)
7	111 (111)	83 (93)	12 (20)	28 (26)	50 (49)	80 (71)	0 (0)	101 (95)
8	84 (102)	114 (105)	25 (24)	27 (24)	53 (52)	43 (49)	91 (81)	0 (0)

Age group: 3

To / From	1	2	3	4	5	6	7	8
1	0 (0)	476 (484)	153 (142)	176 (174)	165 (159)	234 (228)	155 (145)	176 (104)
2	357 (355)	0 (0)	135 (157)	151 (146)	214 (212)	252 (247)	76 (89)	105 (84)
3	105 (84)	112 (129)	0 (0)	174 (170)	150 (158)	32 (33)	10 (12)	19 (16)
4	101 (100)	88 (89)	158 (147)	0 (0)	123 (141)	38 (36)	23 (18)	19 (19)
5	157 (129)	161 (162)	142 (151)	171 (184)	0 (0)	106 (117)	43 (45)	53 (45)
6	108 (123)	215 (197)	33 (31)	32 (34)	119 (117)	0 (0)	52 (58)	36 (35)
7	55 (70)	59 (54)	16 (12)	17 (19)	41 (31)	44 (42)	0 (0)	41 (45)
8	47 (69)	68 (65)	19 (16)	24 (18)	41 (35)	27 (30)	62 (55)	0 (0)

Age group: 4

To / From	1	2	3	4	5	6	7	8
1	0 (0)	577 (635)	190 (184)	250 (241)	200 (215)	265 (280)	166 (125)	153 (120)
2	898 (929)	0 (0)	235 (242)	234 (241)	336 (343)	419 (362)	93 (92)	110 (116)
3	267 (296)	304 (272)	0 (0)	396 (380)	344 (345)	49 (65)	16 (17)	29 (30)
4	222 (276)	169 (146)	257 (240)	0 (0)	255 (241)	49 (56)	27 (19)	29 (28)
5	247 (313)	242 (236)	233 (217)	283 (282)	0 (0)	184 (159)	42 (43)	75 (57)
6	530 (500)	500 (476)	54 (75)	89 (87)	284 (292)	0 (0)	93 (93)	47 (75)
7	363 (344)	147 (159)	47 (35)	53 (58)	117 (95)	97 (116)	0 (0)	100 (117)
8	531 (401)	213 (228)	33 (54)	52 (68)	123 (128)	76 (100)	78 (126)	0 (0)

Age group: 5

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	1512 (1590)	354 (379)	460 (507)	538 (534)	650 (635)	356 (355)	479 (349)
2	2054 (2068)	0 (0)	532 (520)	513 (528)	920 (889)	865 (857)	262 (272)	340 (351)
3	617 (636)	708 (686)	0 (0)	858 (804)	820 (864)	141 (148)	55 (49)	77 (89)
4	615 (690)	441 (429)	658 (579)	0 (0)	685 (700)	147 (149)	68 (64)	93 (96)
5	705 (800)	715 (707)	571 (536)	748 (710)	0 (0)	445 (432)	155 (145)	189 (193)
6	1018 (943)	1081 (1054)	81 (136)	136 (161)	673 (641)	0 (0)	221 (232)	149 (193)
7	682 (626)	361 (339)	48 (61)	98 (103)	193 (201)	223 (224)	0 (0)	239 (290)
8	728 (658)	428 (439)	53 (85)	111 (110)	244 (244)	148 (174)	274 (275)	0 (0)

Age group: 6

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	1897 (1945)	459 (476)	592 (587)	680 (658)	918 (905)	522 (510)	541 (530)
2	1718 (1698)	0 (0)	553 (522)	505 (489)	849 (875)	953 (977)	330 (312)	392 (426)
3	414 (400)	522 (514)	0 (0)	565 (569)	647 (651)	107 (129)	54 (43)	78 (82)
4	618 (600)	442 (445)	582 (615)	0 (0)	758 (730)	185 (180)	59 (77)	127 (123)
5	641 (626)	670 (660)	515 (512)	592 (627)	0 (0)	496 (470)	146 (159)	224 (229)
6	581 (569)	769 (758)	117 (100)	101 (109)	453 (463)	0 (0)	181 (196)	166 (172)
7	320 (338)	228 (218)	39 (40)	76 (63)	113 (130)	145 (168)	0 (0)	267 (232)
8	310 (371)	307 (295)	59 (59)	83 (70)	172 (165)	161 (136)	221 (217)	0 (0)

Age group: 7

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	1067 (1046)	271 (283)	372 (348)	351 (347)	498 (486)	267 (296)	271 (292)
2	795 (781)	0 (0)	309 (289)	244 (269)	437 (429)	480 (486)	173 (168)	202 (218)
3	172 (175)	241 (244)	0 (0)	286 (298)	307 (303)	74 (61)	20 (22)	42 (40)
4	305 (278)	196 (223)	335 (342)	0 (0)	350 (359)	92 (90)	48 (42)	71 (63)
5	337 (313)	366 (358)	296 (308)	372 (375)	0 (0)	231 (255)	110 (93)	119 (127)
6	228 (250)	349 (362)	53 (53)	71 (57)	212 (217)	0 (0)	104 (101)	107 (84)
7	133 (154)	116 (108)	22 (22)	34 (34)	58 (63)	87 (83)	0 (0)	130 (117)
8	140 (158)	141 (136)	41 (30)	37 (35)	78 (75)	61 (63)	108 (108)	0 (0)

Age group: 8

From \ To	1	2	3	4	5	6	7	8
1	0 (0)	593 (556)	153 (143)	211 (214)	177 (187)	246 (270)	155 (152)	128 (141)
2	419 (418)	0 (0)	144 (151)	178 (171)	231 (239)	285 (281)	85 (90)	117 (109)
3	96 (92)	113 (133)	0 (0)	195 (187)	176 (167)	36 (35)	7 (11)	22 (20)
4	155 (138)	114 (114)	153 (165)	0 (0)	192 (186)	43 (48)	22 (21)	22 (29)
5	175 (155)	194 (183)	138 (149)	209 (221)	0 (0)	133 (136)	40 (46)	59 (59)
6	130 (140)	186 (208)	41 (29)	41 (38)	125 (126)	0 (0)	54 (56)	54 (44)
7	70 (82)	53 (59)	18 (12)	17 (22)	42 (35)	57 (48)	0 (0)	60 (59)
8	62 (82)	72 (73)	16 (15)	24 (22)	38 (40)	53 (35)	58 (56)	0 (0)

Age group: 9

To	1	2	3	4	5	6	7	8
From								
1	0 (0)	354 (338)	109 (95)	128 (143)	123 (120)	162 (171)	109 (106)	59 (71)
2	264 (279)	0 (0)	93 (100)	107 (115)	172 (154)	183 (178)	60 (62)	65 (55)
3	70 (70)	81 (92)	0 (0)	154 (143)	120 (122)	31 (25)	3 (9)	14 (11)
4	98 (84)	70 (64)	82 (101)	0 (0)	103 (110)	28 (28)	19 (13)	15 (14)
5	117 (94)	88 (101)	84 (90)	146 (135)	0 (0)	75 (78)	20 (29)	25 (27)
6	93 (101)	137 (137)	33 (21)	31 (28)	84 (88)	0 (0)	40 (42)	22 (24)
7	53 (55)	28 (36)	11 (8)	14 (15)	24 (23)	35 (30)	0 (0)	31 (30)
8	53 (65)	62 (53)	15 (12)	17 (18)	23 (31)	24 (27)	57 (46)	0 (0)

Age group: 10

To	1	2	3	4	5	6	7	8
From								
1	0 (0)	321 (309)	96 (90)	115 (113)	100 (98)	147 (150)	74 (79)	45 (51)
2	230 (228)	0 (0)	83 (86)	97 (82)	124 (114)	128 (142)	37 (42)	38 (43)
3	64 (57)	75 (75)	0 (0)	91 (101)	83 (39)	21 (20)	7 (6)	16 (9)
4	61 (66)	54 (51)	82 (83)	0 (0)	69 (78)	29 (21)	10 (9)	12 (10)
5	109 (83)	82 (90)	71 (83)	93 (104)	0 (0)	70 (67)	20 (21)	27 (23)
6	85 (90)	122 (124)	24 (19)	26 (22)	67 (71)	0 (0)	32 (31)	21 (20)
7	32 (47)	27 (32)	9 (7)	9 (11)	28 (18)	30 (25)	0 (0)	29 (24)
8	41 (51)	41 (42)	13 (10)	13 (12)	18 (22)	22 (20)	39 (30)	0 (0)

Age group: 11

To / From	1	2	3	4	5	6	7	8
1	0 (0)	291 (271)	66 (80)	108 (110)	92 (78)	151 (151)	66 (71)	32 (45)
2	192 (184)	0 (0)	79 (74)	70 (77)	77 (87)	133 (137)	45 (37)	31 (30)
3	41 (42)	55 (58)	0 (0)	83 (86)	59 (62)	20 (17)	5 (5)	12 (6)
4	69 (61)	41 (49)	84 (81)	0 (0)	67 (68)	22 (24)	7 (9)	8 (8)
5	59 (65)	77 (74)	75 (69)	101 (94)	0 (0)	55 (63)	14 (18)	16 (15)
6	82 (77)	108 (112)	13 (18)	27 (22)	53 (58)	0 (0)	32 (29)	16 (15)
7	36 (42)	27 (30)	8 (7)	10 (11)	15 (15)	32 (27)	0 (0)	23 (19)
8	37 (45)	32 (39)	11 (9)	13 (12)	24 (18)	28 (21)	28 (29)	0 (0)

Age group: 12

To / From	1	2	3	4	5	6	7	8
1	0 (0)	242 (239)	67 (67)	135 (104)	65 (71)	112 (124)	62 (66)	21 (33)
2	136 (135)	0 (0)	55 (57)	59 (67)	65 (74)	121 (105)	31 (31)	23 (21)
3	26 (28)	42 (43)	0 (0)	58 (68)	66 (48)	7 (12)	3 (4)	4 (4)
4	46 (38)	33 (34)	51 (55)	0 (0)	52 (50)	10 (16)	5 (6)	7 (5)
5	38 (42)	59 (53)	47 (48)	74 (73)	0 (0)	43 (42)	10 (13)	10 (9)
6	41 (47)	75 (75)	15 (11)	6 (16)	41 (41)	0 (0)	24 (20)	17 (9)
7	44 (32)	15 (25)	2 (5)	10 (10)	14 (13)	23 (21)	0 (0)	12 (13)
8	21 (30)	33 (29)	13 (7)	6 (10)	8 (14)	19 (15)	28 (22)	0 (0)

Age group: 13

To / From	1	2	3	4	5	6	7	8
1	0 (0)	265 (259)	91 (84)	135 (130)	79 (88)	163 (150)	65 (74)	22 (35)
2	118 (121)	0 (0)	57 (62)	70 (73)	81 (79)	105 (109)	32 (31)	31 (19)
3	28 (26)	50 (42)	0 (0)	68 (78)	54 (54)	15 (13)	1 (4)	4 (3)
4	24 (30)	36 (28)	48 (51)	0 (0)	56 (46)	8 (14)	3 (5)	3 (4)
5	53 (41)	47 (55)	53 (56)	92 (86)	0 (0)	42 (48)	12 (14)	11 (9)
6	41 (46)	66 (77)	21 (14)	22 (18)	47 (47)	0 (0)	30 (22)	6 (9)
7	36 (33)	23 (27)	3 (7)	15 (13)	13 (16)	28 (25)	0 (0)	16 (14)
8	27 (29)	29 (29)	9 (8)	7 (11)	15 (16)	16 (17)	31 (24)	0 (0)

Age group: 14

To / From	1	2	3	4	5	6	7	8
1	0 (0)	188 (205)	65 (64)	94 (89)	54 (60)	141 (124)	35 (40)	25 (23)
2	124 (122)	0 (0)	49 (47)	50 (50)	50 (53)	89 (90)	12 (16)	16 (12)
3	25 (28)	37 (36)	0 (0)	47 (57)	48 (39)	15 (12)	4 (2)	0 (2)
4	35 (34)	27 (25)	40 (43)	0 (0)	39 (35)	11 (13)	2 (3)	1 (3)
5	35 (44)	48 (45)	45 (45)	75 (62)	0 (0)	37 (42)	7 (8)	5 (6)
6	49 (54)	77 (71)	11 (12)	13 (15)	41 (38)	0 (0)	15 (14)	4 (7)
7	37 (28)	22 (18)	4 (4)	4 (7)	5 (9)	11 (18)	0 (0)	9 (8)
8	33 (28)	22 (22)	6 (6)	3 (7)	7 (11)	7 (13)	20 (12)	0 (0)

Age group: 15

To / From	1	2	3	4	5	6	7	8
1	0 (0)	80 (87)	29 (29)	37 (43)	33 (23)	44 (42)	19 (18)	9 (8)
2	67 (58)	0 (0)	29 (26)	22 (29)	21 (25)	39 (37)	7 (9)	5 (5)
3	15 (17)	24 (23)	0 (0)	45 (41)	16 (23)	8 (6)	3 (2)	1 (1)
4	14 (19)	22 (15)	24 (28)	0 (0)	20 (19)	7 (6)	3 (2)	0 (1)
5	22 (20)	20 (22)	23 (23)	45 (34)	0 (0)	9 (16)	3 (4)	0 (3)
6	24 (27)	41 (38)	7 (7)	7 (9)	19 (18)	0 (0)	8 (8)	5 (3)
7	16 (18)	16 (12)	3 (3)	6 (6)	2 (6)	11 (10)	0 (0)	6 (5)
8	12 (11)	4 (9)	5 (3)	3 (4)	7 (4)	4 (5)	6 (6)	0 (0)

Age group: 16

To / From	1	2	3	4	5	6	7	8
1	0 (0)	52 (51)	20 (16)	19 (20)	16 (16)	22 (22)	15 (14)	3 (5)
2	49 (42)	0 (0)	11 (13)	10 (13)	18 (16)	13 (18)	3 (6)	9 (5)
3	10 (11)	11 (11)	0 (0)	17 (17)	12 (13)	3 (3)	3 (1)	1 (1)
4	17 (16)	10 (9)	14 (16)	0 (0)	15 (14)	5 (4)	1 (2)	0 (1)
5	13 (13)	9 (10)	9 (10)	16 (13)	0 (0)	9 (7)	1 (3)	1 (2)
6	14 (15)	12 (16)	3 (3)	3 (3)	11 (9)	0 (0)	9 (4)	1 (2)
7	3 (9)	7 (4)	1 (1)	2 (2)	1 (2)	3 (3)	0 (0)	7 (3)
8	5 (6)	4 (3)	1 (1)	2 (1)	1 (2)	3 (2)	1 (3)	0 (0)

Age group: 17

To	1	2	3	4	5	6	7	8
From								
1	0 (0)	35 (37)	14 (10)	16 (14)	8 (8)	17 (20)	7 (7)	2 (2)
2	21 (25)	0 (0)	6 (7)	6 (8)	8 (7)	21 (14)	0 (3)	2 (1)
3	8 (9)	9 (9)	0 (0)	16 (13)	7 (7)	1 (3)	1 (1)	0 (0)
4	16 (10)	5 (6)	8 (10)	0 (0)	7 (7)	1 (3)	1 (1)	0 (0)
5	14 (10)	7 (9)	9 (8)	11 (11)	0 (0)	6 (8)	1 (1)	0 (1)
6	11 (14)	21 (15)	2 (2)	1 (3)	4 (6)	0 (0)	3 (3)	2 (1)
7	7 (7)	4 (4)	1 (1)	0 (1)	1 (1)	5 (3)	0 (0)	1 (1)
8	4 (6)	4 (4)	0 (1)	2 (1)	3 (2)	2 (2)	4 (2)	0 (0)

Age group: 18

To	1	2	3	4	5	6	7	8
From								
1	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
2	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
3	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
4	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
5	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
6	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
7	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
8	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)

SELECTED PAPERS ON MIGRATION AND SETTLEMENT AT IIASA

Rogers, A., ed. (1978) Migration and Settlement: Selected Essays. RR-78-6. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A. and F. Willekens (1978) Migration and Settlement: Measurement and Analysis. RR-78-13. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rogers, A. (1978) The Formal Demography of Migration and Redistribution: Measurement and Dynamics. RM-78-15. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Willekens, F. and A. Rogers (1978) Spatial Population Analysis: Methods and Computer Programs. RR-78-18. Laxenburg, Austria: International Institute for Applied Systems Analysis.