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Does selection of mortality model make a difference in projecting population ageing?

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Abstract

In developed countries where fertility levels are low for a considerable time, assessing future ageing depends to a large extent on our vision of future developments in mortality reduction. In most population projections mortality reduction is implemented via life expectancy increases. Different scenarios of future changes in life expectancy have been developed in this respect. However, as we show in this paper, the selection of models that translate life expectancy into age-specific mortality rates may be of great importance for projecting the age composition of future population and especially ageing. We provide a comprehensive analysis of the sensitivity of different indicators of ageing in respect to the selection of mortality model. For a given level of life expectancy at birth the selection of different models may lead to different levels of life expectancy at age 65 comparable to those obtained in medium and high variants scenarios for life expectancy at age 0 (for example projections prepared by Eurostat). The prospective old-age dependency ratio (POADR), a recently introduced indicator of ageing where the threshold age of being considered old is not static but changes with improvements in life expectancy, is particularly sensitive to different translation models. Our results imply that researchers of population ageing should be as careful about their choice of models of age patterns of future mortality as they usually are when selecting life expectancy scenarios.

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Does selection of mortality model make a difference in projecting population ageing?

Sergei Scherbov and Dalkhat M. Ediev

1 Introduction

Projecting mortality is a crucial first step in studying the prospects of population ageing and possible related threats to public welfare. As life expectancy increases and population ageing is gaining pace worldwide (Lutz et al. 2008), there is a considerable effort in expanding the methodology of mortality projections (Booth & Tickle 2008; Ediev 2011; Mayhew & Smith 2013; Pollard 1987; Raftery et al. 2012; Stoeldraijer et al. 2013). However, authors rarely pay attention to the importance of choosing mortality models and their implications for assessing the future population size and composition, and, consequently, for indicators of population ageing.

Even though many scientists have demonstrated crucial differences between mortality scenarios in terms of life expectancy at birth (often a scenario variable in population projections) and predicted death rates (Bell 1997; Benjamin & Soliman 1993; Cairns et al. 2011; Janssen & Kunst 2007; Pollard 1987; Shang et al. 2011; Stoeldraijer et al. 2013) the impact on projected population ageing was not comprehensively studied. Infinite number of age-specific mortality patterns may produce the same trajectory of life expectancy at birth. Because the mortality level at younger ages in developed countries is already very low and may have only very limited impact on projected population ageing, we focus on mortality at middle ages and above.

We try to quantify the differences in projected indicators of population ageing that result from using different mortality forecasting approaches by assuming the same future trends in life expectancy.

2 Methods and data

Throughout the paper, we rely on data from the Human Mortality Database (University of California, Berkley & Max Planck Institute for Demographic Research 2014). It contains both the time series of mortality rates that are necessary to feed the projection models and the baseline populations by age and sex that is necessary in population projections.

To project populations we use the common cohort-component method (Shryock & Siegel 1973) with age patterns of mortality rates produced with alternative mortality models (see more details to the alternatives below). Our fertility, migration and life expectancy assumptions, however, are similar across alternative projections, which enable us to highlight the effects of mortality models alone. More specifically, we use scenarios from the recent global population projections by the Wittgenstein Centre for Demography and Global Human Capital (Lutz et al. 2014), as outlined in Table 1.

Sweden, Italy and Russia were selected for comparative purposes as these countries represent a range of advanced industrialized countries that are different in their epidemiological transition and have a very different age compositions in the base year.

Table 1. Scenarios for life expectancy at birth, total fertility and net migration assumed in calculations (the same scenarios are applied in all models)

	Life Expectancy at Birth, years						Total Fertility			Net Migration, 000s		
	Italy		Russia		Sweden		Italy	Russia	Sweden	Italy	Russia	Sweden
Year	Women	Men	Women	Men	Women	Men						
2013	85.6	80.3	75.7	64.3	83.8	80.0	1.4	1.7	1.9	293	277	49
2020	86.7	81.3	76.4	65.6	84.9	81.2	1.5	1.6	2.0	144	250	44
2030	88.8	83.4	78.5	68.7	86.9	83.2	1.6	1.5	2.0	148	261	45
2040	90.8	85.5	80.4	71.4	88.9	85.2	1.6	1.6	2.0	150	267	45
2050	92.8	87.5	82.3	74.0	90.9	87.2	1.6	1.7	2.0	149	267	45

Source: Lutz et al., 2014

Note that the scenarios we used are rather optimistic in terms of life expectancy improvements. For Italy and Sweden the life expectancy scenarios are consistent with linear growth of about two years per decade observed in the recent past (Oeppen & Vaupel 2002; White 2002). The expected improvement for Russia is even faster.

We calculate a set of indicators of mortality and population ageing to compare implications of alternative mortality models in each of the country-cases. Our main indicator of population ageing is the *old-age dependency ratio* (OADR), a simple intuitive indicator of population ageing. Its apparent simplicity, however, may be misleading, as the very notion of 'old' may be defined in different ways (Sanderson & Scherbov 2013; Sanderson & Scherbov 2010; Sanderson & Scherbov 2005). The conventional OADR is defined as the ratio of the number of people above the age of 65 years to the number of people between the ages 20 and 64:

$$\text{OADR} = \frac{\text{Number of people 65 years or older}}{\text{Number of people ages 20 to 64}}$$

In some cases the proportion of people aged 60 or older is used in the numerator, sometimes 15 is used as the lower bound of ages in the denominator, or the ratio can be multiplied by 100; but whatever age is used as a threshold for being old, it is always considered fixed in time and space, which can create biased measures.

To overcome this possible misleading, Sanderson and Scherbov (2010) have introduced the *prospective old-age dependency ratio* (POADR) where the threshold of being old is no longer fixed at age 65 but changes with the change in life expectancy. It is based on a constant remaining life expectancy and assumes that people are old when the average remaining life expectancy in their age group is less than 15 years:

$$\text{POADR} = \frac{\text{Number of people older than the old-age threshold}}{\text{Number of people aged between 20 and the old-age threshold}}$$

To project populations we use the common cohort-component method (Shryock & Siegel 1973) with age patterns of mortality rates produced with alternative mortality models.

Regarding the mortality models, we have chosen variants of extrapolative models for the log-mortality rates, the Brass relational model and the Bongaarts shifting model. Those models represent a wide range of possible mortality changes at older ages, from a very limited one as in the Brass relational model to a very strong one, as in the Bongaarts shifting model. These models are briefly described below.

In *the Lee-Carter* (1992) *model*, the log-mortality rate at age *x* at time *t* are extrapolated as

$$\log(M(x,t))=A(x)+B(x)\ k_t$$

We apply no additional adjustments to the mortality level parameter k_t, estimate the model on the most recent thirty-years-long part of the data, and apply monotonicity adjustment to the estimated slopes B(x) to avoid implausible (non-monotone at old age or with men having mortality lower than women) projected age patterns (Ediev 2007).

In *the direct linear extrapolation model* (Ediev 2008):

$$\log(M(x,t))=A(x)+B(x)\cdot t$$

we apply monotonicity adjustment to the estimated slopes B(x) to avoid implausible projected age patterns and estimate the model parameters based on the most recent age-specific periods of linearity in trends on log-mortality rates.

The Brass model (Brass 1971) describes the logits of the life table probabilities to survive to age *x*:

$$\log((1-l(x,t))/l(x,t))=Alpha+Beta\cdot\log((1-l^*(x,t))/l^*(x,t))$$

(the standard probabilities are taken from (smoothed) baseline life table for each of the populations).

The shifting model by Bongaarts (2005) implies for the old-age mortality (at ages 30 years and older):

$$M(x,t)= M(x-S(t),t_{baseline}),$$

where S(t) is the amount of age shift of the baseline profile that are necessary to produce the assumed life expectancy at birth (given its low levels, we assume no background mortality in the model). At ages younger than 30, where mortality is very low and has a minor effect in our study, we link the change in the death rates to that at age 30.

Any of these four models is compatible with practically any level of future life expectancy. In order to separate the effects of mortality models from the expected change in the overall level of mortality, we assume identical scenarios for life expectancy at birth in all models (Table 1). In the projection we fit the parameters of the models (mortality level *k* in the Lee-Carter model, time variable in the direct extrapolation model, mortality level coefficient *Alpha* in the Brass model, and the amount of age shift *S* in the Bongaarts model) to model the assumed life expectancy at birth.

The chosen mortality models, even if assuming similar e_0's, produce a very wide range of mortality forecasts at old age. Extrapolations of the age-specific rates (as in the direct extrapolation and the Lee-Carter methods) tend to overlook the possibility of forthcoming accelerations of mortality declines at the oldest-old age. Similarly, the Brass model tends not to change, as compared to the baseline standard, the death rates at the oldest ages (Ediev 2014). The Bongaarts model, on the other hand, assumes pure (age) shift of old-age mortality and does not account for the compression of period mortality (Cheung & Robine 2007; Cheung et al. 2005; Ediev 2013b; Ediev 2013a; Fries 1980; Kannisto 2000; Tuljapurkar & Edwards 2011; Wilmoth & Horiuchi 1999), that results in a possible overdoing in terms of mortality decline at the oldest-old ages.

3 Results

Projected age profiles of death rates. Not surprisingly, methods differ substantially in how they project the evolution of age-specific death rates (Figure 1). The Bongaarts' shifting model, which explicitly assumes age shifts in old-age mortality, is most optimistic in projecting a strong decline of mortality rates at ages 80 years and older (in the case of Russian males at an even younger age[1]). The extrapolation methods and the Brass model contrast to the Bongaarts' model by not assuming much change of mortality at oldest-old age (except for the case of Russian females, see note 1). Since all our extrapolations are based on the same scenario for life expectancy at birth, the Bongaarts' model also tends to be more pessimistic at young and middle old ages. Regarding differences at old age, the period life expectancy at age 65 shows method-to-method variation of about one year for Sweden and Italy, and more than 4 and 2 years for Russian males and females respectively (Figure 2). The shifting model is most optimistic and the other models stay closer together in their result. These differences are of considerable importance for applications in pension systems and social welfare. The extraordinary difference in the case of Russia may be explained by the combination of a rather optimistic underlying scenario for the life expectancy at birth and the lack of mortality compression in the Bongaarts model. Variation at age when remaining life expectancy is 15 years or less (Figure 3) also shows that the models have very different implications for old-age mortality. Taking this age as a threshold for defining who is old, about 54% of those Russian men who are "old" in 2050 in the Lee-Carter model would still be "young" in the Bongaarts shifting model. The difference would be smaller but still substantial for Russian women (27%) and for Sweden and Italy (about 20%).

Projected population age structure. While methods vary strongly in their projected old-age mortality, effects of these differences on projected population age structures are more modest (Figure 4). That is because on the one hand the mortality is very low at young age in all methods and, on the other hand, numbers of people of old ages are relatively small in cohorts that are subject to largest differences in projected mortality rates. Yet, the Bongaarts' shifting method is producing considerably more people at advanced ages (80-year-olds and older in 2050) and slightly fewer surviving population at younger ages than the other methods. This may have a sizable effect on the dependency ratios (see below). The Russian case, especially for men, is different in showing stronger differences also at younger ages.

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[1] In Russia, the death rates were rather unstable ever since the mid-20th century. They increased at adult ages before 1980s, declined in late 1980s, and followed varying trends ever since. For the lack of consistent long-run trends, we decided not to use the direct extrapolation method in the Russian case. For the same reason, one should also be somewhat critical about the outcomes of the Lee-Carter method in the Russian case and not to generalize the findings for Russia to other higher-mortality countries.

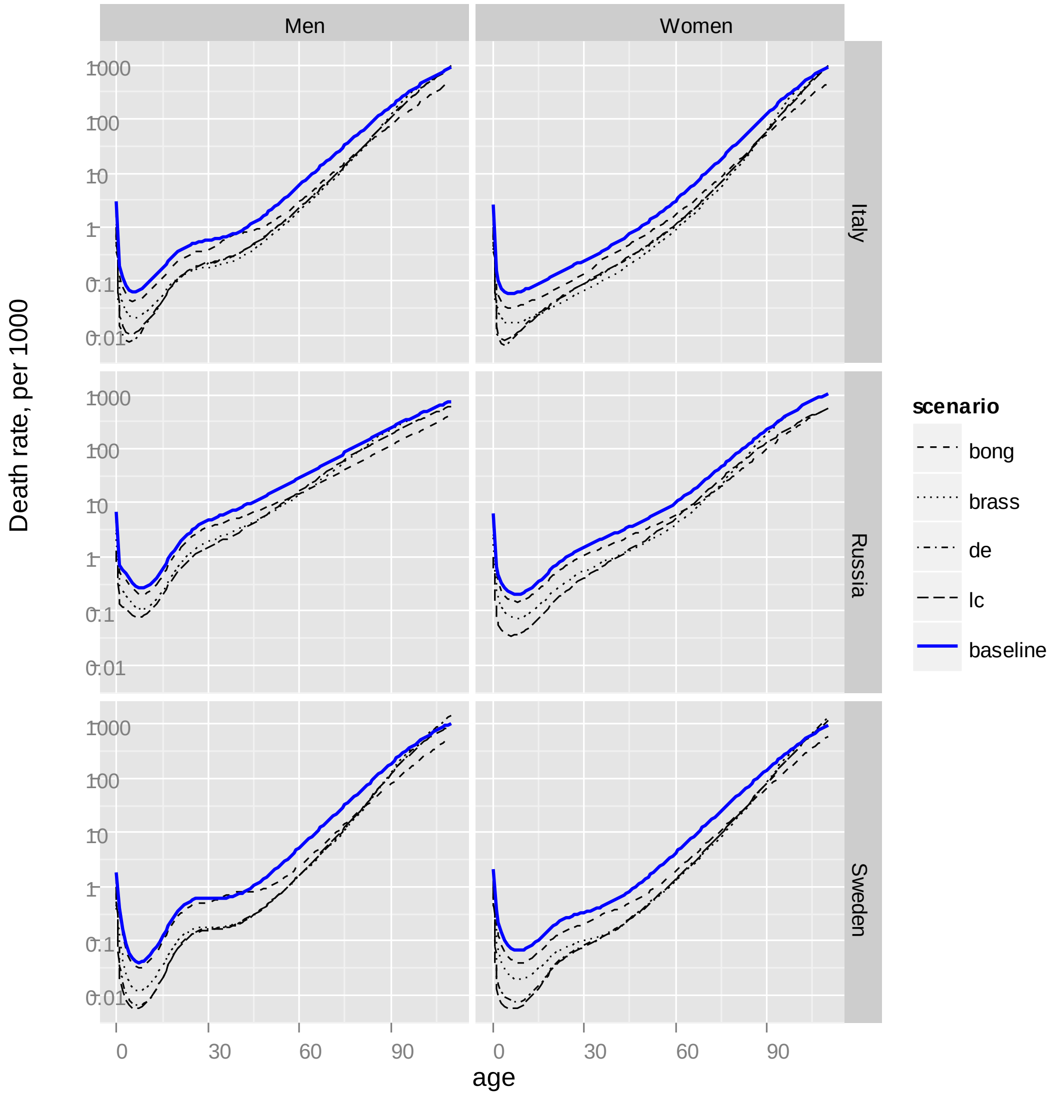

Figure 1. Baseline (2013) and projected (2050) age-specific death rates obtained with alternative projection methods for three countries. Methods: bong=Bongaarts' shifting model; brass=Brass model; de=direct extrapolation method; lc=Lee-Carter method; baseline=the profile of mortality rates in 2013 (obtained from the Bongaarts' model).

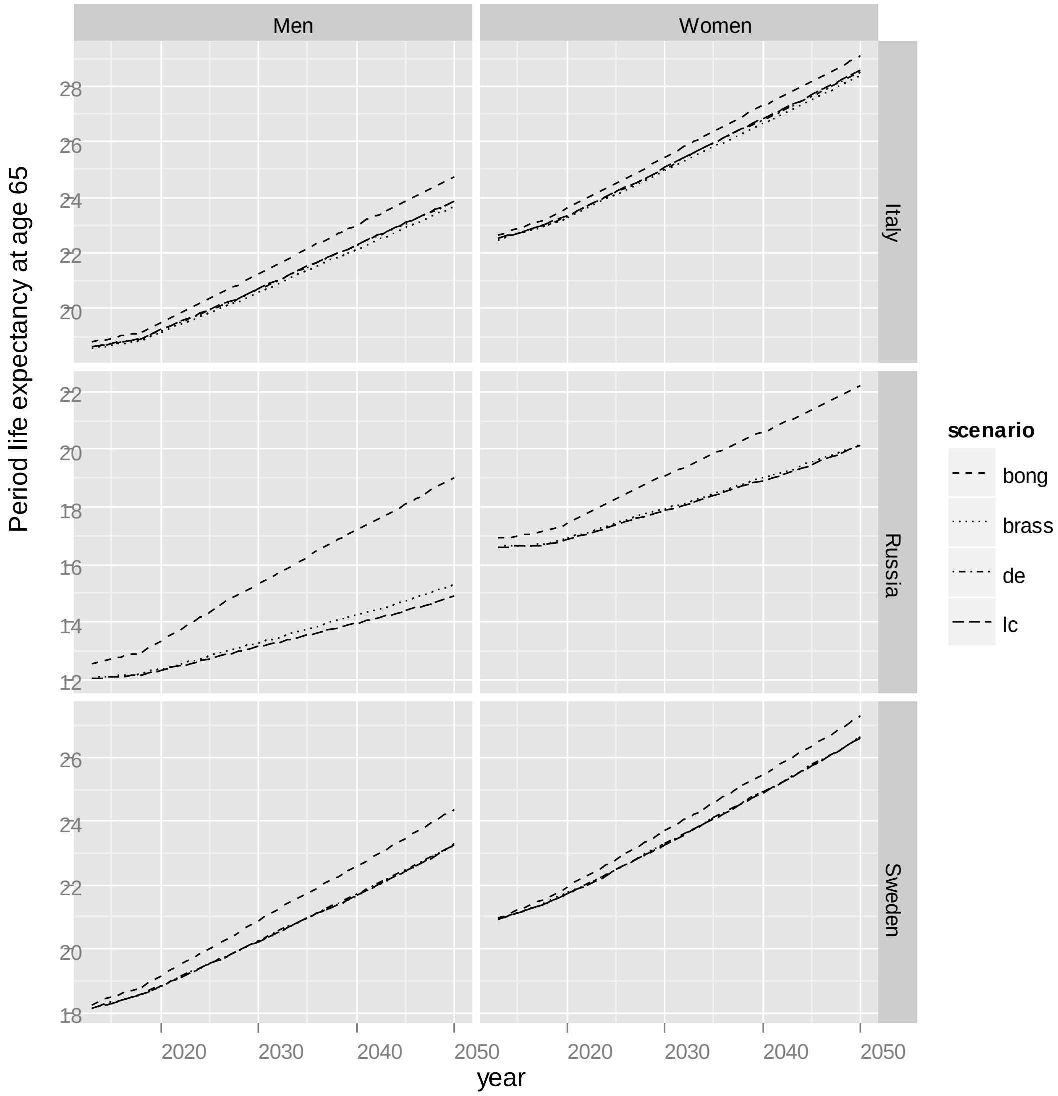

Figure 2. Projected period life expectancies at age 65 obtained with alternative projection methods for three countries. Methods: bong=Bongaarts' shifting model; brass=Brass model; de=direct extrapolation method; lc=Lee-Carter method.

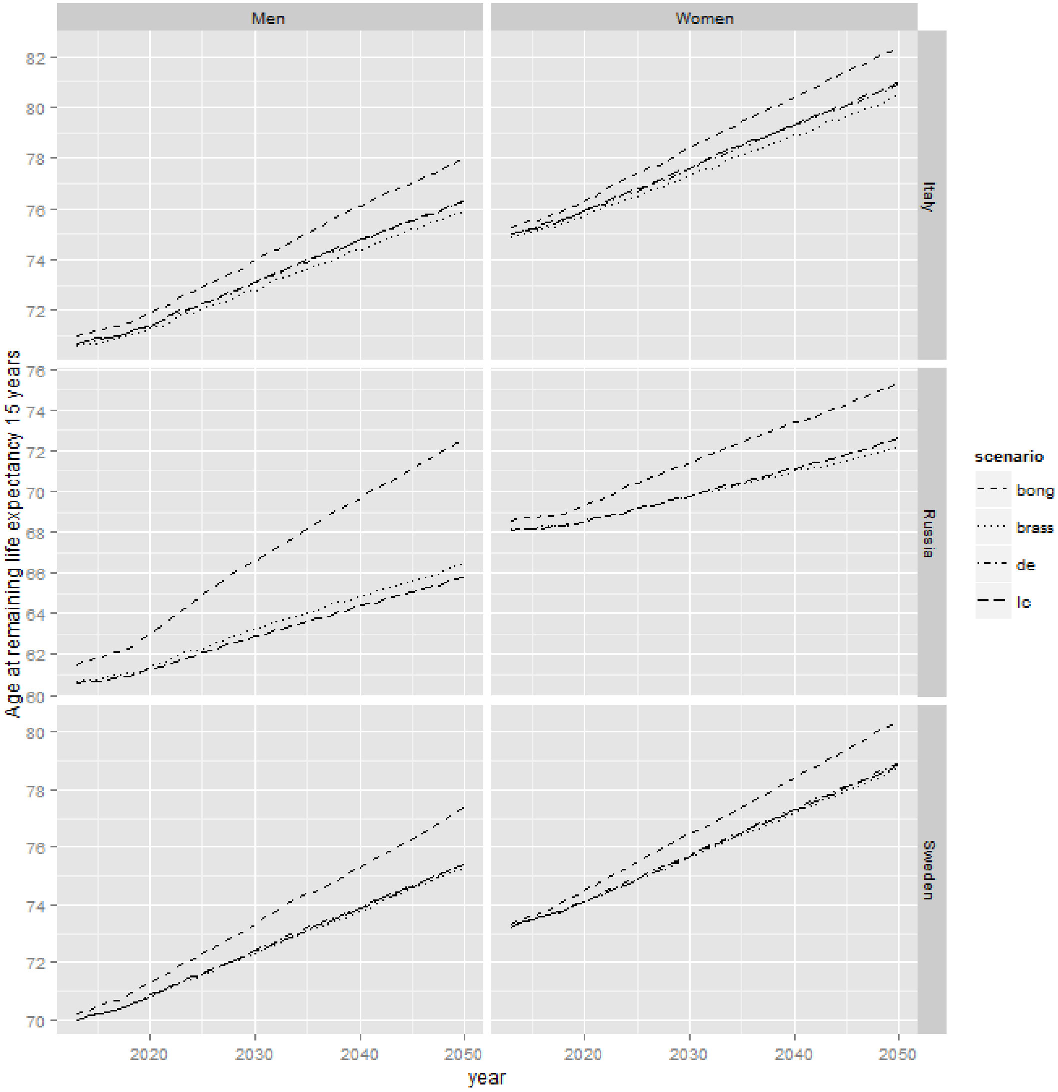

Figure 3. Proportion of people aged 65 and older projected with alternative projection methods for three countries. Methods: bong=Bongaarts' shifting model; brass=Brass model; de=direct extrapolation method; lc=Lee-Carter method.

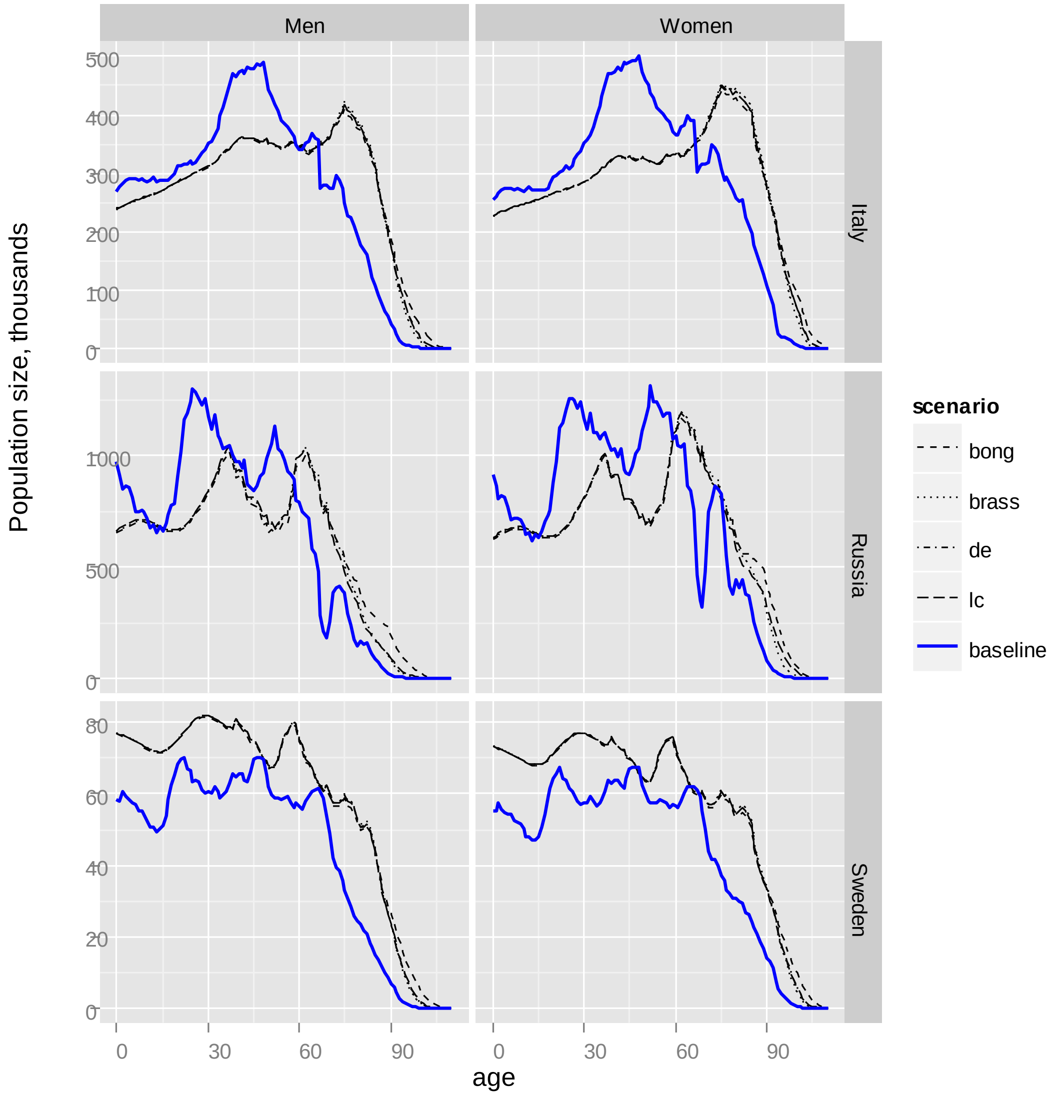

Figure 4. Population age composition in 2050 projected with alternative projection methods for three countries. Methods: bong=Bongaarts' shifting model; brass=Brass model; de=direct extrapolation method; lc=Lee-Carter method; baseline=the profile of the year 2013.

As might have been expected from the above results, *OADR* applying the conventional definition of "old" (aged 65 years and more, the panel to the left in Figure 5) do not vary much from method to method for Sweden and Italy but show more of cross-method variation for Russia. The Brass and the extrapolative methods on the other hand do not show much difference even in the Russian case.

Because the models project steadily increasing age at remaining life expectancy of 15 years (with a different speed for different countries), patterns of POADR differ from those of the conventional OADR (the panel to the right in Figure 5). First, the POADR does not increase as much as the conventional one. For Russia and Sweden, the POADR shows almost no systematic increase in 2013-2050. Second, the methods differ more in the projected POADR. Third, the ranking of the methods also changes: the shifting model shows the lowest POADR, not the highest one as in the conventional case of projecting OADR. The case of Russia is, again, of highest cross-method variation.

Patterns of the proportion old (conventional: aged 65 or older; prospective: with remaining life expectancy shorter than 15 years) are rather similar to those of OADR and POADR and, therefore, not shown here.

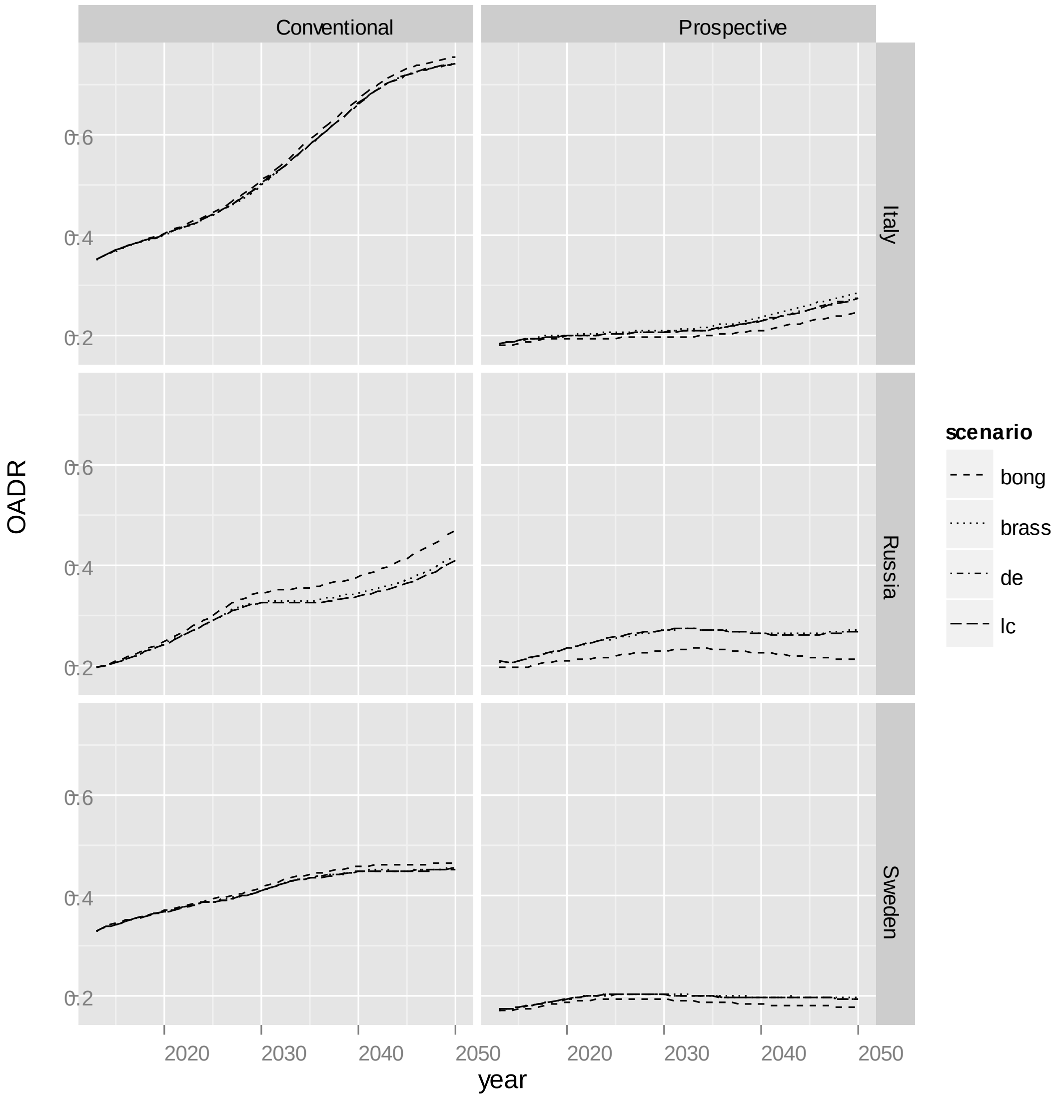

Figure 5. Conventional and prospective old-age dependency ratio (conventional: the ratio of the number of people 65 years or older to the number of people aged 20 through 64; prospective: the ratio of the number of people in the age groups with the remaining life expectancy 15 years or less to the number of people aged 20 through the first age group with the remaining life expectancy 15 years) projected with alternative projection methods for three countries. Methods: bong=Bongaarts' shifting model; brass=Brass model; de=direct extrapolation method; lc=Lee-Carter method.

4 Discussion

By and large, all four models agree in pointing to forthcoming increases of the number of people at older (in conventional sense) age. Despite large systematic differences in projected age patterns of death rates, the models do not differ that much in terms of projected population age structures. The case of Russian males is an important exception. It has high baseline mortality and our scenario assumes fast increase of life expectancy at birth. As a result, stronger relative differences in the projected population size at old age are produced. However, even in the Russian case, the more optimistic Bongaarts model produces conventional OADR not essentially different from OADR in other models (0.47 vs about 0.41 by the year 2050, a relatively small difference when compared to the change from level 0.2 in the baseline year 2013). For lower-mortality Sweden and Italy with more moderate pace of improvement of life expectancy at birth, the differences in OADR are even smaller and may be neglected.

Our projections also indicate that the prospective indicators of ageing will follow a path very different from that of the conventional indicators. In Italy, all methods produce only a gradual increase in POADR (from 0.18 in 2013 to 0.25-0.27, depending on the method, in 2050). In Russia and Sweden, due to the peculiar baseline age structure in the former case and assumed relatively high fertility and net migration in the latter case, gradual increase may be interrupted by periods of decline in POADR. The difference between POADRs produced by the Bongaarts model and other alternatives is somewhat higher than between conventional OADRs in absolute terms and much higher in relative terms when compared to the expected change in the indicators. While yielding the highest OADRs, the Bongaarts model shows the lowest POADR in each of the country cases.

There is also a substantial difference in methods (more specifically, between the Bongaarts and the other three models) in terms of remaining life expectancy at age 65 and age at remaining life expectancy15 years or less that is considered to be the threshold of old age. Both indicators may be important for assessing the consequences of mortality decline for pension systems. Changes in life expectancy at age 65 may show roughly how large a pension obligation may accumulate in systems with rigid age at retirement, while the age at remaining life expectancy of 15 years may indicate how much later people may be supposed to retire in a system with a fixed amount of life-time pension obligations. The inter-method variation in these two indicators is especially wide (about 4 and 7 years, respectively, for the remaining life expectancy at 65 and for and the age at remaining life expectancy 15 in 2050) – due to assumed fast mortality decline – in the case of Russian males. It is also rather wide (about a year or more) in other cases. As was shown above, the number of those considered old according to the old age threshold definition may differ considerably depending on which mortality model was selected.

One may be surprised by the small difference in outputs of the three more conventional methods: the Lee-Carter, the Brass and the direct extrapolation. Despite important differences in their structure and complexity, these models are rather similar in the outputs they produce. The computationally simpler direct extrapolation method is hardly distinguishable (in any aspect of its output) from the more fashionable Lee-Carter model given they fit to a similar future trajectory of the life expectancy at birth. The Brass model shows some differences in the death rates (especially at young age), differences that do not yield a substantially different result in population numbers or any of the population ageing indicators. Similarity of the three conventional methods may be explained by their shared conservatism with regard to the prospects of mortality decline at old-age as discussed above.

The crucial difference is not between relational models (like the Brass) vs extrapolations of age-specific rates (like the direct extrapolation and the Lee-Carter) or statistically sophisticated (as the Lee-Carter model) vs computationally simple mortality models. The difference is in how rigid and flexible model behaves at old age. As outlined in the introduction, the three conventional methods tend to underestimate mortality change at oldest-old age, while the Bongaarts model is prone to produce fast mortality declines at that age. It is widely expected that the future will bring mortality improvements at the oldest ages. The United Nations (2013) has recently built such expectations into its population forecasts by applying a 'robust rotation' adjustment to the Lee-Carter model.

Eurostat (European Commission 2014) mortality scenarios for Italy in 2050 assume uncertainty range (between the main and the high life expectancy scenarios) of 1.3 years in the life expectancy at age 65 for men and 1.4 years for women. For Sweden, Eurostat produces uncertainty ranges of 1.2 (men) and 1.3 (women) years for the same indicator. These ranges are not much different from our model-to-model variation of 1.1 (men) and 0.7 (women) years for life expectancy at age 65 for both countries in 2050. Even in terms of OADR, where we found less of a difference between mortality models, Eurostat's scenario-to-scenario range in 2050 (0.024 for Italy and 0.018 for Sweden) is comparable to the range (0.013 for both countries) we obtained here. For Russia in 2030, the country statistical office (Federal State Statistics Service 2014) projects high-to-low scenario range of OADR of 0.035 that is even narrower that our model-to-model range of 0.02 in the same year.

To answer the question posed in the title, yes, selection of mortality model does make a difference in projecting population ageing. It is as important as the selection of life expectancy scenarios.

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