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Shift and Compression of Mortality at Old Age: A Conservative Scenario

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Abstract

Formal relations are used to demonstrate inability of the relational Brass mortality model to keep up with declining mortality at old age. In order to adjust the model, a descriptive study is undertaken of mortality shifts at old age. To this end, ages X(M) at given levels of the mortality rate are studied. When arranged as functions of the life expectancy at birth, those ages show increasing steepness of the trend. This pattern is explained by approaching, as mortality declines, to upper limits of period mortality compression. In order to take this changing pattern into account, we obtain empirical lower-bound limits to X(M) and fit a quadratic regression lines to the lower bounds. Our models of lower-bound limits may be useful both in examining tendencies in period mortality shift and compression and as a starting point in adjusting the mortality projection models at older age. They may also be useful in the continuing discussion of prospects for further mortality decline.

Acknowledgments

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Shift and Compression of Mortality at Old Age: A Conservative Scenario

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1 Introduction

Motivation to this study comes from the need to improve the forecast accuracy, especially at the old age, of the Brass relational mortality model (Brass 1971), although our findings are wider applicable. Despite its convenience and wide usage (e.g., Benjamin and Soliman 1993; Lutz et al. 1996, 1997, 2008; Lopez et al. 2000, 2001; Sanderson and Scherbov 2005, 2007; Hartmann 2007; Lee et al. 2012), the model is limited in projecting mortality in developed countries due to its tendency to reduce the variability of mortality at oldest old age. This feature was more or less in line with actual developments of mortality in the past and, indeed, was an original advantage of the model. Yet, it may not be adequate for the current and, perhaps, future development of mortality in low-mortality countries.

A particular drawback of the model is its inconsistency with the propagation of mortality declines from younger to older ages (Wilmoth 1997; Willets et al. 2004; Andreev and Vaupel 2005; Rau et al. 2008; Ediev 2011a, 2013a, 2013b) in the process of *mortality inertia* (Ediev 2011a). That process may be driven by several factors, such as changing behavior, educational and income levels, accumulated chronic conditions, etc. from cohort to cohort, with younger cohorts usually being subject to more favorable conditions and having better health in the end. As we explain in the next section, the Brass model fails to capture this dynamic compositional effect upon mortality decline. Based on the analytical study of causes of this limitation (see chapter 2), we aim at adjusting the model in order to capture the mortality shifts at old age. This paper is the first step in this attempt. It empirically explores how old-age mortality is shifting when overall mortality declines. Our research strategy is to examine change over time (and also as a function of life expectancy at age 0) of ages at selected levels of the death rate using (smoothed) age-specific death rates from the Human Mortality Database (HMD) (2014).

The work is structured as follows. In the next section we show analytically why the Brass model tends to underestimate the mortality variation at oldest old age. We then discuss possible ways to improve the models' accuracy at old age and indicate the need to clarify details of mortality shift at old age. In chapter 4 we indicate existence and describe details of the shift in HMD data. This helps us to develop a conservative model of mortality shift at old age.

2 The Brass Model at Old Age

Applying the Brass model to extrapolate mortality as an alternative to separate extrapolation of age-specific mortality rates has certain merits in international population projections: It has a low demand for data, may easily incorporate expert judgment, and produces plausible age schedules of mortality rates. The Brass model relates an observed or projected age schedule l_x of probabilities to survive from birth to age x to another, standard, schedule l_x^* after transforming both schedules into logits:

$$logit(l_x) = \alpha + \beta \cdot logit(l_x^*), \tag{1}$$

where $logit(u) = 0.5\ln\left(\frac{1-u}{u}\right)$; α and β are model parameters. The α parameter may *roughly* be interpreted as reflecting the difference in the level of mortality, while β as reflecting the difference in age pattern of mortality rates (only 'roughly', because this interpretation applies merely to the shapes of logits, not to the original death rates). In projection applications, the most recent baseline mortality schedule would be taken as standard in the model, and the model parameters would be extrapolated based on their trends before the baseline year. In Figure 1 we present an example of the Brass model parameters change over time by using data for mortality among Russian females (HMD 2014). The case of mortality in Russia is particularly challenging in extrapolation due to substantial instability, age irregularities, and non-monotonic patterns of change. Quick short-term improvement by the end of the Soviet era, steep deterioration in the early 1990s with subsequent 'waves', and more recent improvement trends have contributed to the lack of stability in Russian mortality history. Despite these developments, the structural parameter of the model (β) shows a rather smooth trajectory that may be projected with relative ease, e.g., setting the β constant at its baseline value (one). The parameter of the level of mortality, α, may be projected assuming different scenarios about future mortality levels in Russia. For other countries that have more stable trends, the α parameter may also be forecasted using time-series models. Figures 2a-2c show the estimates obtained by assuming different country-specific standards; namely, the earliest available country life table after 1900 (Figure 2a), the country life table closest to 1950 (Figure 2b), and the most recent available country life table (Figure 2c). One may notice high regularity of the change in α, especially in more recent decades in currently low-mortality countries (Figure 2a)[1]. That makes extrapolation of α in low-mortality countries a rather easy task. Even in the higher-mortality countries, despite some irregular changes in α in the past its recent trend seems safe to extrapolate into the near future. Changes in β, on the other hand, are more complex and, at the same time, show no long-term trends. Fixing β at its baseline level, assuming its convergence to a new level or regressing it on

[1] The list of currently 'low-mortality' countries is obtained by excluding Eastern-European countries from the HMD; it includes: Australia, Austria, Belgium, Canada, Chile, Denmark, England and Wales, Finland, France, Total Population, Germany, Ireland, Israel, Italy, Japan, Netherlands, New Zealand (Non-Maori), Northern Ireland, Norway, Portugal, Scotland, Spain, Sweden, Switzerland, Taiwan, USA, UK, West Germany (we also exclude some small countries).

α or e_0[2] may be a good strategy in extrapolating mortality. Patterns of the α parameter become even more regular when, conveniently for projection applications (e.g., Lutz et al. 1996, 1997; Buettner 2002), one takes the life expectancy at birth or its change as compared to the baseline life expectancy and not the calendar time as the input variable (Figures 3, 4). This approach improves the trends even for the higher-mortality countries.

Hence, the Brass model benefits from the possibility of handy extrapolation of its parameters. At the same time, unlike in most other common extrapolations that rely on (occasionally divergent) extrapolations of age-specific mortality by time and may result in rather implausible projected mortality patterns, the Brass model is guaranteed to produce age patterns of mortality that preserve important features of the baseline pattern, such as monotonicity of old-age mortality.

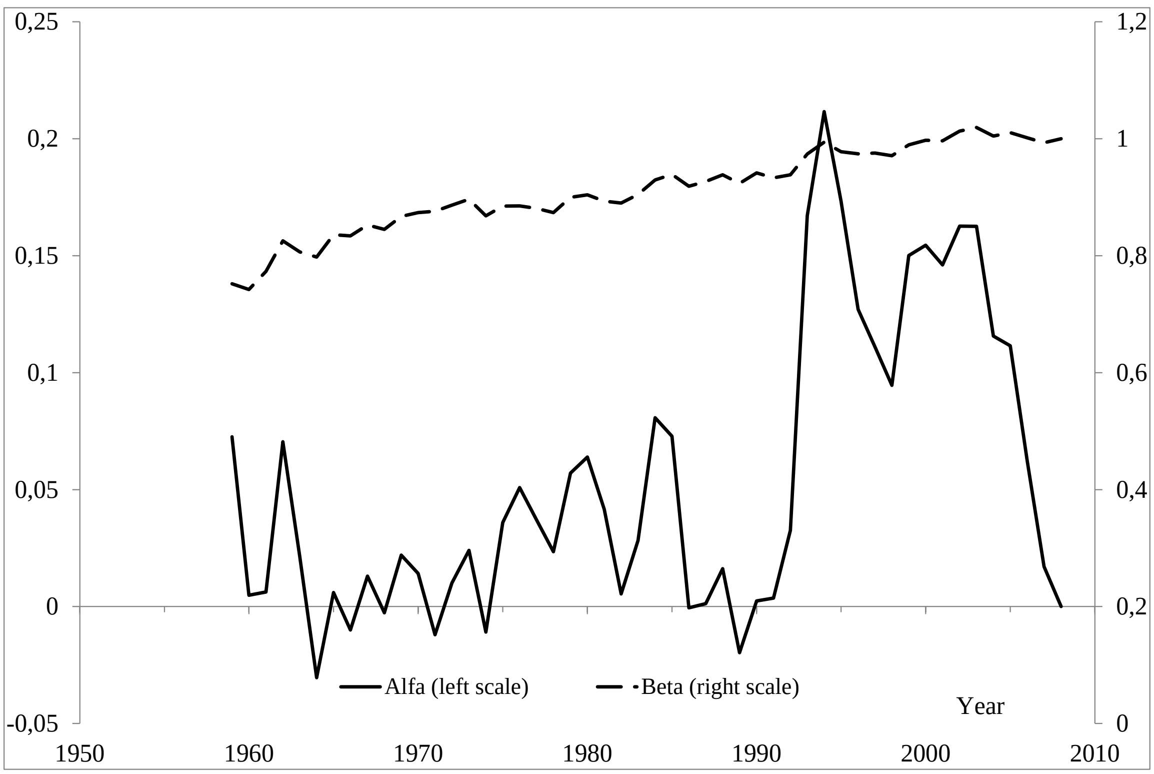


Figure 1. Dynamics Over Time of the Parameters of the Brass Relational Model Estimated for Russian Female Mortality Schedules with 2008 Used as Standard.

[2] Yet, our simulations based on HMD populations show that fixing β at the baseline level may, in fact, produce more robust and accurate projections than assuming a model of its change.

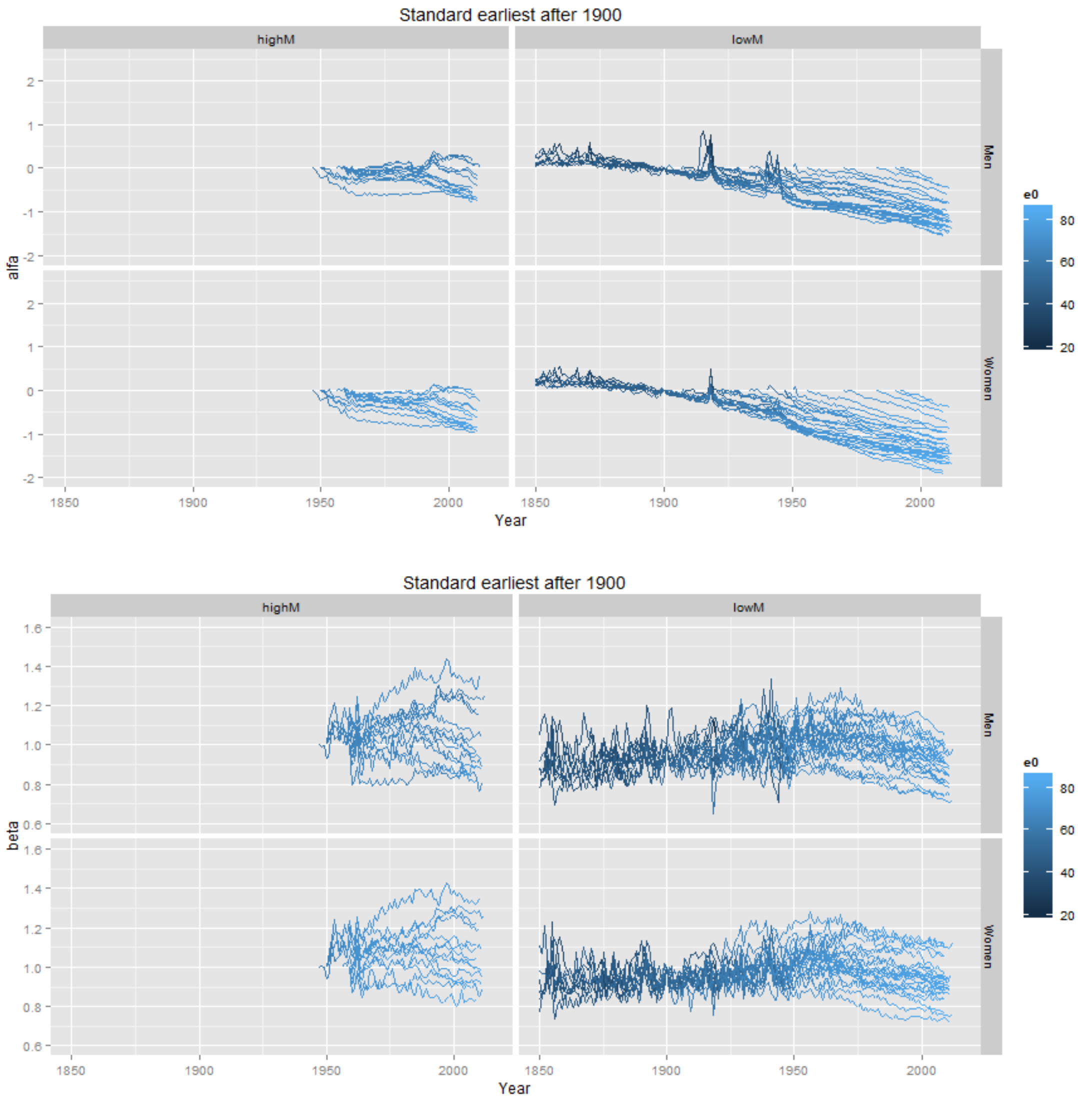
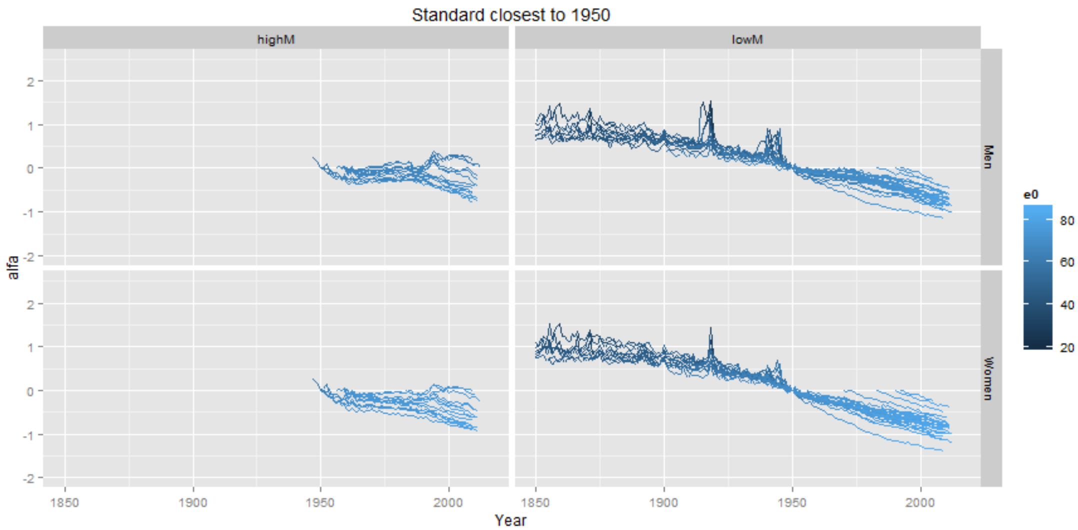


Figure 2a. Dynamics Over Time of the Parameters of the Brass Relational Model Estimated for HMD Populations with the Earliest (after 1900) Country-Specific Schedules Used as Standard. Levels of Life Expectancy at Birth are Indicated by Color; “highM” and “lowM” Panels Show Results for Currently Higher- and Lower-Mortality Countries.



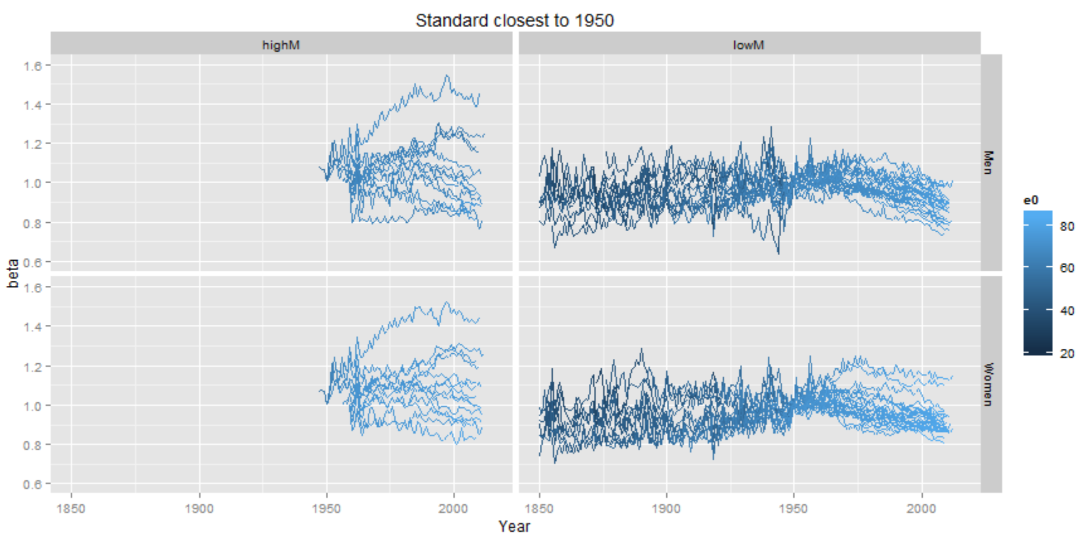


Figure 2b. Dynamics Over Time of the Parameters of the Brass Relational Model Estimated for HMD Populations with the Country-Specific Schedules Closest to 1950 Used as Standard. Levels of Life Expectancy at Birth are Indicated by Color; “highM” and “lowM” Panels Show Results for Currently Higher- and Lower-Mortality Countries.

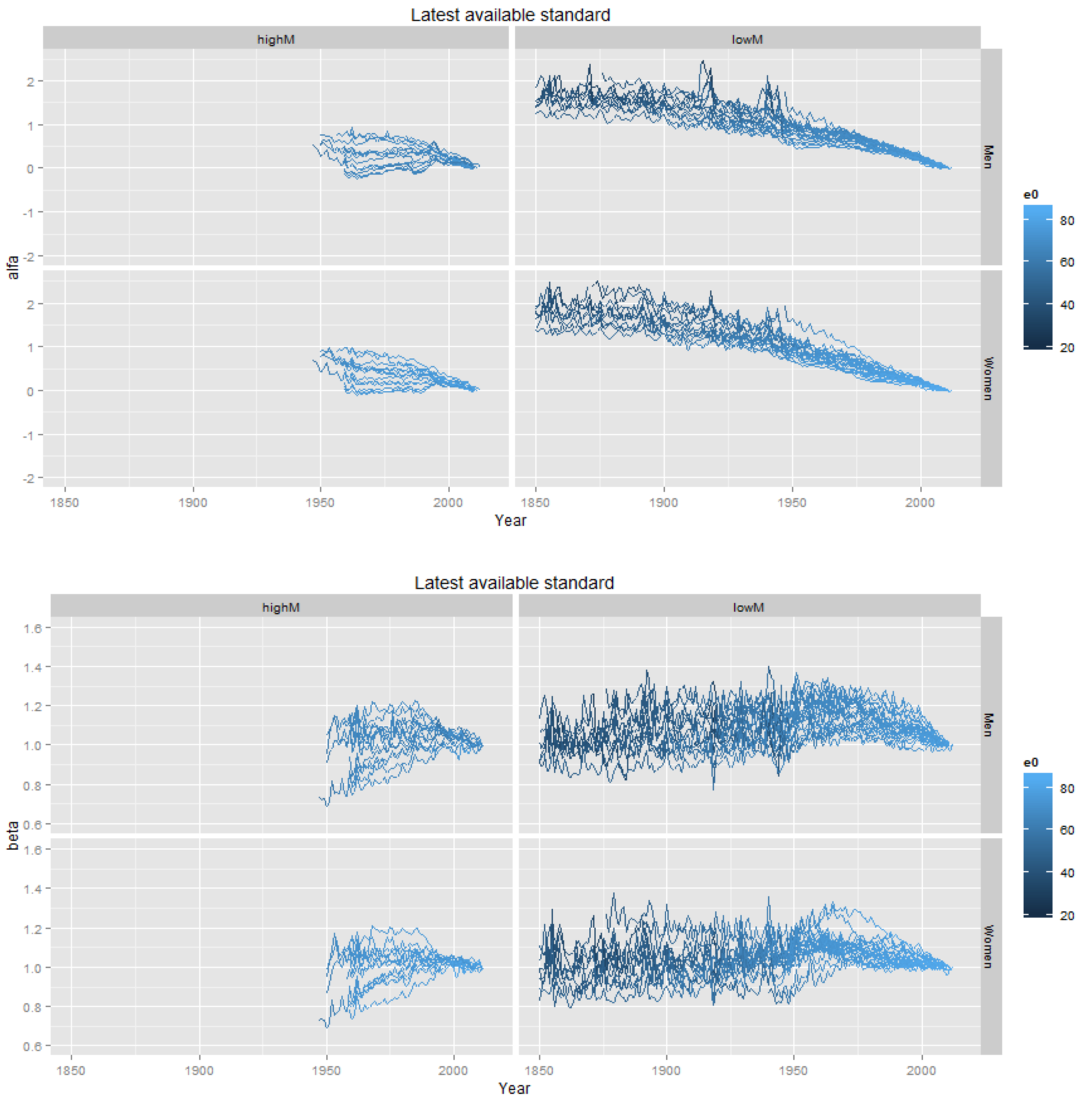


Figure 2c. Dynamics Over Time of the Parameters of the Brass Relational Model Estimated for HMD Populations with the Most Recent Country-Specific Schedules Used as Standard. Levels of Life Expectancy at Birth are Indicated by Color; “highM” and “lowM” Panels Show Results for Currently Higher- and Lower-Mortality Countries.

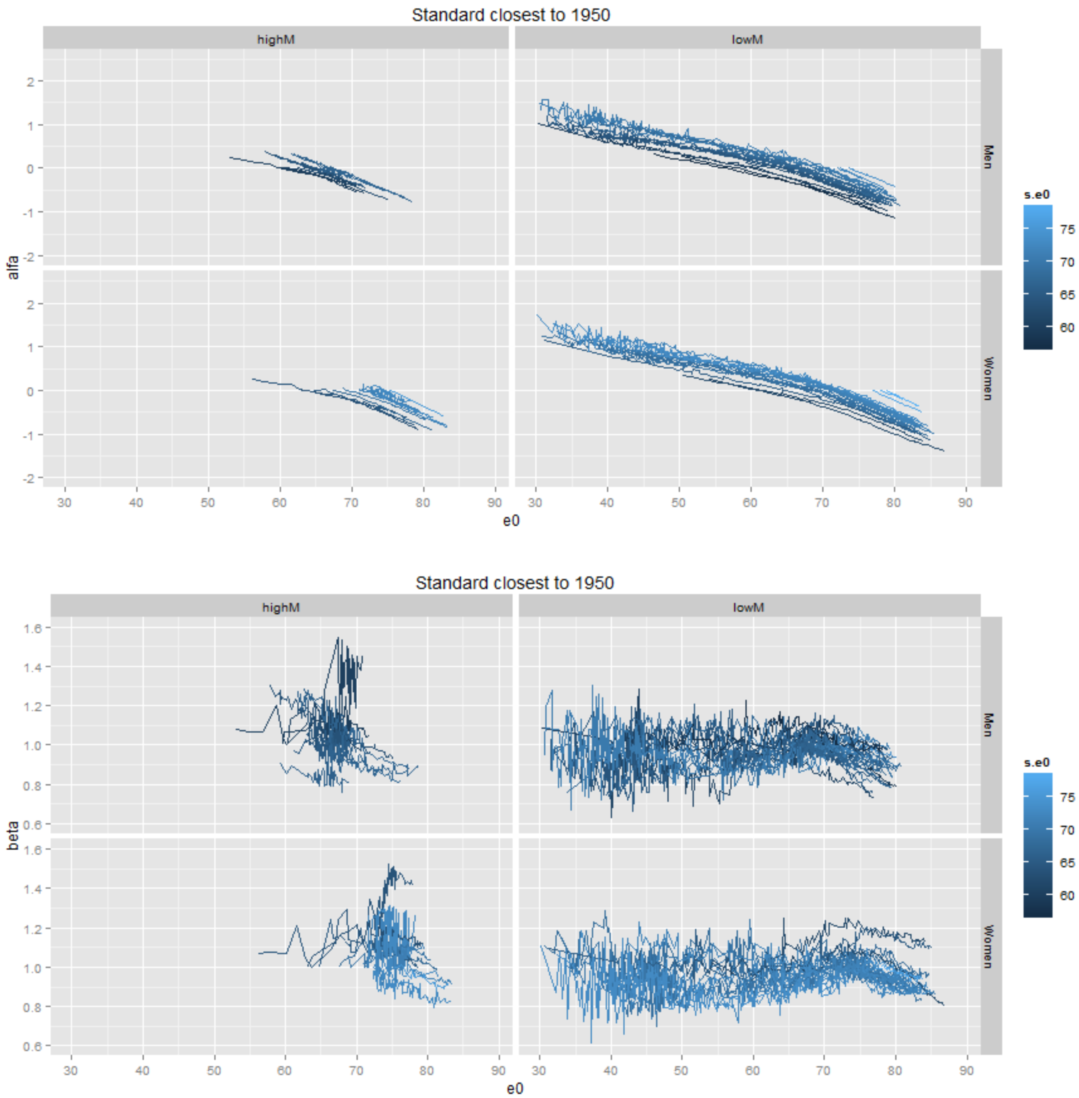


Figure 3. Variation Across Levels of Life Expectancy at Birth of the Brass Relational Model Parameters Estimated for HMD Populations with Country-Specific Schedules Closest to 1950 Used as Standard. Levels of Life Expectancy at Birth are Indicated by Color; “highM” and “lowM” Panels Show Results for Currently Higher- and Lower-Mortality Countries.

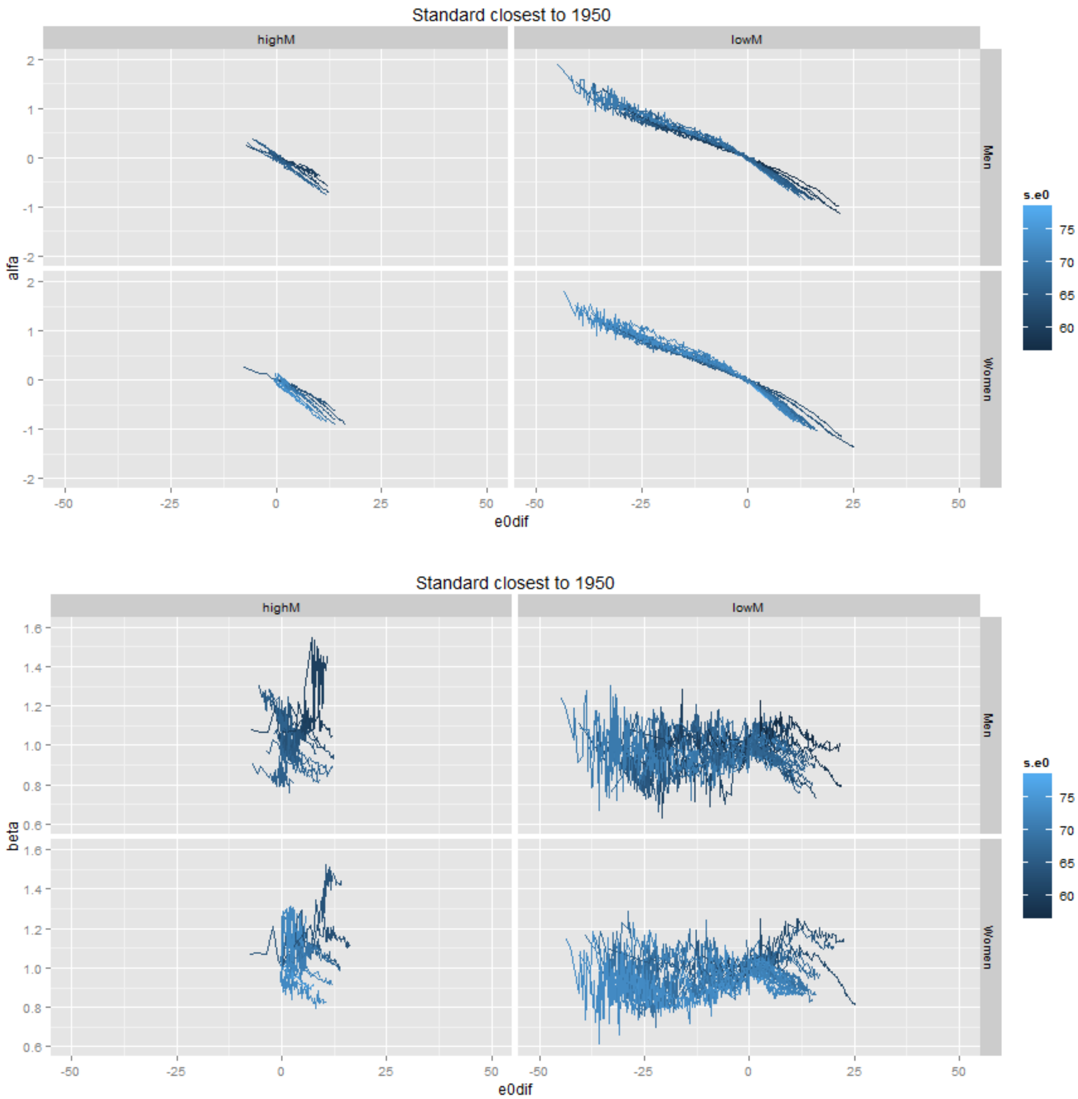


Figure 4. The Parameters of the Brass Relational Model as Function of Change (from the Value in the Model Standard) in Life Expectancy at Birth Estimated for HMD Populations with the Country-Specific Schedules Closest to 1950 Used as Standard. Levels of Life Expectancy at Birth are Indicated by Color; "highM" and "lowM" Panels Show Results for Currently Higher- and Lower-Mortality Countries.

While the Brass model gets away with the common shortcoming of mortality extrapolations, such as unwanted crossovers of the projected mortality rates between ages, it fails to adequately forecast mortality decline at oldest ages. One reason for this shortcoming might be that the standard schedule itself is inadequate to project future mortality. Another and more fundamental reason is that extrapolations of α and β will typically show a continuation of mortality rate decline at younger ages and not much decline at old ages. Similar to other extrapolations of death rates, the Brass model's forecast may be in agreement with past trends but is inconsistent with new developments including the upward age shift of the mortality curve.

Figure 5 illustrates these limitations of the model. It presents UK female patterns of dying probabilities q(x) as of 1922 (the baseline used as standard in the Brass model), 2009, and the (β-fixed) Brass model fit to e_0 in 2009. The Brass model shows clear systematic deviations from the actual data. While it yields substantially lower mortality at ages 30-80 years, it underestimates mortality improvements for children and, more importantly for projecting old-age mortality at ages above 80 years. With mortality in developed countries is already low at child and young adult ages, and declining at 'younger' old age, biasedness of the model at oldest old age will become more and more problematic in projecting future mortality. Among other things, the pattern of biases such as the one presented in Figure 5, indicates 'over-compression' (i.e., steeper increase by age) of the projected mortality – an artifact observed in methods extending the age-specific mortality trends into the future without allowing for newly emerging trends and deeply bedded in the very basics of the period life table model (Ediev 2008b, 2011a, 2013b).

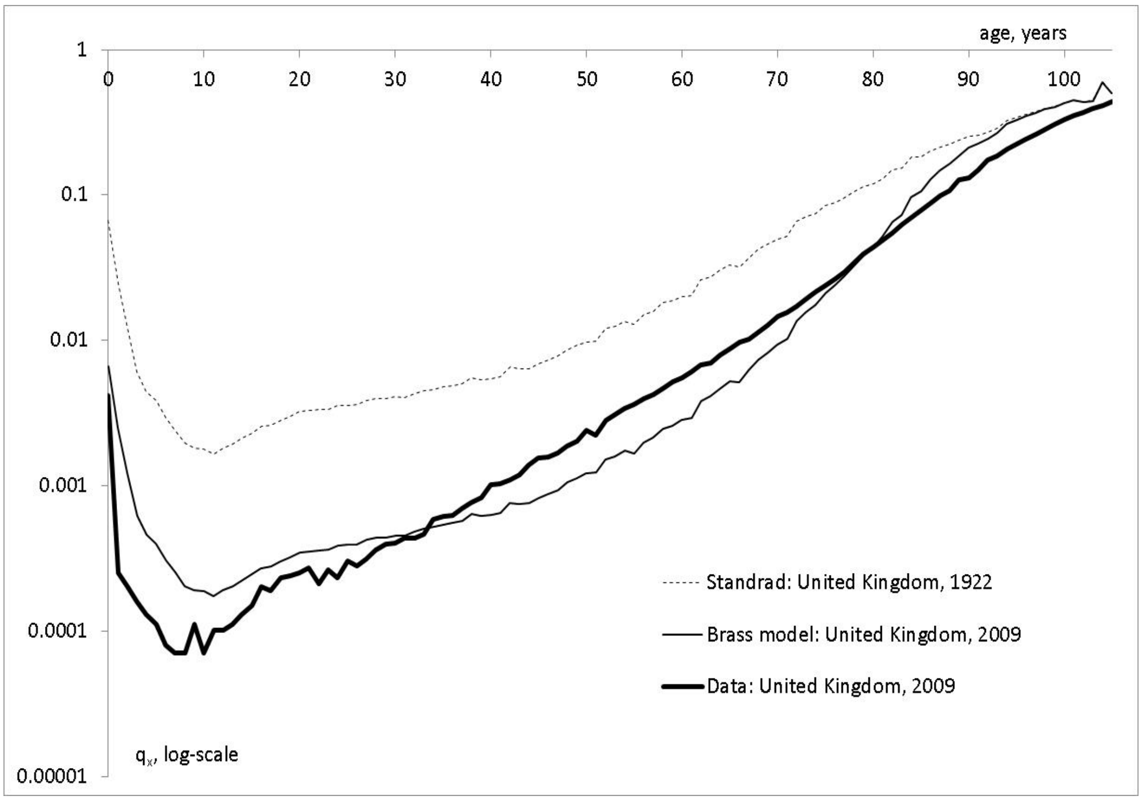


Figure 5. Age Profiles of Dying Probabilities (q(x)): The Baseline (1922), Observed (2009), and Fitted by the Conventional Brass Model with the Baseline Schedule used as Standard; UK, women.

To clarify the origin of the structural problems of the Brass model, its performance at the oldest old ages may be studied analytically. To do so, it has to be noted that the proportion of those who survive to the oldest old ages is small, i.e., $l_x << 1$, $l_x^* << 1$ at the advanced old age. Taking these conditions into account and using Eq. (1), one may obtain for the oldest old ages:

$$ln(l_x) \approx -2\alpha + \beta \cdot ln(l_x^*). \tag{2}$$

Equivalently, for the hazard rates,

$$\mu_x \overset{def}{=} -\frac{d\,ln(l_x)}{dx} \approx \beta\mu_x^*, \tag{3}$$

where $\mu_x^* \overset{def}{=} -\frac{d\,ln(l_x^*)}{dx}$ is the hazard rate of the standard.

It follows from (3), that changing the parameter α and freezing the parameter β, a convenient practice in using the Brass model for projections, induces no change to the death rate later in life, even if it reduces the death rate at younger ages. In this sense, the Brass model is rotating the mortality curve when fixing the parameter β. This would be a typical situation even in the case of extrapolation of both parameters of the model, as β is usually changing only little over time. This explains the pattern of biases shown in Figure 5: the Brass model tends to increase the steepness of the mortality curve.

To overcome the problem described above by changing β faster than suggested by the past trends would, however, also be difficult. At old ages, this would be equivalent to the proportional hazards model as may be seen from (3). That model, however, is not consistent with the observed tendency towards compression and shift of period old-age mortality accompanying the extension of the lifespan (Fries 1980; Wilmoth and Horiuchi 1999; Kannisto 2000; Cheung et al. 2005; Canudas-Romo 2008; Ediev 2013a). Another way to look at the model's performance is to compute the life table ageing rate (LAR) (Horiuchi and Coale 1990):

$$LAR_x = \frac{d\,ln\mu_x}{dx} \approx \frac{d\,ln\mu_x^*}{dx} = LAR_x^*. \tag{4}$$

Hence, the Brass model assumes invariant LARs at older ages, while the data suggests that the LAR has substantially increased at old age in a process of period mortality compression. As mortality decline may accelerate at older ages, LAR may also be pushed up in those ages due to a sort of tempo-effect (Ediev 2008b, 2011a, 2013b).

Mortality inertia (Ediev 2011a) might have been one driver behind both the mortality compression and mortality decline at old age. The idea is that mortality declines at older ages are precluded by mortality declines (indicating overall health improvements) in the same cohorts at younger age. To put it differently, if people become healthier at age 50 (due to smoking less, eating better, becoming better educated, etc.), they are likely to keep the benefit in health and show lower mortality at older ages as well. This model shows good fit to the population-level data (Ediev 2011a) and is supported by differential patterns of mortality compression in period life tables and birth cohorts. In particular, cohort schedules of mortality are rather stable (in terms of age durations between succeeding levels of mortality and, consequently, LARs) and show no persistent time trends (Ediev 2011b, 2013b). This implies that once mortality decline is observed at age x in a birth cohort, mortality at age x+1 in the same cohort should also decline in order for the LAR in the cohort life table to remain stable. At the same time, period life tables do show mortality compression that may be explained by transitional effect: when age at a given death rate M increases by *r* years per calendar year, LAR in the period life table increases by about 1/(1-r) times (Ediev 2008b, 2011, 2013b). Hence, mortality decline, while combined with stable cohort LARs, leads to increasing LAR in period life tables, i.e. to period mortality compression. Mortality decline that originated at young age produces a transition period where younger cohorts show lowered mortality while older cohorts still show pre-transition high mortality. The result is temporarily[3] increasing steepness of the period mortality curve, i.e. mortality compression.

Contrary to these mechanisms, the Brass model, due to the above-presented analytical features, is not capable of producing mortality compression patterns at old ages.

Several important advancements to the Brass model have been proposed in the literature in order to improve the model performance at old age. Zaba (1979) proposed adjusting ($l_N(x)$) the original standard survival function by introducing two additional parameters ψ and χ, and assuming two age schedules of deviations from the original standard $k(x)$ and $t(x)$:

$$l_N(x) = l^*(x) + \psi k(x) + \chi t(x), \quad (5)$$

Ewbank et al (1983) further developed the idea of adding more flexibility to the model by proposing alternative (better interpretable) age schedules of deviation from the original standard. In particular, they propose replacing the logits of the original standard by expression:

$$T\left(l^*(x);\kappa,\lambda\right) = \begin{cases} \dfrac{\left(\dfrac{l^*(x)}{1-l^*(x)}\right)^{\kappa} - 1}{2\kappa}, at\, l^*(x) \geq 0.5 \\ \dfrac{1-\left(\dfrac{1-l^*(x)}{l^*(x)}\right)^{\lambda}}{2\lambda}, at\, l^*(x) < 0.5, \end{cases} \quad (6)$$

that converts to the original standard logits at $\kappa, \lambda \to 0$. While these two models offer more flexibility in *fitting* mortality schedules, their use for projections is less clear. This is partly because their implication for the future development of old-age mortality is not necessarily consistent with empirics. The model by Ewbank, for example, may be reduced at oldest old age, in the same way as presented above for the Brass model:

$$\mu(x) \approx \beta\left(l^*(x)\right)^{-\lambda} \mu^*(x). \quad (7)$$

Hence the future death rates and LARs will be linked to the age pattern of $l^*(x)$ in a rather peculiar way.

Himes et al. (1994) propose improving the Brass models' performance at old age by designing a better, more up-to-date, standard profile based on data for countries advanced in mortality reductions. Being highly valuable for current fitting, this model may well become problematic in dealing with the future profiles due to the above discussed analytical properties of the Brass model.

[3] 'Temporarily' only assuming eventual halt on mortality decline. The compressed pattern may sustain as long as mortality continues declining.

Murray et al. (2003) replaced the modeled logits in (1) by the following adjusted values:

$$\Gamma(l(x)) = Logit(l(x)) + \gamma(x)\left(1 - \frac{Logit(l(5))}{Logit(l^*(5))}\right) + \theta(x)\left(1 - \frac{Logit(l(60))}{Logit(l^*(60))}\right). \quad (8)$$

This approach was reported to have good fit, despite using a single standard, to empirical mortality profiles collected by the World Health Organization, at least at ages below 85 and on data collected by the time the method was published. If two parameters of the empirical life table are used as inputs, the model produces rather accurate 'predictions' of the life expectancy at birth and log-mortality rates (at ages below 85; we could not find more details about age-specific error levels in 'predicted' ln M(x)). Steepness of both adjustment profiles ($\gamma(x)$ and $\theta(x)$) increases with age, however, and the method may become less stable in projections at older ages, where one may approximate:

$$\mu(x) \approx \beta\mu^*(x) - \tfrac{d}{dx}\gamma(x)\left(1 - \frac{Logit(l_5)}{Logit(l^*_5)}\right) - \tfrac{d}{dx}\theta(x)\left(1 - \frac{Logit(l_{60})}{Logit(l^*_{60})}\right). \quad (9)$$

Another potential limitation in projections is that the model relies on two input parameters, which may be problematic in some country contexts. Turning the model into a more convenient single-parameter version (e.g., by using regression of l_5, l_{60} on e_0 and fixing β) may have adverse consequences on predictive accuracy of the model.

Potential limitation of the mentioned improvements to the Brass model is their lack of explicit account of the mortality shift at old age. While the improvements may indirectly reproduce the shift, a better approach might be to model the shift explicitly and incorporate it in the projection model. This is our assumed goal. We aim at improving the efficiency of the model at older age while keeping it practical and simple (in particular, with single input parameter). To this end, we first focus on empirical patterns of mortality shift at old age, with the aim to adjust the Brass model for the observed tendencies in the mortality shift in a second step (we plan to present the details of the model adjustments in a later report).

3 Mortality Shift at Old Age: An Empirical Account

In order to be able to adjust for the limitations of the mortality projection model we examine details of the mortality shifts at old age that the traditional models were shown not to be capable to reproduce. In view of projection applications, where the life expectancy at birth may be a convenient input variable, we study the mortality change at old age as a function of this variable. We do so by examining variation of ages when the death rate (in period life tables) reaches certain levels, so that we may be capable of explicitly describing the mortality shifts at old age. Our study is empirical, based on period mortality data from the Human Mortality Database (2014).

A note is due here about reliability of mortality data at oldest old age. Considerable debate is yet underway in the literature regarding the shape of mortality curve at that age. While some earlier studies, based on selected populations with better data, have suggested that the increase in the force of mortality is decelerating at some old age and the force may go to a plateau at the oldest old age (Horiuchi and Wilmoth 1998), more recent studies indicate that this may not be the case when data quality, cohort heterogeneity and selection effects are taken into account (Gavrilov and Gavrilova 2011, 2014). We do not contribute to this discussion but take the HMD data – showing deceleration in many cases – as it is. One reason for this approach is that even if deceleration might be a statistical artifact due to reliance on discrete age intervals and peculiar within-age distributions of the ages at death (as shown by Gavrilov and Gavrilova 2011, even for the one year-long age intervals), traditional forecasts do rely on such age intervals. Whatever 'distortions' may be introduced by computing the death rates for discrete age intervals, these discrete-age estimates are what can be used in any practical population projection. Another reason is that our main results will be based on studying mortality rates at ages below 100, where the data quality of HMD is better and the difference between deceleration and Gompertzian scenarios is moderate. Yet another reason is that mortality deceleration may well be observed in period data, one that we use here, despite lack of deceleration in cohort data, one considered by Gavrilov and Gavrilova, due to the sort of tempo effects we reported earlier (Ediev 2008b, 2011a,b, 2013b)[4].

To reduce the effects of random variation in the death rates, we smooth the input data. Namely, we apply two-dimensional (7x7 in age-time) moving averaging to the central death rates:

$$\tilde{M}(x,t) = \exp\left(\frac{\sum\limits_{\substack{|x-i|\le s1 \\ |t-j|\le s2}} \ln M(i,j)}{(2s_1+1)(2s_2+1)} \right), \qquad (10)$$

here $2s_1 + 1 \le 7$ and $2s_2 + 1 \le 7$ are the dimensions of the averaging window in age and time respectively (both are set to be 7 years at most and are shortened, as age or time approaches lower/upper limits; lower limit for age is set at 10, which is about the age at minimum mortality).

Using the smoothed death rates, we estimate exact ages X(M), in each period, when the mortality rate reaches levels M spanning from 0.0125 to 0.5 (we excluded few cases, where data shows suspicious round ages at selected round M's). Those levels are selected to cover most of the old-age mortality. The exact ages are approximated by applying the linear regression to three ages in the smoothed life table with values of the log-mortality rate closest to M. We then study shifts in old-age mortality by examining change over time of ages X(M).

Some insights into the old-age mortality shift may be gained from variation of X(M) over time at selected mortality levels (Figure 6, separately for men and women in high- and low-mortality countries). The shift of old-age mortality has been smaller (if not absent, in earlier times) at higher M. That might have been consistent with the Brass model's structure in earlier periods. More recently, however, mortality is shifting at all levels of M, a phenomenon that may not be reproduced by the Brass model with an old standard. One may also notice a sort of convergence of (low-mortality) countries to a narrower range of X(M)'s as time was passing on in a process of compression, both across M's and countries.

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[4] We intend to address details of this hypothesis elsewhere. For now, it may suffice to note that the slope of the period mortality curve increases, as compared to the corresponding slope in cohort mortality curve, when mortality declines. Faster decline of mortality at younger age with slower or lacking decline at older age will then create a pattern where period mortality will increase slower at older age, i.e., show deceleration, despite cohort mortality staying in line with the Gompertz model of similar slope at younger and older age.

In Figure 7, we present X(M)'s as function of period life expectancy at birth. There is large variation among countries in both levels and trends of X(M). Yet, one may also notice general tendency: X(M) are more stable at lower e(0) and start increasing more rapidly at higher e(0). At higher M trends were nearly flat at low e(0) and started shifting to older age only recently (at higher e(0)). Patterns of change in X(M) are more regular when arranged as function of life expectancy at birth rather than time. That feature might be convenient in projection practices that are based on using e(0) as the input variable. Even more regular patterns may be established for association between X(M) and life expectancy at age 30 (Figure 8). Yet, necessity to define the child mortality patterns separately from the older-age mortality may be cumbersome in projections.

The particular patterns of X(M) presented above call for interpretation. It is, in particular, an interesting feature of those patterns that X(M) show a breaking points in their trends: at lower and higher life expectancies e(0), slopes of X(M) are clearly different. Contribution of declines in child mortality may be one explanation to this peculiar pattern. When mortality at young age is high (and life expectancy is low), the life expectancy at birth may be, and used to be, extended without much change at old age mortality[5]. When young-age mortality declines to low levels, however, life expectancy may only be increased by declines of mortality at older age and, consequently, by shifts of death rate's contour lines towards older age. This explanation is supported by considerably less pronounced trend breaks in Figure 8, where X(M) are arranged along the levels of e(30) that excludes child mortality. To further investigate the origin of trend changes in Figure 7, we have computed simplified theoretical lower limits to X(M) at any given e(0) (dotted lines in the Figure). Those are computed under assumptions of high compression (i.e., low levels) of mortality below age X and short remaining life expectancy above that age. Suppose X(M)=X* and assume the following mortality curve with high LAR ($LAR_{\max}(M)$) below X*and short remaining life expectancy ($e_{\min}(M)$) above that age:

$$\begin{cases} M^*\left(x \mid X(M) = X^*\right) = Me^{LAR_{\max}(M)\left(x - X^*\right)}, x \le X^* \\ e\left(X^*\right) = e_{\min}(M) \end{cases} \tag{11}$$

When maximum LAR below age X(M) and minimum e(M) are determined (see below for the details), (11) provides lower estimates to the force of mortality. Therefore, the life expectancy at birth $e^*\left(0 \mid X^*\right)$ calculated under mortality schedule (11) underestimates the true life expectancy:

$$e^*\left(0 \mid X^*\right) \le e(0). \tag{12}$$

Taking into account monotonicity of $e^*\left(0 \mid X^*\right)$ as function of X^* at any given M, (12) implies lower limits to X(M) at given e(0):

$$X\left(M \mid e(0) = \mathrm{e}^*\left(0 \mid X^*\right)\right) \ge X^*. \tag{13}$$

To use the above described lower estimates to X(M), one needs to estimate upper limits to LAR below age X(M) and lower limits to e(X(M)). To this end, we estimate 97-th and 3-rd percentiles, respectively, of average growth rate of the death rate at ages X-20 to X (shortening the frame, if necessary, to preclude from spanning below age 30 years) and e(X) in all HMD period mortality data. At younger ages (M of about 0.05 or lower for men, even lower for

[5] This, of course, does not exclude the possibility that some countries may progress in reducing mortality at old age already at some earlier stage in their epidemiological transition.

women, Ediev 2013b), such estimated LAR may well represent its upper bound, because mortality compression at those ages has recently come to halt or was even replaced by some decompression in countries advanced in mortality decline. Yet, at older age, due to the factors mentioned above and related to possible future acceleration in mortality decline, our estimates for upper-limit LARs may be too conservative. Similar concerns apply to e(X) estimates, but those have more limited impact on our X* estimates. Hence, our estimates may be too conservative at higher ages where mortality decline is still modest, because future accelerations of mortality decline at those ages may push to mortality compression even higher than ever observed so far. We do not follow this possibility, however, and only use our simplified lower limits to X(M) to roughly indicate if mortality regimes are close to their potentially most compressed stage; and also to grasp what happens to mortality at that extreme stage.

Patterns shown in Figure 7 are supportive of the idea that many modern low-mortality countries are indeed close to those extreme limits. At lower mortality levels (M equal to 0.05 or lower), currently, X(M) are not only close to their lower estimates but also show slopes very much consistent with the lower estimates. At higher ages, slopes of X(M) are smaller than those of the lower estimates. This may be explained by the above-noticed deficiency of our upper-bound estimates to LAR at higher M's: in the future, due to acceleration of decline and, consequently, compression of mortality at old age, corresponding LARs may be pushed further up. When factoring this effect in, our lower-bound estimates to X(M) will be shifted down, delaying the time (and extending the e(0) level) at which the observed X(M) may hit their floors. This implies, in particular, that shifts of X(M) at higher M will most likely accelerate in the future when/if life expectancy continues increasing and X(M)'s hit their lower limits. In a different approach (in fact, not considering the possibility of mortality shifts and arguing for approaching limits to human lifespan), Finch et al. (2014) came to a somewhat similar conclusion that further increases of life expectancy may not be possible without new patterns emerging in mortality curves.

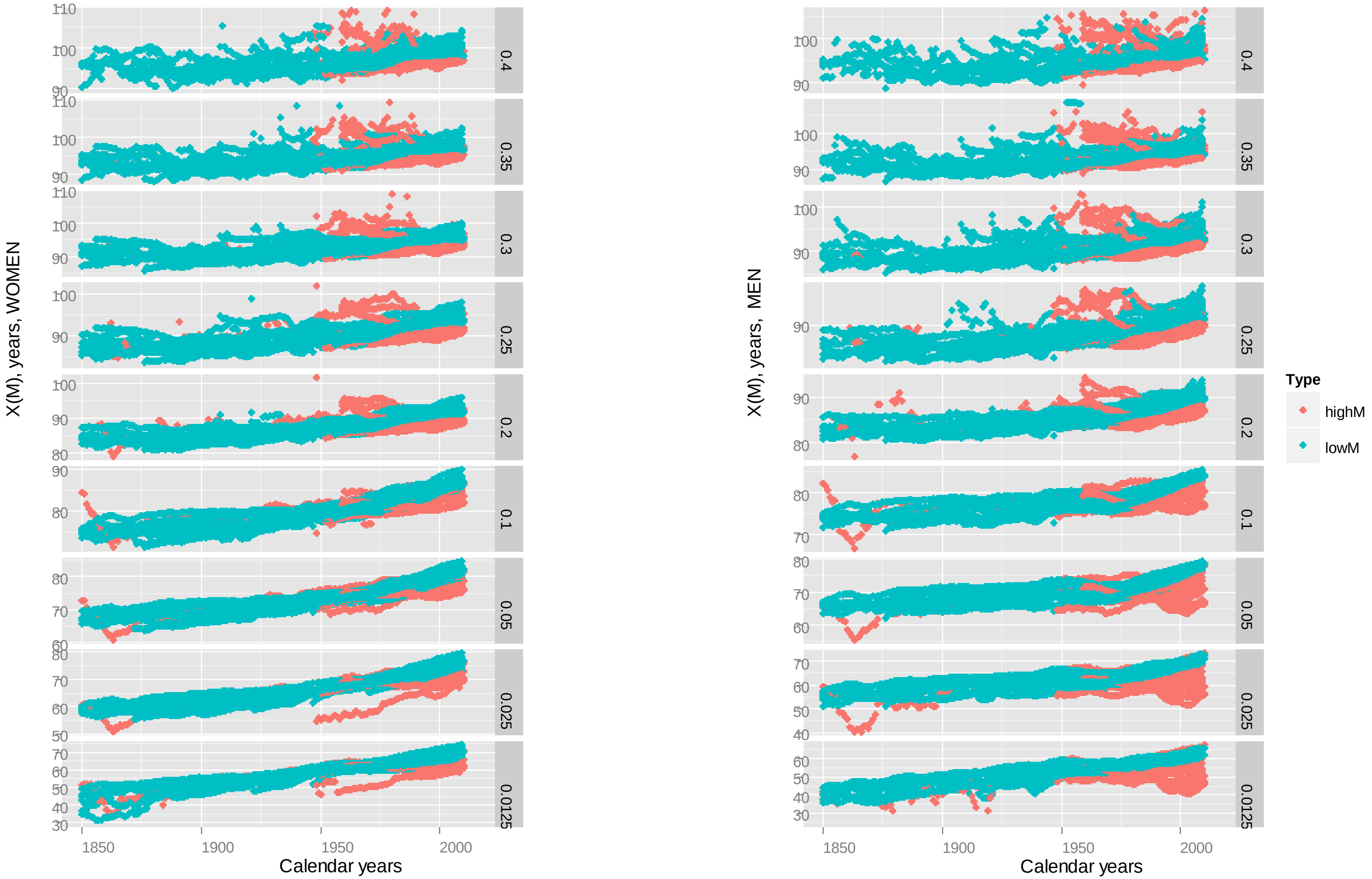

Figure 6. Ages X(M) at Selected Levels of the Death Rate M (as Indicated in the Right-Hand Side Strip of each Panel) as Function of Calendar Year in High and Low-Mortlaity Countries, for Women (the Left-Hand Column) and Men (the Right-Hand Column); 'x min' Denotes Theoretical Approximation from Below.

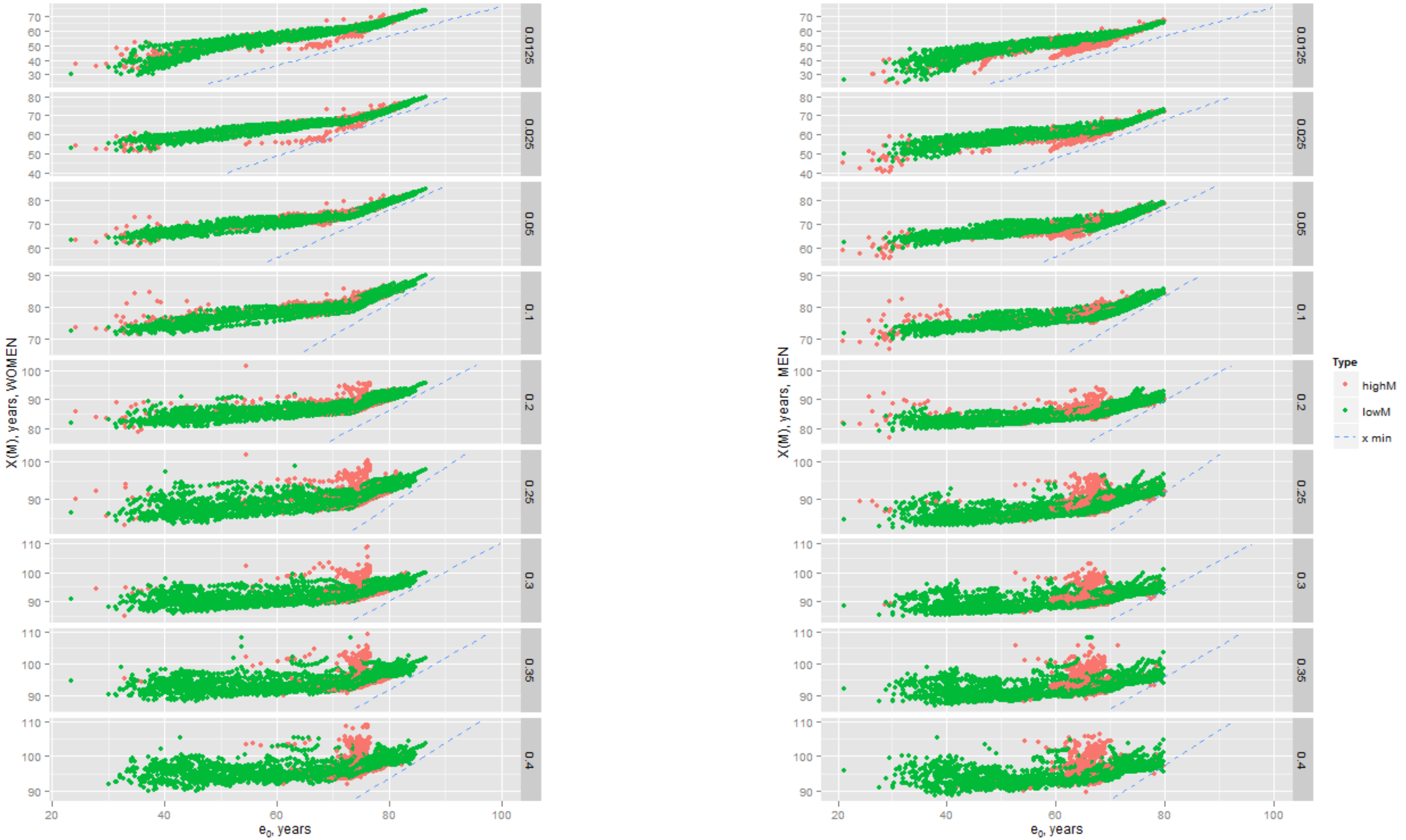

Figure 7. Ages X(M) at Selected Levels of the Death Rate M (as Indicated in the Right-Hand Side Strip of each Panel) as Function of Life Epectancy at Birth in High and Low-Mortlaity Countries, for Women (the Left-Hand Column) and Men (the Right-Hand Column); ‘x min’ Denotes Theoretical Approximation from Below.

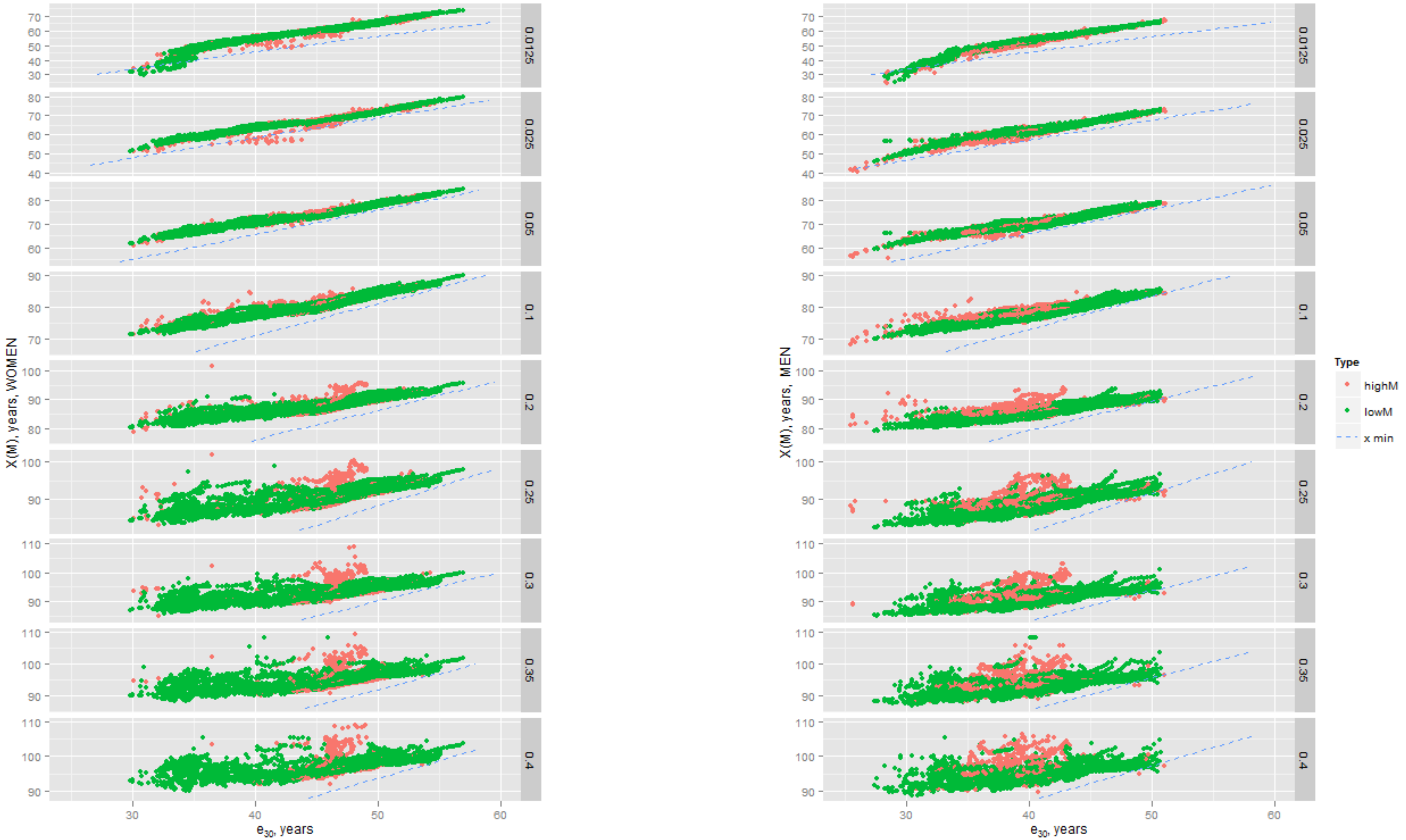

Figure 8. Ages X(M) at Selected Levels of the Death Rate M (as Indicated in the Right-Hand Side Strip of each Panel) as Function of Life Epectancy at Birth in High and Low-Mortlaity Countries, for Women (the Left-Hand Column) and Men (the Right-Hand Column); ‘x min’ Denotes Theoretical Approximation from Below.

4 Mortality Shift at Old Age: The Conservative Scenario

Our idea for taking the above-discussed patterns of old-age mortality into account in mortality projections is to model the empirical lower-bound limits of X(M) curves. We focus on the lower-bounds because: (a) those bounds are likely to represent the limits to mortality compression at any given e_0, so that X(M)'s falling below the limits are unlikely in the future as well; (b) factoring in the lower bounds into projections would subject the projections to the most conservative (of smallest possible mortality shifts) adjustments only.

In order to obtain robust estimates of the lower bounds to X(M) based on empirical data, we apply the following multi-step procedure:

1. At any given M, for any given value of life expectancy e_0, we estimate the minimum of all the X(M) values observed at life expectancy falling within the two-years-long window around e_0:

$$minX(M, e_0) = \underset{it}{MIN}\left(X_{it}(M) \,|\, \left|e_{0,it} - e_0\right| \le h\right), \tag{14}$$

where minimum is taken over all populations observed (i), at all calendar years (t) that fulfil the life expectancy requirement; *h=1* is the tolerance level determining the width of the life expectancy windows within which the minima are taken; but, for the sake of robustness, we do not estimate $minX(M, e_0)$ if the number of observations available for estimation falls below the critical limit n_{min} =5.

2. Having estimated the $minX(M, e_0)$'s, we fit a weighted quadratic regression

$$minX(M, e_0) = C + \alpha e_0 + \beta e_0{}^2 + \varepsilon\,, \tag{15}$$

with weights equal to the number of observations used in (14), at each e_0, and embracing the estimated $minX(M, e_0)$ within three standard errors of X(M). The quadratic form in (15) is aimed to reflect the accelerated shifts of X(M)s as function of the life expectancy. Assuming that the shift may not be faster than the theoretical 'high-compression' limit of one year by each one-year increase in e_0, and also assuming no negative shifts in the model, we impose restrictions to the estimated speed of the shift:

$$0 \le \alpha + 2\beta e_0 \le 1\,. \tag{16}$$

This implies limits to life expectancy within which model (15) may be applied (and fit):

$$\frac{-\alpha}{2\beta} \le e_0 \le \frac{1-\alpha}{2\beta}\,. \tag{17}$$

We observe these limits by iteratively shortening the range in e_0 within that the regression is fit, so that condition (17) is met at the final iteration.

3. Beyond range (17), we replace the quadratic approximation by linear:

$$minX\left(M,e_0\right)-\varepsilon=\begin{cases} C-\dfrac{\alpha^2}{4\beta},\ at\,e_0<-\dfrac{\alpha}{2\beta} \\ C+\dfrac{1-\alpha^2}{4\beta}+\left(e_0-\dfrac{1-\alpha}{2\beta}\right),\ at\,e_0>\dfrac{1-\alpha}{2\beta} \end{cases} \quad (18)$$

(the first line implies X(M) is constant below $e_0=-\frac{\alpha}{2\beta}$; the second line implies X(M) increases by one year per each one-year increase in the life expectancy at birth after it reaches value $e_0=\frac{1-\alpha}{2\beta}$).

We estimate the standard errors in observed X(M)'s by the following procedure:

$$SErr\left(X_{it}\left(M\right)\right)=sd\left(X_{it}\left(M\right)-\frac{X_{it-1}\left(M\right)+X_{it+1}\left(M\right)}{2}\right), \quad (19)$$

where 'sd' stands for the sample standard deviation and is computed over all observations with similar levels of M, gender, and mortality type, see Figure 9 for results. The estimated errors are below 0.5 years in all cases and show considerable variation by M and country type, but they vary less by gender. These errors are substantially lower than the country-to-country or temporal variation in X(M).

Observed, minimum among observed and fitted lower-bound estimates (18) are shown in Figure 10a and 10b: for all countries together (Figure 10a) and separately for low-mortality countries (Figure 10b). Due to similar patterns in X(M), we have put together data for men and women. Apart from some outliers and a lack of fit to the sudden trend change in low mortality countries at life expectancy at birth of about 70 at mortality levels below 0.3, our lower-bound estimates seem to limit the observed data well. Although the clear break in the trend slope around e_0=70 was typical in the low mortality countries, it is not clear if other countries may go through the same peculiar type of change in the future. In any case, our envisaged adjustment procedure to the Brass model will be focusing on mortality levels (starting from 0.3) where the trend change was not that abrupt. Therefore, we will proceed using our obtained lower-bound estimates despite their lack of fit to the abrupt trend change in low mortality countries.

In Figure 11, we show the fitted lower-bound estimates of X(M), in a single plot for all selected mortality levels M (one plot for all countries combined and another one for low mortality countries). That is to see implications for changes in mortality patterns and if extrapolation of our lower-bound estimates, obtained separately at each M, may, in due time, result in undesirable crossovers indicating implausible mortality patterns. Fortunately and conveniently for practical usage, our estimated trends in lower-bound X(M)'s do not crossover all the way until life expectancy at birth reaches beyond the level of 100 years. That implies that the estimated models should produce plausible patterns in the foreseeable future. One may also notice that our models indicate continuation of the compression of period mortality. The compression process looks slightly more regular in the estimates for low-mortality countries. In either case, there seems to be no compression at the highest mortality levels (the lower-bound lines at higher mortality levels move up in parallel). Given that mortality shift at these levels of death rate is only in its beginning stage, our fitted models may well be underestimating forthcoming acceleration in mortality shift at oldest old age, although one may also not discard the possibility that compression below mortality levels M=0.25 may truly be faster than at older age (higher M). In any case, it is worthwhile noting that our lower-bounds are rather conservative and impose only minimum possible mortality shifts consistent with the mortality change that was observed so far. Consequently, adjustments introduced to the projection model on the basis of the estimated lower-bound X(M)'s will be minimalistic and conservative.

In Table 1, we summarize the estimation results of lower bounds to X(M). The last column indicates the range (17) of life expectancy at birth within which the slope of the regression line remains within the plausible limits.

Table 1. Estimated Coefficients of the Quadratic Regression of Lower Bounds of Ages X(M) at Selected Levels of the Death Rate M on Standardized Life Expectancy at Birth: $X(M)=Intercept + a*(e_0-70) + b*(e_0-70)^2$.

Mortality level (M)	Intercept	a	b	R2	Residual stand. error	e_0 range, years
	All countries combined					
0.0125	52.1 (0.43)	1.032 (0.0299)	0.014305 (0.001752)	0.967	5.58	33.9-68.9
0.025	60.43 (0.31)	0.8171 (0.0296)	0.017178 (0.001486)	0.947	3.74	46.2-75.3
0.05	69.34 (0.23)	0.6189 (0.02)	0.013698 (0.000986)	0.96	2.84	47.4-83.9
0.1	77.34 (0.13)	0.4365 (0.0135)	0.009278 (0.000714)	0.961	2.29	46.5-100.4
0.2	84.66 (0.1)	0.3435 (0.0112)	0.007441 (0.000528)	0.959	2.39	46.9-114.1
0.25	87.15 (0.1)	0.3143 (0.0106)	0.006727 (0.000503)	0.959	2.38	46.6-121
0.3	89.3 (0.09)	0.3015 (0.0101)	0.006425 (0.000449)	0.961	1.85	46.5-124.4
0.35	91.31 (0.1)	0.2986 (0.009)	0.006432 (0.000427)	0.966	2.16	46.8-124.5
0.4	92.97 (0.1)	0.2952 (0.0087)	0.006647 (0.000466)	0.966	2.38	47.8-123
	Low-mortality countries					
0.0125	55.16 (0.29)	0.8468 (0.025)	0.006216 (0.00137)	0.974	4.56	1.9-82.3
0.025	62.33 (0.24)	0.6866 (0.0259)	0.011487 (0.00121)	0.958	2.65	40.1-83.6
0.05	69.94 (0.21)	0.5544 (0.0205)	0.01149 (0.00093)	0.955	2.22	45.9-89.4
0.1	77.64 (0.12)	0.434 (0.0118)	0.008408 (0.000564)	0.972	1.53	44.2-103.7
0.2	85.1 (0.1)	0.3523 (0.0078)	0.006849 (0.000396)	0.982	1.38	44.3-117.3
0.25	87.59 (0.09)	0.3245 (0.0077)	0.006315 (0.000362)	0.979	1.56	44.3-123.5
0.3	89.7 (0.08)	0.3088 (0.0075)	0.006018 (0.000343)	0.979	1.35	44.3-127.4
0.35	91.61 (0.08)	0.3011 (0.0069)	0.006026 (0.000346)	0.981	1.7	45-128
0.4	93.28 (0.08)	0.2973 (0.0071)	0.006085 (0.000396)	0.979	2.03	45.6-127.7

Notes: Numbers in the parenthesis indicate standard errors of the estimates; e_0 range refers to the range of applicability of the regression model.

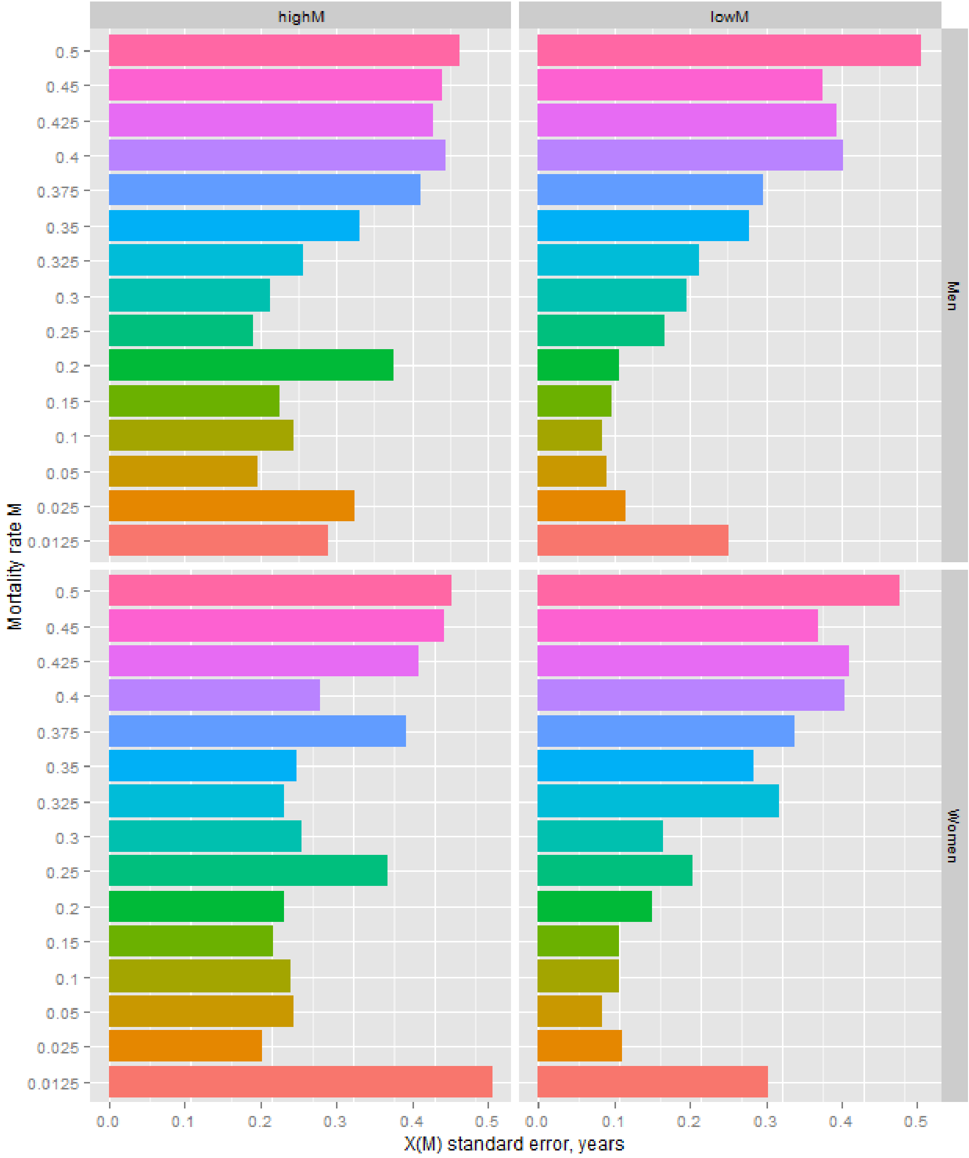

Figure 9. Standard Errors of Ages X(M) at Selected Levels of the Mortlaity Rate M.

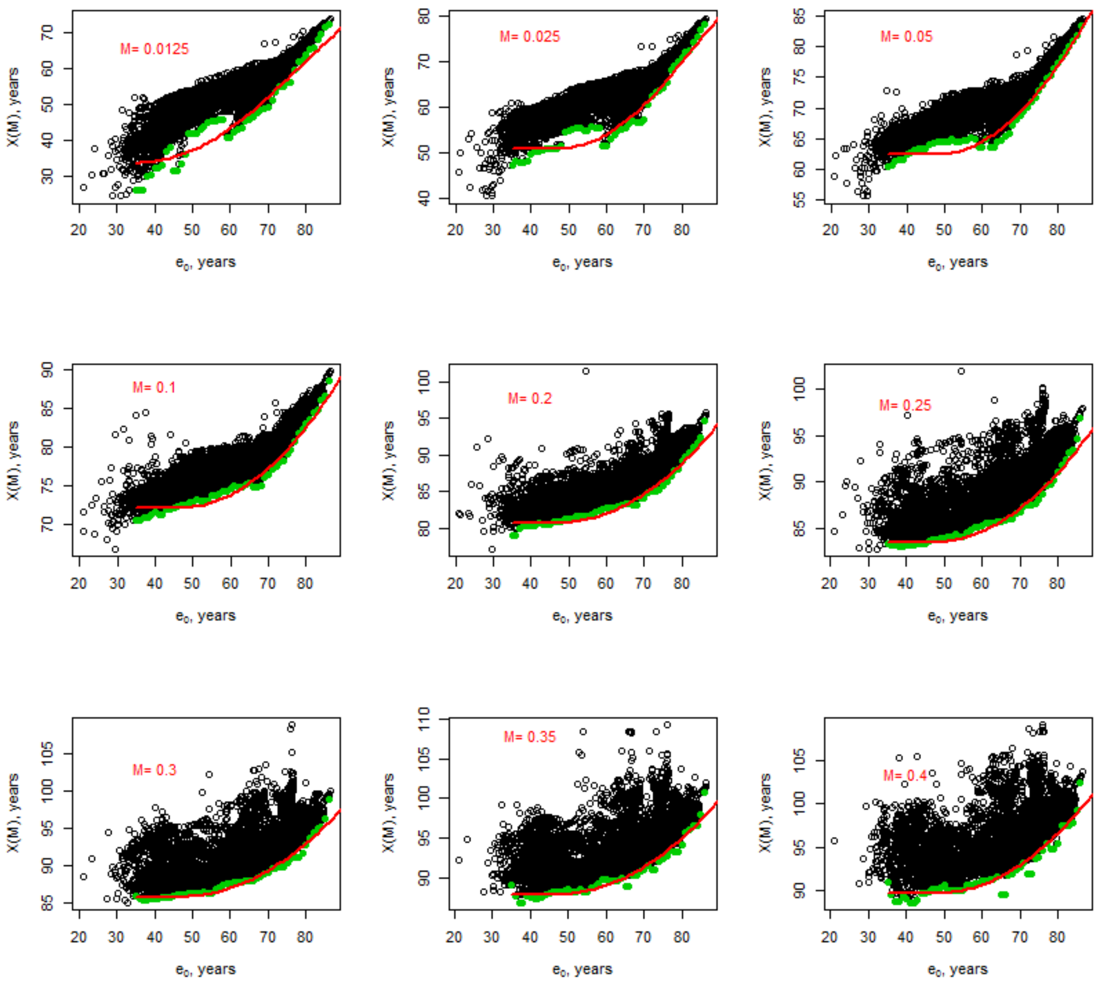

Figure 10a. Observed Values (Black Circles), Minimum Observed Values (Green Points) and Lower-Bound Estimates (Red Lines) of Ages X(M) at Selected Levels of the Death Rate M, Both Genders and all Countries put Together.

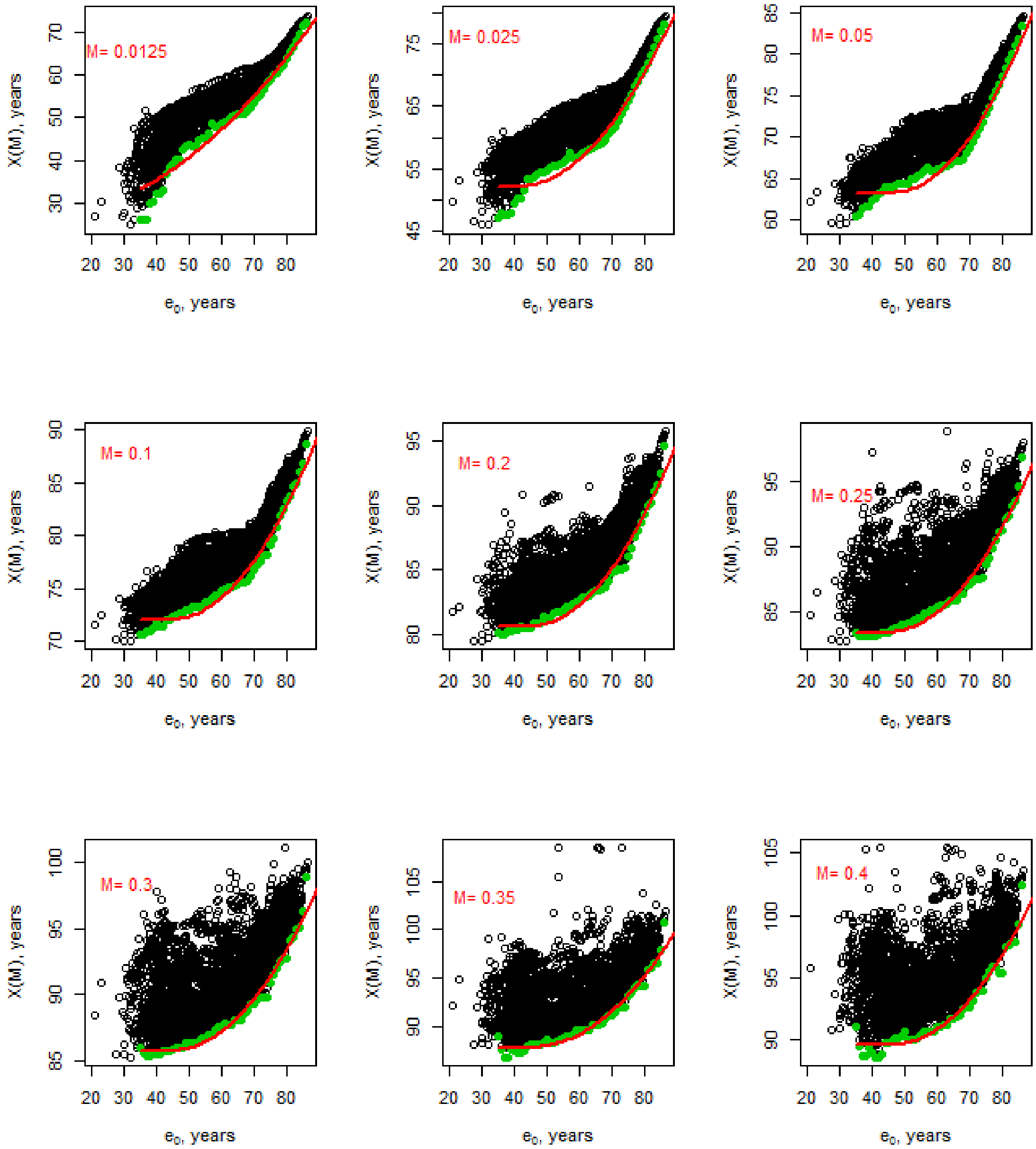

Figure 10b. Observed Values (Black Circles), Minimum Observed Values (Green Points) and Lower-Bound Estimates (Red Lines) of Ages X(M) at Selected Levels of the Death Rate M, Both Genders put Together; Low-Mortality Countries.

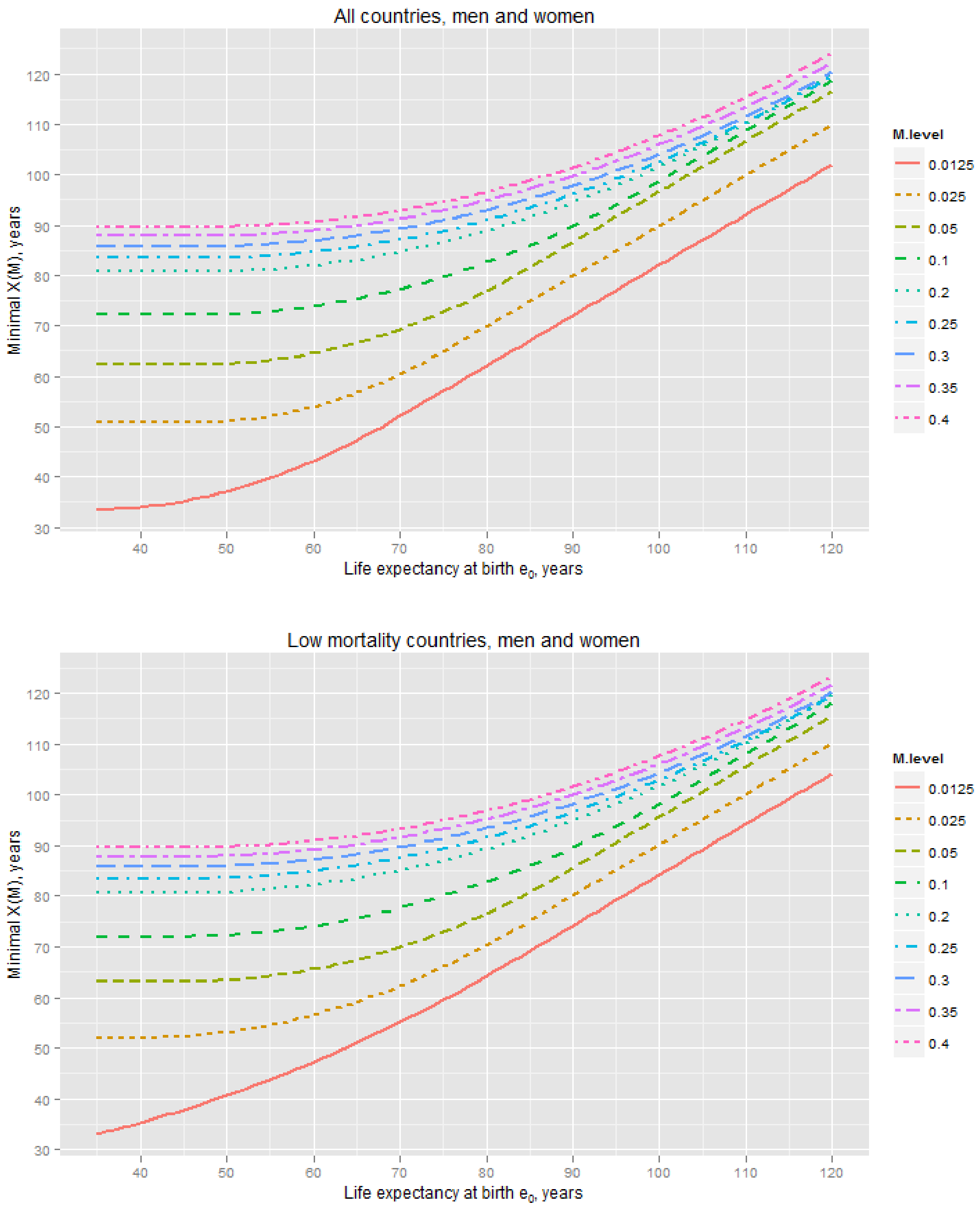

Figure 11. Lower-Bound Estimates, by Country-Type and Life Expectancy at Birth, of Ages X(M) at Selected Levels of the Death Rate M.

Our idea for the projection model adjustment is to start with adjusting the Brass model's standard for the shift in the age at mortality level M=0.3. This level is high enough to represent the oldest-old age; at the same time, it is low enough to enable reliable estimates of X(M) for most of the countries. In Figure 11, M=0.3 separates the part of the mortality where our model, perhaps rightly, predicts continuing mortality compression from the part above, where our model, perhaps wrongly due to lack of observed changes so far, predicts no major compression in the future. Given the projected value of the life expectancy at birth $\hat{e}_0$ (our assumed main input parameter to the model of age profile of mortality rates), we propose to adjust the standard age X(0.3), in a forward projection:

$$X^{**}(0.3) = \max\left(X^{base}(0.3), X^{\min}(0.3)\right), \tag{20}$$

where $X^{base}(0.3)$ is the mentioned age in the baseline year and $X^{\min}(0.3)$ is our estimated lower bound to X(0.3). For $X^{\min}$, the estimates from the previous section for the low mortality countries imply:

$$X^{\min}(0.3) \approx \begin{cases} 85.74, \; at\, e_0 < 44.34 \\ 89.70 + 0.3088(e_0 - 70) + 0.006018(e_0 - 70)^2, \; at\, 44.34 \le e_0 \le 127.43 \\ e_0 - .14, \; at\, e_0 > 127.43 \end{cases}. \tag{21}$$

5 Conclusion

Our formal inquiry into the Brass model at oldest old age shows that the model is incapable of reproducing the mortality decline through compression and shift at old age, a process that has already started in many countries and gains momentum at oldest old age. In this aspect the model is close to other traditional projections, such as the extrapolation of age-specific death rates. In order to be able to adjust these models properly, we offer a deeper view into the shift of mortality at old age, a process that is formalized here as change in ages X(M) at given mortality levels M. Our empirical results show these ages are shifting upwards across old ages and the speed of shift is accelerating at all M's, with a possible upper limit of shift of one year per single year of change in the life expectancy at birth. The models of lower-bund limits to X(M) proposed here are both informative in explicating the processes of shift and compression of mortality at old age, and useful in adjusting the mortality projections to the forthcoming changes in old-age mortality.

We intend to document details of implications for mortality projections of our proposed conservative model of mortality shift in a later report. For an early insight, one may consult Figures 12 and 13 that show selected forecast results for the conventional Brass model and for the model where the standard profile is adjusted for the mortality shift (21)[6]. In Figure 12, we add projections based on the adjusted Brass model to the UK mortality patterns discussed above (Figure 5). Although the adjusted method does not improve the forecast at younger ages, it predicts considerably better at middle and older age. In population projections, where currently low mortality at younger ages matters less, the presented pattern of improvements might be useful. Our example for the UK is indeed extreme in terms of the projection horizon. In a more realistic case of shorter projection horizons, the model (both the original and adjusted versions of it) should work better. This case is addressed in Figure 13, which shows mean absolute

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[6] Both models are essentially single-parametric: We fix the β parameter constant (=1) and change the α parameter so that the projected life expectancy e_0 coincides with the observed value. In the adjusted model, the standard mortality curve is transformed so that X(.3) follows our conservative shift model (21); there is no shift applied at age 30 and below; and shifts at ages in between 30 and X(M) are linearly interpolated from those at ages 30 and X(.3).

percentage errors averaged over recent (observed in 2000 or later) HMD female populations with standard profiles being those closest to 1970 for each of the HMD populations. One may notice clear improvements in the accuracy of projection of old age mortality in the Brass model adjusted for the mortality shift (21). Given the observed tendency towards accelerated shifts of old-age mortality, the contribution of our adjustment to the forecast accuracy may be even more important in the future than one may notice in Figure 13.

Our findings are useful in adjusting the standard of the Brass model for the mortality shift and allow improving the old-age mortality forecast while keeping the model simple with a single input parameter (the life expectancy at birth). This is a valuable feature in mortality projections that, in many cases, rely on formulating the future scenarios in terms of life expectancy at birth.

We have elaborated on the most conservative scenario of mortality shift consistent with the empirical patterns. One may naturally be interested in more optimistic scenarios as well (following either the average X(M) levels observed in low-mortality countries or even the upper limits to them). This may have substantial consequences for projecting mortality in currently higher-mortality countries. Yet, when it comes to predict further declines in old-age mortality, considerable compression of distributions of X(M) in lower-mortality countries at higher end of observed life expectancy values (Figures 7, 8) support the idea that more optimistic scenarios should not, perhaps, be too far away from the conservative scenario considered here.

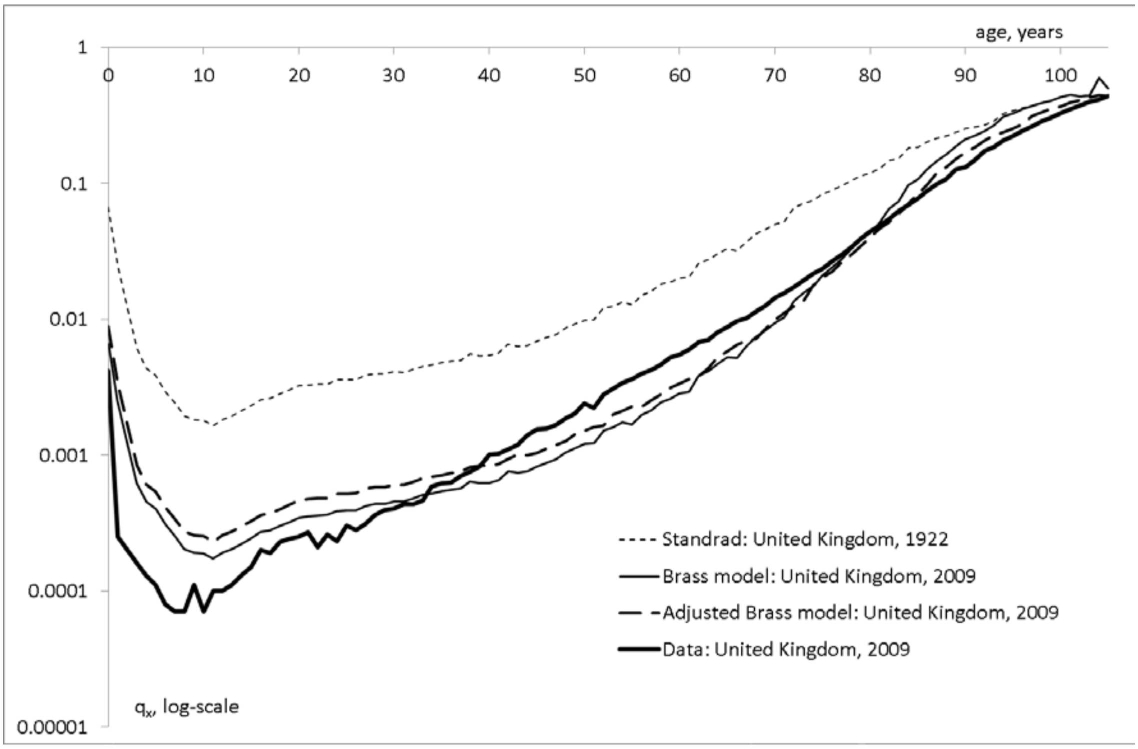

Figure 12. Age Profiles of Dying Probabilities (q(x)): The Baseline (1922), Observed (2009) and Fitted by the Conventional and Adjusted Brass Models with the Baseline Schedule used as the Standard; UK, Women.

Although our original motivation for the study comes from the necessity to improve the Brass relational mortality model as a forecast tool, the proposed scenario of old-age mortality shift may also be valuable in improving other mortality forecasting methods. The extrapolative methods, for example (Pollard 1987; Lee and Carter 1992; Benjamin and Soliman 1993; Ediev 2008a; Hyndman et al. 2013; Stoeldraijer et al. 2013) all tend to underestimate mortality decline at old age and, consequently, to produce gradually slowing improvements in life expectancy despite its persistent linear trend in the past (Oeppen and Vaupel 2002; White 2002). Our account of shift of old-age mortality is quite evident about the cause of such a pessimistic outcome: extrapolative models, lacking account of age shifts in mortality, are not capable of predicting emerging trends of decline or accelerations of such decline at oldest ages. Hence, they result in over-estimation of mortality at those ages, exaggeration of mortality compression prospects and overly pessimistic projection outcomes. In its recent extrapolations, the UN (2013) have improved the traditional extrapolations by applying an ad hoc adjustment of mortality declining rates at ages above 50 ('robust rotation', Li and Gerland 2011). Our explicit account of mortality shift and the conservative shift scenario may be another alternative in addressing the need to improve the forecast efficiency of common extrapolations.

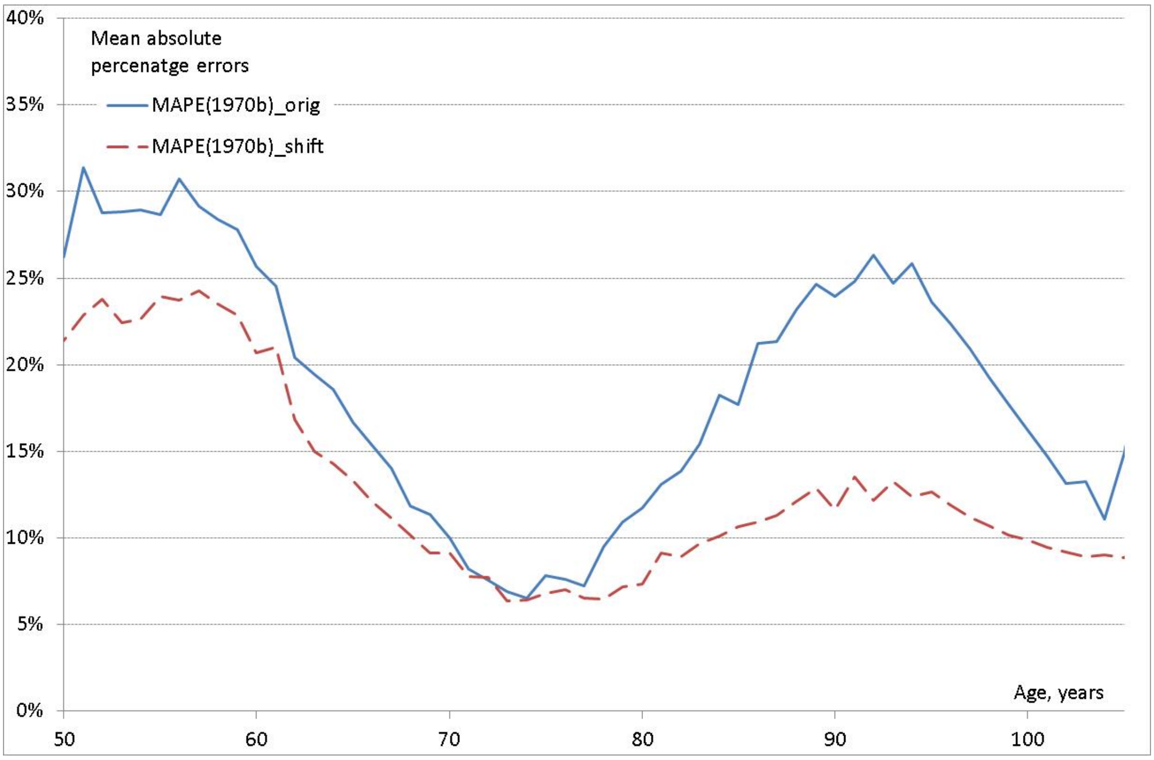

Figure 13. Age Profiles of Mean Absolute Percentage Errors of the Projected Death Rates Mx, Averaged over HMD Populations Observed after Year 2000, with Projections Obtained with the Conventional ("orig") or the Adjusted ("shift") Brass Models with the Country Schedules Closest to Year 1970 used as the Standard in both Models; Women.

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