NOTES ON THE VALUE OF INFORMATION ABOUT THE ARRIVAL DATE OF A NEW TECHNOLOGY
(Revised Version)

Jean-Pierre Ponssard

September 1974 WP-74-54

NOTES ON THE VALUE OF INFORMATION ABOUT THE ARRIVAL DATE OF A NEW TECHNOLOGY [1/]

1. Introduction

We consider an investment problem in which one has to decide on which technology to install to satisfy a growing demand, given that at some date in the future some new technology will become available. More specifically, we assume that there are two existing technologies - the first one with a high capital cost and a low production cost, and the second one with a low capital cost and a high production cost. Presumably the first one would be chosen if the time horison is long and the second one if it is short. Because of uncertainty on the arrival date of the third technology this time horizon is unknown.

The objective of these notes is to provide some insights about the expected value of information on the arrival date of the new technology under simple analytical assumptions.

This question has been discussed at the macroeconomic level by Dasgupta and Heal [D-H] and in the energy context by Manne [M]. For example, the new technology might be fusion whereas the two existing competitive technologies might be nuclear versus fossil fuel plants. The crucial question is whether the initial decisions are significantly affected by the date at which fusion becomes available. A secondary question is the difference between a point estimate and a probability distribution for that date.

[1/] This paper resulted from discussions with A.S. Manne and greatly benefited from his most helpful comments.

These expository notes are organized as follows. The model is described in section 2, and the significance of a deterministic time horizon is then determined. For the case of an uncertain time horizon the value of information is examined in section 3. It is also shown that, as far as present decisions are concerned, a probability distribution may be replaced by a suitably adjusted "discounted" point estimate. Some illustrative numerical examples are included throughout the text. A more rigorous treatment of the subject, including the extension to more than two present technologies may be found in [P-S].

2. The Model (continuous time)

2.1 Existing Technologies

Let k_j and c_j denote capital and production costs per unit for technology j (j = 1,2) with $k_1 > k_2$ and $c_2 > c_1$. These costs are assumed to remain constant in time.

Given a horizon t ($t \geq 0$) and a discount rate ρ, the discounted cost of an investment of one unit in technology j is

$$(2\text{-}1\text{-}1) \qquad C_j(t) = k_j + c_j\int_o^t e^{-\rho\tau}d\tau = k_j + c_j(1-e^{-\rho\tau})/\rho \quad .$$

Let $\delta(t) = C_1(t) - C_2(t)$. For notational simplicity let $k = k_1 - k_2$ and $c = c_2 - c_1$. Then

$$(2\text{-}1\text{-}2) \qquad \delta(t) = k - c(1-e^{-\rho\tau})/\rho \quad .$$

We shall rewrite (2-1-2) in more suggestive terms. For this, it is assumed that $k < c/\rho$ (Otherwise technology 2 would always be preferred to technology 1). Let $T = k/c$ and $T^d = -\rho^{-1}\ln(1-\rho T)$. For ρ=0, note that $T^d=T$. T and T^d may be interpreted respectively as the pay-back and the discounted pay-back periods. These represent the length of time such that the (discounted) cumulative difference in production costs just outweighs the initial difference in capital costs. It can be seen that

$$(2\text{-}1\text{-}3) \qquad \delta(t) = c(e^{-\rho t} - e^{-\rho T^d})/\rho \quad .$$

For $t = T^d$, note that $\delta(t) = 0$.

For minimization of the discounted cost, the optimal rule under certainty takes the following form: if the time horizon is greater or equal to the discounted pay-back period T^d, then invest in technology 1; otherwise, invest in technology 2.

Recall that $\delta(t)$ may be positive, negative or zero. Its absolute value $|\delta(t)|$ may be interpreted as the discounted cost of making the wrong decision.

A numerical example

Suppose that technology 1 represents nuclear plants whereas technology 2 represents fossil fuel plants and let time $t = 0$ be year 1990. Then a reasonable assessement of costs might be:

$$c_1 = \$15/\text{Kw-yr} \quad , \quad c_2 = \$45/\text{Kw-yr} \quad ,$$

$$k_1 = \$500/\text{Kw} \quad , \quad k_2 = \$300/\text{Kw} \quad ,$$

and a possible discount rate would be $\rho = .10/\text{year}$.

Then the pay-back period

$$T = (k_1-k_2)/(c_2-c_1) = 200/30 \simeq 6.7 \text{ yrs}$$

and the discounted pay-back period

$$T^d = -\rho^{-1}\ln(1-\rho T) = 10\text{Log}3 \simeq 11 \text{ yrs.}$$

$$\delta(t) = c(e^{-\rho t} - e^{-\rho T^d})/\rho$$

$$= c\rho^{-1} e^{-\rho T^d} (e^{-\rho(t-T^d)} - 1)$$

$$= 100 \qquad (e^{-.1(t-11)} - 1)$$

$|\delta(t)|$, the discounted cost of making the wrong decision, is given in graphical form in figure 1.

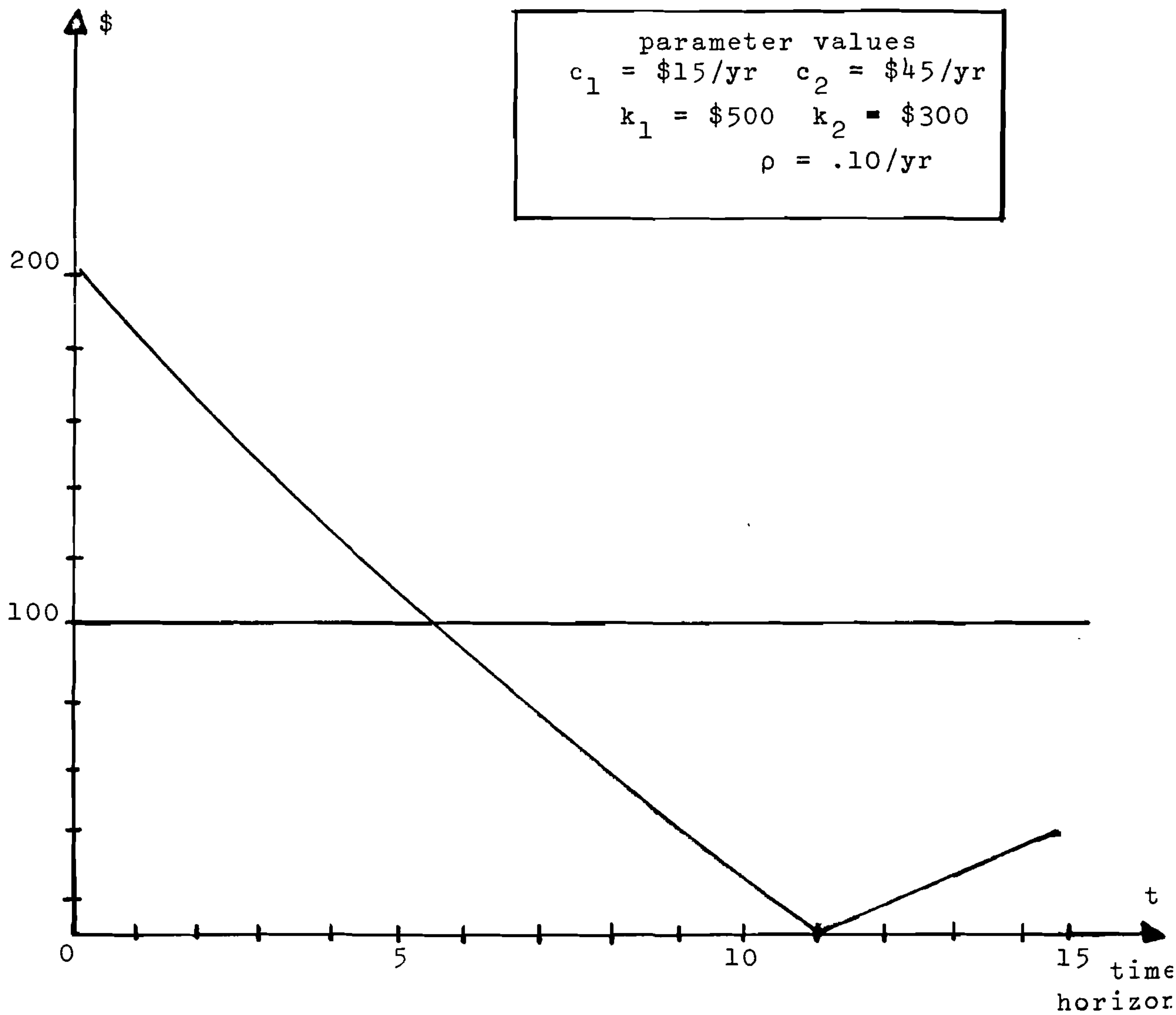

Figure 1

The discounted cost of making the wrong decision

2.2 The New Technology (known costs, probabilistic arrival date)

It is supposed that as soon as the new technology (with respective costs k_3 and c_3) becomes available, then it will be more economical than the existing ones. These will then be taken out of production. Hence the following inequality is assumed to hold:

$$(2\text{-}2\text{-}1) \qquad k_3 < \int_0^\infty (c_1 - c_3) e^{-\rho t} dt \quad ,$$

and so the following holds as well:

$$k_3 < \int_0^\infty (c_2 - c_3) e^{-\rho t} dt \quad .$$

However, the arrival date of the new technology is not known with certainty but only with some probability. Specifically, suppose that the new technology is known for sure not to become available before t_o. After t_o, its arrival date t is distributed according to a negative exponential distribution with arrival probability λ per unit time. The expected arrival date $\bar{t}$ is easily seen to be $t_o + 1/\lambda$.

3. Decision Analysis of the Model

3.1 The Best Decision under Probabilistic Uncertainty

Our decision rule under probabilistic uncertainty is taken as the minimization of the expected discounted cost.

According to the exponential assumption for the arrival date of the new technology, the time horizon t may take any value between t_o and $+\infty$ and its density function is $\lambda e^{-\lambda(t-t_o)}$. Denote by $\Delta(t)$, the difference in expected discounted costs between technologies 1 and 2. Then we have

$$\Delta(t) = \int_{t_o}^{+\infty} [C_1(\tau) - C_2(\tau)] \lambda e^{-\lambda(\tau-t_o)} d\tau = \int_{t_o}^{+\infty} \delta(\tau)\ e^{-\lambda(\tau-t_o)} d\tau \quad .$$

Since

$$\delta(\tau) = c(e^{-\rho\tau} - e^{-\rho T^d})/\rho$$

(see (2-1-3)), by integrating we obtain

$$(3\text{-}1\text{-}1) \qquad \Delta(t) = c[\lambda(\lambda+\rho)^{-1} e^{-\rho t_o} - e^{-\rho T^d}] \quad .$$

Similarly as we defined a discounted pay-back period, let us now define <u>a discounted expected arrival date</u>, $\bar{t}^d$ such that

$$(3\text{-}1\text{-}2) \qquad \bar{t}^d = t_o + \rho^{-1} \mathrm{Log}(1+\rho\lambda^{-1}) \quad .$$

For $\rho = o$, note that $\bar{t}^d = \bar{t} = t_o + 1/\lambda$.

This discounted expected arrival date $\bar{t}^d$ may be interpreted as the certainty equivalent of the uncertain time horizon. Indeed the expression $\Delta(t)$ may then be rewritten as

$$(3\text{-}1\text{-}3) \qquad \Delta(t) = c(e^{-\rho\bar{t}^d} - e^{-\rho T^d})/\rho \quad .$$

For $\bar{t}^d = T^d$, note that $\Delta(t) = 0$.

Then the <u>optimal decision rule under probabilistic uncertainty</u> takes the following form: if the discounted expected arrival date of the new technology is greater or equal to the discounted pay-back period then invest in technology 1; otherwise invest in technology 2. Putting everything together we may now bring out the differences between the deterministic and probabilistic case: the larger the discount rate the smaller the certainty equivalent of the arrival date of the new technology relative to its expected value. Consequently one may prefer the low capital intensive technology (j=2) with an uncertain time horizon t, whereas one would prefer the high capital intensive technology (j=1) with a certain time horizon even if it is smaller than the expected value of t. How much is this reversal of preferences affected by the discount rate and the uncertainty is made graphically precise in the context of our illustration by looking at figure 2.

<u>A Numerical Example</u> (continued).

We shall select as our base case for the probabilistic

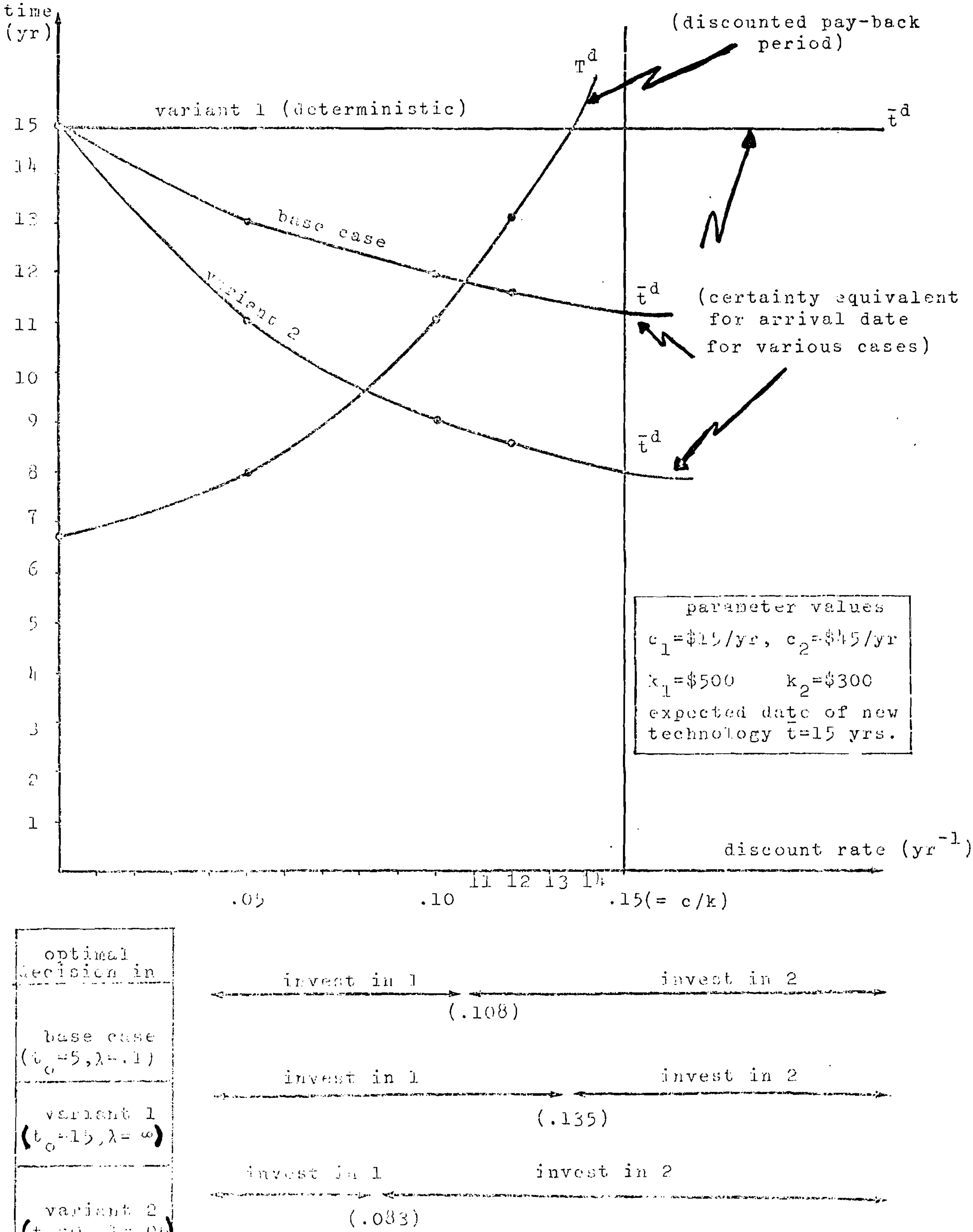

Figure 2

Differences between deterministic and probabilistic arrival date models.

assessment of the arrival date of the new technology $t_o = 5$ years and $\lambda = .1$/year. Thus its expected value is $\bar{t} = 5 + 1/.1 = 15$ years. Two variants will be considered. Variant 1 with $t_o = 15$ years, $\lambda = \infty$ corresponds to the deterministic case. Variant 2 with $t_o = 0$, $\lambda = .06$/year (such that $\bar{t}^d = 15$ years) corresponds to a probabilistic case with high variance. We may now represent graphically T^d and $\bar{t}^d$ as functions of the discount rate ρ and derive the optimal technology for each value of ρ, the base case and the two variants. The expected cost of making the wrong decision, which is now $|\Delta(t)|$, may be readily obtained for $\rho = .1$/year by inserting the corresponding $\bar{t}^d$ in figure 1.

3.2 The Expected Value of Perfect Information on the Arrival Date

3.2.1 Recall of the Definition

The concept of the expected value of perfect information (abbreviated as EVPI) is one of the cornerstones of Decision Ananysis [R]. It is intended to be a guide for the research and development of new strategies and as such it is a creative part of the theory. If in a given decision problem the EVPI is judged significantly high this is an incentive for generating new strategies and in particular strategies which would allow for the gathering of new information on the real state of nature.

We shall study the EVPI in a simplified example in which only two states of the world are possible. Either the arrival date of the new technology is 5 years or 10 years, with respective probabilities 1 - p and p ($0 \leq p \leq 1$). Moreover to encourage the reader to easily reproduce the computations we shall assume that there is no discounting ($\rho = 0$). Then the pay back period is T = 20/3 = 6.7 years and the expected date of arrival is $\bar{t} = 5(1 - p) + 10p = 5 + 5p$. Thus one would prefer to invest in technology 1 if and only if $p \geq 1/3$. The expected costs associated with technologies 1 and 2 as functions of p have been drawn in figure 3, and the minimum of the two lines gives the minimum expected cost associated with the optimal investment.

Now suppose that one had advanced information and could make the decision depend on the arrival date. Then clearly one would invest in 1 if t = 10 yrs and in 2 if t = 5 yrs. For a given p, the difference between the minimum expected cost and the expected cost associated with the perfect information strategy (represented by the dotted line in figure 1) is called the EVPI. It is easily seen that the EVPI is the largest for p = 1/3. That is when one would be indifferent between investing in technology 1 or 2.

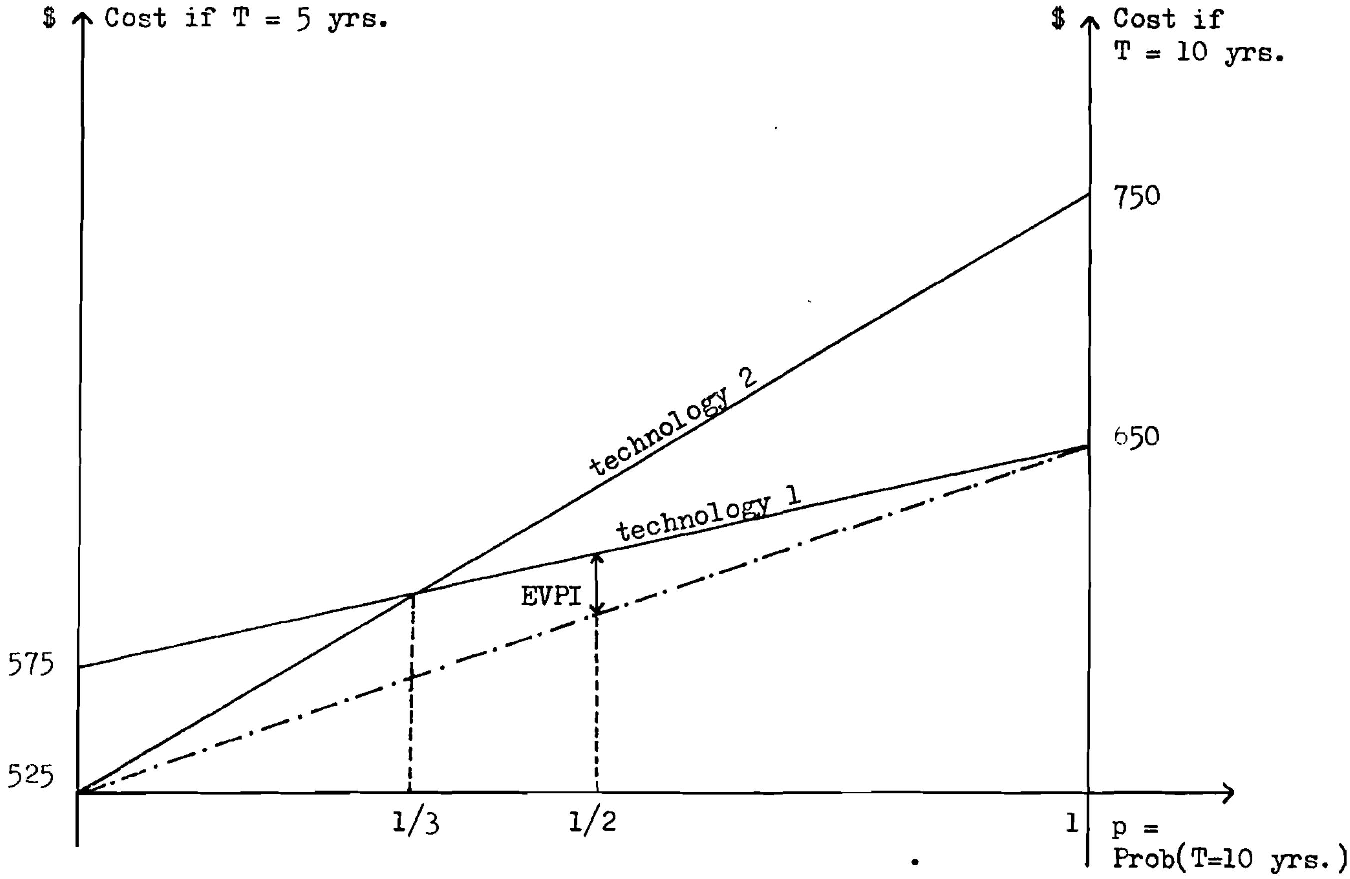

Figure 3

The EVPI in an Example with
only two states of the world

Interpretations of the EVPI

(i) The EVPI represents the minimal expected loss incured from the fact that the decision is taken under uncertainty. Let p = 1/2 and suppose that one would invest in technology 1. If it turns out that the arrival date is 10 years the decision was the best we could possibly have made, no loss incured. However if t turns out to be 5 yrs this was a bad decision which cost \$ 50 more than necessary. A priori the expected loss associated with technology 1 is then \$ 25. Similarly the expected loss associated with technology 2 is easily seen to be 1/2×(750-650) = \$ 50. The minimal expected loss which is associated with the optimal decision (technology 1), \$ 25, is the EVPI.

(ii) The EVPI represents the maximal amount one would be willing to pay to know precisely the arrival date of the new technology. In this numerical example there are 5 or 10 years before the arrival of fusion and if either possibility is equally likely (p = 1/2), one would not pay more than \$ 25 per Kw to know when fusion would be available.

3.2.2 EVPI on the Arrival Date

According to our definition (see 3.2.1), the EVPI may be computed as the minimal expected discounted loss over the two possible technologies. For technology 1, which is optimal when $\bar{t}^d \geq T^d$, the discounted loss is $\delta(t)$ whereas for technology 2, which is optimal when $\bar{t}^d \leq T^d$, it is $-\delta(t)$. Then, the EVPI is obtained as

$$(3.2.2.1) \quad EVPI(t) = \text{Min}\left\{ \int_{t_o}^{\bar{t}^d} \delta(t)\lambda e^{-\lambda(t-t_o)}dt, \int_{\bar{t}^d}^{+\infty} -\delta(t)\lambda e^{-\lambda(t-t_o)}dt \right\} .$$

Note that if $T^d \leq t_o$, that is if the discounted pay-back period is less than the earliest date at which the new technology might be available, then the first term is zero so that the EVPI is zero. Technology 1 is preferable to technology 2 whatever the arrival date of technology 3. (In our numerical illustration assuming a discount rate of 10% per year this would occur if $t_o \geq 11$ yrs).

If $T^d > t_o$, integretating 3.2.2.1 we obtain:

$$(3\text{-}2\text{-}2\text{-}2) \quad EVPI(t) = \text{Min}\left\{ \Delta(t) + K(t),\ K(t) \right\} ,$$

in which

$$\Delta(t) = c(e^{-\rho\bar{t}^d} - e^{-\rho T^d})/\rho$$

$$K(t) = ce^{-\rho T^d} e^{-\lambda(T^d - t_o)}/(\lambda+\rho) \quad .$$

The EVPI is at its maximum when $T^d = \bar{t}^d$, that is when one is indifferent between the two technologies.

<u>A Numerical Example</u> (continued)

We shall compute the expected value of perfect information on the arrival date under the various cases. Also of interest is the relative EVPI that is the EVPI divided by the minimal expected cost of the investment.

Parameter Values. c_1 = \$15/Kw-yr, c_2 = \$45/Kw-yr, k_1 = \$500/Kw, k_2 = \$300/Kw, $\rho = .1 yr^{-1}$			Expected Value of Perfect Information	Relative Value of Perfect Information
	t_o(yr)	$\lambda(yr^{-1})$	EVPI(\$)	$EVPI/\min_j E\, C_j(t)$
base case	5	.1	27.1	.04
variant 1	15	∞	0	0
variant 2	0	.06	28.7	.10

REFERENCES

[D-G] Dasgupta P. and Heal, C., "The Optimal Depletion of Exhaustible Resources", Mimeographed, to appear in Review of Economic Studies, July, 1974.

[M] Manne, A.S., "Waiting for the Breeder", IIASA, January, 1974.

[P-S] Ponssard, J.P. and Strinivasan, T.N., "Selection of Time Independant Technologies under an Uncertain Planning Horizon", IIASA, July, 1974.

[R] Raiffa, H., "Decision Analysis", Addison Wesley, 1968