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REACTOR STRATEGIES AND THE ENERGY CRISIS

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1. Introduction and Historical Review

Reactor strategies as a research topic came up in the early sixties. At that time light water reactors (LWR), heavy water reactors, advanced thermal reactors, and breeders were under development, and it was not obvious what their relative role in satisfying a given demand of electricity would be. In certain quarters, for instance, there was a strong feeling that an intermediate reactor generation would be required to bridge a gap that was felt to be between the capabilities of light water reactors and that of breeder reactors. The heavy water reactor, the spectral shift reactor, and sometimes the high temperature gas cooled reactor (HTGR) were, among others, considered to be candidates for such an intermediate function. Along with it went a reflection on the desired parameters of fast breeder reactors (FBR). Along these lines a traditional attitude was prevailing by asking only for short doubling times of such FBR's. The scheme of a doubling time was introduced by the early pioneers at Argonne National Laboratory and elsewhere. They were under the impression of fairly

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limited uranium resources. In the early sixties it was clear, however, that the uranium resources would be by no means so limited as it was originally assumed in the late forties. Nevertheless, the concept of doubling time prevailed. The doubling time is the time during which a fast breeder reactor (sometimes a fast breeder population is considered instead) has produced, by virtue of its breeding gain, the amount of fissionable material that equals its inventory of such fissionable material. A second breeder reactor thus can be put into operation, and the original breeder has doubled. For the doubling time one finds the following relation:

$$T_D = \frac{10^3}{b\cdot(BR - 1)\cdot K} \cdot \frac{0{,}95}{1 + \alpha} \quad . \qquad (1)$$

The doubling time here refers only to the inpile inventory of one breeder reactor.

T_D = doubling time in days ,

b = rating in MWth/kg fiss ,

$\alpha = \frac{\sigma c}{\sigma f}$, σc = (n,γ) cross section
σf = fission cross section

BR = breeding ratio ,

K = load factor .

Only the product of rating and breeding ratio minus one characterizes the doubling time. One must, however, realize that the concept of doubling time reflects on breeder self multiplication. While this is still the prevailing concept in the USSR, the situation is different if a generation of thermal reactors is used to provide the fissionable material for the first core inventory of fast breeder reactors. In such a situation the role of fast breeder buildup is:

$$\frac{dP_B}{dt} \sim \frac{1}{M\ \text{core}} \sim b \quad , \tag{2}$$

where

P_B = installed fast breeder capacity ,

M core = 1st core inventory of fissionable material .

Relation (2) is valid because in large enough reactors the first core inventory is determined by the required power output of a reactor and the technologically feasible power rating:

$$M\ \text{core} = \frac{Q}{b} \quad (\text{Kg}) \quad . \tag{3}$$

Q is the required power output in MWth. It is therefore no longer the product of $b \cdot (BR - 1)$, but the rating b alone that governs the role of breeder buildup.

Along such lines, F.R. Dietrich [1] studied the interplay of thermal reactors, advanced converters, and fast breeders. He considered, among other parameters, in particular the amount of natural uranium that would be required until a population of fast breeders alone can satisfy the electrical power demand. If that situation has been reached, the supply of natural uranium as the fuel for such a population of fast breeder reactors only is no longer a problem. For fast breeders the utilization of a given amount of natural uranium is better by a factor of 50-80 if compared with a thermal reactor. For this very reason the fraction of the electricity generating costs that goes into the provision of uranium ore is extremely low, about $1^{o}/oo$. Therefore, even extreme ore prices can be afforded. Consequently vast amounts of low grade uranium become accessible, and the amount of energy that is so available is the order of 10^6 Q ($1\ Q \equiv 10^{18}$ BTU or $3.35 \cdot 10^7$ MW year) [2, 3].

Shortly after Dietrich's paper, it was the Report to the President [4] and its discussion during the Third Geneva Conference (1964) that further introduced the research topic of reactor strategies. At the same time it was R. Gibrat [5] who studied the coupling between thermal reactors and fast breeders by a transparent and simply analytical model that very much helped to understand the mechanisms involved. Other authors followed [6, 7]. A major study was presented by the Nuclear Research Center of Karlsruhe [8]. The Karlsruhe investigations came to a preliminary end in a paper by P. Jansen [9]. A comprehensive model for the supply of electrical energy by both nuclear and fossil power plants was presented by Harde and Memmert [10]. A more comprehensive review of the work in Germany is given by H.F. Zech during this conference.

As the research on reactor strategies evolved, it became more and more obvious that the consideration of the consumption of natural uranium served as a heuristic principle only; more and more considerations of the amount of required separative work, the timing for the related fuel cycles, and, above all, cost benefit ratios [11] come to the forefront of attention.

In the following a few significant qualitative results of the investigations on reactor strategies that were made during the sixties shall be summarized:

a) The expected consumption of natural uranium in the western world for the production of electrical power until the year 2000 is a few million tons (~ 4-6 million). This basically assumes that the majority of the electrical power production is taking place in LWR's. In that case the uranium consumption beyond the year 2000 continues to grow heavily.

b) If the LWR's are coupled to FBR's using the oxides as fuel (UO_2/PuO_2), the consumption of natural uranium until 2000 has reached values of 3-4 million tons and decreases only slowly. If the carbides are used as fuel (UC/PuC) instead, about 3 million tons will be consumed until 2000 and the consumption decreases considerably

c) Separative requirements are sharply limited only if the FBR using carbides is developed and deploited.

Starting dates for such deployment could be between 1980 and 1985, and do not influence the results very much.

d) A generation of advanced thermal reactors with a high Pu output such as heavy water reactors do not appear to be a must. Nevertheless, they would speed up the introduction of fast breeders if the Pu requirements for the first core inventory of FBR's comes out to be the limiting factor.

e) As a figure of orientation, it is reasonable to assume that by the year 2000 roughly 50% of the installed capacity could be fast breeders and about 90% of the annual installation at that time would be fast breeders.

f) Benefit/Cost ratios, where the benefit relates to the price advantage of the FBR over the LWR and the cost to the R & D costs of the breeder, are low for the FBR that uses the oxides and comparatively high for the FBR that uses the carbides.

One should recall that the tacit assumptions for these reactor strategies of the sixties were the following:

1) It is only the generation of electrical power that is to be taken into account.

2) Any generation of electrical power in nuclear plants must be justified on strictly economical grounds.

If nuclear power does not meet that criterion, it is fossil power, especially oil and gas, that is then in the business automatically.

3) It is the procurement of Pu from thermal reactors and in particular the LWR's that governs the role of deployment of FBR's.

4) One must envisage a continued economic growth.

2. Changes Since the Sixties: The Energy Problem

During the few years since the late sixties, a number of sometimes-drastic developments have taken place. The developments that are of major relevance here are the following:

a) In the US, the expected LWR capacity for 1980 is at ~140 GWe. The figures for Germany and Japan respectively are ~19 GWe and ~25 GWe, and that for the whole world ~230 GWe (LWR).

b) In the US, Germany, and Japan, i.e. the countries with large LWR populations, the development of FBR is delayed. Commercially significant deployment is now expected only for the nineties, while France and England still expect a significantly earlier date. These two countries have a delayed or no introduction of LWR's.

c) Energy prices have risen generally, sometimes drastically. In 1970 the price for crude oil was

$1/barrel, now it is at $8/barrel.

d) Shortages in the supply of fossil fuel have to be envisaged. Partly this is due to political circumstances and partly to the physical limitedness of such fuels. The time scale for these two types of shortages is quite different.

e) Unlimited economic growth is no longer an unchallenged assumption.

f) Applications for nuclear energy other than the generation of electrical power are now more and more envisaged.

It therefore appears necessary to review and, if necessary, to expand the work on reactor strategies of the sixties. The guideline for doing that must be the consideration of the whole energy supply problem and not only the fraction of the nuclear generated electricity. Further, besides the competitivity, it is now also availability of energy that matters. To that end one has to reflect on the availability of fossil fuel. For the purpose of this paper it will be sufficient to consider the following figures on likely fossil fuel reserves [12]:

	in $Q \equiv 10^{18}$ BTU	%
coal	198	88.8
oil	11	5.2
gas	10	4.7
others	3	1.3
total	216	100.0

The present and expected consumption of the US and the world provides a yardstick for judging on these figures.

in $Q/a \equiv 10^{18}$ BTU/a

	1970	2050
US	0.07	0.2
world	0.25	5(?)

This suggests we consider the problem of the supply of energy in three phases [13]. The near range phase is characterized by the fact that any new technological step requires about 15 years before it can be felt in the commercial domain. One such new technology could be large scale synthetic hydrocarbon production as a substitute for oil, making use of nuclear energy as a source for chemical process heat or not. This could lead to the renewed and extended use of coal, and this would then characterize a medium range phase of the energy problem. A third phase of the energy problem comes into picture when fossil fuel becomes a scarce material, and non-fossil fuel has to take over all phases of production, not only the generation of electricity. It may be useful to recall that the present partition between the primary energy demand for electricity, transport, household, commercial and industry is, as a rule of thumb, 1:1:1:1. The following table summarizes these observations:

The Energy Problem

	time interval	key word
near range	1970-1985 (?)	oil
medium range	1980-2000 (?)	coal/nuclear
long range	1995- (?)	non-fossil fuel

There appear to be four options for the non-fossil supply of all of the energy: nuclear fission, nuclear fusion, solar power, and geothermal energy in the earth's crust [13]. In the rest of this paper we will consider only the option of nuclear fission. Nuclear fission can also provide nonelectrical power [14]. It is in particular the HTGR that has this potential. As a source of chemical process heat at temperatures up to 1000° C, it can be used to split the water molecule by staged chemical processes. Hydrogen has extremely attractive features as a secondary fuel [14]. It is therefore feasible to assume a situation where, in the long range of the energy problem, nuclear fission is the source of primary energy with both hydrogen and electricity as secondary fuel.

3. New Functions of Known Reactor Types

The considerations of section 2, if taken seriously, lead to drastic consequences. In so doing one is first led to consider the long range phase as this establishes the long range target for a consistent approach to the energy problem.

P. Fortescue [15] and work at Karlsruhe have pointed to the possiblity of using the breeding gain of FBR's, not for the doubling of FBR's but, for the supply of U^{233}. To that end two versions of an FBR seem feasible. One version provides for production of U^{233} in the radial blanket of a FBR. The Pu cycle of such an FBR must be, of course, self-sustained. The breeding ratio of the core and its axial blanket must therefore be in operational terms equal to one. The other version is to use the inner portion of the core, roughly one half of the core, for fueling with Pu/Th elements, and to let the radial blanket breed Pu. Work is going on at Karlsruhe to examine both versions in greater detail [16]. It appears that in such a way enough U^{233} can be provided to make up for the annual requirement of an HTGR which operates on the basis of U^{233} and Th. The ratio

$$B = \frac{P_H}{P_B} , \qquad (4)$$

where

P_H = installed HTGR capacity in GW, thermal , and

P_B = installed FBR capacity in GW, thermal , could well be between 1.0 and 1.5 under certain conditions even higher than that.

In such a scheme the FBR does not double any more. The FBR sustains the steady operation of an HTGR instead. This implies a static, non-expanding situation where only U^{238} and Th--i.e. abundant natural isotopes--are consumed as fuel and both types of secondary fuel are produced: electricity from the FBR and hydrogen from the HTGR. Such an approach really employs the genuine advantages of both reactor types: the FBR breeds Th into U^{233}, and the HTGR is there for high temperatures that are a necessary condition for splitting the water molecule. To make electricity from the HTGR does not genuinely require high temperature. In fact, the HTGR is degraded to be only a competitor for the LWR in that case. And under the asymptotic condition of a society with no growth, the doubling of breeders is not desirable anyhow.

Figure 1 characterizes this asymptotic solution of the long range phase of the energy problem.

If that is considered attractive, three questions come up:

- What could be an approach during the medium range phase of the energy problem?
- What is the timing of this approach during the medium range phase?
- What is then the interplay with the finite resources of fossil fuel?

Here it is proposed to appreciate the solid position that the LWR is having now and more so in the future. The fairly drastic assumption is being made that for the next one or two decades all increased demands for electric power will be met by LWR's. LWR's produce roughly 170 Kg/GWe of plutonium. The further drastic assumption shall be made that no Pu recycling in LWR's shall take place. Instead, all Pu produced in LWR's shall be used to establish the first core inventories of new FBR's. If P_L denotes the installed LWR capacity in $GW_{thermal}$, then P_L induces in this way $\frac{dP_B}{dt}$. More than that, the FBR functions as a waste box for the disposal of the Pu produced in the LWR. There the Pu does not increase further, but just stays there as a permanent catalyst for the use of U^{238} and partly Th. The conditions of such a transient phase can therefore be summarized as outlined in Figure 2. Let us recall:

- The FBR does not double any more. The rate of its increase $\frac{dP_B}{dt}$ is proportional to P_L.
- The HTGR can be installed proportionally to the FBR, $P_H = BP_B$. It produces, by virtue of its high temperatures, hydrogen.
- The increase of the whole electricity demand is met in the beginning only by LWR's.
- Pu that is produced in the LWR's goes into the FBR. It stays there and does not double.

4. A Highly Stylized Analytical Model

The approach outlined in section 3 calls for a model. At the International Institute for Applied Systems Analysis work is going on to establish such a computer model. A necessary first step, however, is to have a highly stylized analytical model that allows for the understanding of the mechanisms involved. It serves as a sketch for the above mentioned computer model.

Now we shall describe this analytical model. The demand for power, either electrical or nonelectrical, may be described by a polynomial expression.

$$P(t) = Po + P_O R_O t_1 \, T\left(\frac{t}{t_1}\right) + (P_1 - Po) \, S\left(\frac{t}{t_1}\right) \quad , \qquad (4)$$

$$\text{for } 0 < t < t_1 .$$

Po denotes the value of P(t) at $t = 0$, $P = P_1$ at $t = t_1$, and Ro is the relative yearly increase of P at $t = to$. We assume that at $t = t_1$, not only $P = P_1$ but also $(\frac{dP}{dt})t_1 = 0$, or in other words, a no growth pattern at $t = t_1$.

We then find

$$T\left(\frac{t}{t_1}\right) = \left(\frac{t}{t_1}\right) - 2\left(\frac{t}{t_1}\right)^2 + \left(\frac{t}{t_1}\right)^3 \quad , \qquad (5)$$

and

$$S\left(\frac{t}{t_1}\right) = 3\left(\frac{t}{t_1}\right)^2 - 2\left(\frac{t}{t_1}\right)^3 \quad . \qquad (6)$$

T is of a transient nature that allows for Ro at $t = to$ and S leads into the steady no growth state at $t = t_1$. Figure 3 shows T and S as functions of time.

We now assume a model society G. At t = to G may have $250 \cdot 10^6$ people with 10 KW/capita total power demand. Within 40 years the population shall have leveled off at $362 \cdot 10^6$ people, implying an average growth rate for $0 < t < t_1$ of 0.92%. For Ro we assume 4.5%. Two subcases are considered: in one subcase the per capita demand for power has increased to 20 KW/capita, and in the other subcase it has remained constant at 10 KW/capita. The notation for the total power demand may be P tot (t).

The electrical power demand, Pel(t), shall start at t = 0 with Pel(0) = 0.25 P tot (0) and Ro = 8%. One may recall: Pel is the primary energy demand that goes into the generation of electricity, and it is measured in GW, thermal. At $t = t_1$, we define $Pel(t_1) = 0.5\ P\ tot\ (t_1)$. In other words, the relative share of electricity in the total power production shall double in either of the two subcases considered.

P pr is the notation for the process heat, that is we have

$$P\ tot = Pel + P\ pr \quad . \tag{7}$$

Figure 4 describes the first subcase (20 KW/capita at $t = t_1$), while Figure 5 describes the second subcase (10 KW/capita at $t = t_1$).

We now assume $P_L(0) = 0$, at t = 0 there is only one fossil power. Further, we make the assumption that the capacity of the nuclear industry may follow the function $A_1(t)$. This implies

$$\frac{\delta P_L}{\delta t} + \frac{\delta P_B}{\delta t} = A_1(t) \quad , \qquad 0 \leq t \leq t_{E,1} \tag{8}$$

where $t_{E,1}$ denotes the time when no more LWR's or FBR's are being built. As we will see later, $t_{E,1} > t_1$. For the highly stylized model presented here, we further assume A_1 = const for $0 \leq t \leq t_{E,1}$, and will later vary the value of A_1. A_1 refers to the number of LWR's and FBR's that can be built per year. It is measured in GWth per year. We specialize further by assuming

$$\frac{\delta P_L}{\delta t} = A_1 \quad , \qquad \text{for } 0 \leq t \leq t^*. \tag{9}$$

The amount of Pu that is yearly produced by one GW-LWR shall be denoted by $a_{L,Pu}$, and cumulate amounts of Pu by M_{Pu}. At $t = t^*$ we then have

$$P_L^* = A_1 \cdot t^* \tag{10}$$

and

$$M_{Pu}^* = \frac{A_1 \cdot t^{*2}}{2} \cdot a_{L,Pu} \quad . \tag{11}$$

P_L^* produces $A \cdot t^* \cdot a_{L,Pu}$ of Pu per year. This is sufficient to provide the first core inventories of $\left(\frac{\delta P_B}{\delta t}\right)^*$:

$$\left(\frac{\delta P_B}{\delta t}\right)^* = \frac{A_1 \cdot t^* \cdot a_{L,Pu}}{i_{B,Pu}} \quad , \tag{12}$$

if $i_{B,Pu}$ is the first core inventory (incore + $\frac{1}{3}$ incore for out of pile purposes) for 1 GWe FBR. Here in this analytical model we assume the same thermal efficiency for LWR's and FBR's. We further specialize by assuming that

$$\left.\begin{aligned}\frac{\delta P_1}{\delta t} &= 0 \quad , \\ \frac{\delta P_B}{\delta t} &= A_1 \quad ,\end{aligned}\right\} \quad \text{for } t^* \leq t \leq t_{E,1} \ . \tag{13}$$

Equations (12) and (13) imply that

$$t^* = \frac{i_{B,Pu}}{a_{L,Pu}} \quad . \tag{14}$$

For $a_{L,Pu}$ = 170 Kg/GWe year and $i_{B,Pu}$ = 3.0 to/GWe, we have

$$t^* = 17.6 \text{ years} = 18 \text{ years} \ .$$

Let us assume that 1970 is $t = 0$ and therefore 1988 is t^*. Then the LWR capacity of 1988 is sufficient to fuel all FBR's that are built at a yearly rate of A. If the FBR comes earlier, more LWR's than in operation at that earlier time have to be installed if the annual Pu output $a_{L,Pu}$ is meant to fuel these breeders. But t^* = 18 years is not an unrealistic assumption. We therefore do assume (13) here in this analytical sketch.

Together with the breeders now, HTGR's can be built and operated for the increasing production of nonelectrical power. We assume that their first core inventory is U^{235} and that the annual fueling is U^{233} which comes from the FBR's. We then have (4):

$$P_H = B \cdot P_B \quad ,$$

and we therefore assume

$$\frac{\delta P_H}{\delta t} = A_2 = \text{constans} \qquad \text{for } t^* \leq t \leq t_{E,2} \tag{15}$$

and

$$A_2 = BA_1 \quad , \tag{16}$$

where $t_{E,2}$ denotes the time when no more HTGR's are being built because the demand curve P pr is met by HTGR's and no fossil fuels are necessary any more.

Let t** now be the time when

$$P_L^{**} + P_B^{**} = Pel\ (t^{**}) \quad , \tag{17}$$

or in other words, when all of the electrical power requirements are met by LWR's and FBR's. (Note: $P_1^{**} = P_1^*$ because of (13).) For the purposes of this highly stylized model we now further assume

$$A_1 \leq \frac{P_1 - Po}{t_1} \quad . \tag{18}$$

This leads to

$$t^{**} \geq t_1 \quad .$$

Equation (18) implies for the first subcase (20 KW/capita) $A_1 \leq 36$/year and for the second subcase (10 KW/capita)

$A_1 \leq 18$/year. Such rates for the building of reactors are reasonable for the model society G considered here.

At $t = t^{**}$ we now continue to build FBR's at a rate of A_1. But they are meant to replace LWR's and not to follow demand increases. Due to the symmetry of the model considered here this leads to

$$P_L = 0 \qquad \text{at } t = t_{E,1} \tag{19}$$

and

$$t_{E,1} = t^{**} + t^{*} \quad .$$

We now recall that we have left over the Pu stockpile fuel that was produced before $t^{*} (0 < t < t^{*})$. After t^{**} the LWR's produce again the equivalent amount. We thus have at $t = t_{E,1}$ the following amount of Pu that comes from the LWR during the buildup and reduction phases:

$$M^E_{Pu} = \frac{A_1 t^{*\,2}}{2} \cdot a_{L,Pu} + \frac{A_1 t^{*\,2}}{2} \cdot a_{L,Pu} \quad .$$

The amount of Pu that is required to continue with the installation of the FBR for $t^{**} \leq t \leq t^E$ is

$$A_1 \cdot t^{*} \cdot i_{B,Pu} \quad .$$

Both amounts equal, if t^{*} observes (14). But this was the definition of t^{*}. In the case considered here, no Pu is left over, thus the Pu balance is closed. All Pu ends up in the FBR's.

In the case of $B = 1$, that is $P_H = P_B$, the buildup of HTGR's meets the demand for nonelectrical power, P pr, exactly at $t_{E,1}$. This is the case because for $t \geq t_1$, P pr = Pel, or Pel = 0.5 P tot.

In Figure 6 we now illustrate the highly stylized and symmetrical case considered here which leads to completely closed material balances.

Not so highly symmetrical cases appear if

$$t^* \neq \frac{i_{B,Pu}}{a_{L,Pu}} \quad .$$

LWR's have to be built beyond t* if the breeder comes too early. Such continued LWR buildup follows the function

$$1 - e^{-\frac{a_{L,Pu}}{i_{B,Pu}} t} \quad .$$

A similar observation is true beyond t** when the LWR's are finally replaced by FBR's. In that connection it must be mentioned that the various time delays in the nuclear fuel cycles have not been taken into account. Further, $B > 1$ leads to savings in the U^{235} first core inventories of the HTGR. Values of B that are between one and two seem technically feasible [16] . This leads to values of $t_{E,2} < t_{E,1}$ and again explicit steps have to be taken for $t > t_{E,2}$ during which the production of U^{233} in breeders would be slowed down. $B = 1$ is the one symmetrical case because we had assumed here $P\ pr(t_1)/Pel\ (t_1) = 1$.

If a different asymptotic value of this ratio is envisaged, B should be adjusted accordingly. We also assumed $A_1 \leq \frac{P_1 - Po}{t_1}$. If a larger nuclear construction capacity is considered, then the curve of the electrical demand is met earlier accordingly; the construction of new nuclear power plants has to follow that demand curve for awhile. We also assumed implicitly that the thermal efficiency of the FBR equals that of the LWR, which is obviously an approximation. The logic of all these sub-cases will be taken care of in our computer program. For the heuristic purposes of this outline, it is sufficient to consider the more special case considered here. The computer program will also take care of the various optimizations involved. There are stockpiles of Pu, fossil fuel and natural uranium,and investments in the LWR, in the huge isotope separation plants and uranium mining. These investments will be operative for a limited period of time only. When the asymptotic scheme of Figure 1 becomes operative, no LWR's are in operation any more, no enrichment is required, and uranium mining falls to a different order of magnitude. Despite the highly stylized nature of our analytical model, we did make a numerical evaluation. At $t = t_{E,1}$, no fossil fuel is required any more. It is therefore meaningful to evaluate the amount of fossil fuel that is consumed within $0 \leq t \leq t_{E,1}$. It is equally interesting to evalue the amount of natural uranium that has to be provided for within that time interval. To make quick comparisons, one finds it desirable to express both the amounts of fossil fuel and the natural uranium in terms of Q. While this is straightforward for the fossil fuels, it is a difficulty in the case of uranium. Roughly 20% of the required natural uranium is needed to provide for the first core inventories of the LWR's and HTGR's. These first core inventories have not a fuel but a catalytic function as the critical mass has to be there before the annual reload can be burned. We therefore give only an artificial equivalent. A typical LWR with a given critical mass requires $2.5 \cdot 10^6$ to U_{NAT} to produce 1 Q of heat:

$$2.5 \cdot 10^6 \text{ to nat uranium} = 1\ Q \text{ (LWR equivalence)} \quad .$$

With these clarifications, one can now examine Table 1 and Figures 7 and 8, and Figures 9 and 10 as well. Figures 7 and 8 refer to the separative work requirements as a function of time for cases 1 and 3 of Table 1, and Figures 9 and 10 to the demand for fossil fuel accordingly. Table 2 lists the various numerical assumptions that were made in the calculations.

While keeping in mind that these data refer to a model, it may be still worthwhile to draw a number of conclusions.

1) Case 1 refers to an asymptotic value of 20 KW/cap for $362 \cdot 10^6$ people and an annual installation of 36 nuclear power stations, 1 GWe each. Such a rate

is reasonable. It is close to figures anticipated for the US [17]. Under these conditions it takes 58 years to arrive at the FBR/HTGR all-nuclear energy supply scheme of Figure 1. To arrive there roughly 4 Q's of fossil fuel are required. If a narrow-minded extrapolation from $2.5 \cdot 10^8$ people to a world total of 10^{10} people is attempted, this would then mean 160 Q. This is roughly the amount of fossil fuel that is at all available. A more reasonable approach to the global problem of supplying 10^{10} people with sufficient amounts of energy would provide for more disaggregation. One could think of 5-10 groups of power consumers that all follow principally the same model but with a different phasing, and it would then be interesting to study not only the transitions as described by the model within each group, but also the transitions between the groups. Nevertheless, the observation shall be made here that the fossil reserves of the globe could be just sufficient for transitions, not for long term steady state supplies, and the time scale for such transitions could be between 50 and 80 years. Considerations of the kind that are indicated by the model could also be used for the assessment of future technological developments and the time scales that must be imposed on them. Case 3 gives the figures for the 10 KW/capita case. Simply a factor of two is gained.

2) Attention must be drawn to the large amounts of uranium that are required. For the model society of $250 \cdot 10^6$ people considered here, this uses up all cheap uranium (< \$30/lb) that seems to be available on the globe.[1] Extrapolation to 10^{10} people leads into price classes of uranium as high as \$100-\$200/lb. As the overwhelming amount of this uranium is needed to fuel the LWR's, alternative concepts have probably to be envisaged. There are many possibilities for that. To lower the required fast breeder inventory, for instance, by employing the carbides as fuel is one such possibility. (The value considered here of 3 to/GWe is rather high and refers to the oxide breeders.) To increase the conversion factor of LWR's and thereby to produce more than 170 kg/GWe • a is another such possibility. There are many more. The model envisaged here could help to assess the various priorities for such developments in the new light of "Reactor Strategies and the Energy Crisis."

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[1] According to K. Hubbert [18].

3) Attention must be drawn to the fact that in Table 1 the Q's of fossil fuel and the Q's for natural uranium do not make up for the total energy consumption. The remainder is the share of the FBR's and the operation of the HTGR's.

4) Comparing cases 1 and 2 as well as cases 3 and 4 points to the influence of the capacity of the nuclear industry. For $t_{E,1}$ a difference of 15 years appears and, as the fossil fuel consumption becomes larger, one additional Q is required for the model society G.

5) Cases 5 and 6 do not exactly meet the conditions of a completely closed Pu and U^{233} balance, but they are close to that. They were designed to have the same consumption of fossil fuel, namely 3.97 Q. A reduction from 20 KW/capita to 10 KW/capita increases for such fixed consumptions of fossil fuel the value of $t_{E,1}$ from 61 to 82 years, and requires only one third of nuclear annual installment. Or in other words, such drastic savings of energy per capita stretches the time scale for only 21 years, then with 10 KW/capita the same problem arises as in the case of 20 KW/ capita. Implicit in this reasoning is, of course, that the asymptotic scheme of Figure 1 provides without difficulties even very large amounts of

energy. It should be kept in mind, however, that while the production is not a problem there, the handling of energy (or embedding) may very well pose a major problem [13]. But this is not the point of this paper.

FIG.1 ASYMPTOTIC INTEGRATED REACTOR SYSTEM

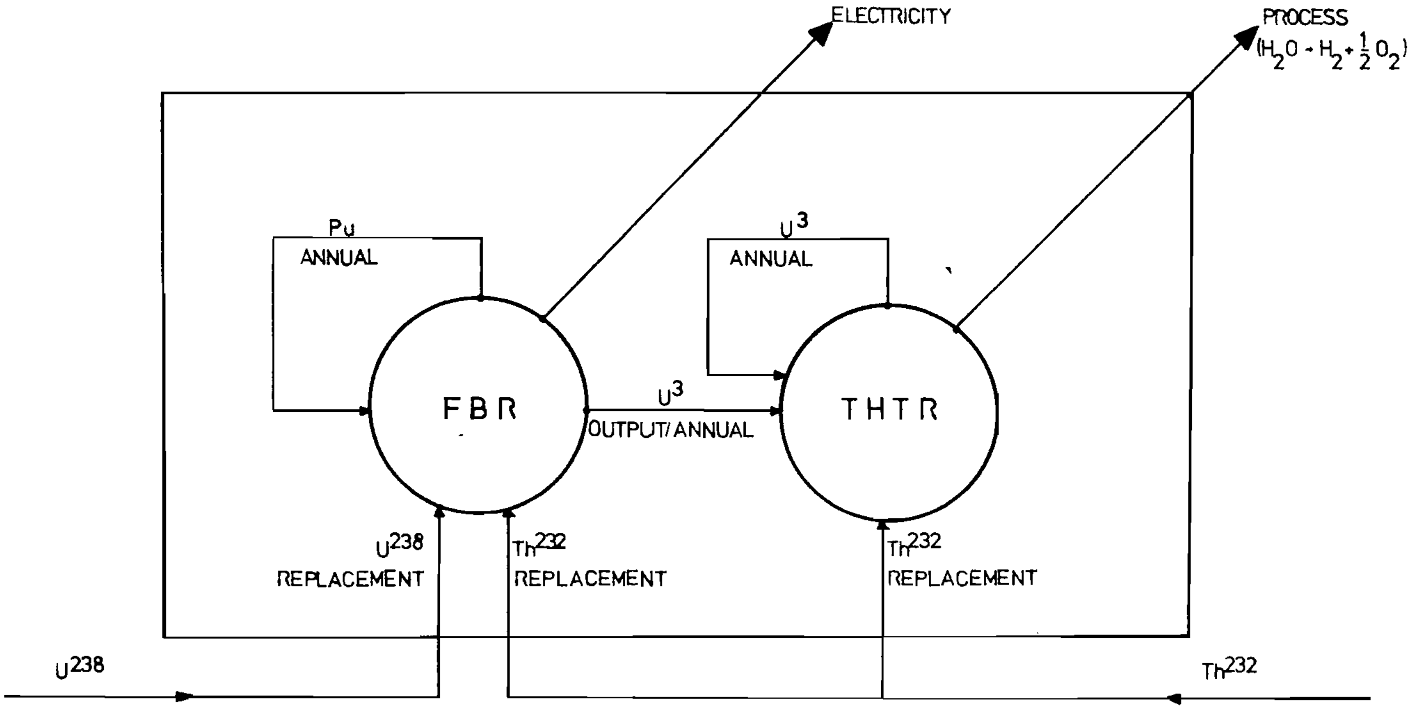

FIG. 2 TRANSIENT REACTOR SYSTEM

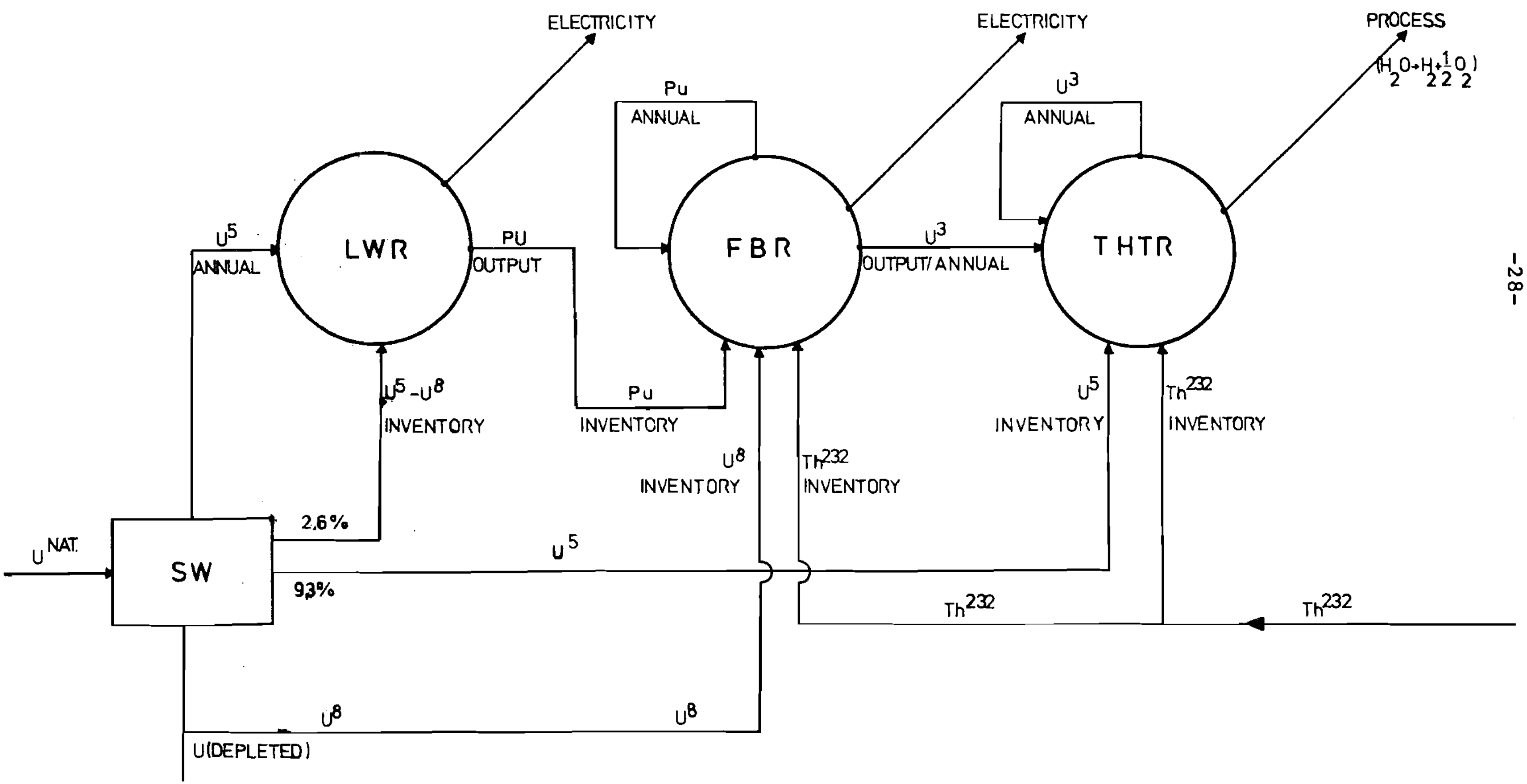

FIG.3 POLYNOMIALS

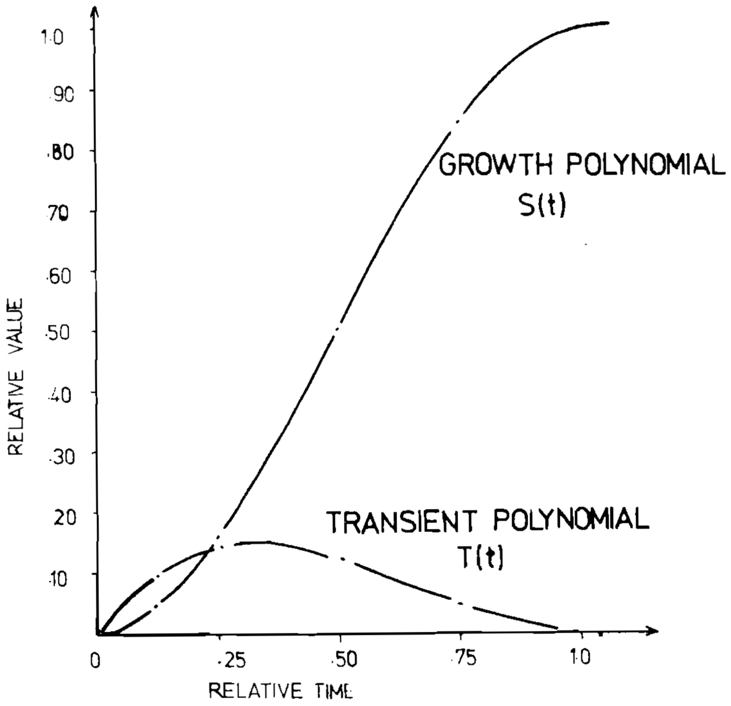

FIG. 4 POWER DISTRIBUTION FOR 20 KW_{th} /CAPITA

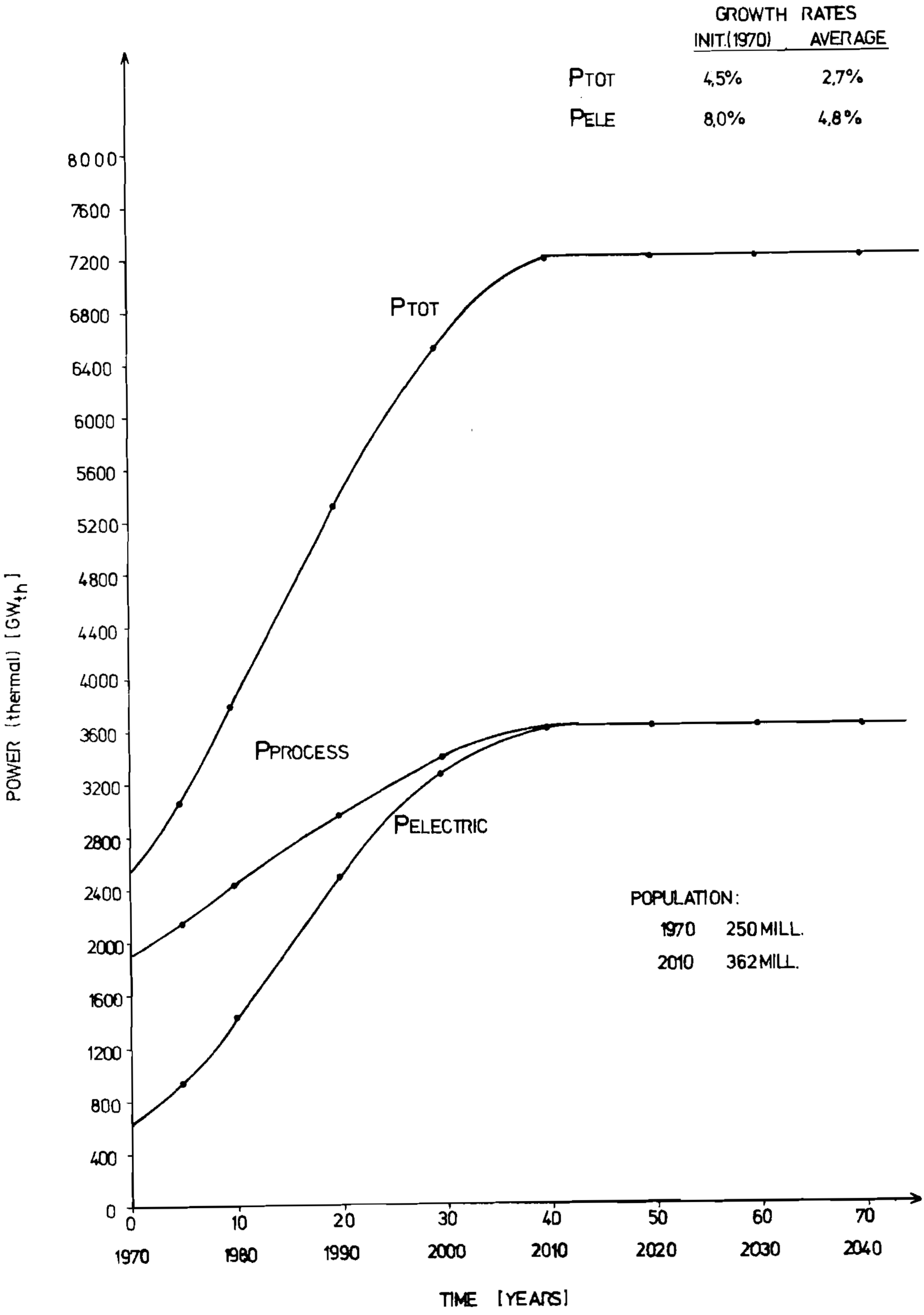

FIG.5 POWER DISTRIBUTION FOR 10KW$_{th}$ /CAPITA

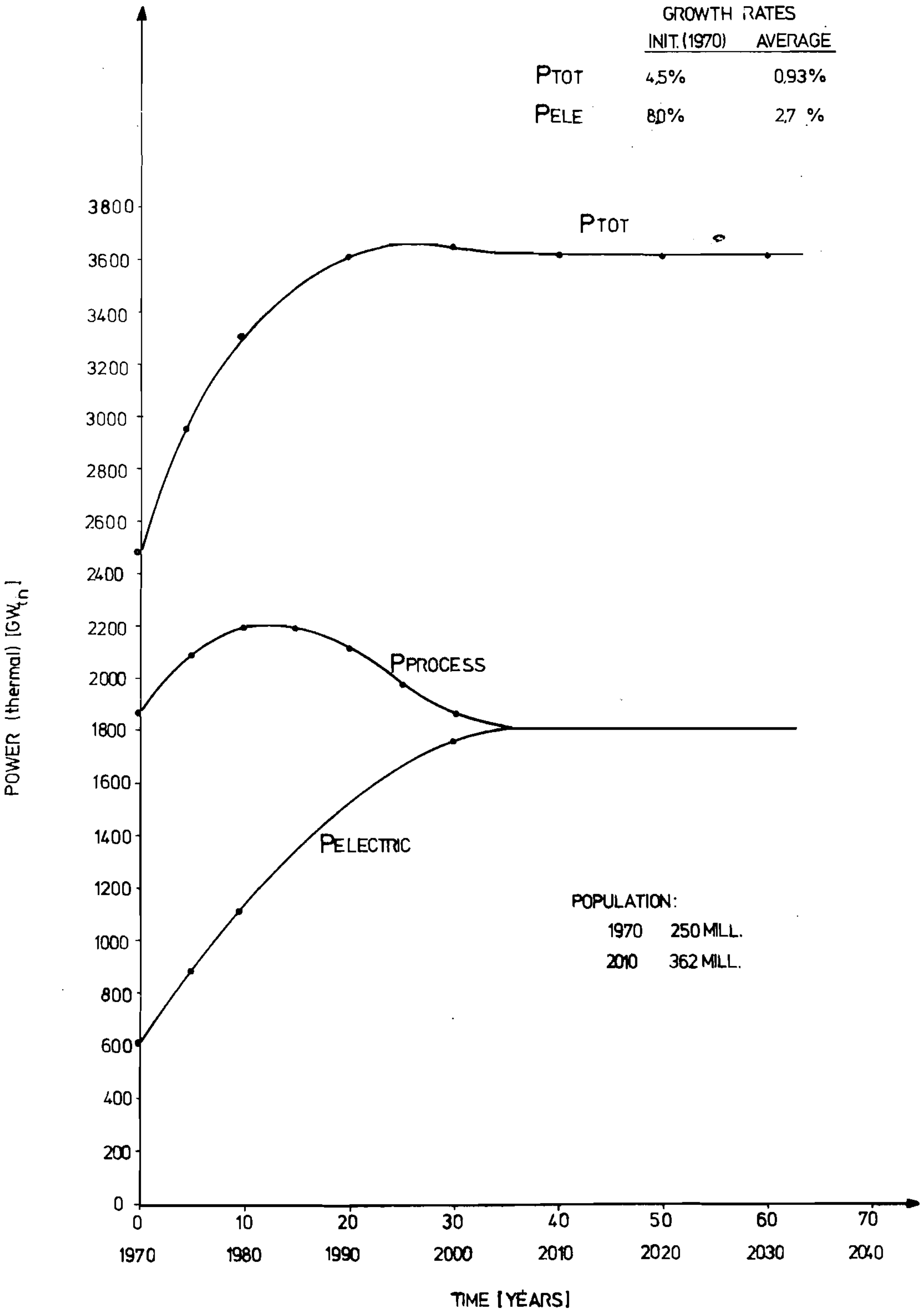

FIG.6 STYLIZED MODEL

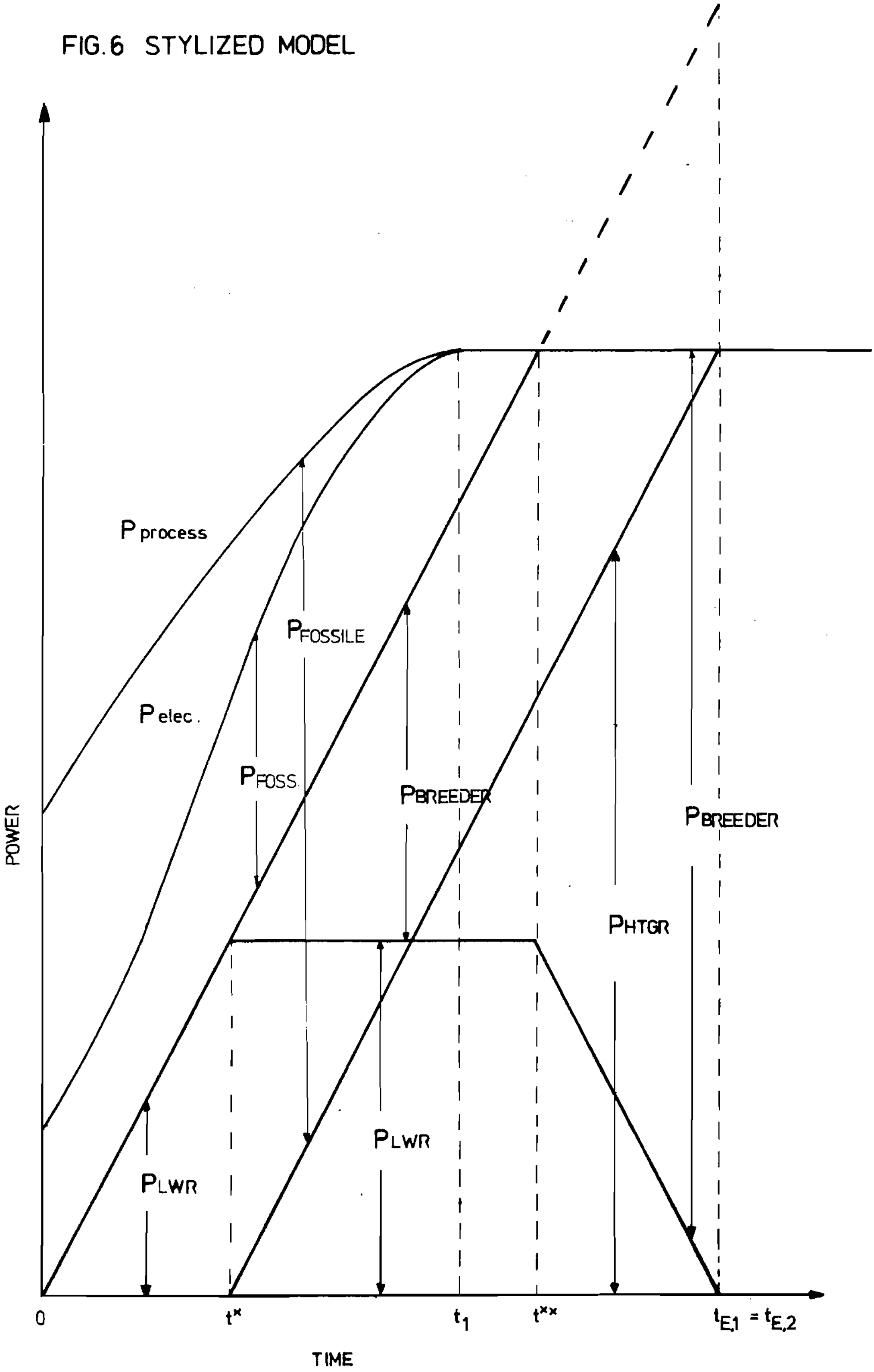

FIG.7 ANNUAL SEPARATIVE WORK

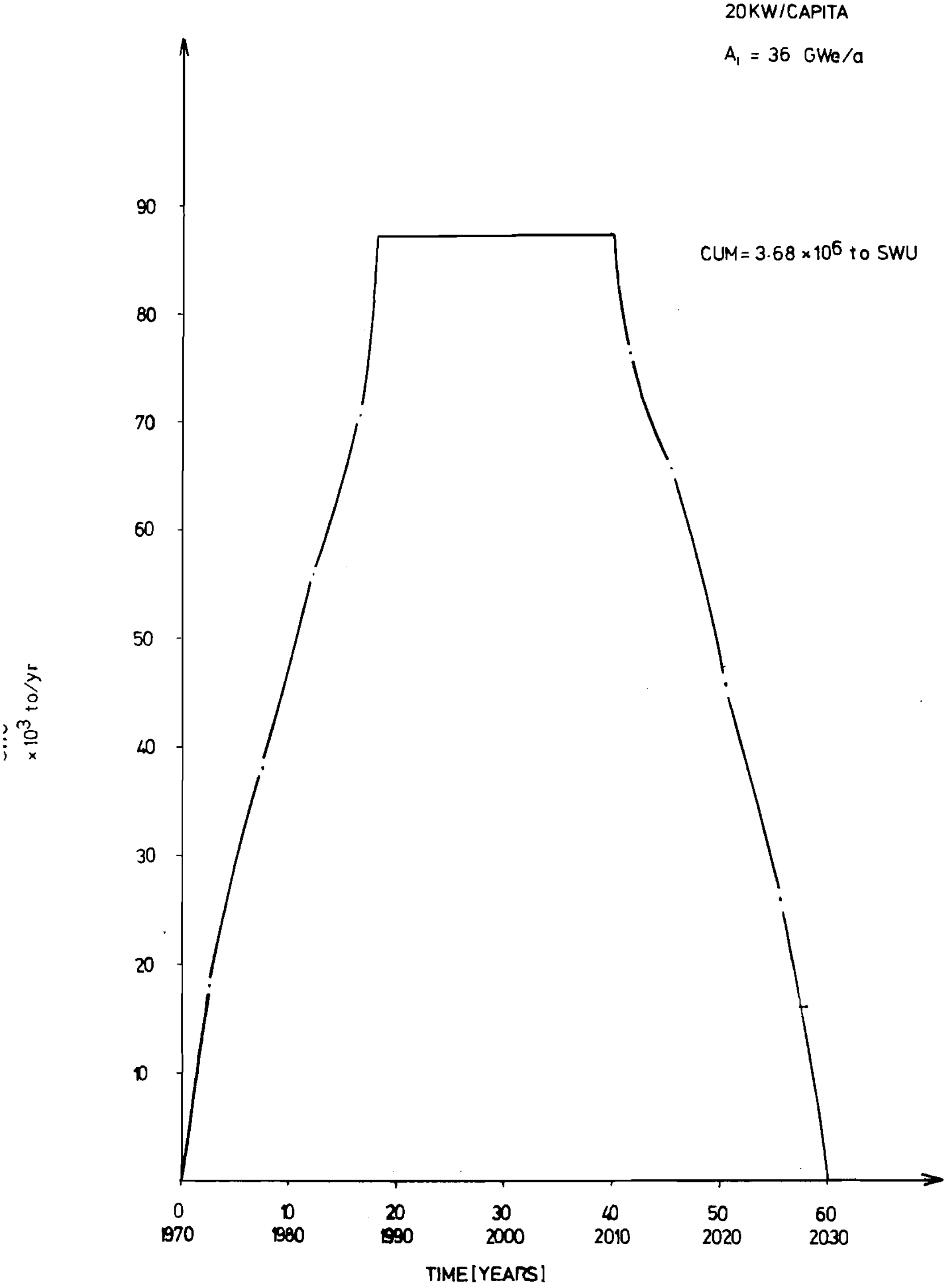

FIG.8 ANNUAL SEPARATIVE WORK

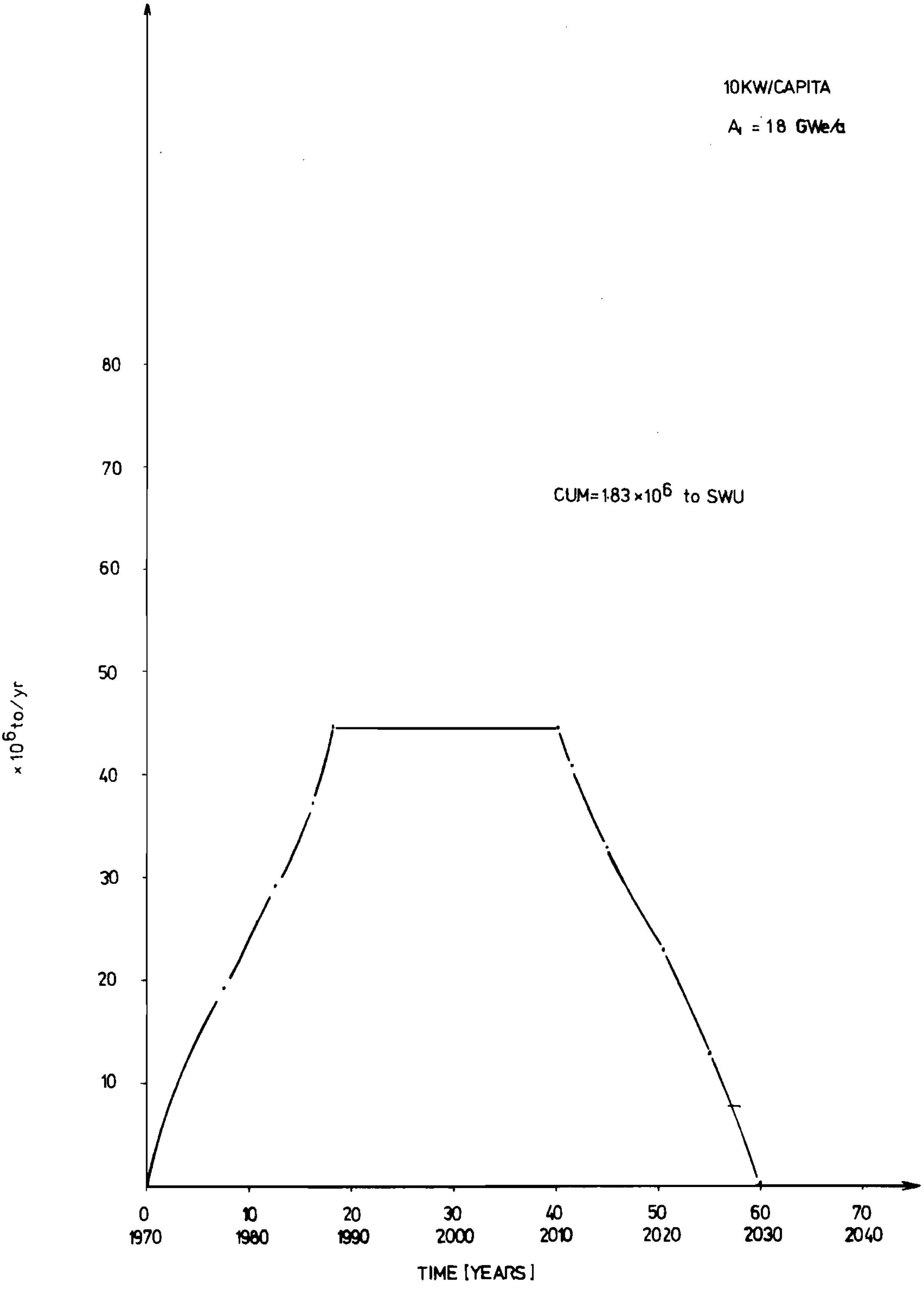

FIG.9 FOSSIL POWER REQUIREMENTS

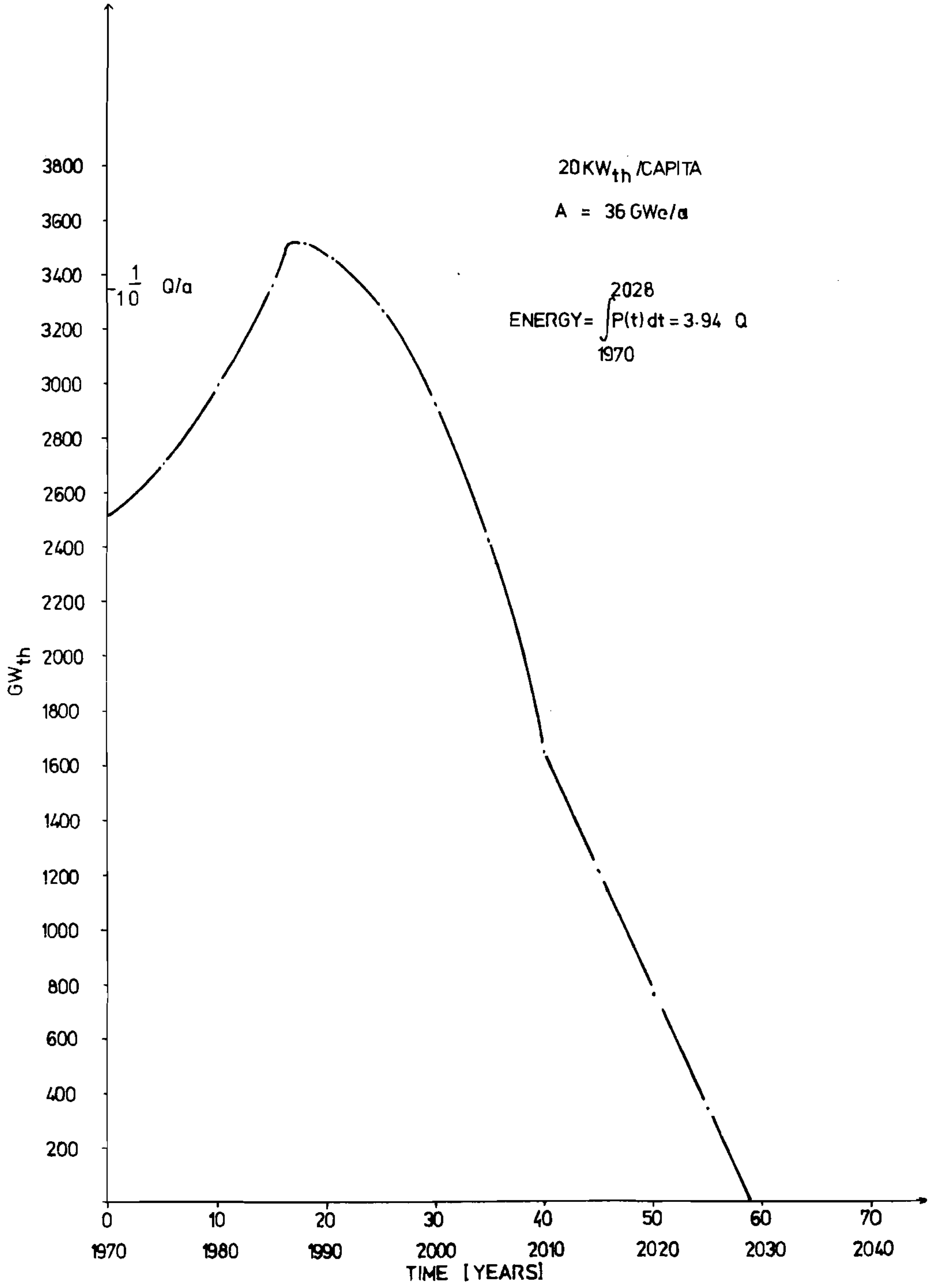

FIG. 10 FOSSIL POWER REQUIREMENTS

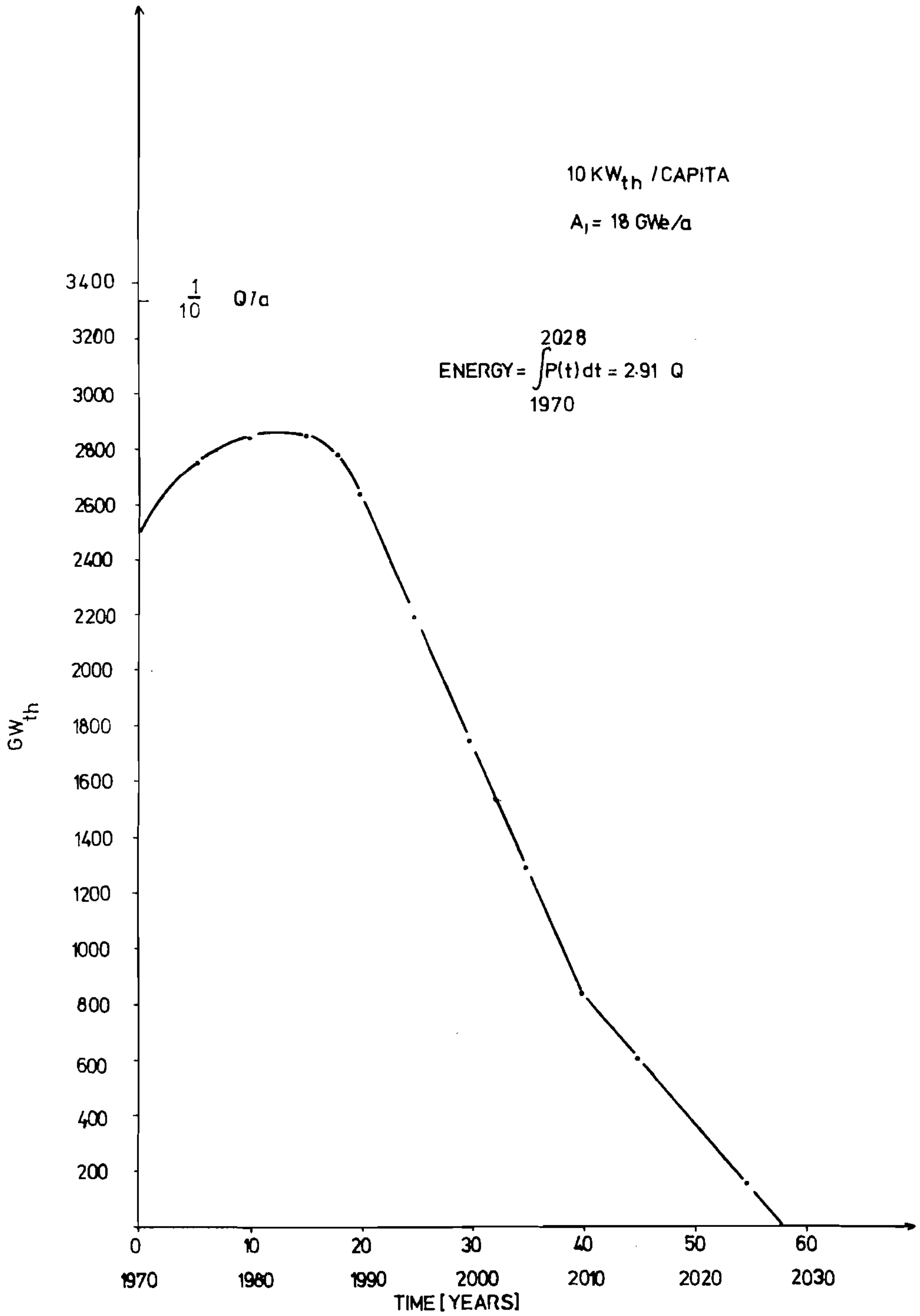

TABLE 1

	N^o	A (GWe/yr)	t^* (yr)	$t_{E,1}$ (yr)	$\int_0^{t_E} P_{tot} dt$, (Q)	(U_3O_8) equiv. cumulative (Q)	Fossil Fuel (Q)	(U_3O_8) cum. ($x10^6$ to)	Sep. Work cum. (10^6 to)
20 KW/cap	1	$A_1 = 36$	18	58	10.2	2.3	3.94	5.78	3.7
	2	$A_1 = 27$	18	72	13.2	2.2	5.25	5.53	3.6
10 KW/cap	3	$A_1 = 18$	18	58	6.0	1.1	2.91	2.78	1.8
	4	$A_1 = 13$	18	73	7.8	1.1	3.75	2.77	1.8
20 KW/cap	5	$A_1 = 36$	20	61	10.8	2.4	3.97	6.02	4.0
10 KW/cap	6	$A_1 = 11,5$	20	82	8.6	1.1	3.97	2.66	1.9

T A B L E 2

MODEL SOCIETY:

POPULATION	TIME (yrs)	ENERGY CONS (KWth / CAPITA)	GROWTH RATES ELEC. POWER	TOTAL POWER	$P_{NON-ELECTRIC}$ $P_{ELECTRIC}$
250×10^6	0 (1970)	10	8 %	4.5 %	3
362×10^6	40 (2010)	20	0 %	0 %	1
362×10^6	40 (2010)	10	0 %	0 %	1

REACTOR DATA:

		UNITS	LWR	FBR	HTGR
INVENTORY (IN+OUT OF PILE)		to/ GWe		3.0	2.025
ANNUAL BREEDING/CONVERSION		to/ GWe.yr	.17 (Pu)	.17 -.24 (U^3)	
SEPARATIVE WORK (CUM.)	INITIAL LOADING	to/ GWe	230		438
	ANNUAL REPLACEMENT	to/ GWe.yr	110		
ORE (U_3O_8) REQUIREMENTS	INITIAL LOADING	to/ GWe	500		540
	ANNUAL REPLACEMENT	to/ GWe.yr	180		

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