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INTEGRATION OF TRANSPORTATION AND LOCATION ANALYSIS - A GENERAL EQUILIBRIUM APPROACH

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Summary

This paper describes an integrated approach to the study of transportation and growth of production in different locations. The model approach is based on a non linear dynamic input-output growth model which is endogenously related to a spatial framework with a transportation model. Both the transportation and the dynamic location model are based on different equilibrium concepts.

The model structure is developed in such a way so as to permit computational possibilities.

Different notions of equilibrium in transportation analysis

The formulation of the equilibrium problem in transportation is in some contexts based on a purely economic reasoning at the micro or macro level. Lefeber (1958) analysed the problem of personal and commodity transportation within such a micro economic framework. The transportation sector is in his analysis looked upon as an intermediate sector in which transportation needs (rather than demands) are seen as functions of the location of production and inputs. Transportation supply is on the other hand seen as an unlocalized production of services regulated by conventional, concave, always differentiable production functions. The network is totally implicit in this kind of transportation equilibrium approach. With suitable assumptions about the individual utility and production functions for the non-transportation sectors it can within this framework be proved that an equilibrium must be such that the difference between FOB- and CIF-prices is equal to the marginal costs of transportation for each one of the consumer commodities. It can also be shown that the marginal value product of each factor must be equal to the scarcity rent of the factor of production plus the marginal cost of transportation of the same factor.

Such a transportation equilibrium is a possible but a very restricted definition of a transportation equilibrium. One of the most important implications is the result that there can be no crosshauling of similar commodities or persons, an implication that is grossly at variance with observations at all statistically possible levels of aggregation.

The concept of equilibrium used in this class of models should not really be viewed as a micro-economic behavioural concept. It is rather formulated within the framework of neoclassical welfare economics. This kind of model presumes the existence of some agent that maximizes a weighted sum of utilities from consumption accruing to all the individual households. There are no real suppliers of commodities and transportation, only production functions acting as constraints. It has been an argument used in micro economic studies without any global maximization function, that individual consumers maximizing their own utility will never in congested situations on the road network act in such a way that a Lefeber-equilibrium is achieved. Instead of looking at the socially relevant marginal costs of transportation, consumers will only take into account the average costs of the system.

It thus seems evident that one can subdivide the equilibrium concepts for the transportation system according to the fundamental level of inquiry. A completely micro-oriented approach would require that each user of the transportation system is looked upon as a decision unit located at every instant of time on some link connecting some pair of nodes. It must also be assumed that the micro decision unit has no incentive to change this decision. It seems natural to assume that decisions can only be taken in the nodes. In order to get a global equilibrium of such a micro specified transportation network the ex ante- and the ex post-evaluation of equilibria at every point in time and space must coincide.

Transportation network analysis is often performed within the framework of the assignment/transportation model approach. This is a very special variant of a Lefeber model. A macro planner is assumed to exist and this macro planner minimizes a total cost function (often assumed to be linear) with all the trips as arguments. The production functions are substituted for with quantitative transportation needs as constraints. The transportation system is represented by estimated minimal costs of movement between each pair of nodes in the network. If a linear goal function is used the transportation pattern regularly turns out to become too concentrated as compared with statistical data, Nijkamp (1975).

Our approach to the transportation problem is macrooriented and yet an equilibrium approach in the microsense. We have taken a dynamic interregional growth and allocation model as an â-priori organizing principle of the flows in space. That model organizes the allocation of production regionally of the different sectors of production in such a way that demand and supply are equilibrated in the different nodes of the network and with a criterion that the rate of capacity use will be maximized for any given expectations of growth of demand for the products. Alternatively it can be used in such a way that it maximizes the rate of growth of the production system as a whole. But such an allocation of production is not the only â-priori information that has to be fulfilled by the pattern of transportation. Politicians do normally require spatial interactions to be such that they are consistent with certain political goals. It is in economically developed societies common to require the economic system to work in such a way that some politically determined full employment level is achieved in each one of the nodes (regions). There is also regularly some requirements that the use of the transportation system would not be excessively resource consuming. Such a goal can be expressed as a constraint for the whole system or in a more specified situation for links connecting pairs of nodes.

We argue that any transportation pattern is in equilibrium if it is such that it preserves a balanced situation on each one of the regionally differentiated markets for commodities, and is consistent with goals like full employment, and some given level of conservation of recourses in the use of the network and will not require any further administration of the flows on the network.

One can consequently argue that an equilibrium of the transportation system should be such that it fulfills all economic and political requirements, while it distributes the traffic over the system in such a way that it requires a minimum amount of organization. We have understood the principle of maximum entropy to be such a minimum organization principle.

Another way to argue about the distribution of trade and traffic on the transportation system is to take the market equilibrium, employment and network constraints as given and regard the formally observed pattern of transportation as the structure that requires the least amount of reorganization of decisions. This approach would then define the equilibrium distribution of transportation flows to be the most conservative in the sense that it gives the minimal deviation of flows from a pattern observed in earlier periods.

These two principles will give similar results under very special assumptions.

The definition of economic equilibria

The idea of the economic system as being in or tending towards an equilibrium is deeply founded in economics. The idea of equilibrium has been much criticized for its lack of realism and yet it remains an important frame of reference for analyzing economic matters. One can, however, imagine a large number of equilibrium concepts. We shall here discuss five possible approaches in order to evaluate the equilibrium assumptions hidden in our own model approach.

The most basic and ultra-micro approach to the definition of an equilibrium in the economic system takes its starting point in characteristics of an à priori given set of decision makers, i.e. consumers and producers. Each one of these elementary decision makers are presumed to have a capacity of choice according to a weak order principle satisfying axioms of completeness and transitivity. Each one of the consumers are furthermore assumed to control some given set of resources. The producers are assumed to be endowed with a choice mechanism and a production technology, which makes it possible for them to transform resource services purchased from the households into consumer commodities to be purchased by the consumers.

An equilibrium called a competitive equilibrium, is then said to exist, if each one of the actors has chosen a macro-consistent structure of purchases and sales of commodities and resource services and - when observing the market signals in the form of prices - has no inducement to change his behaviour at the micro level.

This is a completely micro-oriented definition of an equilibrium and its extensions into a falsifiable set of hypothesis requires a large set of supplementary macro conditions. The basic use of this approach is consequently of a theoretical nature in the sense that it demonstrates the minimal postulate requirements of a theory of general equilibrium of an exonomic system.

The lack of empirical usefulness of the ultra micro approach to the definition of an economic equilibrium was early formulated by Gustav Cassel (1917). Cassel claimed that there is no need for any assumption about a complete preordering of all possible alternatives (or the equivalent utility function) at the level of decision makers. According to

Cassel there is no meaningful economic analysis below the level of a market. These ideas come very close to the interpretation of equilibrium as a state of "balancing forces" used as a fruitful analogy from physics to economics. Cassel and later on, Wald, take the market demand and supply functions as their primitives looking upon them as stimulus-response-mechanisms, which are such that to any set of stimuli (prices), there exists a unique set of responses (quantities of resource-services and commodities). Cassel thus formulates a system of static market equations and Wald was able to prove that a general equilibrium of such a market economy exists. The equilibrium of such a market economy is then a state such that supply equals demand in all markets for commodities and resource services, carrying a non-zero price.

A modern derivation of demand functions deduced from macro assumptions only, has recently been formulated by Warren C. Sanderson in his article "Does the theory of demand need the maximum principle?" (1974). His argument for an equilibrium theory based on testable market relations is quoted below:

"Indeed, the paradigm of the maximizing consumer quite nearly monopolizes the thinking of economists on matters relating to household behaviour. Economic theory would suggest that the result of such a monopoly is likely to have been a reduction in the production of testable hypotheses concerning household behaviour to a level below what it would have been had there existed competing modes of analysis. The same line of thought also leeds us to ponder the persistence of the monopoly and to ask why competing hypothesis concerning household behaviour did not arise? After all, barriers to entry were quite minimal. The answer seems to be that this analysis had a great technological superiority over other modes of explanation. Not only did it produce a product which was pleasing to the eyes of many economists, it appears to elucidate a wider range of phenomena than could be elucidated using any other technique. But this is not to say that there are no alternative modes of analysis in sight. Nor should we agree to shrug off the obviously awkward fact that whereas many of the phenomena 'illuminated' by the paradigm of the maximizing individual consumer are actually collective phenomena, the result of aggregation of many separate market actions, the standard modus operandi is to ignore the aggregation problem by hypothesizing a representative household which consumes at the average indicated by aggregated market transactions data.

Thus, rigorously the two theories are not equally broad. The theory of individual households may in principle be aggregated into a theory of group behaviour, but the theory of aggregate choice, which is immediately suited to the study of market phenomena, is not naturally disaggregated into an explanation of each household's behaviour.

The two definitions mentioned above are not the only equilibrium concepts that are possible in economics. Remaining within the statical framework one can accept the idea that the micro level of individual decision makers is the relevant perspective. One can then argue in two directions. The first one accepts the idea that the individual tries to achieve some aspired level of utility but argues that the level of aspiration is not any theoretical or even practical maximum, but rather a threshold that must be transcended in order for the decision maker to stop his reallocation procedure. This is a position held by Simon and Kornai (1970). Weibull (1977) and Radner (1975) have shown that such a bounded rationalty search leads to predictable response patterns to price signals at some level of aggregation. This principle of bounded rationality is consequently consistent with a market equilibrium approach along the lines proposed by Cassel.

A fourth way of analyzing the micro foundations of equilibrium theory is provided in formulations by Andersson (1978), Scitovsky (1976) and Hägerstrand (1970), These authors take a structural approach to the individual decision making problem. Constraints are assumed to be deducable from a physiological, mental, individual, geographical or social environment of the decision maker. To this is added a technological assumption of the kind used by e.g. Morishima (1959) and Lancaster (1965). No preferences are really needed in this approach. Structural information about society and a price vector are sufficient to create demand - and supply relations at the macro level which relate the individual to the price structure or the quantitative structure of the society being analyzed.

We can conclude from the sections above that a static economic equilibrium model can be formulated as a problem of solving a system of excess demand equations (or inequations) for some suitable aggregates of users and producers of the commodities.

We need not bother about the problems of micro economic foundations of these macro functions. The micro analysis can be totally suppressed as in the case of Cassel and Wald, it can be some assumption about bounded rationality with some predetermined threshold utility level to be achieved, it can be an extensive maximizing postulate as in Walrasian economics or preferences can be aboundoned altogether. In all cases a common ground for the analysis is the formulation of a set of market equations to be solved simultaneously.

The extension of such a market approach becomes much more complicated if we want to consider time as an integral part of our analysis.

It is then important to remember that dynamics presupposes an understanding of the behaviour of the actors, making distinctions between past, present and future. Their past can in the simplest situation be represented by some accumulated stocks. Their future cannot be known but could be represented in the form of quantifiable expectations. There must also be differential equations that represent the mechanical development of the system over time. Our approach to the dynamic economic problem uses an acceleration principle to represent technical possibilities of change over time. Expectations are assumed to be measurable entities reflecting the assumptions about growth of demand for the producers which are aggregated to sectors of supply. The development of the economy is then seen as a dynamic process with certain equilibrium properties under certain assumptions about the investment behaviour and development of expectations. We also assume that there is some quantitatively active government trying to stabilize an otherwise unstable growth process.

The problem of stability of an economic system can also be analyzed within a broader context. In recent years a deep insight has been gained into instability properties of physical and chemical processes, where the dynamics of the system is given by differential equations involving time scales of different magnitudes. A qualitative analysis of an interregional transportation - allocation system exhibits features which might be appropriate to describe in these terms. The time scale of changes in the transportation network is very long and the process of change can be represented by a slow manifold. The pattern of trade and production has a different time scale. It can change the process of adjustment as a **fast** foliation in respect to the slow transportation investment system.

The inner solution over a very short time span corresponds to an equilibrium where the slow variables related to the production and transportation system can be regarded as given, i.e. an adiabatic process. The outer solution for large time periods gives the slow manifold for which the trade and production pattern is always in equilibrium. The asymptotic behaviour of the system depicted in the two extreme time perspectives is abruptly changed when the time scale is allowed to shift continuously from one time scale to the other.

Integration between transportation and growth phenomena in a singularity analysis

Some of the ideas expressed in our discussion of equilibrium problems can now be brought together within a pedagogical model proposed by Alistair Mees (1975). The basic idea of that model is to analyze the qualitative influence of transportation networks on a dynamic allocation process.

It is assumed that a country is subdivided into regions, which can either specialize in agricultural employment or in employment in production of manufacturing products and services. The alternative to specialization in any one of these fields is to have an integrated production, i.e. to be self-sufficient and thus independent of trade and transportation to and from other regions. To perform the analysis we firstly have to specify an elementary differential equation. This equation is given below:

$$\frac{\dot{x}_c}{x_c} = (U_c - U_a)(\bar{x} - x_c) \qquad (1\,a)$$

$$\dot{x}_c = 0 \text{ if } x_c = 0$$

$$\dot{x}_c = 0 \text{ if } x_c = \bar{x}$$

In this equation the total employment is assumed to be given ($\bar{x}$) and the differential equation is thus a quadratic one in production of manufacturing goods and services (x_c).

or $$\dot{x}_c = x_c u_{ca}(x-x_c) = \underbrace{x_c u_{ca} \bar{x}}_{\text{linear}} - \underbrace{x_c u_{ca} x_c}_{\text{quadratic}} \qquad (1b)$$

where $u_{ca} = U_c - U_a$

We have assumed that there is a real income or utility difference for the representative worker (u_{ca}) between the two types of activities i.e. city activities and agricultural activities. This utility difference can be expressed as in equation (2).

$$U_{ca} = U_{ca}(T, X_c, X_a, \bar{K}_c, \bar{K}_a) \qquad (2)$$

where T represents the average friction of the transportation network.

$\bar{K}_c$ = amount of capital in the production of goods and services,

$\bar{K}_a$ = amount of capital available for the production of agricultural commodities.

Equation (2) states that the value of moving from manufacturing and services to agriculture depends on the relative employment in the two sectors and on the frictions in the transportation network. The holdings of capital have an obvious importance also. We can now depict three basic possible differential equations in this case.

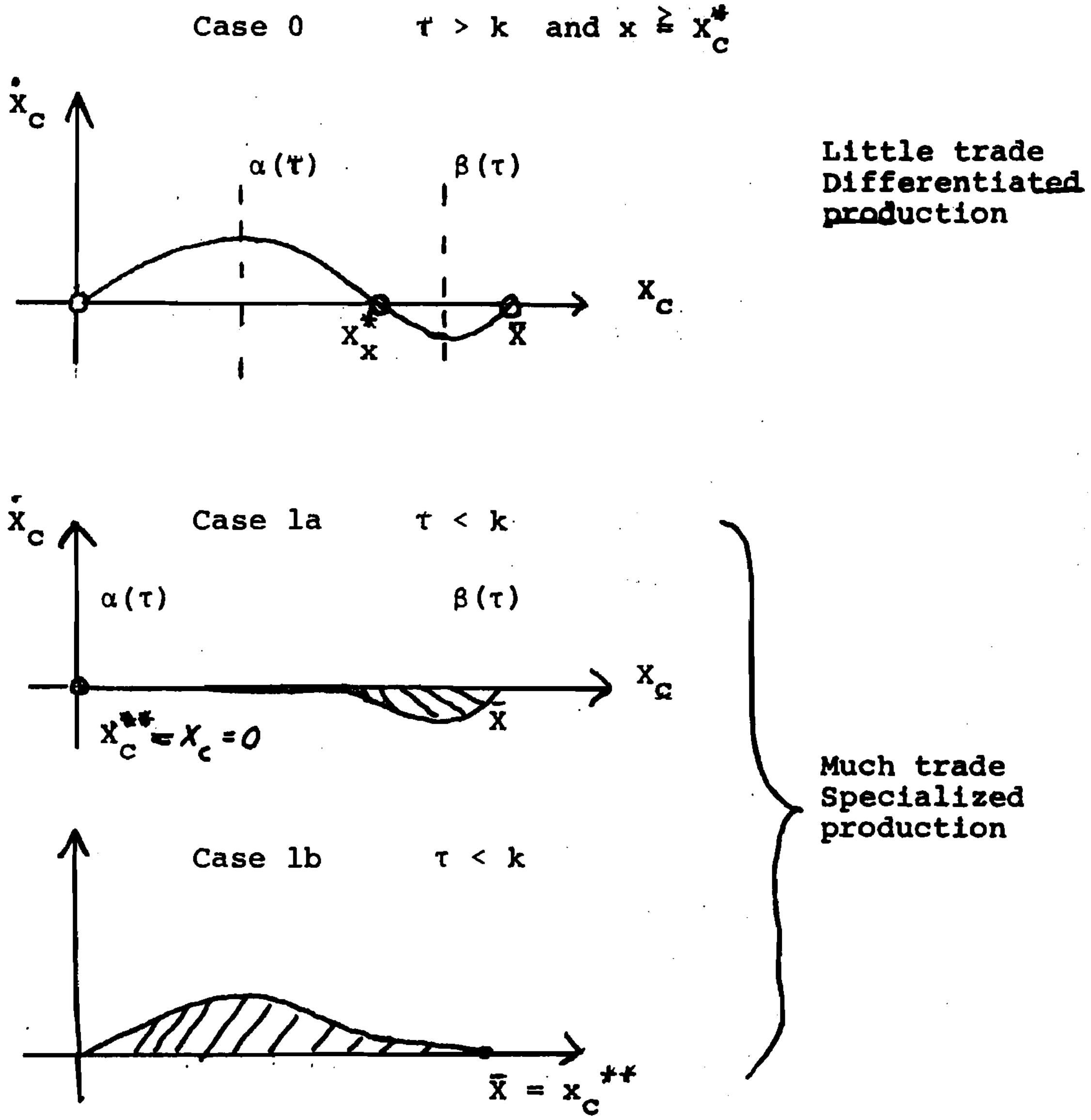

Case 0 τ > k and x ≷ X*c
ẋc
α(τ)
β(τ)
Xc
X*x
X̄
Little trade
Differentiated production
ẋc
Case 1a τ < k
α(τ)
β(τ)
Xc
X̄
X**c = Xc = 0
Much trade
Specialized production
Case 1b τ < k
X̄ = xc**

Figure 1

The first case depicts a situation where the transportation frictions are too large to allow for anything else than a limited amount of trade. This amounts to self sufficiency in the region as depicted in the figure and thus a differentiated production system with production going on both in agriculture, manufacturing and services. In order to have such a solution of our differential equation there must be a stable interior equilibrium, which is depicted by x_c^*. If we now assume that the transportation friction goes down, due to investments in the transportation system, nothing will happen until we get to a singular point where τ becomes less than some value, k. Two possibilities can then occur in this simplified case. The interior solution can disappear in two directions, depending upon the basic comparative advantages for production in the region. It can either result in a differential equation like in case 1a, where the only stable equilibrium is at the zero point, i.e. where the region has specialized in agricultural production. It can also as in case 1 b become a completely positive differential equation, in which the only stable point is one with complete specialization in the production of manufacturing goods and services. The interpretation of this approach, which can be taken into a rather general qualitative analysis, is the following. In a model of this kind, which is a quadratic differential equation, there is a great possibility that a slow increase in the transportation capacity might trigger off a certain and very drastic expansion of trade and transportation at some stage when the change in transportation friction is going through some threshold. Such a drastic structural change is important to forecast but cannot be forecasted with the aid of any extrapolation of experiencies of a statistical nature.

We have seen that the model used in equations (1) and (2) are basically quadratic differential equations. Generalizing from this model into a multidimensional framework we have a generalized differential equation problem as in equation (3).

$$\dot{x} = Q_0 x + x^T Q_1 x \text{, where } x = \{x_i^r\} \text{ and where } Q(x) = Q_1 \qquad (3)$$

Such a model is applied in the context of chemical reactions by Hahn (1974). It is shown by Hahn that such a quadratic dynamic equilibrium model is very informative about the possibilities of complete structural change. We will now preceed to analyze the general transportation-allocation and growth model within a framework that will be shown to be closely related to the formulation in equation (3).

The closed dynamic input-output model

We will specify our closed interregional dynamic input-output model in the following way:

$$x(t) \geqq A(x) \cdot x(t) + B(x) \cdot \dot{x}(t) \qquad (4)$$

The matrix functions A(x) and B(x) indicate that the requirements for current inputs and capital inputs, respectively, can be formulated as matrices of coefficients, with the convention that each coefficient (a_{ij}^{rs} and b_{ij}^{rs}) are functions of the pattern of production. The exact interpretation of this functional relationship will be given in the next section on the transportation system.

We can further assume that a non-negative amount of inputs is always required for a non-negative output. All sectors are structually treated in the same way. Households are aggregated into one or many sectors producing one of different kinds of labour inputs for the other sectors by means of consumer goods delivered to the household sectors from the other sectors of production. Equation (4) states that the scale of production must always be larger than or equal to the needs for current inputs and investment inputs.

A number of approximations are regularly done with respect to (4). In order to provide a solution, it is often assumed that the in-equation form can be transformed into an equation by making the assumption (5):

$$\dot{x}(t) = \lambda x \qquad (5)$$

λ is here an unknown rate of change of the system and the question can then be asked: what is the maximum λ that would provide a solution to equation (6) below and a solution that is also economically meaningful.

$$x(t) = A(\bar{x})\, x(t) + \lambda B(\bar{x})\, x(t) \qquad (6)$$

Equation (6) is linearized, either in the conventional input-output form to origo or within some local interval close to the equilibrium. A linear model can be shown to have one and only one feasible (i.e. semi- positive) solution with a positive eigen-value, provided that all the elements of the matrix Q in equation (7) below are non-negative.

$$\beta x = Qx \qquad (7)$$

where $\beta = 1/\lambda$ and $Q = (I - A(\bar{x}))^{-1} B(\bar{x})$

with $(I - A)^{-1}$ expressible as $I + A + A^2 + \ldots + A^n$

That Q is non-negative can be seen from the fact that A and B are both non-negative. A product of the inverse of $(I - A)$ and B must therefore necessarily be non-negative and thus the Frobenius-Perron theorem applies. The uniqueness of a positive β implies necessarily that λ is unique as well, and thus that a non-negative x-vector also is unique. This is the way that interregional growth-equilibria are normally computed. We will later on show that a linerarization as suggested in most earlier approaches to dynamic interregional input-output theory is not valid under the assumptions about the transportation system that is normally accepted in interregional transportation analysis. With a transportation system an interregional input-output theory must necessarily be non-linear. This point needs not refrain from the construction of a model of interregional growth in the interdependency tradition suggested by input-output theory. We can use a theorem proposed by Nikaido and in a different variant (with a different method of proof) by Morishima and Fujimoto (1974).

Theorem (α) if $H(x) = (H_i(x))$ is defined for all non-negative x in R^n with its values being also non-negative vectors in R^n_+. $H(x) \geq 0$ and (β) $H(x)$ is continuous as a mapping $H: R^n_+ \to R^n_+$ except possibly at $x = 0$, then $\lambda x = H(x)$ is solvable for some $\lambda \geq 0$.

Proof: Let

$$P_n = \{ x \mid x \geq 0, \sum_i x_i = 1 \}$$

The mapping ϕ is given by

$$\phi_i(x) = \frac{x_i + H_i(x)}{1 + \sum_j H_j(x)} \qquad (i = 1, \ldots, n)$$

which carries P_n continuously into P_n by (α), (β) and because

$$\sum_i \phi_i(x) = 1,$$

$$\phi_i(x) \geq 0 \qquad (i = 1, \ldots, n)$$

Hence, by virtue of the Brouwer fixed-point theorem, ϕ has fixed point $\hat{x}$ in P_n, so that

$$\hat{x} = \phi(x)$$

when

$$\hat{x}_i = \frac{x_i + H_i(\hat{x})}{1 + \Sigma H_j(\hat{x})}$$

implying that

$$(\sum_j H_j(\hat{x}))\,\hat{x} = H(\hat{x}) \text{ or } \lambda^* = \sum_j H_j(x) \geq 0$$

If we further assume that the H-functions are homogenous of degree 1, then we can also assure that the solution is unique. Morishima-Fujimoto (1974).

We can analyze this equilibrium problem in another way by introducing more of behavioural characteristics in the dynamic perspective. Equation (7) is solved in such a way that a maximum equilibrium rate of growth is evaluated and this rate of growth is such that it permits the economy to grow in this structure indefinitely. Such an optimum property reveals a rather mechanistic view of the economy. One cannot defend an assumption that investment demands are necessarily approaching such a level and structure that the rate of growth is a balanced maximal one. A more reasonable approach along a generalized Harrodian line is to introduce expectations in the sectors of production as a factor determining their rate of capital accumulation (or investments). We can then make the assumption that firms are forming their expectations in terms of the real growth of demand in different sectors of production located to different regions. We can, in the simplest case, formulate this as a diagonal matrix ($\hat{G}$) giving the expected rate of growth of demand, channelled to a specific sector of production in a specific region. A possible way of modelling the role of expectations on investments and growths can be i illustrated as in figure 2.

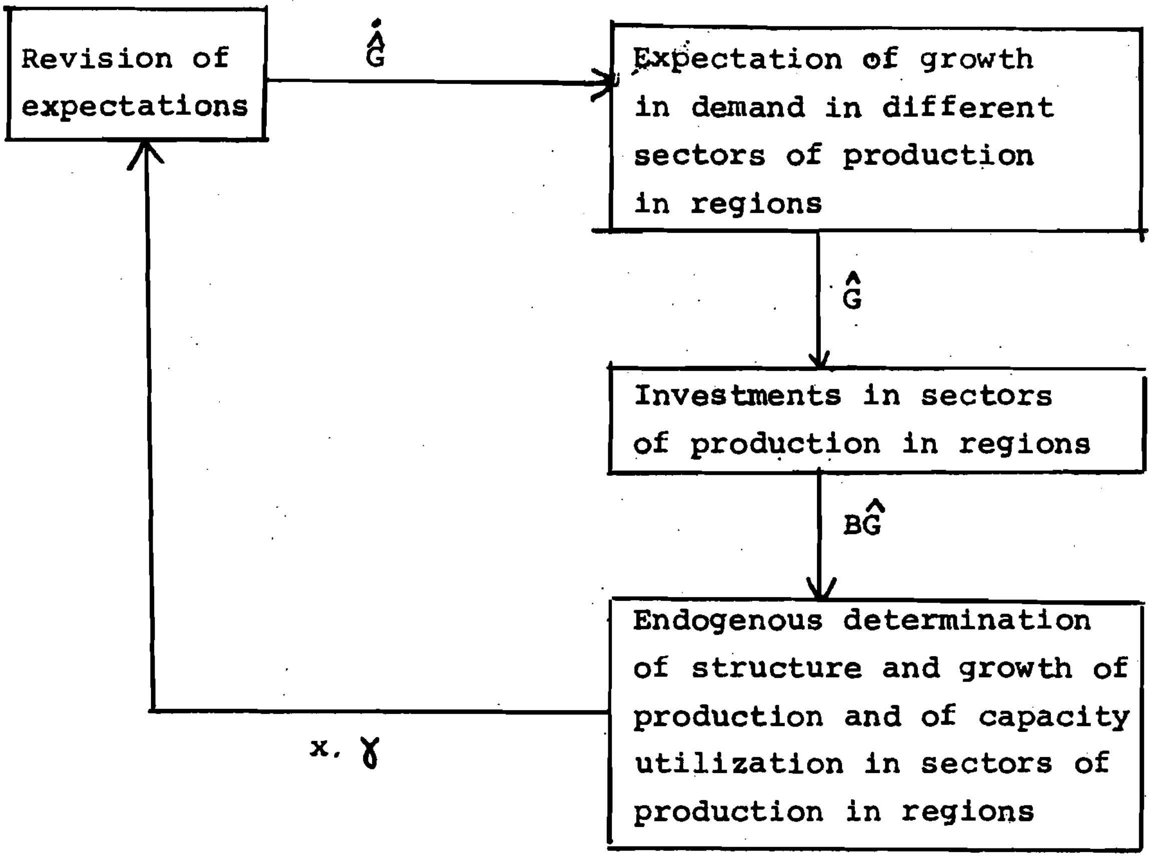
We now get the following formulation:

$$\gamma(t)\ x(t) = A(\bar{x})\ x(t) + B(\bar{x})\ \hat{G}(t)\ x(t)$$

$$\dot{\hat{G}}(t) = F(G(t),\ \gamma(t),\ x(t)) \qquad (8)$$

With a given matrix of expectations of the rate of growth of demand, we can compute a structure of production and a rate of equilibriated use of capacity that would be consistent with the matrices A, B, $\hat{G}$ in an elementary period of time. An equilibriation between the sectors of the rate of capacity utilization ($\gamma(t)$) can be achieved, if some factor of production has an unlimited mobility between the sectors. We could for instance think of a situation, in which labour moves between the unemployment quenes until the relative length of the quenes are the same in all parts of this interregional economy. If we suppose that the expectations of growth in demand is such, that an investment volume is coming about that is too small to give full employment, then we would observe the maximal γ is smaller than 1. The administrators of the decentralized and regionally allocated sectors of production would then observe the rate of capacity utilization and realized rate of growth of production that would come about with the given expectation of growth at the outset.

It is then quite possible that they would revise their expectations according to a revision-of-expectations-function, having realized rate of growth, expected rate of growth and rate of capacity utilization as its arguments. It is then highly probable that if the system would be below full capacity use there would be a decline in the expectation of future growth. A system that is below full use of its capacity might thus have a tendency to slide down towards a level of reproduction without capacity



Revision of expectations
Ĝ
Expectation of growth in demand in different sectors of production in regions
Ĝ
Investments in sectors of production in regions
BĜ
Endogenous determination of structure and growth of production and of capacity utilization in sectors of production in regions
x, γ

Figure 2.

expansion, which in this case means that $\hat{G}$ goes towards zero if the process starts in a position with γ less than 1.

We can of course also assume that the decision makers have stationary expectations equal to $\tilde{G}$. We would in this case generate a long term balanced rate of growth system with the same level of unemployment of resources in all regions and sectors of production and with a rate of growth that would be a weighted average of the expected rates of growth according to $\bar{\hat{G}}$.

Computational Principles in the Interregional Growth Model

In the actual computations we have used a numerical procedure which is closely related to the expectation feed back hypothesis formulated above.

The computational method is illustrated iy figure 3.

The process of solution is given by a set of iterative equations (iteration indicated by k):

$$x^{k+1} = Ax^k + \mu B \hat{G} x^k \tag{9}$$

$$h \cdot \left(\sum_i x_i^{k+1} - \sum_i x_i^k\right) = \dot{\mu} \sum_i x_i^k \tag{10}$$

where μG = actual growth rates

h = feed back control parameter

The transportation model

We have in the former section on the growth model presumed the existence of interdependency relations between sectors located to nodes and other sectors located to other nodes. These relations have been designated $A(x)$ and $B(x)$. This section is devoted to the problem of a determination of such interdependency parameters connection given sector in a certain region with a sector located to another region. We will simplify this problem by making the assumption that there are no frictions on the

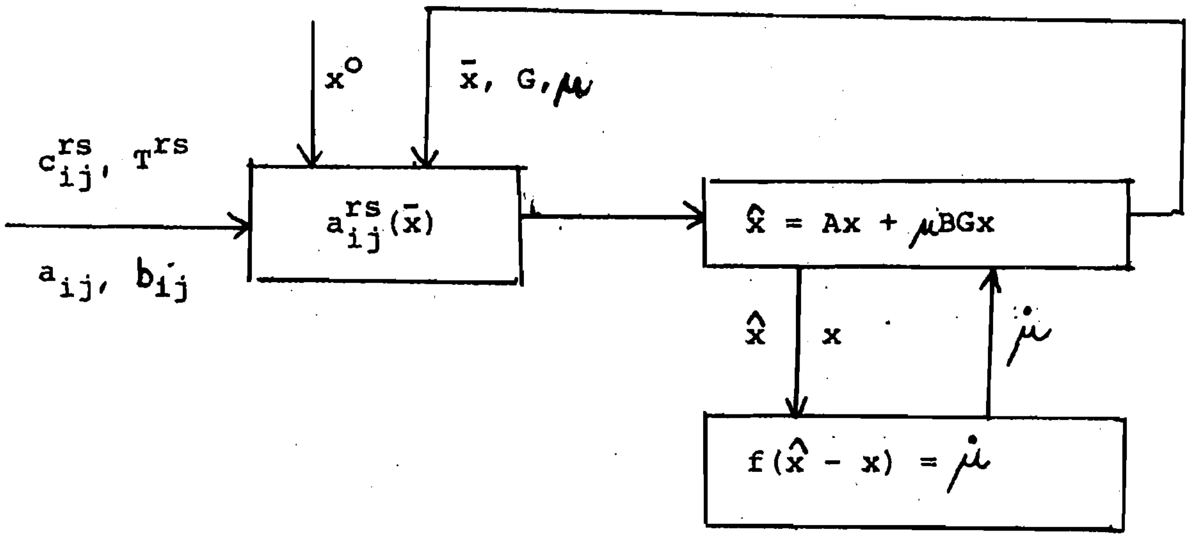


Figure 3.

transportation network nor any political constraints on the location of production. Under these circumstances we would have as a reasonable and consistent allocation of flows the following expression:

$$x_{ij}^{rs} = \frac{x_i^r}{\Sigma x_i^r} (a_{ij} + b_{ij}\, g_j^s)\, x_j^s \tag{11}$$

This formulation is such that it gives a linear relation only in the case of a non-spatial economy.

We will now show that the use of a transportation network with spatial frictions will give rise to a similar equation for the flow of commodities on the network between the nodes. Commodity flow equations will thus in all cases be such that they can be approximated by quadric expressions of a more or less parametrically complex structure. It must also be emphasized that the quadratic transportation pattern is a local property for a given transportation system.

We have in an introductory section discussed principles of equilibria on the transportation and trade network. We have also discussed the problem of the influence of changes on the persistence of trade and production equilibria. It is now time to formulate some of these arguments within the framework of the computable model approach. To simplify the analysis we will from now on take the transportation system as given. This means that the links on the network and the nodes are predetermined both in terms of capacity and in terms of location. What we are discussing now is consequently an equilibrium transportation problem in a somewhat restricted sense. We will further assume, although only as an intermediate step, that the pattern of location of all kinds of production is predetermined. This means that there is a consistency requirement both from the output side as well as from the input side of the economy:

$$\sum_{sj} x_{ij}^{rs} = x_i^r \qquad \text{Output balance}$$

$$\sum_{r} x_{ij}^{rs} = a_{ij} x_j^s + b_{ij}\, \Delta x_j^s = (a_{ij} + b_{ij} g_j^s)\, x_j^s \qquad \text{Input balance}$$

where x_{ij}^{rs} = flow of commodities from production sector i, located in region r to production sector j located in region s

x_i^r = total production in delivery sector i lacated in region r

We also assume that the politicians require a certain level of imployment to be achieved in each one of the regions while they feel completely free to vary the product flows between sectors and regions as long as it is consistent with full employment.

$$\sum_{i} n_i^r \sum_{js}\sum x_{ij}^{rs} = \bar{s}^r \qquad \text{Full employment}$$

n_i^r = labour output ratio for sector i, when located in region r

$\bar{s}^r$ = politically defined level of full employment

Finally, we make the assumption that the transportation system should be used in such a way that flows are compatible with the design, either as defined at the absolute macrolevel or more reasonably with respect to the shortest route links between two nodes. It must be admitted that the node/link capacities ought to be inequalities rather than equations, but this complicates solutions considerably, and we have avoided this formulation for numerical reasons.

Concentrating on a <u>minimal organization</u> or <u>maximum entropy</u> formulation we arrive at the following expression which gives rise to an equilibrium model, which contains an entropy measure to be maximized subject to the preceding constraints.

$$\max_{\{x_{ij}^{rs}\}} -\sum_{ijrs}\sum\sum\sum(x_{ij}^{rs} \ln x_{ij}^{rs} - x_{ij}^{rs}) - \sum_{ijs}\sum\sum\lambda_{ij}^{s} (\sum_{r} x_{ij}^{rs} - (a_{ij} + b_{ij}g_{j}^{s}) x_{j}^{s})$$

$$-\sum_{r}\mu^{r}(\sum_{i} n_{i}^{r} \sum_{js}\sum x_{ij}^{rs} - \bar{s}^{r}) - \sum_{rs}\sum\gamma^{rs}(\sum_{ij}\sum c_{ij}^{rs} x_{ij}^{rs} - T^{rs}) -$$

$$-\sum_{ir}\sum \beta_{i}^{r} (\sum\sum x_{ij}^{rs} - x_{i}^{r}) . \tag{12}$$

Derivation with respect to x_{ij}^{rs} yields:

$$x_{ij}^{rs} = e^{-\lambda_{ij}^{s}} e^{-\mu^{r} n_{i}^{r}} e^{-\gamma^{rs} c_{ij}^{rs}} e^{-\beta_{i}^{r}} \tag{13}$$

which can also be written:

$$x_{ij}^{rs} = A_{ij}^{s} B^{r} D^{rs} F_{i}^{r} e^{n_{i}^{r}} e^{c_{ij}^{rs}} \tag{14}$$

where

$$A_{ij}^{s} = e^{-\lambda_{ij}} \qquad B^{r} = e^{-\mu^{r}} \qquad D^{rs} = e^{-\gamma^{rs}} \qquad F_{i}^{r} = e^{-\beta_{i}^{r}}$$

These parameters may be called the 'correction terms'.

Solving the correction terms with the aid of the corresponding constraints leads to the following expressions:

$$A_{ij}^{s} = (a_{ij} + b_{ij}g_j^s)x_j^s/(\sum_r B^r D^{rs} F_i^r e^{n_i^r} e^{c_{ij}^{rs}})$$

$$B^r = \bar{s}^r/(\sum_i n_i^r \sum_j \sum_s A_{ij}^s D^{rs} F_i^r e^{n_i^r} e^{c_{ij}^{rs}})$$

$$D^{rs} = T^{rs}/(\sum_i \sum_j c_{ij}^{rs} A_{ij}^s B^r F_i^r e^{n_i^r} e^{c_{ij}^{rs}})$$

$$F_i^r = x_i^r/(\sum_j \sum_s A_{ij}^s B^r D^{rs} e^{n_i^r} e^{c_{ij}^{rs}})$$

We can also use the idea of a conservative equilibrium, which we have taken to be similar to the minimum information principle.

$$I = \sum_{ijrs}\sum\sum\sum (x_{ij}^{rs} \ln \frac{x_{ij}^{rs}}{\hat{x}_{ij}^{rs}})$$

where

$\hat{x}_{ij}^{rs}$ is the latest estimated flow.

Minimizing this new function with respect to the same constraints as before yields:

$$x_{ij}^{rs} = \hat{x}_{ij}^{rs}\, e^{-\lambda_{ij}^s}\, e^{-\mu^r n_i^r}\, e^{-\gamma^{rs} c_{ij}^{rs}}\, e^{-\beta_i^r}\, e^{-\varepsilon}$$

We get the same result as in the previous case if we take all $\hat{x}_{ij}^{rs} = 1$

In adding this a priori matrix we can e.g. say that whenever a_{ij} and b_{ij} are equal to zero so are x_{ij}^{rs} for all r and s . This knowledge is taken care of by the input balance relation in the first model so with the new specification that relation can be omitted.

Having the expression for

$$x_{ij}^{rs} = A_{ij}^s B^r D^{rs} F_i^r e^{n_i^r} e^{c_{ij}^{rs}}$$

We can substitute the correction terms into this expression. We then get an expression of the form

$$x_{ij}^{rs} = x_i^r h_{ij}^{rs}(a_{ij} + b_{ij} g_j^s) x_j^s$$

where h_{ij}^{rs} is the element of a matrix $H(\bar{x},\tau)$. This matrix is a function of production, $\bar{x}$, and the transportation system, τ, since it is a function of all the correction terms.

In the calculation of input-output coefficients we have:

$$\{x_{ij}^{rs}/x_j^s\} = A + BG$$

The input-output model can be written in the following way:

$$\mu x = x^T Q_1(\bar{x}, \tau)x + x^T Q_2(\bar{x},\tau)x$$

where the elements of Q_1 are $h_{ij}^{rs} a_{ij}$ and the elements of Q_2 are $h_{ij}^{rs} b_{ij} g_j^s$

The last equation shows that we have finally returned to the formulation of our original general equilibrium growth problem that we discussed on page 14.

We can thus conclude that this model, which is a quadratic growth equilibrium model, will have an equilibrium. We can furthermore be assured that such an equilibrium in the economic system will have a transportation equilibrium in the sense defined above. We can finally add the qualitative observation that if the transportation network is changed there will be smooth changes in the spatial allocation of production up to a certain point of singularity when the system might drastically change its form.

It would be a challenging possibility of development to introduce transportation investment system procedures within this kind of integrated transportation/allocation equilibrium model. Another challenging development that is pursued in another context is the problem of automatic stabilization of this kind of model.

Planning, optimality and equilibrium in an interregional context

We now turn to the problem of equilibrium approaches in planning systems. It is often assumed--especially in earlier theory of planning--that a planned system must by necessity have a global goal function to be maximized subject to certain technological constraints. This attitude has come to be disputed in later years.

Kornai has, for instance, argued that even a centrally planned economy normally lacks a clear cut general optimization procedure. It is rather the case that decentralized planning authorities try to satisfy centrally defined planning constraints, while they, at the same time, pursue their own goals. The discussion on multi-objective programming has been oriented in similar directions, stressing the necessary multiplicity of aspirations in most planning systems.

If we, more specifically, look at the sectoral and regional allocation problem modelled in this paper, an approach with multiple objectives is all the more realistic. Most large countries have sectoral and regional planning agencies having more or less well defined regional and/or sectoral competence in the decision on investments, resource use, production and sales. The g_i^r-coefficients of the $\hat{G}$-matrix should in these cases not be looked upon as "expectation values" as in the market economy versions but rather as planning or aspiration parameters, decided upon locally at the regional / sectoral, regional or sectoral level depending upon the method of decentralization in the planning system.

The fundamental problem of the central authority would then be to achieve a consistency of growth ambitions of the decentralized authorities in such a way that the economy is being run without excess demands for labour and other resources.

One should furthermore in such an approach to planning analyze the consequences for the national growth and development of letting decentralized decisions determine the g_i^r-s.

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