ECONOMIC GROWTH AND THE SOCIAL OWNERSHIP OF OVERHEAD CAPITAL, I

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1. In the last few years, the economist has become keenly aware of the limitations which the environment poses upon the process of economic growth. This is primarily due to the fact that the process of rapid and steady economic growth which many industrialized countries have experienced in the postwar period has brought with it a number of disturbing social and economic problems such as the pollution of air and water, and the destruction of the urban environment. These phenomena are related to the mismanagement of the environment, in its broadest terms including social as well as natural environments, and have resulted in the further inequity in the distribution of real living standard.

However, the traditional economic theory seems to have failed to provide a coherent framework within which the interactions between the environment and ordinary economic activities may be analyzed and effective policy measures formulated in order to remedy such social disturbances. One of the major reasons for this failure, I believe, is due to the fact that the traditional economic theory, of which the neoclassical economic theory in the broadest sense constitutes the basic theoretical framework, is primarily concerned with a decentralized market economy where all resources limitational to economic processes are privately appropriated, while the environment is by its nature not appropriated to individual members of the society but owned and managed collectively by the society as a whole.

In order to analyze the role which the environment plays in the process of economic growth, therefore, one needs to re-examine the basic theoretical framework of the neoclassical economic theory in such a way that those scarce resources which are collectively owned and managed by the society are explicitly treated and analyzed in terms of a coherent theoretical framework. In this paper, I should like to present a preliminary report on the economics of social overhead capital and to indicate a number of propositions concerning the allocation of resources for the construction and maintenance of overhead capital and the optimum rules concerning the use of the services derived from such overhead capital.

The paper will be divided into two parts. The first part concerns itself with the problems of the amount of resources to be devoted for the construction of overhead capital and for the rules to be imposed upon the use of the services derived from such overhead capital. In the second part, I shall be concerned with the criteria by which the society decides to designate some of the scarce resources as social overhead capital not to be privately appropriable, and the rest to be appropriated to individual members of the society.

2. Social overhead capital comprises all those scarce resources which are put in use for the members of the society, either free of charge or with a negligible price. They are either produced collectively by the society or simply endowed within the society.

Thus all the means of production may be classified into two categories: private means of production and social overhead capital. The classification, however, is not absolute, but it depends upon the historical, political, and social aspects of the society in question. The same type of capital goods may be privately appropriated in one society, but not in another, while it is entirely possible that in the same society a capital good may be classified as private at one time and as social at another time, depending upon the stage of economic and social progress. First, I shall not be concerned with the criteria by which means of production are classified into two categories, but instead I shall postulate that such a classification has already been made and will not change throughout the course of the discussion.

Private means of production are appropriated to individual members of the society who are responsible in the management of those private means of production which they own. Individual members are concerned with attaining the maximum amount of profits or pleasure in accordance with the rules prevailing in the society.

On the other hand, social overhead capital in principle is put to use for any member of the society either free of charge or with a negligible cost. For the sake of simplicity, it is assumed that social overhead capital is provided free of charge to every member of the society.

The services provided by social overhead capital belong to the category of public goods or services for which the formal analysis was presented by Samuelson in his classical articles. The Samuelsonian analysis, however, is concerned with pure public goods, which excludes most of the familiar examples of services provided by social overhead capital. I am particularly concerned with two aspects of social overhead capital which are not handled by the Samuelsonian approach. The first is generally concerned with the range of freedom in which each member of the society may use the services of social overhead capital. Most of social overhead capital requires the input of certain amounts of private means of production, and each member of the society uses the services of such social overhead capital to the extent which he thinks most desirable.

I should like to pay particular attention to the second aspect which is related to the phenomenon of conjestion. As typically illustrated by the example of roads, the benefit each individual gets from the use of a certain amount of social overhead capital depends upon the extent to which other members of the society are using the same social overhead capital. Again, the Samuelsonian concept of pure public goods necessarily excludes the possibility of such social overhead capital for which the phenomenon of conjestion arises.

In "Sur la théorie du capital social collectif, cahier d'économetrie et economie mathematique" (1974), I have developed a formulation of social overhead capital where the two aspects as discussed above may be to a certain extent taken care of there.

It is assumed that private means of production and social overhead capital are respectively composed of homogeneous and measurable quantities. Social overhead capital may be used either in the processes of production or directly in the processes of consumption. However, it is assumed that the economy is composed of a large number of economic units, each of which does not exercise any significant influence on the aggregate level of economic activities. Finally, it is assumed that each consumption unit possesses a measurable utility which depends upon the amount of the services of social overhead capital as well as upon the amount of private goods being consumed.

3. To explain the essential nature of the present approach, I should like first to concentrate upon the case where social overhead capital is used as a factor of production only. Social overhead capital is assumed to be composed of a homogeneous and measurable quantity. Hence, it is

possible to measure the amount of social overhead capital existing within the society at every moment of time. Let V be the stock of social overhead capital thus measured.

Production processes of each production unit in the society are affected by the amount of the services derived from social overhead capital as well as those provided by private means of production. Namely, the output Q_β produced by a production unit β depends upon the amount of private means of production K_β and the services X_β derived from social overhead capital. Thus, the production function may be denoted by

$$Q_\beta = F^\beta(K_\beta, X_\beta) \quad .$$

However, as is typically illustrated by the example of highways, the effectiveness of the services of social overhead capital is influenced by the amount of public services being used by other production units as well as by the amount V of social overhead capital existing in the society. Hence, the production function may be rewritten as

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V) \quad ,$$

where X stands for the aggregate amount of the services of social overhead capital used by all other production units existing in the society.

If it is assumed that there is a continuum of production units existing in the society, the aggregate level X of the services of social overhead capital used by all production units may be denoted

$$X = \int X_\beta \, d\beta \quad ,$$

where the integral is always taken from 0 to 1.

It is assumed that social overhead capital becomes congested as more usage is made of it by other production units. The phenomenon of congestion may be explicitly stated by the following properties. First, the amount of output is decreased as the aggregate level X of social overhead capital being used is increased. Namely,

$$F_X^\beta < 0 \quad . \qquad (\ )$$

Secondly, the marginal product of either private means of production or social overhead capital is decreased as X is increased.

On the other hand, an increase in the endowment V of social overhead capital results in a shift upward of the production function. Hence, it may be assumed that

$$F_V^\beta > 0 \quad . \qquad (\ )$$

In addition, it will be assumed that the production function satisfies the standard neoclassical conditions, i.e. the marginal rates of substitution are always diminishing and the law of constant rates of returns prevails when all the variables are taken into account.

Suppose that production units all produce identical goods and that markets for output and private means of production are both perfectly competitive. Each production unit then chooses the combination of private means of production and public services that will maximize the net profit. Let r be the price, quoted in terms of output, of the services rendered by private means of production, prevailing in the factor market. The net profit of the production unit β is given by

$$\Pi_\beta = Q_\beta - rK_\beta \quad , \qquad (\ )$$

and the production unit β chooses the combination of K_β and X_β that maximizes the profit for given levels of the endowment of social overhead capital V and the aggregate level X of the services of social overhead capital being used currently. Since social overhead capital is offered free of charge, the maximum profit is obtained when the following marginal conditions are satisfied:

$$F^\beta_{K_\beta} = r \quad , \quad F^\beta_{X_\beta} = 0 \quad . \qquad (\ )$$

Demand for private capital K_β and social capital X_β by the production unit β is now uniquely determined by the by the rentals rate r. An increase in r results in a decrease in the demand for private capital K_β. If the production processes are complementary, the demand for social overhead capital X_β is shown to be decreased when the rentals rate r for private capital goes up.

The aggregate demand schedule for private capital then is given by summing up individual demand schedules:

$$K^D = \int K_\beta \, d_\beta \quad . \qquad (\ )$$

Thus, in order for a market equilibrium to be obtained, the following two conditions have to be satisfied: first, the rentals rate r for private means of production is so determined as to equate the aggregate demand with the supply of private means of production; and second, the aggregate demand for the services of social overhead capital is equal to the level with respect to which the individual demand both for private means of production and social overhead capital is derived.

If the supply of private means of production is inelastically given at K, then the equilibrium conditions may be explicitly stated as follows:

$$F^\beta_{K_\beta} = r, \quad F^\beta_{X_\beta} = 0 \quad , \qquad (\ )$$

$$K = \int K_\beta \, d_\beta \quad , \qquad (\ )$$

$$X = \int X_\beta \, d_\beta \quad , \qquad (\ )$$

and

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V) \quad . \qquad (\ )$$

The aggregate real output (real net national product) Q then is given by

$$Q = \int Q_\beta \, d_\beta \quad . \qquad (\ )$$

The aggregate level of the services of social overhead capital being used X_β is related to the rentals rate r. In order to derive the aggregate demand schedule for private capital, it is necessary to take into account the adjustment in the aggregate use of social overhead capital. Mathematically, the system of equilibrium conditions (3) - (5) has to be solved with respect to K, X_β, and X, for given levels of the rentals rate r and the endowment of social overhead capital V. It is easily shown that an increase in the rentals rate r is accompanied by a decrease in K_β. X_β, and X. Hence, the demand schedule for the aggregate level of private capital has a downward slope as a function of the rentals rate r.

The equilibrium rentals rate r, therefore, is uniquely determined by the equilibrium condition (4) for the given endowment of private capital K. The aggregate real output Q may be accordingly determined for the given amounts of private capital K and social overhead capital V.

It may be easily shown that the equilibrium rentals rate r is decreased as either the endowment of private capital K or that of social overhead capital being used tends to increase, because of the assumption that private capital and social capital are complementary.

One can easily infer from the existence of external economies with respect to social overhead capital that market allocation is not optimum. The problem then arises if it is possible to devise a rule by which the optimum allocation of private and social means of production may be obtained. To examine this problem, let me next consider the allocation scheme where social overhead capital may be priced for its usage.

Let me consider now the situation where private individuals are charged a price for the use of social overhead capital according to the amount of services being used, where it is assumed that the administrative costs associated with the pricing scheme are negligible. Private means of production are allocated in a perfectly competitive market.

Let θ be the price charged per unit of services derived from social overhead capital. The net profit of the production unit β now becomes

$$\Pi_\beta = Q_\beta - rK_\beta - \theta X_\beta$$

The net profit thus defined is maximized if the following marginality conditions are satisfied:

$$F^{\beta}_{K_\beta} = r \quad , \quad F^{\beta}_{X_\beta} = \theta$$

Other equilibrium conditions are identical with those obtained for the previous situation; namely,

$$K = \int K_\beta \, d_\beta \quad ,$$

$$X = \int X_\beta \, d_\beta \quad , \text{ and}$$

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V) \quad .$$

It is assumed that private capital is inelastically supplied at the level K and the endowment of social overhead capital is given at V. For a given price θ for the use of social overhead capital, the system of equilibrium conditions are solved to determine the equilibrium allocations of private capital and social overhead capital, K_β and X_β, together with the aggregate level of the services of social overhead capital X being used. It is easily shown that the equilibrium rentals rate r for private capital is also uniquely determined for a given price θ.

Let $K_\beta(\theta)$, $X_\beta(\theta)$, and $X(\theta)$ be respectively the equilibrium allocations of private capital and social capital, and the aggregate level of social capital being used, all corresponding to the imputed price θ. The resulting aggregate real output $Q(\theta)$ may be denoted by

$$Q(\theta) = \int Q_\beta(\theta)\, d_\beta \quad ,$$

where $Q_\beta(\theta)$ stands for the equilibrium output of production unit β.

It may be interesting to see if the aggregate real output $Q(\theta)$ is increased or not when the imputed price θ is increased. Differentiating (12) with respect to the imputed price θ, one obtains the following relationships:

$$\frac{dQ(\theta)}{d\theta} = \int |F^\beta_{K_\beta} \frac{dK_\beta(\theta)}{d\theta} + F^\beta_{X_\beta} \frac{dX_\beta(\theta)}{d\theta} + F^\beta_X \frac{dX}{d\theta}|\, d\beta \quad ,$$

Hence, relationships may be reduced to the following:

$$\frac{dQ(\theta)}{d\theta} = |\theta + \int F^\beta_X\, d\beta|\, \frac{dX}{d\theta} \quad ,$$

which may be rewritten as

$$\frac{dQ(\theta)}{d\theta} = (MSC - \theta)\,(- \frac{dX}{d\theta}) \quad ,$$

where

$$MSC = - \int F^\beta_X\, d\beta \quad .$$

The expression (13) corresponds to the concept of the marginal social costs associated with the use of social overhead capital. It represents the loss in the aggregate real output due to the marginal increase in the use of social overhead capital.

Since an increase in the imputed price Θ reduces the aggregate usage X of social overhead capital,

$$\frac{dX}{d\Theta} < 0 \quad .$$

Hence, whether an increase in the imputed price increases the aggregate real output $Q(\Theta)$ or not depends upon the difference between the marginal social costs, MSC, and the imputed price Θ. Namely, if the imputed price Θ is less than the marginal social costs MSC, then the aggregate real output is $Q(\Theta)$. The maximum aggregate real output $Q(\Theta)$ then may be obtained when the imputed price Θ is just equal to the marginal social costs MSC.

Starting with the market solution which corresponds to the case where $\Theta = 0$, the aggregate real output $Q(\Theta)$ is increased until the imputed price Θ is equated to the marginal social costs MSC. Hence, it is possible to devise an iterative procedure by which the maximum aggregate real output may be obtained, provided the marginal social costs may be calculated from the known allocation of private and social capital among individual units.

The procedure discussed above relies upon the price mechanism for the allocation of both private and social capital. Suppose now that it is possible, without incurring any costs, to make a centrally controlled plan as for the allocation of scarce means of production. It is supposed for the moment that a central planning board possesses a complete knowledge about the production processes of each production unit. What would then be the allocation of private and social capital among individual production units that maximizes the aggregate real output? This problem may be mathematically stated as follows:

Let K and V be the given endowments of private capital and social overhead capital. Then find the allocation of private capital among production units K and the levels of individual and aggregate uses of social overhead capital, X_β and X, so as to maximize the aggregate real output

$$Q = \int Q_\beta \, d\beta$$

subject to the constraints:

$$K = \int K_\beta \, d\beta \quad ,$$

$$X = \int X_\beta \, d\beta \quad ,$$

and

$$Q_\beta = F^\beta(K_\beta, X_\beta, X, V) \quad .$$

Such a maximization problem may be easily solved in terms of Lagrange multipliers. Let r and θ be respectively the Lagrange multipliers associated with the constraints (19) and (20), and introduce the Lagrangian form:

$$\int Q_\beta \, d\beta + \eta | K - \int K_\beta \, d\beta | + \theta | X - \int X_\beta \, d\beta | \quad .$$

The optimum allocation may now be obtained by finding the allocation for which the Lagrangian may be maximized without any constraints.

Therefore, the optimum allocation may be obtained by solving the following equations:

$$F^\beta_{K_\beta} = r$$

$$F^\beta_{X_\beta} = \theta$$

$$\theta = - \int F^\beta_X \, d\beta \quad ,$$

together with the constraints (19-21).

These conditions are identical with those which have been obtained for the case where the imputed price for social overhead capital θ is equated to the marginal social costs MSC.

Hence, the allocative process discussed in the previous section results in an optimum allocation of scarce resources.

In the analysis presented in the previous sections, it has been assumed that the economy is composed of producers only, without having consumers to play any role in the process of resource allocation. I should like to consider the general case where consumers are involved with the allocative process of both private capital and social overhead capital.

Let the consumers be denoted by the generic symbol α, ranging continuous numbers from 0 to 1, as has been the case with producers. Namely, it is assumed that the economy is composed of a large number of consumers each of whom plays a role which is negligible from the aggregative point of view. The process of aggregation again will be denoted by the integral.

The level of utility each consumer may enjoy is related to the amount of the services derived from social overhead capital as well as private consumption. Again as has been with the case for producers, it may be assumed that there is only one kind of private consumption goods and that the services from social overhead capital are measurable. By adopting the Benthamite utility concept, it may be assumed that consumer α's utility U_α is a function of the level of private consumption C_α and the amount X_α of the services derived from social overhead capital. In view of the presence of the congestion phenomenon, the effectiveness of the services of social overhead capital to consumer α depends upon the aggregate level X of the services of social capital being used as well as upon the stock of social overhead capital V. Namely, it may be written as

$$U_\alpha = U^\alpha(C_\alpha, X_\alpha, X, V) \quad ,$$

where the aggregative level X has to be defined by

$$X = \int X_\alpha \, d\alpha + \int X_\beta \, d_\beta \quad .$$

It is assumed that the utility function U is concave with respect to the variables C_α, X_α, X, and V, and that the marginal utility of private consumption is positive, while that of the services of social overhead capital is merely decreasing.

If the services of social overhead capital are rendered to consumers free of charge, then each consumer will use them up to the level where the marginal rate of substitution between social overhead capital and private consumption equals zero and all of his income Y_α will be spent on private consumption C_α.

Market equilibrium will be attained when these conditions concerning consumers' equilibrium are satisfied together with producers' equilibrium conditions discussed in the previous section. It is obviously seen that the resulting pattern of resource allocation is neither efficient nor optimum.

In the general situation where consumers are present, one may have to be careful in defining the concept of optimum resource allocation. However, if the Benthamite concept of measurable and comparable utility is presupposed, tnen the social utility U is simply defined by the aggregate of individual levels U_α; namely,

$$U = \int U_\alpha \, d\alpha \quad ,$$

where the integral ranges over all the consumers in the society.

A pattern of resource allocation and the accompanying income distribution may be defined as optimum if the social utility U is maximized among the feasible set of resource allocation. This statement may be put in a more precise form as follows:

At each moment of time, let the amounts of private and social capital be given at K and V, respectively. A pattern of resource allocation $(C_\alpha, X_\alpha, K_\beta, X_\beta)$ is defined optimum if the social utility U is maximized among the set of all feasible resource allocations:

$$\int C_\alpha \, d\alpha = \int Q_\beta \, d_\beta \quad ,$$

$$Q_\beta = F_\beta(K_\beta, X_\beta, X, V) \quad ,$$

$$X = \int X_\alpha \, d\alpha + \int X_\beta \, d_\beta \quad ,$$

$$K = \int K_\beta \, d_\beta \quad .$$

Let p, p_0, pr, be respectively the Lagrange multipliers associated with the constraints (28), (30), and (31). Then a simple calculation will show that an optimum allocation has to satisfy the following conditions:

$$U^\alpha_{C_\alpha} = p \quad , \text{ or } \quad U^{\alpha'}_{C_{\alpha'}} / U^{\alpha''}_{C_{\alpha''}} = 1 \text{ for all pairs } \alpha' \text{ and } \alpha''$$

$$U^\alpha_{X_\alpha} / U^\alpha_{C_\alpha} = \theta$$

$$\theta = \int - U^{\alpha}_{X}/U^{\alpha}_{C_{\alpha}} \; d\alpha + \int - F^{\beta}_{X} \; d\beta$$

$$F^{\beta}_{K_{\beta}} = r \quad , \quad F^{\beta}_{X_{\beta}} = 0$$

The marginal social costs associated with the use of social overhead capital in the present context becomes:

$$MSC = \int - U^{\alpha}_{X}/U^{\alpha}_{C_{\alpha}} \; d\alpha + \int - F^{\beta}_{X} \; d\beta$$

The optimum conditions (32-35) suggest that, in order to obtain an optimum resource allocation, it is necessary to introduce a transfer mechanism in such a manner that the marginal rate of distribution between any pair of two consumers becomes unity

$$MRD_{\alpha'\alpha''} \left(\equiv \frac{U^{\alpha'}_{C_{\alpha'}}}{U^{\alpha''}_{C_{\alpha''}}} \right) = 1 \quad ,$$

in addition to the pricing scheme for the use of social overhead capital according to the marginal social cost principle.

The analysis so far has been concerned with the allocation of scarce resources where the stock of private and social capital has been assumed to be given. The analysis may be extended to the situation where one is concerned with the process of capital accumulation for both private and social capital, and try to examine the pattern of resource allocation over time which is optimum from a dynamic point of view. It will be shown that the principle of the marginal social costs may be extended to this dynamic case and the criteria for optimum allocation of investment between private and social capital will be obtained within the framework of the Ramsey theory of optimum growth.

In order to simplify the exposition, it will be assumed, throughout the rest of this paper, that the rate of discount by which consumers discount their future levels of utility is constant and identical for all consumers in the society. Let δ be the rate of discount. The level of social utility U may now be expressed by

$$U = \int_0^\infty U(t)e^{-\delta t}\, dt$$

where the utility level 1 U(t) at a point of time t may be given y

$$U(t) = \int U_\alpha(t)\, d\alpha \quad ,$$

with

$$U_\alpha(t) = U^\alpha(C_\alpha(t), X_\alpha(t), X(t), V(t)) \quad .$$

Let V_0 be the stock of social overhead capital existing at the initial point of time 0. I am concerned with the problem of finding a path of private consumption for each consumer, of allocation of private and social capital between various economic units, and of capital accumulation for both private and social capital over time such that the resulting level of social utility is maximized over all feasible paths. In order to discuss this optimum problem, I should like to pay a particular attention to the difference between private and social capital with regard to the extent to which investment is used to increase the stock of capital (to be measured in the efficiency unit). In general, social overhead capital is difficult to reproduce in the sense that a rather significant amount of scarce resources have to be used in order to increase the stock of capital, while, for private capital, investment without much difficulty, converted into the accumulation of capital. It may be possible to formulate the relationships between the amount of investment and the resulting increase in the stock of capital in terms of a certain functional relationship.

Let I_V be the amount of real investment devoted to the accumulation of social overhead capital. If social overhead capital V is measured in a certain efficiency unit, the amount of real investment I_V may not necessarily result in the increase in the stock of capital by the same amount. Instead, there exists a certain relationship between the amount of real investment I_V and on the one hand the corresponding increase $\dot{V}$ in the stock of social overhead capital and the current stock of social overhead capital V on the other:

$$I_V = \phi_V(\dot{V},V) \quad .$$

This relationship may be interpreted as follows: i.e. in order to increase the stock of social overhead capital V by the amount $\dot{V}$, real investment I_V has to be spent on the accumulative activities for social overhead capital. In what follows it will be assumed that the function ϕ exhibits a feature of constant returns to scale with respect to $\dot{V}$ and V, thus

$$I_V/V = \phi_V(\dot{V}/V) \quad .$$

Since it may be assumed that the marginal costs of investment are increasing as the level of investment is increased, the function ϕ_V satisfies the following conditions:

$$\phi_V'(\cdot) > 0 \quad , \quad \phi_V''(\cdot) > 0 \quad .$$

Similar relationships may be postulated for the accumulation of private capital for each producing unit; namely, for each producer β, the amount of real investment I_β required to increase the stock of capital K_β by the amount $\dot{K}_\beta$ may be determined by the following Penrose function:

$$I_\beta/K_\beta = \phi_\beta(\dot{K}_\beta/K_\beta) \quad ,$$

where the Penrose function ϕ_β again satisfies the conditions:

$$\phi_\beta'(\cdot) > 0 \quad , \quad \phi_\beta''(\cdot) > 0 \quad .$$

Furthermore, it is assumed that the rate of depreciation of social overhead capital depends upon the extent to which it is used. Hence, the rate of depreciation μ may be written as

$$\mu = \mu(X/V) \quad ,$$

$$\mu'(\cdot) > 0 \quad , \quad \mu''(\cdot) > 0 \quad .$$

The optimum problem may now be more precisely stated as follows:

A path of resource allocation over time, $(C_\alpha(t), I_\beta(t), I_V(t), X_\alpha(t), X_\beta(t), K_\beta(t), V(t))$, is defined as feasible if it satisfies the following consistency conditions:

$$Q(t) = \int C_\alpha(t)\, d_\alpha + \int I_\beta(t)\, d\beta + I_V(t) \quad ,$$

$$Q(t) = \int F^\beta(K_\beta(t), X_\beta(t), X(t), V(t))\, d\beta \quad ,$$

$$X(t) = \int X_\alpha(t)\, d\alpha + \int X_\beta(t)\, d\beta \quad ,$$

$$\frac{I_\beta(t)}{K_\beta(t)} = \phi_\beta(z_\beta(t)) \quad , \quad \frac{\dot{K}_\beta(t)}{K_\beta(t)} = z_\beta(t) \quad ,$$

$$\frac{I_V(t)}{V(t)} = \phi V(z_V(t)) \quad , \quad \frac{\dot{V}(t)}{V(t)} = z_V(t) - \mu\left|\frac{X(t)}{V(t)}\right| \quad ,$$

$$K_\beta(0) = K_\beta^o \quad , \quad V(0) = V^o \quad \text{given.}$$

I am then interested in finding a feasible path of resource allocation over time which maximizes the social utility.

This optimum problem is in general extremely difficult to solve, and I shall be instead concerned with finding a path of resource allocation which approximates the optimum path to a reasonable extent. Among such an approximated path, the one with the simplest structure will be obtained by examining the conditions which the imputed prices of private and social capital have to satisfy.

Let $p_\beta(t)$ and $p_V(t)$ respectively be the imputed prices at time t, of private capital K_β and social overhead capital V, and let p(t) and $\theta(t)$ be the imputed prices of output Q and the use of social overhead capital X. These imputed prices correspond to the Lagrange multipliers associated with the constraints for the optimum problem. The Euler-Lagrange conditions which the optimum path has to satisfy may be rearranged to yield the following conditions:

$$U^\alpha_{C_\alpha} = p \quad , \quad U^\alpha_{X_\alpha} / U^\alpha_{C_\alpha} = \theta \quad ,$$

$$F^\beta_{X_\beta} = \theta \quad ,$$

$$\Theta = \int \frac{(-\,U_X^\alpha)}{U_{C_\alpha}^\alpha}\, d\alpha + \int (-\,F_X^\beta)\, d\beta + \frac{p_V}{p}\,\mu'(X/V) \quad ,$$

$$\frac{\dot{p}_\beta}{p_\beta} = \delta - z_\beta - \frac{r_\beta - \phi_\beta(z_\beta)}{\Theta_\beta'(z_\beta)} \quad ,$$

where

$$\phi_\beta'(z_\beta) = \frac{p_\beta}{p} \quad , \quad r_\beta = F_{K_\beta}^\beta \quad ,$$

$$\frac{\dot{p}_V}{p_V} = \delta - z_V - \frac{r_V - \phi_V(z_V)}{\phi_V'(z_V)} \quad ,$$

where

$$\phi_V'(z_V) = \frac{p_V}{p} \quad , \quad r_V = \int \frac{U_V^\alpha}{U_{C_\alpha}^\alpha}\, d\alpha + \int F_V^\beta\, d\beta \quad .$$

It may be noted that the marginal costs associated with the depreciation of social overhead capital are evaluated in terms of its imputed price p_V/p measured in real terms. The quantity r_β defined in (42) is nothing but the marginal product of private capital, while the r_V defined in (44) is the marginal social product of social overhead capital measured in real terms. Namely, the r_V represents the marginal gain to the society measured in real terms due to the marginal increase in the stock of social overhead capital V.

The conditions (40-42) suggest that, in order to attain an optimum allocation of scarce resources in the short run, one has to impose the charges equal to the marginal social costs for the use of social overhead capital, with the marginal social costs being defined in the modified sense (42). On the other hand, the pattern of accumulation of private and social capital may be described by the conditions (41-44) describing the rules by which the imputed prices change over time. In order to approximate the structure of the optimum path of capital accumulation, I should like to consider the case where the imputed prices are assumed as if they were not to change at each moment of time. Namely, the rates of accumulation of private and social capital are obtained by assuming that equations (41) and (43) are equated to zero. It can be shown that the path of capital accumulation obtained by such a procedure reasonably approximates the optimum path, although the sense in which reasonable approximation is used needs a more complicated formalization.

If the imputed prices were assumed as if they were not to change over time, then the rates of capital accumulation z_β and z_V, may be obtained by solving the following conditions:

$$\frac{r_\beta - \phi_\beta(z_\beta)}{\delta - z_\beta} = \phi_\beta'(z_\beta) \quad ,$$

$$\frac{r_V - \phi_V(z_V)}{\delta - z_V} = \phi_V'(z_V) \quad .$$

It is easily seen that the rates of accumulation of private and social capital are uniquely determined, that the higher the marginal product of private capital, the higher is the corresponding rate of accumulation for private capital, and that the higher the marginal social product of social overhead capital, the higher is the rate of accumulation. On the other hand, an increase in the social rate of discount δ will lower the rate of accumulation both for private and social capital.

Thus, the approximate optimum rates of accumulation for private and social capital will be determined once the marginal private or social product of these capital are known. However, the marginal products of both private and social capital depend upon the extent to which social overhead capital is used by the member of the society. The amount of the services of social overhead capital used is in turn related to the imputed price p_V/p of social overhead capital, as is seen from the definition of the marginal costs.

In this part, an introductory analysis of social overhead capital has been presented, with an emphasis upon the implications of the presence of such an overhead capital upon the process of resource allocation and the ensuing pattern of real income distribution. I have emphasized two aspects of social overhead capital which are not readily covered by the standard Samuelsonian concept of pure public goods. Namely, each individual member of the society is free to use the services of social overhead capital to the extent to which he desires, but the effectiveness of the services he uses of social overhead capital crucially depends upon the way other individuals are using the same services, incorporating the phenomenon of congestion.

The main conclusions of this part have been concerned with the pattern of resource allocation which results in an optimum allocation of social as well as private resources, both from a static and dynamic point of view. From the static point of view, the given stock of social overhead capital may be efficiently used if each individual member is charged a price equal to the marginal social costs for the use of social overhead capital, provided the administrative costs associated with such a pricing scheme are negligible. For the optimum allocation from the dynamic point of view, one has first to modify the concept of the marginal social costs for the use of social overhead capital, by taking into account the value of the marginal depreciation of social overhead capital due to the marginal increase in the use of social overhead capital. The evaluation of the marginal depreciation of social overhead capital has been based upon the imputed price of social overhead capital. The imputed price of social overhead capital, being the discounted present value of the marginal social product or benefits due to a marginal increase in the stock of social overhead capital, is also a crucial factor in the determination of the optimum rate of accumulation and corresponding investment in social overhead capital. The optimum rate of accumulation of social overhead capital (although only an approximately optimum pattern has been discussed in the paper) is closely related to the ease or difficulty with which such a social overhead capital may be reproduced. It can be shown, as is expected, the more difficult and the more costly it is to reproduce social overhead capital, the smaller the amount to be devoted to the accumulation of such a social overhead capital. These propositions have been discussed in terms of the Penrose type relationships which relate the amount of real investment to the rate by which social overhead capital is accumulated.

The analysis has been presented for the case where there is only one kind of social overhead capital. However, most of the propositions obtained above may be extended, with slight modifications, to the general case where there are a variety of social overhead capitals--one has merely to replace V by a vector of the stock of social overhead capital having a number of components as many as there are of various kinds of social overhead capital. In particular, it is possible to extend the analysis to the case where