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A PROCESS MODEL OF TECHNOLOGICAL EFFICIENCY AND IMPROVEMENT IN SOCIO-ECONOMIC SYSTEMS

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PREFACE

Professor Philip M'Pherson worked in the Management and Technology Area on sabbatical leave from the Systems Department of the City University, London, UK, from September 1976 to August 1977. He worked on developing models to explore the technological efficiency of socio-economic systems but was unable at the time to obtain sufficient empirical data for adequate testing of his hypotheses. The time series of available data is now long enough for such testing to begin and this paper is the first publication of his results. It was presented at the Fifth European Meeting on Cybernetics and Systems Research and will be published in the proceedings of this meeting by the Hemisphere Publishing Corporation. Fuller and more detailed research reports will follow in the near future.

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It is argued that orthodox economic analysis of resource conversion should take account of the efficiency of resource conversion processes, particularly with respect to the constraints applied by the laws of thermodynamics. If a process model is added to an economic model the resultant "entropic depletion effect" induces most of the familiar symptoms of inflation. The natural trend towards entropy increase that is inherent in every material resource conversion process cannot be legislated away: it imposes limits to growth. The paper develops a general process model, and aggregates it to a simple "engine model" of a nation with primary energy consumption as input and GNP as output. The energy and GNP data for 29 developed nations for 1950-77 are shown to have a remarkably good fit to the "Gompertz" logistic function that is typical of many forms of natural and technological growth. So, it is argued, national economic growth follows this well known development pattern as well. Economic growth does obey thermodynamic laws.

INTRODUCTORY ESSAY : THE ECONOMICS AND THERMODYNAMICS OF TECHNOLOGICAL RESOURCE CONVERSION

Perhaps a few definitions would be useful as a start:

Economics. The science that studies the production, distribution and consumption of wealth (with the related problems of labour, finance, taxation and government control).

Technology-as-product. The range of machinery and procedures that result from technological action that are required for....

Technology-as-process. The system by which society transforms material (through the application of scientific knowledge) to provide the goods and services that it needs and desires.

Entropy. A measure of the disorder of the structure of natural material. In thermodynamics entropy increases as energy is consumed to do work or dissipates to the ambient state; in information theory entropy increases as a system moves from an improbable state to a more probable state.

Added value. The additional utility provided to a good by an agency that converts the good into a more readily useful form as the good passes through the agency.

These definitions demonstrate the close relationship between Economics and Technology, although one may be forgiven for thinking that a conspiracy of silence has been joined by economists and engineers when it comes to their mutual interest: resource conversion. Material wealth originally stems from the conversion of natural resources into a good that is useful or consumeable. The added value of the good is what the consumers pay for ... so wealth is created. But first the technologists have been busy creating the means whereby the natural resource can be extracted, processed, manufactured and distributed. During this resource conversion process the added value appears as a result of technological action that lowers the entropy of the materials that are being transformed into highly structured goods. But every change in nature must also be covered by an energy flow, consequently, the upgrading of material into useful artifacts and services implies the downgrading of the energy used in the process. The natural balance, as prescribed by the Laws of Conservation of Energy and Matter, are illustrated in figure 1 in which

FIGURE 1

- the upward value-adding entropy-decreasing flows of energy and material conversion are balanced by
- the downward value-decreasing entropy-increasing flows of energy and material consumption accompanied by
- the wastes that inevitably result from energy and material conversion

Note that energy has two components:

- waste due to inefficiency in the conversion process
- the unavailable energy resident in every energy conversion process because it is impossible to exhaust to an absolute zero temperature

Following the paths through, one observes that they all end up as waste products returning to a natural high entropic state: there is nothing permanent about the highly ordered states that result from man's technological activity. An intricate and exquisitely operating engine, if left to itself in a field, will rust to a solid lump of metal. The engine's maintainers have to work hard and expend much energy and effort to counteract that natural decay; in fact they cannot win, as the engine - however well maintained - will eventually wear out. The only really steady state is the dead state: dust to dust, ashes to ashes is the natural order of things. There is however a transient benefit obtainable from consumption. The parent society extracts the benefit inherent in the added value as it uses and consumes the goods. And there is also a transient capital accumulation in the form of the stock of the technological plant needed in the various parts of the process. But the benefit passes with the consumer, and the wealth represented by the capital stock decays along with the plant. It is not surprising that man ascribes the highest value to diamonds and gold - the one a hard rock, the other a metal that does not corrode readily.

It becomes apparent that economics is the value adding and distributing dimension of resource conversion to wealth and benefit. As such, the economic process must also be bounded by the physical laws that govern material and energy conversion. But a consideration of the thermodynamic balances and constraints that enter economic process do not seem to be in the mainstream of orthodox economic texts. This omission means that economics tends to be unaware of the importance of the energy balance and conversion efficiency to the economic process; prices do not always reflect thermodynamic factors correctly. Resource conversion must be accompanied by energy use, consequently unavoidable wastes are also generated. The consequent discharge of rubbish and pollutants may degrade slowly back to raw material again, but during each and every conversion process there is also a permanent loss of energy to the system. The cycles cannot go on for ever. Money, however, can keep on circulating and - if any gets "lost" due to inflation - governments attempt fiscal control of some sort, even to the extent sometimes of printing more money. But no politico-economic action can counteract the physically necessary entropic deficit at the end of each conversion process. Money cannot redeem this permanent loss even if the money balance ends up as a positive profit.

The Effect of Entropic Depletion on an Economy

The complete analysis of technology-economy requires an accurate recording of all inputs and outputs (including wastes) across the boundaries of the system and a proper balance struck between resource flows and funding. A conceptual and simplified form of such a model is shown in FIGURE 2. The matarial resource flows, aided by energy flows, are shown moving from Material Conversion through Industry, Goods and Services to Consumption. The raw materials and energy are drawn from depleting natural resource reservoirs. Each conversion process is accompanied by the attendant energy and material waste outputs. So far the figure repeats figure 1. But there are three additions: Human Resource Flows, R + D and Renewal of Capital Stock, and Money Flows. These money flows travel in a direction opposite to the resource for which they are paying. The paths of money flow form a network with nodes where the flows in and out ought to balance: a Kirchoff network for money in fact. Take the resource flows away and one is left with a rudimentary input-output (Leontief) economic model.

It is useful to ponder this simple model for a while to examine the interaction between resource conversion and the money flow. Assume for argument a zero growth population with everything in balance to begin with: the money flow is balanced by the added value of resource flow. The following sequence of events can be argued:

1. Society's steady consumption requires the continuing production of goods and the consequent extraction of resources. Society happily pays for these at the prevailing prices from its wages paid to it by Industry. Those prices are initially steady in this model economy.
2. Because of the continuing consumption resources deplete and become harder to extract. Moreover the quality of raw material degrades because the first resources to be exploited are usually the best quality ores.
3. Because of resource depletion more energy is required to extract the less accessible ores, and more energy is required to convert those lower quality ores to engineering grade material.
4. This model is not demanding more or different goods, so the production of final goods can remain constant. But the rate of energy consumption has now to increase steadily to balance the degradation of the depleting raw materials to produce the same output. In turn this depletes the energy stocks faster.
5. The increased energy consumption has to be paid for. So the price of goods has to go up. Society's wages can no longer buy the same amount of goods as before, so it demands quite reasonably a wage increase.
6. To meet the well-behaved society's request for higher wages to maintain their standard of living:

Either (a) the production of goods must be raised to balance the increase in the money supply,
Or (b) the efficiencies of energy and material use in the conversion process must be increased to lower the energy demand,
Or (c) the production/energy balance must be restored by (i) finding new energy sources, (ii) substituting materials that are less energy demanding, or (iii) introducing alternative technologies to escape the constraints
Or (d) society has to be told that times are hard and that they must accept a lower standard of living
Or (e) some people are sacked to reduce the wage bill
Or (f) money is printed to provide the extra wages
Or (g) additional value-adding activity is planned (eg road building) to generate extra employment and wages.

Consider now the likely consequences of these solutions, and the likelihood that they might provide a cure to the "entropic depletion effect":

(a) is the "increase productivity" strategy, but it accelerates the rate of energy and resource depletion, hence prices rise faster.

Consequently (a) offers no cure.

(b) the efficiency increases in energy use will provide a short to mid-term cure, but the depletion effect is bound to catch up in the end. Also increased efficiency usually means pricier higher technology! No long-term cure.

(c) this has been the usual technological escape route by innovation or substitution: the "technological fix". But the R + D and the new plant have to be paid for first, inducing further price rises and wage demands. Eventually feasible substitute technologies may be found and implemented. But they will have to cost more per unit of production to pay for the investment in new technology and the higher wage labour force - even though the model society continues to ask only for the original standard of living. Technological fixes can provide a partial cure, but it is a race between technological efficiency improvement and cost increases due to higher technology and resource depletion. In the very long-term, resource depletion is bound to win the race.

(d) a dictatorship could, or a very docile or enlightened population might, accept this solution. A difficult solution to impose.

(e) is the same as (d) really: the in-work population must share its wages with the unemployed. An historic alternative has been to substitute energy for people, eg the increase of energy consumption in agriculture replacing labourers with tractors and machinery. Works only if energy is undervalued. No long-term cure.

(f) the extra printed money in circulation is not matched by an increase of goods, consequently the value of money has to fall. Classic inflation is induced: no cure.

(g) is the Keynesian solution. The trouble is that it does not get round the underlying cause: resource and energy depletion. In fact it will probably make matters worse as energy use is increased to build the roads. No cure, except in the short-term if energy is cheap.

The simple model of a resource-linked but energy constrained model of an economy obeying the laws of thermodynamics has demonstrated that the entropy deficit of energy-using resource cycles produces inflation - for two very straight forward reasons:

- to give the appearance of constant real value to wages the money circulation has to be increased unsupported by an increase in production
- increasing production requires an increase in energy and material use which only makes matters worse when resources are depleting.

This inflation is induced by that entropic depletion effect. It is the price that nature exacts to pay for the entropy increase consequent on continuing resource consumption. The inflation will not be too noticeable if energy is undervalued (as it was up to 1973 prior to the OPEC action). But the effect on economies will become more marked the tighter the material and energy constraints become. The only solutions that have any palliative effect are:

(b) and (c) which are "technological fixes" and offer some hope of escape - for a while anyway

(d) controls the money supply (Monetarism), but is only useful for the short-term while an escape route is engineered - providing society accepts the discipline.

(e) and (g) which are coupled as the Keynesian solution to high unemployment. It is no solution if the unemployment is induced by high costs due to energy and resource depletion, as increased public works do not correct the entropic effect.

There is one further comment on solution (d): it is also the "go backwards" solution favoured by protagonists of the simple "green" life uncluttered by high technology.

All these arguments are very familiar. The interesting point here is that almost the entire vocabulary of the contemporary discussion on inflation in industrially developed countries has been reproduced, not by discussion of complex economic theory, but by following the logic of a simple model in which an elementary (initially stable) economy reaches the point where the second and third laws of thermodynamics intervene to exact their price. Inflation is of course no simple phenomenon: there are many economic and psychological forces at work. But it does seem that an economic model that is also constrained by thermodynamic laws homes into at least one important underlying problem of inflation rather nicely. And it does seem that the present public discussion on inflation control misses the significance of the entropic depletion effect. Engineers would have no difficulty in following the argument. Unfortunately, however, there is no way that an economist or politician can legislate the laws of thermodynamics out of existence. They have to be accepted, and - as has been hinted at above - the long-term implications of that acceptance are none too happy. Economists tend to argue that economic progress need not be constrained as technology will always be able to invent substitutions to get round constraints (eg 1). Engineers - always optimists - tend to agree, but would point out that arriving at suitable technological fixes becomes more difficult, and the price for the entropic depletion effect has to be paid continuously, however efficient or inefficient technology may be. An economic "perpetual motion machine" is not possible.

The coupling of thermodynamics to economics is not new. The name of the eminent economist Georgescu-Roegen is, perhaps, most closely associated with the development. His writings on the theme began during the 1950s, culminating in his book "The Entropy Law and the Economic Process" (9). His 1975 paper (10)provides a critique of orthodox economic theory. Gallagher (8) writes a useful and concise summary. Georgescu-Roegen has his critics of course - just about every other trained economist, so he has been in the wilderness somewhat. But orthodoxy is shifting under the sustained attack of events and the protagonists of energy-economics or energy analysis (24,25). There have also been some earlier forays, for example Sir Frederick Soddy FRS wrote in 1926, "... the flow of energy should be the primary concern of economics" (26, quoted 8). Sir Frederick was a chemist.

SUMMARY OF THE RESEARCH REPORTED IN THIS PAPER

This paper does not intend to pursue the arguments between the two extreme camps: that steady economic growth can continue indefinitely or that limits to growth are inevitable. The purpose of the introductory essay is two-fold:

(i) to demonstrate that economic models should contain a section that subjects material resource conversions to thermodynamic constraints
(ii) to provide some arguments that will link up with the later analysis of national economic growth patterns.

The trail to Georgescu-Roegen was stumbled over while searching for economic theory to support a notion born of an earlier study on the modelling and dynamics of technological change (18). The notion is that the economic growth of at least the industrialized nations might be profitably studied as if they were technological resource conversion process - or "engines" for short. The engines convert natural and human energy into wealth. The thought was born of two observations:

1. For long periods the rate of technical progress has been fairly constant as measured by economists (eg 27).
2. The correlation between GNP and primary energy consumption is remarkably good over time for a large ensemble of nations (eg 25). Figure 3 presents a spot sample for 1973.

FIGURE 3

If this simple "engine" model of a nation holds up to validatory tests it would mean that national economic development can be treated as if it were a technological process with an overall conversion efficiency that changes in time. The efficiency would increase as a result of technological improvement, and would be constrained by thermodynamic limits. Which suggests that the relationship between energy and GNP should have a logistic form in common with technological improvements in general. The original sample consisted of data from GNP and primary energy consumption over 1950-1973 for 24 developed nations (both market and planned economies). The logistic test made on this sample gave encouraging but not too well correlated results. This paper reports further work using an enlarged sample of 29 nations over the period 1950-1977. A more sophisticated logistic test is employed with good results. Consequently it will be argued that:

(i) the logistic form of national development is arguably the case
(ii) a process model of national economic growth is worth developing
(iii) economic development does "obey" thermodynamic criteria.

THE PROCESS MODEL OF RESOURCE CONVERSION

The Conversion Process in Growth Economics

The typical Production Function model for economic growth is illustrated in Figure 4.

FIGURE 4

Such models are used to investigate the capital requirement K to give a continuous growth in national product Y while ensuring full employment L under different types of technical progress A. The model is of the general form (13):

$$Y = F\ (A, K, L) \qquad 1$$

This becomes for various types of technical change (TC):

Hicks-neutral TC ... $Y = AF(K,L) = e^{mt}K^{\alpha}L^{1-\alpha}$ 2a

Harrod-neutral TC .. $Y=F(K,AL) = e^{m(1-\alpha)t}K^{\alpha}L^{1-\alpha}$ 2b

Solow-neutral TC ... $Y=F(AL,L) = e^{m\alpha t}K^{\alpha}L^{1-\alpha}$ 2c

The right hand expression is the Cobb-Douglas production function with constant returns to scale where:

$$\alpha = \frac{dY}{dK} \cdot \frac{K}{Y} > 1, \text{ the output elasticity of capital} \qquad 3a$$

$$1-\alpha = \frac{dY}{dL} \cdot \frac{L}{Y}\ , \text{ the output elasticity of labour} \qquad 3b$$

$$A = e^{mt}\ , \quad m \text{ is a constant rate of technical change} \qquad 4$$

Solving equations 2 for, say, a given rate of labour increase $L = L_o e^{nt}$, Hicks-neutral TC and full employment gives

$$Y(t) = Y_o\, e^{\,n + \left(\frac{m}{1-\alpha}\right)t} \qquad 5$$

Which is interesting as it suggests that the long-run growth in Y is a function only of the growth rates of labour supply and technological improvement, and is independent of the proportion of national product sY that is diverted to investment rather than consumption! The labour supply may increase for free. But technological improvement results only after heavy investment in education, R + D and new plant incorporating the improved processes. Other assumptions with respect to TC give different solutions. This illustration is representative of text-book analysis rather than real applications. Complex multisector production function models of the national economy are used in national central planning and Global models to determine the optimal allocation of capital between sectors to provide a desired strategy for growth rate in GDP (eg 5, 15, 3, 21). But the inherent defects in the production function - however complicated - remain.

Three peculiarities of the production function should be noted:

1. It deals only in terms of capital, labour and a factor representing rather disembodied technological change. The capital stock K stands for technology.
2. The details of the technological processes producing Y are ignored. Only final products

are counted. Intermediate products are neglected as it is assumed that their values are properly represented by the prices of the final goods.

3. The rate of technical change is found by differentiating 2a (say):

$$\frac{\dot{Y}}{Y} = m + \alpha \frac{\dot{K}}{K} + (1-\alpha) \frac{\dot{L}}{L} \qquad 6$$

All the rate terms can be quantified from economic statistics, hence m is the residual (ie error!) between the LHS and RHS. Its value has stayed steady at between 1% and 2% per annum. for Europe and North American over the last 50 years or so (27, 6). The efficiency increase is attributed to technological and managerial improvement in general terms.

This all too brief study of the production function in economic growth theory should be enough to show that it is somewhat removed from the thermodynamic balances of the resource conversion process itself. It can only reflect the efficiency of energy and material conversion properly if prices are an exact reflection of the amount of total energy, material and effort consumed in adding value. There is certainly some evidence to show that the smoothed long-term price trends of technological product have a good correlation with the amount of energy sequestered in forming them (22). But the price of energy itself is often determined by political considerations as well as by market forces.

Prices also have to reflect the two forms of technological improvement that occur in the process:

- efficiency improvement in the production process resulting from more efficient managerial and operational procedures, and more effective use of energy and material because of improved technological design of the production processes
- quality improvement in the product that comes from better design and better quality control of the product during manufacture. The rise in quality is represented by higher capability, reliability, maintainability, useability, internal efficiency, better style etc.

A good example of how prices can completely miss the quality component in goods is to be found in the present computer market where real prices are falling rapidly but the quality of the good is rising by leaps and bounds.

And finally, it has to be admitted that a formulation that determines technological change as the residual term in a broad brush analysis of historical data is less than perfect.

A Technological Model of the Resource Conversion Process

A general model of the resource flows in a national technological "engine" is represented by figure 5. The intricacies of the flows of energy and intermediate products within the Production Sector are outlined. The pattern of flows is clearly related to a dynamic Leontief open model:

$$(I-A)\hat{X}(t) - B\hat{X}(t) - \hat{X}(t-1) = \hat{Y}(t) \qquad 7$$

where $\hat{X}$ and $\hat{Y}$ are output and final demand vectors, A and B are matrices of flow and capital coefficients.

FIGURE 5

The economist quantifies the various types of flow (raw materials, energy, chemicals, motor cars ....) as money flows via prices. It is difficult to see how one might do otherwise as money is the unit of transaction between different parts of the process. But prices are not a true reflection of thermodynamic transactions between and within the process. Energy flows throughout the system because each and every technological conversion process requires an energy transaction. The energy flows are quantifiable in detail as the energy consumption of each user is carefully measured either as energy supplied or fuel bought in. Thus the energy content carried forward by each product into the next part of the process can be accumulated to provide an overall "energy cost" of the final products. The energy cost gives a far more accurate measure of technological work and efficiency than money cost. Energy audits on technological activity are becoming important aspects of design and management. Energy Analysis provides the methodology for conducting the audit (25, 2).

A model is under development to provide an explicit account of (i) the thermodynamic transaction in conversion processes and (ii) the improvements in the efficiency and quality of the process with time as a result of technological change. A model outline for the general process is shown in figure 6. Any national engine will have many such general processes linked together by matrices of resource and product flows, energy flows, capital and money flows, (but the model is assumed to link in to an appropriate economic and capital allocation model via prices of the input and output flows). The general processes will cover such technosectors as resource extraction, conversion, energy generation, manufacture, construction ..., distribution ..., services, domestic sector. Each sectoral model is constructed along the lines of figure 6 with three compartments: the conversion process itself, the supporting energy utilisation and manpower (labour) systems. The formulation is summarised in the Appendix. It will be seen that the model is described in terms of inputs, outputs and internal state (see matrix over the page).

FIGURE 6

The model operates as a resource demand: given a desired output P_{x2}, the prevailing state of Technology $T_x(t)$ and the associated conversion efficiences $\eta_{(\cdot)}(T_x, t)$, the inputs necessary to provide the output are calculated subject to constraints from capacity, resource availability and quality. Capacity is partly a function of the capital stock K. Material resource input availability and quality will have to be provided by an external resource data base and depletion model (21, 11).

Technological improvement is felt through

Model Compartment	Input	Output	Internal State		
			Efficiency	Quality	Productivity
Process	P_{X1}	P_{X2}, W_{XP}	η_{XPU}	γ_{XP}	e_{XP} π_{XL} Π_{XL}
Energy	E_{X1}	E_X, W_{XE}	η_{XEU}		
Labour	L_X	Outputs, Tech Change	η_{XLU}	γ_{XL}	
(Money)	K_X	C_X, K_X			

changes in the efficiencies $\eta_{(.)}$ and in the product and labour qualities $\gamma_{(.)}$. The material and energy-use efficiencies are calculated along straightforward lines (Appendix). "Efficiency" is used loosely in terms of output/input ratios. The more efficient the process the less the wastes, but for a given capacity the more the capital charges will be due to the deployment of better technology. The quality of the product γ_{XP} is quantified in terms of effectiveness - the indicator of system quality used in Systems Engineering - calculated in terms of capability, reliability, maintainability availability, utility, worth etc. The "efficiency" of labour use η_{XLU} is defined as the product of labour mix λ_X (ratio of managerial and technical staff to operations), per-man product value added π_{XL}. The quality of labour γ_{XL} is difficult: it is a function of educational and training levels, experience, initiative and psychological attitude to work and risk taking. The change in time in the efficiencies due to technological improvement is not entirely equivalent to the changes in the elements of the A and B matrices of an input-output model. Structural change between the sectors in an advanced economy would appear to be small and slow - in spite of considerable advances in technological efficiency and quality (4).

The State of Technology

The efficiencies cover the thermodynamic transactions and resource useage in the process. They are sensitive to technological changes, improvements and substitutions, and to human efficiency improvement due to better management, design and operation. Figure 7 tracks the various kinds of efficiency improvement.

FIGURE 7

From a resource-use point of view the efficiencies provide a useful indicator of the state of technology $T_X(t)$. The three efficiencies of material, energy and labour use are plotted as elements of a vector in a three-dimensional euclidean space. The coordinate (0, 0, 0) represents a completely useless technology, and (1, 1, 1) signifies the ideal but impossible technology. Technological improvement causes the vector to approach the (1, 1, 1) point, but it will begin to shy away when a resource, energy or labour constraint is approached. Analysis of the change of length and direction of the vector in time yields useful information for the management of technological change (18). For example increasing efficiencies will reduce the material and energy demand. But continuous resource useage will tend to increase the energy requirement due to depletion and quality degradation of the raw materials, and energy use will begin to rise lowering the overall process efficiency in spite of the process improvements. This signals to the economic model that an increase in price is ripe, and to the technology manager that resource constraints are being approached - so it is time to research and plan for substitution.

The Simple "Engine" Model of a National Technology and Economy

The process model just described is complex and would require considerable effort to implement in full. Thus it is desirable to test for the validity of the concepts on which it is built before embarking on major modelling exercises. The main objectives of the model are to introduce thermodynamic constraints and more explicit forms of technological change to studies of economic growth and development. Consequently its validity would be demonstrated if it could be shown that economic growth follows the same logistic pattern as that of an improving technology within ultimate resource and efficiency constraints.

The test decided on was to discover if the well known relationship between increases in GNP and energy consumption (figure 3) correlated well with the 'S' shaped logistic curve of technological improvement. This test requires the postulation that the "engine" model with primary energy input and GNP output (figure 8) is equivalent to an overall aggregation at national level of the process model. A process model enlarged to cover all the national sectors of technological and economic activity would have as inputs primary raw materials, primary energy and labour, while the outputs would be the national product and wastes. The efficient or inefficient use of improving or degrading raw materials is accurately reflected in the energy cost of the product. The efficient or inefficient use of the product generate further energy costs. Those energy costs also reflect improvements in the

FIGURE 8

efficiency of energy generation and utilisation, as well as the improvements or otherwise in the management, design and operation of all technological activities overall. Thus it can be argued that the primary energy consumed is a fair indicator of the energy cost of the production of the GNP for that year. Wastes are "paid for" in those energy costs, and human activity is implied. Hence we assume ' that the engine model of figure 8 is a reasonable aggregation of the process model-at least for industrially developed nations where high grade industrial energy is by far the major share of energy consumption.

THE LOGISTIC FUNCTION FOR GROWTH

It is well known that biological and technological growth patterns tend to follow a logistic curve. Two such curves are shown in figure 9A. The F + P curve is the Fisher and Pry curve which is symmetrical about the half-way point $f = \frac{1}{2}$, $t = t_o$, (7). The model assumes:

(a) a competitive technological substitution process
(b) once a new product has penetrated a market by a few percent, its superiority is demonstrated and it will proceed to take the market over
(c) the rate of fractional substitution is proportional to the extent of market left to be penetrated.

The F + P curve is given by:

$$\frac{1}{f}\ \frac{df}{dt} = \alpha\ (1-f) \qquad 8$$

where f = fraction of market substitution, α = constant

On integration: $$\frac{f}{1-f} = e^{\alpha(t-t_o)} \qquad 9$$

where t_o is the time when $f = \frac{1}{2}$

The curve may be stretched into a straight line by plotting it on semi-log coordinates (figure 9B) where ν is non-dimensional time

$$\nu = \frac{t-t_o}{\Delta t}\ ;\ \Delta t = t(f = \cdot 9) - t(f = \cdot 1) \qquad 10$$

Fisher and Pry analyzed seventeen cases of substitution and found extremely good correlations with the curve. The remarkable assumption (b) has been validated over a large number of historical samples with the take-over time predicted accurately when only two percent of the final substitution had occurred (16). This simple logistic form followed the earlier studies of biologists, agricultural economists and economists in the logistic patterns of growth and diffusion (20, 12 19). And it was followed by considerable research into the logistic pattern of substitution, particularly to predict future substitutions and growth. Many of these studies are collected in (17).

In spite of its undoubted success, the Fisher and Pry model has a number of limitations, one of which is its questionable assumption (c) that the fractional rate of substitution is proportional to the remaining penetration potential (23). Many researchers into biological, information, transportation and urbanisation growth have noticed a non-symmetrical pattern of logistic growth that is better fitted to what is known as the "Gompertz" function:

$$y = \alpha \exp\ \{-\beta \exp\ (-\gamma t)\} \qquad 11$$

where α, β, are constants to be found. The form is shown as curve G in figure 9A. Its inflexion point is at the point $y = \alpha/\exp$, $t = \beta/\gamma$. It can also be stretched into a straight line with the transformation

$$\ln\{|\ln \frac{\alpha}{y}|\} = \ln\beta - \ln\gamma t \qquad 12$$

The "Gompertz" curve has a faster acceleration during early growth representing the ability of small unencumbered organizations to develop rapidly. Writing the LHS of 12 as Y_i, for a trial value of $\alpha = \alpha_i$, the coefficients β, γ are found by minimizing

$$\sum_i (Y_i - (\ln\beta - \gamma t_i)^2) \qquad 13$$

THE LOGISTIC VALIDATION OF THE ENGINE MODEL

The data used for the test are the per capita GNP (g) and primary energy consumption in tons coal equivalent (e) for a family of 29 developed nations. One has to be wary of cross-sectional GNP comparisons. The absolute figures as listed in the sources contain all the anomalies of currency exchange rates which are unreliable indicators of the real purchasing power of the currency in the parent nation. Additionally, the rate of inflation varies with time and country, and the base currency may also be subject to inflation. We have selected the 1970 US dollar as reference and allowed for differential inflation as best we can. International GNP data is also usually related to the US dollar. But the selection of 1970 also allows us to at least start within the framework of the most extensive comparison of cross-sectional GNP available (14)

FIGURE 10

A typical example of the data is shown in figure 10 for 11 of the 29 nations examined. Each nation is represented by a (g-e) trajectory over the period 1950-1977. Notice that these trajectories maintain a fairly steady slope up to 1970, after that they steepen due to the increasing price of oil energy. The curving lines across the trajectory are the best regression fits to the exponential power model for the indicated year:

$$g_t = \alpha_t\, e^{\beta_t} \qquad 14$$

where α_t is the g/e ratio and β_t is the slope of the logarithmic fit for the year t. Notice the variations in β which is the overall "efficiency" of energy conversion to GNP: technological improvement is at work. Notice also that the effect of the OPEC cartel is to improve the efficiencies (increasing β) as the nations attend to energy conservation. Sweden demonstrates

TABLE 1 : The Constants in Power and Gompertz Curves

	$g = \alpha_1 e^{\beta_1}$			$y = \alpha\exp(\beta\exp(\gamma\nu))$				
	rr	α	β	α min	α max	β	γ	RR
Austria	·98	15·5	1·0	150	200	·71	·17	·977
Belgium	·92	5·5	1·5	144	164	·27	·75	·926
Denmark	·95	25·0	0·7	260	-	-·40	·40	·940
Finland	·98	27·8	0·5	120	180	·51	1·20	·981
France	·97	13·6	1·2	170	240	-·03	·58	·969
East Germany	·96	2·0	1·7	220	-	·50	·40	·971
West Germany	·94	7·0	1·6	120	170	·63	1·00	·970
Greece	·98	20·6	0·6	40	70	6·03	5·77	·950
Iceland	·91	7·3	1·3	95	115	·81	1·00	·914
Ireland	·93	16·6	0·8	250	-	-·48	·51	·933
Italy	·98	20·1	0·5	52	100	3·36	4·03	·980
Netherlands	·94	21·5	0·7	100	140	·84	1·18	·945
Norway	·97	27·6	0·7	250	-	·20	·50	·980
Portugal	·98	24·0	0·7	30	70	5·72	6·77	·957
Spain	·94	20·8	0·8	42	58	4·56	3·73	·957
Sweden	·97	32·3	0·7	290	1900	-·67	·24	·961
Switzerland	·97	71·0	0·4	165	230	·09	·47	·960
United Kingdom	·69	0·4	2·9	285	-	·20	·50	·680
United States	·97	7·5	1·1	400	900	-·68	·27	·964
Australia	·98	25·7	0·7	125	290	·19	·70	·980
Canada	·96	0·3	1·1	175	184	·07	·67	·974
Japan	·99	18·9	0·9	100	148	·78	1·11	·987
Bulgaria	·97	10·6	0·7	167	485	·58	1·51	·986
Czechoslovakia	·96	3·1	1·4	260	-	·20	·60	·964
Hungary	·86	7·3	1·1	290	-	·05	·80	·920
Poland	·97	3·0	1·6	100	200	·38	·75	·985
USSR	·98	4·7	1·2	200	440	-·20	·60	·996
Romania	·98	3·1	1·2	67	87	2·56	2·01	·952
Yugoslavia	·98	11·5	1·0	30	96	3·98	2·89	·976
For the Whole Sample	0·5604	1·70	0·751			-0·387	-0·902	·9408

that one may reduce energy consumption and still increase GNP. Table 1 lists the constants for the 29 nations in the sample. Individual national data fit the power model very well, the average of the correlation coefficients rr being 0·95, the average slope during 1950-77 was 1·04. The United Kingdom is different, of course! It is off on some law of its own with a very high slope of 2·9(!). and a lousy correlation coefficient of 0·69. While individual nations fit the power model well, figure 10 shows that the national trajectories have a wide dispersion in (g-e) space. Consequently the correlation coefficient for the best power model fit to the whole sample of 29 nations during 1950-77 is low, rr for the family is only 0·5604.

The earlier study fitted a smaller 24 nation sample to the Fisher-Pry logistic model for 1950-1973: the result was encouraging rather than giving a good correlation (18). The larger sample of 29 nations, extended to 1977 to cover the "OPEC effect", has been fitted to the Gompertz function with very satisfactory results. Figures 11, 12 plot the data for the sample against the linear and logarithmic forms of the Gompertz function. (The energy index ν is given by

$$\nu = \cancel{\exp}\,\frac{e(g) - e(\cdot 5)}{e(\cdot 9) - e(\cdot 1)} \qquad 15$$

where e(g) is e at GNP/cap = g).
Table 1 also lists the coefficients for the best national fits and the best sample fit to the Gompertz curve. Note:

- the correlation coefficients for individual nations are high, the average being 0·953
- that the correlation coefficient for the whole sample of 29 nations over 1950-77 is also high, RR = 0·941.

For the Gompertz fit, α is the final maximum value of g when the curve finally flattens off (100 α = $g + g_{USA}$(1970)). This saturation means that each national "technology" in its 1950-77 structure has a maximum g/e ratio "built into its engine design". In other words there comes a time when the engine is up against its design limits and no more wealth (power) can be squeezed out of extra energy consumption. The actual value of the limiting α is sensitive to the correlation coefficient, the actual values of α_{max}, α_{min} listed give the range of α that keeps the correlation coefficient above the national RR listed.

CONCLUSIONS

It would seem that the very good results obtained when fitting national GNP and energy data to the Gompertz function demonstrates quite strongly that national economic development and growth is energy-use dependent, and that its evolvement in time follows a logistic pattern closely. This implies that economic development is constrained to thermodynamic efficiency considerations, and provides encouragement for the detailed development of process type models to bring thermodynamic constraints into economic analysis.

All the evidence suggests that logistic trends are stable over long periods in time. Thus national planners could use the Gompertz fits to provide some indication as to where the design of their national engine is going to take the nation to, what the limits to growth are and when the limit is reached. This is not as final as it sounds, because the limit refers to the technological design in use. The present national engines depend on high-grade energy consuming industry. A change in design could provide an escape from the present constraints ... but there is never any escape from limits. Each design has its necessary thermodynamic limits. Finally the evidence of the good Gompertz fits lends weight to the arguments of the introductory essay. The entropic depletion effect is a vital matter for economic analysis. Perhaps Sir Frederick Soddy was right.

APPENDIX : FORMULATION FOR GENERAL PROCESS (see figure 6)

MATERIAL CONVERSION PROCESS

$$P_{X1} = P_{X2} + W_{P1}$$

$$W_{XP1} = \rho\, P_{X1} \quad ; \quad W_{XP} = (1-\sigma)\, W_{XP1}$$

$$\eta_{XPU} = \frac{P_{X2}}{P_{X1}} = 1 - (1-\sigma)\rho$$

$$\rho = f\,(T_X,\ XP1,\ XLU,\ t)$$

$$\sigma = f\,(T_{XS},\ XPU,\ XLU,\ t)$$

Quality of product γ_{XP}

$$= f\,(CE_{XP},\ T_X, \gamma_{XL})$$

ENERGY CONVERSION PROCESS

$$E_{X1} = E_{X1P} + E_{X1S} = E_{X2P} + E_{X2S} + W_{XE}$$

$$E_{X1S} = \sigma_{XE}\, E_{X1P} \quad \therefore E_{X1} = (1+\sigma_{XE})\, E_{X1P}$$

$$\eta_{EUXP} = \frac{E_{X2P}}{E_{X2P}+W_{EXP}} = \frac{E_{X2P}}{E_{X1P}}$$

$$\eta_{EUXS} = \frac{E_{X2S}}{E_{X2S}+W_{EXS}} = \frac{E_{X2S}}{E_{X1S}}$$

$$E_{X1} = f(\dot{P}_{X2},\ \rho,\ \eta_{EUXP}, \eta_{XLU},\ T_{XE},\ t)$$

$$E_{X2} = f\,(\sigma\dot{W}_{P1}, \eta_{EUXS}, \eta_{XLU},\ T_{XS},\ t)$$

$$\eta_{XEU} = \frac{E_{X2P}+E_{X2S}}{E_{X1}} = 1 - \frac{W_{XE}}{E_{X1}}$$

$$E_X = E_{X2} + E_{XS} = (\eta_{EUXP} + \sigma_{EX}\, \eta_{EUXS})\, E_{X1P}$$

$$E_{X1} = (1 + \sigma_{EX})\, E_{X1P}$$

$$\bar{\eta}_{XEU} = \frac{E_X}{E_{X1}} = \frac{\eta_{EUXP} + \sigma_{EX}\, \eta_{EUXS}}{1+\sigma_{EX}}$$

Energy cost of the product

$$e_{XP} = \frac{E_X}{P_{X2}}$$

LABOUR

$$L_X = L_{XM} + L_{XQ} + L_{XO}$$

$$\lambda_X = \frac{L_{XM} + L_{XQ}}{L_X} = 1 - \frac{L_{XO}}{L_X} < 1$$

$$\eta_{LUX} = \lambda_X \cdot \frac{\pi_{XL}}{\pi_{XL}^{+}} \cdot \frac{\Pi_{XL}}{\Pi_{XL}^{+}}$$

Quality of labour γ_{XL}

$$= f\,(\gamma_{LXM}, \gamma_{LXQ}, \gamma_{LXO})$$

Product value added per unit labour

$$\pi_{XL} = \frac{XP\,P_{X2}}{L_X} \quad ; \quad \pi_{XL}^{+} = \text{a high referenc}$$

COSTS

$$C_X = C_{XP} + C_{XE} + C_{XL} + C_{XM} + C_{XK}$$

Money value added per unit product

$$\Pi_{XL} = \frac{R_X - C_X}{P_{X2}} \ ; \ \Pi_{XL}^{+} = \text{a high referenc}$$

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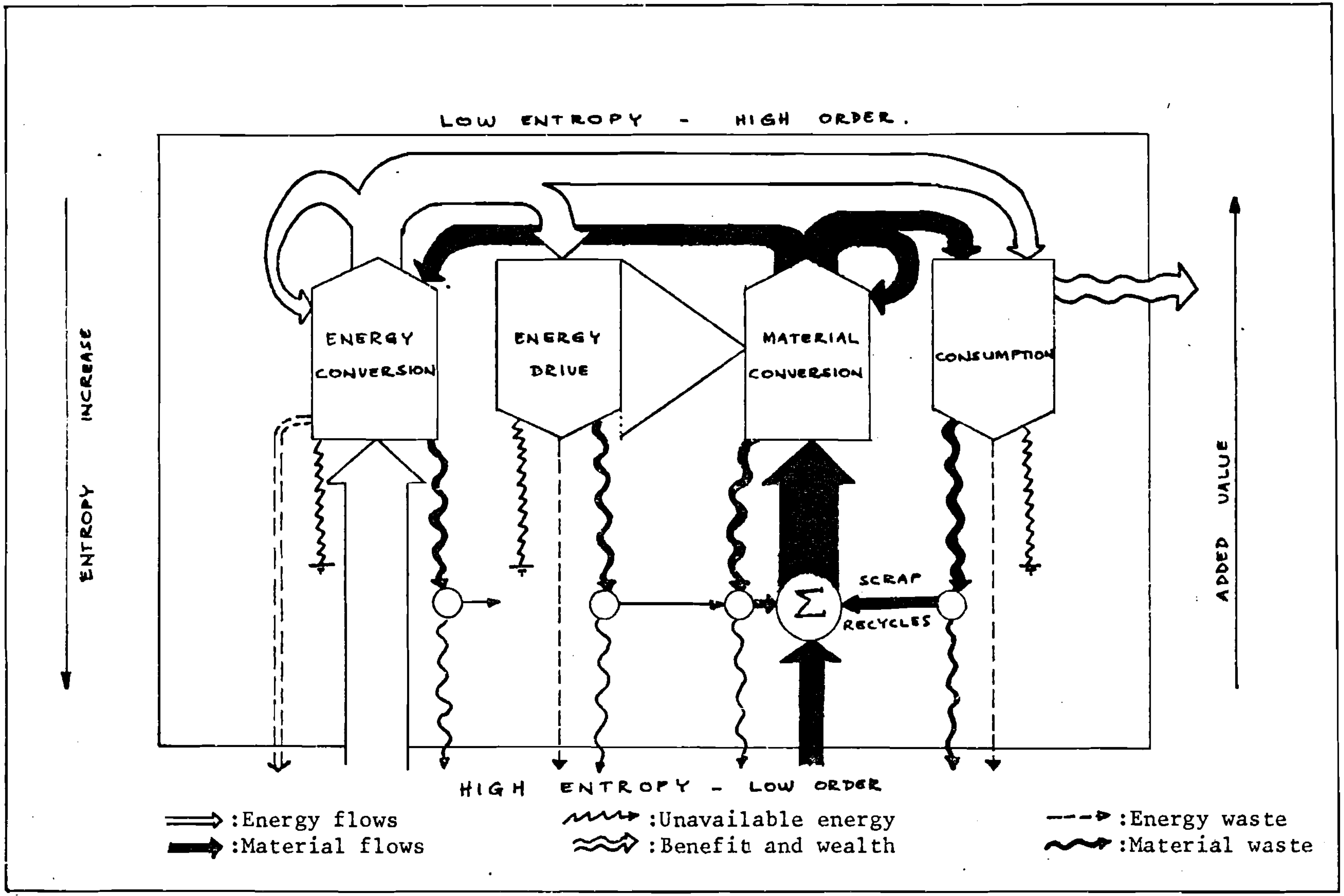
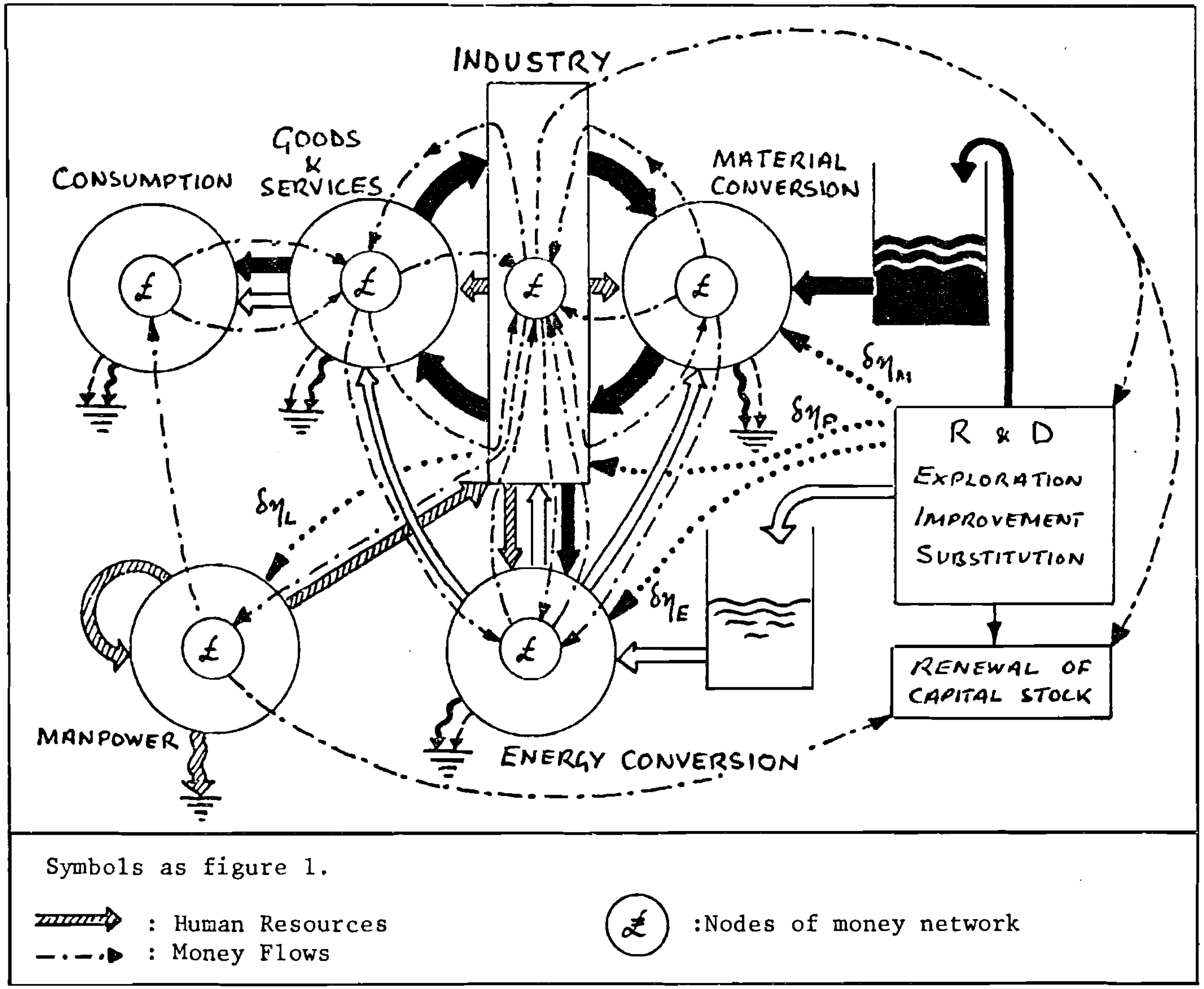


Figure 1 : INDUSTRY, CONSUMPTION AND ENTROPY



Symbols as figure 1.

: Human Resources

: Money Flows

:Nodes of money network

Figure 2 : RESOURCE CONVERSION AND MONEY FLOW

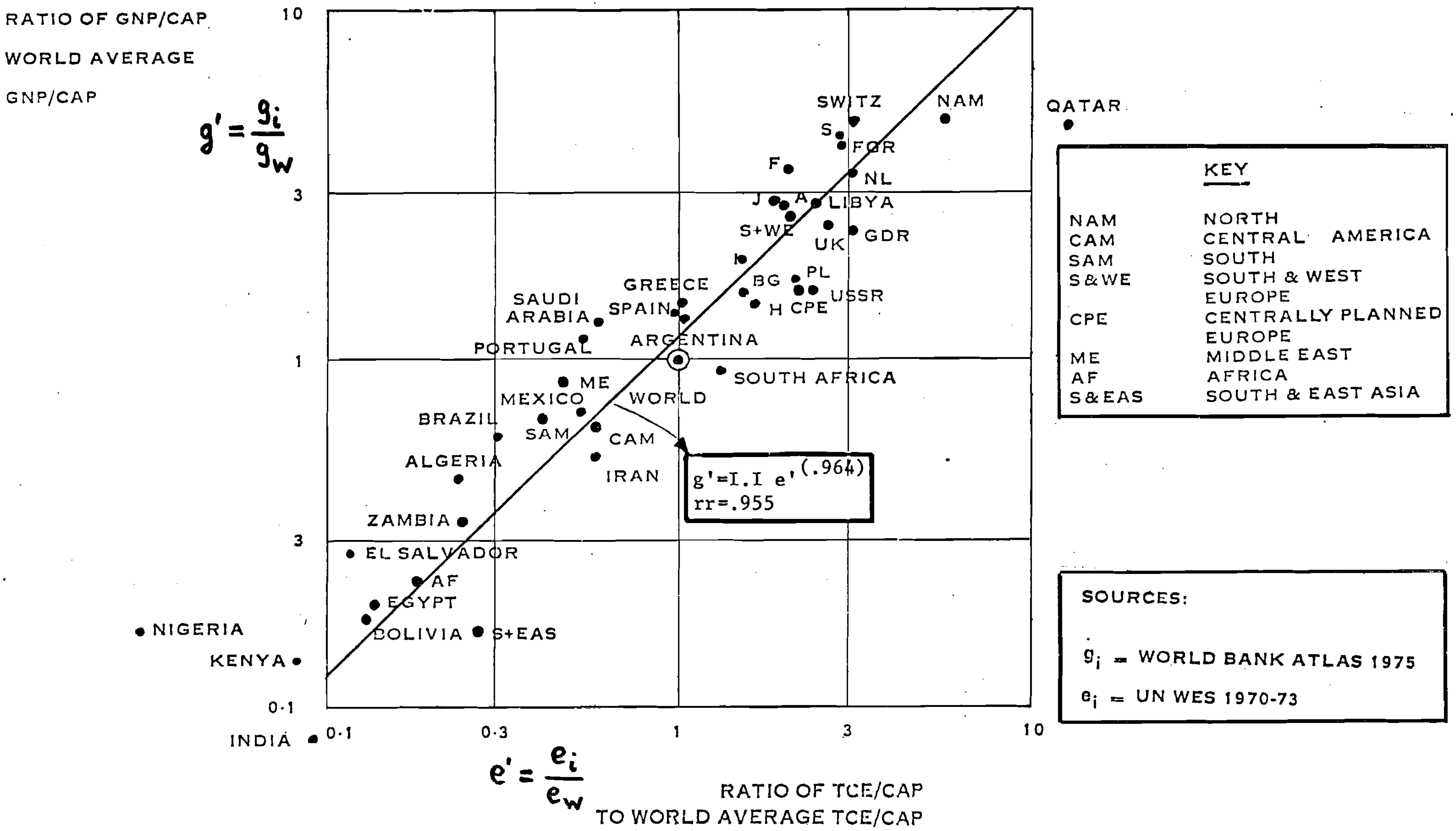


Figure 3 : RELATIVE TECHNOLOGICAL WEALTH FOR WORLD REGIONS

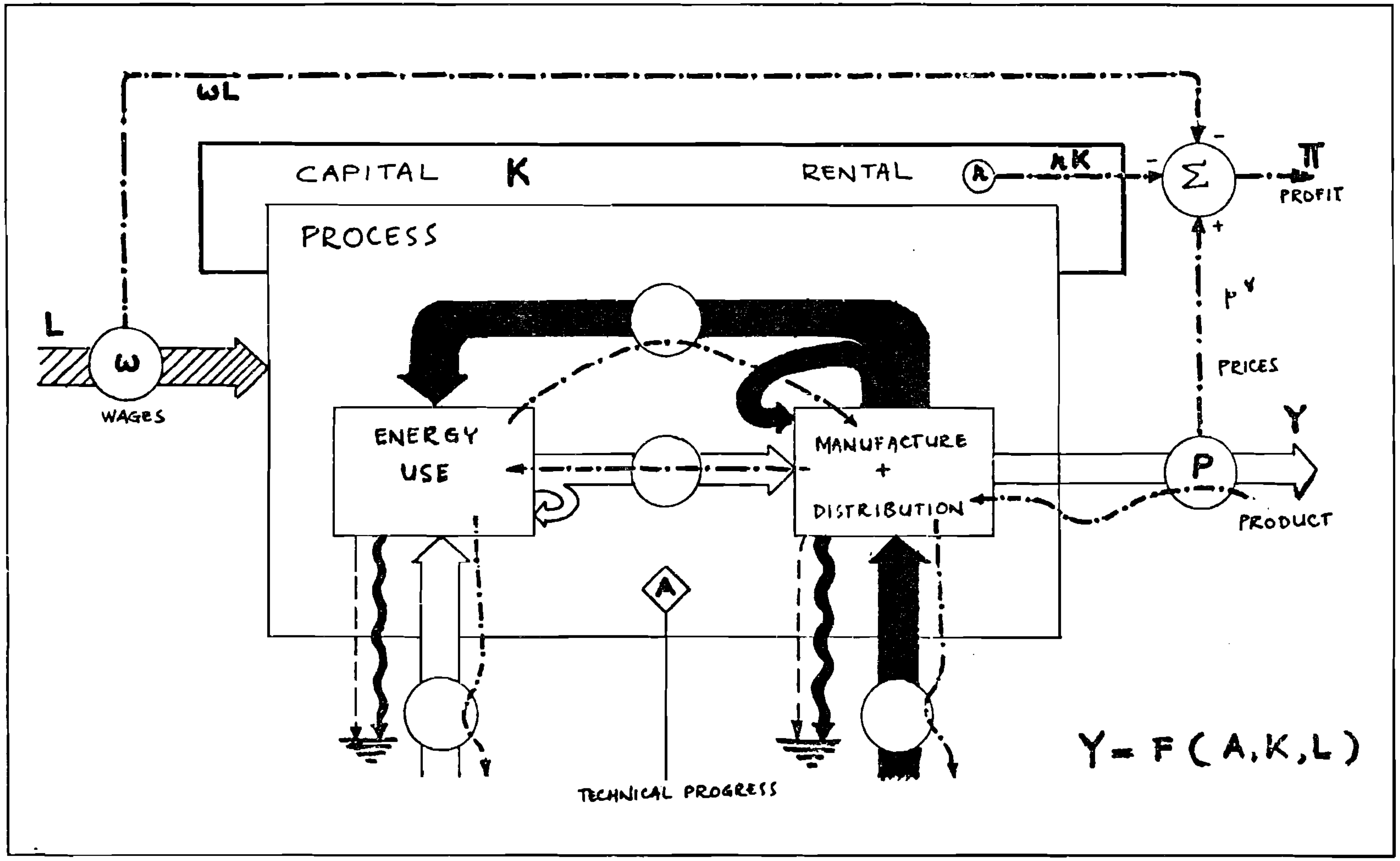


Figure 4 : PRODUCTION FUNCTION MODEL

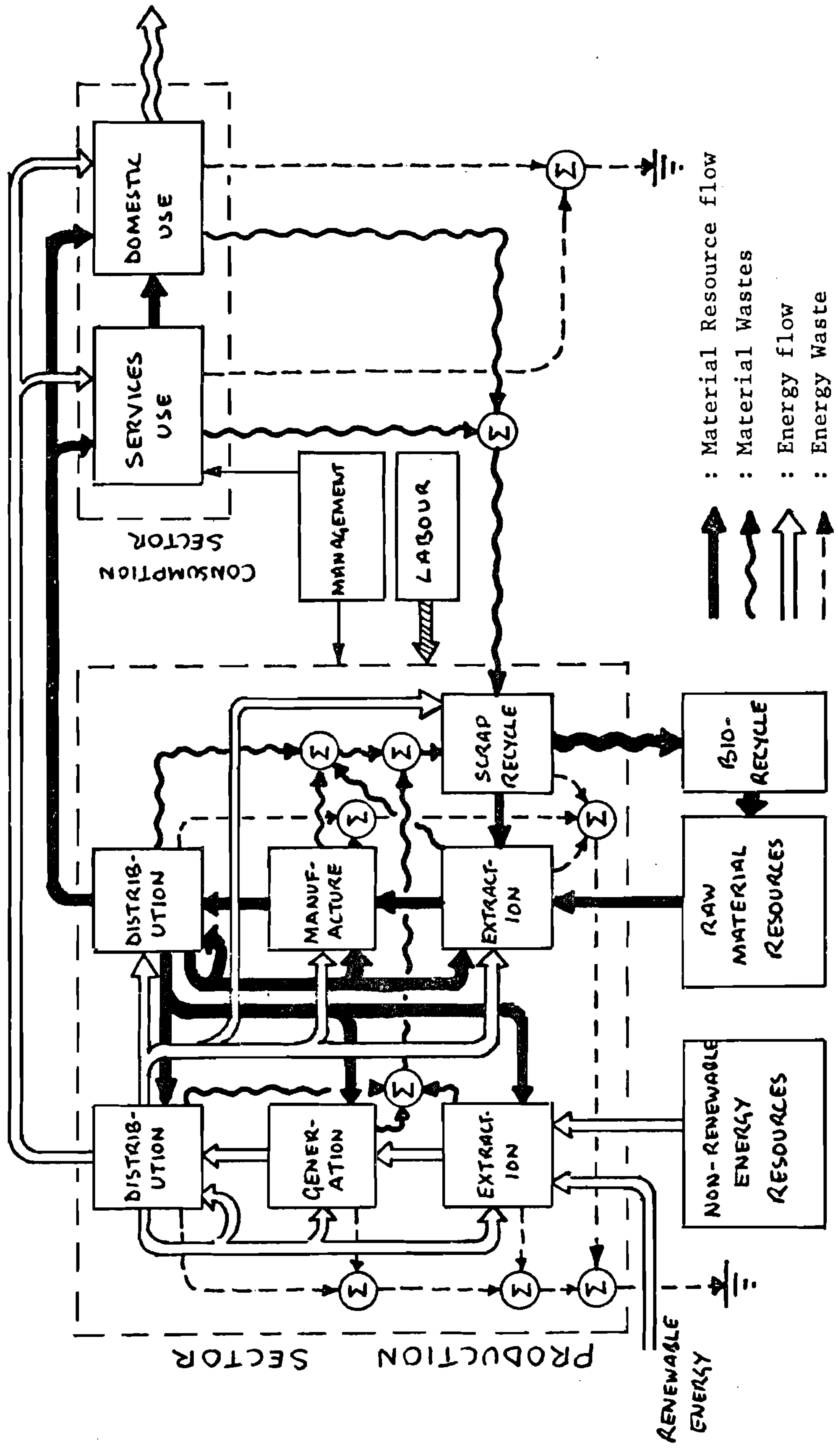
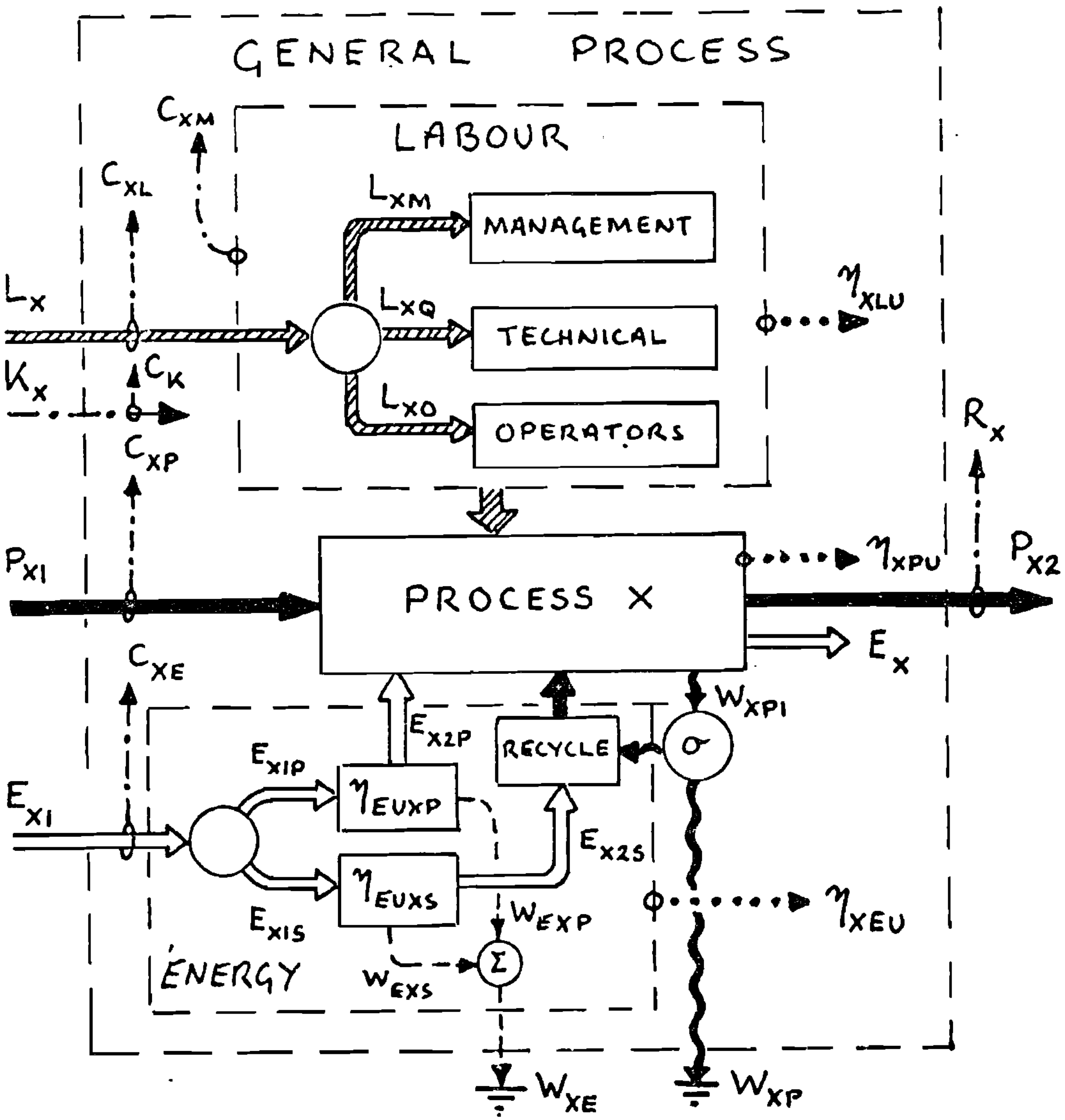


Figure 5 : PROCESS MODEL AT NATIONAL LEVEL (SIMPLIFIED)



NOTATION

Process

P_{X1} material input; P_{X2} product

W_{XP1} material wastes; W_{XP} final wastes

η_{XPU} efficiency of material use

E_X energy content of product

σ_{XP} scrap recovery factor; ρ wast ratio

Energy Use

E_{X1} total energy supplied to operation

E_{X1P}, E_{X1S} total energy inputs to process, recycling

E_{X2P}, E_{X2S} energy consumed by process, recycling

W_{EXP}, W_{EXS} energy wastes from process, recycling

W_{XE} total energy wastes

σ_{XE} proportion of energy to recycling

η_{EUXP}, η_{EUXS} efficiency of process, recycle energy use

η_{XEU} overall efficiency of energy use

Labour

L_{XM}, L_{XQ}, L_{XO} management, technical, operator labour

λ labour mix

η_{XLU} efficiency of labour use

Financial

$C_{X(.)}$ costs of x(.) operations

R_X revenue from product

K_X capital stock of X

Figure 6 : MODEL OF THE GENERAL PROCESS

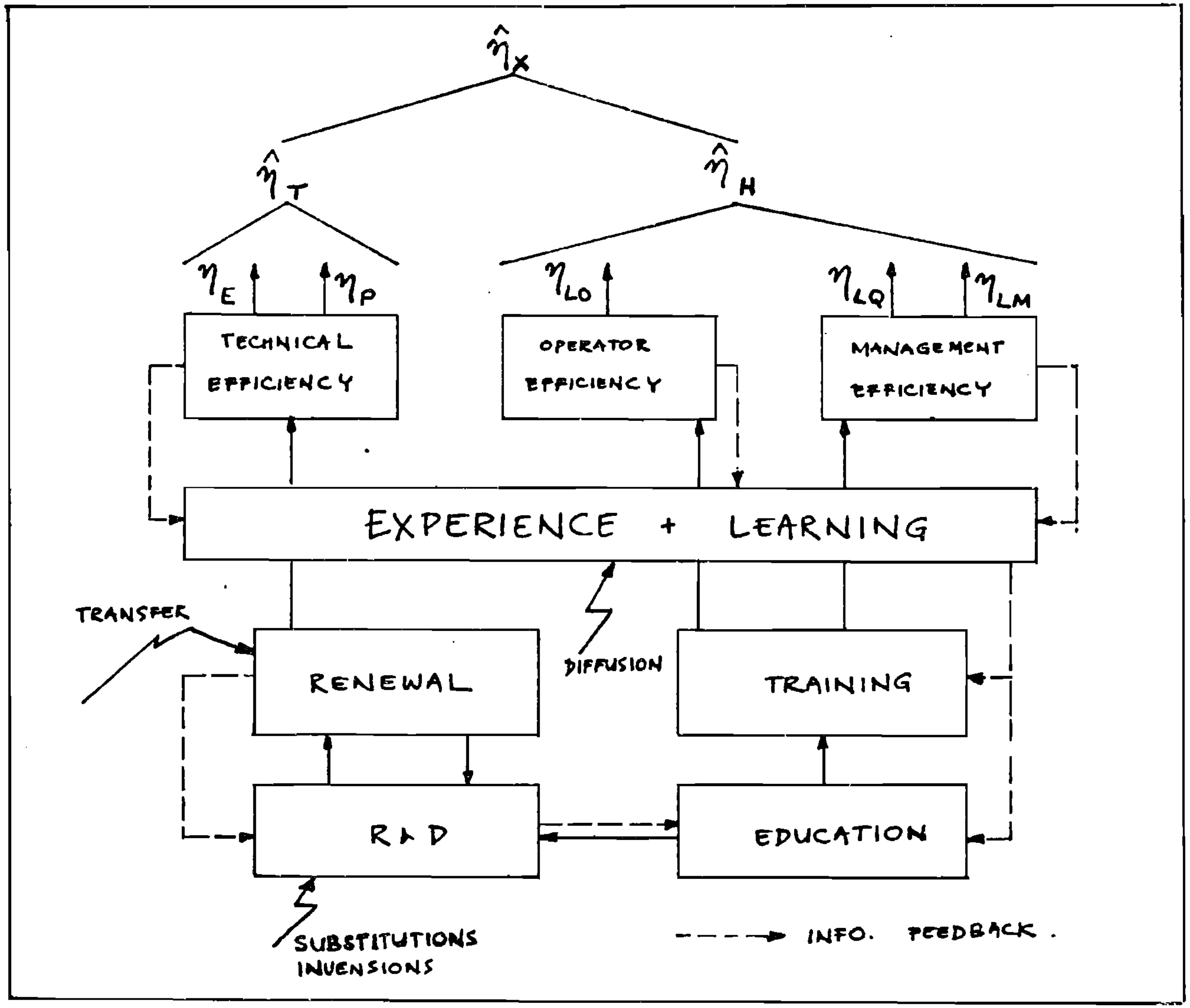


Figure 7 : SOURCES OF TECHNOLOGICAL IMPROVEMENT

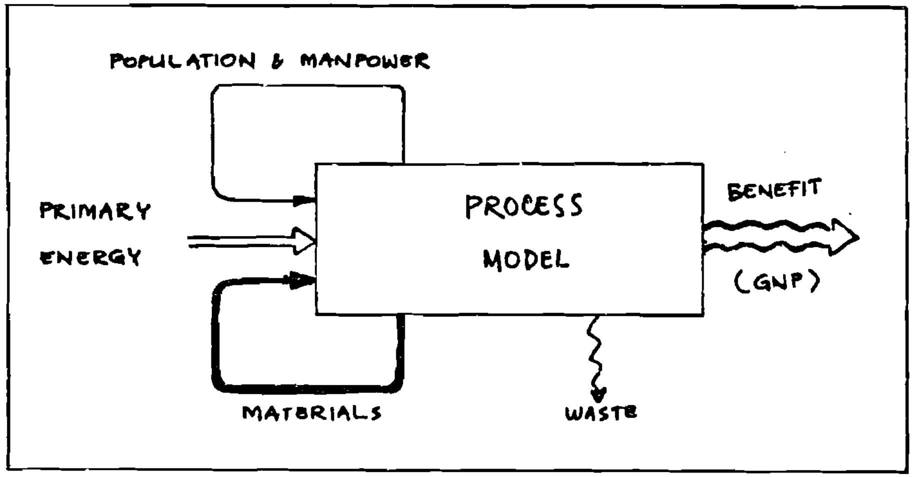


Figure 8 : THE PROCESS AS AN "ENGINE"

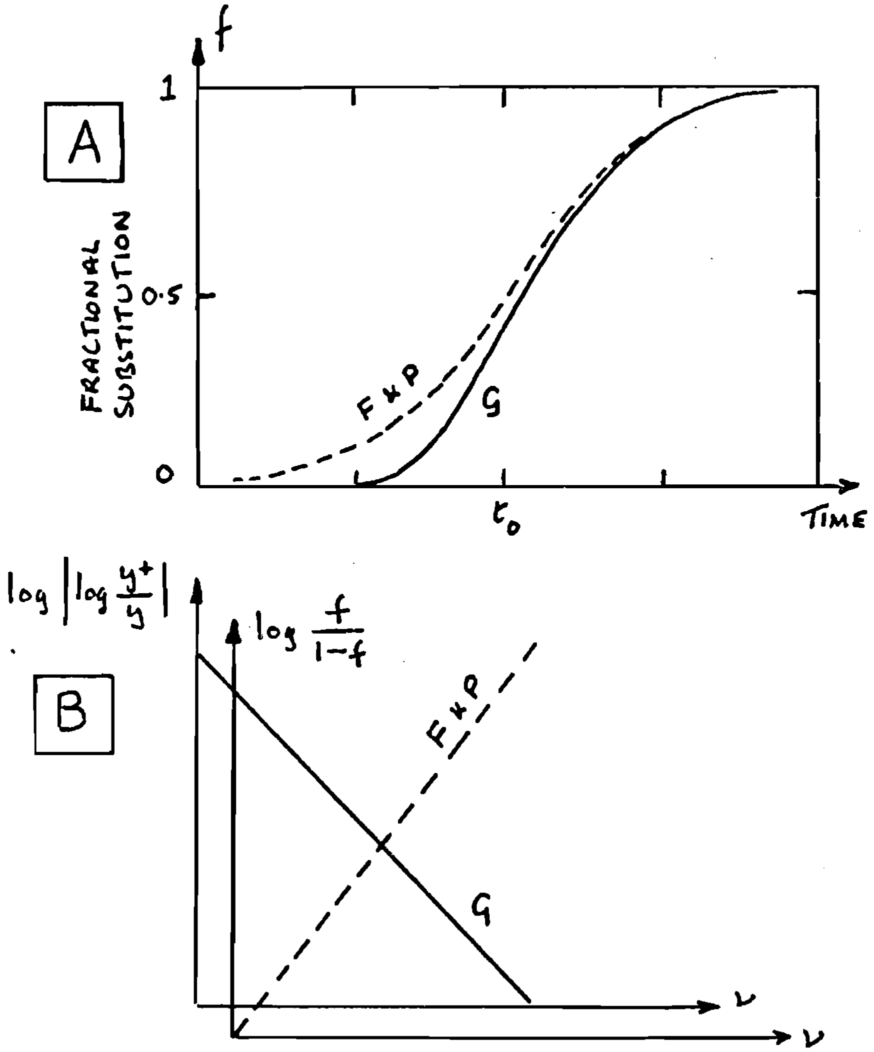
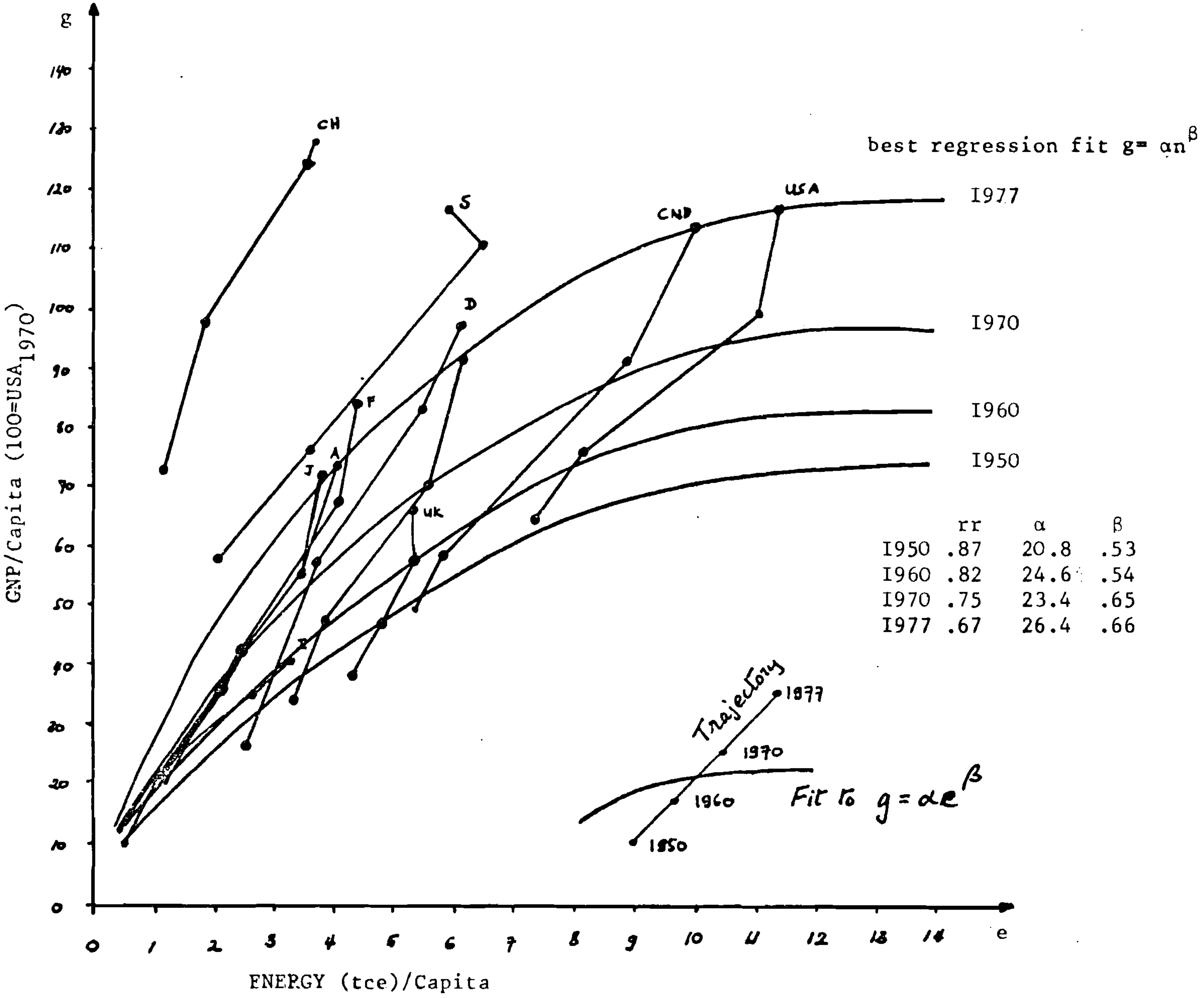


Figure 9 : LOGISTIC FUNCTIONS

A linear form
B logarithmic form
F + P : Fisher and Pry
G : "Gompertz"



	rr	α	β
I950	.87	20.8	.53
I960	.82	24.6	.54
I970	.75	23.4	.65
I977	.67	26.4	.66

Figure 10 : g - e TRAJECTORIES FOP 1970-1977

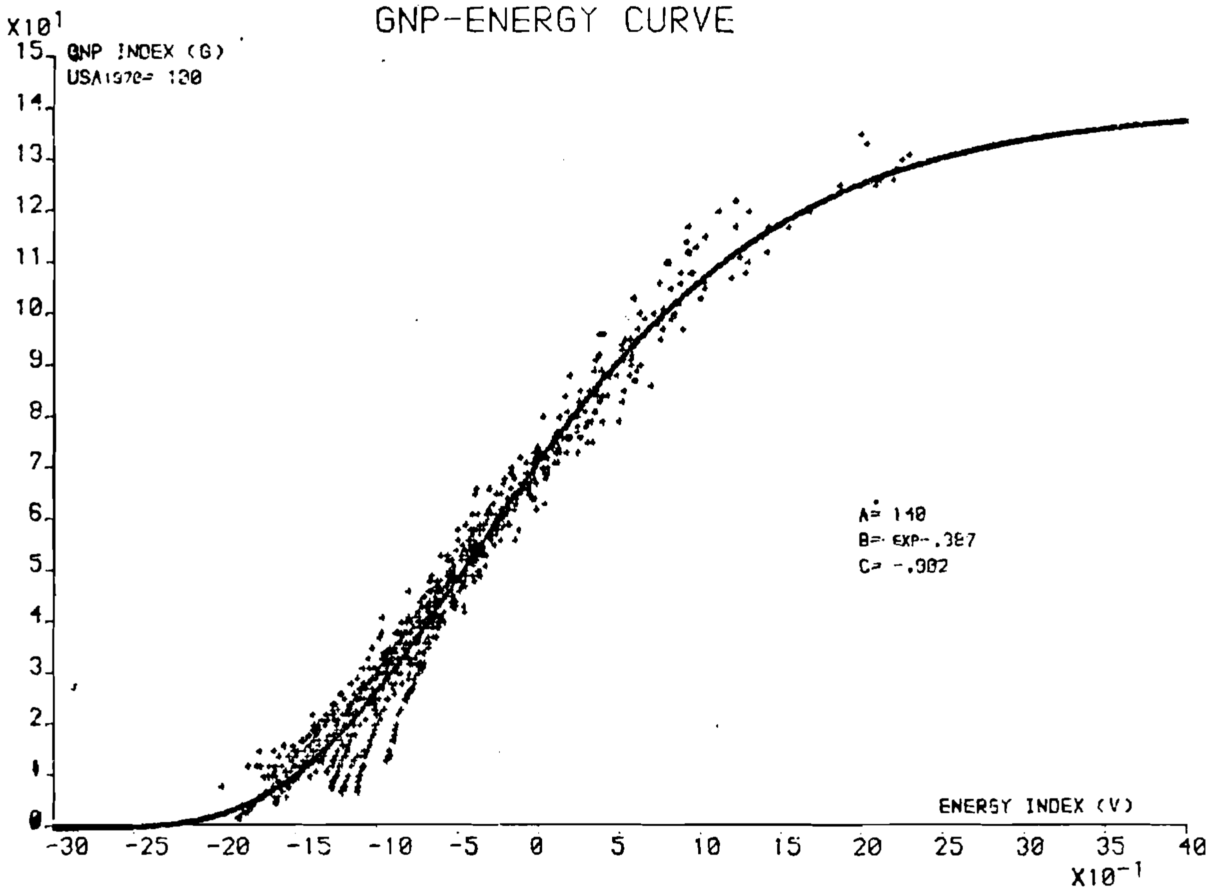


Figure 11 : NATIONAL SAMPLE FITTED TO GOMPERTZ CURVE (linear form)

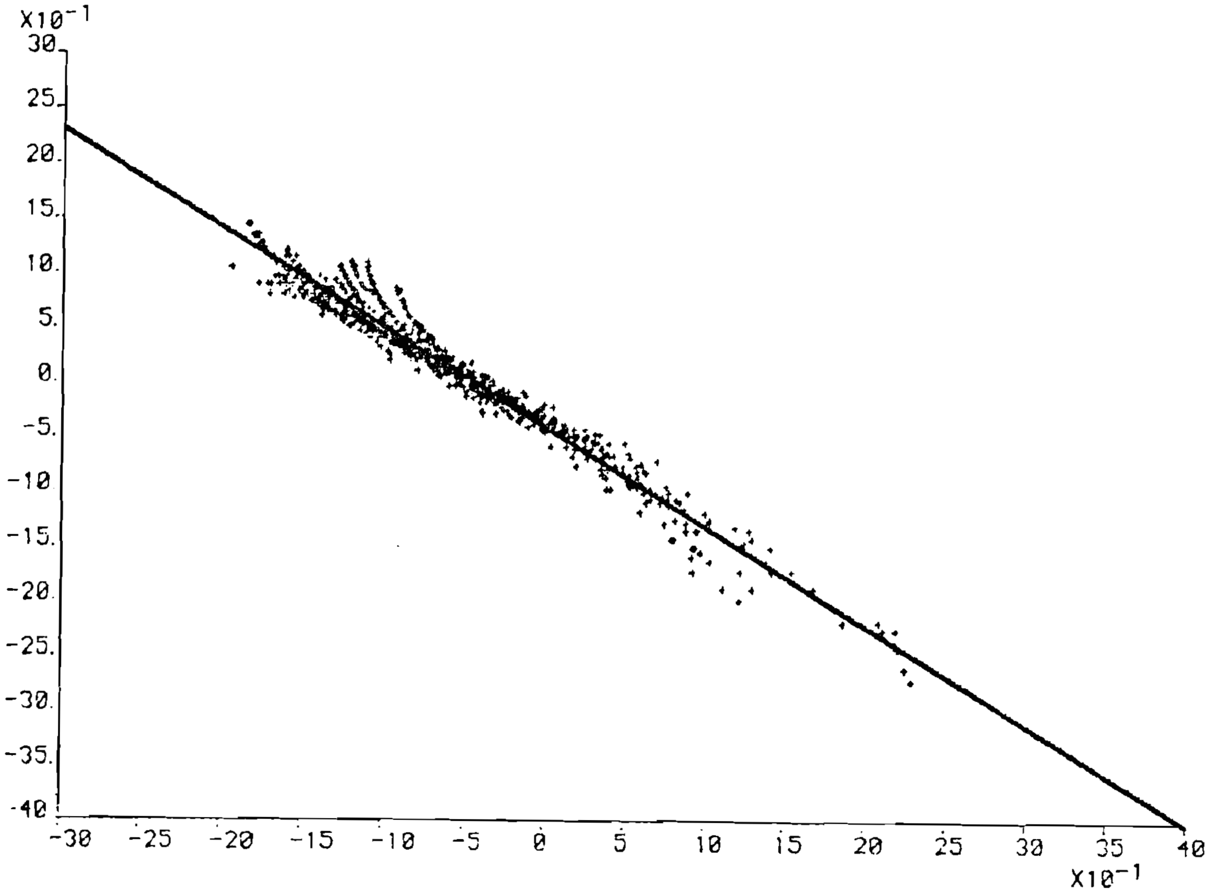


Figure 12 : NATIONAL SAMPLE FITTED TO GOMPERTZ CURVE (logarithmic form)