# DYNAMIC GROWTH ANALYSIS AND PROJECTIONS FOR THE SILISTRA REGION IN BULGARIA: Construction of Models for Creating and Analyzing Development Scenarios

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## PREFACE

The purpose of regional economic models in the context of the Silistra study is to serve as a tool of empirical analysis and predictions for the material aspects and operations of the economy of the Silistra region. The field of study concentrates on economic activities both at the micro and macro scale. Therefore, much information is needed at a disaggregated level. In the following, a short review of the theoretically possible regional growth and equilibrium models is presented, which could be implemented when regional development is analyzed.

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## Neoclassical Model of Regional Allocation

The neoclassical growth theory assumes that entrepeneurial decisions about the use of labor and capital are guided by profit maximization. Given a certain stage of economic growth of an economy (country or region), the production factors must be combined in various quantities in order to achieve a maximum profit for the activity in question.

Furthermore, the neoclassical theory of regional growth is based on the openness of regions, which implies free mobility of production factors and resources between regions. The relative availability of production factors in various regions, however, is in general not equal, involving different relative prices. Mostly a relative scarcity of one factor does exist. If the economy is growing, this relative disequilibrium will induce forces tending towards an equilibrium at which the relative factor prices in all regions are equal. This situation will be attained only if there is free interregional mobility of labor and capital.

The formal way of stating a neoclassical hypothesis of regional growth process is the following:

-- similar regional production functions;
-- equal regional output;
-- negligible transportation costs;
-- given total amount of production factors;
-- capital is decomposable and can be shifted to, and constructed in, all regions.

This kind of regional growth model is typically a multiregional model. For each region, the production function is equal to:

$$Y = f(K,L) \quad ,$$

where Y is the regional output and where K and L are, respectively, equal to the amount of regional capital and labor. The following "classical" assumptions are made:

$$f'_K < 0 \quad ,$$

$$f'_L < 0 \quad ,$$

which implies positive marginal products of capital and labor and:

$$f''_{KK} < 0 \quad ,$$

$$f''_{LL} < 0 \quad ,$$

$$f''_{LK} = f''_{KL} > 0 \quad ,$$

where the first two conditions indicate decreasing marginal productivities and where the last condition indicates that an addition of capital to labor brings on rising labor productivity and vice versa. The foregoing conditions can be checked for a well-known Cobb-Douglas production function:

$$f = \gamma K^{\alpha} L^{\beta} \qquad \text{with} \qquad \alpha + \beta = 1 \quad .$$

Neoclassical theory states that in the equilibrium point each productive factor is rewarded according to the value of its marginal product, or:

$$f'_{K} p = r = \alpha \gamma K^{\alpha - 1} L^{\beta} \quad ,$$

and

$$f'_{L} p = w = \beta \gamma K^{\alpha} L^{\beta - 1} \quad , \qquad \frac{L}{K} = \frac{\beta}{\alpha} \frac{r}{w} \quad ,$$

where p represents the price of output, and r and w, respectively, the payments to capital and labor.

Assuming different regional economic structures one may evaluate the various interregional effects. For instance, if a region possesses a surplus of labor accompanied with low wages and a rather small stock of capital, in general the profitability of capital (in terms of labor) will be positive. The effects are, of course, strongly dependent on the hypotheses made a priori. A slight modification in any of the hypotheses may lead to quite different conclusions.

In principle a neoclassical growth model can be suited to analyzing interregional development of market economies. However, empirical application and verification of the model seems not to be very easy.

## The Lefeber Equilibrium Model of Allocation

The first general spatial equilibrium analysis was specified by Lefeber. This static analysis allows one to determine the spatial allocation of factors and the distribution of goods, and to investigate the pattern of industrial location. Lefeber's analysis can be considered as a generalization of the neoclassical equilibrium theory by taking account of interactions of spatially dispersed economic activities. The assumptions of the model are the following:

- existence of a fixed number of discrete location points, suitable for both production and consumption;
- each location point is endowed with a given assortment of productive factors; the latter may be transported to each other by making use of the transportation services;
- transportation services are generated by completely mobile production factors;
- production functions are linear and homogenous, and for the same good equal in all locations;
- no individual supplier can influence the price of goods or factors;
- prices of consumption goods are given or determined by a welfare function provided by a central planning board.

On the basis of these assumptions a spatial equilibrium analysis is performed in three parts: the allocation of productive factors, the distribution of final goods and the choice of production locations. The system as a whole needs an objective function, for instance defined as the maximization of the value of production. The a priori given prices of final commodities can be determined endogenously within the model by including social and individual welfare functions as a representation of the whole system.

Lefeber's static model is actually not a regional growth model and it does not describe or simulate the growth process of a region. Conversely, it describes the location of productive factors and the spatial distribution of final goods. This kind of model, like neoclassical equilibrium models in general, is difficult to implement empirically, except as in the linear programming versions proposed by Lefeber and others.

## Growth Models and the Center-Periphery Concept

One way of looking at regional growth originates from the concept of polarization, attraction and externalities. The general idea lying behind these concepts and the center-periphery notion was first introduced by Pother (1963), and revised by Hihorst (1972). In the general notions about center-peripheral phenomena the concept of a development axis plays a crucial role. Such a development axis can be considered as the spatial representation of a set of geographical points, which constitute the dominating lines in a communications network such that they can act as transmittors of growth effects. These development axes constitute essentially a spatial diffusion mechanism for development processes. The center-periphery notion is frequently used as an analytical tool for studying divergencies in growth rates between central regions and peripheral regions. Closely related to the center-periphery notion is the growth center concept. This theory assumes that selected geographical points can act as promoters of accelerated growth, both for the region itself and for its surroundings.

In addition to a growth center, one may distinguish an attraction center, which attracts the activities from adjacent regions to the center itself, so that the center undergoes a positive effect of these attraction forces, but the surroundings, a negative effect. In spite of the abundant quantity of literature in the field of growth center theory, only a few attempts have been made to integrate the growth center theory in a formal dynamic model.

The foregoing sections give a few of possible starting points for an explanation of differences in regional growth, especially in market economics. However, a more coherent approach seems to be desirable for regional planning and policy analysis.

## REGIONAL STATIC AND DYNAMIC I-O ANALYSIS: GENERAL CHARACTERISTICS

Regional analysis and projection for development planning of the region in concern requires much information at a disaggregated level. This disaggregation relates especially to the economic activities, a division according to sectors, industries, etc. Input-output analysis is a powerful instrument for analyzing intersectoral relationships.

Input-output analysis is an empirical approach to sectoral phenomena in a region or between several regions. It consists of a set of technical and definitional relationships between economic phenomena. The reason why the use of I-O analysis has undergone a rapid growth is the fact that this tool of applied economic analysis is based on production statistics, that can actually be collected. Furthermore, the various classifications made and the degree of disaggregation is rather flexible, since they can easily be adapted to the problem at hand and to the information available. Input-output analysis also provides a useful framework of a spatially dispersed production system. It is capable of a quantitative analysis and of a theoretical approach to optimization techniques.

Input-ouput tabulation has the advantage of presenting a surveyable and consistent table of relations between the various sectors of an economy both in relation to the productive structure as well as in relation to the cost structure.

The usefulness of I-O analysis is mainly based on a set of simple linear relationships between inputs and outputs remaining approximately constant during the period of analysis. This assumption of constant coefficient is on the other hand the most criticized point of I-O analysis.

In classical I-O analysis final demand (consumption, investment, export) is assumed to be exogenous. By introducing behavioral assumptions both for the investment and for the consumption sector I-O analysis can be extended in a more flexible way. A dynamic model can be created by introducing time lags in the relationships in the consumption and investment equations.

For the moment, many regional scientists agree that input-output analysis is an indispensable instrument for regional analysis and planning, provided this analysis is used in a flexible way and complemented with a set of additional methods. Finally, I-O analysis can be applied both to planned and market economies.

In the context of regional policy analysis, like the Silistra study, different kinds of impact analysis are needed. The input-output models are useful tools, when economic impacts are to be analyzed. It may refer to the introduction of new plants into the region, the growth of a new industry, or an agricultural-industrial complex, expansion of a dominant factor, a public investment project or an inflow central government spending in the region. In most of these cases economic impact analysis shows the relevance of I-O models as a tool of analysis for regional planning and policy. Of course the input-output model is only a tool, and is no substitute for a regional development strategy.

Originally the Input-Output model was not designed to be an optimization model, but it can quite easily be converted into such a form. Since planners are often faced with problems of scarcity and with how to economize on scarce resources in order to achieve their objectives most effectively, the usefulness of an approach which allows for the possibility of economic choice is obvious. A programming model recognizes that there are many feasible production possibilities and enables the analyst to choose one that either maximizes a desired benefit or minimizes losses. It is also a flexible approach, since there are many possible general or specific, which could be optimized. The objective function could refer to maximization of gross regional product, minimization of investment in particular sectors, subject to satisfying future demands, minimization of labor costs, etc. The structure of the objective function depends on the uses of the model, i.e., it is for economic forecasting, resource allocation in space and an optimal investment program or for analysis of resource utilization and investment requirements in particular industries.

A programming approach offers the considerable advantage of being able to feed policy goals and objectives directly into the analysis.

An Input-Output table is formally presented in the following chapter, with some versions of I-O models.

## The Input-Output Table

An input-output table fulfills two separate functions. First it is a descriptive framework for showing the relationship between industries and sectors and between inputs and outputs.

Second, given certain economic assumptions about the nature of production functions it is an analytical tool for measuring the impact of autonomous disturbances on an economy's output and income.

The distinction between an input-output account (table) and an operational input-output model is an important one. The former is an account framework, the latter an analytical tool.

Figure 1 represents an input-output transactions table for one region (or nation).

Row i in the table shows the sales of sector (industry) i to all other sectors (industries, households, investment, government spending and exports). Thus in an n sector table:

$$X_i = \sum_{j=1}^{n} X_{ij} + \underbrace{(C_i + I_i + G_i + E_i)}_{Y_i} \quad ; \tag{1}$$

$(i = 1,\ldots,n)$ ,

gross output = intermediate demand + final demand .

Conversely, column j shows the purchases of industry j from all other industries (intermediate inputs), from primary inputs (labor, capital, etc.) which are value added entries. Summing down column j yields:

$$X_j = \sum_{i=1}^{n} X_{ij} + L_j + V_j + M_j \quad ; \qquad (j = 1,\ldots,n) \quad . \tag{2}$$

Then,

$$\sum_{j=1}^{n} X_j = \sum_{i=1}^{n} X_i = \text{total gross output of the economy} \quad . \tag{3}$$

| From \ to | | Purchasing sectors 1, 2, ... ,j, ... ,n | Local final demand: Households | Investment | Government | Exports | Total gross output |
|---|---|---|---|---|---|---|---|
| Producing sectors | 1 | $X_{11}$ $X_{12}$ ... $X_{1j}$ ... $X_{1n}$ | $C_1$ | $I_1$ | $G_1$ | $E_1$ | $X_1$ |
| | 2 | $X_{21}$ $X_{22}$ ... $X_{2j}$ ... $X_{2n}$ | $C_2$ | $I_2$ | $G_2$ | $E_2$ | $X_2$ |
| | ⋮ | ⋮ ⋮ ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | i | $X_{i1}$ ... $X_{ij}$ ... $X_{in}$ | $C_i$ | $I_i$ | $G_i$ | $E_i$ | $X_i$ |
| | ⋮ | ⋮ ⋮ ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | n | $X_{n1}$ ... $X_{nj}$ ... $X_{nn}$ | $C_n$ | $I_n$ | $G_n$ | $E_n$ | $X_n$ |
| Labor | | $L_1$ ... $L_j$ ... $L_n$ | $L_C$ | $L_J$ | $L_G$ | $L_E$ | L |
| Other Primary Factors | | $V_1$ ... $V_j$ ... $V_n$ | $V_C$ | $V_I$ | $V_G$ | $V_E$ | V |
| Imports | | $M_1$ ... $M_j$ ... $M_n$ | $M_C$ | $M_I$ | $M_G$ | – | M |
| Total Gross Outlay | | $X_1$ ... $X_j$ ... $X_n$ | C | I | G | E | X |

Figure 1: Simplified input-output table.

Simple Model

Input-output accounts of Figure 1 can be transformed into an analytical model if certain assumptions are made concerning the sectoral production functions.

If the amount of industry 1's output purchased by each of the purchasing industries is a constant function of the latter's output we may write

$$X_1 - a_{ii}X_1 - a_{12}X_2 - \ldots - a_{in}X_n = Y_1 \quad , \qquad (4)$$

where

$$a_{11} = \frac{X_{11}}{X_1} \;; \qquad a_{12} = \frac{X_{12}}{X_2} \;; \qquad \ldots \qquad a_{1n} = \frac{X_{1n}}{X_n} \quad ,$$

are input coefficients. In an n-sector model they represent the direct requirements of input of any sector i per unit of output of any other purchasing sector j.

The crucial assumption for equation (4) to hold is that the money value of goods and services delivered by an industry i to other production sectors is a linear, homogeneous function of the output level of the purchasing sector j. If the linear input coefficients remain constant over time, they link final demand to gross output. So input-output analysis describes the interaction of the elements of an economic system: final demands, the input requirements of each industry, and their gross outputs. The main analytical purpose of open static input-output model is to determine the effects of specified changes in final demand upon gross output, given the input coefficient matrix.

To capture all the direct and indirect effects we can express the model in matrix form:

$$x = Ax = y \quad , \qquad (5)$$

where x and y are column vectors of gross output and final demand, and A is an $n \times n$ matrix of direct input coefficients, $a_{ij}$. If I is the identity matrix we can rewrite (5) as:

$$(I = A)x = y \quad . \tag{6}$$

Under the condition that (I - A) has an inverse we may use the inverse matrix to express gross output as a function of (exogenous) final demand:

$$x = (I - A)^{-1}y \quad . \tag{7}$$

Let $(I - A)^{-1} = Q$. Then

$$x = Qy \quad ; \tag{8}$$

The coefficients $q_{ij}$ (i, j = 1,...,n) of the matrix Q represent the direct and indirect requirements of sector i per unit of final demand for the output of sector j.

We can multiply the inverse matrix Q by any size and composition of final demand in order to obtain the level of gross output for each industry. So it is possible to simulate changes in impact on the economy of exogenous changes in final demand (investments, government, exports).

## DYNAMIC MODELS

### Closed Dynamic Models

If it is desired to use an input-output framework for long-run regional forecasting, it is necessary to emply a dynamic model. For short-run projections, it is sometimes permissible to use the standard static model by deriving forecasts for regional gross outputs by using the original inverse matrix and by projecting changes in final demand. For medium-term forecasting the interindustry matrix could be adjusted by allowing for changes in the input coefficients and possibly for shifts in regional trade coefficients. A truly dynamic model must allow for structural relations between stocks and flows and take explicit account of the fact that substantial increases in output will create additional capacity requirements so that projected changes in final demand will not only require more intermediate goods but also investment goods from all appropriate sectors in the economy.

The difference between open, static and closed dynamic models is quite profound. The dynamic models resemble the static ones in that they use all the data in the same form. It also requires additional data for the variables endogenous in its dynamic version only.

The closed dynamic model can be written in the form:

$$x = Ax + B\dot{x} \quad , \tag{9}$$

where

$x$ = vector of total outputs;

$A$ = matrix of flow coefficients including labor and consumption coefficients;

$B$ = matrix of stock coefficients (investments);

$\dot{x} = \frac{dx}{dt} = \{\frac{dx_i}{dt}\}$ .

Formally we can define the stock coefficient matrix B to correspond to the flow coefficient matrix A. While $a_{ij}$ stands for the amount of product i used up to produce one unit of product j, let $b_{ij}$ stand for the amount of product i tied u' in the same process.

Product flows, represented by the matrix A, and product stocks represented by the matrix B, do not come into being independently. Flows and stocks are two aspects of the same economic transaction. The phenomenon observed is that some buyer, sector j for example, buys a certain amount of product from sector i. This exchange is motion, because the product moves from one sector to the other. It is described by the flow coefficient $a_{ij}$. But the same transaction also changes the state of the product. It will stay in the new sector until its use-value is used up entirely in the production process, until its value is transferred to the product of the process. The ratio of the product i requires as stock per unit of output per year of product j is the stock coefficient $b_{ij}$ (see Brody, 1970:36-37).

So the notion of turnover time connects the notions of flow and stock, motion and state, it establishes a mathematical relation between the matrixes A and B. If, for example, the amount $a_{ij}$ is tied up in the sector j for a given turnover time $t_{ij}$, then it is possible to express the stock coefficient $b_{ij}$ by:

$$\{b_{ij}\} = \{a_{ij}t_{ij}\} \quad .$$

The essential assumptions of the model (9) are:

-- Output can be increased only by investing.
-- New investment is made according to the same coefficient as the old technology. There are no technological improvements.
-- Every branch of production, every sector, is augmented by the same factor, the universal growth rate, $\dot{x} = \lambda x$.

Using the notation $(1 - A)^{-1} = Q$ we transform equation (2) to the form $(1/\lambda I - QB)X = 0$ (equation (2) is premultiplied by the factor $1/\lambda \cdot Q$). This now is an eigenequation for the matrix QB. This matrix is strictly positive as Q is positive and B is non-negative and irreducible. A stationary solution of the model, yielding an average rate of profit and securing a uniform growth rate, can be interpreted as an equilibrium where supply, x, is equal to demand for flows and increments to stock. The stationary state of the economy is thus given by the positive eigenvector x, belonging to the maximal, positive eigenvalue of the matrix QB. $\lambda$ is the reciprocal of the maximal eigenvalue. No greater growth rates than $\lambda$ are accompanied by economically meaningful output proportions. Therefore if the economy deviates from the stationary path towards an apparently faster one it cannot be followed for long without endangering future growth. The stationary growth path can be momentarily the slowest, but it secures the fastest growth in the long run (Prody, 1970:114).

Here $\lambda$ is the uniform rate of growth in every sector. In practice it will however not be optimal to have the same growth rate in every branch. Branches whose products are substituted for others should grow faster than others. Those that are becoming obsolete should grow at less than average rates. The constant technology assumption of the model is a problem of long range planning. Therefore, coefficient projection, however difficult, is central to the planning process.

## Closed or Open Models?

The choice between closed and open models is quite important in the planning context. The logic of the open system makes exogenous factors decisive. As independent variables they are the objectives of the economic process. In practice planning work--as opposed to theory--there are some questions of analysis and (regional) planning that can be handled more readily by the open model. For example, the impacts of governmental regional policy on the regional economic process can be better appraised by the open model.

In the following some versions of open dynamic models are introduced.

## Open Dynamic Models

When a region has a relatively high proportion of trade with other regions or with other nations, trade may be one of the major problems of input-output analysis. One approach to this problem is to handle "foreign" trade as any other sector in the closed system, exports being its output, resulting from import inputs. Even the subdivision of various foreign markets is possible.

A more practical approach to trade may be to open the model. Then the export is the sector of the final demand of the open dynamic model. When we are interested in the outcome of some decision we might use an open model because it is suitable for analyzing the impact of the policy decison, for example on the structure of foreign trade.

Kossov (1975, 185) gives an explicit presentation of an open, dynamic input-output planning model for Soviet economy:

$$AX_t + B\Delta M_t + Y_t = X_t \quad ,$$

$$F_t X_t - C_t \Delta M_t = \Phi_t \quad ,$$

where B is the coefficient matrix of investments,

F, the coefficient matrix of funds intensity,

C, the coefficient matrix for omission of activated production capacity,

$\Delta M$, the growth of production capacity, and

$\Phi_t$, the funds for consumption available at the beginning of planning period and which are used during it.

The first equation of this model is the balance of outputs and inputs of products, and the second one the balance of basic funds. X and $\Delta M$ can be solved from the form

$$X = (I - A)^{-1} B\Delta M + (I - A)^{-1} Y$$

$$F(I - A)^{-1} B\Delta M + F(I - A)^{-1} Y - (C\Delta M) = \Phi$$

$$\Rightarrow \Delta M = [F(I - A)^{-1} B - C]^{-1} [\Phi - F(I - A)^{-1} Y]$$

In this model Y is the exogneous final demand.

More simple is Leontief's formulation of open dynamic input-output models:

$$X_t = AX_t + B(X_t - X_{t-1}) + Y_t$$

$$\Rightarrow X_t = (I - A)^{-1}Y_t + (I - A)^{-1}B\Delta K \quad ,$$

where $\Delta K$ represents the difference between required capacity in year t and actual capacity in t-1 if we assume a direct correspondence between output and capacity.

The above equation represents a set of n linear difference equations from which the system can be solved. In regional models it is necessary to take notice of the fact that regional economies are very open. This means that investments may take place with the aid of imported capital goods. In other words,

$$I_e = (I - A)^{-1}B\Delta K + M_c\Delta K \quad ,$$

where

$I_e$ = total net investment, and
$M_c$ = diagonal matrix of capital import coefficients.

The Leontief type of open model assumes that investments can be put in to immediate use to increase production capacity. The model of Kossov makes a difference between investment, activation and putting the new capacity in to operation.

Data difficulties may arise when attempts are made to implement a dynamic I-O model at the regional level, i.e., data scarcities with respect to capital stock measures; how to separate net and replacement investment etc.

The dynamic model has, however, the advantage compared with static models, not only that it is more satisfying theoretically, but also that there is a consistency check on investment available from the identity between capital sales and capital purchases, whereas the static model merely includes sales on capital account.

## SPECIFIC ECONOMIC CHARACTERISTICS OF THE SILISTRA REGION

The Silistra region is located in the north-west border part of Bulgaria. It comprises a territory of 2870 $km^2$, 2.6% of the total area of Bulgaria. The population of Silistra consists of 176,000 inhabitants, which is 2% of the country's population.

The land utilization in Silistra is characterized by a high percentage of arable land, and by an insignificant acreage of forestry of no economic value. The area enjoys a moderate, continental climate with a marked maximum precipitation in the summer period but lacks surface water.

The state of the economy of Silistra region is determined by the location of the region and the targets set within the national economic system. Agriculture is the dominant industry in the Silistra region. It is well developed, due to appropriate agroclimate conditions. The specialization is based on cereals and grain fodder crops.

The agriculture is organized in agricultural-industrial complexes, the boundaries of which conincide with the region's boundaries and include 15 cooperative farms with a close production cycle. The main goal of development of agriculture in the future is to increase livestock production, specialized cultivation, and modernized livestock enterprises.

Manufacturing is comparatively underdeveloped in the Silistra region. The main specialized branches of manufacturing industry are machine-building, metal-producing and food processing industries. The territorial distribution of productive activities is unstable. The largest manufacturing center is the town of Silistra where 74% of the gross industrial production, and 69% of the labor resources are concentrated. The towns of Tutracan, Dulovo, Alfatar, and the village of Glavanissa, also have some industrial significance. It is planned in the future to set up four territorial productive complexes in the above-mentioned territorial centers.

The transportation system of the Silistra region is comprised of truck and automobile, railway, water, and air transport. The most developed is truck and automobile transportation. Important structural changes in the economic and technical aspect are expected in the transportation system with the development of water transport.

The human settlement system of the region is defined mostly by the agricultural character of the region. The number of settlements is 116 (4 towns and 112 villages and 47% rural population). The further improvement of the regional human settlement system has to be founded on the vicinity of the Danube river and on agricultural development. Accounting for the territorial production allocation, three comparatively stable centers with regard to commuting of the labor force are available: Silistra, Tutracan, and Dulovo.

The Bulgarian government has approved six human settlement systems: Silistra, Tutracan, Dulovo, Glavanissa, Sittovo, and Srediste.

Taking into account the existing tendencies in regional development and the government's decisions, the following three basic targets concerning the socio-economic mechanism improvement have been set up:

1st: leading economic functions have to be developed and specialized

2nd: the territorial distribution of the activities has to be organized in a suitable way

3rd: the optimal living and working conditions for the population have to be created on the basis of: reducing and eliminating the deficit in labor man-power, optimization of migration processes, and satisfaction of the social needs of the population.

## SNAPSHOT INPUT-OUTPUT TYPE EQUILIBRIUM MODEL FOR CREATING AND ANALYZING SCENARIOS FOR THE SILISTRA REGION

As was noted before regional input-output model is a relevant tool of analyzing and simulating effects of regional policy, expansion of dominant economic functions or the creation of new industry. The so-called "snapshot model" is one version of I-O models, which suits well for analyzing different regional development scenarios and is quite easy to "handle" empirically. The structure of the model is the following:

$$X_t = AX_t + B\Delta K_t + C_t(X_{t-1}) + E_t \quad ; \quad \text{where}$$

$X_t$ = regional gross output vector in period t
$A$ = flow coefficient matrix
$B$ = stock coefficient matrix
$\Delta K_t$ = difference between required capacity in period t and actual capacity in period t-1
$C_t$ = consumption vector of households in period t
$E_t$ = net export vector in period t

$E_t$ is the exogenous variable of the model. The investment function is of an usual acceleration type:

$$\Delta K_{ji,t} = I_{ji,t} = b_{ji}(X_{i,t-1} - X_{i,t-2}) \quad , \quad \text{where}$$

$I_{it}$ = the investment requirements of sector i on period t. The coefficient $b_i$ is a marginal capital coefficient. The explicit form of the consumption function of the model is the following:

$$C_{it} = \alpha_0 + \alpha_1 \underbrace{w_{t-1} \cdot X_{L,t-1}}_{\text{labor income}} + \beta B_{t-1} \quad ,$$

where

$$X_L = \sum_i x_{li} = \sum_i \alpha_{li} x_i = \text{total demand for labor} \quad ,$$

and

$$w = \sum_i (w_i \cdot \alpha_{Li} \cdot X_i) = \text{total sum of wages} \quad .$$

Then the consumption function can be written in the form:

$$C_{it} = \alpha_0 + \alpha_i [\sum_j w_{j,t-1} \alpha_{Lj} x_{j,t-1}] + \beta B_{t-1} \quad ,$$

where

$\alpha_0$ = constant,

$\beta$ = constant,

B = constant ( is the "non-earned" income factor or transfers to households)

$\alpha_i$ = sectoral propensity to consume, and

$\alpha_{Lj}$ = a factor which transforms the demand for labor from sectoral outputs.

In this model sectoral demand of consumption goods is a function of labor inputs used in production. The consumption function is a dynamic element of the model and so it is possible to simulate with the model the development of the economy periodically.

## Sectoral Division of the Model

The number of sectors in an input-output table and model will be determined by factors as costs and resources, research objectives, economic structure of the region in question, and data availability. The main criterion should, however, be the homogenity of sectors, in the sense of sectors having similar purchases and sales patterns. In the case of Silistra region the agriculture has a leading economic significance. Therefore, its role in the model is most important.

The preliminary division of sectors is as follows:

| | | |
|---|---|---|
| Animal husbandry : | 1. | meat |
| | 2. | milk |
| | 3. | wool |
| | 4. | eggs |
| Grain production : | 5. | grain |
| | 6. | seed |
| | 7. | forrage |
| | 8. | fruits and vegetables |
| Industrial crops : | 9. | tobacco |
| | 10. | bean |
| | 11. | sunflower |
| Manufacturing industry: | 12. | meat products |
| | 13. | milk products |
| | 14. | leather processing |

| | | |
|---|---|---|
| | | 15. other food processing ind. |
| | | 16. textiles |
| | | 17. other |
| Industries | : | 18. machine and metal producing |
| | | 19. wood processing |
| | | 20. clothing and footwear |
| | | 21. fertilizers and chemicals |
| | | 22. forestry |
| | | 23. construction |
| | | 24. energy |
| | | 25. water |
| | | 26. environmental protection |
| | | 27. trade |
| | | 28. communications |
| | | 29. social service |
| | | 30. industrial service |
| Households | : | 31. low education |
| | | 32. high education |
| Import-Export | : | 33. other regions in Bulgaria |
| | | 34. comecon (SEV) |
| | | 35. other countries |