A MODEL OF THE EQUILIBRIUM BETWEEN DIFFERENT LEVELS OF TREATMENT IN THE HEALTH CARE SYSTEM: PILOT VERSION

David J. Hughes

March 1979
WP-79-15

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

PREFACE

The aim of the IIASA Health Care Systems Modelling Task is to build a family of submodels for the National Health Care System (HCS), as an aid to Health Service planners. The modelling work is proceeding along the lines proposed in earlier papers. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

This paper introduces a submodel of how different levels of the HCS achieve equilibrium by balancing the flows of patients between them. The pilot version described here can be used to study the consequences of different resource allocations for patients and physicians. It is complementary to the other IIASA HCS resource allocation submodel DRAM.

Recent related publications on resource allocation by the IIASA Health Care Systems Modelling Task are listed at the end of this paper.

Evgenii N. Shigan
Leader
Health Care Systems Task

ABSTRACT

Health Care Systems manage to balance competing demands for care with limited supplies of resources. They achieve an equilibrium. This paper describes a resource allocation model that represents this equilibrium as the equalising of pressures between different levels of treatment. A pilot version of the model is formulated, solved, and programmed; and an illustrative example is given. Work towards a more sophisticated model is proposed.

CONTENTS

A MODEL OF THE EQUILIBRIUM BETWEEN DIFFERENT LEVELS OF TREATMENT IN THE HEALTH CARE SYSTEM: PILOT VERSION

David J. Hughes

INTRODUCTION

A feature of health care systems (HCS) which is observed almost everywhere is that patients with different needs are referred to physicians in different specialisations. For example, a patient who needs surgery is not treated by a general practitioner but is referred by the GP to a hospital surgeon. Furthermore, the decision about whether or not to refer the patient is made not by health service planners or managers, but by medical staff acting on their own clinical judgement. This paper formulates a model of how equilibrium is established between the levels of treatment under certain assumptions about aggregate referral behaviour. Such a model is useful to planners who must set future resource levels. It helps to answer questions like "Where will patients be referred if general practitioners, outpatient clinics, inpatient hospitals, etc. are available at prescribed levels?"

This model is a new member of the family of submodels of the HCS being developed by a group of scientists from different countries working at IIASA. Like the existing models, it is designed for application with collaborating national research centres as an aid to health service planning. Figure 1 shows the five groups of models which have been developed so far, and which are described in more detail in a recent status report (Shigan et al, 1979).

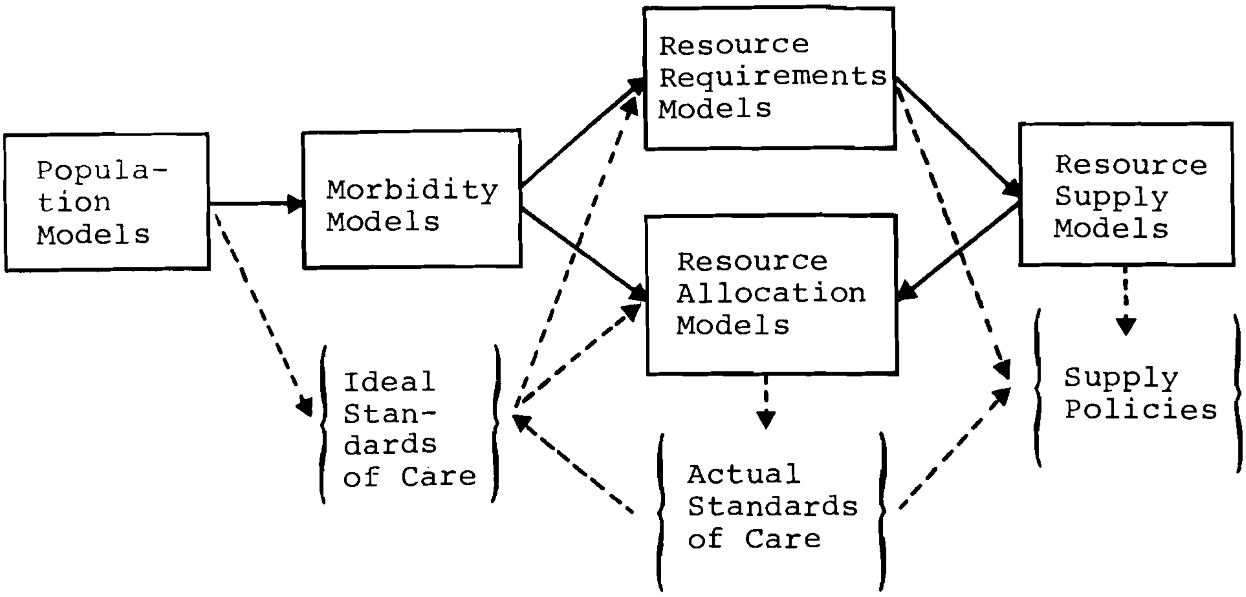

Figure 1. Family of HCS submodels constructed at IIASA.

The model described below belongs in the group of resource allocation models. Although this paper describes only a pilot version of the model, it is helpful to give it a name, and we have called it METL--model of the equilibrium between treatment levels.

Numerous models for resource allocation in the HCS were reviewed by Fleissner and Klementiev (1977) and Gibbs (1977). Furthermore, one such model, DRAM, has already been established at IIASA by Gibbs (1978) and Hughes (1978a). Why then do we need another one? The first reason is that the model DRAM is designed primarily for applications where different modes of treatment share the same resources. Although DRAM recognises that there may be substitution between (for example) inpatient and outpatient care, it concentrates on the shared resources, and it does not explicitly recognise that substitution may be influenced by effects like bottlenecks in outpatient clinics, or the blocking of acute inpatient beds by elderly patients who cannot be discharged. Yet such phenomena are observed in many countries and are a common concern. A second reason for new modelling is to improve upon the simulation flow models of the HCS which already exist. Many authors have defined states of sickness, patients, or facilities, so as to be able to simulate the transitions between these states according to historic rates. Clayden (1977), for example, has developed a sophisticated model of this type in the UK. However, such models share the disadvantage that historic transition rates may change in unforeseen ways. METL recognises that transition rates are set through equilibrium between different parts of the HCS.

A third reason for considering a new model is that the procedure of patient referral is common to many countries, and is therefore particularly appropriate for modelling at an international institute. The referral procedure is fixed in the structure of the model, but the different model parameters can reflect the different important influences which affect referral in different countries. In the same way as DRAM was based initially on the econometric analysis of Feldstein (1967), METL is related to the studies in the Netherlands by Rutten (1978). Following his analysis, we expect referral flows to be influenced by

-- demands for treatment,
-- supplies of resources,
-- preferences of physicians,
-- external controls applied by physician colleagues, hospital managers, sickness funds, health ministries, etc.,

and the first three of these influences are represented in the model described below. It might be possible to include the fourth group of influences in a country-specific model. For us, however, it is more important to model the concept that the HCS achieves equilibrium by reacting to change in a certain way. Still less is our intention to prescribe optimal behaviour for the HCS. METL is a simulation model designed to help answer "what-if" questions facing decisionmakers at national and regional levels of health care management.

MODEL FORMULATION

In order to develop a model of the equilibrium between treatment at different levels, we must evidently define the index

$$i = \text{Treatment Level} \quad , \qquad i = 1,2,\ldots,N \quad .$$

Three such levels could be, for example, treatment in a general practitioner's clinic, at an outpatient department, or in an inpatient hospital. Figure 2 depicts these levels as facilities arranged so that admission to a "higher" facility depends upon prior admission to a "lower" facility. Equally, however, we could consider the levels to represent sequential modes of treatment, as for example: prevention, hospital, and community care. The principal model variables are

x_i = the number of individuals who are treated at level i, per head of population per year, and

y_i = the amount of resources received by each individual treated at the i-th level.

It is these variables that the model seeks to predict, within certain resource constraints, and under certain assumptions about referral behaviour.

The model equations fall into three main groups: the equations which define the resource constraints, the equations which express the equilibrium between levels, and the equations which express the equilibrium across levels. The resource constraints are rather simple. We define

R_i = the total resources at level i available to the HCS, per year, per head of population

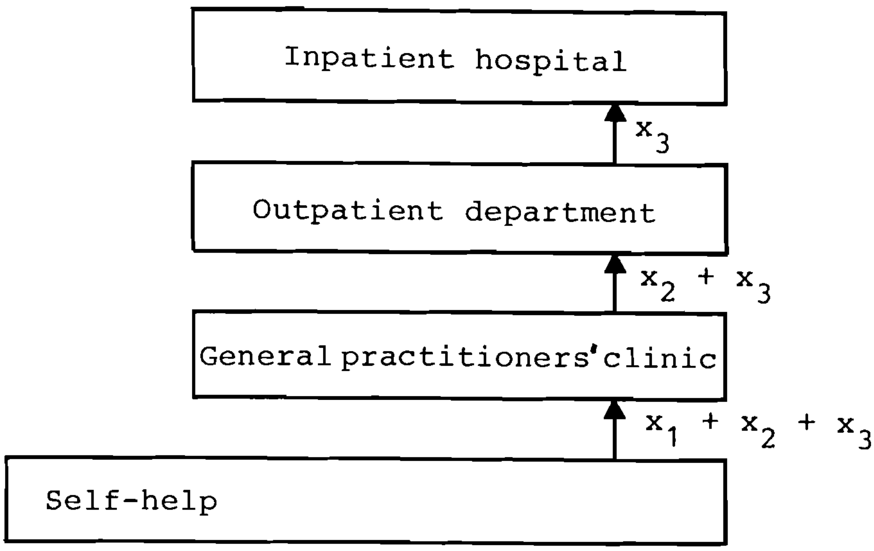

Figure 2. Patient flows to three levels of treatment.

and assume that all the available resources are allocated for treatment

$$x_i y_i = R_i \qquad \forall\ i \quad . \tag{1}$$

This statement implies that the demand for HCS resources always exceeds the supply, a finding widely observed and used explicitly in METL.

Next we must specify how equilibrium is attained between treatment levels. The physician at each level is subject to two opposing tendencies. On the one hand, he wants to use more resources for each patient in order to improve the quality of care. On the other hand, the available resources are limited, and not too many patients can be referred to other levels of treatment because the resources there are also limited. We assume that these two tendencies balance when the upward pressure p_{i-1} to refer patients from level i - 1 to level i equals the downward pressure q_i not to refer patients to level i from level i - 1. Specifically

$$p_{i-1} = q_i \quad . \tag{2}$$

One advantage of an equilibrium model such as this is that we do not need to specify whether any particular individual will be referred up, down, or out of the system altogether. On the other hand, we assume that however equilibrium is attained, the characteristics of individuals leaving the system are similar to those entering it. More simply, more people who are admitted are cured.

The final group of equations express the equilibrium across treatment levels. Rather than equalising the pressures, which would be a rather strong assumption, we assume that the equilibrium is stable to small changes in the numbers of individuals treated, by setting

$$\frac{\partial p_i}{\partial x_i} = \frac{\partial q_i}{\partial x_i} \qquad i = 1,2,\ldots,N-1 \quad . \tag{3}$$

This implies that if a small change is made in the number of individuals at a certain level of treatment, the change in the upward referral pressure will be equal to the change in the downward pressure opposing referrals. One individual discharged from the system is just as likely to be replaced from the level above as from the level below. There are just N - 1 of these equations because there are no referrals beyond the Nth level of treatment.

There are various ways in which the functions p and q might be defined. Here we define the upward pressures as

$$p_i = Sx_i^2 \qquad i = 1,2,...,N-1 \tag{4}$$

$$p_o = S\chi^2 \tag{5}$$

where

$$\chi = (X - f \sum_{i=1}^{N} x_i) \tag{6}$$

and where

S = the seriousness of individuals requiring referral,

X = the total number of individuals needing treatment, per head of population, per year,

f = a fraction which corrects the total number of individuals apparently treated, to account for those treated at two or more different levels.

With this definition, we assume that the desire to refer patients to a higher level depends only upon the number and characteristics of the patients, and is independent of other factors (such as the physician's income). Figure 3 shows how the pressure to refer patients is proportional to the square of the number of individuals. The pressure to give any treatment at all, p_o, naturally depends upon how many of those needing treatment, X, are already receiving treatment in the system.

There are also various ways in which the functions q_i might be defined. Here we let

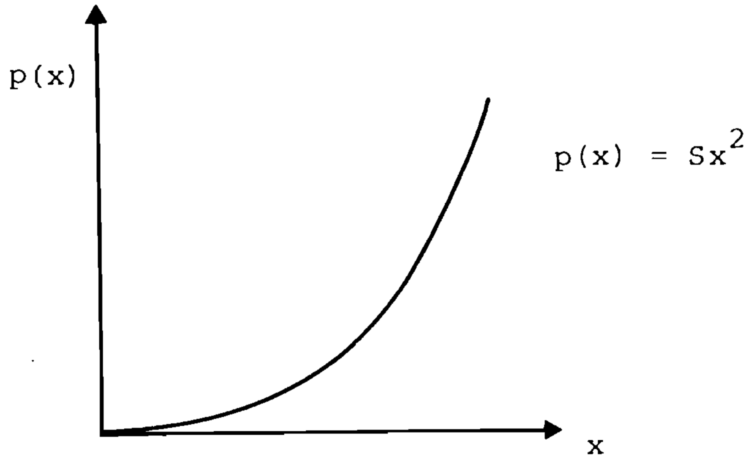

Figure 3. A function which measures the pressure to refer patients.

$$q_i = k_i \left\{ \left(\frac{y_i}{Y_i} \right)^{-\beta_i} - 1 \right\} \qquad \forall\, i \; . \tag{7}$$

Figure 4 shows how this implies infinite pressure on resources when y_i is zero, and zero pressure on resources when y_i equals some ideal level Y_i. Beyond this value there is negative pressure, and referral is encouraged from above! The power parameter β_i is high when it is more important to be close to the ideal levels Y_i, and low in the opposite case. With this definition we assume that the only factor influencing admission is the availability of resources. Other factors (such as the ability to pay for treatment) are neglected here, although they might be reflected in other definitions of the functions q_i.

Finally, we must specify which of the quantities defined above are to be regarded as model parameters and which as controllable variables. The quantities β_i, Y_i are model parameters which define how the pressure on resources varies with their utilisation. So also are the quantities S,X,f which together represent the total pressure applied to, and hence the workload accepted by, the system. This leaves the resource levels R_i which are regarded as controllable or experimental variables. As indicated in Figure 5 the remaining quantities are output variables to be determined by the equations (1) to (7).

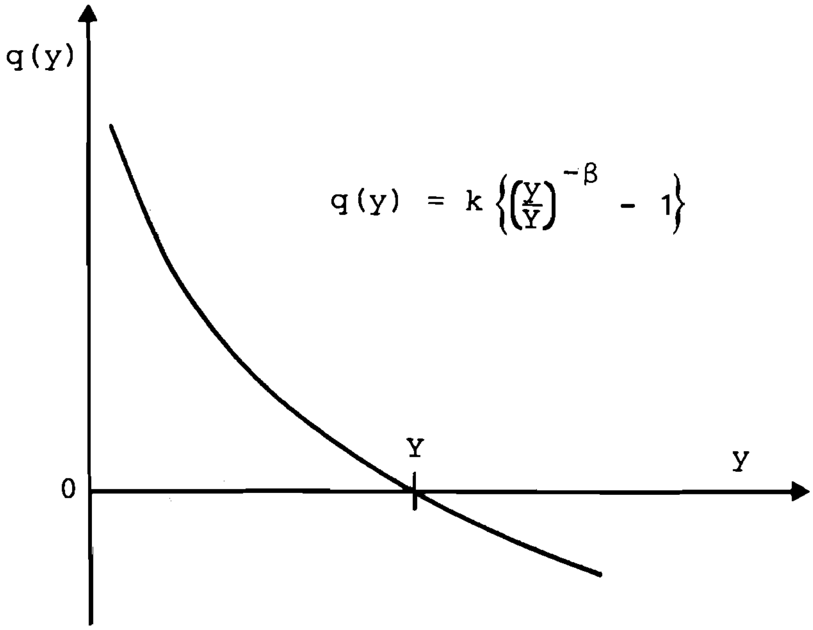

Figure 4. A function which measures the pressure on resources.

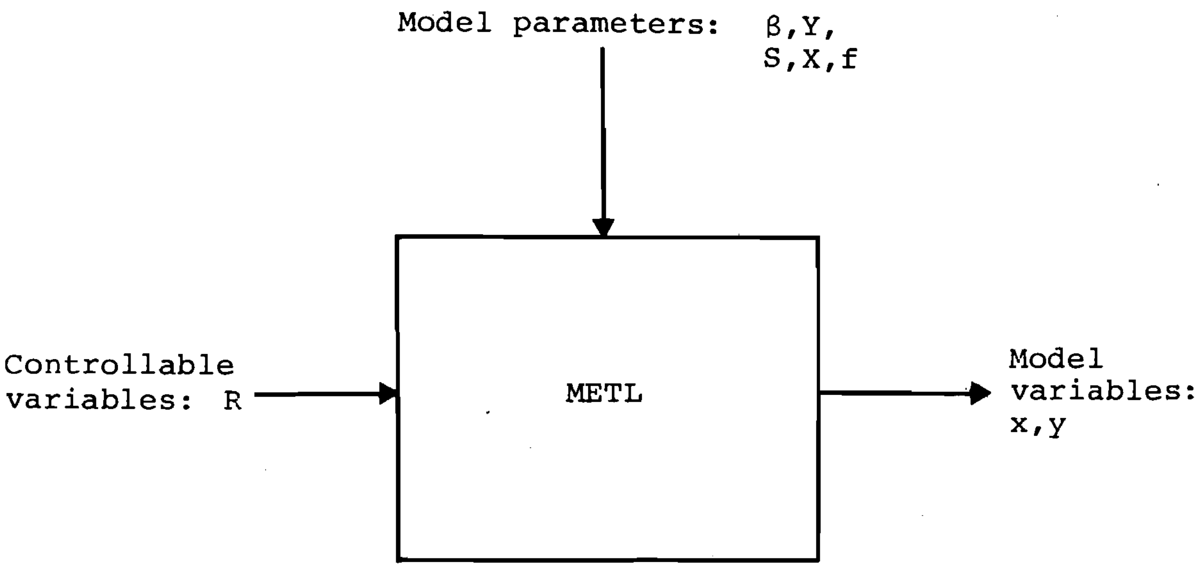

Figure 5. Inputs and outputs of METL.

MODEL SOLUTION

The model as described above is rather elementary. In particular, it makes no distinction between patients in different categories (e.g., with different diseases) who might be given greater referral priority than others. Its assumptions about referral behaviour are also rather simple. Nevertheless, it is useful to investigate the properties of a simple model before introducing further complications.

The first step in solving the model is to substitute the specific expressions for p_i and q_i given by equations (4) to (7) into the general equations of the model (1),(2), and (3). We obtain

Constraints: $x_i y_i = R_i \qquad \forall\ i \qquad (1)$

Between levels: $$k_i\left\{\left(\frac{y_i}{Y_i}\right)^{-\beta_i} - 1\right\} = \begin{cases} Sx_{i-1}^2 & \forall\ i > 1 \\ S\chi^2 & i = 1 \end{cases} \qquad (8)$$

Across levels: $$k_i\beta_i\left(\frac{y_i}{Y_i}\right)^{-\beta_i} = Sx_i^2 \qquad \forall\ i < N \qquad (9)$$

Equations (1),(8),(9) are 3N - 1 equations in 3N unknowns x,y,k. (Henceforth, we use x to denote $\{x_i, i=1,\ldots,N\}$ with similar notation for other variables.) However, the units of k and S are arbitrary, so that without loss of generality, we may divide them through by k_N. Equivalently, we set k_N equal to one in equation (8). We then have the same number of unknowns as equations.

In real life, the different levels of the HCS achieve equilibrium simultaneously. However, for computational solution of the model we must employ an iterative technique. The variable chosen to be constant at each iteration is χ, the number of individuals still outside the system. By eliminating k_i and x_i between equations (1),(8), and (9) we obtain

$$f(y_i) = 0 \qquad \forall\, i \tag{10}$$

where

$$f(y_i) = \begin{cases} y_1^2 + \dfrac{R_1^2}{\beta_1 \chi^2}\left\{\left(\dfrac{y_1}{Y_1}\right)^{\beta_1} - 1\right\} & i = 1 \\[2ex] y_i^2 + \dfrac{R_i^2}{\beta_i x_{i-1}^2}\left\{\left(\dfrac{y_i}{Y_i}\right)^{\beta_i} - 1\right\} & i = 2,\ldots,N-1 \\[2ex] 1 + \dfrac{1}{Sx_{N-1}^2}\left\{1 - \left(\dfrac{y_i}{Y_i}\right)^{-\beta_N}\right\} & i = N \end{cases} \tag{11}$$

If we know x_{i-1}, and equation (10) has a solution for y_i, then equation (1) may be used to determine x_i. For i = 1, we know χ, and so equations (10),(1) may be solved successively for $i = 1,2,\ldots,N$ to determine all values of x and y. Then an improved value of χ may be found from Equation (6).

Unfortunately, equation (10) is itself nonlinear in y_i. We may show, however, that

$$f(y_i) < 0 \quad , \qquad \text{when } y_i = 0, \quad \forall\, i < N \quad ,$$

and

$$f(y_i) > 0 \quad , \qquad \text{when } y_i = Y_i, \quad \forall\, i < N \quad .$$

Furthermore,

$$\frac{\partial f(y_i)}{\partial y_i} = \begin{cases} 2y_1 + \dfrac{R_1^2}{Y_1 \chi^2}\left(\dfrac{y_1}{Y_1}\right)^{\beta_1 - 1} & i = 1 \\[2ex] 2y_i + \dfrac{R_i^2}{Y_i x_{i-1}^2}\left(\dfrac{y_i}{Y_i}\right)^{\beta_i - 1} & i = 2,\ldots,N-1 \end{cases} \tag{12}$$

is always positive between these values. Therefore, equation (10) has a unique solution within the desired range of y_i, for $i < N$, and this solution may be found by a simple numerical technique. For $i = N$, we find that equation (10) has the solution

$$y_N = Y_N/(1 + Sx_{N-1}^2)^{\frac{1}{\beta_N}} \tag{13}$$

which is always positive and less than Y_N.

Given an improved value of χ, $g(\chi)$ say, a second problem is to use values of $g(\chi)$ to solve equation

$$g(\chi) = \chi \quad . \tag{14}$$

We know, however, that $g(\chi)$ is a continuous function and that

$$g(\chi) < \chi \qquad \text{when } \chi = X$$

$$g(\chi) > \chi \qquad \text{when } \chi = 0$$

provided that resources do not exceed ideal requirements. These conditions guarantee a solution for equation (14) within the range $0 < \chi < X$. Figure 6 depicts a procedure which finds χ by subdividing the feasible interval. This procedure is easy to implement as a computer program and is successful in finding the solutions.

CALIBRATION OF THE MODEL AND AN ILLUSTRATIVE EXAMPLE

Whether the model described above is useful depends upon whether it can be calibrated to apply to particular problems. There are two groups of parameters to be estimated which we shall briefly discuss in turn.

The first group of parameters (Y,β) measure the pressure on resources at various levels of treatment. The ideal resource

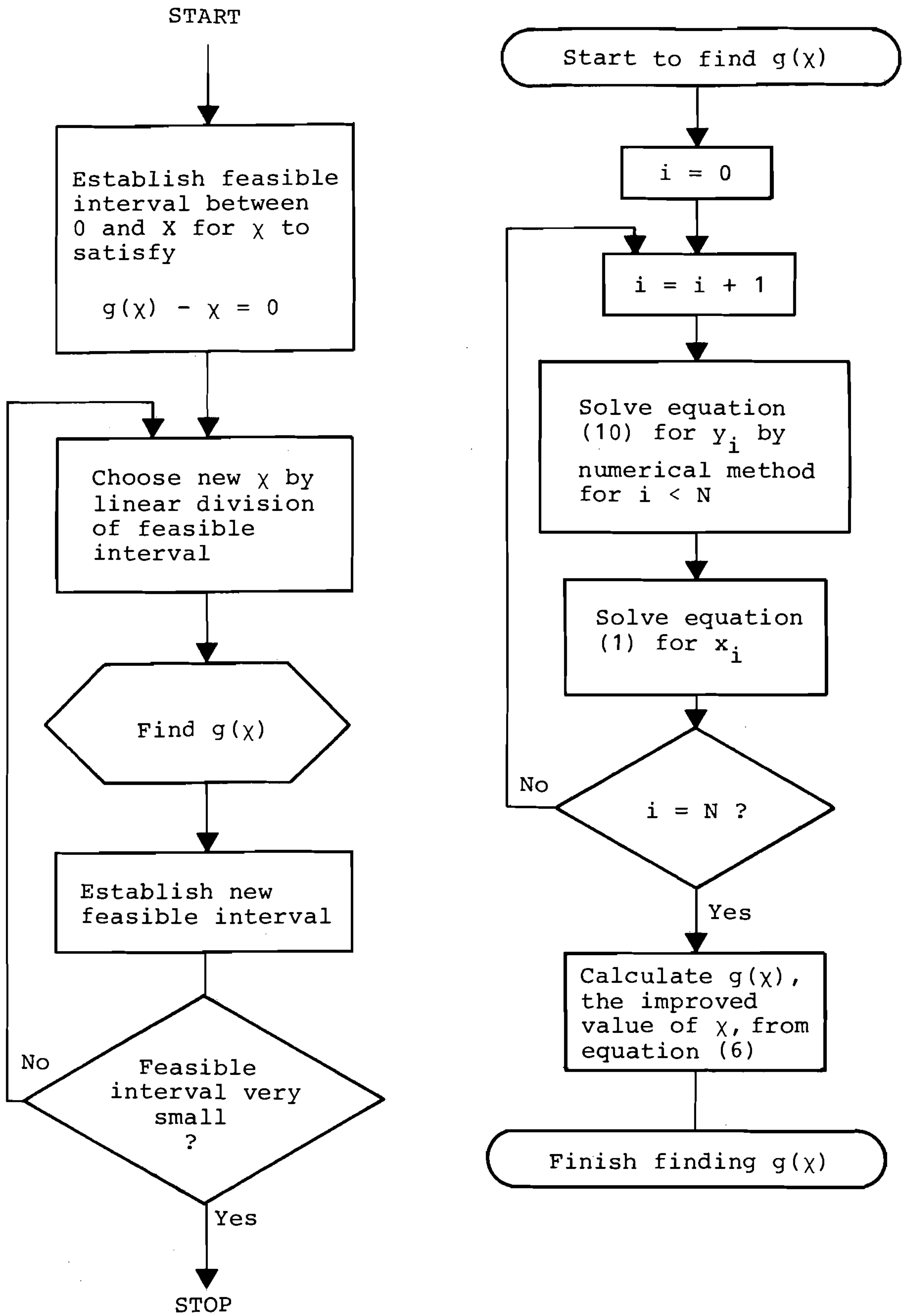

Figure 6. Iterative procedure for solving the model equations.

levels Y are clearly analogous to similar variables in the other IIASA HCS resource allocation model DRAM (Gibbs, 1978). As in DRAM, they may be specified directly in countries where such levels are planned, or expert judgement can be used for their estimation. Other procedures enable the estimation of Y from historical data (Hughes, 1978b). The power parameters β are harder to estimate, but they are related to the observed elasticities of resource utilisation to resource supply sometimes revealed by empirical studies. (Note that they are *different* from the power parameters in DRAM). In order to explain this relation, consider a treatment facility in equilibrium with a group of potential patients as shown in Figure 7. The equation which represents the equilibrium

$$k\left\{\left(\frac{y}{Y}\right)^{-\beta} - 1\right\} = S\left(X - \frac{R}{y}\right)^2 \qquad (15)$$

holds also if differentiated through by R

$$k\left(\frac{-\beta}{Y}\right)\left(\frac{y}{Y}\right)^{-(\beta+1)}\frac{\partial y}{\partial R} = 2S\left(X - \frac{R}{y}\right)\frac{\left(y - R\frac{\partial y}{\partial R}\right)}{-y^2} \qquad (16)$$

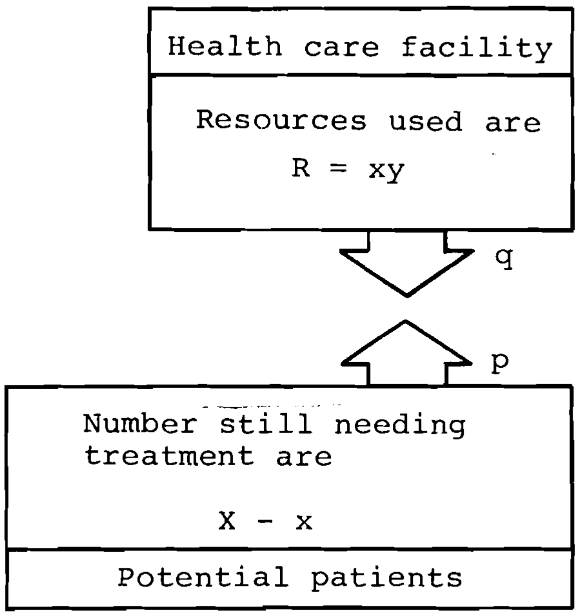

Figure 7. The pressures for (p), and opposing (q), referral to a health care facility.

and combining equations (15) and (16) gives the empirical utilisation elasticity

$$\eta = \frac{\partial \ln y}{\partial \ln R} = \left[1 + \frac{\frac{\beta}{2}\left(\frac{X}{x} - 1\right)}{1 - \left(\frac{y}{Y}\right)^{\beta}}\right]^{-1} \qquad (17)$$

as a function of the power parameter β. We find, if $\left(\frac{x}{X}\right),\left(\frac{y}{Y}\right) < 1$, that η varies between 0 and η_{max}, as β varies between ∞ and 0, where

$$\eta_{max} = \left[1 + \frac{\left(\frac{X}{x} - 1\right)}{2 \ln \left(\frac{Y}{y}\right)}\right]^{-1} < 1 \quad .$$

When empirical elasticities greater than η_{max} are observed, it suggests that the pressure functions assumed above are incorrect. When utilisation elasticities less than η_{max} are observed in empirical studies, equation (17) may be useful in estimating β.

The second group of parameters (S,X,f) measure the pressure applied to the system. The seriousness parameter, S, is not so easy to choose in advance and we regard it here as a tuning variable, to be chosen during calibration in order to control the number of patients admitted to the system. Equation (13) shows that as S tends to zero, the model solution is obtained by setting y_N equal to Y_N. The parameter X reflects the total potential workload on the system. Like Y, it is analogous to similar variables in DRAM, and similar comments apply. We interpret the parameter f as a structural parameter, ranging from zero, when the treatment levels are alternatives, to higher values when the treatment levels are progressive.

In order to illustrate a possible application of the model, we use some of the data collected by Rutten (1978) on referrals to outpatient and inpatient treatment from general practitioners in the Netherlands in 1973. Table 1 gives the model parameters used for this illustration. The ideal levels Y are rather arbitrarily set at five standard deviations above the observed values. The power parameters β derive from empirical elasticities of $\eta_1 = 0.29$, $\eta_2 = 0.21$ estimated by Rutten, and equation (17). The total potential demand is assumed to be everybody (X = 1000 individuals per 1000 population), and we assume the two treatment to be alternatives (f = 1).

Table 2 shows the flows found by the model for various values of the parameter S for 1973 resource levels, and compares them with the actual flows observed in the Netherlands. As S decreases, so does the number of patients admitted to the system but never quite to the number actually observed. Most probably this is

Table 1. Parameters for illustrative run.

Y_1	Ideal level of OP attendances/OP referral	3.85
Y_2	Ideal level of IP days/IP referral	24.6
β_1	OP Power parameter	0.79
β_2	IP Power parameter	1.0
X	Potential patients/1000 population	1000
R_1	Available OP attendances/1000 population	823.0
R_2	Available IP bed-days/1000 population	1656.7
f	(see text)	1

OP = outpatient
IP = inpatient

Table 2. Output of illustrative run compared with actual values.

		Actual values in Netherlands in 1973	Values predicted by model using parameters in Table 1 and S = 0.5×10^{-5},	10^{-6},	10^{-7}
Referrals to OP	x_1	342.2	481.8	510.4	517.9
Av. attendances/OP	y_1	2.41	1.71	1.61	1.59
Referrals to IP	x_2	99.5	145.3	84.7	69.0
Av. length of stay	y_2	16.7	11.4	19.5	24.0
Total number treated	$\sum x$	441.7	627.1	595.1	584.9

because we have assumed everyone to be a potential patient. This pessimistic assumption inflates the number of patients treated as outpatients (x_1) and deflates the resources received by each (y_1). Otherwise, the degree of agreement between the observed and predicted results (when $S = 10^{-6}$, say) is reasonable enough in an illustrative run with proxy values for some of the parameters, although a real application would require an improvement on this.

In order to see how the model might be used to explore outcomes from alternative policies, we halve the number of outpatient sessions, and double the number of inpatient beds available to the model. Will more or fewer patients be treated, and how? Table 3 shows that the model predicts slightly higher numbers of total treatments. Many more individuals are treated as inpatients with slightly longer lengths of stay. Slightly fewer individuals are treated as outpatients, with almost half as many attendances per referral.

Table 3. Two illustrative runs with different resource levels.

		Values predicted by model (using $S = 10^{-6}$) for two different resource levels	
OP attendances/1000 pop.	R_1	823.0	411.5
IP bed days/1000 pop.	R_2	1656.7	3313.4
Referrals to OP	x_1	510.4	437.0
Av. attendances/OP	y_1	1.61	0.95
Referrals to IP	x_2	84.7	160.1
Av. length of stay	y_2	19.5	20.7
Total number treated	$\sum x$	595.1	597.1

DIRECTIONS FOR FURTHER DEVELOPMENT

Although this pilot version of the model has many simplifying assumptions, it can be developed in various ways. We can, for example, disaggregate patients into different categories with different pressures for referral. It is easy to see that patients in surgical specialties are more likely to be referred to hospital, and that elderly patients may not be discharged when insufficient convalescent care is available. The model would show how these effects influence the numbers of admissions earlier in the system.

A second sort of development would be to elaborate the pressure functions. These might be set *prescriptively*, for example, so as to maintain a minimum resource per patient at each level. Alternatively, they might be set *descriptively*, using information about doctors' and patients' behaviour. Different pressure functions from the ones used here may operate in countries where payment systems have some influence on referral.

A more radical form of development would be to allow branching flows, waiting between states, and other features which are common in simulation flow models. We deliberately avoided these features here because we wanted to use the equilibrium mechanism in an application which is apparently new. (Microeconomic theory applied to health care systems might be similar). It would be interesting to investigate how these flow simulation features could be incorporated in an equilibrium approach.

Even without these developments, METL has several attractive features. It is sufficiently similar to DRAM to share some of the same parameters. It can therefore be applied in conjunction with DRAM. Indeed, this is part of our general aim of developing and applying different submodels of the HCS within a single conceptual framework. The idea used by METL that the HCS is in some sort of equilibrium, is easy to accept in different countries, and yet seems not to have been previously exploited. Our view of the HCS has close links with that of Rutten (1978). It might also relate to that of Cantley (1978) who depicts the care of the elderly in the UK as a system with flows between components of the HCS. Finally, the model has attractive analytic features which make it easy to solve and understand. Further study will determine how these properties can be further exploited.

REFERENCES

Cantley, Mark F. (1978) Strategic Control for a UK Regional Health Authority--A Conceptual Framework. RM-78-54. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Clayden, A.D. (1977) A Decision Simulation Model for Health Services Management. Operational Research Quarterly 28:505-515.

Feldstein, M.S. (1967) Economic Analysis for Health Service Efficiency. North-Holland, Amsterdam.

Fleissner, P., and A. Klementiev (1977) Health Care System Models: A Review. RM-77-49. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Gibbs, R.J. (1977) Health Care Resource Allocation Models--A Critical Review. RM-77-53. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Gibbs, R.J. (1978) The IIASA Health Care Resource Allocation Sub-Model: Mark 1. RR-78-8. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978a) The IIASA Health Care Resource Allocation Sub-Model: Mark 2--The Allocation of Many Different Resources. RM-78-50. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978b) The IIASA Health Care Resource Allocation Sub-Model: Estimation of Parameters. RM-78-67. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Rutten, F.F.H. (1978) The Use of Health Care Facilities in the Netherlands--An Econometric Analysis. Doctoral dissertation, University of Leiden, Netherlands.

Shigan, E.N. et al. (1979) Health Care Systems Modelling at IIASA A Status Report. WP-79-8. Laxenburg, Austria: International Institute for Applied Systems Analysis.

IIASA REPORTS AND PAPERS ON RESOURCE ALLOCATION IN HEALTH CARE SYSTEMS MODELLING

Gibbs, R.J. (1977) Health Care Resource Allocation Models--A Critical Review. RM-77-53. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Gibbs, R.J. (1978) The IIASA Health Care Resource Allocation Sub-model: Mark 1. RR-78-8. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Gibbs, R.J. (1978) A Disaggregated Health Care Resource Allocation Model. RM-78-1. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Gibbs, R.J. (1978) Computer Programs for the IIASA Health Care Resource Allocation Sub-Model, Mark 1--A User's Guide. WP-78-15. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978) The IIASA Health Care Resource Allocation Sub-Model: Mark 2--The Allocation of Many Different Resources. RM-78-50. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978) The IIASA Health Care Resource Allocation Sub-Model: Formulation of DRAM Mark 3. WP-78-46. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Hughes, D.J. (1978) The IIASA Health Care Resource Allocation Submodel: Estimation of Parameters. RM-78-67. Laxenburg, Austria: International Institute for Applied Systems Analysis.